

# Holographic superconductors and spatial modulation

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## Introduction: AdS/CMT

Use the gauge/gravity correspondence as a tool to investigate the dynamics of strongly coupled CFTs at finite temperature and charge density and/or placed in an external magnetic field.

- Attempt to understand universal features of strongly coupled condensed matter systems found in the vicinity of 'quantum critical points'.
- Explore black hole physics: construct and study novel charged black hole solutions that asymptote to AdS.

# Introduction: Top-Down Vs Bottom-Up

## Top-Down Approach:

- Consider theories obtained by consistent truncations of the  $D=10,11$  supergravities.
- Difficult to obtain CFTs of interest; involved calculations.
- CFT guaranteed to be well defined.

## Bottom-Up Approach:

- Consider phenomenological gravity theories with only few additional d.o.f. that are dual to CFTs with the desirable features.
- No guarantee to have a string embedding.
- Simple calculations.

# Introduction: Questions

Specific questions that can be addressed:

- What type of phases are possible and what are the transport properties of each phase?
- What kind of ground states are possible? classification of IR geometries, investigation of new emergent IR scaling behaviours?
- How do these phases compete? What is the dual phase diagram?

## Holographic superconductors

High  $T_c$ -superconductors constitute one of the most challenging problems in condensed matter physics. Remain mysterious due to their strongly coupled nature.

Minimal ingredients:

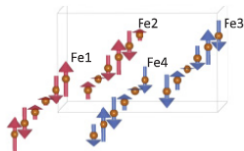
- finite temperature  $\rightarrow$  black holes in the bulk
- finite charge density  $\rightarrow$   $U(1)$  gauge field
- an “order parameter” that spontaneously acquires an expectation value, e.g. a charged scalar for s-wave superconductors.

*Superconducting instabilities:* As the temperature is reduced, a new branch of black holes supported by non-vanishing hair emerges at some critical temperature. The  $U(1)$  symmetry is now spontaneously broken.

## Spatially modulated phases

In condensed matter, phases with *spontaneously* broken translational invariance are very common. Realised in various configurations, e.g. stripes, checkerboards and helices.

- Spin Density Wave
- Charge Density Wave
- Current density wave



The modulation is fixed by an order parameter with non-zero momentum and it's not related to the underlying lattice of the material. Holographically, SM phases dual to BHs with broken translational invariance.



Examples, at finite charge density:

- in  $D=5$  [Nakamura, Ooguri, Park] and in  $D=4$  [Donos, Gauntlett] with the mixing introduced by CS terms and axions respectively.
- PT does not necessarily need to be broken [Donos, Gauntlett].

Examples, in magnetic field:

- in  $D=4,5$  with mixing term  $\phi * F \wedge G$  [Donos, Gauntlett, CP]
- in  $U(1)^3$  and  $U(1)^4$  sugra with metric components mixing as well [Donos, Gauntlett, CP]. Interesting interplay with susy solutions.

→ Spatially modulated phases are more the rule, than the exception.



## Plan for this talk

### Question:

Is it possible to have spatially modulated superconducting states in holography? [Donos, Gauntlett]

- The FFLO state describes an s-wave superconductor in which the Cooper pair has non-vanishing momentum.
- This was conjectured to exist in the '60s and possibly has been seen experimentally in heavy fermion materials and some organic superconductors, eg  $\text{CeCoIn}_5$ .

## The model of interest

*Aim:* Study p-wave superconductors in  $D=5$ . The order parameter can be either an  $SU(2)$  vector or a two-form. Both cases give similar results; we focus on the second case.

Consider a theory of gravity in  $D=5$  coupled to a  $U(1)$  gauge field and a complex two-form

$$\mathcal{L} = (R + 12) * 1 - \frac{1}{2} * F \wedge F - \frac{1}{2} * C \wedge \bar{C} - \frac{i}{2m} C \wedge \bar{H}$$

where  $F = dA$  and  $H = dC + ieA \wedge C$ .

- For particular values of  $(e, m)$ , this theory can be obtained as a consistent truncation from  $D = 10, 11$ ; corresponds to a subsector of the  $D = 5$  Romans theory.

- This theory admits a unit radius  $AdS_5$  vacuum solution with  $A = C = 0$  which is dual to a  $d=4$  CFT with a conserved  $U(1)$  current and a tensor operator with charge,  $e$ , and scaling dimension  $\Delta = 2 + m$ .
- Another solution of the theory is the electrically charged AdS-RN black hole.

$$ds^2 = -gdt^2 + g^{-1}dr^2 + r^2(dx_i^2), \quad A = \mu\left(1 - \frac{r_+^2}{r^2}\right)dt.$$

This corresponds to the high temperature, spatially homogeneous and isotropic phase of the dual CFTs when held at finite chemical potential  $\mu$ .

## Step 1: Instabilities in the NHL

Study perturbations around the near-horizon limit of the electric AdS-RN:  $AdS_2 \times R^{D-2}$  .

- Modes tend to be more unstable in this region.
- Use the  $AdS_d$  BF bound criterion to check for instabilities: if the bound is violated, the theory is unstable (converse not always true)

$$L^2 M^2 \geq -\frac{(d-1)^2}{4}.$$

**In our model:**

In the NHL, perturbations of the two-form,  $\delta C$ , decouple; consider the ansatz [Donos, Gauntlett]

$$\delta C = \dots + dx_1 \wedge (u_3 dx^3 + v_3 dx^2),$$

where  $u_3 = d_3 \cos(kx_1)$ ,  $v_3 = d_3 \sin(kx_1)$   $\rightarrow$  p-wave

or  $u_3 = d_3 e^{ikx_1}$ ,  $v_3 = id_3 e^{ikx_1}$   $\rightarrow$  p+ip-wave.

- For both cases, if  $e^2 > \frac{m^2}{2}$ , the BF bound is violated for a certain  $k$ . Most unstable mode has  $k \neq 0$ ; when heated up, we expect the preferred branch to be modulated.



**(p+ip)-wave:**  $\delta C = \dots + e^{-ikx_1} id_3(r) dx_1 \wedge (dx_2 - id_3)$

$k = 0$ : order parameter pointing in  $dx_2 + id_3$

3 translations and  $(x_2, x_3)$  rotations upto const gauge transf.

$k \neq 0$ : same as before, but  $x_1$  translations are compensated by a gauge transf.

No helical structure, no symmetry reduction.

## Step 2: Perturbations around $AdS_5$ -RN

Consider linearised perturbations around the full AdS-RN black hole. [Donos, Gauntlett]

- Specify the critical temperature at which the instability sets in.
- Allows to search for instabilities localised far from the horizon.

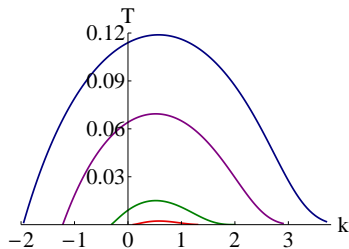
### In our model:

- Two-form perturbations,  $\delta C$ , decouple again. Consider the same ansatz as before.
- All the action is included in the last term  $\sim d_3(r)$ :  
*regular* at the horizon and *spontaneously* breaking the  $U(1)$ .

$$d_3 = d_{3+} + \mathcal{O}(r - r_+), \quad d_3 = c_{d_3} r^{-|m|} + \dots$$



- Obtain an one-parameter family of solutions as expected. Plot the critical temperatures  $T_c$  versus  $k$  for the existence of normalisable static perturbations of the two-form for fixed  $(m, e)$ .



- For fixed  $m$ ,  $T_c$  increases as  $e$  increases. For fixed  $e$ ,  $T_c$  decreases as  $m$  increases.
- Depending on  $(e, m)$  this plot may not cross the  $k = 0$  axis.
- p- and  $(p+ip)$ -wave set in at the same temperature.

## Step 3: Backreacted solutions

Construct the backreacted BHs to get information about the thermodynamics: is the new branch of black holes preferred?

- In principle, one needs to solve PDEs to study spatial modulation.
- Here, we can get away with solving ODEs: the three-dimensional Euclidean group breaks down to Bianchi VII<sub>0</sub>.
- Use the left-invariant one-form of this Lie algebra when constructing the ansatz.

$$\omega_1 = dx_1 ,$$

$$\omega_2 = \cos(kx_1) dx_2 - \sin(kx_1) dx_3 ,$$

$$\omega_3 = \sin(kx_1) dx_2 + \cos(kx_1) dx_3 .$$

## p-wave helical superconductors

Consider the following ansatz:

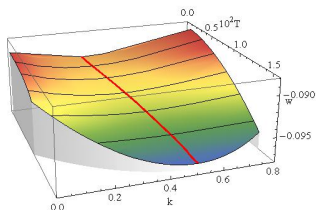
$$ds^2 = -g f^2 dt^2 + g^{-1} dr^2 + h^2 \omega_1^2 + r^2 (e^{2\alpha} \omega_2^2 + e^{-2\alpha} \omega_3^2) ,$$

$$A = a dt ,$$

$$C = (i c_1 dt + c_2 dr) \wedge \omega_2 + c_3 \omega_1 \wedge \omega_3 ,$$

- E.o.m. boil down to a set of ODEs which is solved subject to boundary conditions: regularity at the horizon and  $AdS_5$  asymptotics compatible with spontaneous symmetry breaking - no sources.
- Obtain a two parameter family of solutions, labeled by  $(k, T)$ .

- All the solutions have a lower free energy than AdS-RN: the CFT undergoes a second order phase transition to a helical superconducting phase.
- The preferred ones lie along the red locus: as the temperature is lowered, the pitch is increasing.  $(e, m) = (2, 2)$
- Interestingly, preferred solutions (red line) satisfy  $c_h = 0$ . This is understood by varying the free energy with respect to  $k$ :  $\partial_k w = 0$ .



$$\langle T_{tt} \rangle = 3M + 8c_h,$$

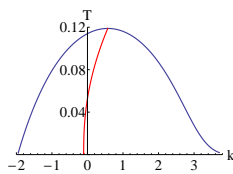
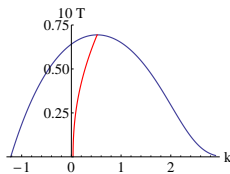
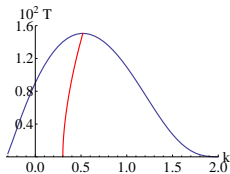
$$\langle T_{x_1 x_1} \rangle = M + 8c_h,$$

$$\langle T_{x_2 x_2} \rangle = M + 8c_\alpha \cos(2kx_1),$$

$$\langle T_{x_3 x_3} \rangle = M - 8c_\alpha \cos(2kx_1),$$

$$\langle T_{x_2 x_3} \rangle = -8c_\alpha \sin(2kx_1).$$

- For large enough  $e$ , the preferred locus exhibit inversion of the pitch of the helix - this point is completely regular. This phenomenon was seen experimentally in e.g. helimagnets.



## T=0 limit

The system admits the following scaling solutions in the IR:

- If  $k = 0$ , the IR fixed point is invariant under the anisotropic scaling: similar to [\[Taylor\]](#)

$$r \rightarrow \lambda^{-1}r, \quad t \rightarrow \lambda^z t, \quad x_{1,3} \rightarrow \lambda^{1-\gamma} x_{1,3}, \quad x_2 \rightarrow \lambda^{1+\gamma} x_2.$$

The entropy density scales like  $S \sim T^{(3-\gamma)/z}$ .

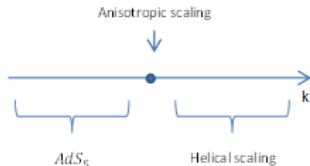
- If  $k \neq 0$ , there is a helical scaling symmetry: similar to [\[Kachru et al.\]](#)

$$r \rightarrow \lambda^{-1}r, \quad t \rightarrow \lambda^z t, \quad x_{2,3} \rightarrow \lambda x_{2,3}, \quad x_1 \rightarrow x_1.$$

The entropy density scales like  $S \sim T^{2/z}$ .

- If  $k \neq 0$ , (same)  $AdS_5$  in the IR, similar to [\[Sachdev et al.\]](#)

The  $T=0$  limit of our black holes approaches smooth domain walls that interpolate between  $AdS_5$  in the UV and the following IR:



### Question:

Both, the  $AdS_5$  and the helical scaling, exist at  $k \neq 0$  but emerge at  $k < 0$  and  $k > 0$  respectively. Is this related to thermodynamics or is there a problem with constructing the domain walls?

## (p+ip)-wave superconductors

A very similar story for the (p+ip)-case:

$$ds^2 = -gf^2 dt^2 + g^{-1} dr^2 + h^2(dx_1 + Qdt)^2 + r^2(dx_2^2 + dx_3^2),$$

$$A = a dt + w dx_1,$$

$$C = (ic_1 dt + c_2 dr + ic_3 dx_1) \wedge (\omega_2 - i\omega_3),$$

- The ansatz is now stationary, but not static.
- By solving a set of seven coupled ODEs subject to boundary conditions, we obtain a 2-parameter family of (p+ip)-wave superconductors. Label solutions by  $(k, T)$  as before.



- These solutions have lower free energy than the AdS-RN as well; p-wave and (p+ip)-wave phases compete.
- The preferred locus of solutions satisfy  $c_w = 0 \Rightarrow c_h = c_Q = 0$  (due to additional smarr formulas). Also understood by varying the action with respect to  $k$ .

$$\langle T_{tt} \rangle = 3M + 8c_h,$$

$$\langle T_{tx_1} \rangle = 4c_Q,$$

$$\langle T_{x_1x_1} \rangle = M + 8c_h,$$

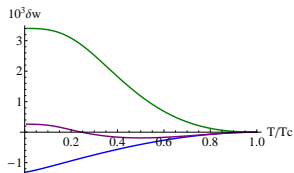
$$\langle T_{x_2x_2} \rangle = M,$$

$$\langle T_{x_3x_3} \rangle = M,$$

- For large  $e$ ,  $k$  becomes slightly negative as the temperature is lowered.
- Have not yet been able to pin down the precise behaviour of the solution when  $T \rightarrow 0$ ; scenarion of  $AdS_5$  in the IR.
- The entropy of the ground states goes to zero. For small values of the charge, there is an interesting crossover in its behaviour.

## Competition between $p$ and $p+ip$

Both instabilities set in at the same  $T_c$ . Which is preferred?



- $p$ -wave is preferred for small  $e$  and  $(p+ip)$ -wave for large  $e$ .
- For intermediate values of  $e$ , there is a first order transition between the two at  $T_*$ ; the black holes have the same free energy, but do not intersect on field space.  
c.f. [Gubser,Pufu]

## Further remarks

- The discussion of homogeneous p-wave superconductors has been generalised to include spatial modulation.
- Can you find two forms with  $e, m$  in top down setting, perhaps with addition of extra fields, that are unstable?
- Spatially modulated phases are more fundamental than expected.
- The field is moving towards solving PDEs [Donos],[Withers],[Rozali et al.]

# Thank you!