A Holographic Model for the Fractional Quantum Hall Effect

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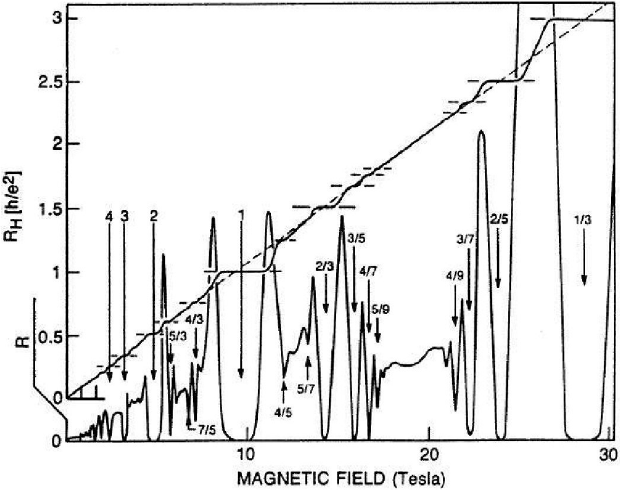
February 5, 2013

WIP with Elias Kiritsis, Matthew Lippert and Tassos Taliotis

Outline

The Fractional Quantum Hall Effect

- In systems with 2D electron gases, at very low temperatures, high magnetic fields, clean samples :



Stormer (1992)

The Fractional Quantum Hall Effect

- FQHE states are gapped and incompressible states with quantized Hall conductvitiy

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- Physics of pseudoparticle excitations invariant under Modular Group Action : $\sigma = \sigma_{xy} + i\sigma_{xx}$

$$\sigma \mapsto \frac{a\sigma + b}{c\sigma + d}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(2) \subset SL(2, \mathbb{Z}), \quad c \text{ even}$$

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- $\Gamma_0(2)$ action maps between Phases of 2D Electron Gas

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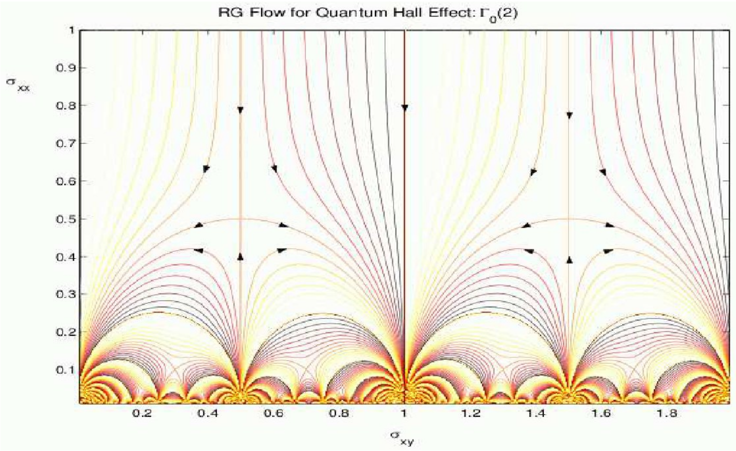

[Burgess+Lutken 1997, Dolan 1999, Lutken+Ross 2009]

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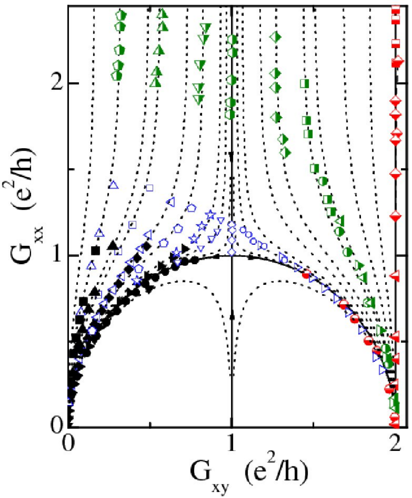
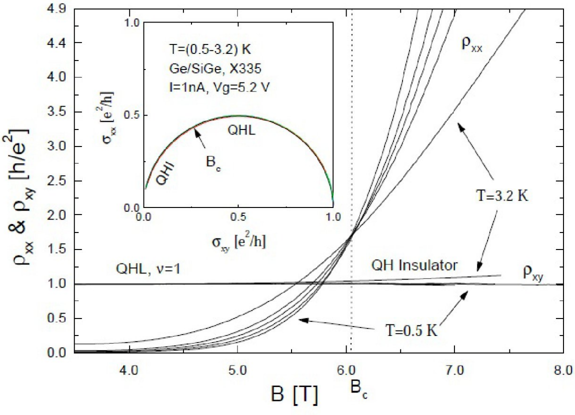


[S.S. Murzin et al 2002]

The Fractional Quantum Hall Effect

- Semicircle law: Conductivity sweeps out a semicircle in σ plane during QH transitions [e.g. Burgess etal 1008.1917]



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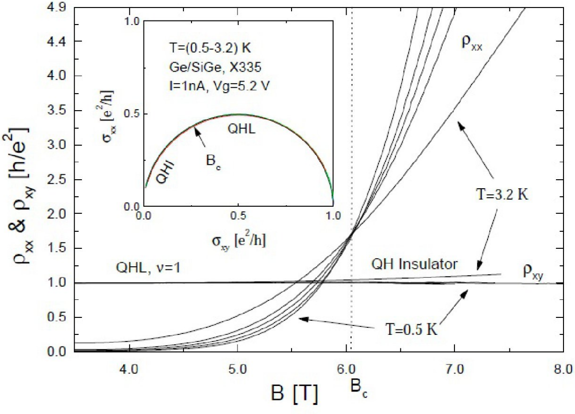
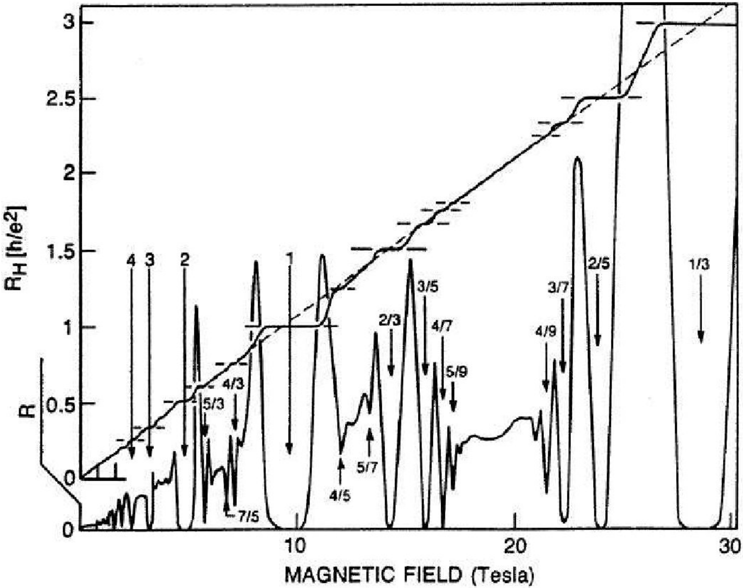


- Any p/q can be reached from the $\sigma : 0 \to 1$ transition by a $\Gamma_0(2)$
  $\Rightarrow$ Selection Rule: $p'q - pq' = 1$ [Dolan 1998]

The Fractional Quantum Hall Effect



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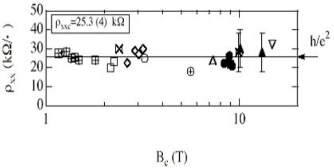
The Fractional Quantum Hall Effect

- QHL-QHI Transition: B_c is temperature independent and $\rho_{xx}(B_c)$ largely sample-independent



[cond-mat/9805143]

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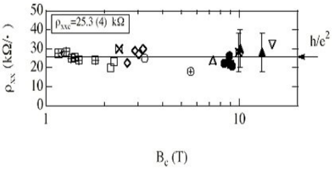

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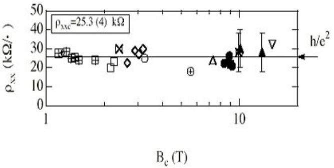

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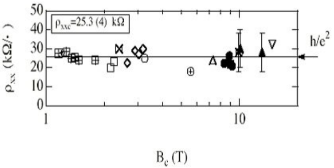

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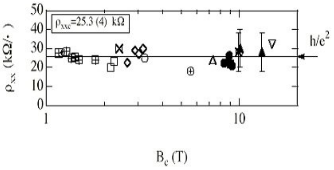

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- CAN WE REPRODUCE THIS IN A SINGLE HOLOGRAPHIC MODEL?

Outline

$SL(2,R)$ and Black Hole Charges

- The main idea of [1007.2490,1008.1917,WIP] is to use an SL(2,R) or SL(2,Z) invariant gravity action. These groups act on the electric and magnetic charges of the black hole solutions, which label the QH plateaux with charge density and external magnetic field. The filling fraction n/B inherits the group action, as do the conductivity and other observables.

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- The starting point of [1007.2490] is the SL(2,R) invariant action

$$S = \int d^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}(\partial a)^2 - \frac{1}{4}(e^{-\phi}F^2 + aF\tilde{F})\right]$$

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- SL(2,R) acts as

$$\tau = a + ie^{-\phi} = \tau_1 + i\tau_2\,, \quad \tau \to \frac{a\tau + b}{c\tau + d} \quad \text{and}$$

$$F \to F' = (c\tau_1 + d)F - c\tau_2\tilde{F}$$

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the SL(2,R) action generates dyonic black branes with [1007.2490]

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- Any filling fraction can be generated in this way:

$$\nu = \frac{Q_e'}{Q_m'} = \frac{a}{c}$$

$SL(2,R)$ and Black Hole Charges

- The electric solution flows in the IR to $\tau_{1*} = 0\,, \tau_{2*} = +\infty$, which after SL(2,R) becomes $\tau'_{1*} = \frac{a}{c}$ and $\tau'_{2*} = {\tau_{2*}}^{-1} = 0$. The filling fraction in the IR is hence equal to the value of the (transformed) axion

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- Are these black branes the QH plateaux?

  Evidence 1: The attractor mechanism shows that these are the unique IR attractors (in the absence of a potential $V(a,\phi)$). The effective potential
$$V_{eff} = \frac{(Q_e - Q_m\tau_1)^2}{\tau_2} + Q_m\tau_2$$
is minimized by either $Q_m = 0$, $\tau_2 \to \infty$ (electric solution),

  or by $Q_e = \tau_1 Q_m$ and $\tau_2 = 0$ (dyonic solution).

$SL(2,R)$ and Black Hole AC Conductivities

- Evidence 2: Hall Conductivity [1007.2490] used the known AC conductivity of the purely electric solution,

$$\sigma_{xx} = C' \frac{T^2}{\mu^2} + iC'' \frac{\mu}{\omega} + \dots, \quad \sigma_{yx} = 0$$

and the action of $SL(2,R)$ on the AC conductivity

$$\sigma_\pm \to \frac{a\sigma_\pm + b}{c\sigma_\pm + d}, \quad \sigma_\pm = \sigma_{yx} \pm i\sigma_{xx}$$

to show that at low frequencies the AC conductivity of the dyonic attractor solution behaves as

$$\sigma'_{yx} = \frac{a}{c}\left(1 + O(\omega^2)\right), \quad \sigma'_{xx} = \frac{16}{i(Q'_m)^2 C''} \frac{\omega}{\mu}\left(1 + O(\omega)\right).$$

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The Hall conductivity thus agrees with the filling fraction and also with the attractor value of the axion in the IR. σ_{xx} has no ω^{-1} pole and no delta function any more, as momentum is not conserved in a magnetic field.

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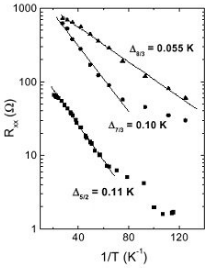

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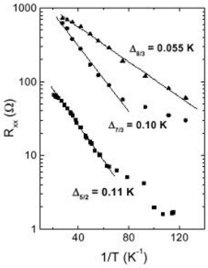
    2. Performing a SL(2,R) trafo from one filling fraction to another, $\sigma_{DC}(T=0)=0$ along the way, while the Hall conductivity changes. This is not the experimentally observed behavior.



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Gapped Spectra in Charged Systems and the FQHE

[WIP with Elias Kiritsis, Matthew Lippert, Anastasios Taliotis]

- In string theory, SL(2,R) is usually broken to SL(2,Z) by stringy or nonperturbative effects. This typically will generate a SL(2,Z) invariant potential for the axio-dilaton $(\tau = \tau_1 + i\tau_2 = a + ie^{\gamma\phi})$

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- A simple choice is the real-analytic Eisenstein series

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- For large τ_2 there is a expansion

$$E_s = 2\zeta(2s)\tau_2^s + 2\sqrt{\pi}\frac{\Gamma(s-1/2)}{\Gamma(s)}\zeta(2s-1)\tau_2^{1-s} + \text{instanton contributions}$$

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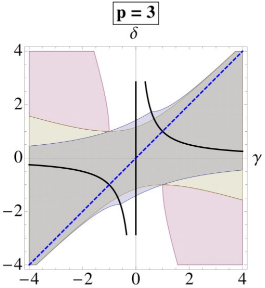
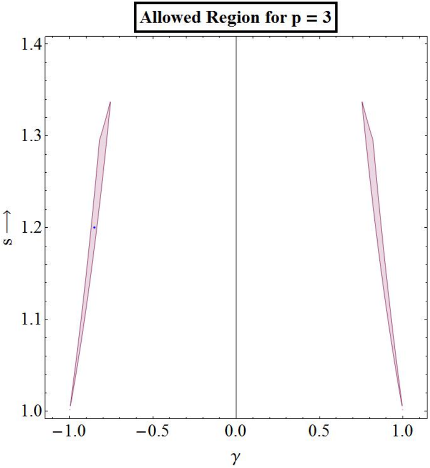
The system has two parameters, (γ, s). The leading behavior of V for large τ_2 is $e^{\gamma s\phi}$, and falls into the general class of scaling solutions of [1005.4690] with $\delta = -\gamma s$.
Depending on (γ, δ) the spectrum of charge excitations can be discrete and gapped: (for $\Delta_\phi < 1$)



Gapped Spectra in Charged Systems and the FQHE

- Imposing Gubser's constraint, thermodynamic instability of small black holes, consistency of the spin 1 fluctuation problem and existence of a discrete and gapped spectrum narrows down the allowed region:



Gapped Spectra in Charged Systems and the FQHE

- QH Plateaux? Since E_s is SL(2,Z) invariant it has runaway minima at $\tau_1 = \frac{p}{q}$, $\tau_2 = 0$, the images of the CDBH at $\tau_2 = \infty$. Their charges fulfill

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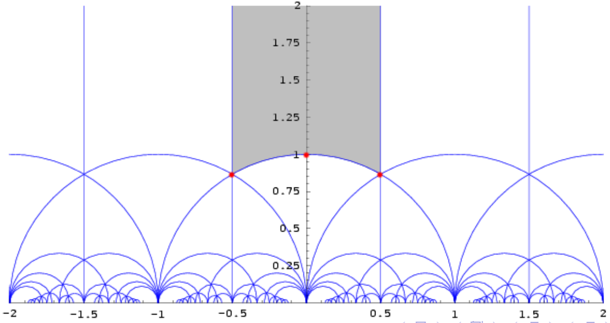
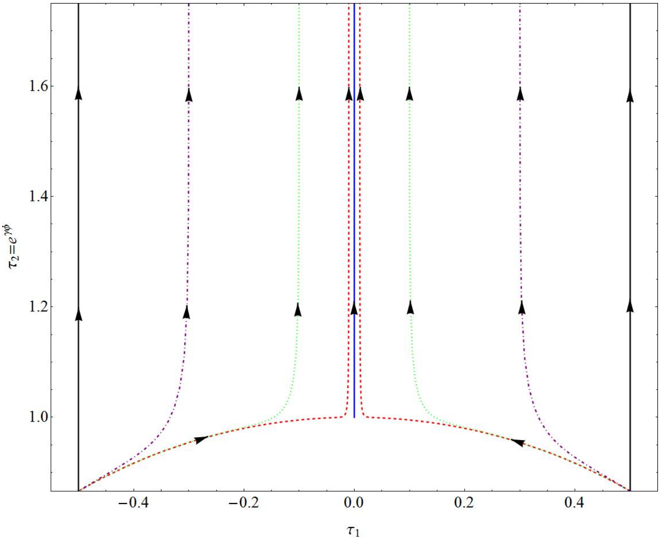
- RG Flows: E_s is stationary in the fundamental domain at the SL(2,Z) fixed points $\tau_*^{(1)} = i \, , \quad \tau_*^{(2)} = e^{\pm 2\pi i/3}$:



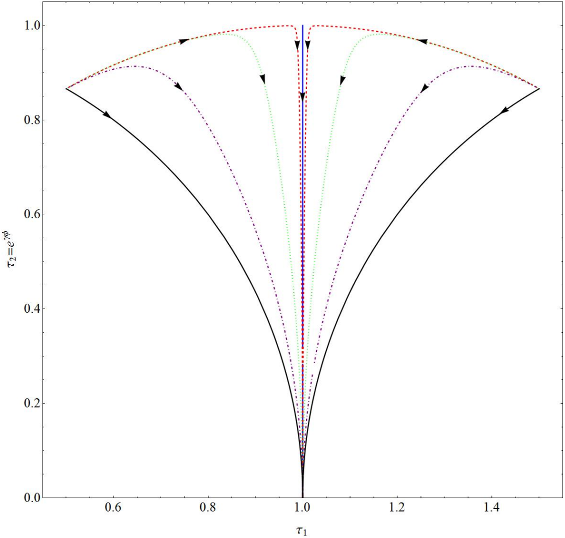
Gapped Spectra in Charged Systems and the FQHE

- Since SL(2,Z) commutes with the RG flow it suffices to construct the RG flows inside the fundamental domain:



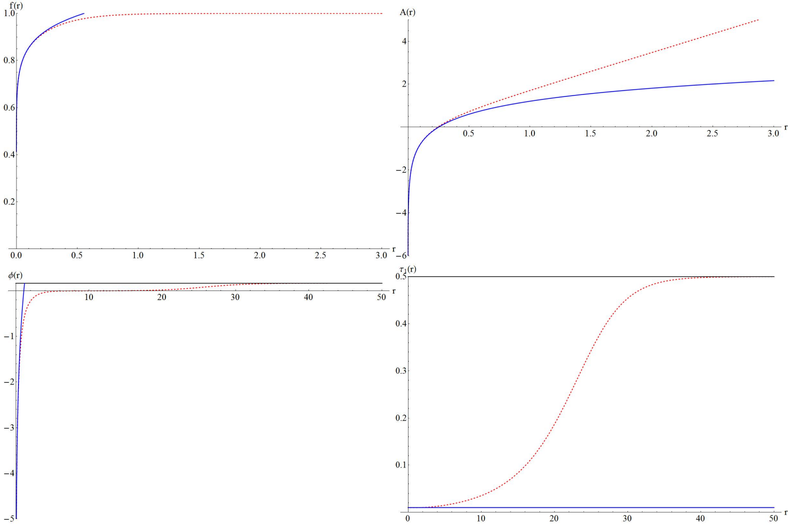
Gapped Spectra in Charged Systems and the FQHE

- By SL(2,Z) we can generate flows to any QH plateaux $\tau_1 = p/q$. E.g. $\nu = 1$:



Gapped Spectra in Charged Systems and the FQHE

- Our flows are the IR scaling geometries of [1005.4690]



Gapped Spectra in Charged Systems and the FQHE

- Conductivities: At low enough temperatures the purely electric state is discrete and gapped. Hence the conductivity at small ω is dominated by the contribution from translation invariance:

$$\sigma_{xx}(\omega) \simeq \frac{iC''\mu}{\omega} + O(1)\,, \quad \sigma_{xy} = 0$$

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- We calculated the conductivity at small frequencies by numerically solving the gauge field fluctuation equations in our flow geometries. The appropriate (real) boundary conditions are given by the Schrödinger problem

$$-\Psi''(z) + \frac{c}{z^2}\Psi(z) = \omega^2\Psi(z)\,, \quad c > 0\,, \quad z \to 0_+\,.$$

We find $Re\sigma_{xx} = C''\delta(\omega)$ with $C'' = O(1)$.

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- Conductivities: At low enough temperatures the purely electric state is discrete and gapped. Hence the conductivity at small ω is dominated by the contribution from translation invariance:

$$\sigma_{xx}(\omega) \simeq \frac{iC''\mu}{\omega} + O(1)\,, \quad \sigma_{xy} = 0$$

- We calculated the conductivity at small frequencies by numerically solving the gauge field fluctuation equations in our flow geometries. The appropriate (real) boundary conditions are given by the Schrödinger problem

$$-\Psi''(z) + \frac{c}{z^2}\Psi(z) = \omega^2\Psi(z)\,, \quad c > 0\,, \quad z \to 0_+\,.$$

  We find $Re\sigma_{xx} = C''\delta(\omega)$ with $C'' = O(1)$.

- Thus the QH plateaux have vanishing σ_{xx} (w.out the $1/\omega$ pole), and the correct Hall conductivity

$$\sigma_{xy} = \frac{a}{c}$$

But are they gapped as well?

Gapped Spectra in Charged Systems and the FQHE

- In electrically charged black branes (and with varying axion) a consistent fluctuation Ansatz needs two gauge field fluctuations $A_{x,y}$, sourcing two metric perturbations $\delta g_{tx,y}$. One can use the constraint equation [1005.4690] to decouple the metric from the gauge field fluctuations by directly replacing $\delta g'_{tx,y}(r)$ in the Maxwell equation.

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- In dyonic solutions the equations can be decoupled into a single second order equation after taking linear combinations [0910.0645]

$$E_z = \omega(A_x + iA_y) + hg_{rr}(\delta g^x{}_t - i\delta g^y{}_t)\,.$$

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$$E_z'' + F(r)E_z' + G(r)E_z = w^2 H(r) E_z$$

- This is equivalent to the Schrödinger problem

$$-\Psi'' + V(z,w)\Psi = w^2\Psi$$

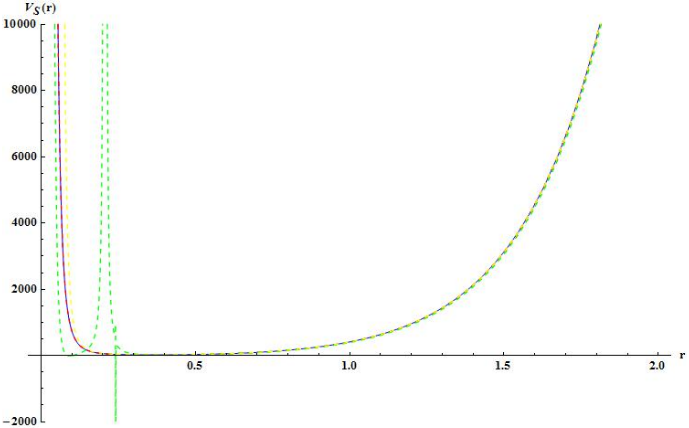
if we set $\frac{dz}{dr} = \sqrt{H(r)}\,, \quad \Psi(z) = E_z(z) e^{\frac{1}{2}\int dz \sqrt{\frac{g_{tt}}{g_{rr}}} F(z)}$

Gapped Spectra in Charged Systems and the FQHE

- For our choice of γ, s the potential

  $$V(z,w) = \frac{1}{4}\left(F^2 - 4\left(\frac{G}{H} - w^2\right) + 2F'\right)$$

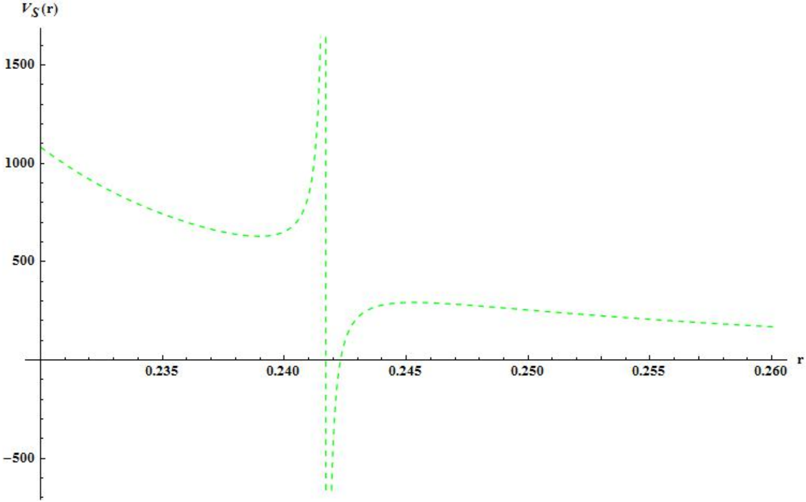
  diverges in the IR. In the UV we can establish the existence of a gap at small enough ω . E.g. Flow to $\tau = i$:



$w = 0$, $w = 10^{-3}$, $w = 10^{-1}$, $w = 1$

Gapped Spectra in Charged Systems and the FQHE

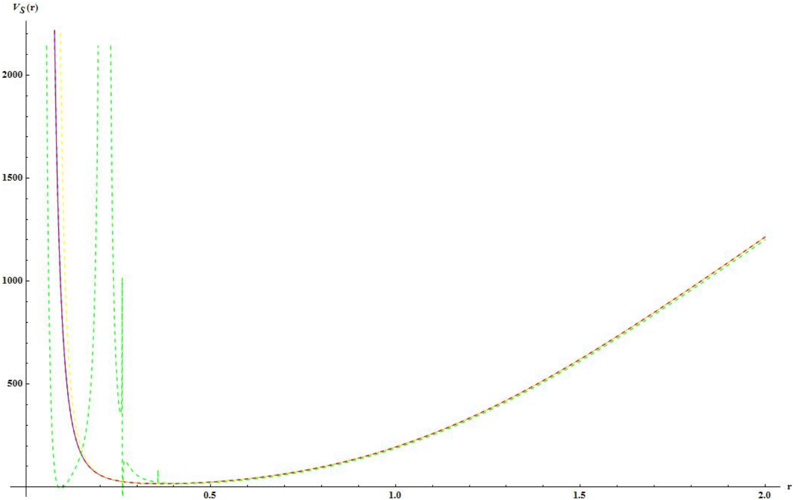
- The behaviour at $w \approx O(1)$ is more complicated:



Interpretation unclear so far

Gapped Spectra in Charged Systems and the FQHE

- There is little difference for the flows with varying axion:



Outline

Conclusions

- Several holographic bottom-up models of the FQHE so far have employed SL(2,R) transformations to infer the properties of the QH state from an ungapped state at zero magnetic field. The resulting QH state was ungapped. [1007.2490(,1008.1917)]

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- We use a SL(2,Z) invariant Eisenstein potential which allows us to tune the electric state to have a gapped and discrete charge spectrum at low temperatures. We constructed the RG flows to CDBHs in the fundamental domain, and hence all RG flows to QH plateaux states, and showed that the QH states have the correct Hall conductivity, and a real gap (no $\delta(\omega)$ pole).

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- STAY TUNED!

SL(2,R) invariant probe branes [1008.1917]

- [Burgess etal 1008.1917] realized that the DC conductivity in the QH state of [1007.2490] vanishes due to the momentum-conservation pole in $\Im \sigma_{xx}$ of the purely electric solution.

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- [Burgess etal 1008.1917] realized that the DC conductivity in the QH state of [1007.2490] vanishes due to the momentum-conservation pole in $\Im\sigma_{xx}$ of the purely electric solution.
- They introduce dissipation by separating the sector that generates the gravity background of [1007.2490] from the sector of charge carriers, which they model using a SL(2,R) invariant probe brane

$$
\begin{aligned}
S &= M_{Pl}^2 \int d^4x \sqrt{-g} \left[R - 2\Lambda - \frac{1}{2} \left((\partial\phi)^2 + e^{2\phi} (\partial a)^2 \right) \right] + \\
&\quad + M_{Pl}^2 S_{Lifshitz} + S_{gauge}
\end{aligned}
$$

The first two terms are assumed to be separately SL(2,R) invariant, and $S_{Lifshitz}$ to be chosen such as to generate the metric of the $z = 5$ Lifshitz black hole of [1007.2490], together with an appropriate axio-dilaton profile.

Backup Slides

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- S_{gauge} is taken to be a SL(2,R) invariant version of the DBI action, treated in the probe limit:

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S_{gauge} &= -T\int d^4x\left[\sqrt{-det\left(g_{\mu\nu}+\ell^2 e^{-\phi/2}F_{\mu\nu}\right)}-\sqrt{-g}\right]\\
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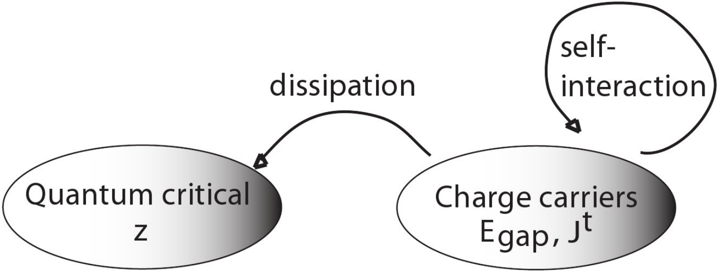
- This describes self-interacting charge carriers coupled to a large reservoir of quantum critical excitations into which they can loose energy via dissipation:



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$$\sigma_{xx} = \frac{\sigma_0}{d^2 + c^2\sigma_0^2}\,, \quad \sigma_{xy} = \frac{ac\sigma_0^2 + bd}{d^2 + c^2\sigma_0^2}\,,$$

with $\sigma_0(T/\mu)$ the DC conductivity of the probe brane in the purely electric state (with $\sigma_{yx} = 0$). For probe branes in Lifshitz backgrounds like

$$ds_z^2 = L^2\left[-h(r)\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2 h(r)} + \frac{dx^2 + dy^2}{r^2}\right]$$

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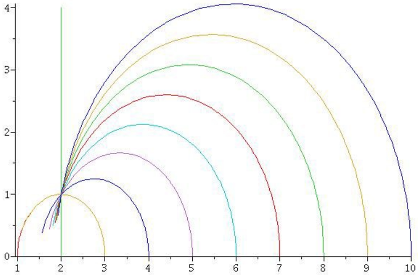
- This temperature flow commutes with SL(2,R) or any subgroup.

SL(2,R) invariant probe branes [1008.1917]

- The four parameters of the necessary SL(2,R) transformation are fixed by the data of the endpoint $(Q'_e, Q'_m, a, e^{-\phi})$. The temperature flow of the conductivities then trace out semi-circles in the σ plane, and for small T asymptote to (in linear response)

$$\sigma^{xx} \sim \frac{\rho T^{2/z}}{B^2} \to 0$$
$$\sigma^{xy} = \nu = \frac{a}{c}$$



This also predicts the superuniversality exponents $\kappa \approx \frac{2}{z} = \kappa'$ close to the measured value if $z = 5$ as in [1007.2490] .

However there is still no hard gap. .