PROBE BRANES ON FLAVORED ABJM BACKGROUND

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Niko Jokela, J. M., Alfonso V. Ramallo & Dimitrios Zoakos arXiv: 1211.0630
based on Eduardo Conde & Alfonso V. Ramallo 1105.6045

PLAN OF THE TALK

- The ABJM theory
- The flavored ABJM background
- Probes on the flavored ABJM background
- The flavored thermal ABJM background
- Probes on the flavored thermal ABJM background
- Conclusions

- **The ABJM theory**
- The flavored ABJM background
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The ABJM theory

field theory

Chern-Simons-matter theories in 2+1 dimensions

gauge group: $U(N)_k \times U(N)_{-k}$

field content (bosonic)

-Two gauge fields $A_\mu,\ \hat{A}_\mu$

-Four complex scalar fields: $C^I \ \ (I = 1, \cdots, 4)$

bifundamentals $(N, \bar{N})$

action

$$S = k\,CS[A] - k\,CS[\hat{A}] - k\,D_\mu C^{I\,\dagger} D^\mu C^I - V_{\rm pot}(C)$$

$V_{\rm pot}(C) \rightarrow$ sextic scalar potential

Aharony, Bergman, Jafferis & Maldacena 0806.1218

The ABJM theory

The ABJM model has $\mathcal{N} = 6$ SUSY in 3d

it has two parameters

$N \rightarrow$ rank of the gauge groups

$k \rightarrow$ CS level $(1/k \sim$ gauge coupling$)$

't Hooft coupling $\lambda \sim \frac{N}{k}$

it is a CFT in 3d with very nice properties

- partition function and Wilson loops can be obtained from localization

Gaiotto&Jafferis 0903.2175

Drukker, Mariño & Putrov 1003.3837

- has many integrability properties (Bethe ansatz, Wilson loop/amplitude relation, ...) T. Klose, 1012.3999

- connection to FQHE? Fujita, Li, Ryu & Takayanagi, 0901.0924

it is the 3d analogue of N=4 SYM

The ABJM theory

sugra description in type IIA :

Effective description for $N^{\frac{1}{5}} << k << N$

$$AdS_4 \times \mathbb{CP}^3 + \text{ fluxes} \qquad \mathbb{CP}^3 = \mathbb{C}^4/(z_i \sim \lambda z_i)$$

$$ds^2 = L^2 ds^2_{AdS} + L^2 ds^2_{\mathbb{CP}^3}$$

$$F_4 = \frac{3\pi}{\sqrt{2}} \left(kN \right)^{\frac{1}{2}} \Omega_{AdS_4}$$

$$F_2 = 2k\, J$$

$$e^{\phi} = \frac{2L}{k} = 2\sqrt{\pi} \left(\frac{2N}{k^5} \right)^{\frac{1}{4}}$$

$$L^4 = 2\pi^2 \frac{N}{k} = 2\pi^2 \lambda$$

$J \rightarrow$ Kahler form of $\mathbb{CP}^3$

$$\frac{1}{2\pi} \int_{\mathbb{CP}^1} F_2 = k$$

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- Probes on the flavored ABJM background
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- Conclusions

Flavors in the ABJM background

D6-branes extended in AdS_4 and wrapping $\mathbb{RP}^3 \subset \mathbb{CP}^3$

Hohenegger&Kirsch 0903.1730

Gaiotto&Jafferis 0903.2175

Introduce quarks in the $(N,1)$ and $(1,N)$ representation

$Q_1 \to (N,1)$ $\quad Q_2 \to (1,N)$ $\quad \tilde{Q}_1 \to (\bar{N},1)$ $\quad \tilde{Q}_2 \to (1,\bar{N})$

coupling to the vector multiplet

$Q_1^\dagger e^{-V} Q_1 + Q_2^\dagger e^{-\hat{V}} Q_2 + \text{antiquarks}$ $\quad V, \hat{V}$ vector supermultiplets for $A\,,\hat{A}$

coupling to the bifundamentals → $C^I = (A_1, A_2, B_1^\dagger, B_2^\dagger)$

$\tilde{Q}_1\, A_i B_i\, Q_1\ ,$ $\quad \tilde{Q}_2\, B_i\, A_i\, Q_2$ $\quad$ plus quartic terms in $Q, \tilde{Q}$'s

Flavors in the ABJM background

$$ds^2 = L^2 ds^2_{AdS} + L^2 ds^2_{\mathbb{CP}^3}$$

Write $\mathbb{CP}^3$ as an $\mathbb{S}^2$-bundle over $\mathbb{S}^4$ $\quad (\underbrace{x^0, x^1, x^2, r}_{AdS_4}, \ \underbrace{\xi, \hat{\theta}, \hat{\psi}, \hat{\varphi}}_{\mathbb{S}^4}, \ \underbrace{\theta, \varphi}_{\mathbb{S}^2})$

$$ds^2 = L^2 ds^2_{AdS_4} + L^2 \left[\frac{4}{(1+\xi^2)^2} \left(d\xi^2 + \frac{\xi^2}{4} \left((\omega^1)^2 + (\omega^2)^2 + (\omega^3)^2 \right) \right) + \left(dx^i + \epsilon_{ijk} A^j x^k \right)^2 \right]$$

where

$$\begin{aligned} \omega^1 &= \cos\hat{\psi} d\hat{\theta} + \sin\hat{\psi} \sin\hat{\theta} d\hat{\varphi} \\ \omega^1 &= \sin\hat{\psi} d\hat{\theta} - \cos\hat{\psi} \sin\hat{\theta} d\hat{\varphi} \\ \omega^3 &= d\hat{\psi} + \cos\hat{\theta} d\hat{\varphi} \end{aligned}$$

$$\begin{aligned} x^1 &= \sin\theta \cos\varphi \\ x^2 &= \sin\theta \sin\varphi \\ x^3 &= \cos\theta \end{aligned}$$

$$A^i = -\frac{\xi^2}{1+\xi^2} \, \omega^i$$

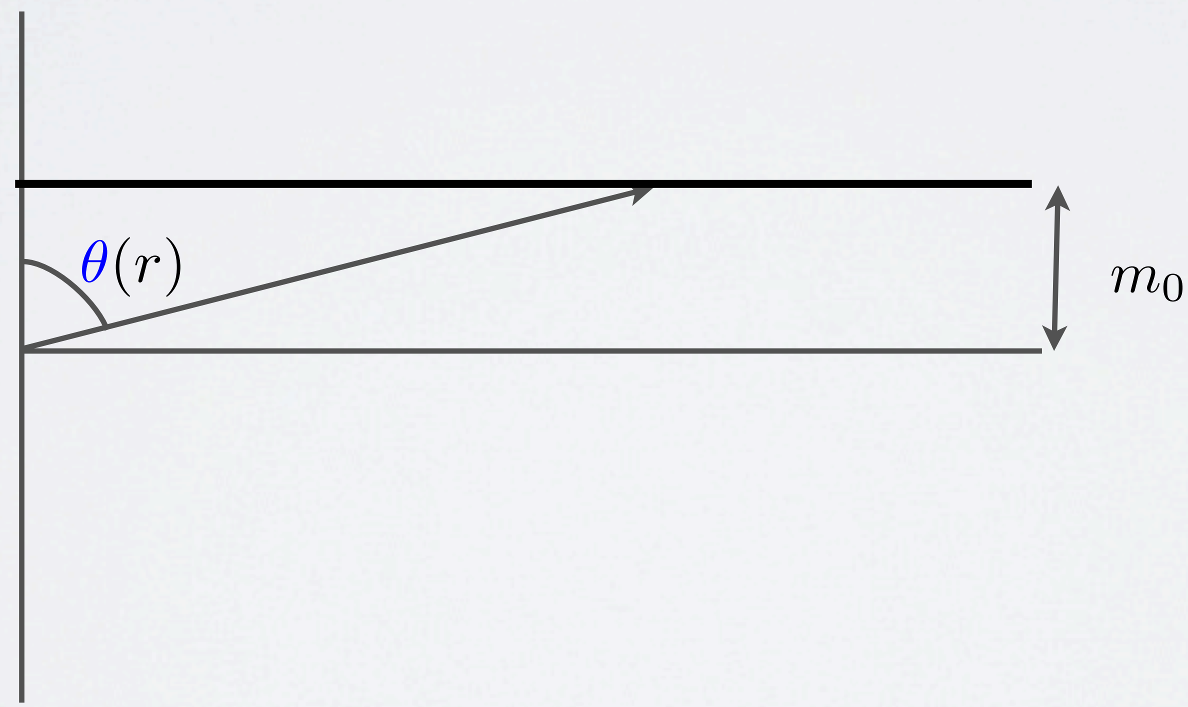
$SU(2)$ instanton on $\mathbb{S}^4$

Flavors in the ABJM background

D6-branes extended in AdS_4 and wrapping $\mathbb{RP}^3 \subset \mathbb{CP}^3$

$$(\underbrace{x^0, x^1, x^2, r,}_{AdS_4} \ \underbrace{\xi, \hat\theta = 0, \hat\psi, \hat\varphi = 0,}_{\mathbb{S}^4} \ \underbrace{\theta = \theta(r)\,, \varphi}_{\mathbb{S}^2} \ )$$

$$S = S_{DBI} + S_{WZ} = -T_{D_6} \int d^7\zeta \, e^{-\phi} \sqrt{-\det \hat g_7} + T_{D_6} \int \hat C_7$$



$$\theta(r) = \pi/2 \quad \Rightarrow \quad m_0 = 0$$

The ABJM flavored background

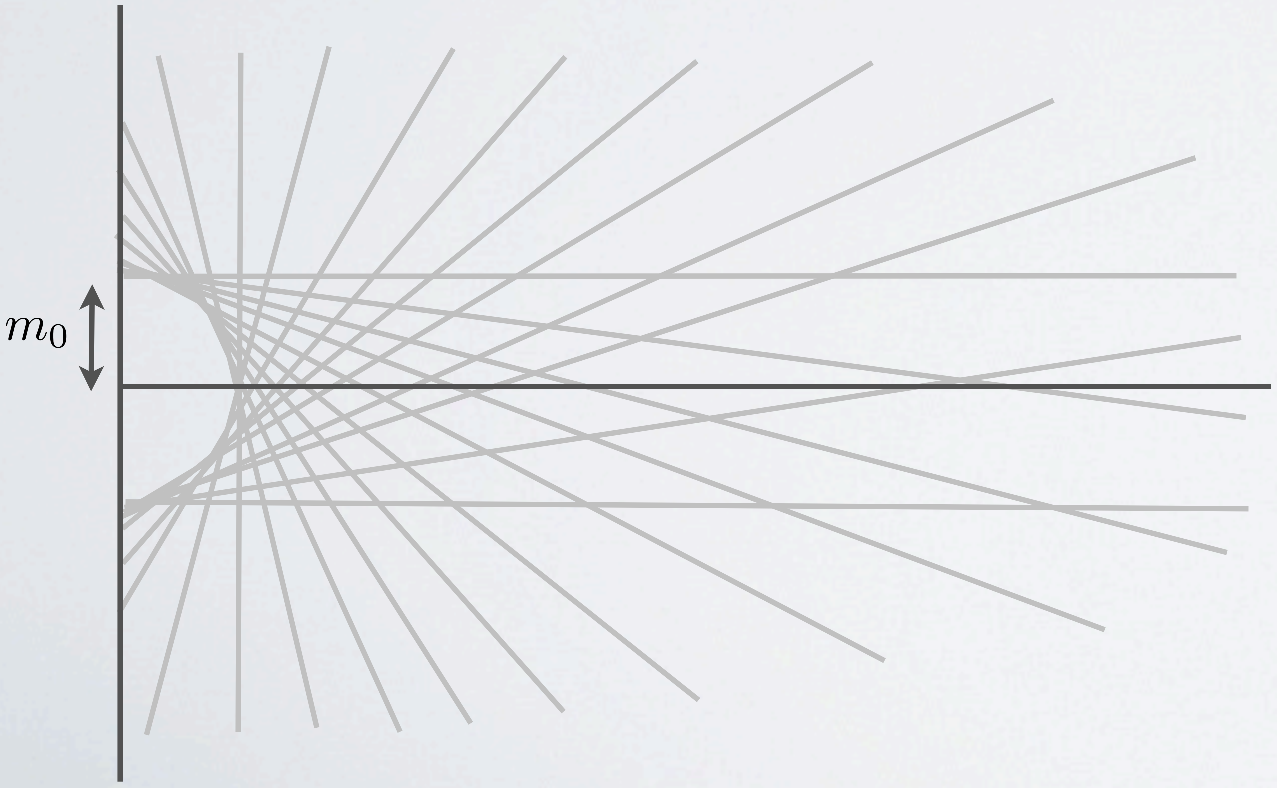
the idea is now to smear over positions and orientations

E. Conde & A.V. Ramallo 1105.6045

$$\sim \; (x^0, x^1, x^2, r, \; \xi, \int \hat{\theta}, \hat{\psi}, \int \hat{\varphi}, \; \int \theta, \varphi)$$

Backreaction $\quad S_{flav} = \sum_{i=1}^{N_f} \left(-T_{D_6} \int_{\mathcal{M}^{(i)}} d^7\zeta \, e^{-\phi} \sqrt{-\det \hat{g}_7} + T_{D_6} \int_{\mathcal{M}^{(i)}} \hat{C}_7 \right)$

$$\rightarrow \quad \frac{1}{\kappa_{10}^2} \left(- \int d^{10}x e^{3\phi/4} \sqrt{-\det g_{10}} |\Omega| + \int d^{10}x \, C_7 \wedge \Omega \right)$$



Ω is a charge distribution 3-form

$C_7 = e^{-\phi} \mathcal{K}$ is the calibration form

preserve N=1 SUSY

- no delta-function sources

- much simpler (analytic) solutions

$-$flavor symmetry : $U(1)^{N_f}$

The ABJM flavored background

modified Bianchi identity $$dF_2 = 2\pi \, \Omega$$

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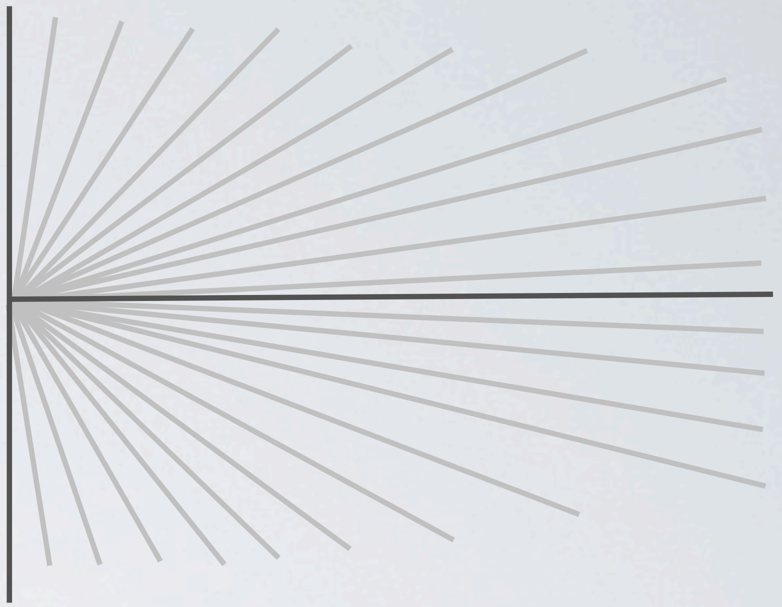
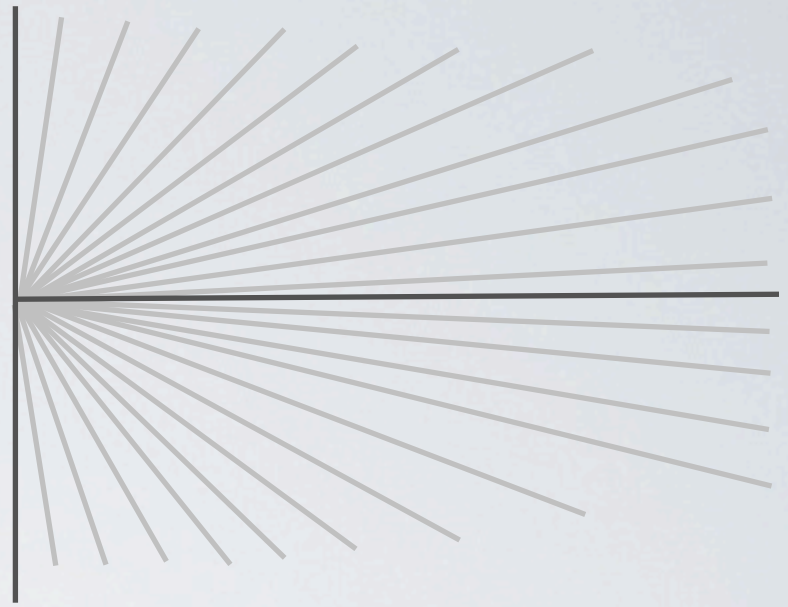


solution for massless flavors $$\theta(r) = \pi/2$$

The ABJM flavored background

modified Bianchi identity $$dF_2 = 2\pi\ \Omega$$



solution for massless flavors $$\theta(r) = \pi/2$$

go to vielbein basis

$$(d\xi, \omega^1, \omega^2, \omega^3, d\theta, d\varphi) \rightarrow$$

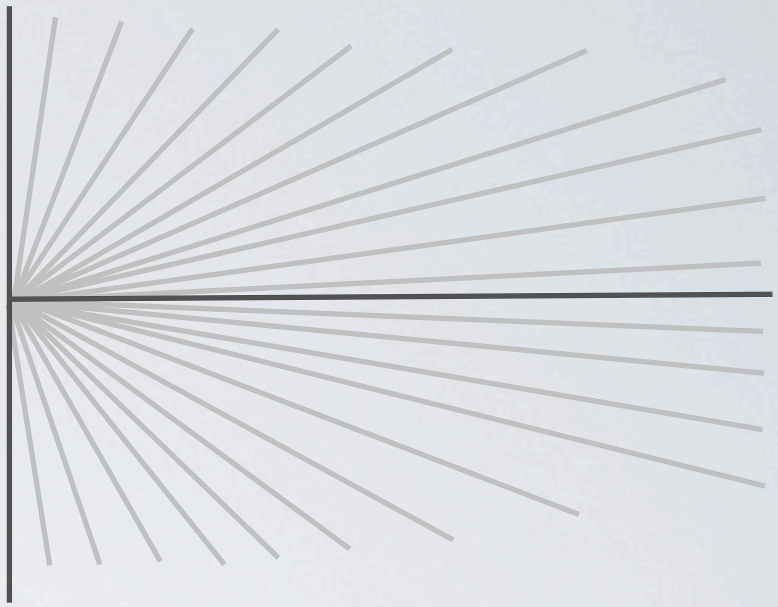
$\mathcal{S}^i = (\mathcal{S}^1, \mathcal{S}^2, \mathcal{S}^3, \mathcal{S}^4)$ along the $\mathbb{S}^4$ base

$E^a = (E^1, E^2)$ along the $\mathbb{S}^2$ fiber

The ABJM flavored background

modified Bianchi identity $dF_2 = 2\pi\ \Omega$

solution for massless flavors $\theta(r) = \pi/2$



go to vielbein basis

$$(d\xi, \omega^1, \omega^2, \omega^3, d\theta, d\varphi) \ \rightarrow \ \begin{array}{ll} \mathcal{S}^i = (\mathcal{S}^1, \mathcal{S}^2, \mathcal{S}^3, \mathcal{S}^4) & \text{along the } \mathbb{S}^4 \text{ base} \\ E^a = (E^1, E^2) & \text{along the } \mathbb{S}^2 \text{ fiber} \end{array}$$

$$\begin{aligned} S^\xi &= \frac{2}{1+\xi^2} d\xi \\ S^1 &= \frac{\xi}{1+\xi^2}\left(\sin\varphi\,\omega^1 - \cos\varphi\,\omega^2\right) \\ S^2 &= \frac{\xi}{1+\xi^2}\left(\sin\theta\,\omega^3 - \cos\theta\left(\cos\varphi\,\omega^1 + \sin\varphi\,\omega^2\right)\right) \\ S^3 &= \frac{\xi}{1+\xi^2}\left(-\cos\theta\,\omega^3 - \sin\theta\left(\cos\varphi\,\omega^1 + \sin\varphi\,\omega^2\right)\right) \end{aligned}$$

$$\begin{aligned} E^1 &= d\theta + \xi S^1 \\ E^2 &= \sin d\varphi - \xi S^2 \end{aligned}$$

The ABJM flavored background

$$ds^2 = L^2 ds^2_{AdS} + L^2 \left(\sum_{i=1}^{4} (\mathcal{S}^i)^2 + \sum_{a=1}^{2} (E^a)^2 \right)$$

$$F_2 = \frac{k}{2} \Big[E^1 \wedge E^2 - (\mathcal{S}^4 \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2) \Big]$$

$$F_4 = \frac{3k}{2} L^2 \, \Omega_{AdS_4}$$

The ABJM flavored background

Flavor backreaction

$$ds^2 = L^2 ds^2_{AdS} + L^2 \left(\sum_{i=1}^{4} (\mathcal{S}^i)^2 + \sum_{a=1}^{2} (E^a)^2 \right)$$

$$F_2 = \frac{k}{2}\left[E^1 \wedge E^2 - \eta(\mathcal{S}^4 \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2)\right]$$

$$F_4 = \frac{3k}{2} L^2\, \Omega_{AdS_4} \qquad \eta = 1 + \frac{3}{4}\frac{N_f}{k}$$

The ABJM flavored background

Flavor backreaction

$$ds^2 = L^2 ds^2_{AdS} + L^2 \frac{1}{b^2}\left(q \sum_{i=1}^{4} (\mathcal{S}^i)^2 + \sum_{a=1}^{2} (E^a)^2 \right)$$

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$$F_4 = \frac{3k}{2} L^2\, \Omega_{AdS_4} \qquad \eta = 1 + \frac{3}{4}\frac{N_f}{k}$$

$$q = 3 + \frac{9}{4}\frac{N_f}{k} - 2\sqrt{1 + \frac{3}{4}\frac{N_f}{k} + \left(\frac{3}{4}\right)^4 \left(\frac{N_f}{k}\right)^2} \qquad \begin{cases} 1 + \frac{3}{8}\frac{N_f}{k} - \ldots \\ q \to \frac{5}{3} \qquad (\frac{N_f}{k} \to \infty) \end{cases}$$

$$b = \frac{4 + \frac{39}{16}\frac{N_f}{k} - \sqrt{1 + \frac{3}{4}\frac{N_f}{k} + \left(\frac{9}{16}\frac{N_f}{k}\right)^2}}{3 + \frac{3}{2}\frac{N_f}{k}} \qquad \begin{cases} 1 + \frac{3}{16}\frac{N_f}{k} - \ldots \\ q \to \frac{5}{4} \qquad (\frac{N_f}{k} \to \infty) \end{cases}$$

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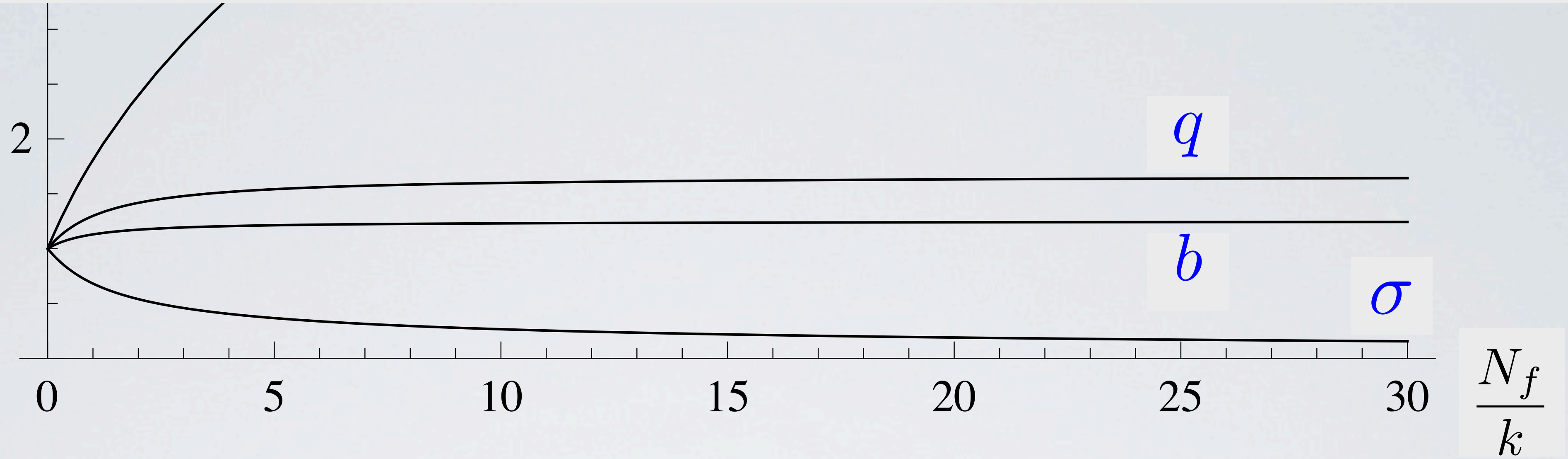
$L^2 = \pi\sqrt{2\lambda}\,\sigma$ where σ is related to the quark-antiquark potential screening

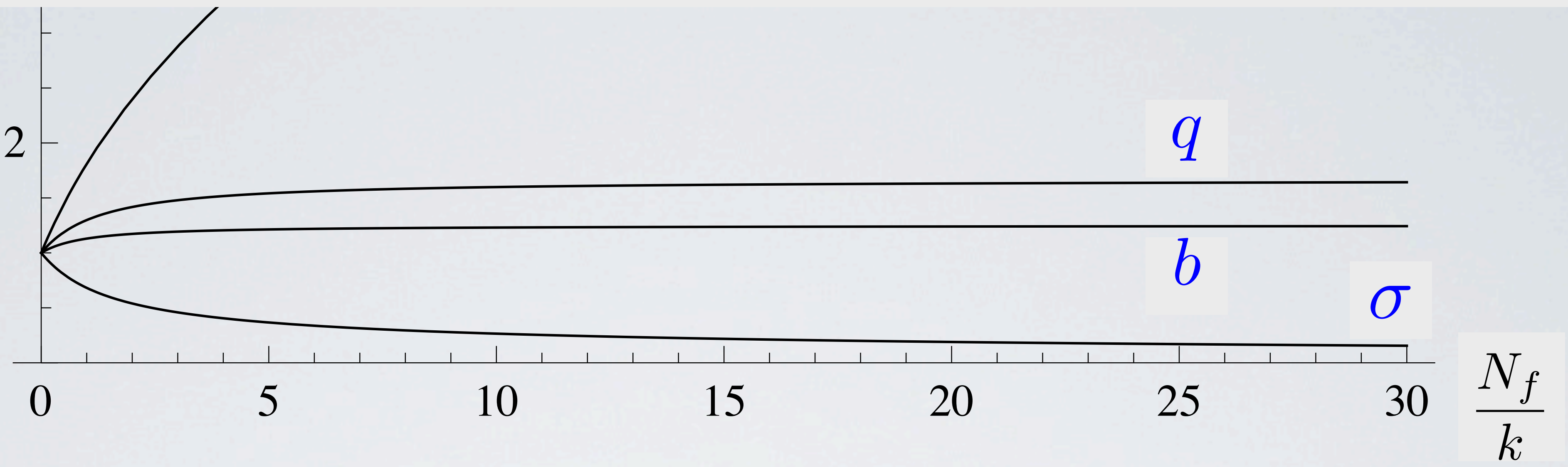
The ABJM flavored background



potential screening $$V_{q\bar{q}} = -\frac{Q}{l} \;\; ; \qquad \rightarrow \qquad Q = \frac{4\sqrt{2}\pi^3}{\Gamma(1/4)^4}\sqrt{\lambda}\,\sigma$$

$$\sigma = \frac{1}{4}\frac{q^{3/2}(\eta+q)^2(2-q)^{1/2}}{(q+\eta q-\eta)^{5/2}} \begin{cases} 1 - \frac{3}{8}\frac{N_f}{k} - \dots \\ \rightarrow \sqrt{\frac{k}{N_f}} \qquad\qquad (\frac{N_f}{k} \rightarrow \infty) \end{cases}$$

The ABJM flavored background



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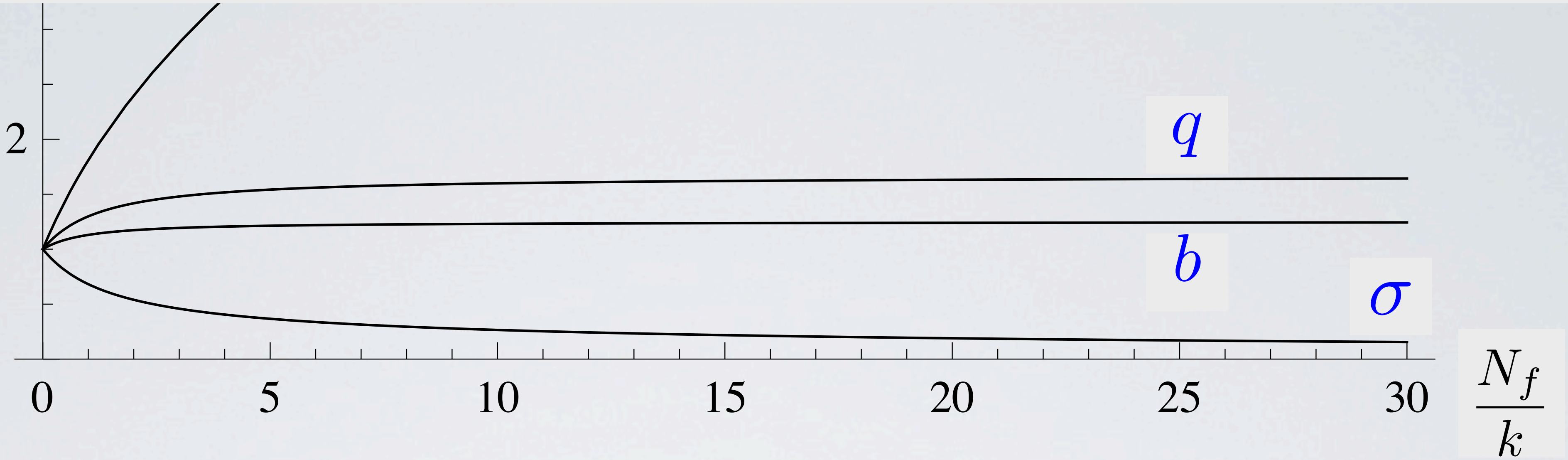
dilaton shifts

$$e^{\phi} = 4\sqrt{\pi}\left(\frac{2N}{k^5}\right)^{1/4}\frac{(2-q)^{5/4}}{(\eta+q)[q(q+\eta q-\eta)]^{1/4}}$$

The ABJM flavored background



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dilaton shifts $$e^{\phi} = 4\sqrt{\pi}\left(\frac{2N}{k^5}\right)^{1/4}\frac{(2-q)^{5/4}}{(\eta+q)[q(q+\eta q-\eta)]^{1/4}}$$

regime of validity $$N^{1/5} \ll N_f \ll N$$

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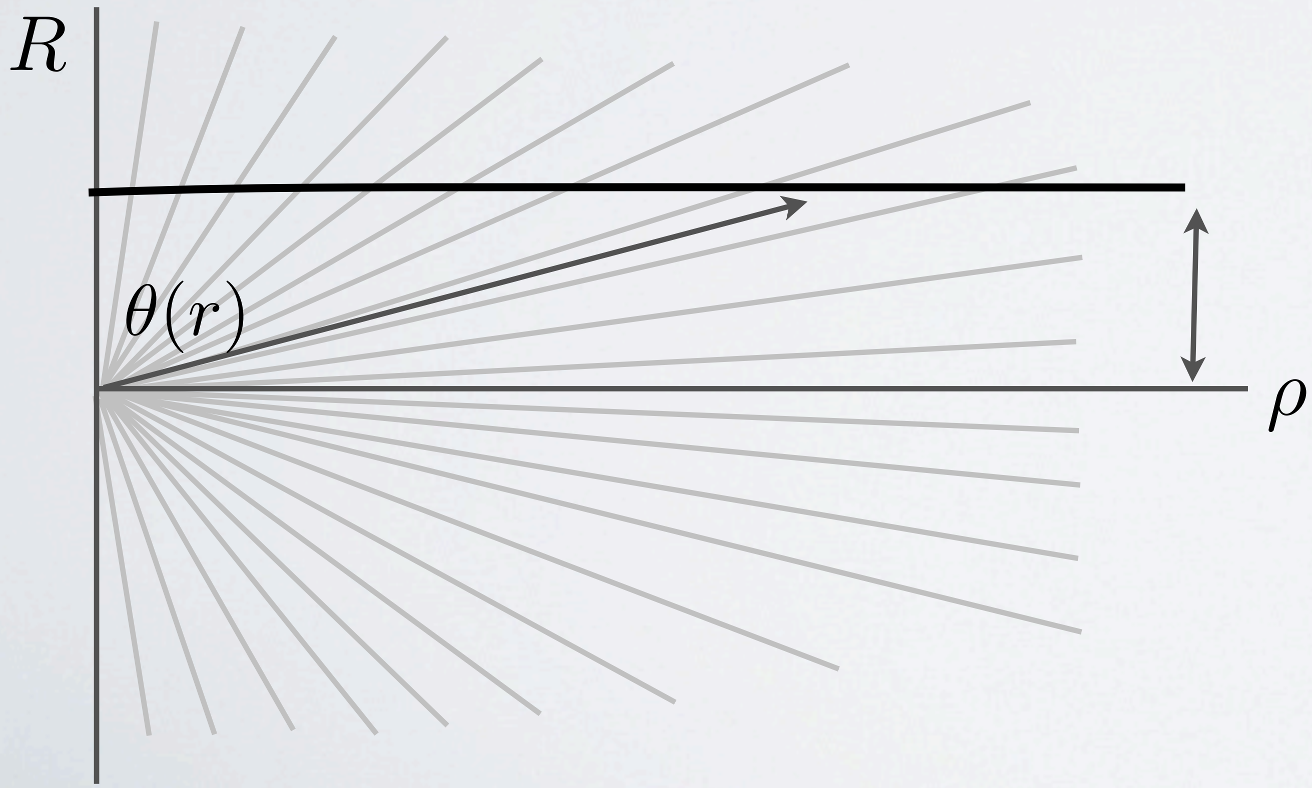
Probe Branes

D6-branes extended in AdS_4 and wrapping $\mathbb{RP}^3 \subset \mathbb{CP}^3$

$$(\underbrace{x^0, x^1, x^2, r,}_{AdS_4}\ \underbrace{\xi,\ \hat\theta = 0,\ \hat\psi,\ \hat\varphi = 0,}_{\mathbb{S}^4}\ \underbrace{\theta(r)\,,\varphi}_{\mathbb{S}^2}\ )$$

new cartesian-like coordinates $\quad u = r^{b}$

$$R = u\cos\theta \quad ; \quad \rho = u\sin\theta \quad \Rightarrow \quad L^2\left(\frac{dr^2}{r^2} + \frac{d\theta^2}{b^2}\right)$$



$$\frac{L^2}{b^2(\rho^2+R^2)}\left[d\rho^2 + dR^2\right]$$

profile $\quad R = R(\rho)$

Probe Branes

DBI+WZ action $S = S_{DBI} + S_{WZ} = T_{D6}\left(-\int d^7\zeta e^{-\phi}\sqrt{-\det \hat{g}_7} + \int d^7\zeta \hat{C}_7\right)$

embedding $R(\rho)$ → $\sim \int_0^\rho d\rho\, \rho(\rho^2 + R^2)^{\frac{3}{2b}-1}(\sqrt{1+R'^2} - 1)$

asymptotic behavior $\rho \to 0 \quad \Rightarrow \quad \partial_\rho\left(\rho^{3/b}\partial_\rho R\right) = 0$

$$R \sim m + \frac{c}{r^{3-2b}}$$

compare with $\phi \sim \phi_0\, r^{\Delta-3} + \frac{\langle\mathcal{O}\rangle}{r^\Delta}$

ϕ_0 is the source of $\mathcal{O}$

$$\Delta = 3 - b$$

$\Delta \to$ dimension of $\mathcal{O}$

Probe Branes

anomalous dimension

In our case $\mathcal{O} \sim \bar{\psi}\psi$

$$\dim(\bar{\psi}\psi) \; = \; 3 - b$$

$$\longrightarrow \quad 2 - \frac{3}{16}\frac{N_f}{k} + \frac{63}{512}\left(\frac{N_f}{k}\right)^2 + \ldots$$

$$\hookrightarrow \frac{7}{4} \qquad \left(\frac{N_f}{k} \to \infty\right)$$

Probe Branes

anomalous dimension In our case $\mathcal{O} \sim \bar{\psi}\psi$

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$$\hookrightarrow \frac{7}{4} \qquad \left(\frac{N_f}{k} \rightarrow \infty\right)$$

on shell action

$$\rho \rightarrow \infty \quad \Rightarrow \quad S \sim \rho^{2-3/b}$$

is automatically finite

Depends on the gauge for C_7!!

$C_7 \rightarrow C_7 + d\Lambda_6$ generates boundary conterterms

$C_7 = e^{-\phi}\mathcal{K} \quad \rightarrow \quad$ SUSY scheme

SUSY solution $\rightarrow$ $R =$ constant $\rightarrow$ $S = 0$

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The ABJM flavored thermal background

replace AdS by Schwarzschild-AdS

$$ds^2 = L^2 \left(-r^2 h(r) dt^2 + \frac{dr^2}{r^2 h(r)} + r^2 (dx_1^2 + dx_2^2) \right) + L^2 \frac{1}{b^2} \left(q \sum_{i=1}^{4} (\mathcal{S}^i)^2 + \sum_{a=1}^{2} (E^a)^2 \right)$$

blackening factor $h(r) = \left(1 - \frac{r_h^3}{r^3} \right)$ → $T = \frac{3\, r_h}{4\pi}$

entropy $s_{back} = \frac{2\pi}{\kappa_{10}^2} \frac{A_8}{V_2} = \frac{1}{3\sqrt{2}} \left(\frac{4\pi}{3} \right)^2 \sqrt{k}\, N^{3/2}\, \xi \left(\frac{N_f}{k} \right) T^2$

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$$ds^2 = L^2\left(-r^2h(r)dt^2 + \frac{dr^2}{r^2h(r)} + r^2(dx_1^2 + dx_2^2)\right) + L^2\frac{1}{b^2}\left(q\sum_{i=1}^{4}(\mathcal{S}^i)^2 + \sum_{a=1}^{2}(E^a)^2\right)$$

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entropy $s_{back} = \frac{2\pi}{\kappa_{10}^2}\frac{A_8}{V_2} = \frac{1}{3\sqrt{2}}\left(\frac{4\pi}{3}\right)^2\sqrt{k}\,N^{3/2}\,\xi\left(\frac{N_f}{k}\right)T^2$

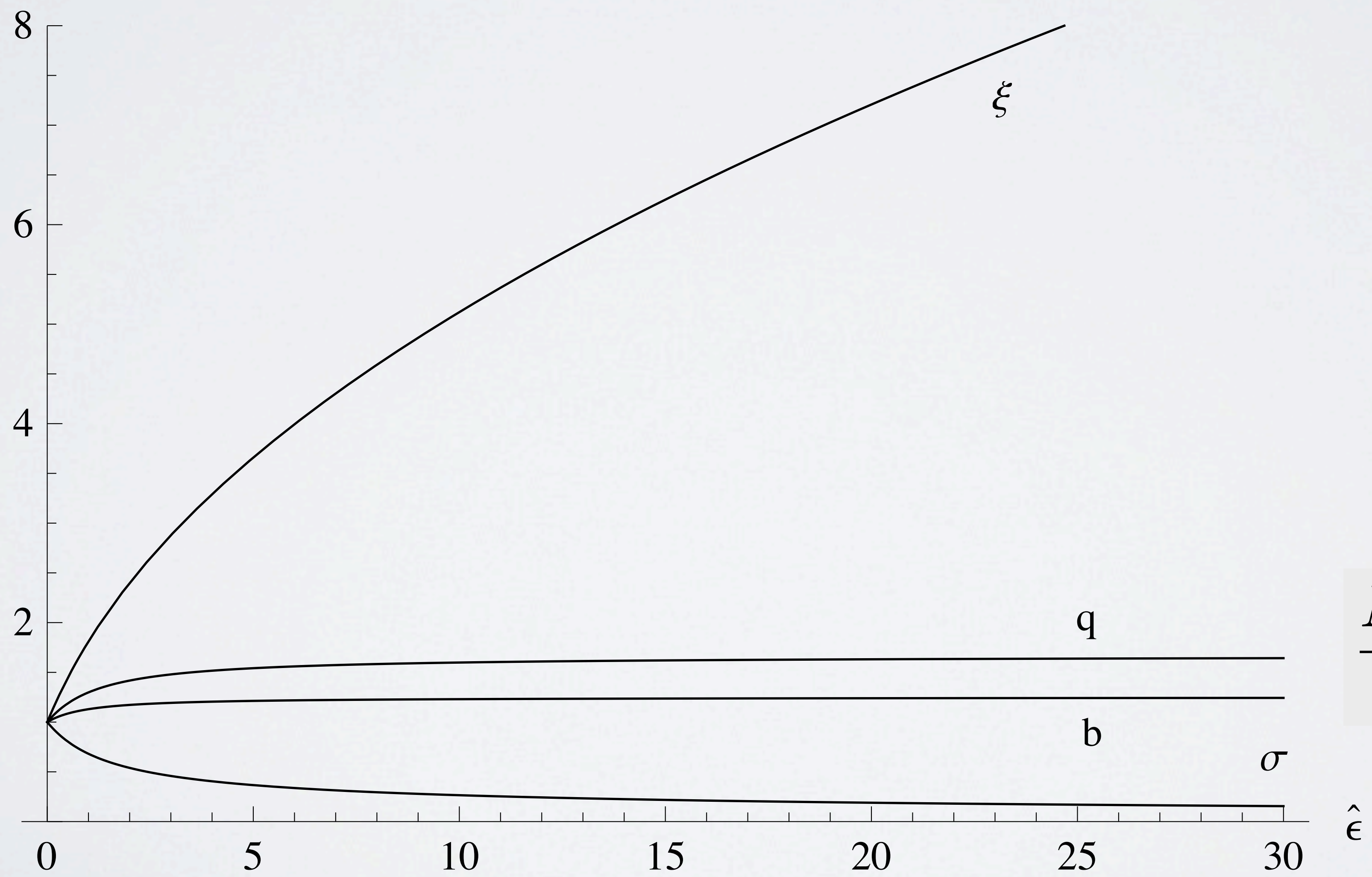
energy density $E_{ADM} = -\frac{1}{\kappa_{10}^2}\sqrt{|G_{tt}|}\int_{t_0,r_\infty}\sqrt{\det G_8}\,(K_T - K_0)$

free energy $F_{back} = E_{ADM} - T s_{back} = -\frac{1}{9\sqrt{2}}\left(\frac{4\pi}{3}\right)^2\sqrt{k}\,N^{3/2}\,\xi\left(\frac{N_f}{k}\right)T^3$

unflavored term $\sim N^{\frac{3}{2}}$ $\longrightarrow$ field theory match by Drukker et al. (1007.3837) !

The ABJM flavored thermal background

$$\xi\Big(\frac{N_f}{k}\Big) \equiv \frac{1}{16}\;\frac{q^{\frac{5}{2}}\,(\eta+q)^4}{(2-q)^{\frac{1}{2}}\,(q+\eta q\,-\,\eta)^{\frac{7}{2}}}\qquad \begin{cases} = 1+\dfrac{3}{4}\dfrac{N_f}{k}-\dfrac{9}{64}\left(\dfrac{N_f}{k}\right)^2+\dots & N_f\to 0\\[2mm] \sim 1.389\sqrt{\dfrac{N_f}{k}} & N_f\to\infty \end{cases}$$



$$\frac{N_f}{k}$$

The ABJM flavored background

Comparison with 3-Sasakian

$(U(N_f), \mathcal{N}=3$ flavors$)$

Localized solution in 11d for coincident massless flavors

$AdS_4 \times \mathcal{M}_7$ with $\mathcal{M}_7$ a hyperkahler 3-Sasakian manifold

$\mathcal{N}=3$ with $U(N_f)$ flavor symmetry

$$\xi\left(\frac{N_f}{k}\right) = \frac{1}{16}\frac{q^{5/2}(\eta+q)^4}{\sqrt{2-q}(q+\eta q-\eta)^{7/2}}$$

$$\xi^{3S}\left(\frac{N_f}{k}\right) = \frac{1+\frac{N_f}{k}}{\sqrt{1+\frac{N_f}{2k}}}$$

Gaiotto&Jafferis 0903.2175

Couso-Santamaria et al. 1011.6281

The ABJM flavored background

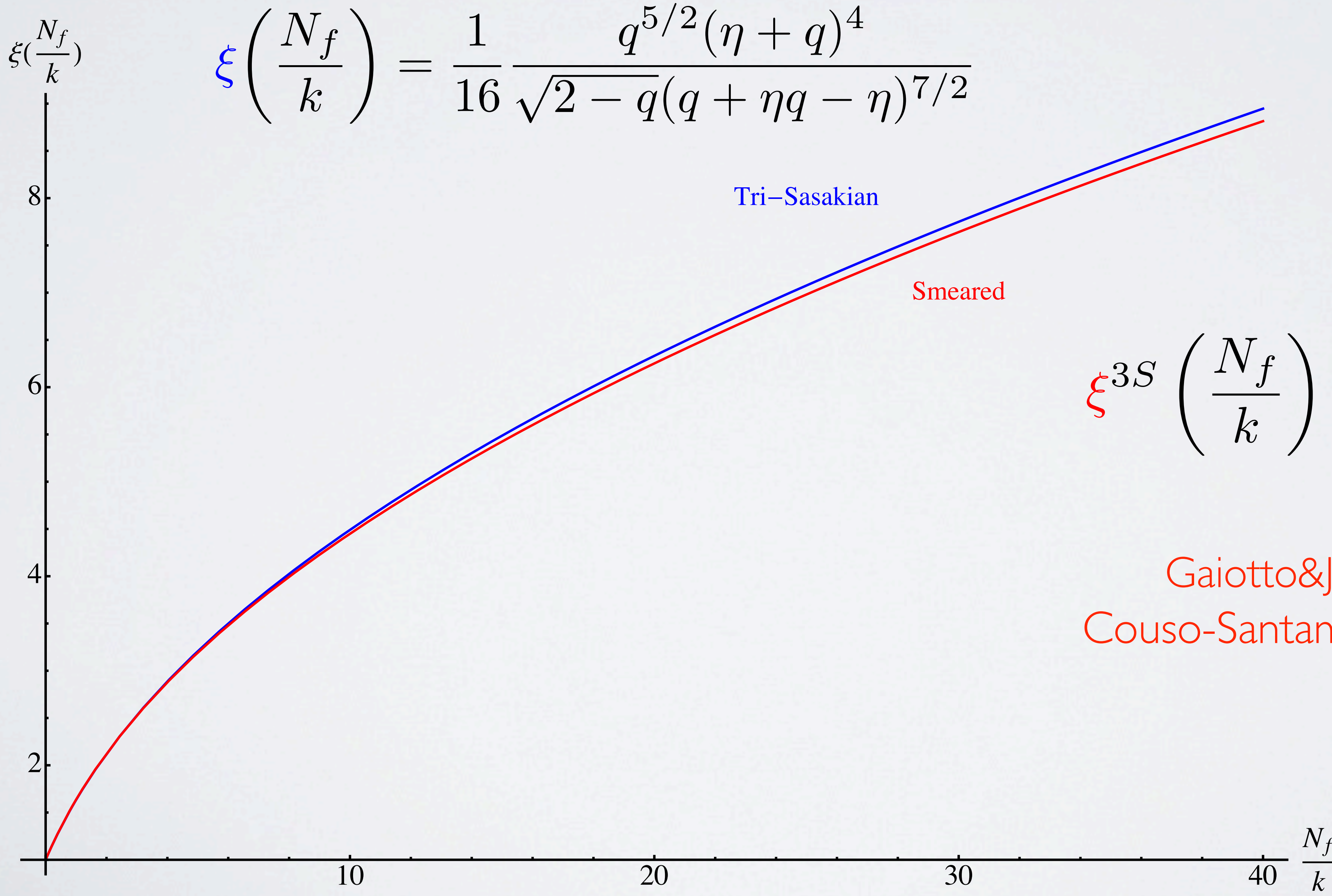
Comparison with 3-Sasakian $(U(N_f), \mathcal{N}=3$ flavors$)$

Localized solution in 11d for coincident massless flavors

$AdS_4 \times \mathcal{M}_7$ with $\mathcal{M}_7$ a hyperkahler 3-Sasakian manifold

$\mathcal{N}=3$ with $U(N_f)$ flavor symmetry

$$\xi\left(\frac{N_f}{k}\right) = \frac{1}{16}\frac{q^{5/2}(\eta+q)^4}{\sqrt{2-q}(q+\eta q-\eta)^{7/2}}$$



$$\xi^{3S}\left(\frac{N_f}{k}\right) = \frac{1+\frac{N_f}{k}}{\sqrt{1+\frac{N_f}{2k}}}$$

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- The ABJM theory
- The flavored ABJM background
- Probes on the flavored ABJM background
- The flavored thermal ABJM background
- **Probes on the flavored thermal ABJM background**
- Conclusions

Probes on the ABJM flavored thermal background

the embeddings are governed by the DBI+WZ action

$$S = S_{DBI} + S_{WZ} = T_{D6}\left(- \int d^7\zeta e^{-\phi}\sqrt{-\det \hat{g}_7} \; + \; \int d^7\zeta \hat{C}_7 \right)$$

$$S_{DBI} = \mathcal{N} \int d^3x \int_{r_{min}}^{\infty} dr \frac{r^2}{r_h^3} \; \sin\theta \sqrt{1 + \frac{r^2 h}{b^2}\dot{\theta}^2}$$

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$$\mathcal{N} = \frac{2\sqrt{2}\pi^2}{27} N^{3/2}\sqrt{k}\; T^3 \;\zeta\left(\frac{N_f}{k}\right)$$

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$$\zeta\left(\frac{N_f}{k}\right) = \frac{1}{32}\frac{\sqrt{2-q}(\eta+q)^4 q^{5/2}}{(q+\eta q-\eta)^{9/2}} \quad\rightarrow \begin{cases} 1-\frac{3}{8}\frac{N_f}{k}+\ldots & (N_f\to 0) \\ \sqrt{\frac{k}{N_f}}+\ldots & (N_f\to\infty)\end{cases}$$

Probes on the ABJM flavored thermal background

$$S_{WZ} = T_{D6} \int d^7\zeta\, \hat{C}_7 = T_{D6} \int d^7\zeta \left(e^{-\phi}\hat{\mathcal{K}} + \delta C_7\right)$$

C_7 must be improved to get a consistent thermodynamics

$$e^{-\phi}\hat{\mathcal{K}} = \frac{L^7 q}{b^3} e^{-\phi} d^3x \wedge \left[\frac{r^3}{b} \sin\theta\cos\theta d\theta + r^2 \sin^2\theta dr\right] \wedge \Xi_3$$

represent the improvement term as follows

$$\delta C_7 = \frac{L^7 q}{b^3} e^{-\phi} d^3x \wedge \left[L_1(\theta)d\theta \,+\, L_2(r)dr\right] \wedge \Xi_3 \qquad \Rightarrow \qquad d\,\delta C_7 = 0$$

the angular part of C_7 must vanish at the horizon   Jensen 1006.3066

$$L_1(\theta) = -\frac{r_h^3}{b} \sin\theta\cos\theta$$

Probes on the ABJM flavored thermal background

define (a zero point energy) $$\int dr L_2(r) = \Delta_0$$

the total action is now

$$S = \mathcal{N} \int d^3x \left[-\frac{4b}{r_h^3} \int dr\, r^2 \sin\theta \left(\sqrt{1 + \frac{r^2 h(r)}{b^2} \dot{\theta}^2} - \sin\theta - \frac{r h(r)}{b} \cos\theta\, \dot{\theta} \right) + \Delta_0 \right]$$

$$\mathcal{N} = \frac{2\sqrt{2}\pi^2}{27} N^{3/2} \sqrt{k}\, T^3\, \zeta\left(\frac{N_f}{k}\right)$$

and satisfies that the canonical momentum vanishes at the horizon

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \Big|_{r=r_h} = 0$$

in order to fix Δ_0 let us compare the free energy (density) of the probe and the background

$$F = T\, S_E \longrightarrow F = -\frac{S}{\int d^3x}$$

Probes on the ABJM flavored thermal background

Consistency check:

a) infinite mass limit ⇒ decoupling

$$\lim_{m\to\infty} F = \mathcal{N}\,(1-\Delta_0) = 0 \quad \longrightarrow \quad \Delta_0 = 1$$

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$$F_{back}\left(\frac{N_f}{k}\right) + F_{probe}(m=0) = F_{back}\left(\frac{N_f+1}{k}\right) = F_{back}\left(\frac{N_f}{k}\right) + \frac{1}{k}F'_{back} + ...$$

hence

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$$\frac{1}{\sqrt{2}}\left(\frac{4\pi}{9}\right)^2 \frac{N^{3/2}}{\sqrt{k}}\, T^3\, \frac{3}{4}\zeta\left(\frac{N_f}{k}\right)\Delta_0 = \frac{1}{\sqrt{2}}\left(\frac{4\pi}{9}\right)^2 N^{3/2}\sqrt{k}\, T^3\, \frac{1}{k}\xi'\left(\frac{N_f}{k}\right)$$

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$$\zeta\left(\frac{N_f}{k}\right) = \frac{1}{64}\frac{\sqrt{2-q}(\eta+q)^4 q^{5/2}}{(q+\eta q-\eta)^{9/2}}$$

$$\xi\left(\frac{N_f}{k}\right) = \frac{1}{16}\frac{q^{5/2}(\eta+q)^4}{\sqrt{2-q}(q+\eta q-\eta)^{7/2}}$$

$$\longrightarrow \quad \frac{3}{4}\zeta = \xi' \quad \longrightarrow \quad \Delta_0 = 1$$

Probes on the ABJM flavored thermal background

the same mechanism for the entropy yields

$$s_{total} = s_{back} + s \approx \frac{1}{3} \left(\frac{4\pi}{3}\right)^2 \frac{N^2}{\sqrt{2\lambda}} \xi \left(\frac{N_f + 1}{k}\right) T^2 \ , \qquad\qquad (m \to 0)$$

hence, massless probe entropy $\approx$ increase in area of the horizon

Now that the action is completely fixed

$$S = \mathcal{N} \int d^3x \left[-\frac{4b}{r_h^3} \int dr\, r^2 \sin\theta \left(\sqrt{1 + \frac{r^2 h(r)}{b^2} \dot{\theta}^2} - \sin\theta - \frac{r h(r)}{b} \cos\theta\, \dot{\theta} \right) + 1 \right]$$

we may derive the correct equations of motion and the solutions, as well as the thermodynamics

Probes on the ABJM flavored thermal background

isotropic coordinates

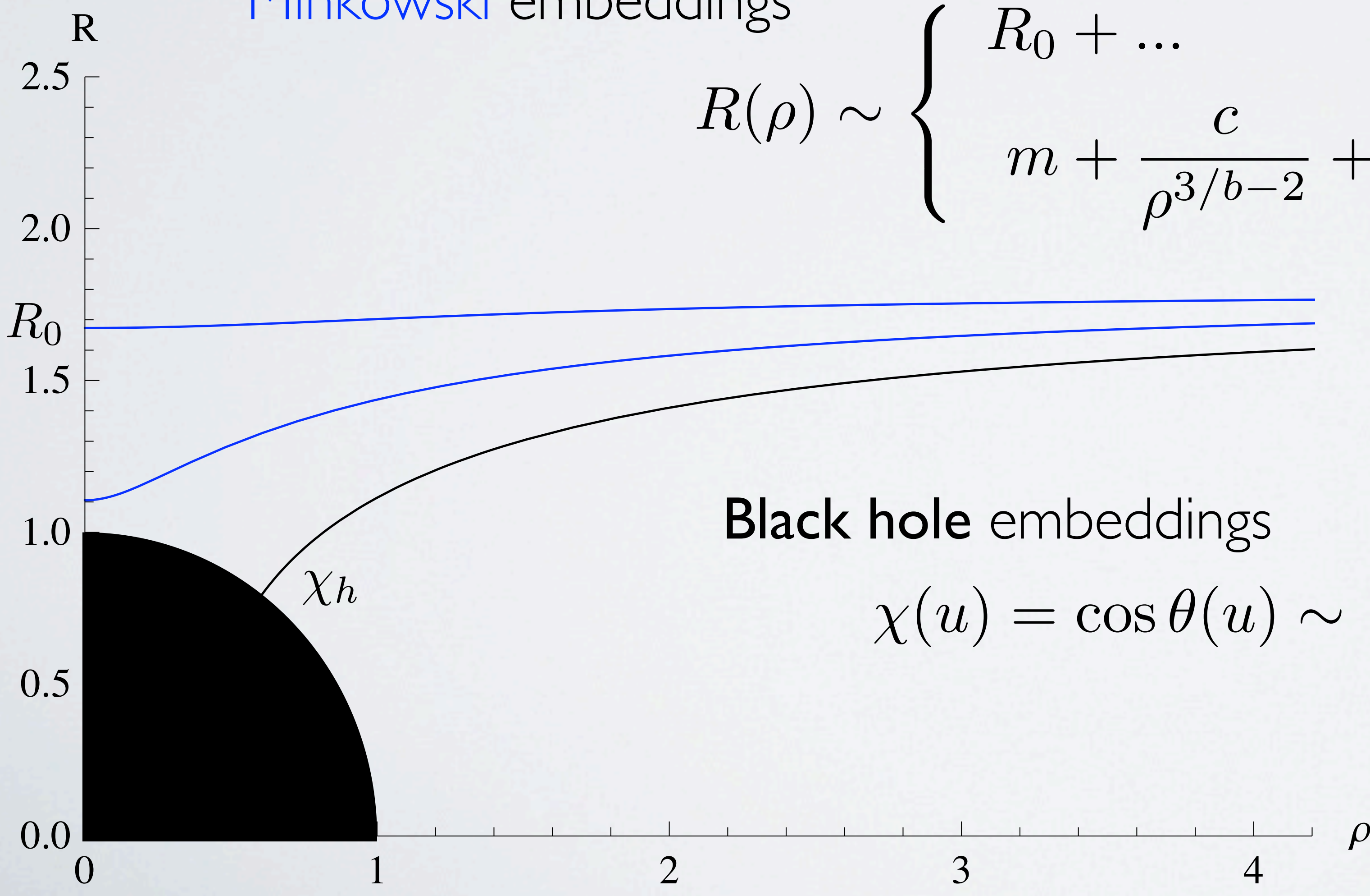
$$\frac{dr^2}{r^2 h(r)} + \frac{d\theta^2}{b^2} = \frac{1}{u^2 b^2}\left(du^2 + u^2 d\theta^2\right) \quad \rightarrow \quad u = \left[\left(\frac{r}{r_h}\right)^{\frac{3}{2}} + \left(\left(\frac{r}{r_h}\right)^3 - 1\right)^{\frac{1}{2}}\right]^{\frac{2b}{3}}$$

$$R = u\cos\theta \quad ; \quad \rho = u\sin\theta$$

embeddings

Minkowski embeddings

$$R(\rho) \sim \begin{cases} R_0 + \ldots & (\rho \to 0) \\ m + \dfrac{c}{\rho^{3/b-2}} + \ldots & (\rho \to \infty) \end{cases}$$



Black hole embeddings

$$\chi(u) = \cos\theta(u) \sim \begin{cases} \chi_h + \ldots & (u \to u_h) \\ \dfrac{m}{u} + \dfrac{c}{u^{3/b-1}} + \ldots & (u \to \infty) \end{cases}$$

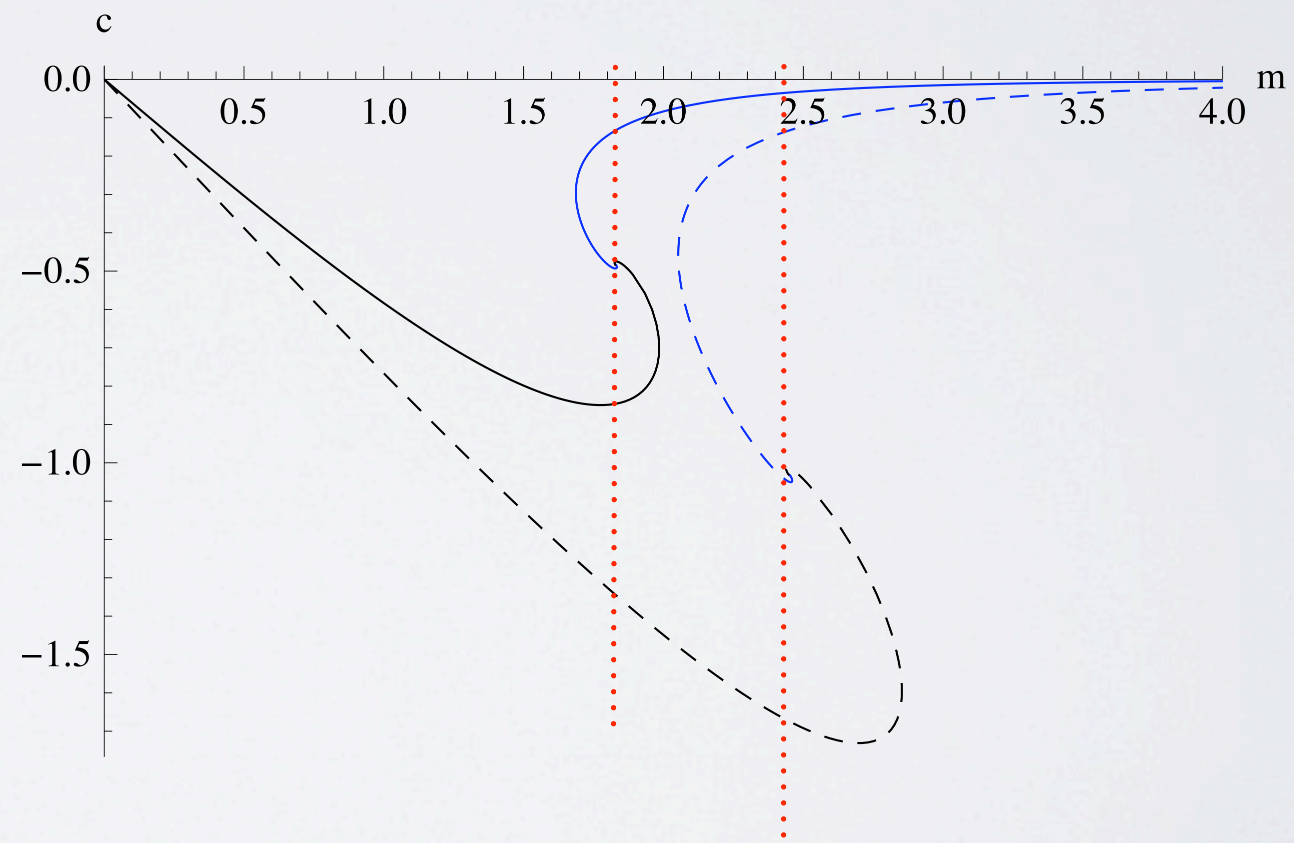
Probes on the ABJM flavored thermal background

mass and condensate

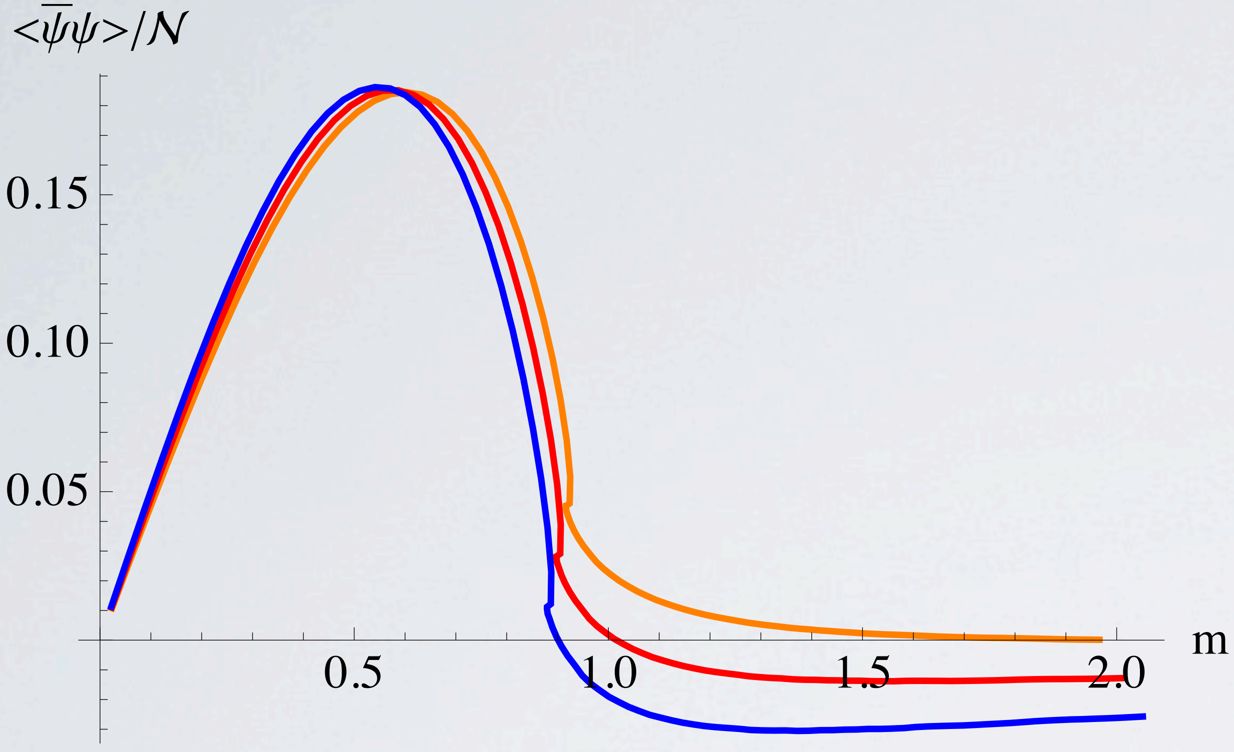
$$M_q = \frac{2^{1/3}\pi}{3}\sqrt{2\lambda}\,\sigma\,T\,m^{1/b}$$

$$\langle \mathcal{O}_m \rangle = -\frac{2^{\frac{2}{3}}\,\pi}{9}\;\frac{(3-2b)\,b}{q}\;\sigma\;N\,T^2\,c$$

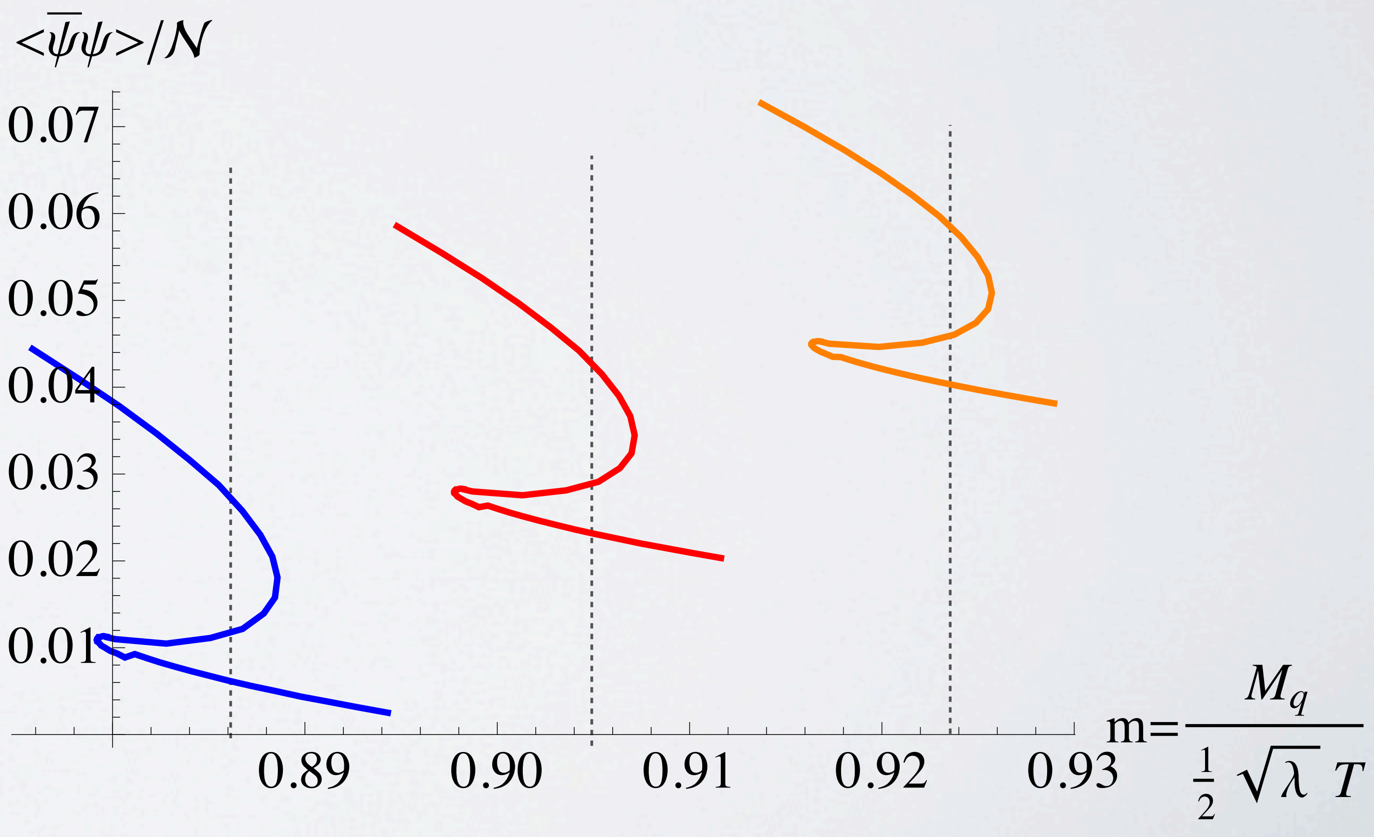
Numerical $c = c(m)$ →



D7 massive probes: condensate



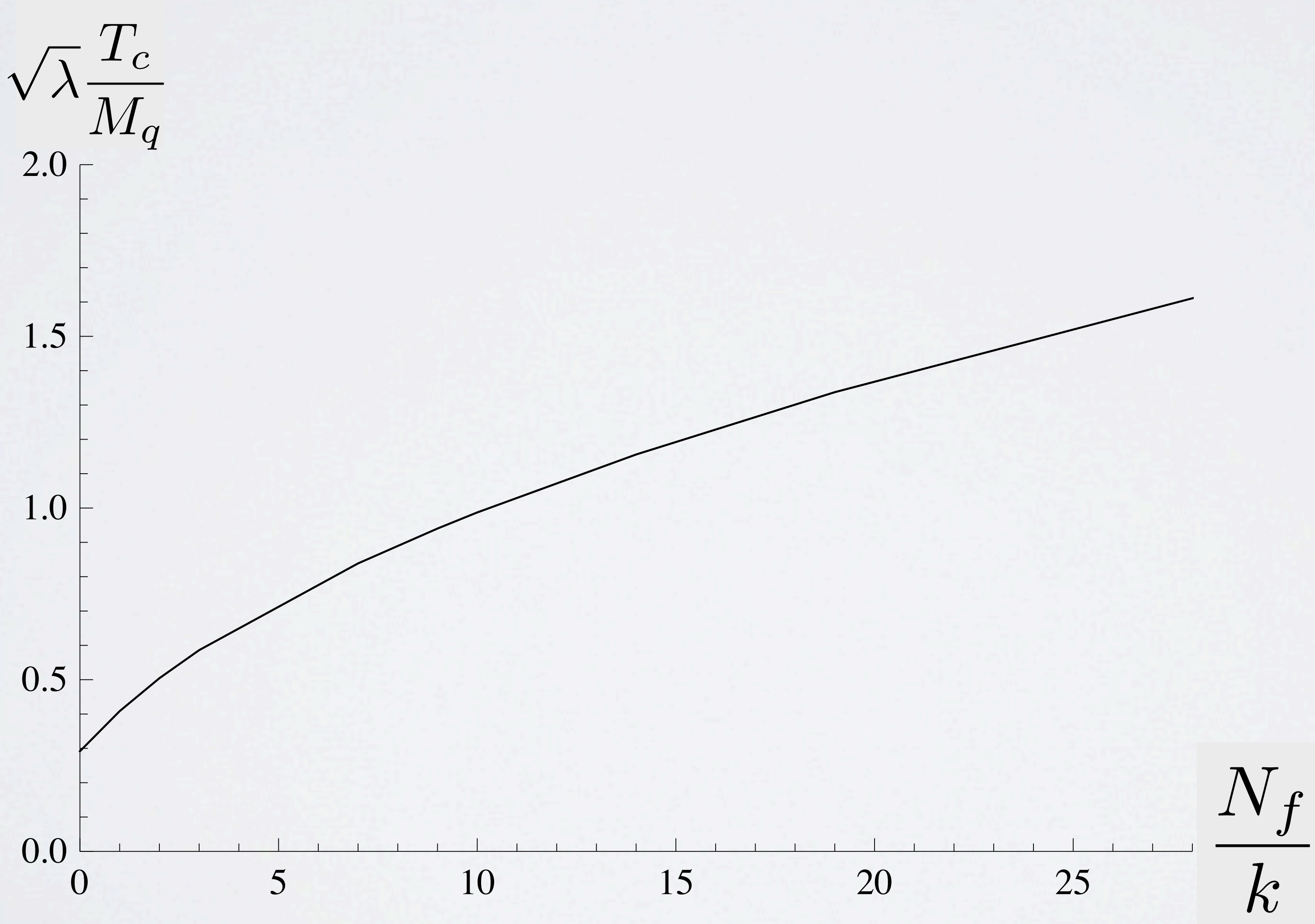
$\epsilon_h = $ 0, 0.2 , 0.4



Probes on the ABJM flavored thermal background

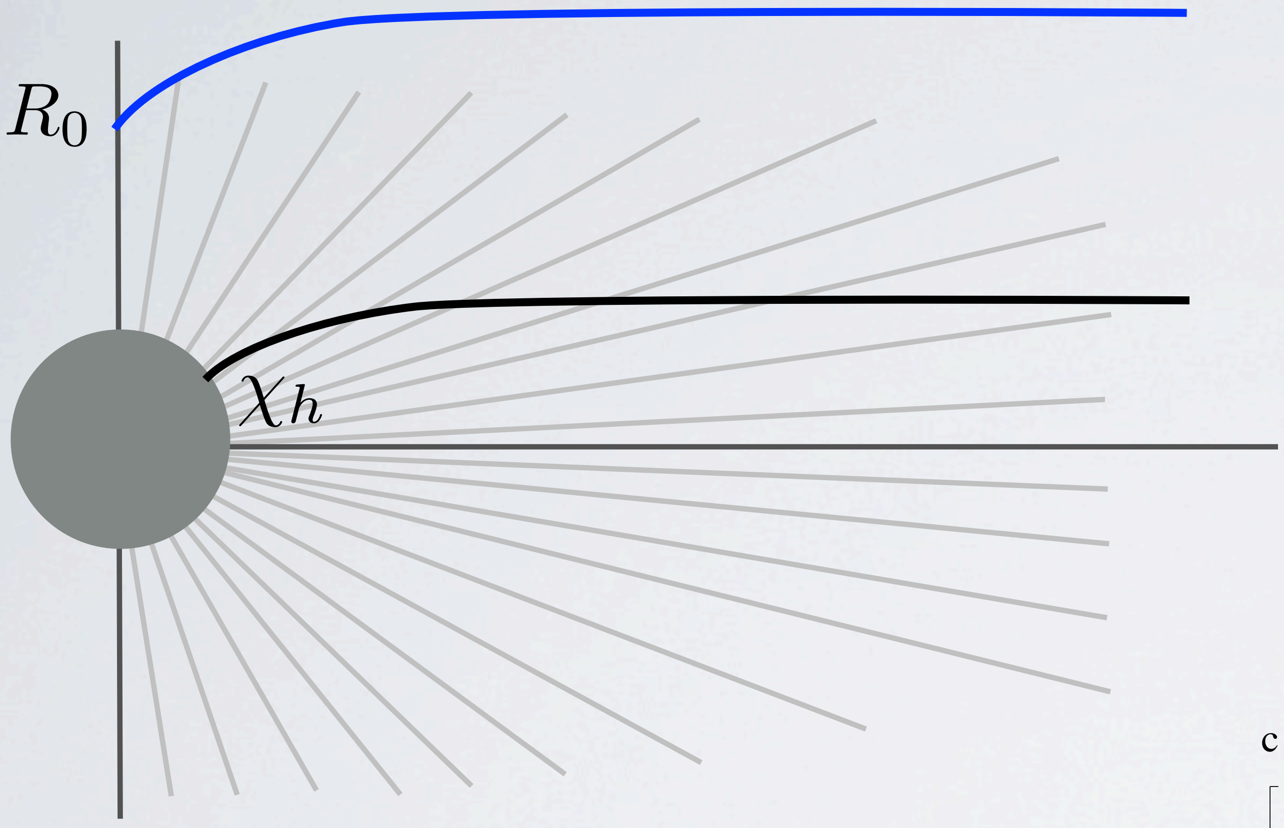
meson melting phase transition temperature increases

$$\sqrt{\lambda}\frac{T_c}{M_q} \sim \frac{1}{m_c^{1/b}\,\sigma} \sim \sqrt{N_f}$$



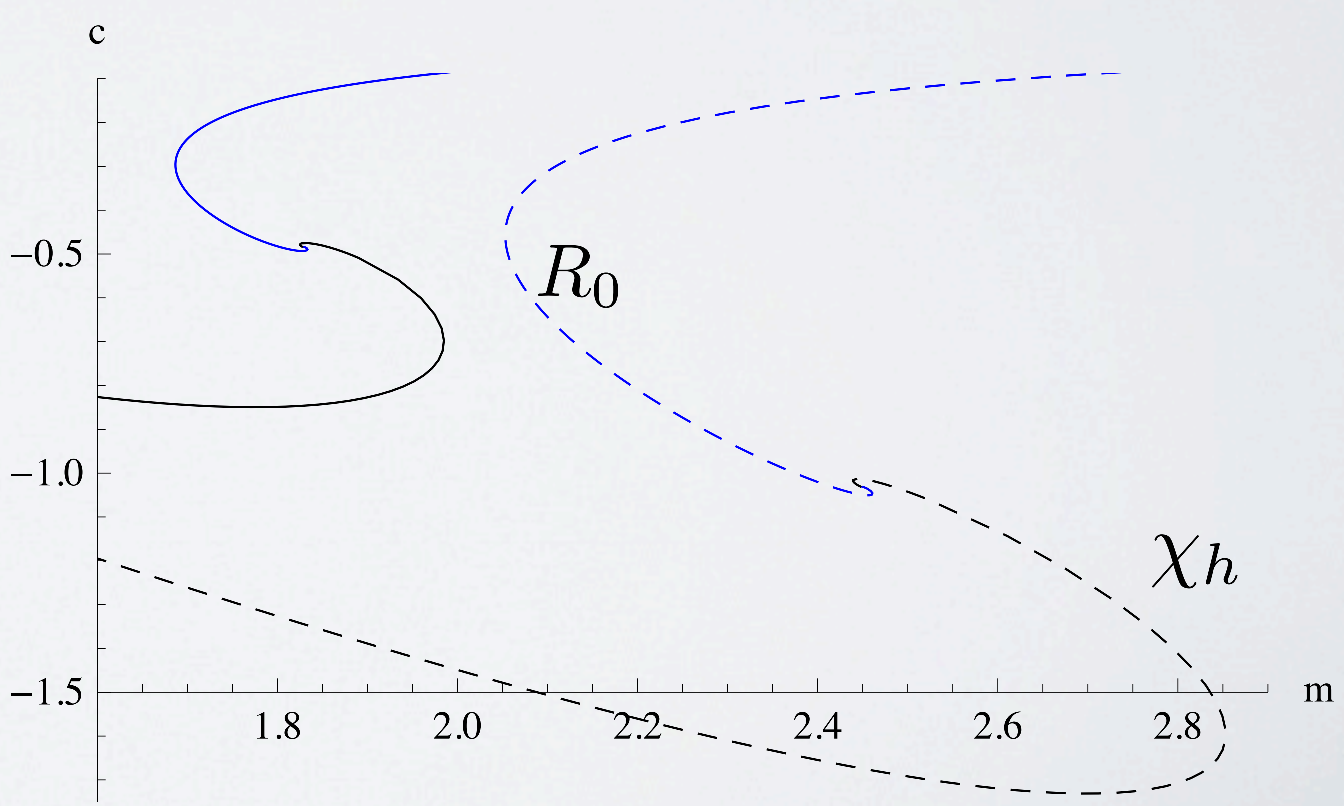
Probes on the ABJM flavored thermal background

critical embedding



- self-similar behavior
- c and m oscillate

Mateos, Myers & Thomson hep-th/0701132



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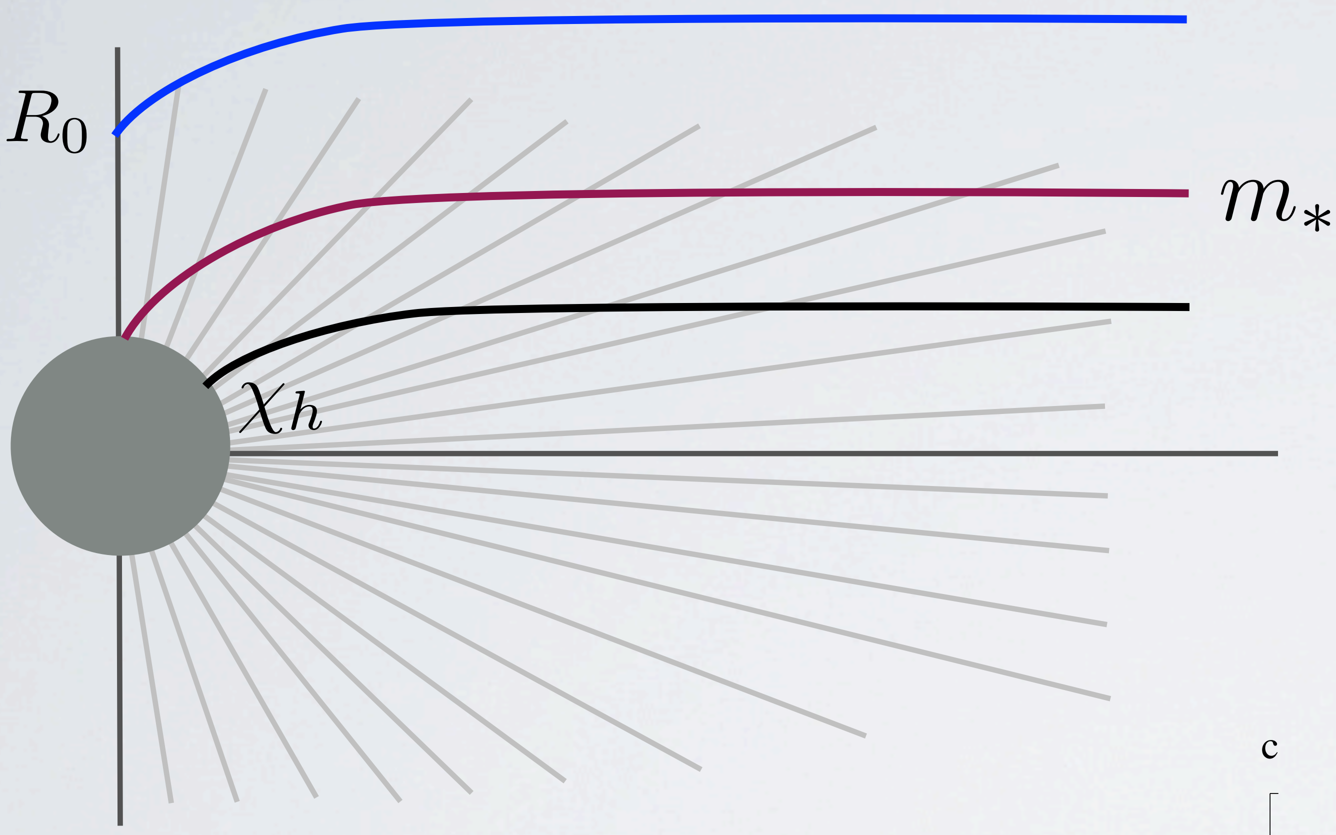

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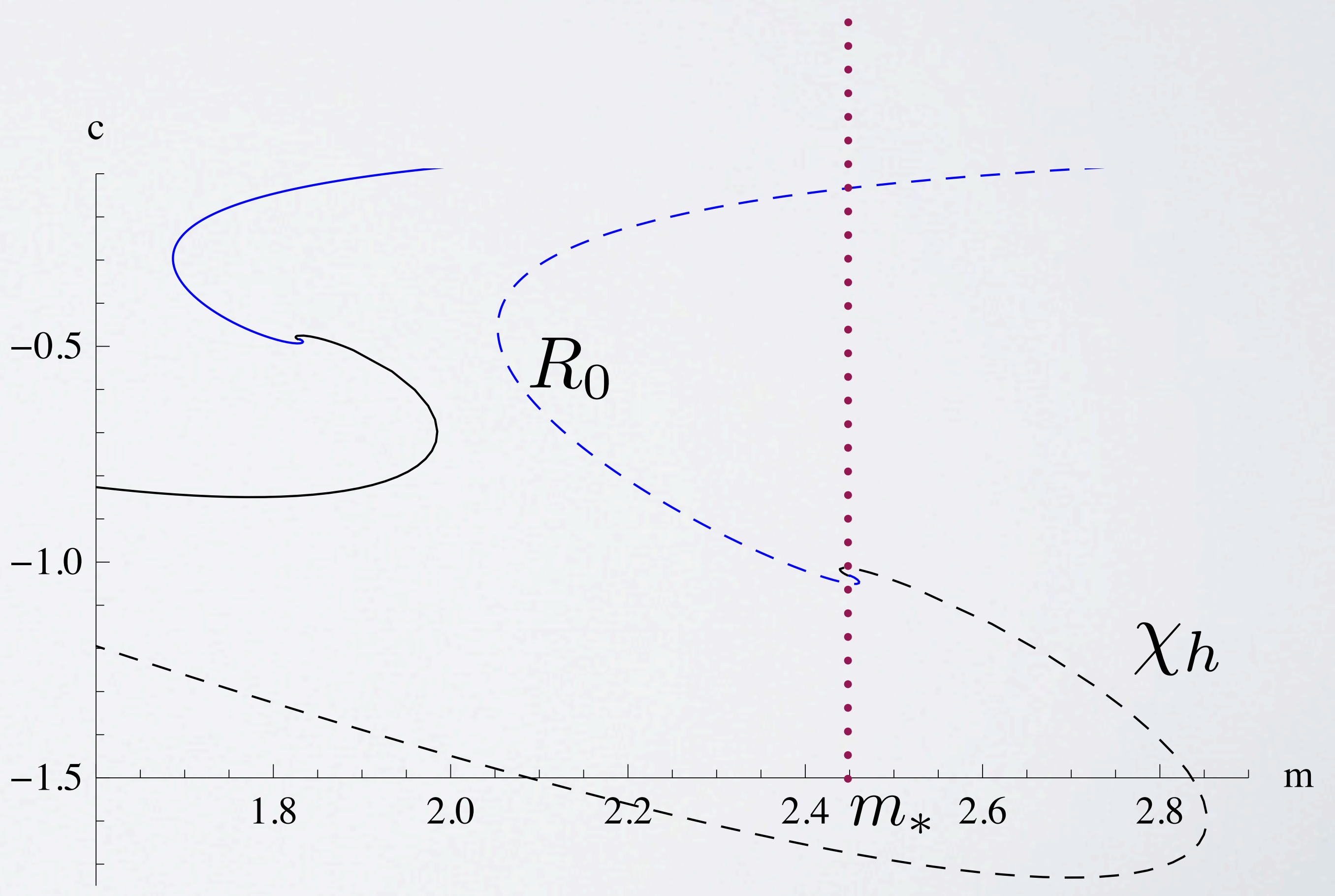
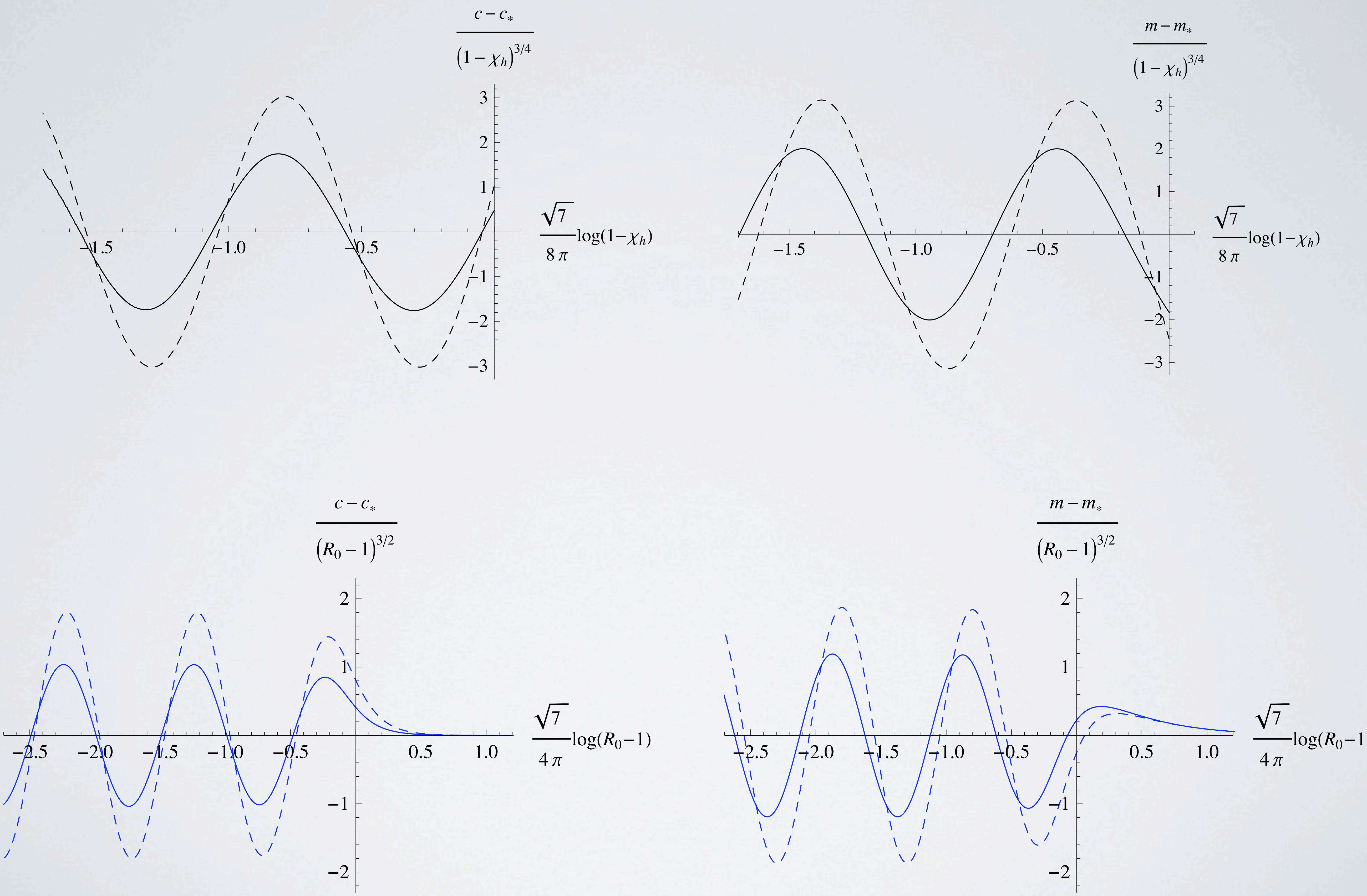


Probes on the ABJM flavored thermal background

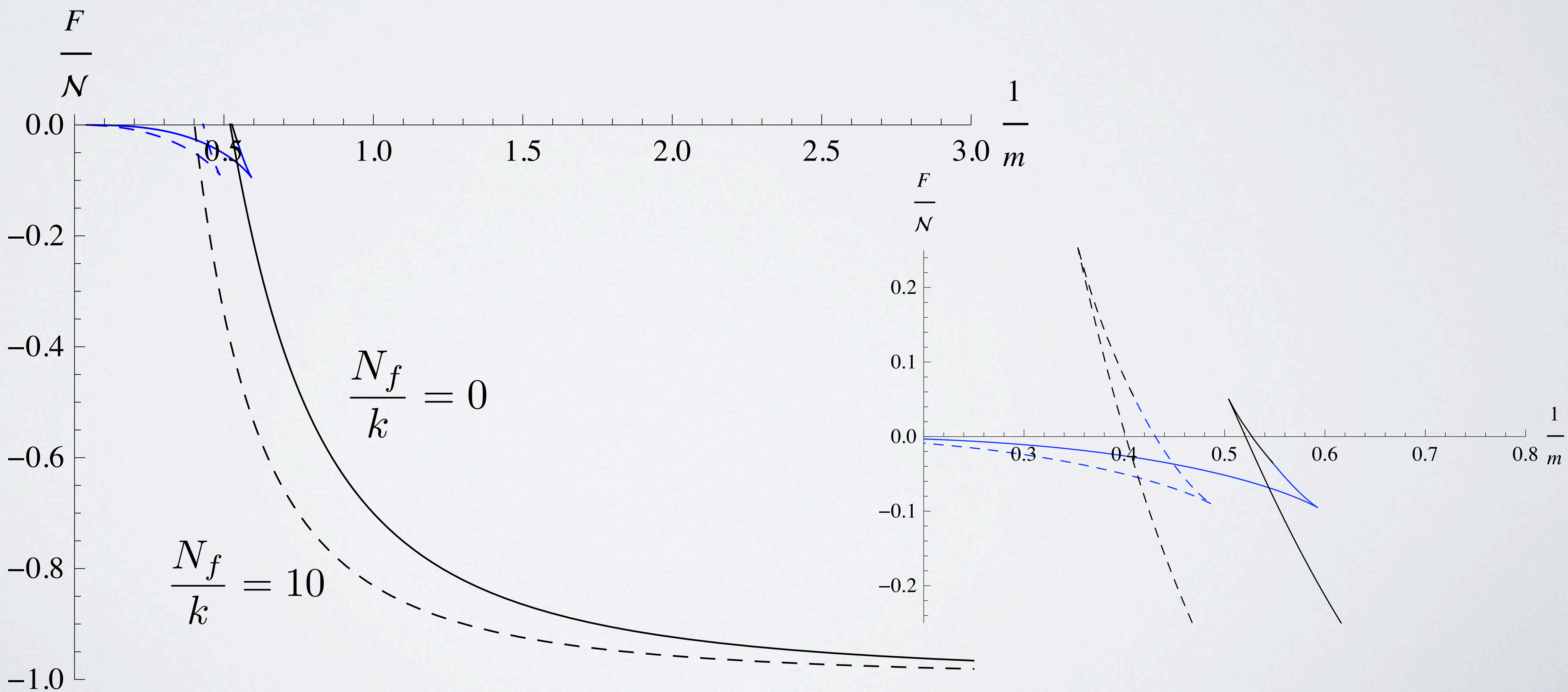


Probes on the ABJM flavored thermal background

free energy density

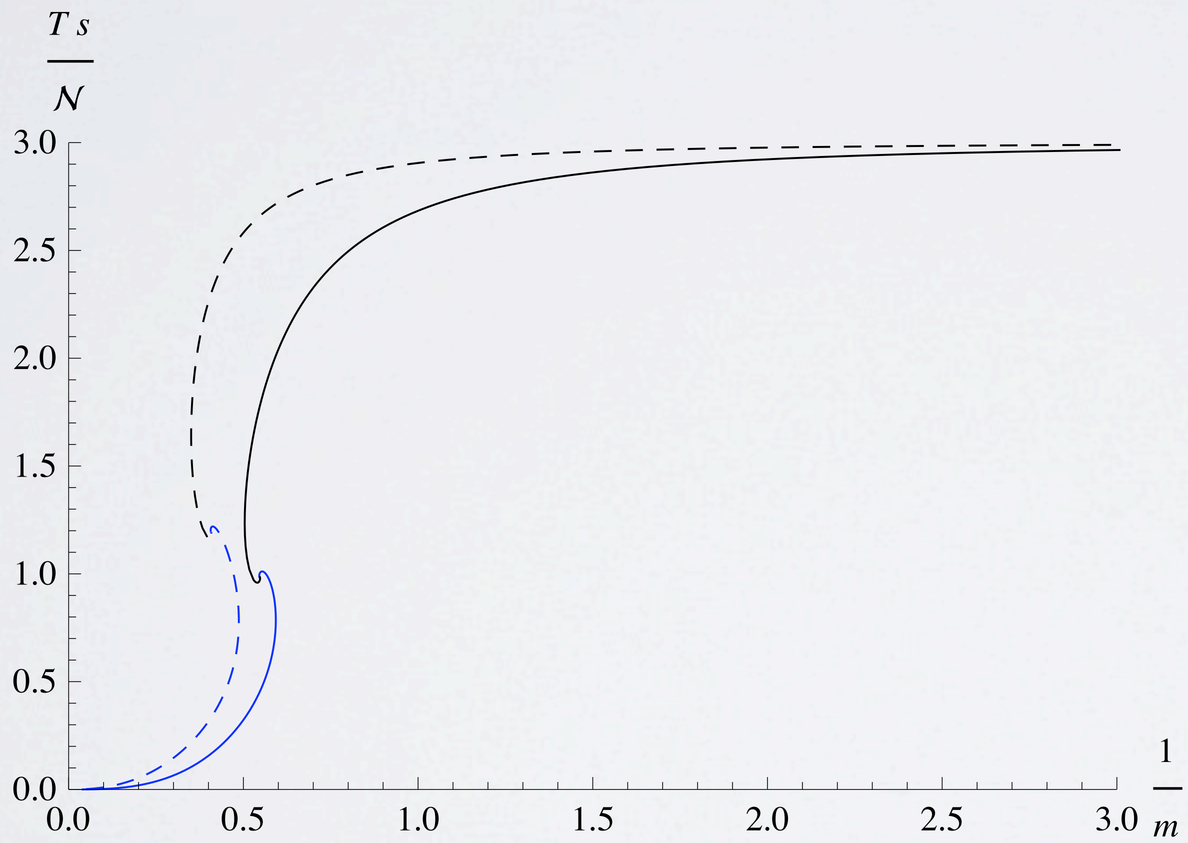
$$F = \frac{S_E}{\int d^3x} = \mathcal{N} \int d^3x \left[\frac{4b}{r_h^3} \int_{r_{min}(m)}^{\infty} dr\, r^2 \sin\theta \left(\sqrt{1 + \frac{r^2 h(r)}{b^2} \dot{\theta}^2} - \sin\theta - \frac{r h(r)}{b} \cos\theta\, \dot{\theta} \right) - 1 \right]$$

$$= \mathcal{N} \Big(\mathcal{G}(m) - 1 \Big) \quad = \left(\frac{2\sqrt{2}\pi^2}{27}\, N\sqrt{\lambda}\, T^3 \right) \zeta\!\left(\frac{N_f}{k} \right) \Big(\mathcal{G}(m) - 1 \Big)$$

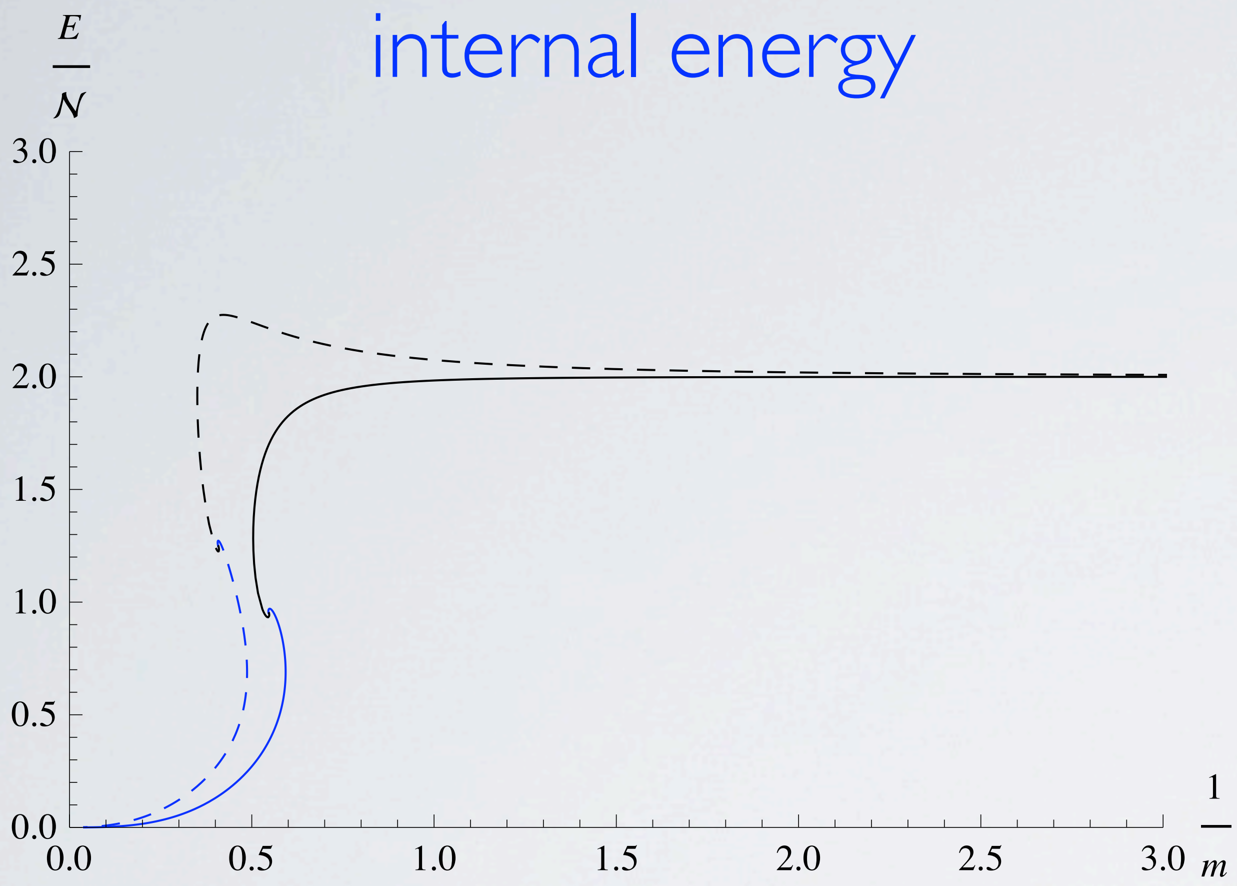


entropy

$$s = -\frac{\partial F}{\partial T} = -\frac{F}{\mathcal{N}}\frac{\partial \mathcal{N}}{\partial T} - \mathcal{N}\frac{\partial}{\partial T}\left(\frac{F}{\mathcal{N}}\right) = -\frac{3}{T}\frac{F}{\mathcal{N}} - \frac{m}{T}(3-2b)c$$

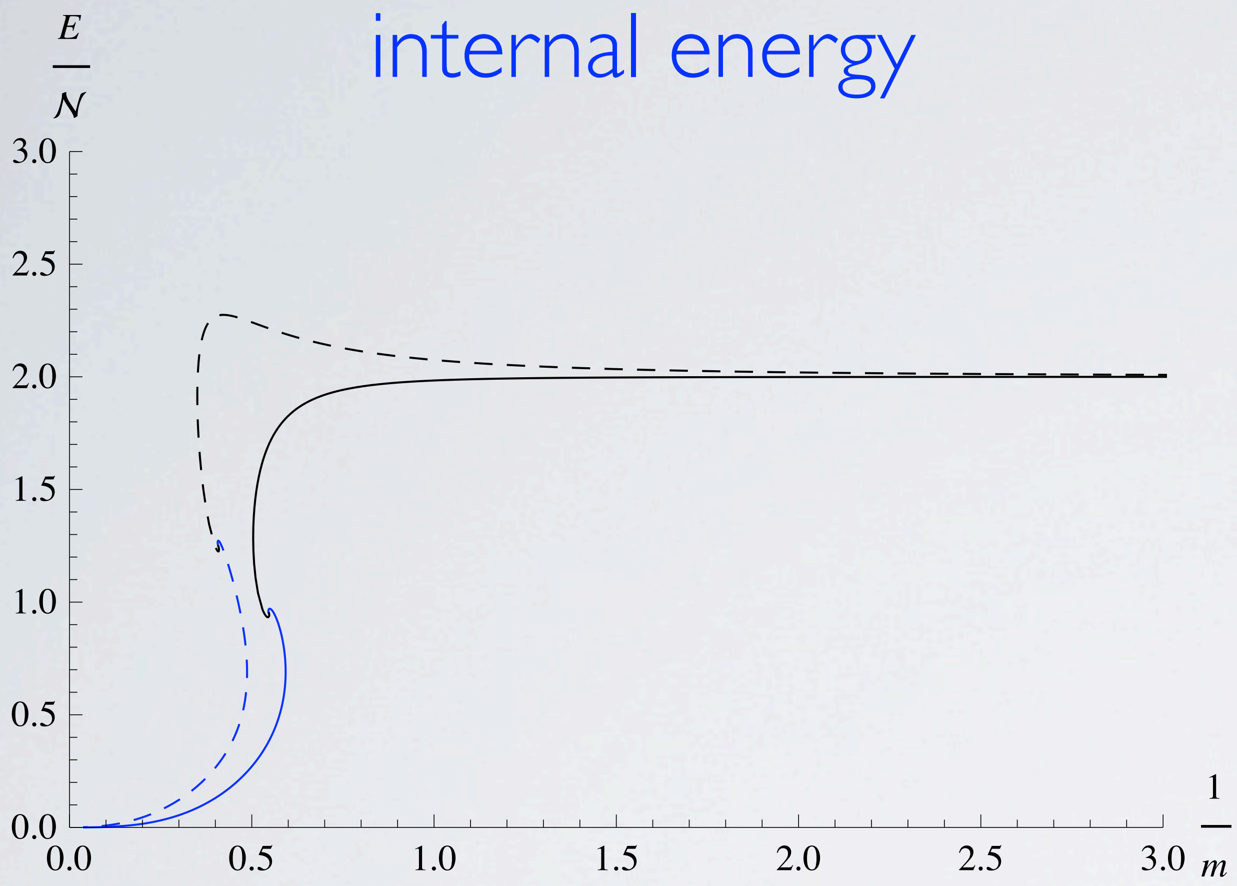


internal energy



$$E = -2F - \mathcal{N}(3-2b)\,c\,m$$

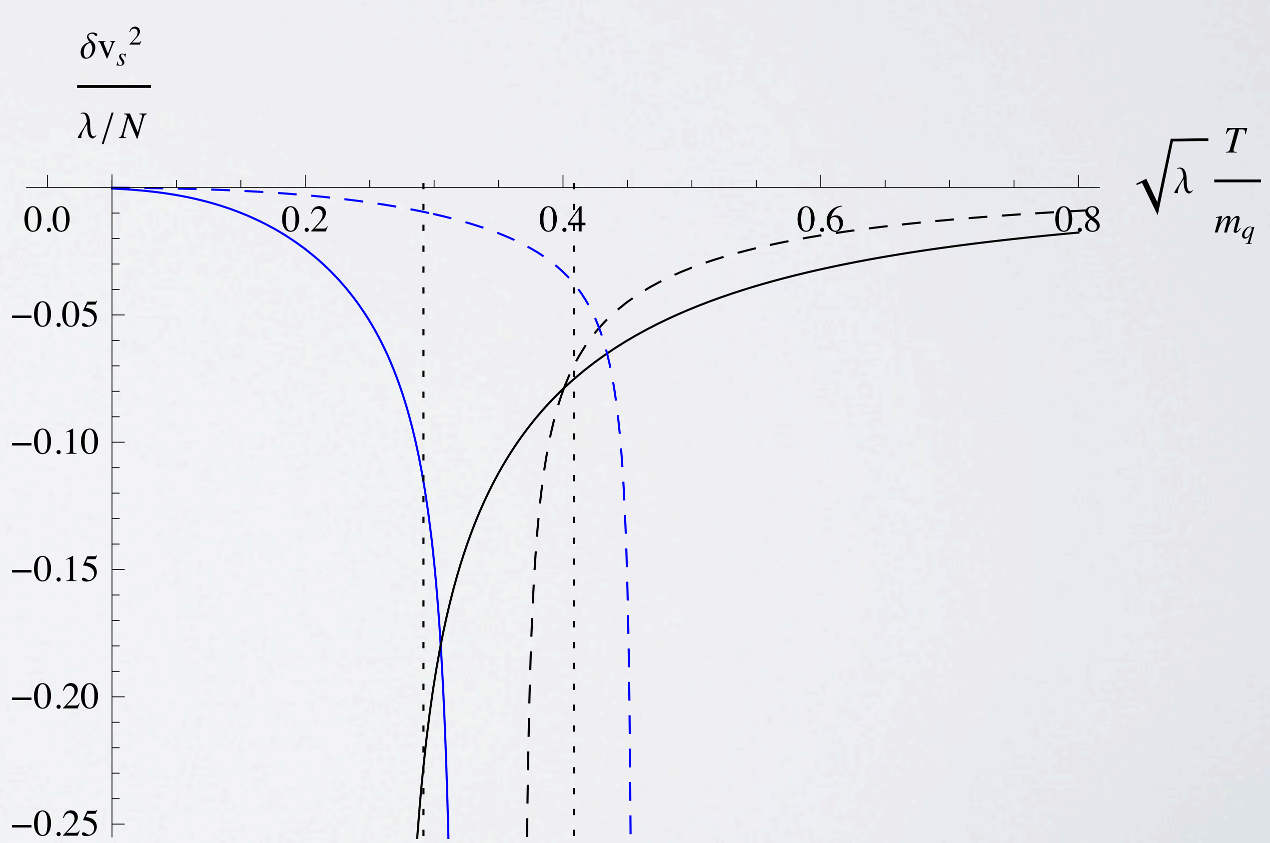
internal energy



$$E = -2F - \mathcal{N}(3-2b)\, c\, m$$

speed of sound

$$v_s^2 = \frac{\partial P}{\partial E} = \frac{s}{c_v} = \frac{1}{2} + \delta v_s^2$$



Conclusions

- the flavored ABJM theory dual is a conformal field theory
- the thermal deformation is analytic and fully under control.
- we have added massive probe flavors to the theory and examined the thermodynamics
- the scheme dependence can be fixed by demanding a compatibility of the UV and IR behavior of the probe brane
- The flavors introduce cuantitative shifts but no cualitative change in the picture. For example T_c rises like $\sqrt{N_f}$

Further work

- add chemical potential to the probe brane and study transport properties (conductivity etc.)
- constructing the flavored thermal ABJM theory with chemical potential (dilaton stops being constant, and H_3 enters the game)
- adding B field, could study magnetic catalysis
- smearing massive flavors in ABJM at zero T
- ....

THANK YOU FOR LISTENING