Universal response in anomalous cold holographic superfluids

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Goal: understand the role of anomalies in superfluids

- In particular: response to magnetic field and vorticity
- Quantum anomalies $\Rightarrow$ Chiral Magnetic and Chiral Vortical Effects
  - Normal fluids: transport fixed by anomalies
  - Superfluids: generically unconstrained $\Leftarrow$ extra d.o.f.'s
- Can we make any prediction for the superfluid case?

Outline

Anomalous normal fluid in 3+1

- Charge current in Landau frame

$$J^\mu = \rho u^\mu + \frac{\kappa}{T}(E^\mu - TP^{\mu\nu}\partial_\nu \frac{\mu}{T}) + \tilde{\kappa}_\omega \omega^\mu + \tilde{\kappa}_B B^\mu$$

- Anomaly

$$\partial_\mu J^\mu = -\frac{c}{8}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

- Chiral conductivities

$$\tilde{\kappa}_\omega = c\left(\mu^2 - \frac{2}{3}\frac{\rho}{\epsilon + P}\mu^3\right), \qquad \tilde{\kappa}_B = c\left(\mu - \frac{1}{2}\frac{\rho}{\epsilon + P}\mu^2\right)$$

- Entropy current

$$J_s^\mu = su^\mu - \frac{\mu}{T}(J^\mu - \rho u^\mu) + \sigma_\omega \omega^\mu + \sigma_B B^\mu$$

$$\sigma_\omega = c\frac{\mu^3}{3T}, \qquad \sigma_B = c\frac{\mu^2}{2T}$$

[Banerjee et al., Son et al., Bhattacharya et al., ...]

Anomalous superfluid

- Extra hydro dof from Goldstone boson: $\xi_\mu = -\partial_\mu \phi + A_\mu$
- Charge current:

$$J^\mu = \left(parity \quad preserving \quad terms \right) + \tilde{\kappa}_\omega \omega^\mu + \tilde{\kappa}_B B^\mu$$

- Entropy current:

$$J_s^\mu = \left(parity \quad preserving \quad terms \right) + \left(\sigma_\omega - \frac{\mu}{T} \tilde{\kappa}_\omega \right) \omega^\mu + \left(\sigma_B - \frac{\mu}{T} \tilde{\kappa}_B \right) B^\mu$$

- Only constraint

$$\frac{1}{2} \sigma_\omega - \mu \sigma_B = -c \frac{\mu^3}{3T}$$

[Bhattacharya et al., Chapman et al.]

Here:

- If parity broken only by an anomaly
- Generic isotropic holographic superfluids
- $T \to 0 \Rightarrow$ Universal chiral response: fixed by the anomaly

$$\tilde{\kappa}_\omega = 0 \qquad \tilde{\kappa}_B = \frac{c}{3}\mu \qquad \sigma_\omega = 0 \qquad \sigma_B = \frac{\mu}{T}\tilde{\kappa}_B = c\frac{\mu^2}{3T}$$

Anomalous holographic superfluid

- Minimal superfluid model in 3+1 dim:
  - Einstein-Maxwell-Higgs on asymptotically AdS_5
  - Abelian gauge field A_M and charged scalar field ψ
  - SSB of U(1) by condensation of the charged scalar
- $U(1)^3$ anomaly: parity odd topological term

$$S = S_{\rm EH} + S_{matter} + S_{\rm CS}$$

$$S_{\rm EH} = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\,(R+12)$$
$$S_{matter} = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(-\frac{1}{4}V_F(|\psi|)F_{MN}F^{MN} - V_\psi(|\psi|)(D_M\psi)(D^M\psi)^* - V(|\psi|)\right)$$
$$S_{\rm CS} = \frac{c}{24}\int d^5x\sqrt{-g}\epsilon^{MNPQR}A_M F_{NP} F_{QR}$$

- $D_M = \partial_M - iqA_M$
- $V_F(0) = V_\psi(0) = 1$ and $V(0) = 0$
- $c \equiv$ anomaly strenght

[Bhattacharya et al. 1105.3733]

Stationary superfluid

- Ansatz

$$ds^2 = -r^2 f(r)dt^2 + r^2 d\vec{x}^2 + 2h(r)dtdr\,, \qquad \psi = \varrho(r)e^{iq\varphi(r)}$$
$$A_M = (A_0(r), 0, 0, 0, A_4(r))\,, \qquad G_M = A_M - \partial_M\varphi$$

- Thermodynamic properties from asymptotics

$$T = \frac{r_h^2 f'(r_h)}{4\pi h(r_h)}\,, \qquad s = \frac{2\pi r_h^3}{\kappa^2}$$
$$f = 1 - \frac{2\kappa^2 P}{r^4} + \mathcal{O}(r^{-5})\,, \qquad h = 1 - \frac{\Delta C_\Delta^2 |\langle O_\psi\rangle|^2}{6r^{2\Delta}} + \mathcal{O}(r^{-2\Delta-2})$$
$$\varrho = \frac{C_\Delta |\langle O_\psi\rangle|}{r^\Delta} + \mathcal{O}(r^{\Delta-2})\,, \qquad G_0 = \mu - \frac{\kappa^2\rho_t}{r^2} + \mathcal{O}(r^{-3})$$

- Noether charge and Gibbs Duhem relation

$$Q_1 = \frac{r^5 f' - r^3 V_F G_0 G_0'}{2\kappa^2 h} = sT = 4P - \mu\rho_t$$

Small superfluid velocity

- Linear perturbations

$$G_i = -g(r)\partial_i\phi \qquad g_{ti} = -r^2\gamma(r)\partial_i\phi$$

- Asymptotics

$$\gamma = \frac{1}{2}\frac{(\rho_t - \rho)\kappa^2}{r^4} + \mathcal{O}(r^{-5}), \qquad g = 1 - \frac{(\rho_t - \rho)\kappa^2}{\mu r^2} + \mathcal{O}(r^{-3})$$

- Conserved charges

$$Q_2 = \frac{r^5\gamma' + r^3 V_F g G_0'}{2\kappa^2 h} = \rho$$

$$f\, Q_2 = \gamma Q_1 + \frac{f r^3 V_F (g G_0' - g' G_0)}{2\kappa^2 h}$$

Transport coefficients from fluid-gravity correspondence

- Boost solution $\Rightarrow$ constant normal component velocity
- Allow for space-time dependence of thermo variables
- Add corrections to metric and matter fields to satisfy eoms
- Solve order by order in gradient expansion of thermo variables
- Apply AdS/CFT dictionary to compute the (covariant) current
- Read off the transport coefficients

Chiral conductivities

$$\tilde{\kappa}_B = c \int_{r_h}^{\infty} g^2 G_0' + R(G_0 - g\mu) g G_0' dr$$

$$\tilde{\kappa}_\omega = -2c \int_{r_h}^{\infty} (G_0 - \mu g) g G_0' + R(G_0 - \mu g)^2 G_0' dr$$

$$\sigma_B = \frac{c}{T} \int_{r_h}^{\infty} g G_0 G_0' dr$$

$$\sigma_\omega = -\frac{2c}{T} \int_{r_h}^{\infty} (G_0 - \mu g) G_0 G_0' dr$$

with

$$R = \frac{\rho}{4P - \mu(\rho_t - \rho)}$$

- For $T > T_c$: no condensate $\Rightarrow \psi = 0$ and $\rho_t = \rho$
- RN background $+\ g = 1,\ \gamma = 0 \Rightarrow$ Exact chiral conductivities:

$$\tilde{\kappa}_\omega = c\left(\mu^2 - \frac{\rho}{6P}\mu^3\right) \qquad \tilde{\kappa}_B = c\left(\mu - \frac{\rho}{8P}\mu^2\right)$$

$$\sigma_\omega = c\,\frac{\mu^3}{3T} \qquad \sigma_B = c\,\frac{\mu^2}{2T}$$

- For $T < T_c$: condensate $\Rightarrow \rho_t > \rho \Rightarrow$ In general $\tilde{\kappa}$, σ model dependent
- But! for $T \to 0$: $\rho \to 0 \Rightarrow$ Universal $\tilde{\kappa}$, σ

Low temperature

- At low T (small Q_1)

$$\frac{\mu g}{G_0} = 1 + \mu \int_r^\infty \frac{2\kappa^2 Q_2 h}{V_F(\psi) G_0^2 r'^3} dr' + \mathcal{O}(Q_1)$$

- Finite g/G_0 at horizon implies $Q_2 \to 0 \Rightarrow g = G_0/\mu$

- Zero temperature chiral conductivities

$$\tilde{\kappa}_\omega = 0 \qquad \tilde{\kappa}_B = \frac{c}{3}\mu \qquad \sigma_\omega = 0 \qquad \sigma_B = \frac{\mu}{T}\tilde{\kappa}_B = c\frac{\mu^2}{3T}$$

- Zero temperature $\equiv$ zero normal charge density $\rho = 0$

Ground state of isotropic superfluids

following [Gubser-Nellore, Horowitz-Roberts]

- Zero temperature limit of the BH dual to the condensed phase ?
- Domain wall between aAdS in the UV and an IR stationary configuration
- UV asymptotic AdS: $ds^2 = r^2(-dt^2 + d\vec{x}^2) + 2dtdr$
- Two different possible IR emergent behaviors
  - If ψ sits at a minimum of $V(\psi) \Rightarrow$ AdS geometry
  - If ψ sits at a different constant value $\Rightarrow$ Lifshitz like geometry

AdS to AdS domain wall

- IR: $\psi = \psi_{IR}$ minimum of $V \Rightarrow$ IR AdS solution:

$$ds^2 = \frac{r^2}{L_{IR}^2}(-f_0 dt^2 + d\vec{x}^2) + 2\frac{\sqrt{f_0}}{L_{IR}} dtdr$$

$$G_0 = p_0 r^{\Delta_G - 3}, \qquad g = g_0 r^{\Delta_G - 3}, \qquad \gamma = 0$$

$$\Delta_G = 2 + \sqrt{1 + \frac{2q^2|\psi_{IR}|^2 V_\psi(\psi_{IR})}{V_F(\psi_{IR})} L_{IR}^2}$$

- AdS to AdS stable if current operator irrelevant, i.e. $\Delta_G > 4$
- g/G_0 finite $\Rightarrow$ normal charge density has to vanish

AdS to Lifshitz domain wall

- IR: $\psi = \psi_0 \Rightarrow$ IR Lifshitz solution:

$$ds^2 = -\frac{z p_0^2 V_F(\psi_0)}{2(z-1)} r^{2z} dt^2 + r^2 d\vec{x}^2 + \frac{\sqrt{3} z p_0 V_F(\psi_0)}{q\psi_0 \sqrt{(z-1) V_\psi(\psi_0)}} r^{z-1} dr dt$$

$$G_0 = p_0 r^z \,, \qquad g = g_0 r^z \,, \qquad \gamma = -\frac{z g_0 p_0 V_F(\psi_0)}{2(z-1)} r^{2z-2}$$

- z is fixed given the couplings and potential: V_ψ, V_F and V
- Reality of the solution demands $z > 1$
- again g/G_0 finite $\Rightarrow$ normal charge density vanishes

Numerics

- Study temperature dependence of $\tilde{\kappa}$'s and σ's
- Construct explicit AdS to AdS and AdS to Lifshitz DW.
- Choose particular potential and couplings

$$V(|\psi|) = m^2|\psi|^2 + \frac{u}{2}|\psi|^4$$

$$V_\psi = 1\,, \qquad V_F = 1$$

with $m^2 < 0$ and $u > 0$.

- Admits both AdS and Lifshitz ground states depending on $\{q, m, u\}$

- Conformal fixed point:

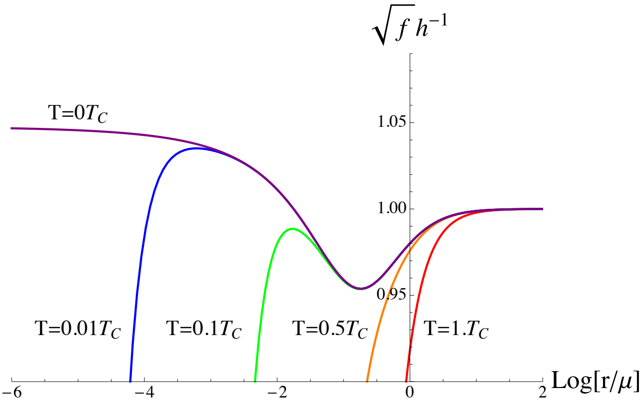
$$\psi_{IR} = \sqrt{-\frac{m^2}{u}} \qquad L_{IR} = \sqrt{\frac{24u}{m^4 + 24u}}$$

- Lifshitz fixed point:

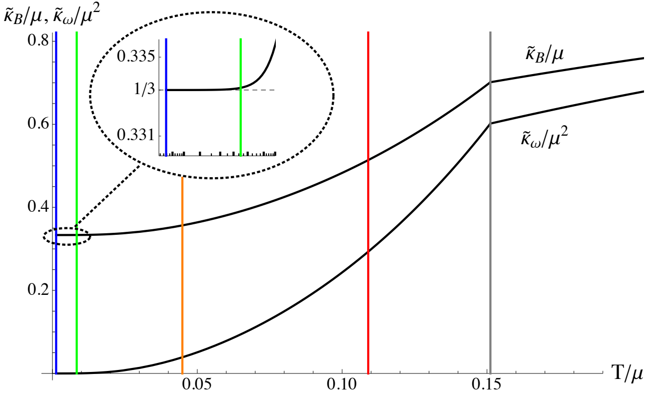
$$\psi_0 = \sqrt{-\frac{m^2}{u} + \frac{2q^2(z-1)}{zu}} \qquad \frac{u}{2}\psi_0^4 + m^2 + \frac{2q^2(9 + z(2+z))}{3z} - 12 = 0$$

- Might be that both solutions are possible ⇒ Stability

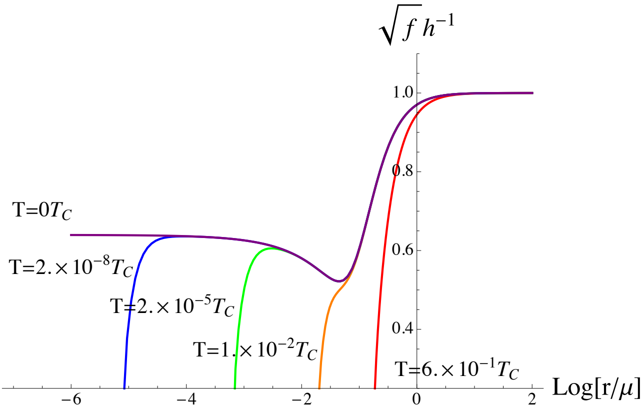
AdS to AdS: $m^2 = -15/4$, $q = 2$ and $u = 6$



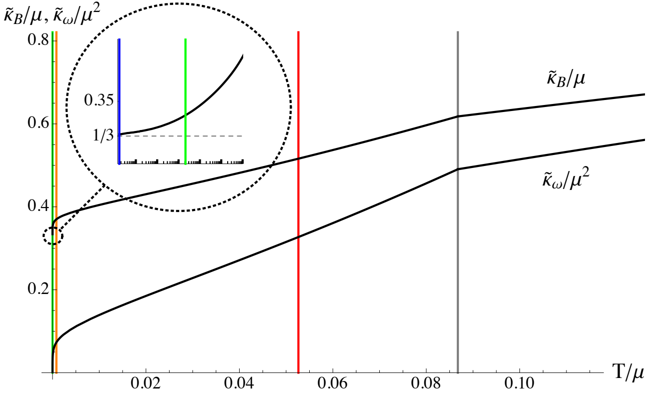
AdS to AdS: $m^2 = -15/4$, $q = 2$ and $u = 6$



AdS to Lifshitz: $m^2 = -15/4$, $q = 3/2$ and $u = 7$



AdS to AdS: $m^2 = -15/4$, $q = 3/2$ and $u = 7$



Convengerce gets worse the larger the z

Conclusions and outlook

- At $T \to 0$: the chiral conductivities are universal $\Leftarrow \rho \to 0$
- The chiral vortical parameters vanish: no normal component to support the vorticity
- General validity ?
  - Other dimensions
  - Other anomalies
  - Other parity breaking effects
  - Other ground states
- Tip from C. Hoyos:
  possible to fix the coefficients in effective action in [Chapman et al.]

Thanks