A Holographic Model of the Kondo Effect

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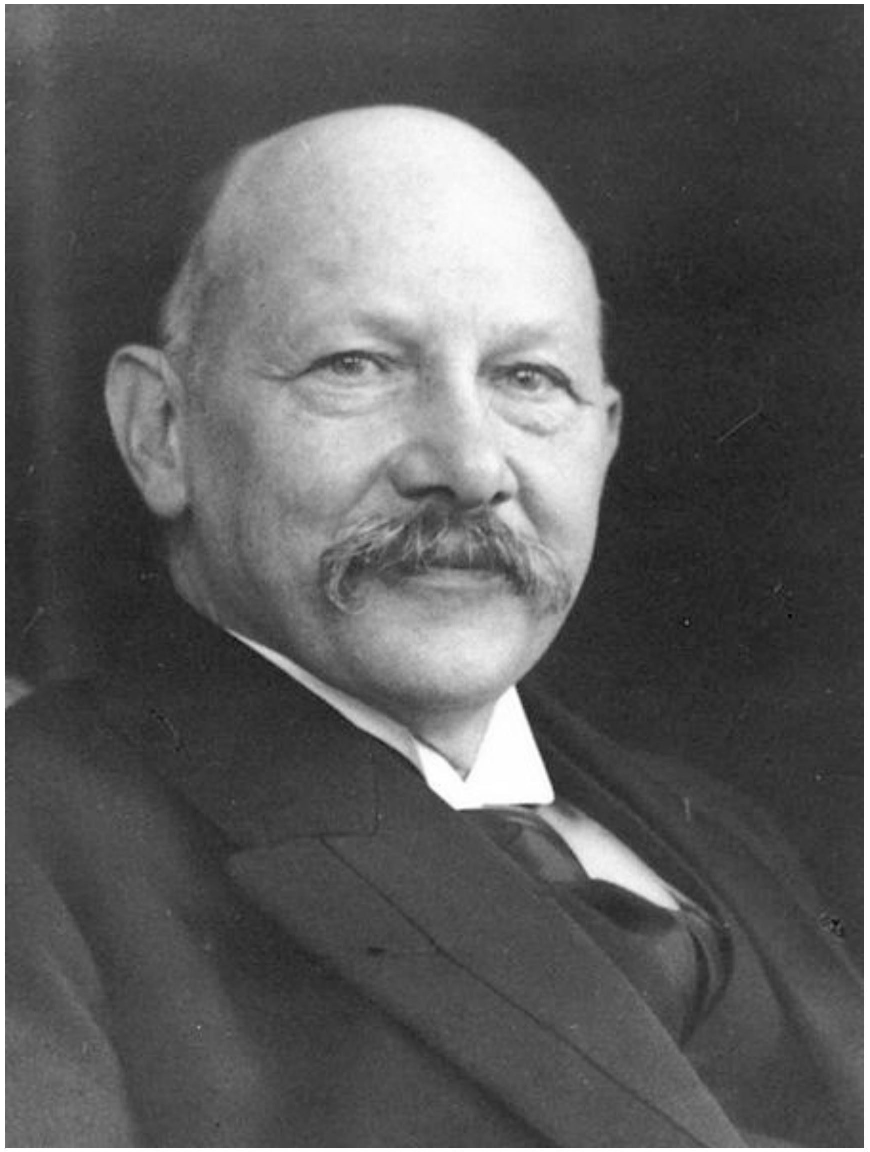
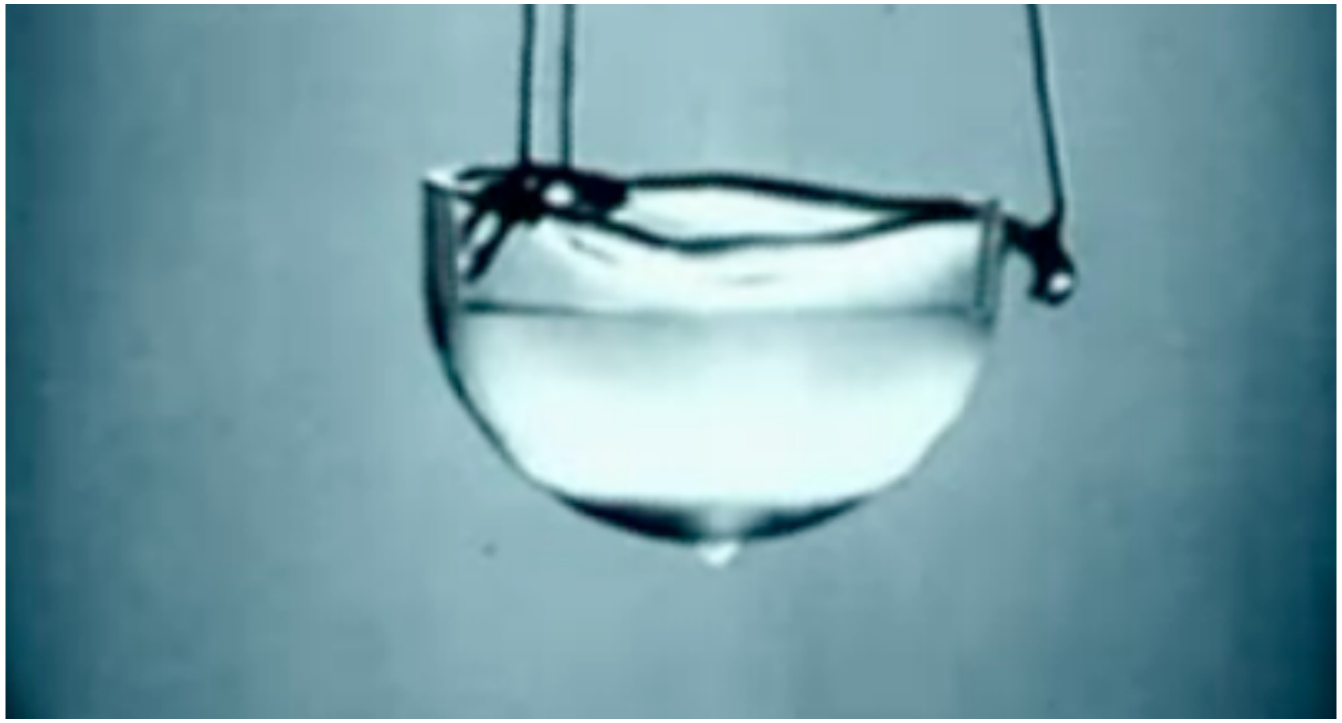
Outline:

- The Kondo Effect
- The CFT Approach
- A Top-Down Holographic Model
- A Bottom-Up Holographic Model
- Summary and Outlook

July 10, 1908

Leiden, the Netherlands



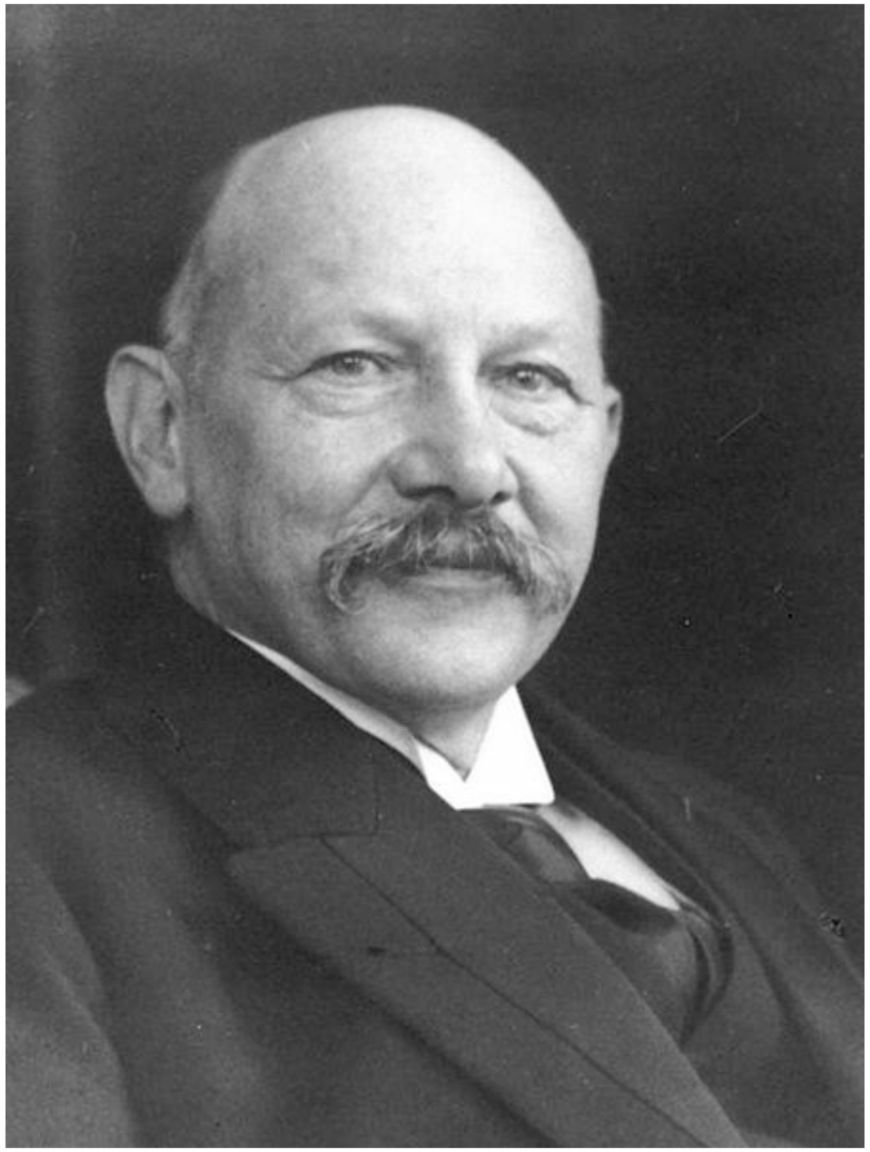


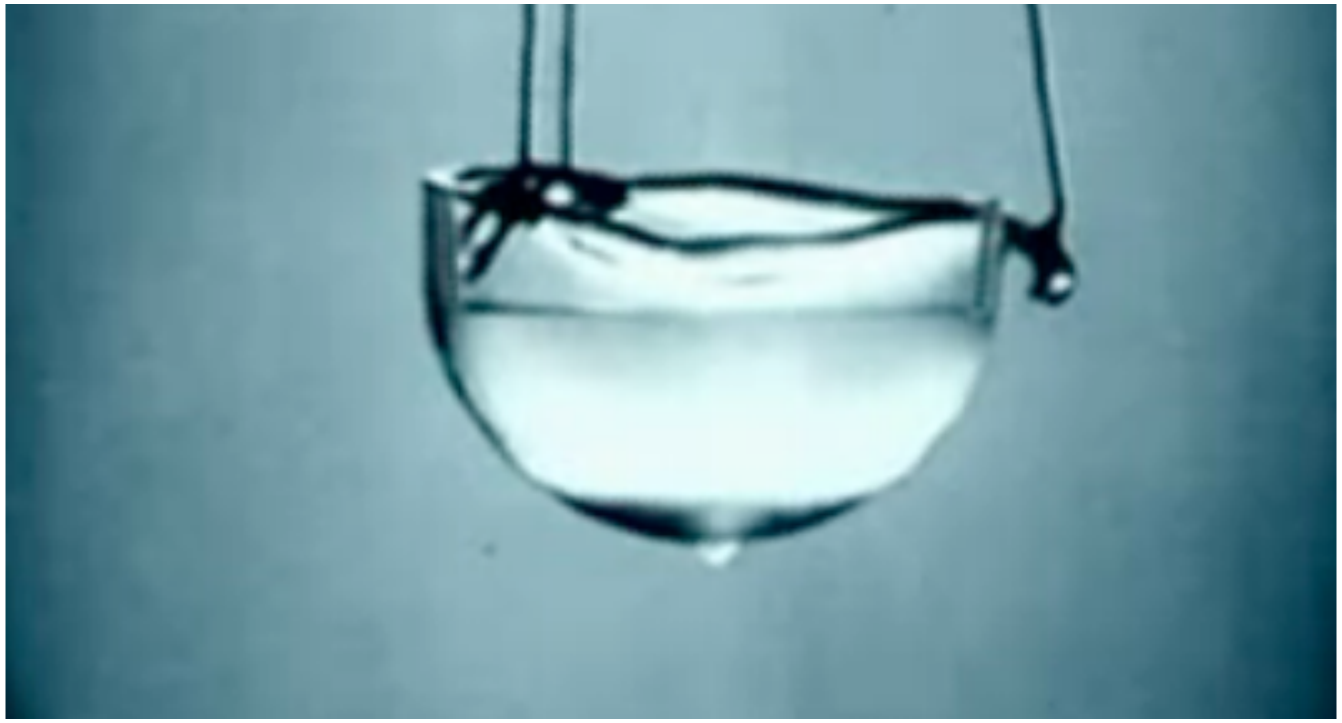
Heike Kamerlingh Onnes liquifies helium

$$T \approx 4.2 \text{ K} \quad (1 \text{ atm})$$

Shortly Thereafter

Leiden, the Netherlands



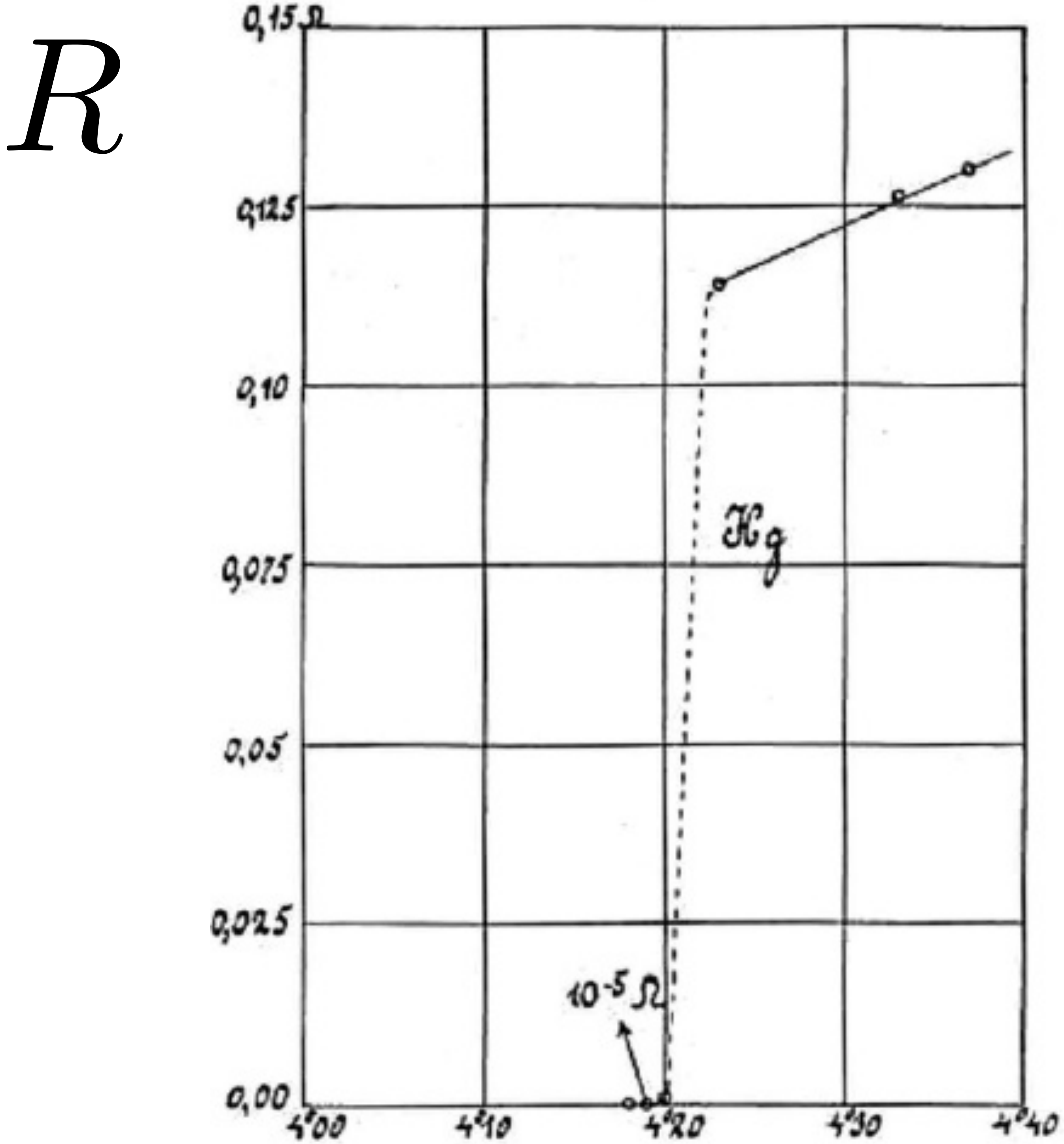


Begins studying low-temperature properties of metals

$$T \approx 1 \text{ to } 10\ K$$

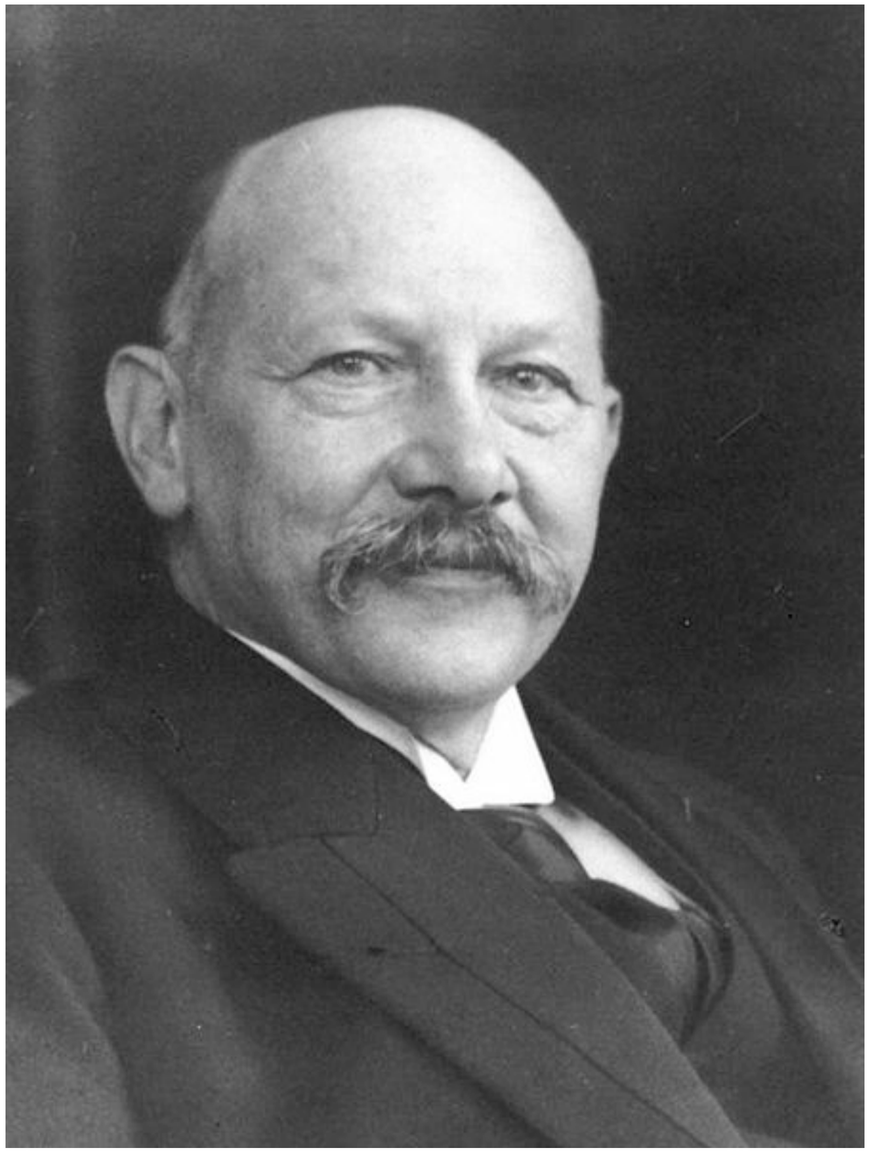
April 8, 1911

Heike Kamerlingh Onnes discovers superconductivity

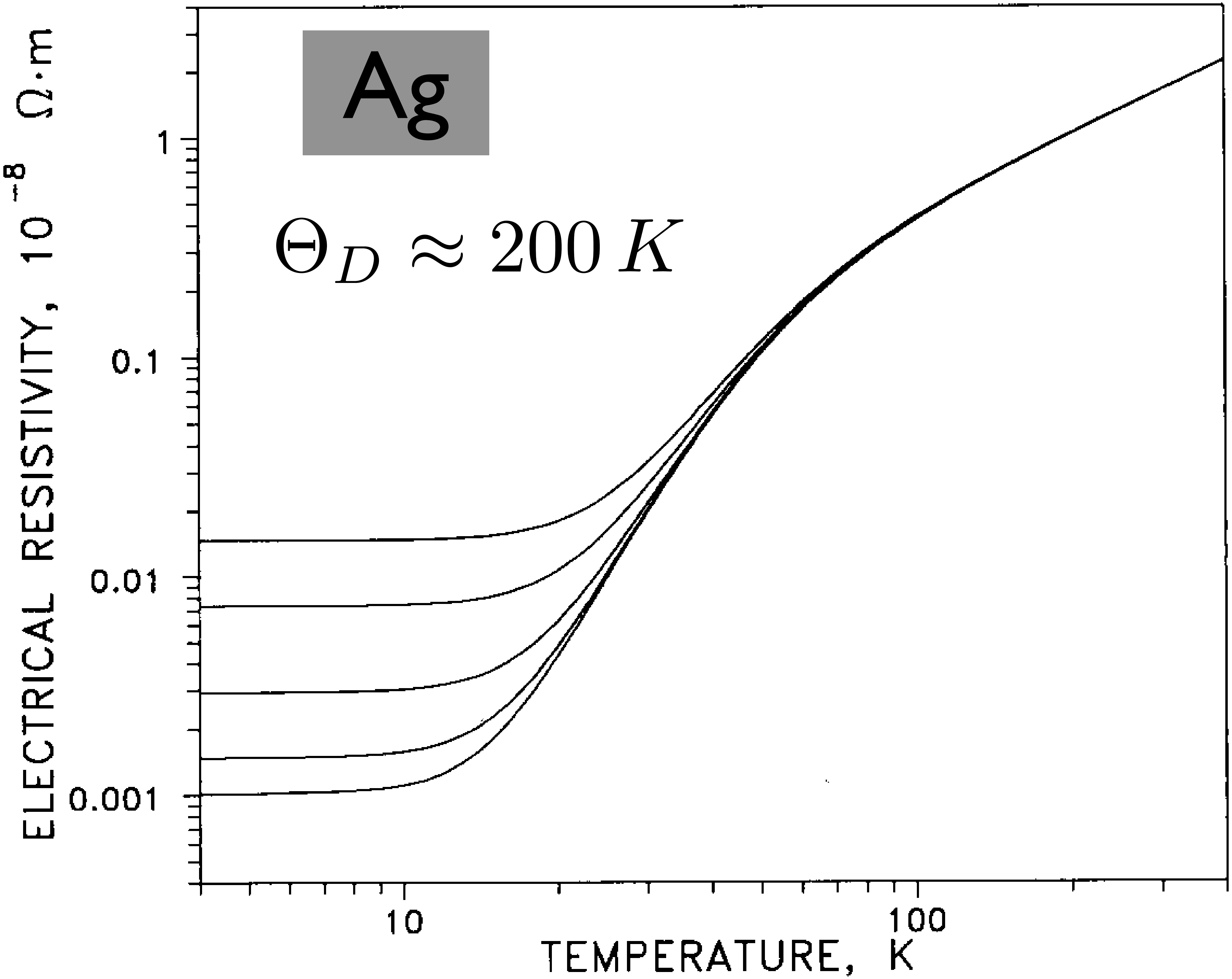


1913

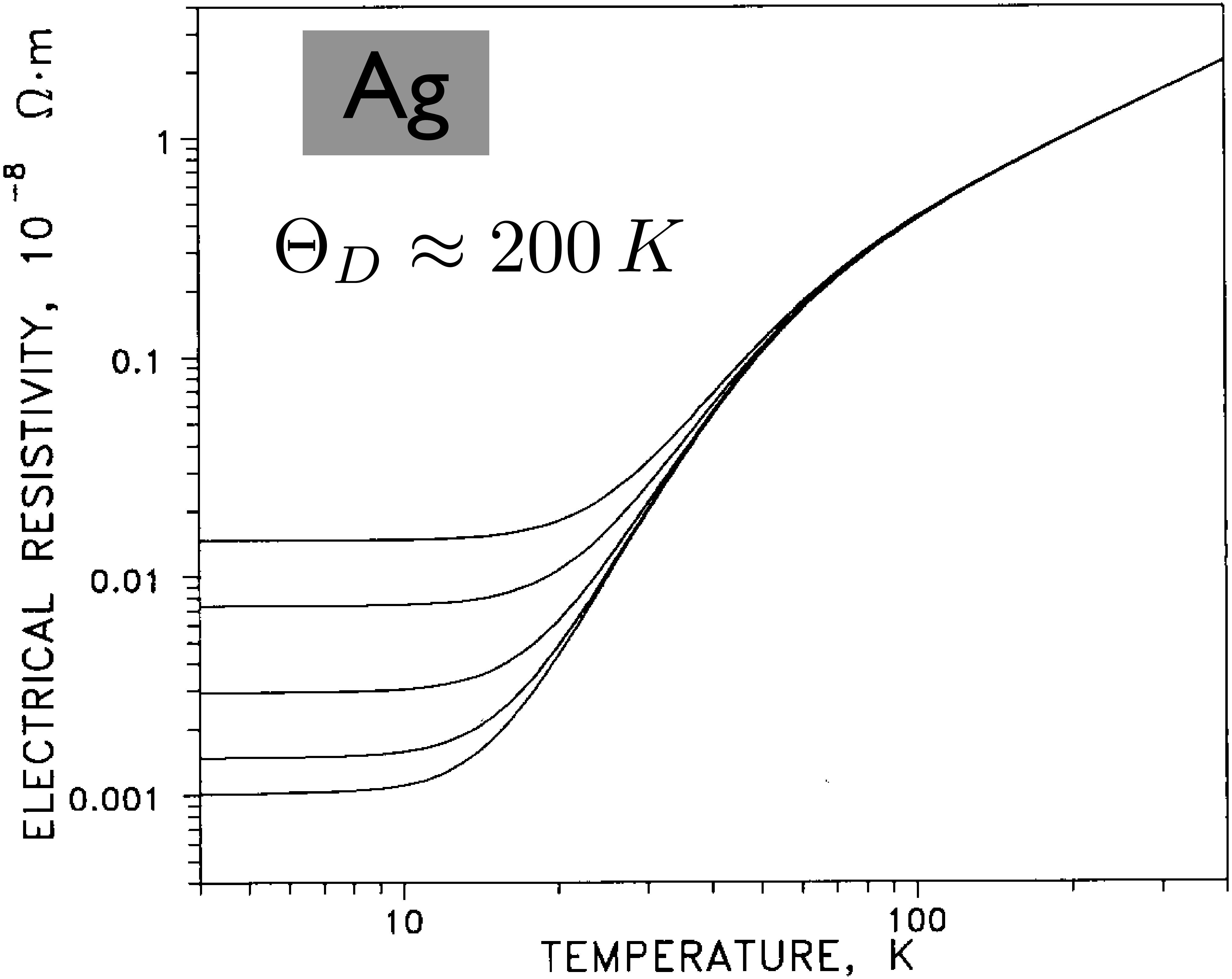
Onnes receives the Nobel Prize in Physics



"for his investigations on the properties of matter at low temperatures which led, *inter alia*, to the production of liquid helium"



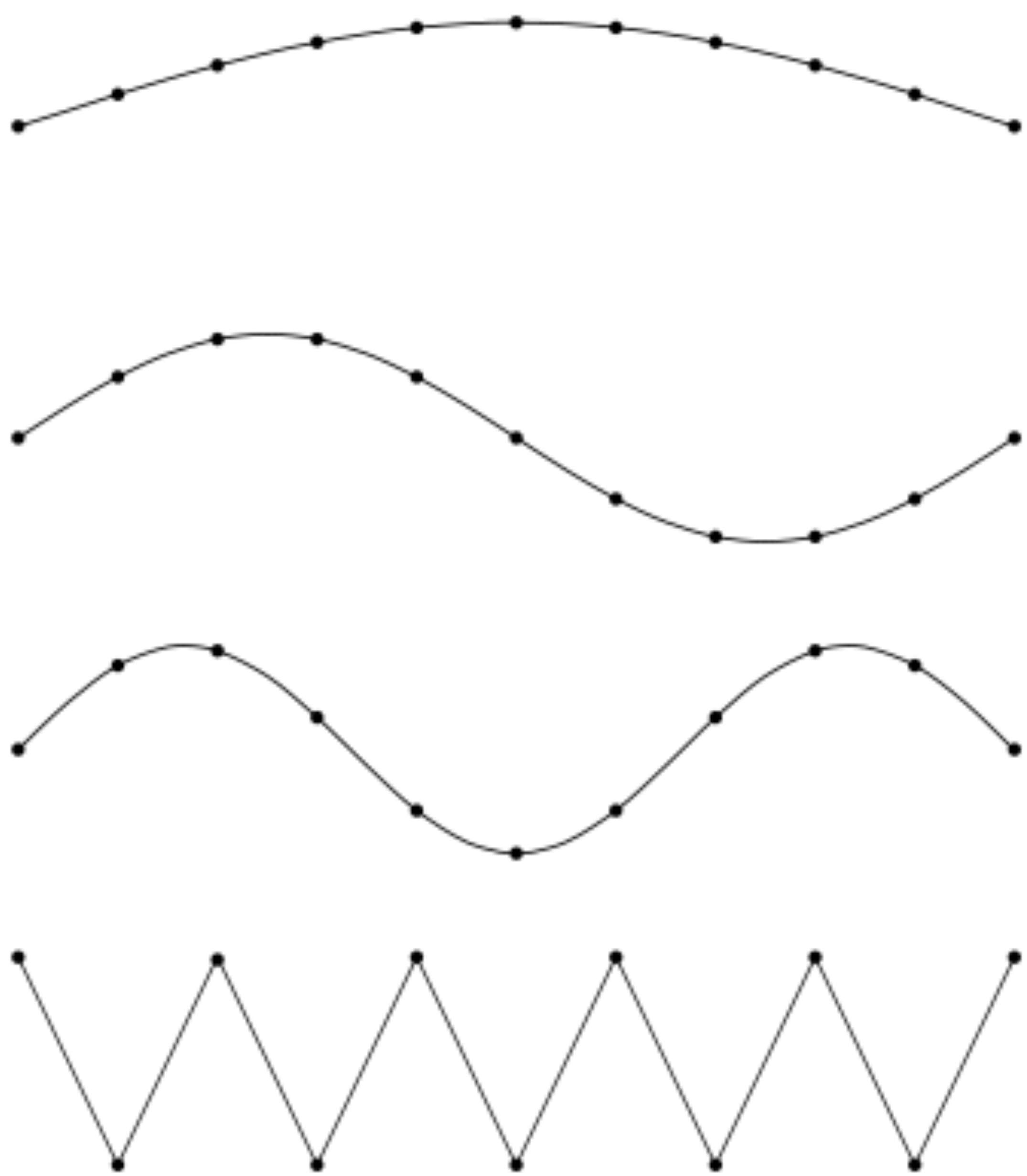
Smith and Fickett, J. Res. NIST, 100, 119 (1995)



Resistivity measures electron scattering cross section

Debye Temperature

Quantized vibrational modes of a solid = Phonons

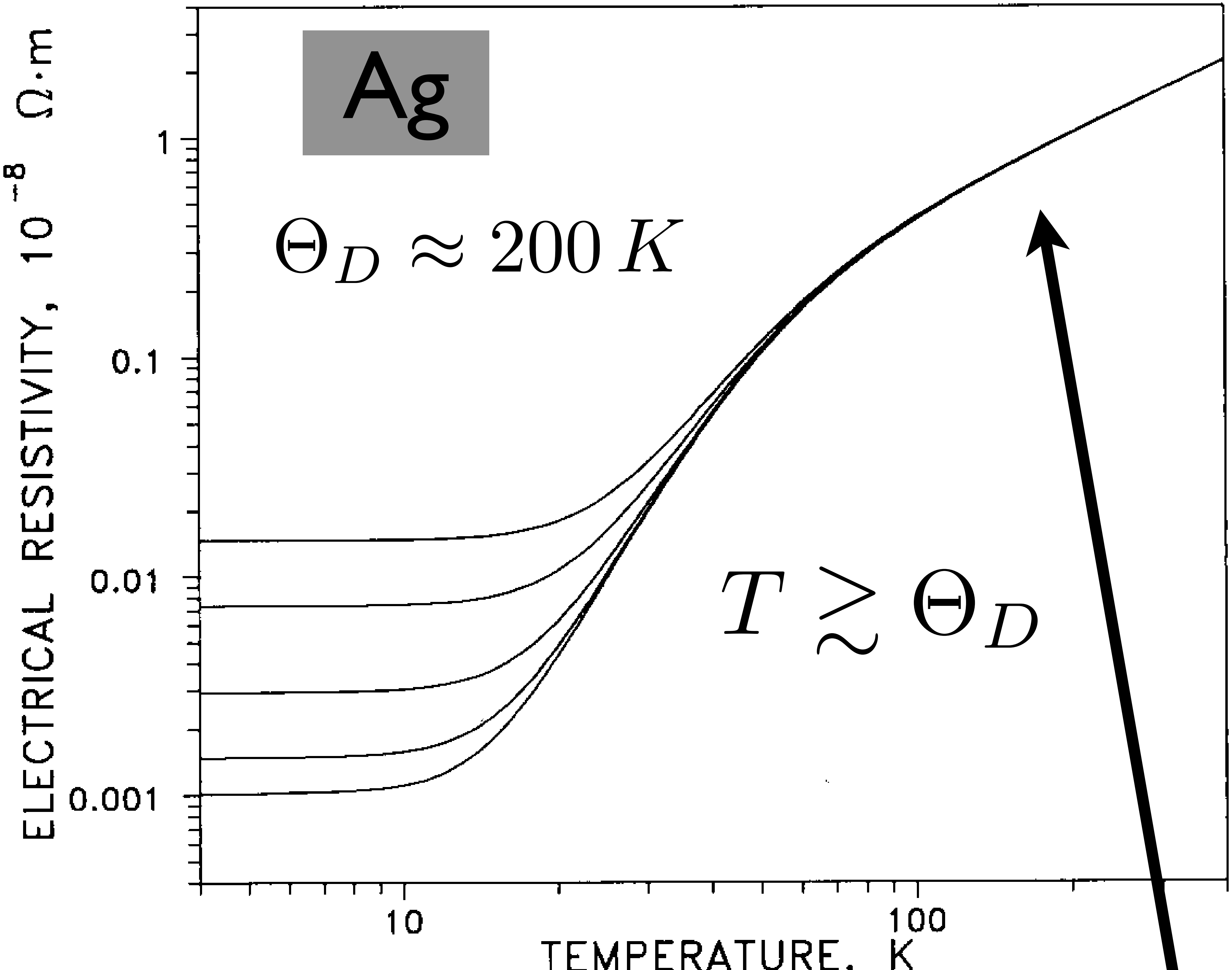


Minimum wavelength:
2 x (lattice spacing)

Maximal energy

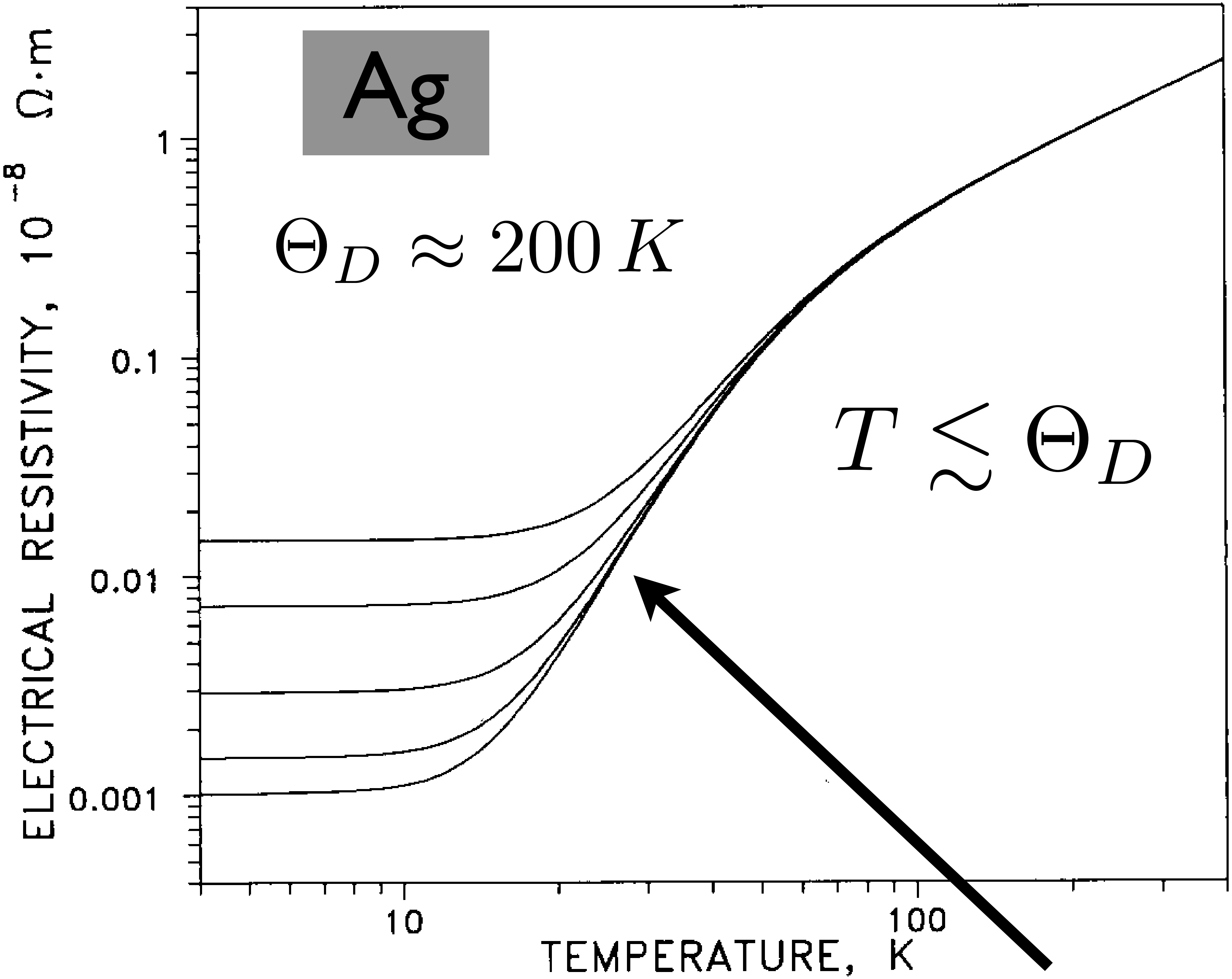
$$\Theta_D$$

lowest temperature at which maximal-energy phonon excited



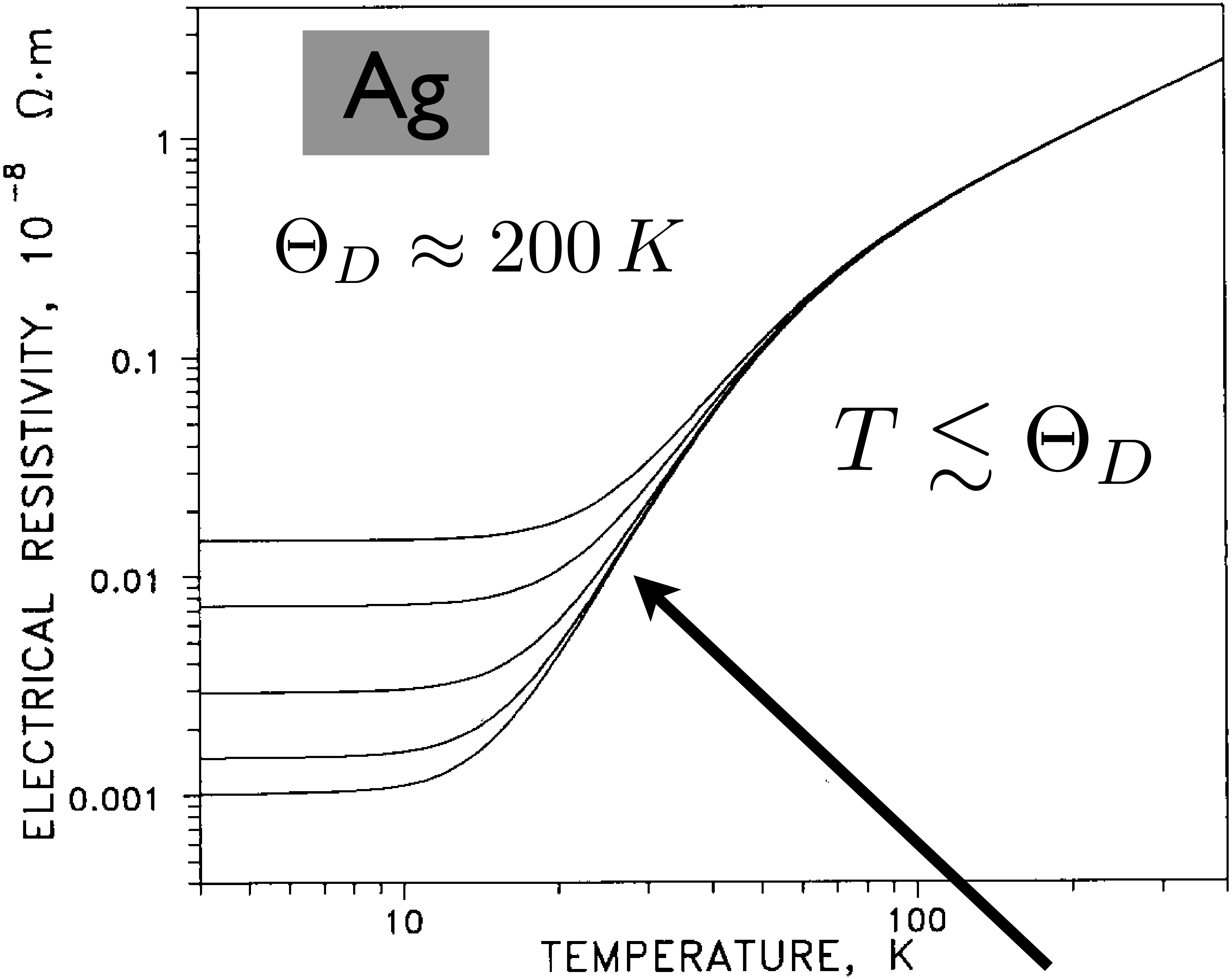
electron-phonon scattering

$\rho(T) \propto T$



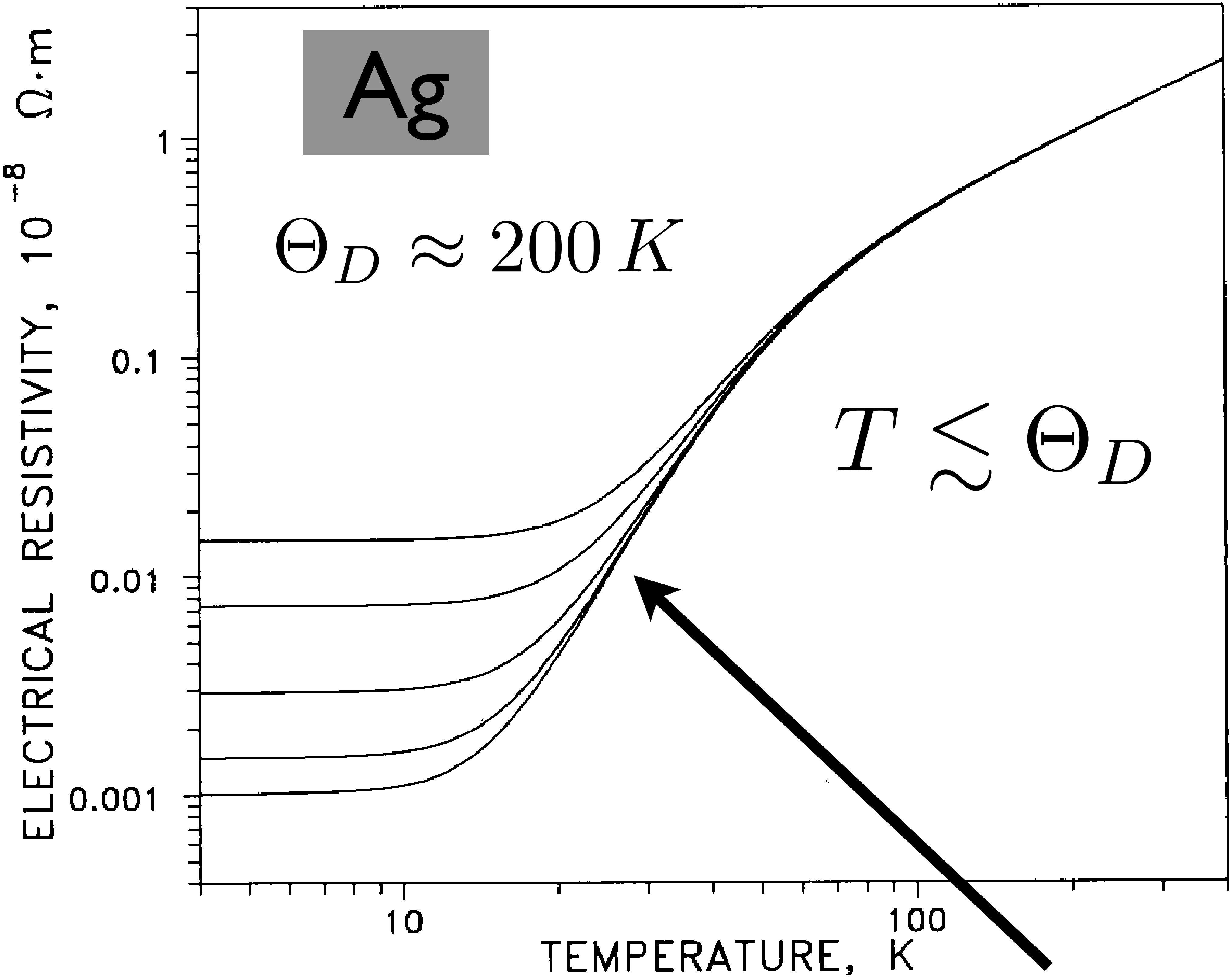
electron-phonon scattering

$$\rho(T) = \rho_0 + a\,T^2 + b\,T^5$$



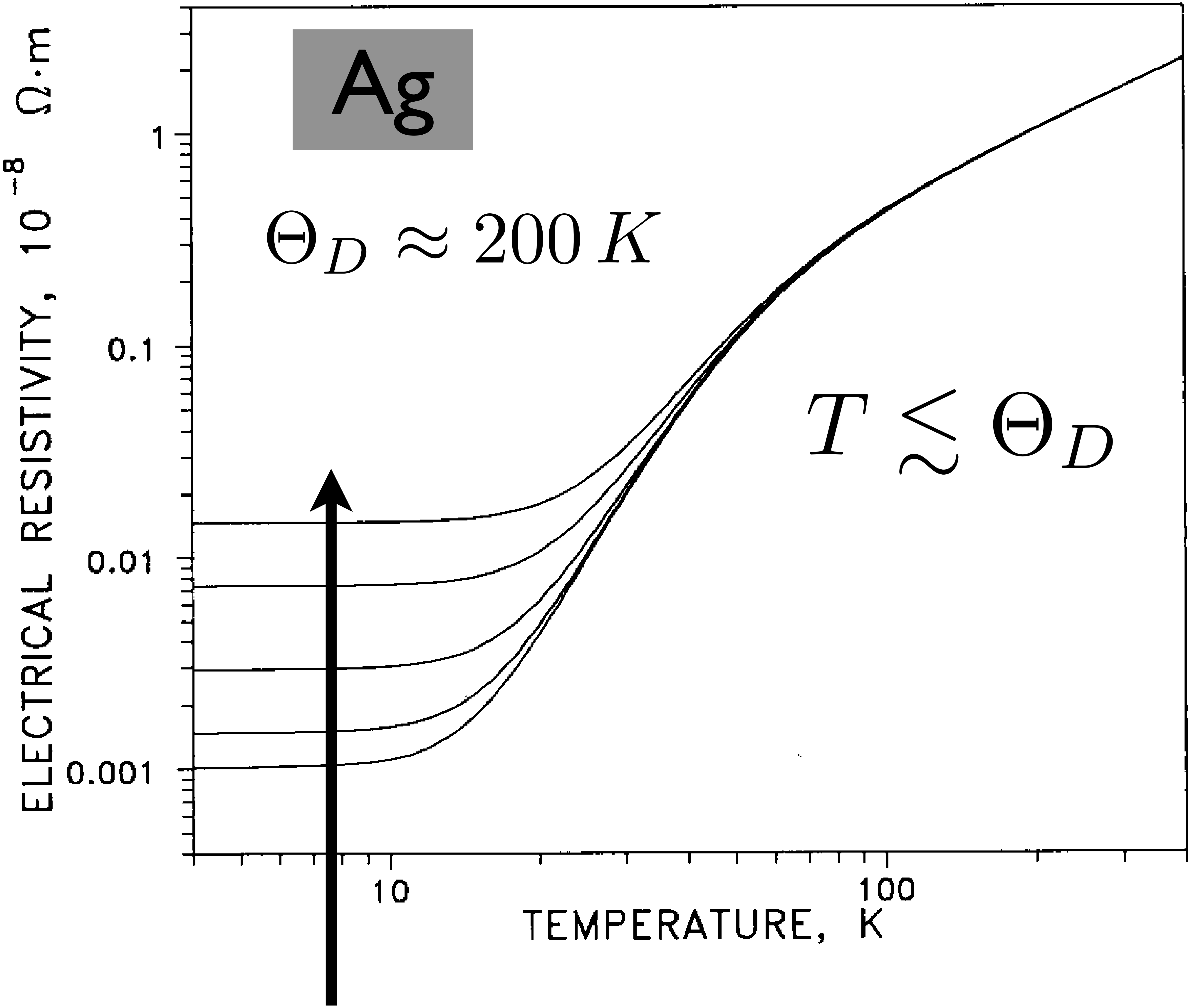
electron-electron scattering

$$\rho(T) = \rho_0 + a\,T^2 + b\,T^5$$



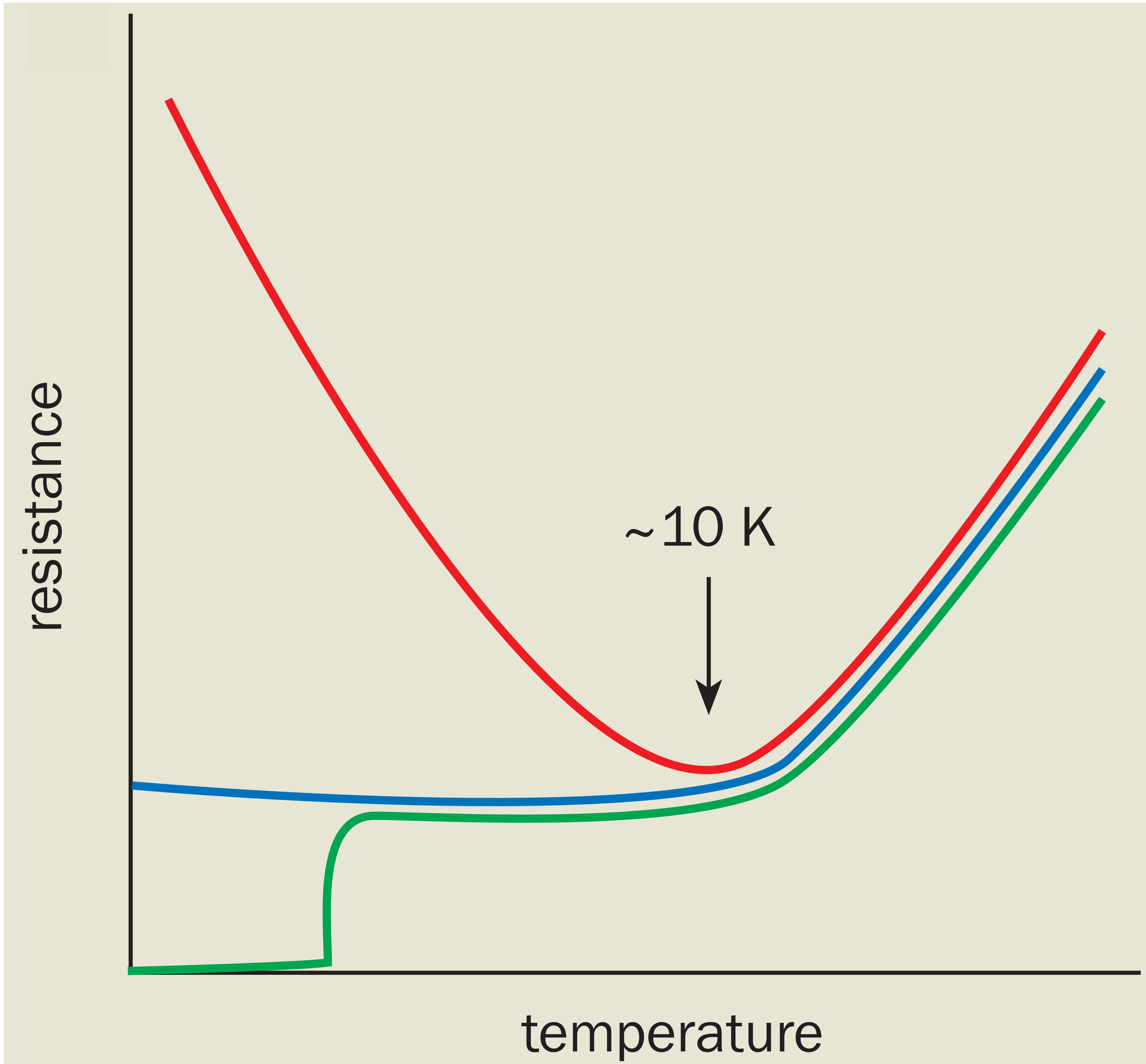
electron-impurity scattering

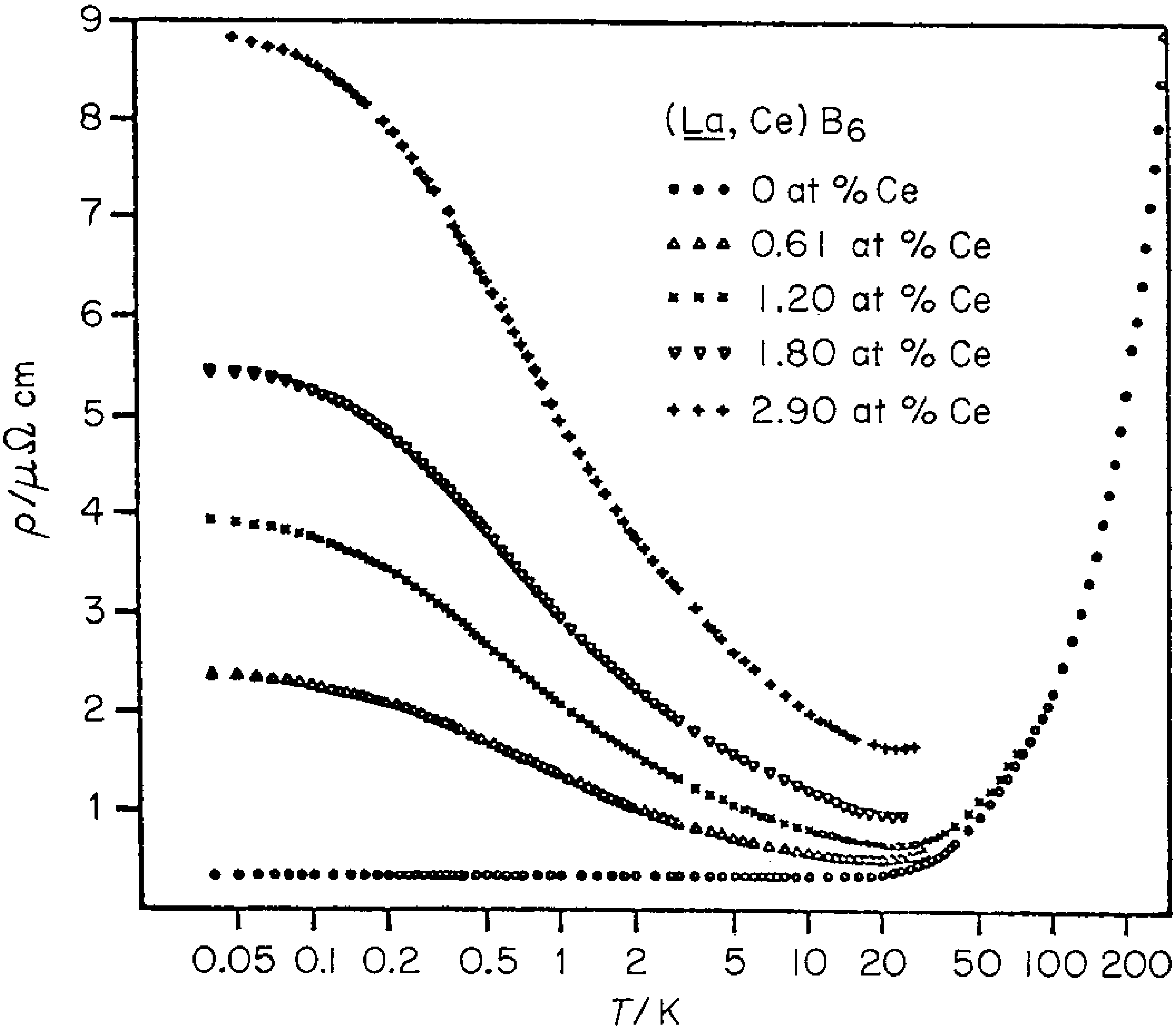
$$\rho(T) = \rho_0 + a\,T^2 + b\,T^5$$



increasing concentration of impurities

The Kondo Effect

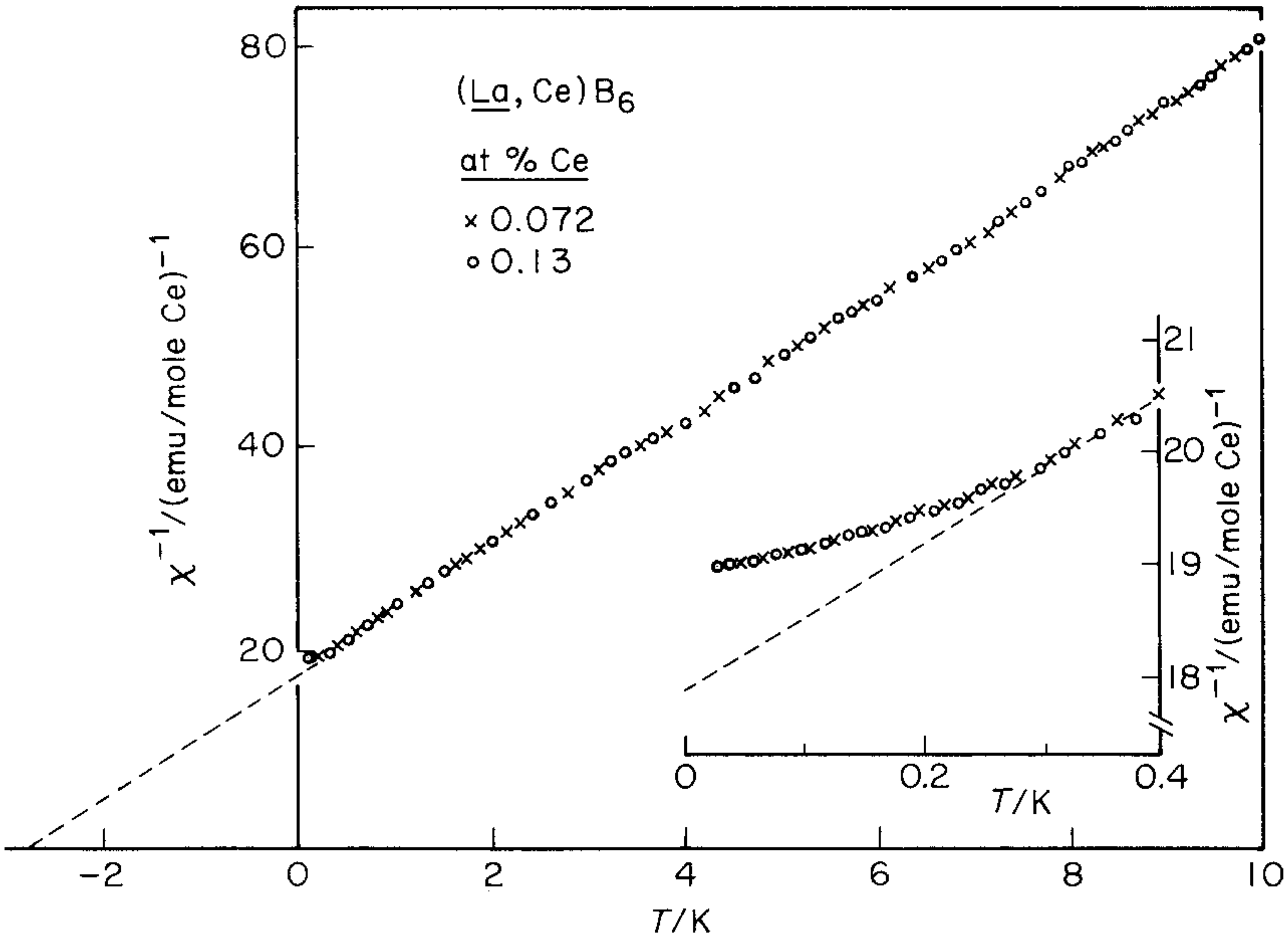




Samwer and Winzer, Z. Phys B, 25, 269, 1976

MAGNETIC Impurities

$$\chi = \left.\frac{\partial^2 F}{\partial B^2}\right|_{B=0}$$



Felsch, Z. Phys B, 29, 211, 1978

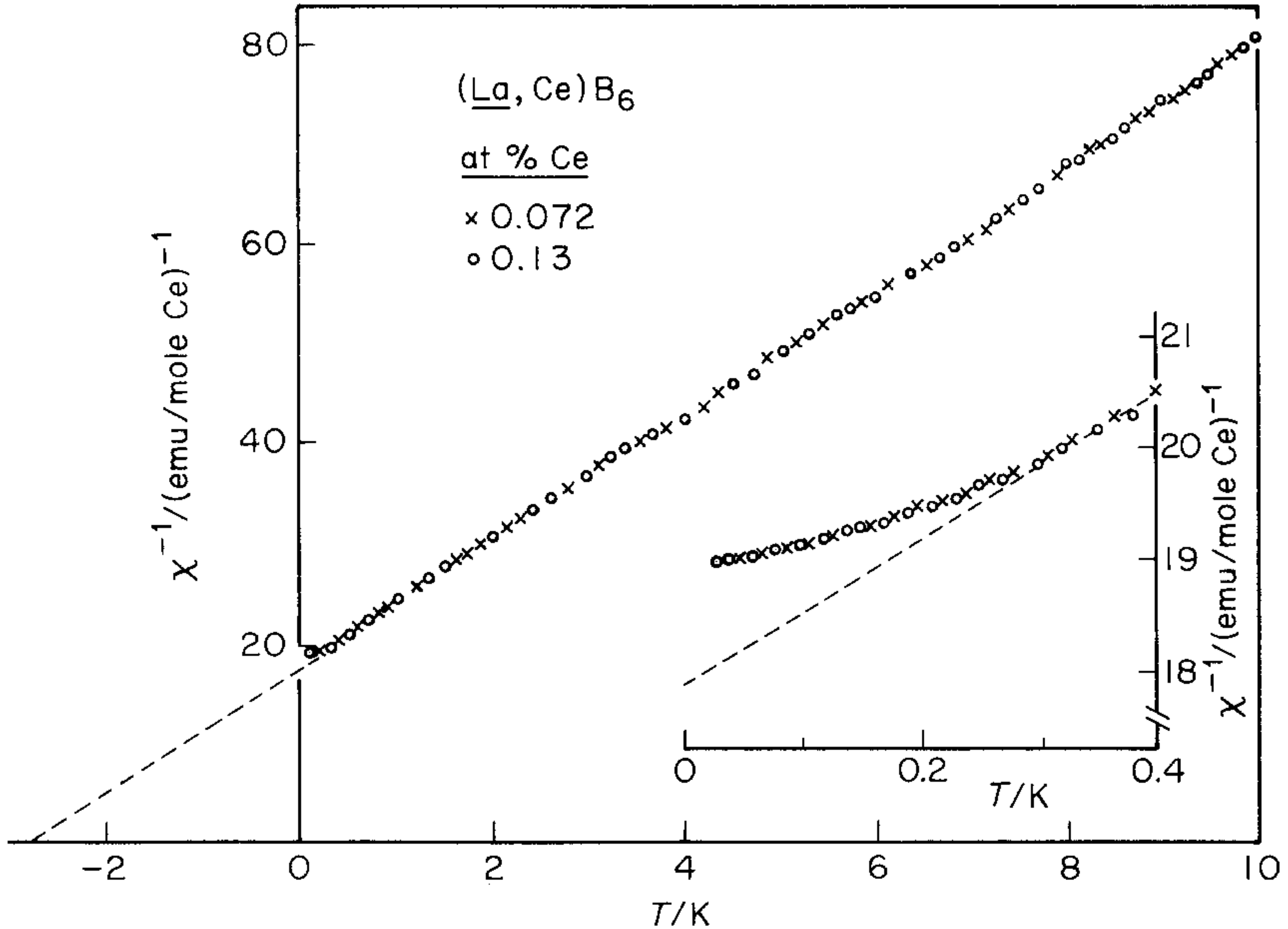
MAGNETIC Impurities

Fermi liquid

Pauli: $\chi \propto T^0$

Free magnetic moment

Curie: $\chi \propto T^{-1}$



Felsch, Z. Phys B, 29, 211, 1978

Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

Jun KONDO

The Kondo Hamiltonian

$$H_K = \sum_{k,\sigma} \varepsilon(k)\, c^\dagger_{k\sigma} c_{k\sigma} + g_K \vec{S} \cdot \sum_{k\sigma k'\sigma'} c^\dagger_{k\sigma} \frac{1}{2} \vec{\tau}_{\sigma\sigma'} c_{k'\sigma'}$$

$$c^\dagger_{k\sigma}\,,\, c_{k\sigma}$$

Conduction electrons

$$\sigma = \uparrow, \downarrow$$

Spin $SU(2)$

$$c_{k\sigma} \longrightarrow e^{i\alpha} c_{k\sigma}$$

Charge $U(1)$

$$\varepsilon(k) = \frac{k^2}{2m} - \varepsilon_F$$

Dispersion relation

The Kondo Hamiltonian

$$H_K = \sum_{k,\sigma} \varepsilon(k)\, c^\dagger_{k\sigma} c_{k\sigma} + g_K \vec{S} \cdot \sum_{k\sigma k'\sigma'} c^\dagger_{k\sigma} \frac{1}{2} \vec{\tau}_{\sigma\sigma'} c_{k'\sigma'}$$

$\vec{S}$ — Spin of magnetic impurity

$\vec{\tau}$ — Pauli matrices

g_K — Kondo coupling

$g_K < 0$ — Ferromagnetic

$g_K > 0$ — Anti-Ferromagnetic

$$\rho(T) = \rho_0 + a\,T^2 + b\,T^5 + c\,g_{_K}^2 - \tilde{c}\,g_{_K}^3\,\ln\left(T/\varepsilon_F\right)$$

$c, \tilde{c} > 0$ $\propto$ concentration of impurities

ε_F $=$ UV cutoff

$g_{_K} < 0$
Ferromagnetic $\Rightarrow$ as T decreases $\rho(T)$ DECREASES

$$\rho(T) = \rho_0 + a\,T^2 + b\,T^5 + c\,g_{_K}^2 - \tilde{c}\,g_{_K}^3 \ln\left(T/\varepsilon_F\right)$$

$c, \tilde{c} > 0 \quad \propto$ concentration of impurities

$\varepsilon_F \quad = \quad$ UV cutoff

$g_{_K} < 0 \quad \Longrightarrow$ as T decreases $\rho(T)$ DECREASES

Ferromagnetic

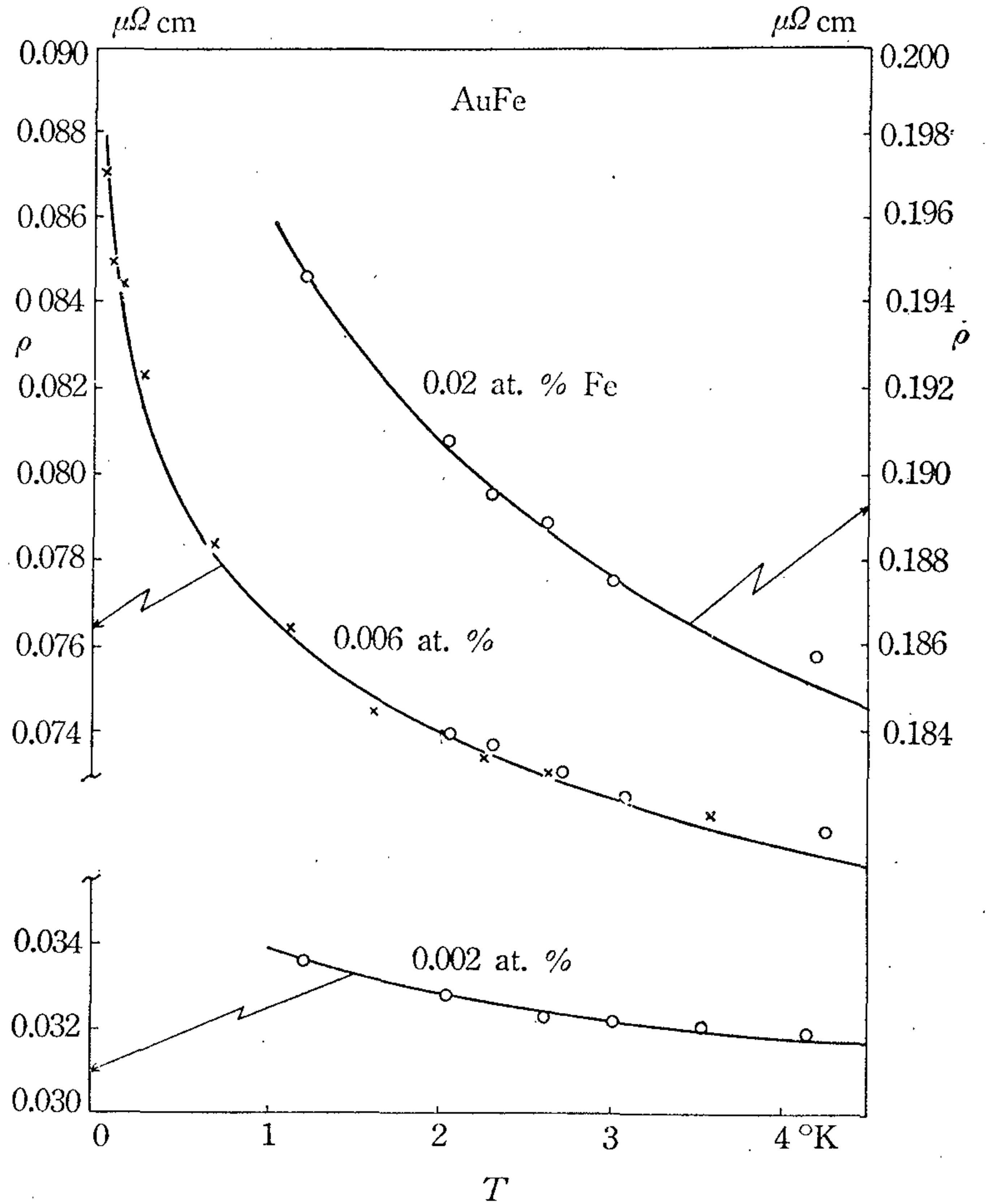
$$\rho(T) = \rho_0 + a\,T^2 + b\,T^5 + c\,g_{_K}^2 - \tilde{c}\,g_{_K}^3 \ln\left(T/\varepsilon_F\right)$$

$c, \tilde{c} > 0 \quad \propto$ concentration of impurities

$\varepsilon_F \quad = \quad$ UV cutoff

$g_{_K} > 0$ Anti-Ferromagnetic $\Rightarrow$ as T decreases $\rho(T)$ INCREASES

$$\rho(T) = \rho_0 + a\,T^2 + b\,T^5 + c\,g_K^2 - \tilde{c}\,g_K^3 \ln\left(T/\varepsilon_F\right)$$



$$\rho(T) = \rho_0 + a\,T^2 + b\,T^5 + c\,g_{_K}^2 - \tilde{c}\,g_{_K}^3 \ln\left(T/\varepsilon_F\right)$$

Breakdown of Perturbation Theory

$\mathcal{O}(g_K^3)$ term is same order as $\mathcal{O}(g_K^2)$ term when

$$T_K \approx \varepsilon_F\, e^{-\frac{c}{\tilde{c}}\frac{1}{g_K}}$$

"Kondo temperature"

$$\rho(T) = \rho_0 + a\,T^2 + b\,T^5 + c\,g_{_K}^2 - \tilde{c}\,g_{_K}^3 \ln\left(T/\varepsilon_F\right)$$

Cross section for electron scattering off a **MAGNETIC** impurity **INCREASES** as energy **DECREASES**

$$\beta_{g_{_K}} \propto -g_{_K}^2 + \mathcal{O}(g_{_K}^3)$$

Asymptotic freedom!

$$T_K \sim \Lambda_{\mathrm{QCD}}$$

The Kondo Problem

What is the ground state?

The coupling diverges at low energy!

We know the answer!

Solutions of the Kondo Problem

Numerical RG (Wilson 1975)

Fermi liquid description (Nozières 1975)

Bethe Ansatz/Integrability
(Andrei, Wiegmann, Tsvelick, Destri, ... 1980s)

Large-N expansion
(Anderson, Read, Newns, Doniach, Coleman, ...1970-80s)

Quantum Monte Carlo
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Conformal Field Theory (CFT)
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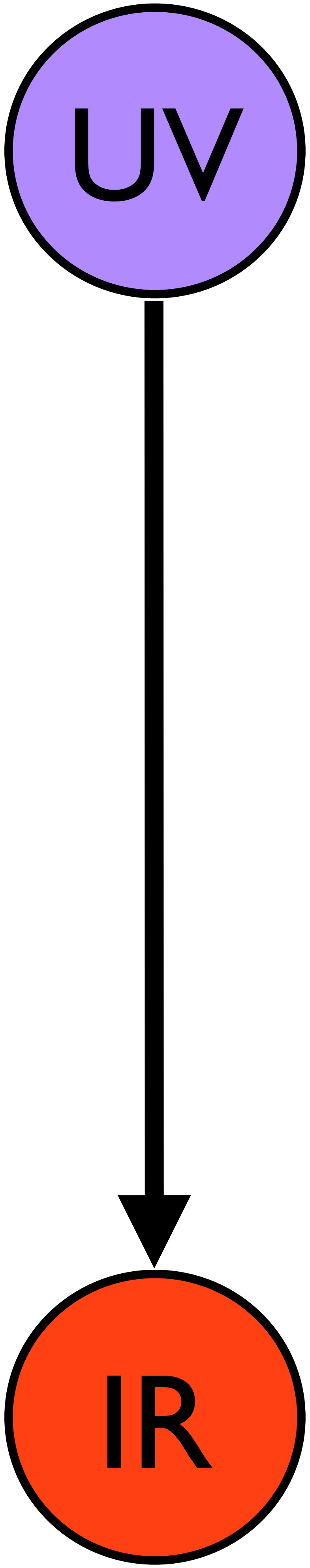


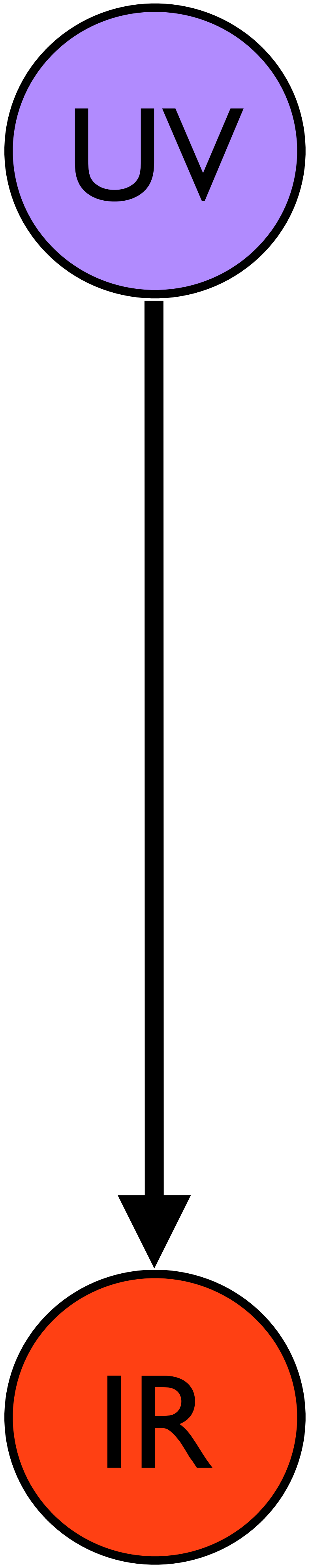
Fermi liquid
+
decoupled spin

Intuitive description

SINGLE-BODY physics

Exact description

MANY-BODY physics



Fermi liquid
+
decoupled spin

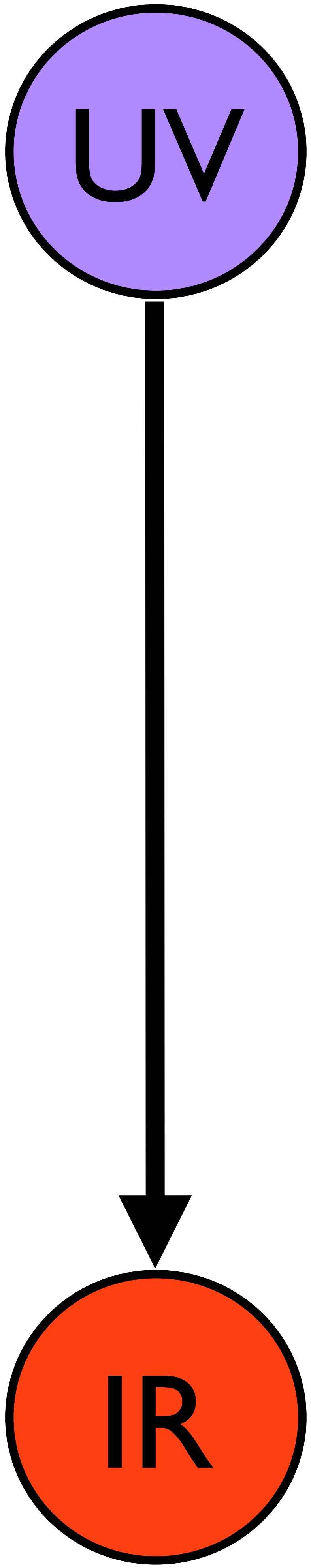
Intuitive description

An electron binds with the impurity

Anti-symmetric singlet of $SU(2)$

$$\frac{1}{\sqrt{2}}\left(|\uparrow_i \downarrow_e\rangle - |\downarrow_i \uparrow_e\rangle\right)$$

“Kondo singlet”



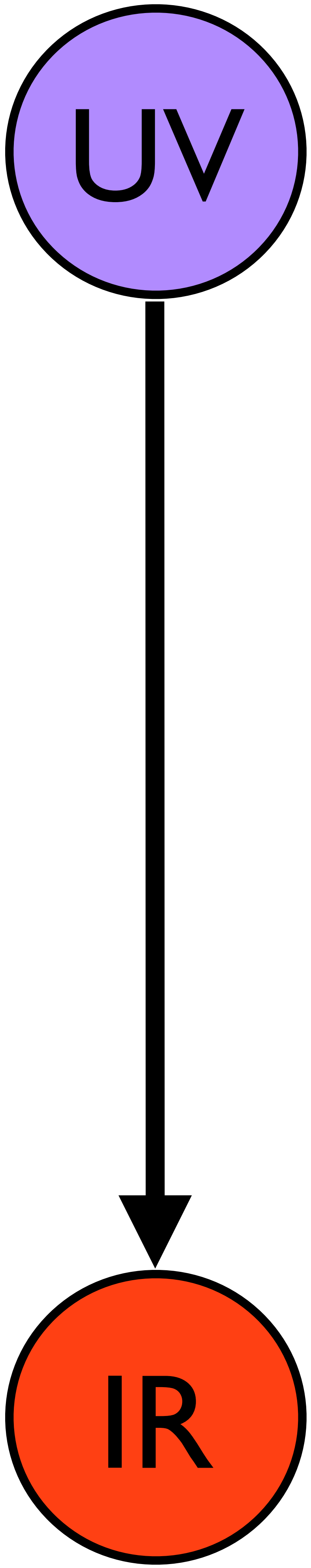
Fermi liquid
+
decoupled spin

Exact description

The electrons work collectively
to SCREEN the impurity's spin

Produces a MANY-BODY RESONANCE

"Kondo resonance"



Fermi liquid
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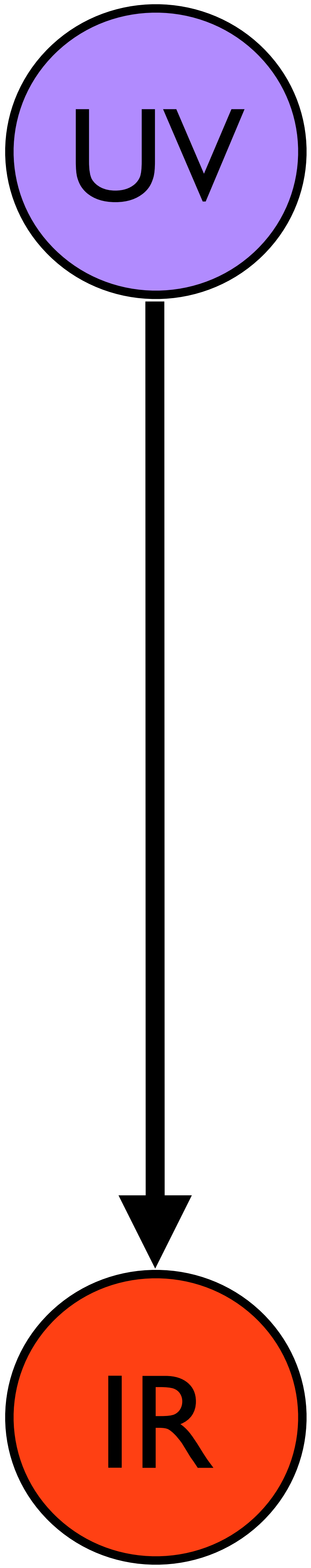
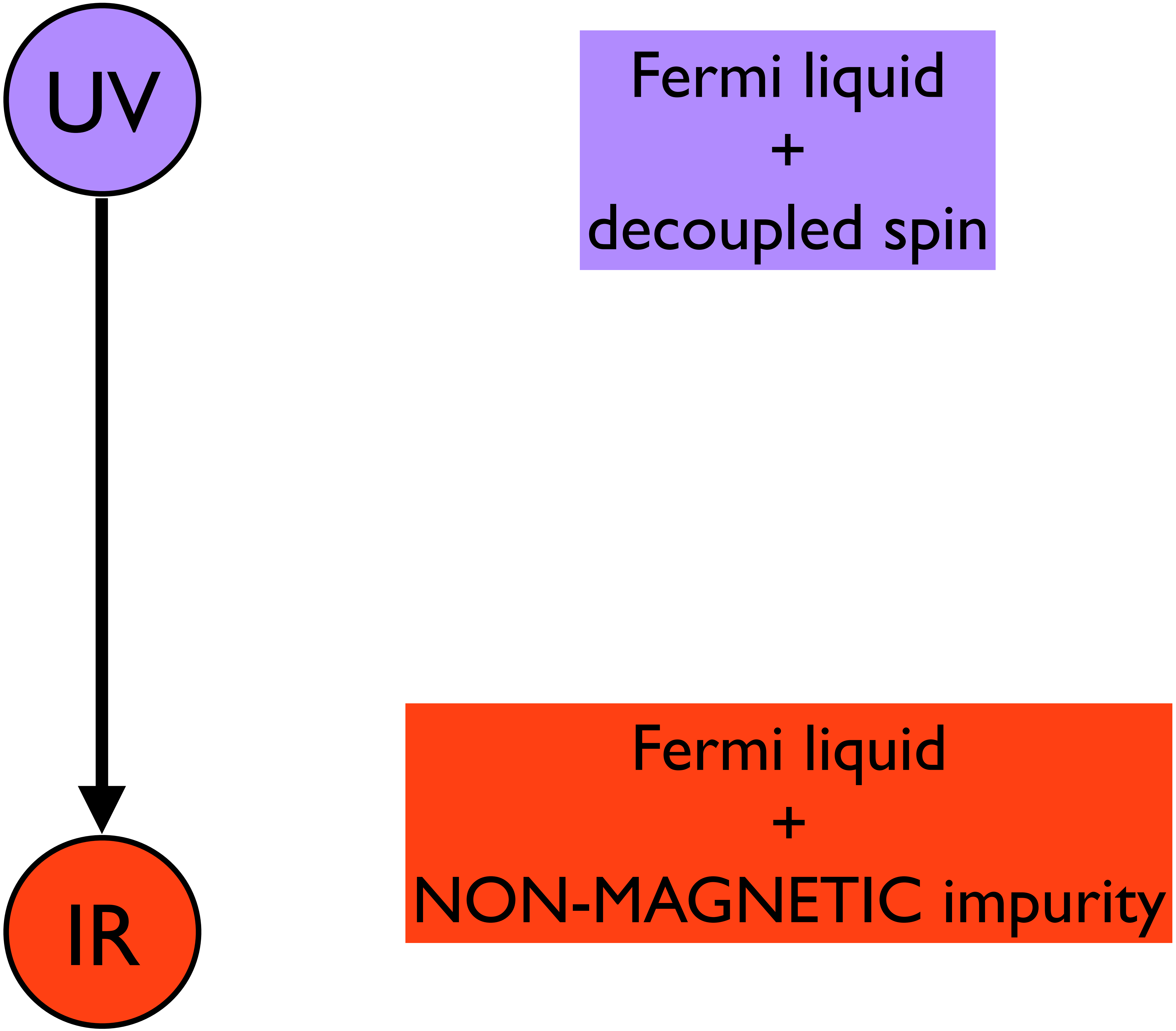


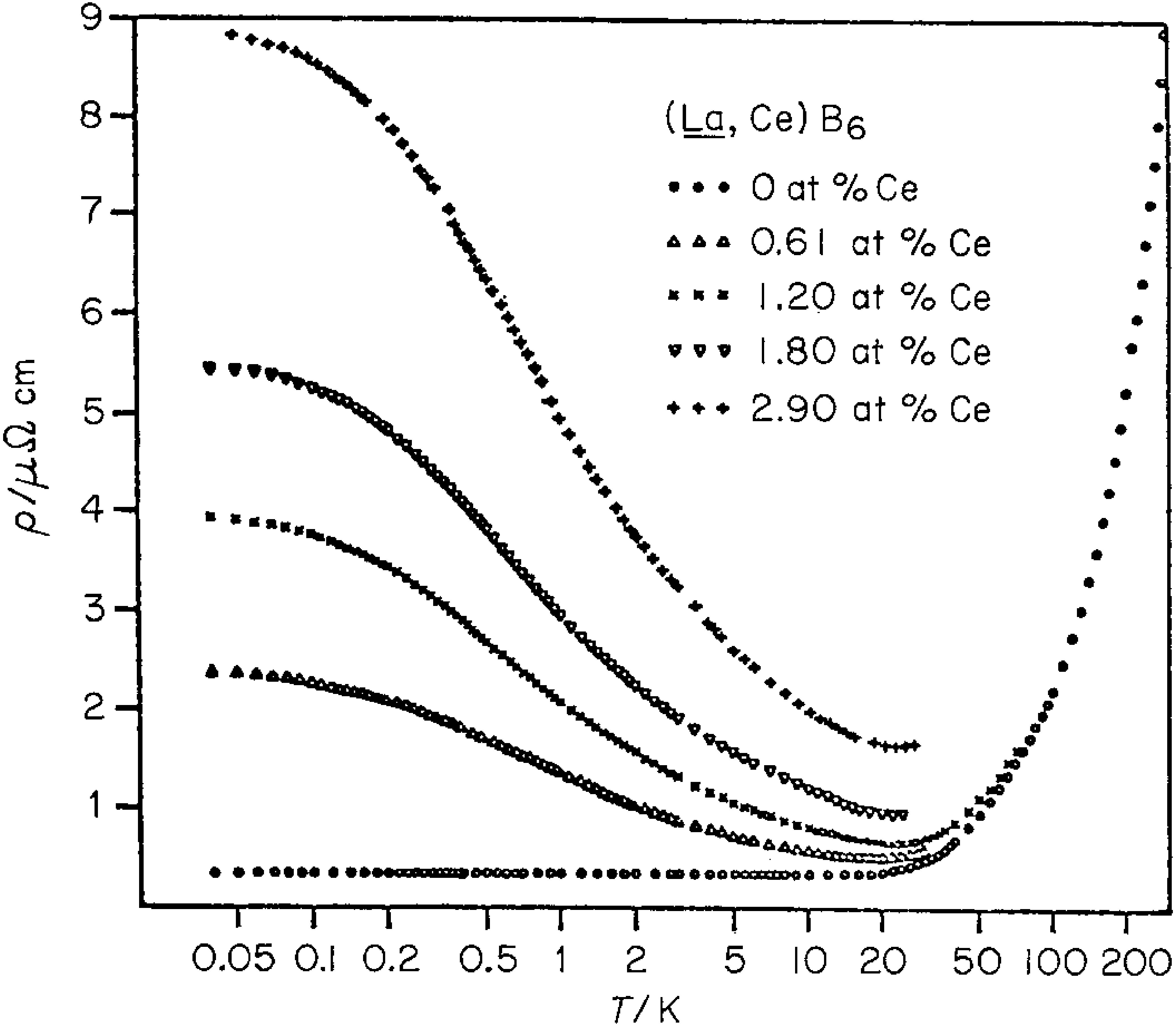
Fermi liquid
+
decoupled spin

NO spin

Fermi liquid

+ electrons EXCLUDED
from impurity location


UV
IR
Fermi liquid
+
decoupled spin
Fermi liquid
+
NON-MAGNETIC impurity



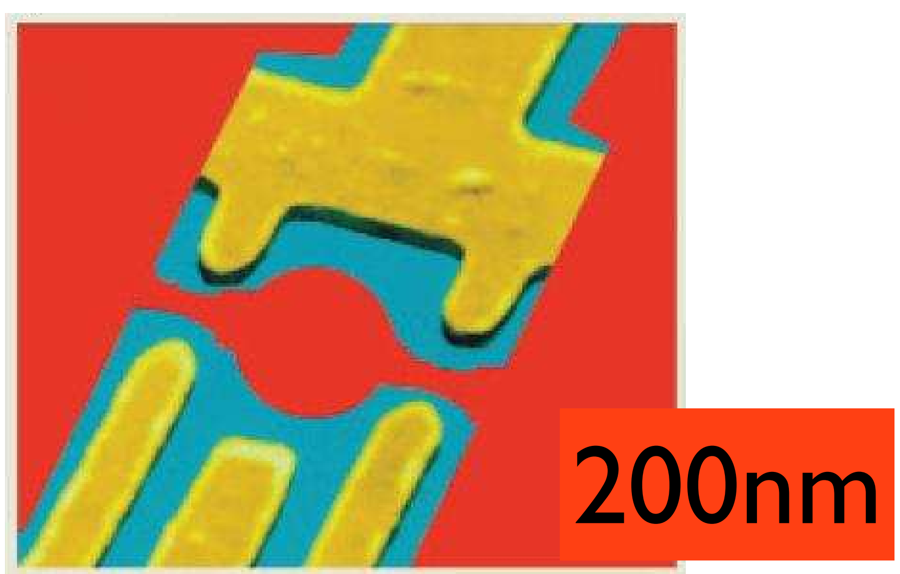
Samwer and Winzer, Z. Phys B, 25, 269, 1976

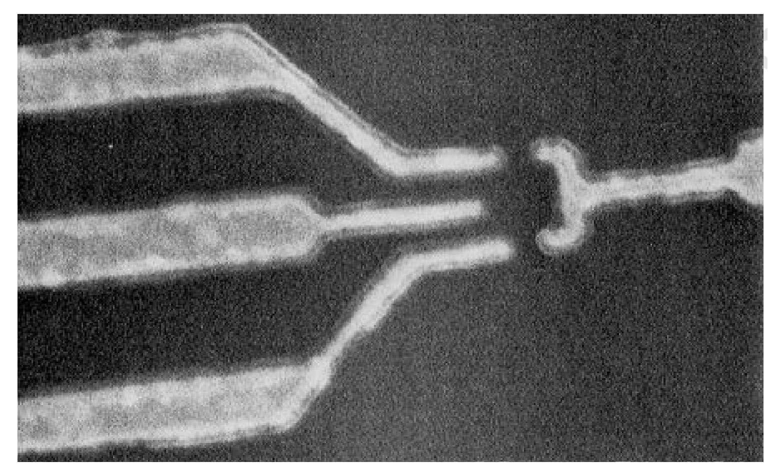
Kondo Effect in Many Systems

Alloys

Cu, Ag, Au, Mg, Zn, ... doped with Cr, Fe, Mo, Mn, Re, Os, ...

Quantum dots





Goldhaber-Gordon, et al., **Nature** 391 (1998), 156-159.

Cronenwett, et al., **Science** 281 (1998), no. 5376, 540-544.

Generalizations

Enhance the spin group

$$SU(2) \to SU(N)$$

Representation of impurity spin

$$s_{\text{imp}} = 1/2 \quad \longrightarrow \quad R_{\text{imp}}$$

Multiple "channels" or "flavors"

$$c \to c^{\alpha} \qquad \alpha = 1, \ldots, k$$

$$U(1) \times SU(k)$$

Generalizations

Kondo model specified by

$$N, \;\; k, \;\; R_{\mathrm{imp}}$$

Apply the techniques mentioned above...

IR fixed point:

NOT always a fermi liquid

"Non-Fermi liquids"

Open Problems

Entanglement Entropy

Quantum Quenches

Multiple Impurities

Kondo: Form singlets with electrons

$\vec{S}_i \cdot \vec{S}_j$ Form singlets with each other

Competition between these can produce a

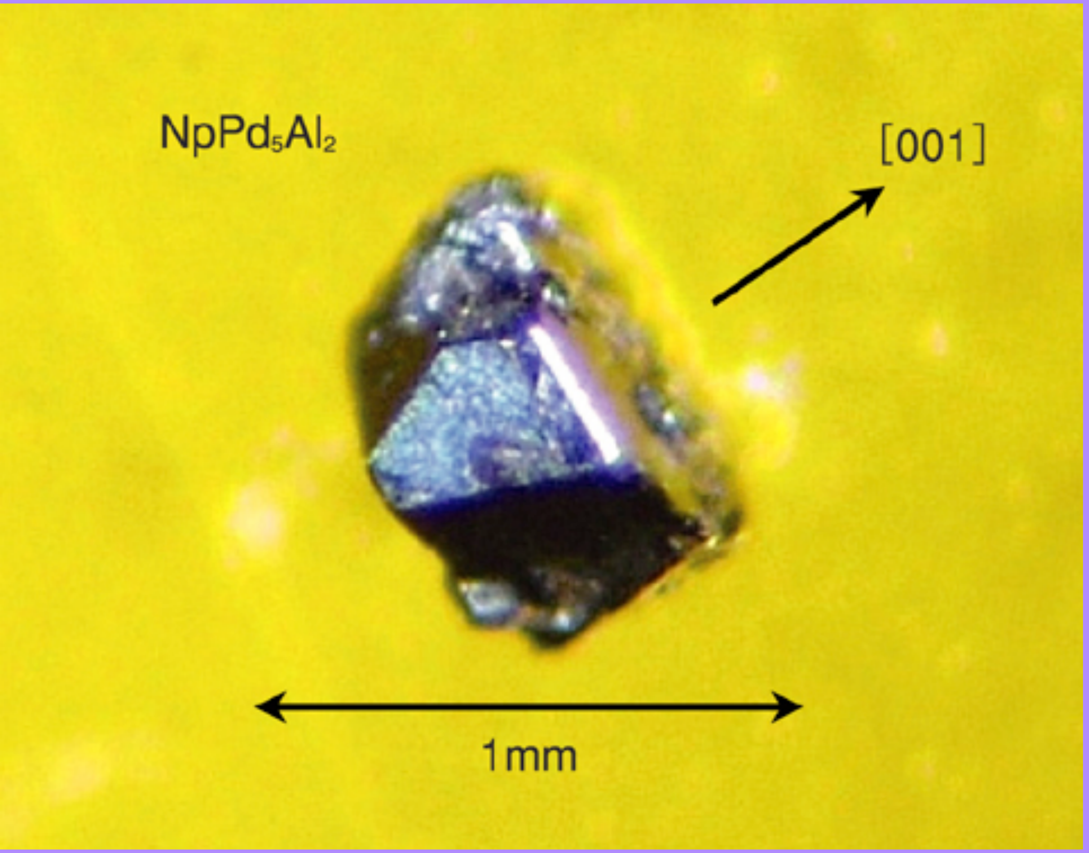
QUANTUM PHASE TRANSITION

Open Problems

Multiple Impurities

Heavy fermion compounds

$NpPd_5Al_2$	$CeCoIn_5$
$CeCu_6$	$CePd_2Si_2$
$YbAl_3$	$YbRh_2Si_2$
UBe_{13}	UPt_3



Open Problems

Multiple Impurities

Heavy fermion compounds

$NpPd_5Al_2$	$CeCoIn_5$
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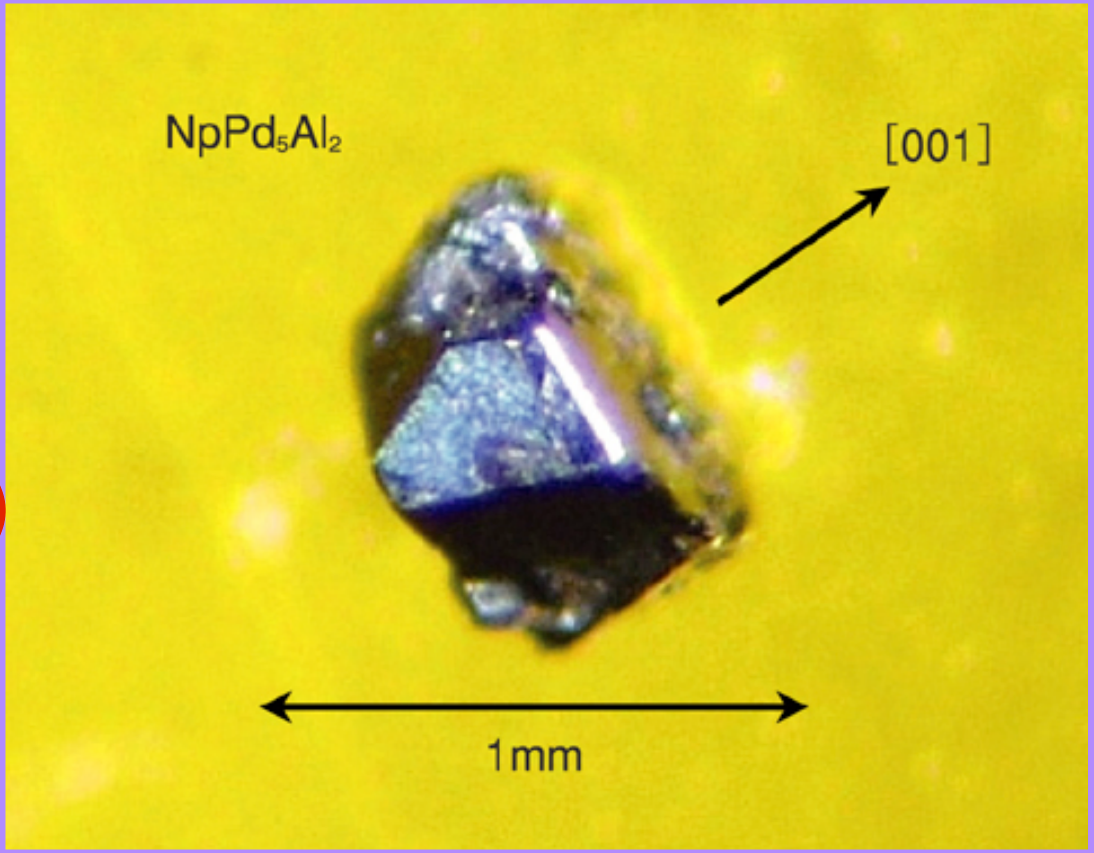


Open Problems

Multiple Impurities

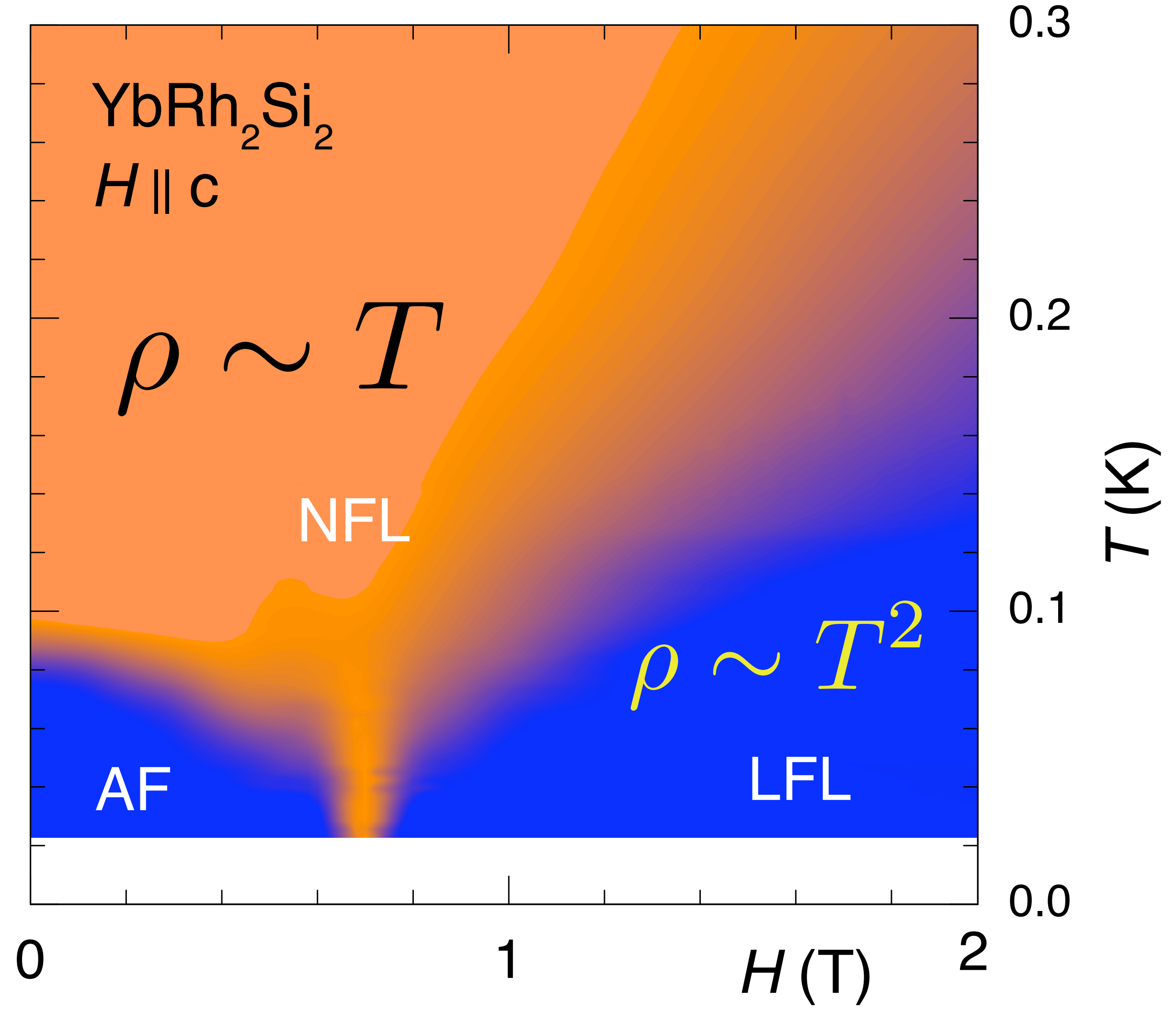
Heavy fermion compounds

Example

$$\mathrm{YbRh_2Si_2}$$

Kondo lattice

J. Custers et al., Nature 424, 524 (2003)



$$\Theta_D \approx 380\,K$$

Solutions of the Kondo Problem

Numerical RG (Wilson 1975)

Fermi liquid description (Nozières 1975)

Bethe Ansatz/Integrability
(Andrei, Wiegmann, Tsvelick, Destri, ... 1980s)

Large-N expansion
(Anderson, Read, Newns, Doniach, Coleman, ...1970-80s)

Quantum Monte Carlo
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Conformal Field Theory (CFT)
(Affleck and Ludwig 1990s)

The Kondo Lattice

The Kondo Lattice

Alexei Tsvelik

"... remains one of the biggest unsolved problems in condensed matter physics."

Alexei Tsvelik
QFT in Condensed Matter Physics
(Cambridge Univ. Press, 2003)

The Kondo Lattice


Alexei Tsvelik

“... remains one of the

ms

ics.”

Let’s try AdS/CFT!

QFT in Condensed Matter Physics

(Cambridge Univ. Press, 2003)

GOAL

Find a holographic description
of the
Kondo Effect

Solutions of the Kondo Problem

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CFT Approach to the Kondo Effect

Affleck and Ludwig 1990s

Reduction to one dimension

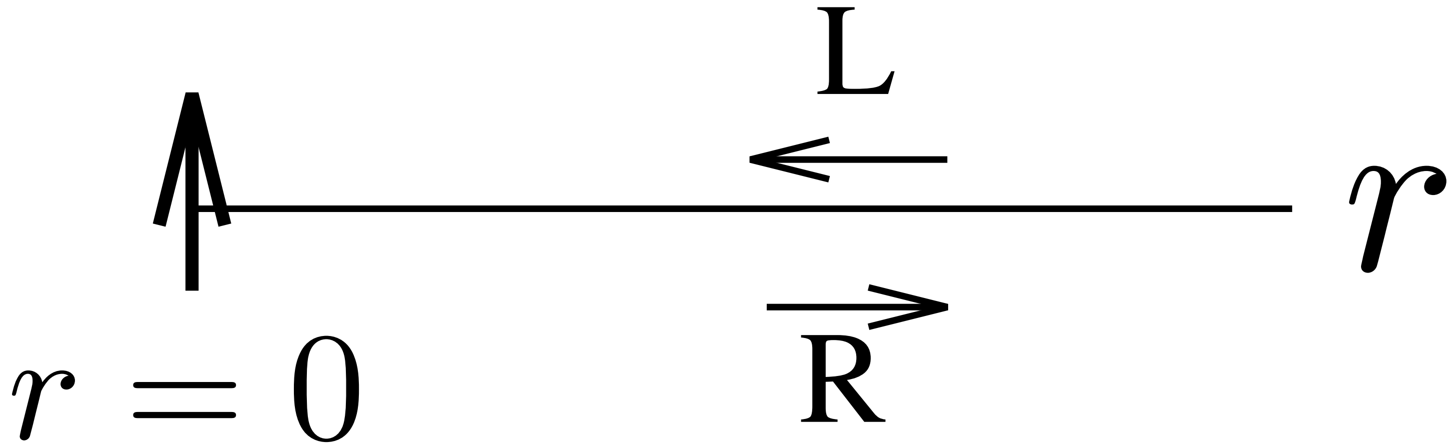
Kondo interaction preserves spherical symmetry

$$g_K \delta^3(\vec{x})\, \vec{S} \cdot c^\dagger(\vec{x})\, \frac{1}{2}\vec{\tau}\, c(\vec{x})$$

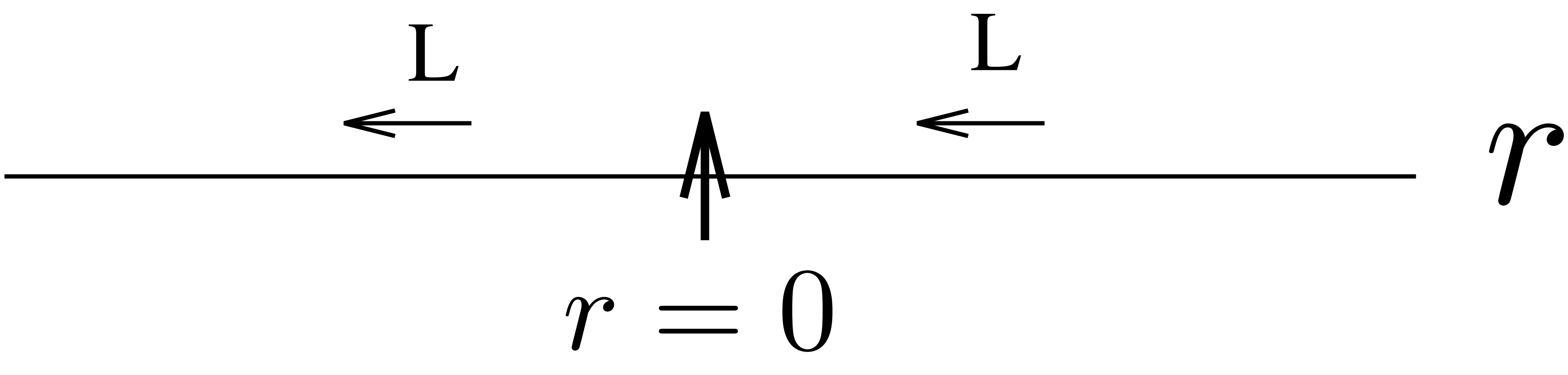
restrict to s-wave

restrict to momenta near k_F

$$c(\vec{x}) \approx \frac{1}{r}\left[e^{-ik_F r}\psi_L(r) - e^{+ik_F r}\psi_R(r)\right]$$



$$\psi_R(+r) \equiv \psi_L(-r)$$



CFT Approach to the Kondo Effect

$$H_K = \frac{v_F}{2\pi}\int_{-\infty}^{+\infty} dr \left[\psi_L^\dagger i\partial_r \psi_L + \delta(r)\,\tilde{g}_K\,\vec{S}\cdot\psi_L^\dagger \vec{\tau}\,\psi_L\right]$$

$$\tilde{g}_K \equiv \frac{k_F^2}{2\pi^2 v_F} \times g_K$$

RELATIVISTIC chiral fermions

v_F = "speed of light"

chiral CFT!

Spin $SU(N)$

$k \geq 1$

$$J = \psi_L^\dagger \psi_L$$

$U(1)$

$$\vec{J} = \psi_L^\dagger \, \vec{\tau} \, \psi_L$$

$SU(N)$

$$J^A = \psi_L^\dagger \, t^A \, \psi_L$$

$SU(k)$

$$z \equiv \tau + ir$$

$$J^A(z) = \sum_{n \in \mathbb{Z}} z^{-n-1} J_n^A$$

$$[J_n^A, J_m^B] = if^{ABC} J_{n+m}^C + N \frac{n}{2} \delta^{AB} \delta_{n,-m}$$

$SU(k)_N$ Kac-Moody Algebra

N counts net number of chiral fermions

CFT Approach to the Kondo Effect

$$H_K = \frac{v_F}{2\pi} \int_{-\infty}^{+\infty} dr \left[\psi_L^\dagger i\partial_r \psi_L + \delta(r)\, \tilde{g}_K\, \vec{S} \cdot \psi_L^\dagger \vec{\tau} \psi_L \right]$$

Full symmetry:

$(1+1)d$ chiral conformal symmetry

$$SU(N)_k \times SU(k)_N \times U(1)_{kN}$$

CFT Approach to the Kondo Effect

$$H_K = \frac{v_F}{2\pi}\int_{-\infty}^{+\infty} dr \left[\psi_L^\dagger i\partial_r \psi_L + \delta(r)\,\tilde{g}_K\,\vec{S}\cdot\psi_L^\dagger \vec{\tau}\,\psi_L\right]$$

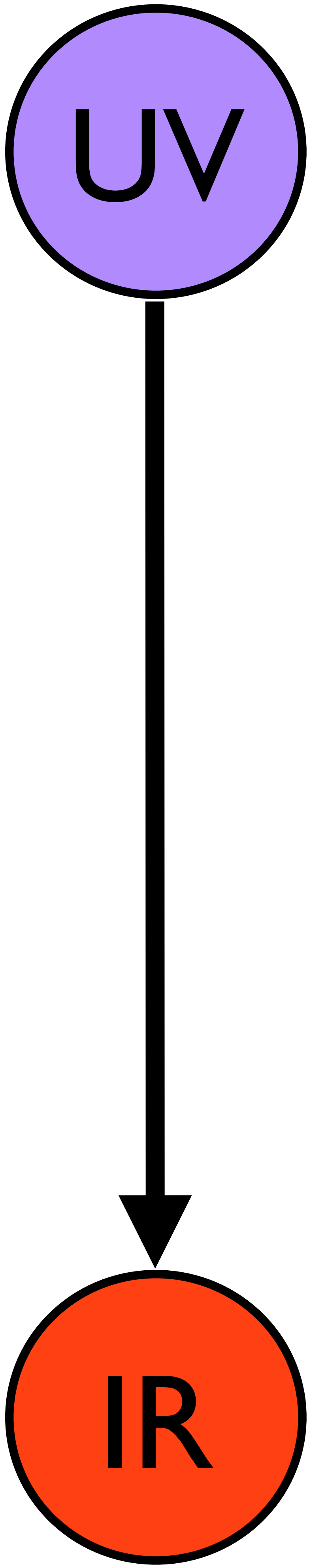
$$J = \psi_L^\dagger \psi_L \qquad U(1)$$

$$\vec{J} = \psi_L^\dagger\,\vec{\tau}\,\psi_L \qquad SU(N)$$

$$J^A = \psi_L^\dagger\, t^A\, \psi_L \qquad SU(k)$$

Kondo coupling: $\vec{S}\cdot\vec{J}$

marginal



$$SU(N)_k \times SU(k)_N \times U(1)_{Nk}$$

Eigenstates are representations of the Kac-Moody algebra

Determine how representations re-arrange between UV and IR

$$R^{UV}_{\text{highest weight}} \otimes R_{\text{imp}} = R^{IR}_{\text{highest weight}}$$

$$SU(N)_k \times SU(k)_N \times U(1)_{Nk}$$

CFT Approach to the Kondo Effect

Take-Away Messages

Central role of the Kac-Moody Algebra

Kondo coupling: $\vec{S} \cdot \vec{J}$

marginal

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GOAL

Find a holographic description
of the
Kondo Effect

What classical action do we write
on the gravity side of the correspondence?

How do we describe holographically...

① The chiral fermions?

② The impurity?

③ The Kondo coupling?

Holography

Top-down:

AdS solution to a string or supergravity theory

Bottom-up:

AdS solution of some *ad hoc* Lagrangian

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

Open strings

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

$3\text{-}3$ and $5\text{-}5$ and $7\text{-}7$

$3\text{-}7$ and $7\text{-}3$

$3\text{-}5$ and $5\text{-}3$

$7\text{-}5$ and $5\text{-}7$

CFT with holographic dual

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

Decouple

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

(1+1)-dimensional chiral fermions

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

the impurity

7-5 and 5-7

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

Kondo interaction

Previous work

Kachru, Karch, Yaida 0909.2639, 1009.3268

Mück 1012.1973

Faraggi and Pando-Zayas 1101.5145

Jensen, Kachru, Karch, Polchinski, Silverstein 1105.1772

Karaiskos, Sfetsos, Tsatis 1106.1200

Harrison, Kachru, Torroba 1110.5325

Benincasa and Ramallo 1112.4669, 1204.6290

Faraggi, Mück, Pando-Zayas 1112.5028

Itsios, Sfetsos, Zoakos 1209.6617

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

Absent in previous constructions

The D3-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						

3-3 strings

$\mathcal{N}=4$ SYM

$N_c \to \infty$

$\lambda \to \infty$

$=$

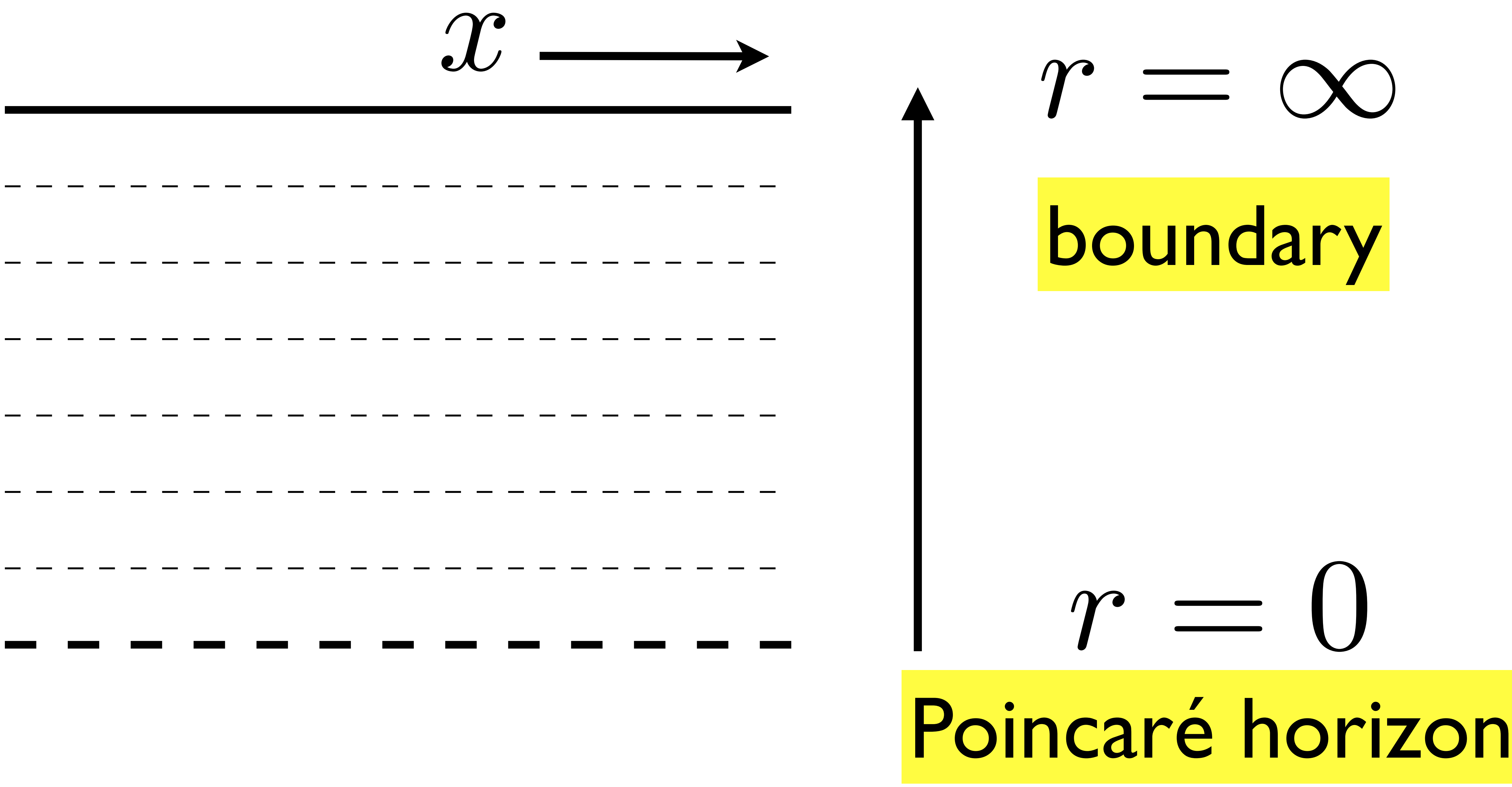
Type IIB Supergravity

$AdS_5 \times S^5$

$$\int_{S^5} F_5 \propto N_c \qquad F_5 = dC_4$$

Anti-de Sitter Space

$$ds^2 = \frac{dr^2}{r^2} + r^2\left(-dt^2 + dx^2 + dy^2 + dz^2\right)$$



Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

Decouple

Probe Limit

$$N_7/N_c \to 0 \text{ and } N_5/N_c \to 0$$

$U(N_7) \times U(N_5)$ becomes a global symmetry

Total symmetry:

$$\underbrace{SU(N_c)}_{\text{gauged}} \times \underbrace{U(N_7) \times U(N_5)}_{\text{global}}$$

(plus R-symmetry)

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

(1+1)-dimensional chiral fermions

The D7-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X

Skenderis, Taylor hep-th/0204054

Harvey and Royston 0709.1482, 0804.2854

Buchbinder, Gomis, Passerini 0710.5170

(1+1)-dimensional chiral fermions ψ_L

$$SU(N_c) \times U(N_7) \times U(N_5)$$

$$N_c \qquad \overline{N}_7 \qquad \text{singlet}$$

The D7-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X

Skenderis, Taylor hep-th/0204054

Harvey and Royston 0709.1482, 0804.2854

Buchbinder, Gomis, Passerini 0710.5170

(1+1)-dimensional chiral fermions ψ_L

Kac-Moody algebra

$$SU(N_c)_{N_7} \times SU(N_7)_{N_c} \times U(1)_{N_c N_7}$$

The D7-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X

$(1+1)$-dimensional chiral fermions ψ_L

Differences from Kondo

Do not come from reduction from $(3+1)$ dimensions

Genuinely relativistic

The D7-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X

(1+1)-dimensional chiral fermions ψ_L

Differences from Kondo

$SU(N_c)$ is gauged!

$$\vec{J} = \psi_L^\dagger \vec{\tau} \psi_L$$

The D7-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X

$SU(N_c)$ is gauged!

Gauge Anomaly!

Harvey and Royston 0709.1482, 0804.2854

Buchbinder, Gomis, Passerini 0710.5170

Probe Limit

$$N_7/N_c \to 0$$

In the probe limit, the gauge anomaly is suppressed...

$$SU(N_c)_{N_7} \to SU(N_c)$$

... but the global anomalies are not.

$$SU(N_7)_{N_c} \times U(1)_{N_c N_7} \to SU(N_7)_{N_c} \times U(1)_{N_c N_7}$$

$\mathcal{N} = 4$ SYM

$N_c \to \infty$

$\lambda \to \infty$

$=$

Type IIB Supergravity

$AdS_5 \times S^5$

Probe ψ_L

$=$

Probe D7-branes

$AdS_3 \times S^5$

$\mathcal{N} = 4$ SYM

$N_c \to \infty$

$\lambda \to \infty$

$=$

Type IIB Supergravity

$AdS_5 \times S^5$

Probe ψ_L

$=$

Probe D7-branes

$AdS_3 \times S^5$

$$ds^2 = \frac{dr^2}{r^2} + r^2 \left(-dt^2 + dx^2 + dy^2 + dz^2\right) + ds^2_{S^5}$$

$\mathcal{N} = 4$ SYM

$N_c \to \infty$

$\lambda \to \infty$

$=$

Type IIB Supergravity

$AdS_5 \times S^5$

Probe ψ_L

$=$

Probe D7-branes

$AdS_3 \times S^5$

$U(N_7)$ Current J

$=$

$U(N_7)$ Gauge field A

Current J	$=$	Gauge field A
Kac-Moody Algebra	$=$	Chern-Simons Gauge Field
rank and level of algebra	$=$	rank and level of gauge field

Gukov, Martinec, Moore, Strominger
hep-th/0403225

Kraus and Larsen
hep-th/0607138

Probe D7-branes along $AdS_3 \times S^5$

$$S_{D7} = +\frac{1}{2} T_{D7} (2\pi\alpha')^2 \int P[C_4] \wedge \mathrm{tr}\, F \wedge F + \ldots$$

$$= -\frac{1}{2} T_{D7} (2\pi\alpha')^2 \int P[F_5] \wedge \mathrm{tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) + \ldots$$

$$= -\frac{N_c}{4\pi} \int_{AdS_3} \mathrm{tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) + \ldots$$

$U(N_7)_{N_c}$ Chern-Simons gauge field

Answer #1

The chiral fermions:

Chern-Simons Gauge Field in AdS_3

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3 ← the impurity

7-5 and 5-7

The D5-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_5 D5	X				X	X	X	X	X	

Skenderis, Taylor hep-th/0204054

Camino, Paredes, Ramallo hep-th/0104082

Gomis and Passerini hep-th/0604007

(0+1)-dimensional fermions χ

$$SU(N_c) \times U(N_7) \times U(N_5)$$

$$N_c \qquad \text{singlet} \qquad \overline{N}_5$$

The D5-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_5 D5	X				X	X	X	X	X	

$SU(N_c)$ is "spin"

$$\vec{S} = \chi^\dagger \vec{\tau} \chi$$

"slave fermions"

"Abrikosov pseudo-fermions"

Abrikosov, **Physics** 2, p.5 (1965)

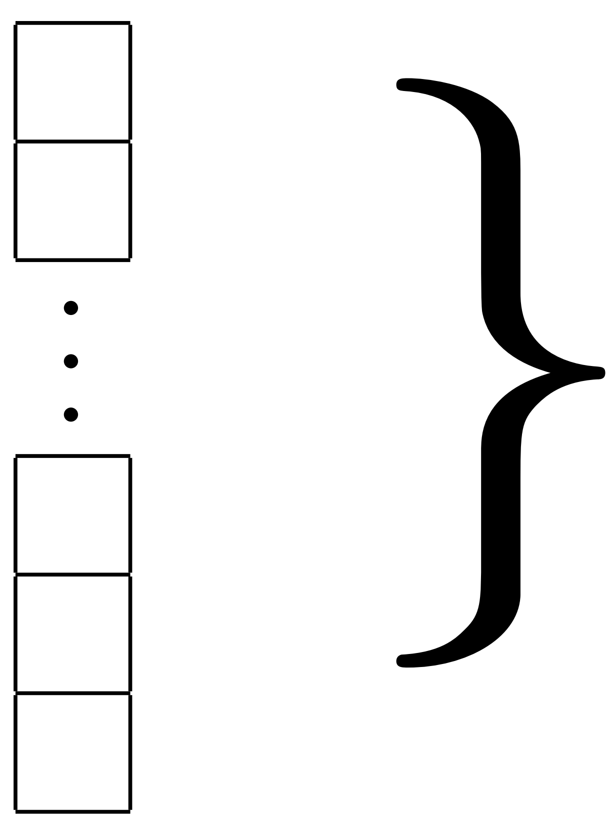
Gomis and Passerini hep-th/0604007

$$N_5 = 1$$

Integrate out χ

$$\mathrm{Det}\,(\not{D}) = \mathrm{Tr}_R P \exp\left[i\int dt\, A_t\right]$$

$$R =$$



$$Q = \chi^\dagger \chi$$

$U(N_5) = U(1)$ charge

$\mathcal{N} = 4$ SYM

$N_c \to \infty$

$\lambda \to \infty$

$=$

Type IIB Supergravity

$AdS_5 \times S^5$

Probe χ

$=$

Probe D5-branes

$AdS_2 \times S^4$

$\mathcal{N} = 4$ SYM

$N_c \to \infty$

$\lambda \to \infty$

$=$

Type IIB Supergravity

$AdS_5 \times S^5$

Probe χ

$=$

Probe D5-branes

$AdS_2 \times S^4$

$$ds^2 = \frac{dr^2}{r^2} + r^2 \left(-dt^2 + dx^2 + dy^2 + dz^2\right) + ds^2_{S^5}$$

$\mathcal{N} = 4$ SYM $N_c \to \infty$ $\lambda \to \infty$	$=$	Type IIB Supergravity $AdS_5 \times S^5$
Probe χ	$=$	Probe D5-branes $AdS_2 \times S^4$
$U(N_5)$ Current J	$=$	$U(N_5)$ Gauge field a
Q	$=$	Electric flux

Probe D5-brane along $AdS_2 \times S^4$

Camino, Paredes, Ramallo hep-th/0104082

Dissolve Q strings into the D5-brane

AdS_2 electric field $f_{rt} = \partial_r a_t - \partial_t a_r$

$$\sqrt{-g} f^{tr} \Big|_{\partial AdS_2} = Q = \chi^\dagger \chi$$

Answer #2

The impurity:

Yang-Mills Gauge Field in AdS_2

$R_{\rm imp}$ $=$ electric flux

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

Kondo interaction

The Kondo Interaction

	0	1	2	3	4	5	6	7	8	9
N_5 D5	X				X	X	X	X	X	
N_7 D7	X	X			X	X	X	X	X	X

Complex scalar!

$$SU(N_c) \times U(N_7) \times U(N_5)$$

singlet	$\overline{N}_7$	N_5

$$\mathcal{O} \equiv \psi_L^\dagger \chi$$

The Kondo Interaction

	0	1	2	3	4	5	6	7	8	9
N_5 D5	X				X	X	X	X	X	
N_7 D7	X	X			X	X	X	X	X	X

TACHYON

$$m^2_{\text{tachyon}} = -\frac{1}{4\alpha'}$$

D5 becomes magnetic flux in the D7

The Kondo Interaction

$SU(N_c)$ is "spin"

$$\vec{J} = \psi_L^\dagger \vec{\tau} \psi_L \qquad \vec{S} = \chi^\dagger \vec{\tau} \chi$$

$$\vec{S} \cdot \vec{J} = \chi^\dagger \vec{\tau} \chi \cdot \psi_L^\dagger \vec{\tau} \psi_L$$

$$\vec{\tau}_{ij} \cdot \vec{\tau}_{kl} = \delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}$$

$$\vec{S} \cdot \vec{J} = |\psi_L^\dagger \chi|^2 + \mathcal{O}(1/N_c)$$

"double trace"

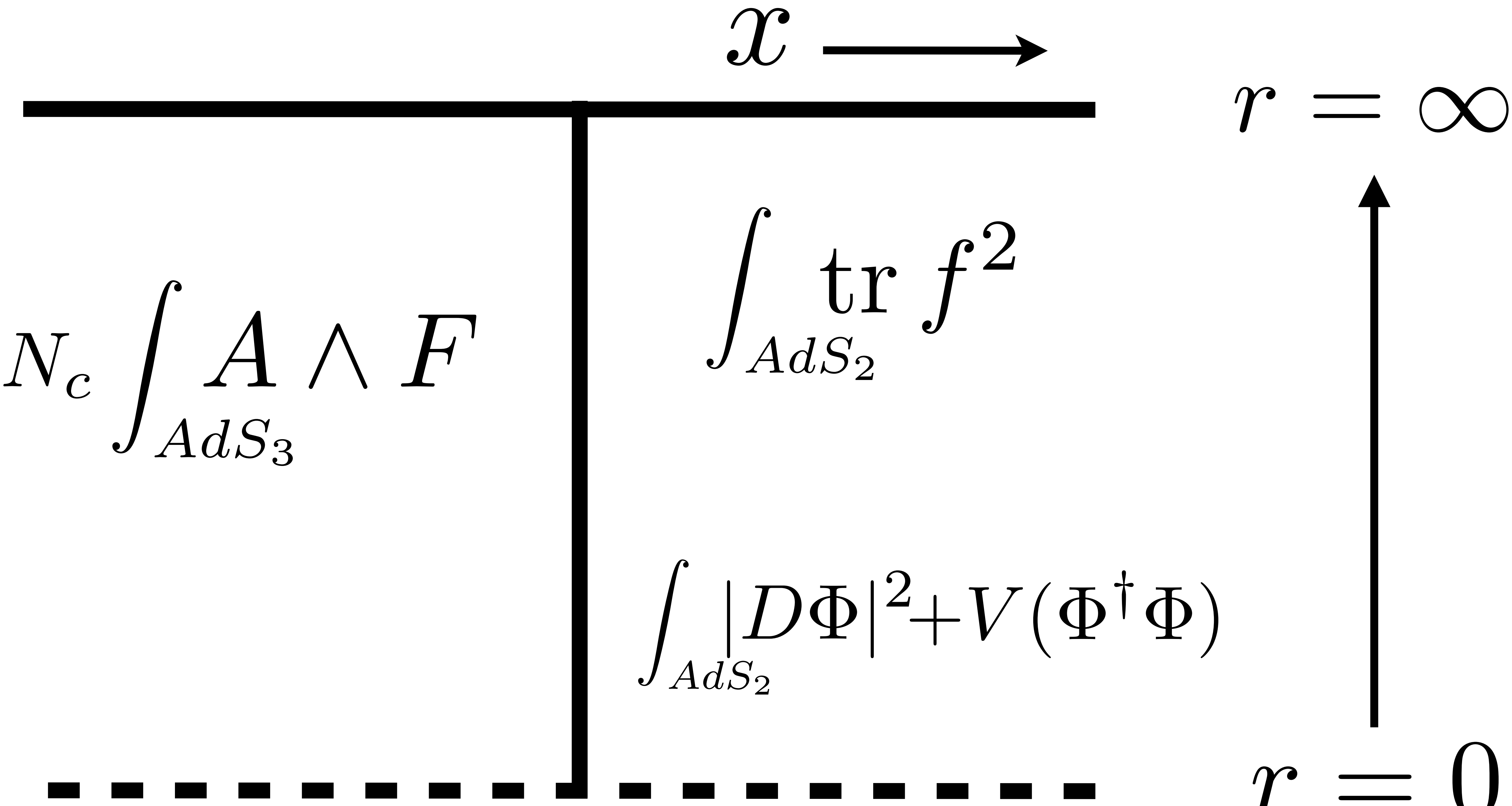
$\mathcal{N}=4$ SYM $N_c \to \infty$ $\lambda \to \infty$	$=$	Type IIB Supergravity $AdS_5 \times S^5$
Probe ψ_L	$=$	Probe D7-branes $AdS_3 \times S^5$
Probe χ	$=$	Probe D5-branes $AdS_2 \times S^4$
$\mathcal{O} \equiv \psi_L^\dagger \chi$	$=$	Bi-fundamental scalar $AdS_2 \times S^4$

Answer #3

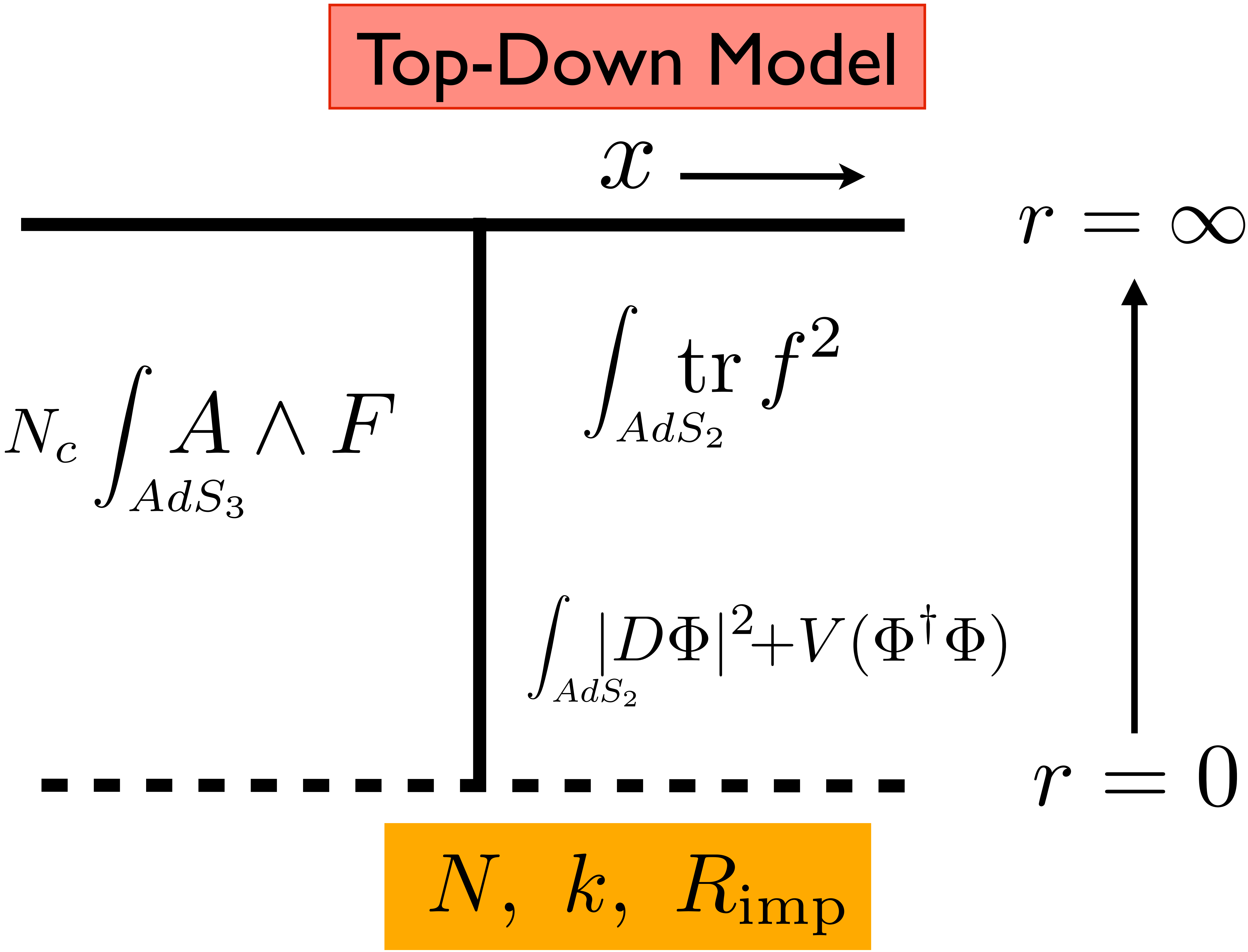
The Kondo interaction:

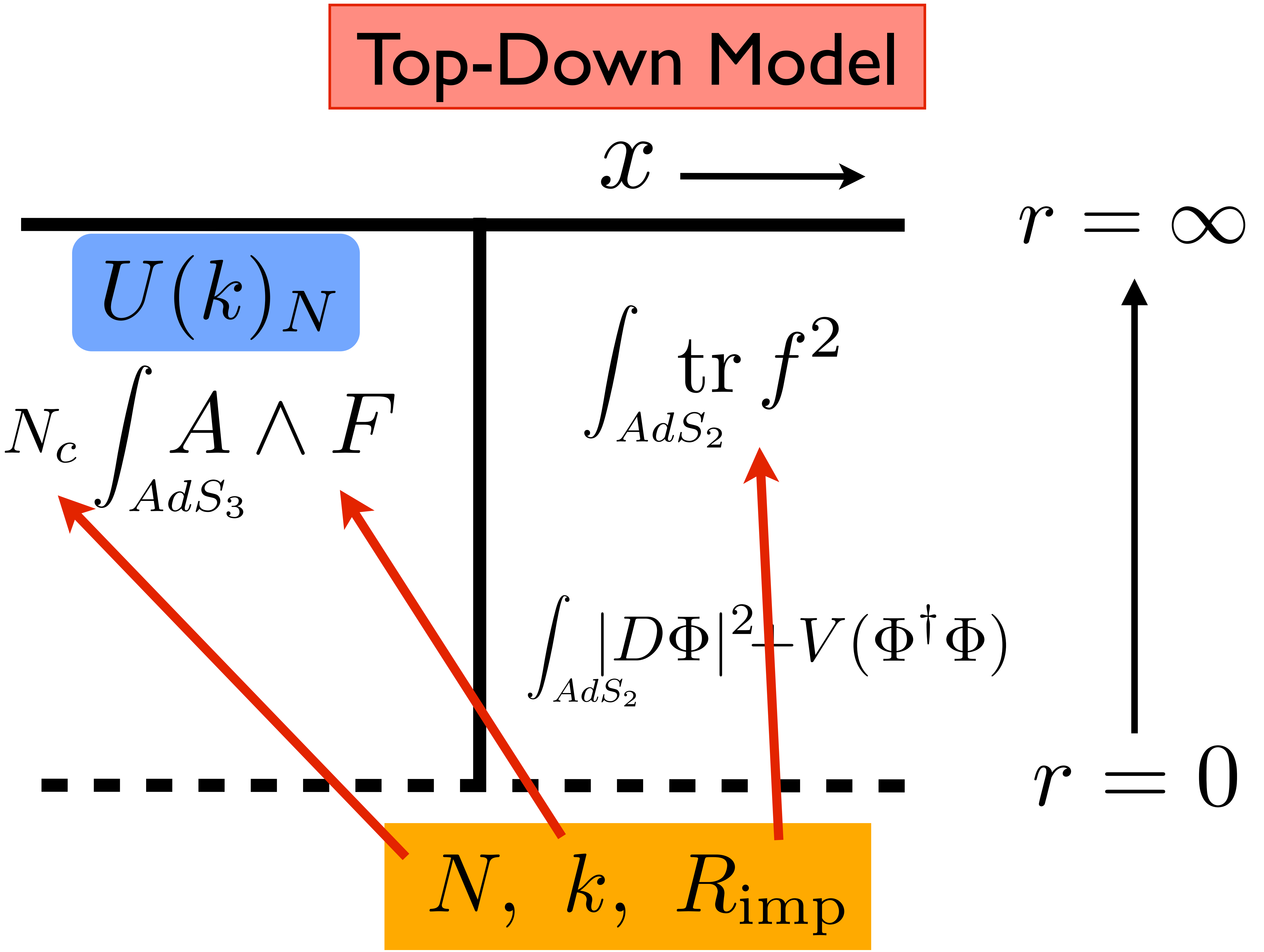
Bi-fundamental scalar in AdS_2

Top-Down Model



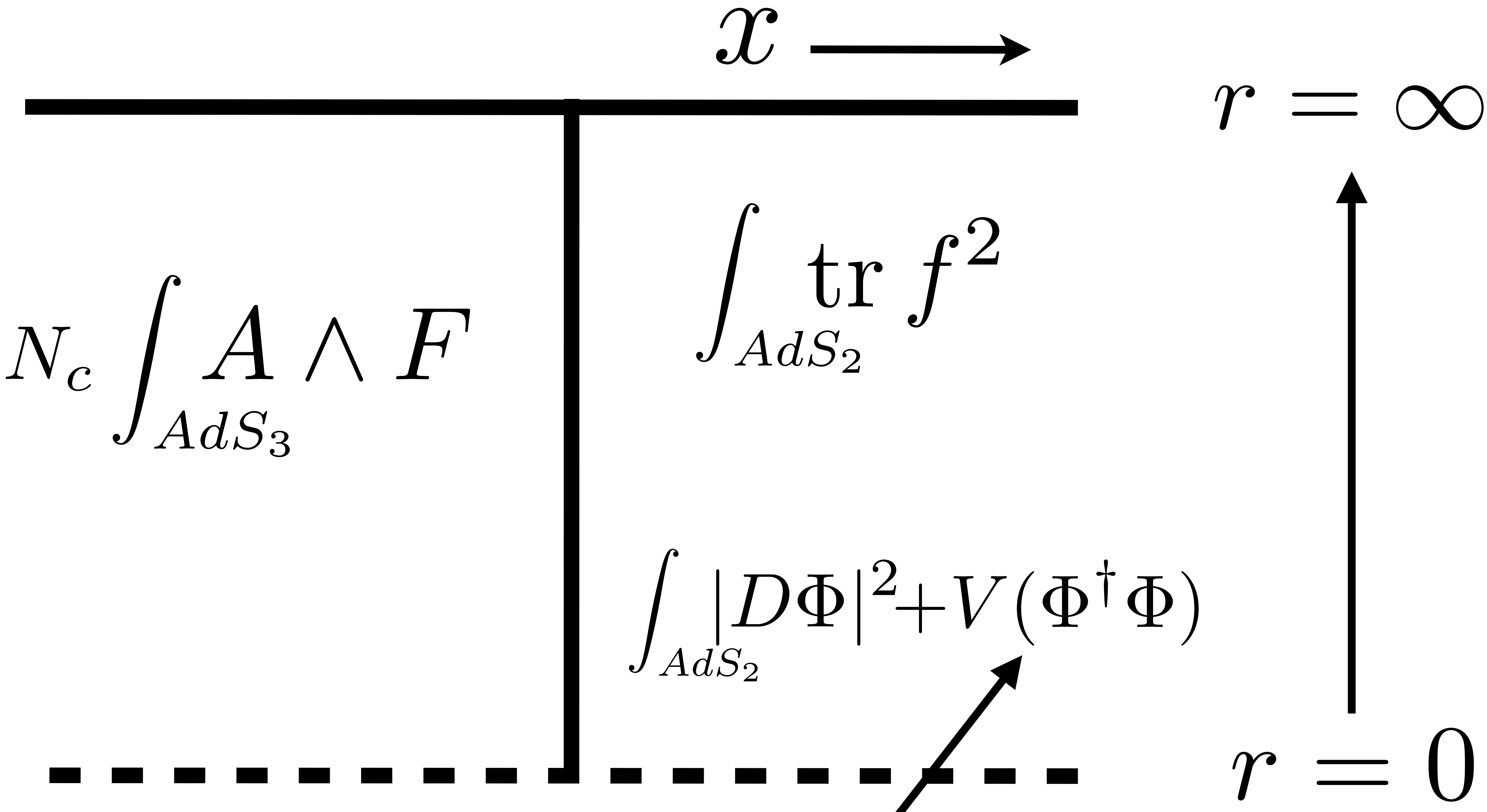
$$D\Phi = \partial\Phi + iA\Phi - ia\Phi$$


Top-Down Model
x
$r = \infty$
$N_c \int_{AdS_3} A \wedge F$
$\int_{AdS_2} \mathrm{tr}\, f^2$
$\int_{AdS_2} |D\Phi|^2 + V(\Phi^\dagger \Phi)$
$r = 0$
$N, \ k, \ R_{\mathrm{imp}}$



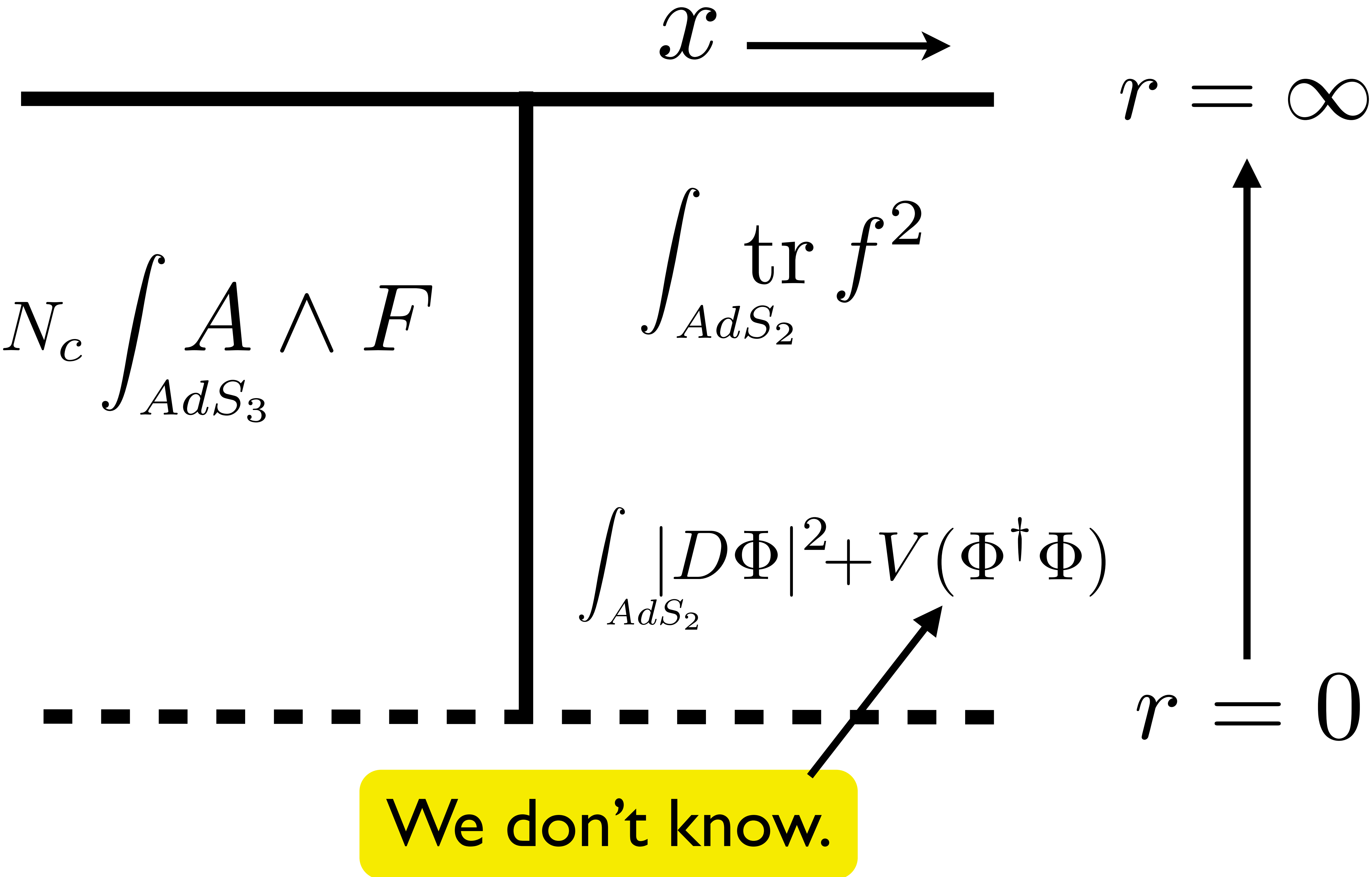
Top-Down Model
x
$r = \infty$
$U(k)_N$
$N_c \int_{AdS_3} A \wedge F$
$\int_{AdS_2} \mathrm{tr}\, f^2$
$\int_{AdS_2} |D\Phi|^2 + V(\Phi^\dagger \Phi)$
$r = 0$
$N, \; k, \; R_{\mathrm{imp}}$

Top-Down Model



What is $V(\Phi^\dagger \Phi)$?

Top-Down Model



Top-Down Model

What is $V(\Phi^\dagger \Phi)$?

Calculation in $\mathbb{R}^{9,1}$

Gava, Narain, Samadi hep-th/9704006

Aganagic, Gopakumar, Minwalla, Strominger hep-th/0009142

Difficult to calculate in $AdS_5 \times S^5$

Top-Down Model

What is $V(\Phi^\dagger \Phi)$?

Calculation in $\mathbb{R}^{9,1}$

Gava, Narain, Samadi hep-th/9704006

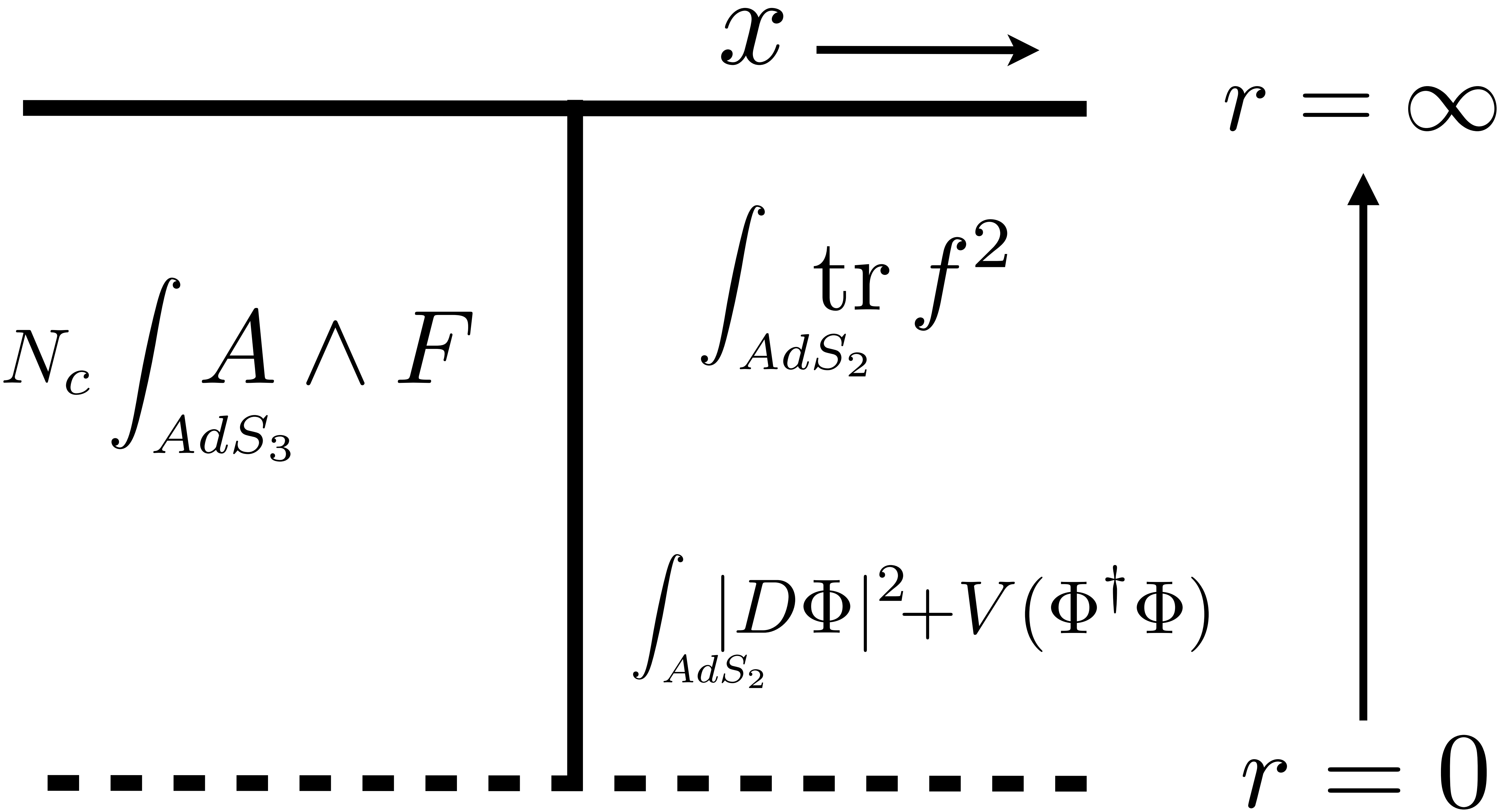
Aganagic, Gopakumar, Minwalla, Strominger hep-th/0009142

Switch to bottom-up model!

Outline:

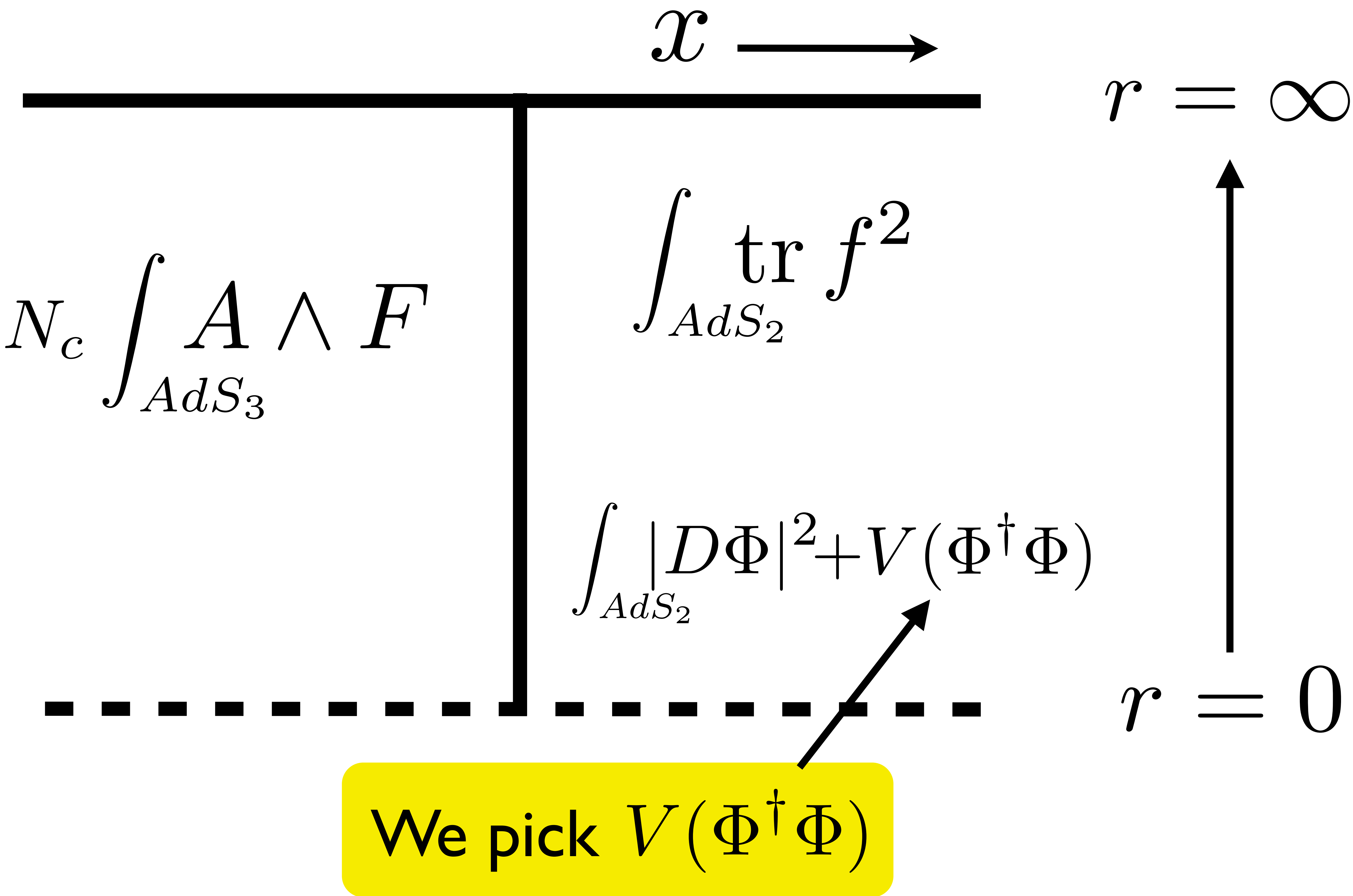
- The Kondo Effect
- The CFT Approach
- A Top-Down Holographic Model
- A Bottom-Up Holographic Model
- Summary and Outlook

Bottom-Up Model

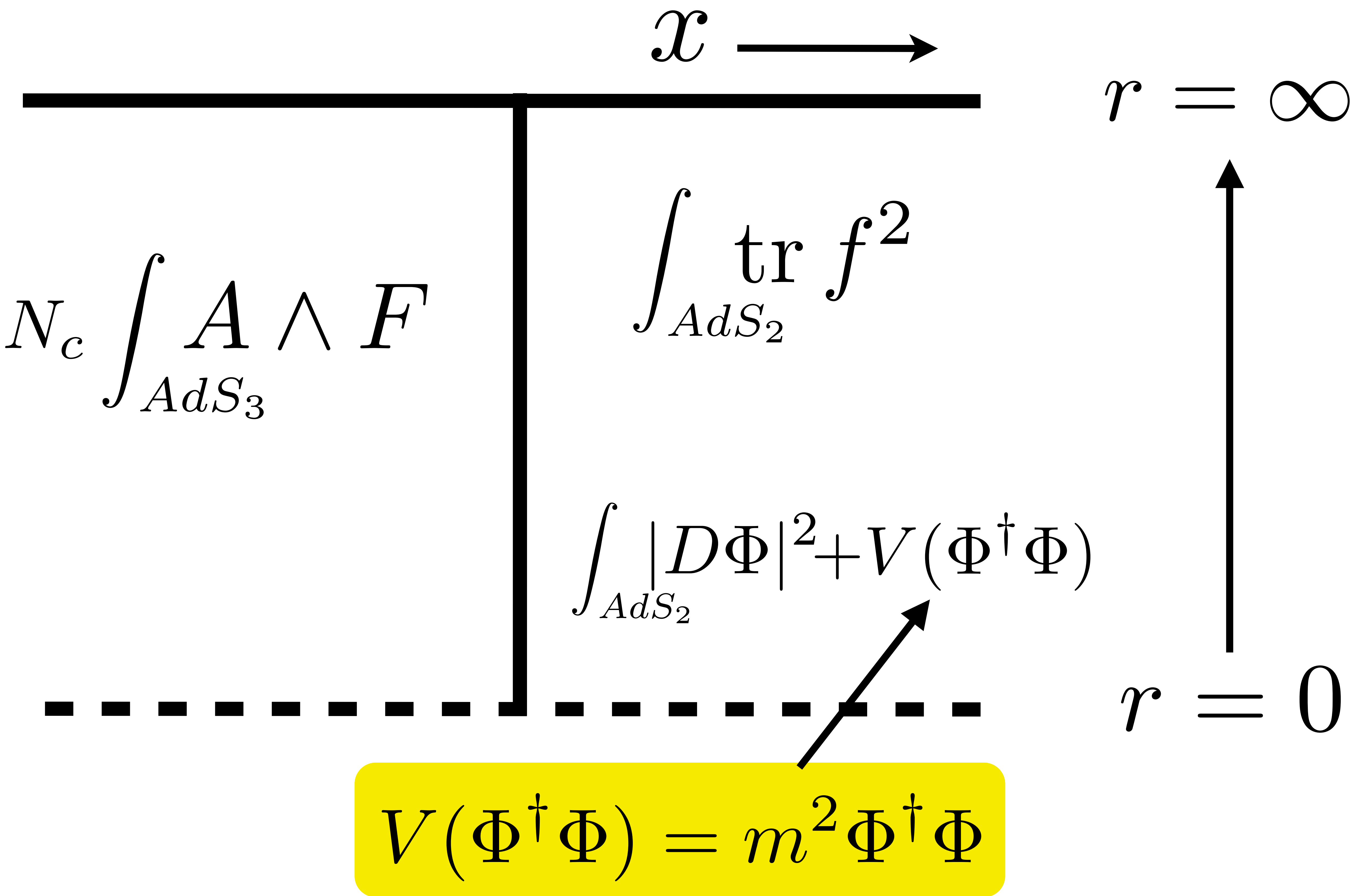


$$D\Phi = \partial\Phi + iA\Phi - ia\Phi$$

Bottom-Up Model



Bottom-Up Model



Bottom-Up Model

$$S = S_{CS} + S_{AdS_2}$$

$$S_{CS} = -\frac{N}{4\pi}\int \mathrm{tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$$

$$S_{AdS_2} = -\int d^3x\, \delta(x)\sqrt{-g}\left[\frac{1}{4}\mathrm{tr} f^2 + |D\Phi|^2 + V(\Phi^\dagger \Phi)\right]$$

$$D\Phi = \partial\Phi + iA\Phi - ia\Phi$$

$$V(\Phi^\dagger\Phi) = m^2 \Phi^\dagger \Phi$$

Probe limit

$U(1)$ gauge fields

Chern-Simons

$$F = dA$$

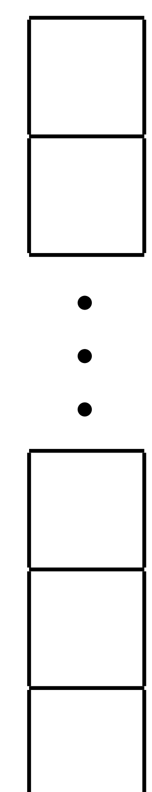
AdS_2

$$f = da$$

Single channel

$$U(1)_{N_c}$$

$$R_{\mathrm{imp}} =$$



Boundary Conditions

$$\sqrt{-g} f^{rt}\Big|_{\partial AdS_2} = Q$$

We choose $m^2 =$ Breitenlohner-Freedman bound

$$\Phi(r) = \tilde{c}\, r^{-1/2} + c\, r^{-1/2} \log r + \ldots$$

Our double-trace (Kondo) coupling:

$$c = \tilde{g}_K\, \tilde{c}$$

Witten hep-th/0112258

Berkooz, Sever, Shomer hep-th/0112264

AdS-Schwarzschild black hole

Hawking temperature $=$ T

$T > T_c$ $\quad \sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \quad \Phi(r) = 0$

$\langle \psi_L^\dagger \chi \rangle = 0$

$T < T_c$ $\quad \sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \quad \Phi(r) \neq 0$

$\langle \psi_L^\dagger \chi \rangle \neq 0$

A holographic superconductor in AdS_2

AdS-Schwarzschild black hole

Hawking temperature $=$ T

$T > T_c$ $\quad \sqrt{-g} f^{tr}|_{\partial AdS_2} \neq 0 \quad \Phi(r) = 0$

$\langle \psi_L^\dagger \chi \rangle = 0$

$T < T_c$ $\quad \sqrt{-g} f^{tr}|_{\partial AdS_2} \neq 0 \quad \Phi(r) \neq 0$

$\langle \psi_L^\dagger \chi \rangle \neq 0$

Superconductivity???

AdS-Schwarzschild black hole

Hawking temperature $=$ T

$T > T_c$ $\quad \sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \quad \Phi(r) = 0$

$\langle \psi_L^\dagger \chi \rangle = 0$

$T < T_c$ $\quad \sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \quad \Phi(r) \neq 0$

$\langle \psi_L^\dagger \chi \rangle \neq 0$

The large-N Kondo effect!

Solutions of the Kondo Problem

Numerical RG (Wilson 1975)

Fermi liquid description (Nozières 1975)

Bethe Ansatz/Integrability
(Andrei, Wiegmann, Tsvelick, Destri, ... 1980s)

Large-N expansion
(Anderson, Read, Newns, Doniach, Coleman, ...1970-80s)

Quantum Monte Carlo
(Hirsch, Fye, Gubernatis, Scalapino,... 1980s)

Conformal Field Theory (CFT)
(Affleck and Ludwig 1990s)

Large-N Approach to the Kondo Effect

Spin $SU(N)$

$R_{\text{imp}} = \text{anti-symm.}$

$k = 1$

$N \longrightarrow \infty$ with $N g_K$ fixed

$$\vec{S} = \chi^\dagger \vec{\tau} \chi$$

$$\mathcal{O}(\tau) \equiv c^\dagger(0,\tau)\chi(\tau)$$

$$\underbrace{SU(N)}_{\text{singlet}} \times \underbrace{U(1) \times U(1)}_{\text{bi-fundamental}}$$

Large-N Approach to the Kondo Effect

Spin $SU(N)$ | $R_{\text{imp}} = \text{anti-symm.}$ | $k = 1$

$N \longrightarrow \infty$ with $N g_K$ fixed

Coleman PRB 35, 5072 (1987)

Senthil, Sachdev, Vojta PRL 90, 216403 (2003)

$$T > T_c \qquad \langle \mathcal{O} \rangle = 0$$

$$T < T_c \qquad \langle \mathcal{O} \rangle \neq 0$$

$$T_c \simeq T_K$$

Large-N Approach to the Kondo Effect

Spin $SU(N)$

$R_{\mathrm{imp}} = \text{anti-symm.}$

$k = 1$

$N \longrightarrow \infty$ with $N g_K$ fixed

Coleman PRB 35, 5072 (1987)

Senthil, Sachdev, Vojta PRL 90, 216403 (2003)

$T > T_c$ $\quad \langle \mathcal{O} \rangle = 0$

$T < T_c$ $\quad \langle \mathcal{O} \rangle \neq 0$

Represents the binding of an electron to the impurity

Large-N Approach to the Kondo Effect

Spin $SU(N)$ | $R_{\text{imp}} = \text{anti-symm.}$ | $k = 1$

$N \longrightarrow \infty$ with Ng_K fixed

Coleman PRB 35, 5072 (1987)

Senthil, Sachdev, Vojta PRL 90, 216403 (2003)

$T > T_c$ $\quad \langle \mathcal{O} \rangle = 0$

$T < T_c$ $\quad \langle \mathcal{O} \rangle \neq 0$

$U(1) \times U(1) \rightarrow U(1)$

"(0+1)-DIMENSIONAL SYMMETRY BREAKING"

Large-N Approach to the Kondo Effect

Spin $SU(N)$

$R_{\text{imp}} = \text{anti-symm.}$

$k = 1$

$N \longrightarrow \infty$ with $N g_K$ fixed

Coleman PRB 35, 5072 (1987)

Senthil, Sachdev, Vojta PRL 90, 216403 (2003)

$T > T_c$ $\langle \mathcal{O} \rangle = 0$

$T < T_c$ $\langle \mathcal{O} \rangle \neq 0$

The phase transition is an ARTIFACT of the large-N limit!

The actual Kondo effect is a crossover

AdS-Schwarzschild black hole

Hawking temperature $=$ T

$T > T_c$ $\quad \sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \quad \Phi(r) = 0$

$$\langle \psi_L^\dagger \chi \rangle = 0$$

$T < T_c$ $\quad \sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \quad \Phi(r) \neq 0$

$$\langle \psi_L^\dagger \chi \rangle \neq 0$$

The large-N Kondo effect!

Work in Progress...

- Entropy?
- Heat capacity?
- Magnetic susceptibility?
- Resistivity?

Outline:

- The Kondo Effect
- The CFT Approach
- A Top-Down Holographic Model
- A Bottom-Up Holographic Model
- Summary and Outlook

Summary

What is the holographic dual of the Kondo effect?

Holographic superconductor in AdS_2

with a special boundary condition on the scalar

coupled as a defect

to a Chern-Simons gauge field in AdS_3

Outlook

- Multi-channel?
- Other impurity representations?
- Spin as global symmetry?
- Entanglement entropy?
- Quantum Quench?
- Multiple impurities?
- Suggestions welcome!

Thank You.