Non-Abelian T-duality in supergravity and the AdS/CFT correspondence

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Based on:

- K Sfetsos, DCT: Nucl.Phys. **B846** (2011) 21-42, arXiv:1012.1320 [hep-th].
- G. Itsios, C. Núñez, K. Sfetsos & DCT, arXiv:1212.4840 [hep-th], arXiv:1301.6755 [hep-th], ongoing.
- A. Barranco, J. Gaillard, N. Macpherson, C. Núñez, DCT forthcoming

General settings and context – Motivation

Generating solutions in the context of AdS/CFT

- There exist a wide variety of solution generating techniques in supergravity.
- These techniques provide a powerful tool:
  - Matching β-deformations in gauge theory to the TST transformation in gravity.
  - U-duality transformations.
  - Fermionic T-duality & regular T-duality to explain dual superconformal symmetry at strong coupling.
- Some are well understood as exact string symmetry ($O(d, d; \mathbb{Z})$ dualities), others, like Fermionic T-duality, are probably not - but evidently can be very useful.

Strings and T-dualities in curved backgrounds

- Abelian T-duality is well understood and explored.
- Non-Abelian T-duality? [Fridling-Jevicki 84, Fradkin-Tseytlin-85], [de la Ossa-Quevedo 93, KS-94, Alvarez–Alvarez-Gaume–Lozano 94...]
  Not as well understood and not likely to be an exact symmetry.
- Major advances in the last two years:
  - Understanding how it acts on RR fluxes.

Generating solutions in type-II supergravities

- Reducing symmetry/supersymmetry (generically) controllably.
- New examples and features in AdS/CFT
  - It caprures generic features of $\mathcal{N} = 2$ superconformal field theories corresponding to quiver (moose-type) diagrams.
  - New (non-singular) flows with $\mathcal{N} = 1$ supersymmetry.

Outline

- Constructing the backgrounds: General strategy
  - The tranformation of the NS fields.
  - The induced Lorentz tranformation.
  - The tranformation of the RR fluxes.
- Type-II backgrounds with $SO(4)$ isometry:
  - In Principal Chiral Models (intermediate step).
  - The general transformation rules.
  - Supersymmetry.
- Non-Abelian T-duality and AdS/CFT: The road map.
  - $\mathcal{N} = 2$ backgrounds, from D3-brane near horizon, M-theory lift and interpretation.
  - $\mathcal{N} = 1$ backgrounds from conifolds, Conformal and non-conformal cases, fate of charges (central and brane).
- Concluding remarks.

Constructing the backgrounds: General strategy

Consider a type-II background with an isometry group G.

The tranformation of NS fields

Write the 2-dim σ model action for the NS fields

$$S(X) = \int d^2\sigma (G_{\mu\nu} + B_{\mu\nu}) \partial_+ X^\mu \partial_- X^\nu \ , \quad \text{also a } \Phi \ .$$

- Gauge $H \subset G$: gauge fields $A_\pm \in \mathcal{L}(H)$ and $\partial \to \nabla$,

$$S_{\text{T-dual}}(X, v, A_\pm) = S_{\rm g}(X, A_\pm) - i \int d^2\sigma \underbrace{\mathrm{Tr}(vF_{+-})}_{\text{Lagr.mult.}} \ , \quad S_{\rm g}(X, 0) = S(X) \ .$$

  Invariant under the transformation of the X^μ's and

$$A_\pm \to \Lambda^{-1}(A_\pm - \partial_\pm)\Lambda \ , \quad v \to \Lambda^{-1} v \Lambda \ .$$

- Gauge fix $\dim(H)$ parameters in X^μ and in the v's.

The gauge fields $A_{\pm}$ appear quadratically and non-dynamically. Integrating them out (in a [Buscher 87]–like procedure) gives the T-dual σ-model.

- Some of the Langrange multipliers become σ-model variables.
- The dilaton transformation is a 1-loop quantum effect.
- This procedure is a canonical transformation in phase space.

Induced Lorentz transformation

Using the frame of the original background construct

$$e^a_\pm = \epsilon^a_\mu \partial_\pm X^\mu \ ,$$

- Left and right world-sheet derivatives transform differently and give rise to different T-dual frames $\tilde{e}^a_{\mu\pm}$.
- A Lorentz transformation relates the frames in the vector rep

$$\tilde{e}^a_{\mu+} = \Lambda^a{}_b \tilde{e}^a_{\mu-} \ .$$

- Then compute the Lorentz transform in the spinor rep using

$$\Omega^{-1}\Gamma^a\Omega = \Lambda^a{}_b\Gamma^b \ ,$$

which preserves the Clifford algebra $\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}$.

Transformation of Ramond fluxes

Use the democratic formulation of type-II [Bergshoeff, Kallosh, Ortin, Roest, Van Proeyen 01]; All F_p's $p = 0, 1, \ldots, 9$ appear.
For Minkowski signature

$$\text{Reduction conditions}: \quad F_p = (-1)^{[\frac{p}{2}]} \star F_{10-p} \ ,$$

reduce to the right degrees of freedom.

- We combine the forms RR-forms into a bi-spinor as

$$\text{IIB}: \ P = \frac{e^{\Phi}}{2} \sum_{n=0}^{4} \frac{\not{F}_{2n+1}}{(2n+1)!} \ , \qquad \text{IIA}: \ \hat{P} = \frac{\hat{e}^{\Phi}}{2} \sum_{n=0}^{5} \frac{\hat{\not{F}}_{2n}}{(2n)!} \ ,$$

  where $\not{F}_p = \Gamma_{\mu_1 \ldots m_p} F_p^{\mu_1 \mu_2 \ldots m_p}$.
- The above fluxes transform according to

$$\boxed{\hat{P} = P \Omega^{-1}} \ ,$$

  using the pure spinor superstring formalism of [Berkovits 00].

General comments on the T-dual background

Abelian T-duality:

- For d successive ones

$$\Lambda = \mathrm{diag}(\underbrace{-\mathbb{1}_d}_{\text{Duality}}, \underbrace{\mathbb{1}_{10-d}}_{\text{Extras}}) \ .$$

  The spinorial representation is [Hassan 99]

$$\Omega = \prod_{i=1}^{d} (\Gamma^i \Gamma_{11}) \ ,$$

  where $\Gamma_{11} = \Gamma^0\Gamma^1 \cdots \Gamma^9$, obeying $\Gamma_{11}^2 = \mathbb{1}$.

- A single Abelian factor changes the chirality form type-IIA to type IIB and vice versa.

Non-Abelian T-duality:

- $\dim(G)$ =odd: Ω starts with Γ_{11} followed by a linear combination of products of an odd number of Γ-matrices.
- $\dim(G)$ =even: Then Γ_{11} is omitted and the linear combination has products of an even number of Γ-matrices.
- Hence, chirality might change or stay the same.
- Massive IIA solutions, when $\dim(G)$ equals the rank of a form.
- Equations of motion should be automatically satisfied.
- Supersymmetry:
  - This may be reduced to a fraction of the original.
  - Later we develop a criterion based on the action of the Lie-Lorrentz or Kosmann derivative on the Killing spinors of the original background.

Type-II backgrounds with $SO(4)$ isometry

Original background with $SO(4)$ isometry

We consider (for concreteness) type-IIB backgrounds:

- The NS fields are

$$\boxed{ds^2 = ds^2(M_7) + e^{2A} ds^2(S^3) \ , \quad B \ , \quad \Phi} \ .$$

- The factor A, the 2-form B and Φ may depend on M_7.
- The RR fluxes respecting the symmetry of the round S^3 are

$$\begin{aligned} F_5 &= G_2 \wedge \mathrm{Vol}(S^3) - e^{-3A} \star_7 G_2 \ , \\ F_3 &= G_3 - m\mathrm{Vol}(S^3) \ , \\ F_1 &= G_1 \ . \end{aligned}$$

  The G_i's are lying entirely on M_7.
- The T-duality will be performed w.r.t. the $SU(2)_L$ subgroup of $SO(4) \sim SU(2)_L \times SU(2)_R$.

Non-Abelian T-duality in Principal Chiral Models (PCM)

- The 2-dim σ-model action for a PCM is

$$S(g) = -\int d^2\sigma \, \mathrm{Tr}(g^{-1}\partial_- g g^{-1}\partial_+ g) \ , \qquad g \in G \ .$$

- It has an $G_L \times G_R$ global symmetry.
- Gauge the symmetry G_L by introducing gauge fields $A_\pm$.
- The corresponding action is

$$S_{\mathrm{nonab}} = -\int d^2\sigma \, \mathrm{Tr}(g^{-1}D_- g g^{-1}D_+ g) + \overbrace{i\mathrm{Tr}(vF_{+-})}^{\text{Lagr. Multip.}} \ ,$$

with $D_\pm g = \partial_\pm g - A_\pm g$ and F_{+-} the field strength.
- Invariance under G_L (local) $\times G_R$ (global).

- Gauge fix $g = \mathbb{1}$and integrate out the $A_\pm$'s

$$\boxed{S_{\text{T--dual}}(v) = \int d^2\sigma\, \partial_+ v_a (M^{-1})^{ab} \partial_- v_b}\ ,$$

where

$$M_{ab} = \delta_{ab} + f_{ab}\ , \qquad f_{ab} \equiv f_{ab}{}^c v_c\ .$$

Contribution to the Dilaton: $\Phi = -\frac{1}{2} \ln \det(M)$.

- Original isometry $G_L \times G_R$ is broken to G_R.
- Introduce coordinates $X^\mu \in g$ and the left-invariant forms L^a.
- Then, the relation of world-sheet derivatives is

$$\partial_+ v_a = M_{ba} L^b_\mu \partial_+ X^\mu\ , \qquad \partial_- v_a = -M_{ab} L^b_\mu \partial_- X^\mu\ .$$

- These define two frames related by the Lorentz transformation

$$\boxed{\Lambda_{ab} = -(M M^{-1\,T})_{ab}}\ .$$

Then

$$\Omega = \exp\left(\frac{1}{2} f_{ab} \Gamma^{ab}\right) \prod_{i=1}^{\dim(G)} (\Gamma_{11}\Gamma_i)\ .$$

T-dual background w.r.t. $SU(2)$; NS-sector

- We specialize to the case of a PCM for $SU(2)$ and
  - use spherical coordinates and
  - take into account e^{2A}.
- The NS-sector fields are given by

$$
\begin{aligned}
d\hat{s}^2 &= ds^2(M_7) + e^{-2A}dr^2 + \frac{r^2 e^{2A}}{r^2 + e^{4A}} ds^2(S^2) \ , \\
\hat{B} &= B + \frac{r^3}{r^2 + e^{4A}} \mathrm{Vol}(S^2) \ , \\
e^{-2\hat{\Phi}} &= e^{-2\Phi} \ e^{2A}(r^2 + e^{4A}) \ ,
\end{aligned}
$$

- The round S^2 sphere appears; Manifest $SU(2)$ symmetry.

T-dual background w.r.t. $SU(2)$; RR-sector

- The Lorentz transformation matrix Ω acting on the spinors is

$$\boxed{\Omega = \Gamma_{11} \frac{e^{2A}\Gamma_{789} + \mathbf{v}\cdot\Gamma}{\sqrt{r^2 + e^{4A}}}} \quad \Longrightarrow \quad \Omega^{-1} = \Gamma_{11} \frac{e^{2A}\Gamma_{789} - \mathbf{v}\cdot\Gamma}{\sqrt{r^2 + e^{4A}}} \ .$$

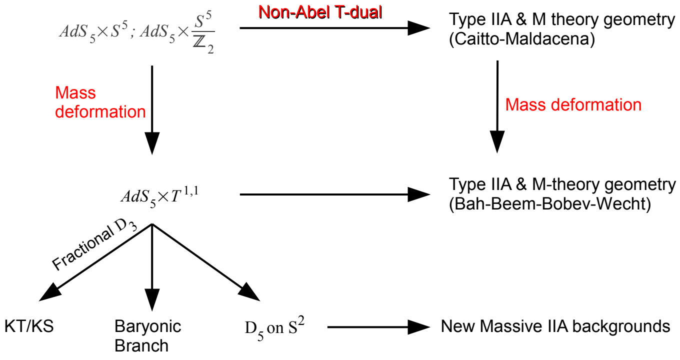
- The isometry group is 3-dim and has "legs" along F_3.
  Hence we expect a massive IIA solution.
- Indeed, the massive IIA fluxes are

$$\begin{aligned}
\hat{F}_0 &= m \ , \\
\hat{F}_2 &= \frac{mr^3}{r^2 + e^{4A}} \mathrm{Vol}(S^2) + rdr \wedge G_1 - G_2 \ , \\
\hat{F}_4 &= \frac{r^2 e^{4A}}{r^2 + e^{4A}} G_1 \wedge dr \wedge \mathrm{Vol}(S^2) - \frac{r^3}{r^2 + e^{4A}} G_2 \wedge \mathrm{Vol}(S^2) \\
&\quad + rdr \wedge G_3 + e^{3A} \star_7 G_3 \ .
\end{aligned}$$

Supersymmetry

- Is any/how much Supersymmetry preserved? what conditions? *Preserved susy when Lie-Lorentz or Kosmann derivative vanishes*
$$\boxed{\mathcal{L}_\xi \epsilon = \xi^\mu D_\mu \epsilon + \frac{1}{4} D_\mu \xi_\nu \Gamma^{\mu\nu} \epsilon} \;,$$
- Is there a mapping of the Killing spinor eqs/spinors?
$$\boxed{\hat{\epsilon}_1 = \Omega \epsilon_1 \;, \quad \hat{\epsilon}_2 = \epsilon_2}$$
- The fraction of supersymmetry preserved is determined by the independent extra conditions needed to make the Kosmann derivative vanish.

Non-Abelian T-duality and AdS/CFT: The Road Map



$\mathcal{N} = 2$ *backgrounds*

D3 near horizon

- We write the S^5 of the original type-IIB background as

$$ds^2(\mathrm{S}^5) = 4(d\theta^2 + \sin^2\theta \; d\phi^2) + \cos^2\theta \; ds^2(\mathrm{S}^3) \; .$$

- The NS-part of the T-dual background has

$$ds^2 = ds^2(\mathrm{AdS}_5) + 4(d\theta^2 + \sin^2\theta d\phi^2) + \frac{dr^2}{\cos^2\theta} + \frac{r^2\cos^2\theta}{\cos^4\theta + r^2} d\Omega_2^2 \, ,$$

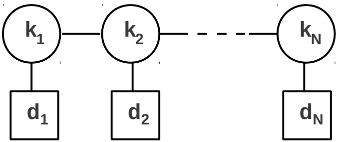
  a dilaton and an NS 2-form, as well as

$$F_2 = -8\cos^3\theta\sin\theta \; d\theta\wedge d\phi \; ,$$
$$F_4 = -8\frac{r^3\cos^3\theta\sin\theta}{\cos^4\theta + r^2} \; d\theta\wedge d\phi\wedge \mathrm{Vol}(S^2) \; ,$$

- A type-IIA solution with $SO(4,2)\times SU(2)\times U(1)$ symmetry.
- Solution is singular at $\theta = \pi/2$ and 1/2 supersymmetric.

M-theory lift and gauge theory interpretation

$\mathcal{N} = 2$ superconf. quiver theories using D4,D6 and NS5-branes.



- Notation:
  Circular: $SU(k_n)$ gauge group. Square: $SU(d_n)$ global (d_n fundamentals), Horizontal lines: bi-fundamental $(\mathbf{k_{n-1}}, \bar{\mathbf{k}}_\mathbf{n})$.
- They admit 11-dim dual geometries containing AdS_5 factors and possessing $SU(2) \times U(1)$ isometry [Gaiotto-Maldacena 09],
- The details of the solution are fed up using solutions of

$$(\partial_x^2 + \partial_y^2)\Psi + \partial_z^2 e^{\Psi} = 0 \ ,$$

the continual Toda eq. [Boyer-Finley 82, Saveliev 89].

The relation to the quiver theories is very clear in the subclass having an additional $U(1)$ symmetry:

- Corresponds, via dim reduction, to a type-IIA solution.
- There is a mapping of the continual Toda to the Laplace eq.
- Electrostatic problem: Find the potential $V(\rho, \eta)$ of a semi-infinite charged line with density $\lambda(\eta)$.

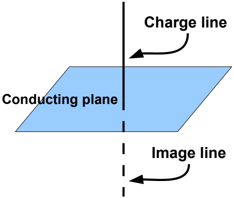


Figure: A charged line perpendicular to an infinite conducting plane.

- $\lambda(\eta)$ is composed of linear segments with integer slopes.
- The rank of the gauge group $k_n = \lambda(n)$, with $n = 1, 2, \ldots$. Changes in slope correspond to extra fundamentals d_n.

The non-Abelian T-dual captures generic features:

- Our solution can be cast into that form with

$$\lambda(\eta) = \eta \ , \qquad V(\rho,\eta) = \underbrace{\eta \ln \rho}_{\text{source}} + \underbrace{\eta \left(\frac{\eta^2}{3} - \frac{\rho^2}{2} \right)}_{\text{1st harmonic}} ,$$

- Describes the general geometry near the origin (small η).
- Change of the gauge group:
  The original $SU(N)$ has changed to $\prod_{i=2} SU(i)$.

$\mathcal{N} = 1$ *Backgrounds*

Remarks/Quastions:

- Can the same amount of supersymmetry be preserved?
- Are there non-singular (non)-Abelian T-duals?

The answer is yes!

- D3-brane on a conifold singularity [Klebanov-Witten 98].
  N=1 SCFT with $SU(N) \times SU(N)$ gauge group.
  - The gravity dual is $AdS_5 \times T_{1,1}$, with

$$T_{1,1} = \frac{SU(2) \times SU(2)}{U(1)} \; .$$

The symmetry group is $SO(4,2) \times SU(2) \times SU(2) \times U(1)$.

Up lift of the non-Abelian dual of the KW

The non-Abelian T-dual is a type-IIA with g, B, Φ, F_2 and F_4.

Uplift to 11-dims:

- The metric

$$ds^2 = \Delta^{1/3}\left(ds^2_{AdS_5} + \lambda_1^2(\sigma_1^2+\sigma_2^2)\right) + \Delta^{-2/3}\Big[(x_1^2+\lambda^2\lambda_1^2)dx_1^2 + (x_2^2+\lambda_1^4)dx_2^2 + 2x_1x_2\,dx_1\,dx_2 + \lambda^2\lambda_1^2x_1^2\sigma_{\hat{3}}^2 + \left(dx_\sharp + \frac{\sigma_3}{27}\right)^2\Big]\ ,$$

  where $\Delta = \lambda_2^2x_1^2 + \lambda^2(x_2^2+\lambda_2^4)$ and $\lambda = \frac{1}{3}$, $\lambda_1 = \lambda_2 = \frac{1}{\sqrt{6}}$.
- There is also an F_4 flux.
- Preserves $\mathcal{N} = 1$ superconformal. In the class of [Gauntlett-Martelli-Sparks-Waldram 04] and [Bah-Beem-Bobev-Wecht 12].

Susy preserved and non singular

- The Kosman derivative of the original $T^{1,1}$ Killing spinors vanishes

$$\mathcal{L}_{k_1}\epsilon \sim \mathcal{L}_{k_2}\epsilon \propto (\Gamma^{12}+\Gamma^{34})\epsilon \sim 0 \ , \quad \mathcal{L}_{k_3}\epsilon = 0 \ ,$$

- Singularities associated to fixed points of the isometry (c.f. polar $U(1)$ duality in R^2), points where the norms of the killing vectors vanish. But

$$|k_i|^2 > 0$$

- Subtlety: Removable bolt singularity dictates a halving of the range of angular coordinate ψ: possible sign of an orientifold?
- Preserves $\mathcal{N}=1$ superconformal.

Can we brake conformal invariance but retain supersymmetry?

- Include M fractional D3-branes by turning H_3, F_3 and a Φ [Klebanov-Nekrasov 99, Klebanov-Tseytlin 00, Klebanov-Strassler 00].
- Theory becomes non-conformal $\mathcal{N} = 1$ with $SU(N+M) \times SU(N)$ gauge group.
- An RG flow which is non-singular at all scales.
- For simplicity concentrate on the UV regime in which

$$ds^2 = e^{-5q} ds_5^2 + \frac{1}{6} e^{2f+3q} \sum_{i=1}^{2} (d\theta_i^2 + \sin^2\theta_i d\phi_i^2) + \frac{1}{9} e^{3q-8f} (d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2 ,$$

where

$$ds_5^2 = du^2 + e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu .$$

Also, there is a self-dual F_5, $\Phi = \mathrm{const.}$ and

$$B = P\,T(u) [\mathrm{Vol}(S_1^2) - \mathrm{Vol}(S_2^2)] , \quad F_3 \sim P [\mathrm{Vol}(S_1^2) - \mathrm{Vol}(S_2^2)] \wedge (d\psi + \cdots) .$$

The constant $P \sim M/N$.

Important characteristics:

- f, q, T and A satisfy a non-linear system of eqs.; Solvable.
- The R-symmetry corresponds to shifts of ψ.
- The Killing spinors do not dependent on ψ.

What happens when T-duality acts?

- Abelian T-duality: Will be a solution of type-IIA.
  - w.r.t. ψ will break supersymmetry and
  - w.r.t. to any ϕ_i will lead to a singular background.
- Non-Abelian T-duality: Will be a solution of massive type-IIA.
  - Will have F_0, F_2 and F_4 turned on. In particular,

    $$F_0 \sim M \ ,$$

    the Romans' mass is quantised.
  - The global symmetry is $SU(2) \times U(1)_\psi$.
  - Non-singular with unbroken Supersymmetry.

Fate of the central charge

- The central charge: For the metric

$$ds^2 = \alpha(u)dx_{1,3}^2 + du^2 + g_{ij}(x,u)dx^i dx^j \ ,$$

  it is of the form

$$c \sim \frac{H^{7/2}}{\alpha^{3/2}(H')^3}\ , \qquad H = \alpha^3 V_{\rm int}^2\ , \quad V_{\rm int} = \int d^5x e^{-2\Phi}\sqrt{\det g}\ .$$

  Non-increasing towards the IR [Girardello et. al. 98].
- Invariant of T-duality up to a single RG scale background independent coefficient.
- Invariance of the ratio

$$\frac{c_{\rm IR}}{c_{\rm UV}} = \frac{27}{32}\ ,$$

  for the RG flow from $AdS_5 \times S^5/\mathbb{Z}_2$ to $AdS_5 \times T_{1,1}$ as well as for their T-duals.

Fate of the brane charges

There are three type of charges (for a review, see [Marolf 00]).

	Gauge invariant	localized	conserved	quantized
Brane sources	Yes	Yes	No	No
Maxwell	Yes	No	Yes	No
Page	No	Yes	Yes	Yes

Maxwell:

$$\text{Before}: \quad Q_{D3} \sim \int_{\theta_i,\phi_i,\psi} F_5 \sim N \ln r \ , \quad Q_{D5} \sim \int_{\theta_1,\phi_1,\psi} F_3 = M \ ,$$

$$\text{After}: \quad Q_{D6} \sim \int_{\theta_1,\phi_1} F_2 \sim N \ln r \ , \quad Q_{D8} = M \ .$$

Page:

$$\text{Before}: \quad Q_{D3} \sim \int_{\theta_i,\phi_i,\psi} (F_5 - B \wedge F_3) = N_0 \ , \quad Q_{D5} \sim \int_{\theta_1,\phi_1,\psi} (F_3 - B \wedge F_1) = M$$

$$\text{After}: \quad Q_{D6} \sim \int_{\theta_1,\phi_1} (F_2 - F_0 B) = N_0 \ , \quad Q_{D8} = M \ .$$

Non-singular number of "mobile" D3 $N_0 = 0$

Seiberg duality

To compute the change in the gauge group along the RG flow one computes the effective brane charges. Following a procedure as in [Benini-Canoura-Cremonesi-Nunez-Ramallo 07]:

- This can be done by realizing that a shift in the NS charge as

$$b_0 \to b_0 + n \,, \qquad b_0 = \frac{1}{4\pi^2} \int_{\Sigma_2} B \,,$$

  leaves the string theory invariant.
- Can be compensated by a shift in the holographic coordinate which changes the Maxwell charges.
- At a fixed energy scale perform a large gauge transformation by changing $B \to B + \Delta B$. This affects the Page charges.
- Both procedures give the same result. It turns out that a change of $\Delta Q_{D3} = M$ units in the KT case, induces a change of $\Delta Q_{D6} = 2M$ after the transformation.

Other probes

We can consider further observables and also probe IR physics (using dual of full KS or KS+bb or Wrapped D5)

- Domain walls in IR : matching of effective tension (wrapped $D5 \rightarrow unwrapped D2$)
- Wilson loop/$q\bar{q}$ potential preserved
- 't Hooftline/monopole potential not preserved but still becoming effective tension goes to zero in IR (wrapped $D3 \rightarrow$ wrapped $D4$ on Σ_2)
- Gauge coupling (instanton action of euclidean $D1$ brane on Σ_2 $\rightarrow$ euclidean $D3$ brane on Σ_3) $1/g^2 \rightarrow \rho$ in IR

G-structures

- KW, KT etc. $\mathcal{N}=1$ backgrounds: Ω_3 and J_2 or $SU(3)$ structure.

$$\Phi_1 = \Omega_3 \ , \quad \Phi_2 = e^{-iJ} \ ,$$
$$(d+H)\Phi_1 = 0 \ , \quad (d+H)\Phi_2 = F_{RR}$$

- These pure spinors transform like RR fluxes:

$$e^{\phi}\Phi_i = e^{\hat{\phi}}\hat{\Phi}\cdot\Omega^{-1}$$

- Shows the duals produced above are $SU(2)$ structure and $\mathcal{N}=1$ SUSY

$$\hat{\Phi}_1 = e^{-v\wedge w}\wedge\omega_2 \ , \quad \hat{\Phi}_2 = e^{-ij}\wedge(v+iw)$$

Flavours

- Add D7 flavour branes to KW (susy callibration conditions [Areán et al. , Martucci and Smyth])
- Beyond probe/quenched limit $N_f \sim N_c$ by smearing encoded by a smearing form

$$\Xi_2 = -N_f \left(\sin\theta d\theta \wedge d\phi + \sin\tilde{\theta} d\tilde{\theta} \wedge d\tilde{\phi}\right)$$

- Modified Bianchi identities $dF_1 = \Xi_2$ and accommodate back reaction with this ansatz

$$\begin{aligned} ds^2 &= \frac{e^{\frac{\Phi}{2}}}{\sqrt{h}} dx^2_{1,3} + e^{\frac{\Phi}{2}} \sqrt{h}\Big(dr^2 + \lambda_1^2 e^{2g} (\sin^2\theta d\varphi^2 + d\theta^2) + \lambda_2^2 e^{2g} (\sigma_1^2 + \sigma_2^2) + \lambda^2 e^{2f} (\sigma_3 + \cos\theta d\varphi)^2 \Big) \ , \\ F_1 &= \frac{N_f}{4\pi}(\sigma_3 + \cos\theta d\varphi) \ , \qquad F_5 = (1+\star) dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge K dr \ . \end{aligned} \tag{1}$$

- 1^{st} order BPS equations which can be solved!

Flavours: The T-dual

- Can T-dualise as before, complicated geometry but $SU(2)$ structure and SUSY
- Source D7s $\rightarrow$ source D6 and D8.
- T-dual smearing forms transform similar to RR fluxes:

$$e^{\Phi}\Xi = e^{\hat{\Phi}} e^{B\wedge}\hat{\Xi}$$

- T-dual Bianchi Identities

$$(d - \hat{H}) \wedge \hat{F} = e^{\hat{B}\wedge}\hat{\Xi}$$

- Can now add flavours to all the interesting IIA backgrounds described.
- Side remark - change in B redistributes $D4$, $D6$ charges

$Y^{p,q}$ *and its T-dual*

- $Y^{p,q}$ are an infinite class of Sasaki-Einstein manifolds [Gauntlett et al.]
- $AdS_5 \times Y^{p,q}$ are an infinite class of AdS-CFT dual pairs in type IIB [Martelli and Sparks, Benvenuti et al., Franco et al.]
- Isometry group $SU(2) \times U(1) \times U(1)$ so can dualise w.r.t. $SU(2)$
- Kosmann vanishes and Killing norm non-zero $\Rightarrow$ susy and smooth
- Result: a new infinite class of solutions in type IIA with dynamic $SU(2)$ structure and M-theory lifts!
- Metric retains many of the features of the dual KW and supported by B, $F2$, $F4$

Concluding remarks

1. Non-Abelian T-duality extended to type II supergravity
2. $AdS_5 \times T_{11}$ and its deformations give new smooth solutions with $\mathcal{N} = 1$ in (massive) IIA & M-theory
3. Probes indicate duality cascade and confining
4. Classified solutions in terms of $SU(2)$ structure
5. Showed how to add flavour branes beyond the quenched approximation
6. A new infinite class of M-theory solutions from dualising $Y^{p,q}$

- Opens Qs: better handle on gauge theory; integrability; global issues; generalised geometry