Crete Center for Theoretical Physics, April 1st, 2013

New transport properties of holographic superfluids

Daniel Fernández

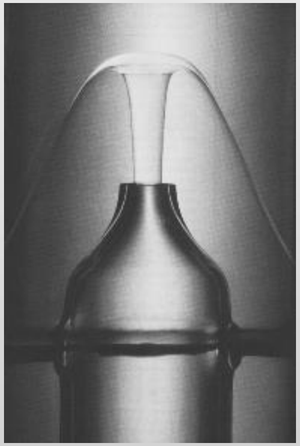
University of Barcelona

work in collaboration with Johanna Erdmenger and Hansjörg Zeller

s-wave and p-wave superfluids

Superfluid:

- State of matter with zero viscosity at very low temperatures.
- Gauge theory with spontaneous breaking of global symmetry.

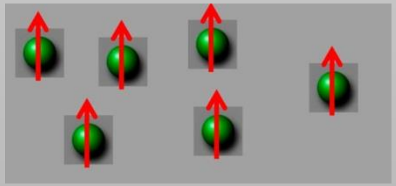


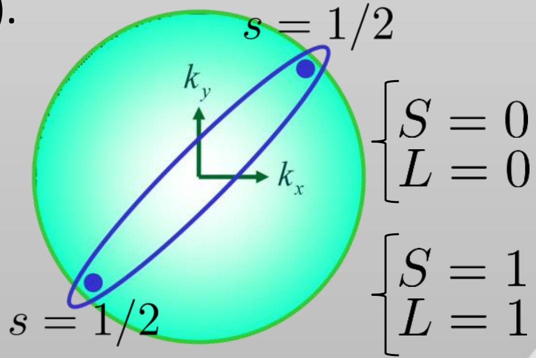
Conventional superfluids:

- Helium-4: Bose-Einstein condensation of atoms.
- New hydrodynamic mode: Superfluid velocity

“*p*-wave” SFs, like Helium-3:

- Cooper pairs of ions form bosonic states (like in BCS).
- Rotational symmetry is broken: more modes.
- Superconductivity with new pairing states.
- Much lower temperature than conventional.
- Several different phases.





Liquid crystals:

- Flow like liquids, but molecules are oriented.
- Related to high temperature SCs (d-wave).

[Lee, Osheroff, C. Richardson, Leggett]

Hydrodynamics of superfluids

Condensed-matter analog of the Higgs phenomena

$$\Psi = |\Psi|\, e^{i\varphi}$$

Spontaneous Symmetry Breaking of continuous symmetry

→ Nambu-Goldstone boson in the spectrum

→New hydrodynamic mode

$$\{\mu,\ T,\ u_\mu\},\ v_\mu = \partial_\mu \varphi$$

(superfluid velocity)

- Bosons form a highly collective state.
- Wavefunction Ψ is expectation value. Phase φ, coherent superposition in condensate.
- In our case:

$$SU(2) \xrightarrow[\text{Expl.B}]{} U(1)_3 \xrightarrow[\text{SSB}]{} \mathbb{Z}_2$$

$$SO(3) \xrightarrow[\text{SSB}]{} SO(2)$$

- 3 Goldstone modes! We can expect different hydrodynamics.

Outlining the duality

IIB Supergravity on AdS_5 $\Longleftrightarrow$ Conformal Field Theory at large N_c and strong coupling

- Energy Scale $\longleftrightarrow$ Radial Coordinate
- Temperature $\longleftrightarrow$ Black Hole
- Global currents J_μ $\longleftrightarrow$ Gauge Fields A_μ

In particular,

$$\mu = \lim_{r\to\infty} A_t, \quad T = \frac{\kappa_{\mathrm{BH}}}{2\pi} \quad \text{and if} \quad A_\mu(x,r) \to A_\mu^{(0)}(x) + A_\mu^{(2)}(x)\frac{1}{r^2} + \ldots,$$

- Expected Value $A_\mu^{(2)}(x) = \langle J_\mu(x) \rangle$
- Source $A_\mu^{(0)}(x)$

Outlining the duality

IIB Supergravity on AdS_5 $\Longleftrightarrow$ Conformal Field Theory at large N_c and strong coupling

$$\left\{\begin{array}{lcl} \text{Energy Scale} & \longleftrightarrow & \text{Radial Coordinate} \\ \text{Temperature} & \longleftrightarrow & \text{Black Hole} \\ \text{Global currents } J_\mu & \longleftrightarrow & \text{Gauge Fields } A_\mu \end{array}\right.$$

In particular,

$$\mu = \lim_{r\to\infty} A_t, \quad T = \frac{\kappa_{\mathrm{BH}}}{2\pi} \quad \text{and if} \quad A_\mu(x,r) \to A_\mu^{(0)}(x) + A_\mu^{(2)}(x)\frac{1}{r^2} + \ldots,$$

$$\left\{\begin{array}{ll} \text{Expected value} & A_\mu^{(2)}(x) = \langle J_\mu(x)\rangle \\ \text{Source} & A_\mu^{(0)}(x) \end{array}\right.$$

And to be precise,

$$Z_{\mathrm{SUGRA}}[\phi(x,r)|_{r\to r_{\mathrm{bdy}}} = \phi_0(x)] = \left\langle e^{\int \mathrm{d}^4x\, \phi_0(x)\mathcal{O}(x)} \right\rangle$$

The Field/Operator Correspondence

Field-Operator dictionary: $\Phi(x,r) \leftrightarrow \mathcal{O}(x)$

If the action for bulk field is

$S \propto \int \mathrm{d}r\,\mathrm{d}^4x\,\sqrt{-g}\,\left(\partial_M\Phi\partial^M\Phi + m^2\Phi^2\right) + \ldots,$ the asymptotic solution is

$$\Phi(x,r) \to \phi_0(x)\,r^{\Delta-4} + \phi_2(x)\,r^{-\Delta}$$

where

$$\Delta = 2 + \sqrt{m^2L^2+4}$$

Stability requires real Δ, otherwise exponential growth.
→ Mass term not "too negative" (BF bound)

The Field/Operator Correspondence

Field-Operator dictionary: $\Phi(x,r) \leftrightarrow \mathcal{O}(x)$

If the action for bulk field is

$S \propto \int \mathrm{d}r\,\mathrm{d}^4x\,\sqrt{-g}\,\left(\partial_M\Phi\partial^M\Phi + m^2\Phi^2\right) + \ldots,$ the asymptotic solution is

$$\Phi(x,r) \to \phi_0(x)\,r^{\Delta-4} + \phi_2(x)\,r^{-\Delta}$$

where

$$\Delta = 2 + \sqrt{m^2L^2+4}$$

Stability requires real Δ, otherwise exponential growth.
→ Mass term not "too negative" (BF bound)

If $m^2L^2 \geq -3$,

- $\phi_0(x)$ is non-normalizable, enters boundary theory.

$$S_{\text{bdry}} \to S_{\text{bdry}} + \int \mathrm{d}^4x\,\phi_0(x)\mathcal{O}(x)$$

- $\phi_2(x)$ is normalizable, belongs to bulk Hilbert space.

Hilbert spaces of dual theories identified:
Normalizable modes ↔ states of bdry theory

$$\phi_2(x) \propto \langle\mathcal{O}(x)\rangle_{\phi_0}$$

The Field/Operator Correspondence

Field-Operator dictionary: $\Phi(x,r) \leftrightarrow \mathcal{O}(x)$

If the action for bulk field is

$S \propto \int \mathrm{d}r\,\mathrm{d}^4x\,\sqrt{-g}\,\left(\partial_M\Phi\partial^M\Phi + m^2\Phi^2\right) + \ldots,$ the asymptotic solution is

$$\Phi(x,r) \to \underbrace{\phi_0(x)}_{\text{source}} r^{\Delta-4} + \underbrace{\phi_2(x)}_{\text{vev}} r^{-\Delta}$$

where

$$\Delta = 2 + \sqrt{m^2L^2+4}$$

Stability requires real Δ, otherwise exponential growth.
→ Mass term not "too negative" (BF bound)

If $m^2L^2 \geq -3$,

- $\phi_0(x)$ is non-normalizable, enters boundary theory.

$$S_{\mathrm{bdry}} \to S_{\mathrm{bdry}} + \int \mathrm{d}^4x\,\phi_0(x)\mathcal{O}(x)$$

- $\phi_2(x)$ is normalizable, belongs to bulk Hilbert space.

Hilbert spaces of dual theories identified:
Normalizable modes ↔ states of bdry theory

$$\phi_2(x) \propto \langle\mathcal{O}(x)\rangle_{\phi_0}$$

AdS/CFT and Green's functions

Retarded Green's function = Correlator:

$$G^{\mathrm{R}}_{\mathcal{O}_\mathrm{A}\mathcal{O}_\mathrm{B}} = -\mathrm{i}\int \mathrm{d}^{d-1}x\, dt\, e^{\mathrm{i}\omega t - \mathrm{i}kx}\,\theta(t)\,\langle[\mathcal{O}_\mathrm{A}(t,x),\, \mathcal{O}_\mathrm{B}(0,0)]\rangle$$

Time-dependent perturbation in the action includes a source for B:

$$S(t) = \ldots + \int \mathrm{d}^{d-1}x\,\phi_{\mathrm{B}(0)}(t,x)\,\mathcal{O}_\mathrm{B}(x)$$

Expectation value for observable A in its presence is

$$\langle\mathcal{O}_\mathrm{A}\rangle(t,x) = \mathrm{Tr}\,\rho(t)\,\mathcal{O}_\mathrm{A}(x) \quad \text{where} \quad \mathrm{i}\,\partial_t\rho = [H_0 + \delta H, \rho].$$

The increase due to a δH is $\delta\langle\mathcal{O}_\mathrm{A}\rangle$. The perturbation comes from the source:

$$\phi_\mathrm{B}(r) \to \phi_\mathrm{B}(r) + \delta\phi_\mathrm{B}(r)e^{-\mathrm{i}\omega t + \mathrm{i}kx}$$

Linear response around equilibrium:

$$\boxed{\delta\langle\mathcal{O}_\mathrm{A}\rangle(\omega,k) = G^{\mathrm{R}}_{\mathcal{O}_\mathrm{A}\mathcal{O}_\mathrm{B}}(\omega,k)\,\delta\phi_{\mathrm{B}(0)}(\omega,k)}$$

[Son, Starinets]

The correspondence allows for a simple calculation!

The gravity model

SU(2) Einstein-Yang-Mills theory

$$S = \frac{1}{2\kappa_5^2}\int \mathrm{d}^5x\,\sqrt{-g}\,\left[R - \Lambda - \frac{\alpha^2}{2}\,F^a_{MN}F^{aMN}\right] + S_{\mathrm{bdy}}$$

$$\left(\alpha = \frac{\kappa_5}{\hat{g}},\ \Lambda = -\frac{12}{L^2}\right)$$

Ansatz for gauge field:

$$A = \phi(r)\,\tau^3\,\mathrm{d}t + w(r)\,\tau^1\,\mathrm{d}x$$

The gravity model

SU(2) Einstein-Yang-Mills theory

$$S = \frac{1}{2\kappa_5^2}\int \mathrm{d}^5x\,\sqrt{-g}\left[R - \Lambda - \frac{\alpha^2}{2}F^a_{MN}F^{aMN}\right] + S_{\mathrm{bdy}}$$

$$\left(\alpha = \frac{\kappa_5}{\hat{g}},\ \Lambda = -\frac{12}{L^2}\right)$$

Ansatz for gauge field:

$$A = \phi(r)\tau^3\,\mathrm{d}t + w(r)\tau^1\,\mathrm{d}x$$

Chemical potential
→ explicit breaking

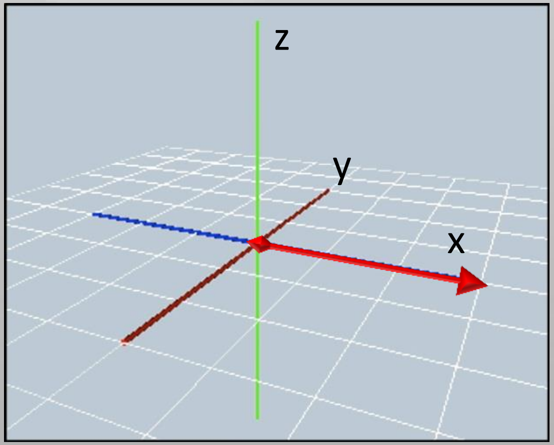
Spontaneous value $w(r) \to w_1^b/r^2$ acquired in broken phase:

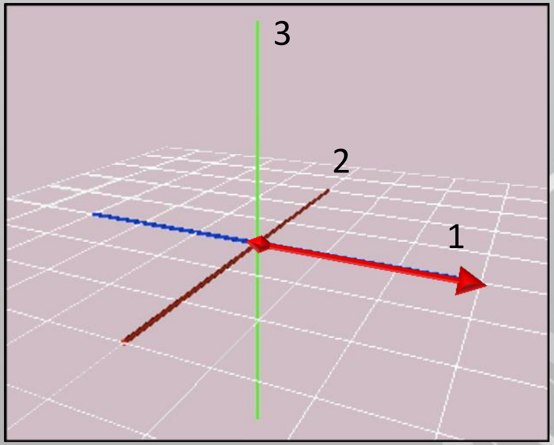
$$w_1^b \propto \langle J_1^x\rangle \neq 0$$

[Ammon, Erdmenger, Grass, Kerner, O'Bannon]

$$SU(2) \xrightarrow[\text{Expl.B}]{} U(1)_3 \xrightarrow[\text{SSB}]{} \mathbb{Z}_2$$

$$SO(3) \xrightarrow[\text{SSB}]{} SO(2)$$





Possible backgrounds

Ansatz for the metric:

$$\mathrm{d}s^2 = -N(r)\sigma(r)^2\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + r^2 f(r)^{-4}\mathrm{d}x^2 + r^2 f(r)^2\left(\mathrm{d}y^2 + \mathrm{d}z^2\right)$$

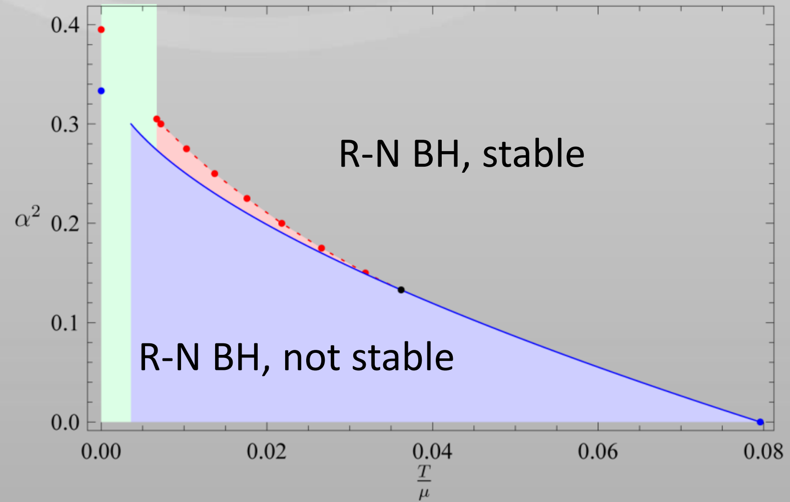
Solution 1

- Reissner–Nordström BH (asymptotically AdS)
- $w(r) = 0$
- Ground State for $\frac{\mu}{T} < \left(\frac{\mu}{T}\right)_c$

Solution 2

- Charged BH with vector hair (asymptotically AdS)
- $w(r) \neq 0$
- Ground State for $\frac{\mu}{T} > \left(\frac{\mu}{T}\right)_c$

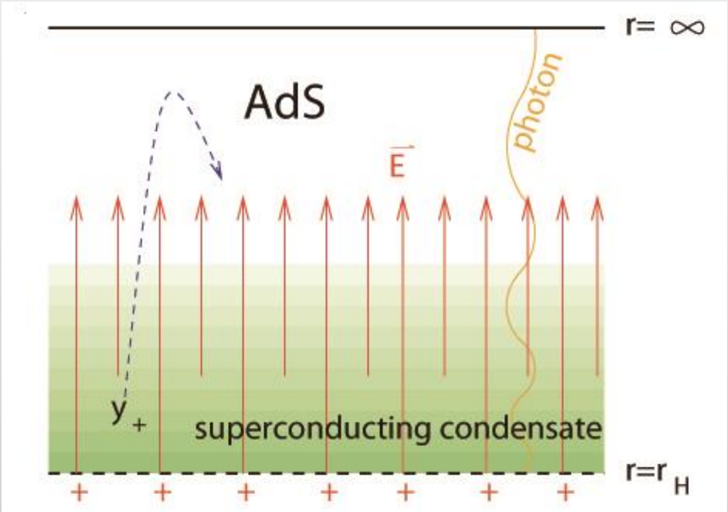
Phase diagram:



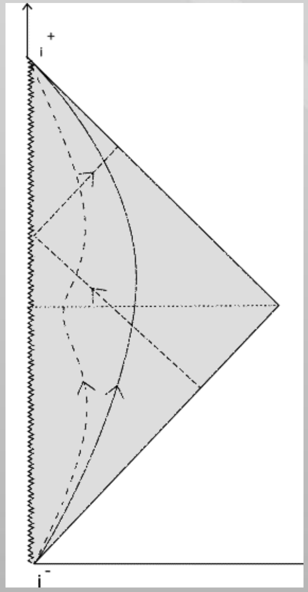
[Erdmenger, Grass, Kerner, Hai Ngo]

Vector hair

In solution 2,
a condensate layer floats above the horizon.



- In asympt. **flat** spacetime,
Electrostatic repulsion sends it to infty.
- In asympt. **AdS** spacetime,
Massive particles do not reach bdry.



Action for A^1_x:

[Gubser, Pufu]

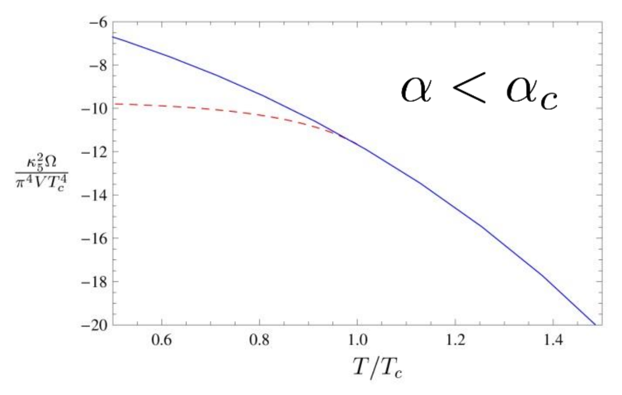
$$S_{A^1_x} \sim \partial_M A^1_x \partial^M A^1_x + \underbrace{2 g^{tt} g^{xx} \left(A^3_t\right)^2}_{m^2_{\rm eff}} \left(A^1_x\right)^2$$

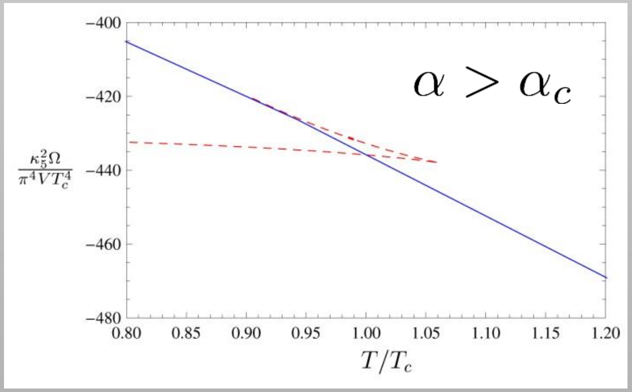
- Since $g_{tt}(r_{\rm H}) = 0$, A^1_x is tachyonic near the horizon...
- It condenses in a normalizable profile ($w = 0$ at bdry.)
- This translates into $\langle J^x_1 \rangle \neq 0$ in the dual field theory.
- The action can be embedded into M-theory.

Thermodynamics

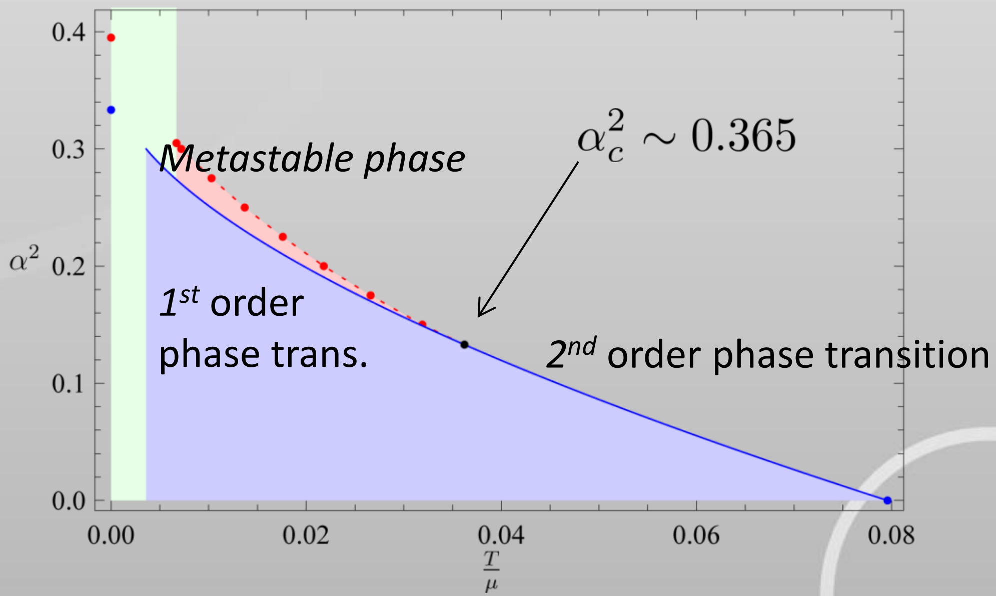
Solution to the EOM in gravity theory ⟺ Thermal equilibrium state in field theory

Central quantity: Free Energy $\Omega = I_{\text{on-shell}}/T$



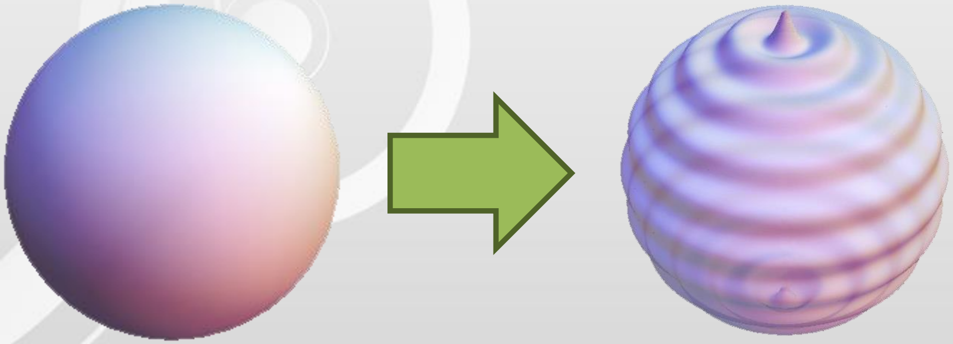


Besides thermodynamic calculations, ask if solution stable under perturbations…



$$\alpha^2 = \frac{\kappa_5^2}{\hat{g}^2} \sim \frac{\#\ \text{charged d.o.f.}}{\text{Total}\ \#\ \text{d.o.f.}}$$

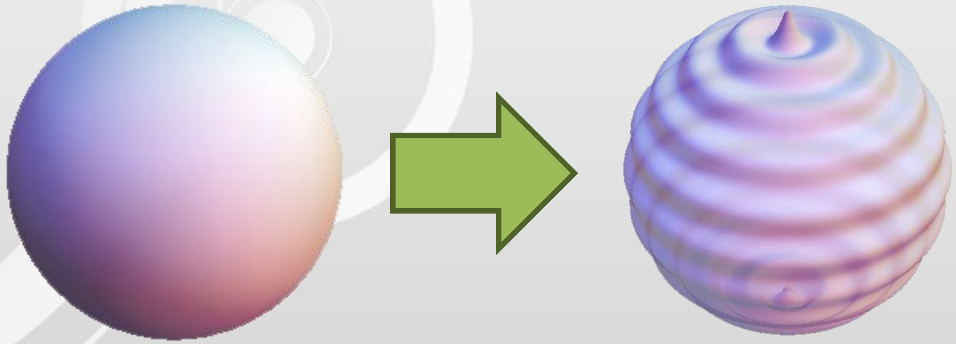
Perturbations



$$\hat{g}_{MN}(t,\vec{x},r) = g_{MN}(r) + \int \frac{\mathrm{d}\omega\,\mathrm{d}^3\vec{k}}{(2\pi)^4}\, h_{MN}(\omega,\vec{k},r)e^{-\mathrm{i}\omega t+\mathrm{i}\vec{k}\cdot\vec{x}}$$

$$\hat{A}^a_M(t,\vec{x},r) = A^a_M(r) + \int \frac{\mathrm{d}\omega\,\mathrm{d}^3\vec{k}}{(2\pi)^4}\, a^a_M(\omega,\vec{k},r)e^{-\mathrm{i}\omega t+\mathrm{i}\vec{k}\cdot\vec{x}}$$

Perturbations



$$\hat{g}_{MN}(t,\vec{x},r) = g_{MN}(r) + \int \frac{\mathrm{d}\omega\,\mathrm{d}^3\vec{k}}{(2\pi)^4} h_{MN}(\omega,\vec{k},r)e^{-\mathrm{i}\omega t+\mathrm{i}\vec{k}\cdot\vec{x}}$$

$$\hat{A}^a_M(t,\vec{x},r) = A^a_M(r) + \int \frac{\mathrm{d}\omega\,\mathrm{d}^3\vec{k}}{(2\pi)^4} a^a_M(\omega,\vec{k},r)e^{-\mathrm{i}\omega t+\mathrm{i}\vec{k}\cdot\vec{x}}$$

- Gauge fixing:

$$h_{Mr} = 0\ , a^a_r = 0$$

- Longitudinal momentum:

$$k^\mu = (\omega, k_|, \cancel{k_\perp}, 0)$$

so that perturbations preserve *SO(2)*.

The classification of perturbation fields

Helicity 2, helicity 1, helicity 0:

$$a^a_M = \begin{pmatrix} a^a_t \\ a^a_x \\ a^a_y \\ a^a_z \\ 0 \end{pmatrix}, \quad h_{MN} = \begin{pmatrix} h_{tt} & h_{xt} & h_{ty} & h_{tz} & 0 \\ h_{xt} & h_{xx} & h_{xy} & h_{xz} & 0 \\ h_{yt} & h_{xy} & h_{yy} & h_{yz} & 0 \\ h_{zt} & h_{xz} & h_{yz} & h_{zz} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h_{yy} = \frac{1}{2}\left(h_{yy} + h_{zz} + h_{yy} - h_{zz}\right)$$

$$\xi_y = g^{yy} h_{yy}, \quad \xi_x = g^{xx} h_{xx}, \quad \xi_t = g^{tt} h_{tt}, \quad \xi_{tx} = g^{xx} h_{tx}$$

Parity:

If k=0, also classifiable by change under $P_{||}$:

- $U(1)_3 \to \mathbb{Z}_2 \Longrightarrow$ flip sign index 2
- $A = \phi(r)\,\tau^3\,\mathrm{d}t + w(r)\,\tau^1\,\mathrm{d}x \Longrightarrow$ flip indices 1,x

even	*odd*
a^3_t	a^1_t, a^2_t
a^1_x, a^2_x	a^3_x
ξ_t, ξ_x, ξ_y	ξ_{tx}

The Physical Fields

Helicity zero, k=0:

- There are 10 perturbation modes.
- Einstein's and Yang-Mills's eqs. give 10 DEs and 6 constraints → 14 d.o.f. at bdry.
- Ingoing condition (for retarded GF) at the horizon takes away 10 d.o.f.
- Remaining: 4 physical fields, invariant under residual gauge freedom.

$$\Phi_1(\omega, r) \longrightarrow \left(a_x^1\right)_0^b ,$$
$$\Phi_2(\omega, r) \longrightarrow \left(a_x^2\right)_0^b ,$$
$$\Phi_3(\omega, r) \longrightarrow \left(\xi_x\right)_0^b - \left(\xi_y\right)_0^b ,$$
$$\Phi_4(\omega, r) \longrightarrow \left(a_x^3\right)_0^b .$$

It is convenient to change into:

$$a_x^\pm = a_x^1 \pm \mathrm{i} a_x^2$$
$$\xi_{p,m} = \xi_x \pm \xi_y$$

The action cannot be written in terms of physical fields only.

$$S_{\text{o.s.(I)}} = \int d^d x \left[\alpha_{\text{IJ}} \left(\Phi_{\text{I}}\right)_0^b \left(\Phi_{\text{J}}\right)_0^b + \beta_{\text{IJ}} \left(\Phi_{\text{I}}\right)_0^b \left(\varphi_{\text{J}}\right)_p^b + \zeta_{\text{IJ}} \left(\varphi_{\text{I}}\right)_p^b \left(\varphi_{\text{J}}\right)_p^b\right] ,$$
$$S_{\text{o.s.(II)}} = \int d^d x \left[\kappa_{\text{I}j} \left(\Phi_{\text{I}}\right)_0^b \left(\varphi_j\right)_0^b + \lambda_{ij} \left(\varphi_i\right)_0^b \left(\varphi_j\right)_0^b\right] ,$$

Replace those perturbations by physical fields, so that

$$S_{\text{o.s.1}} = \int \mathrm{d}^d k \left[\Phi_{\text{I}}(-k, r) A(k, r)_{\text{IJ}} \partial_r \Phi_{\text{J}}(k, r) + \Phi_{\text{I}}(-k, r) B(k, r)_{\text{IJ}} \Phi_{\text{J}}(k, r)\right]_{r=r_b}$$

Transverse thermoelectric effect

[Erdmenger, Kerner, Zeller]

Simultaneous transport of electric charge and heat:

$$\begin{pmatrix} \langle J^y \rangle \\ \langle Q^y \rangle \end{pmatrix} = \begin{pmatrix} \sigma^{yy} & T\alpha^{yy} \\ T\alpha^{yy} & T\bar{\kappa}^{yy} \end{pmatrix} \begin{pmatrix} E_y \\ -(\nabla_y T)/T \end{pmatrix}$$

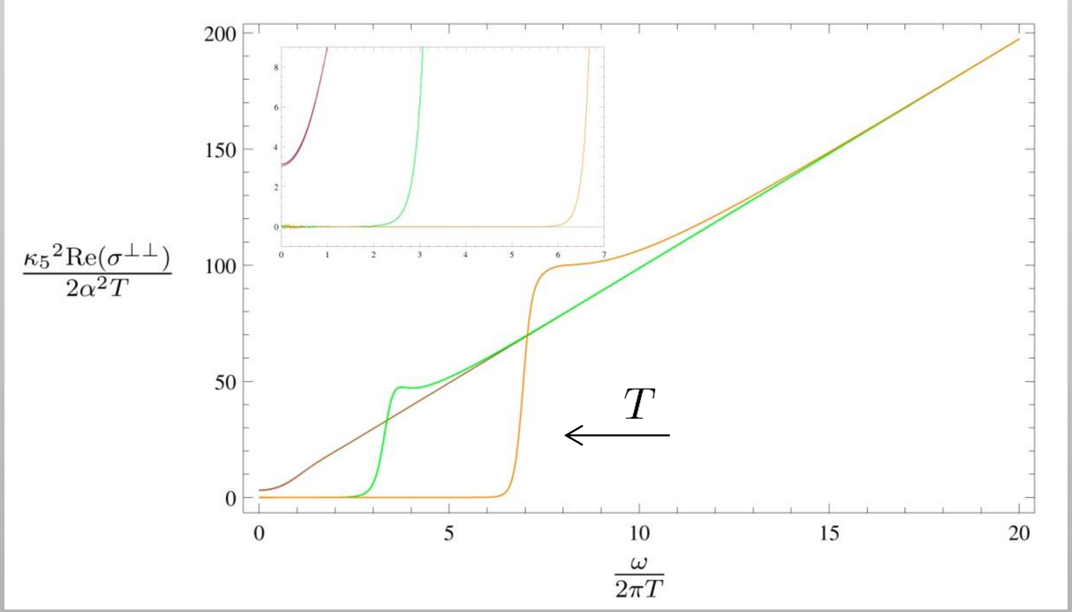
- Generation of electric current due to thermal gradient.
- Generation of thermal transport due to an external electric field.

Heat flux

$$Q^y = T^{ty} - \mu J^y$$

Thermal gradient

$$\nabla_y T = -\mathrm{i}\omega\, T\, (\xi_{ty})_0^b$$



Transverse thermoelectric effect

[Erdmenger, Kerner, Zeller]

Simultaneous transport of electric charge and heat:

$$\begin{pmatrix} \langle J^y \rangle \\ \langle Q^y \rangle \end{pmatrix} = \begin{pmatrix} \sigma^{yy} & T\alpha^{yy} \\ T\alpha^{yy} & T\bar{\kappa}^{yy} \end{pmatrix} \begin{pmatrix} E_y \\ -(\nabla_y T)/T \end{pmatrix}$$

- Generation of electric current due to thermal gradient.
- Generation of thermal transport due to an external electric field.

Heat flux

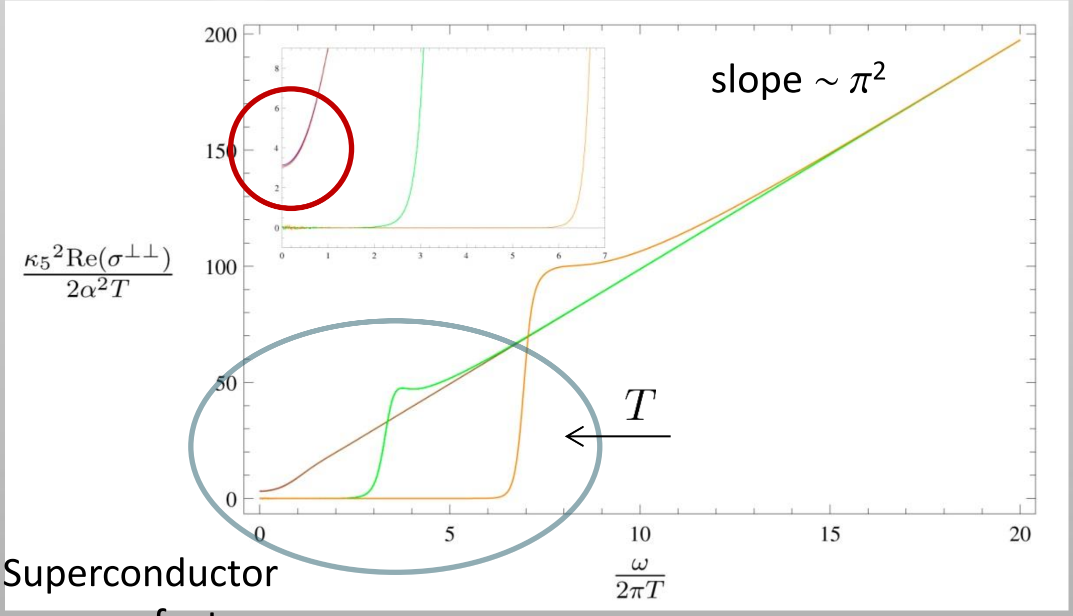
$$Q^y = T^{ty} - \mu J^y$$

Thermal gradient

$$\nabla_y T = -\mathrm{i}\omega\, T\, (\xi_{ty})_0^b$$

Electric field

$$E_y = \mathrm{i}\omega \left((a_y^3)_0^b + \mu\, (\xi_{ty})_0^b \right)$$



Superconductor feature

- Curves almost overlap for $T > T_c$
- Overlap of all curves asymptotically:

$$\mathrm{Re}(\sigma^{yy}) \propto \omega$$

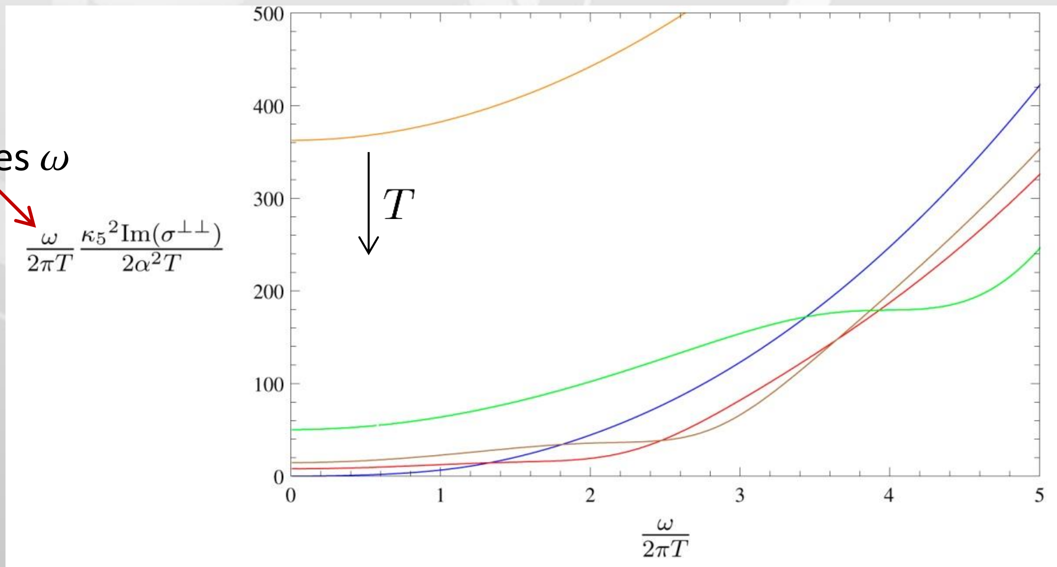
- Consequence of conformal symmetry.

Transverse thermoelectric effect

[Erdmenger, Kerner, Zeller]

Imaginary part:

times ω



- Pole at the origin $\Longrightarrow$ Real part has delta peak (K-K relation)
- Delta peak due to sum rule, observed here.
- Anticipated behavior:

$$\omega\,\mathrm{Im}\,(\sigma) \simeq A_{\mathrm{D}}(\alpha, T) + A_s(\alpha)\left(1 - \frac{T}{T_c}\right)$$

Drude peak ∀T

Appears in superfluid phase

Longitudinal thermoelectric effect

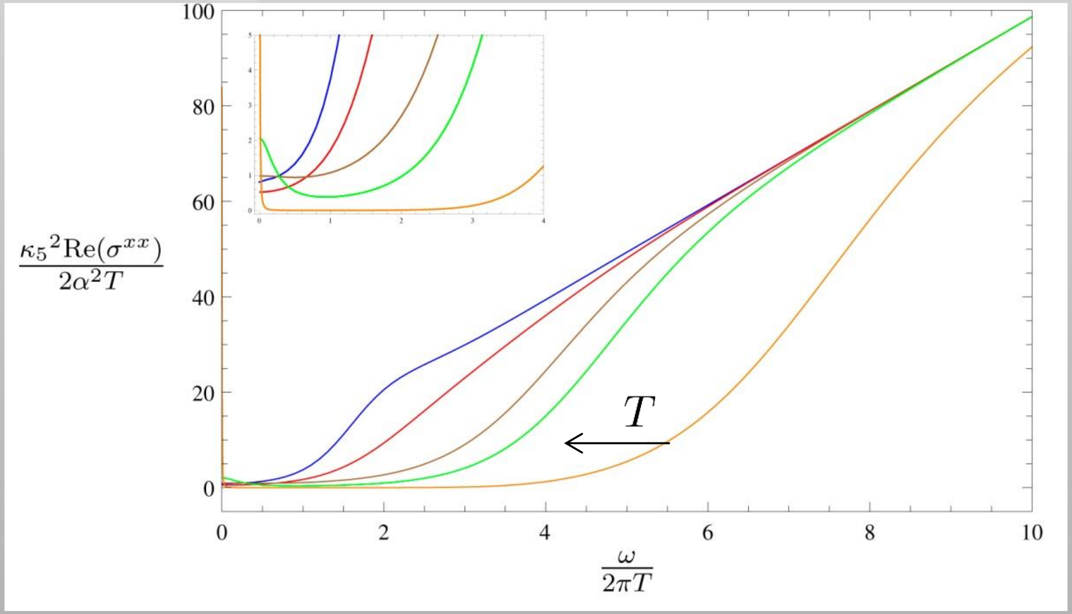
[Erdmenger, DF, Zeller]

$$\begin{pmatrix} \langle J^x \rangle \\ \langle Q^x \rangle \end{pmatrix} = \begin{pmatrix} \sigma^{xx} & T\alpha^{xx} \\ T\alpha^{xx} & T\bar{\kappa}^{xx} \end{pmatrix} \begin{pmatrix} E_x \\ -(\nabla_x T)/T \end{pmatrix}$$

Additional couplings:

$$\begin{pmatrix} \langle J_1^t \rangle \\ \langle J_2^t \rangle \\ \langle J^x \rangle \\ \langle Q^x \rangle \end{pmatrix} = \begin{pmatrix} \sigma_{1,1}^{t,t} & \sigma_{1,2}^{t,t} & \sigma_{1,3}^{t,x} & -\mu\sigma_{1,3}^{t,x} \\ \sigma_{2,1}^{t,t} & \sigma_{2,2}^{t,t} & \sigma_{2,3}^{t,x} & -\mu\sigma_{2,3}^{t,x} \\ \sigma_{3,1}^{x,t} & \sigma_{3,2}^{x,t} & \sigma^{xx} & T\alpha^{xx} \\ -\mu\sigma_{3,1}^{x,t} & -\mu\sigma_{3,2}^{x,t} & T\alpha^{xx} & T\bar{\kappa}^{xx} \end{pmatrix} \begin{pmatrix} i\omega a_t^1 \\ i\omega a_t^2 \\ E_x \\ -\frac{\nabla_x T}{T} \end{pmatrix}$$

Interpretation: a_t^1, a_t^2 rotate charge density in directions 1, 2 without changing its magnitude.



Longitudinal thermoelectric effect

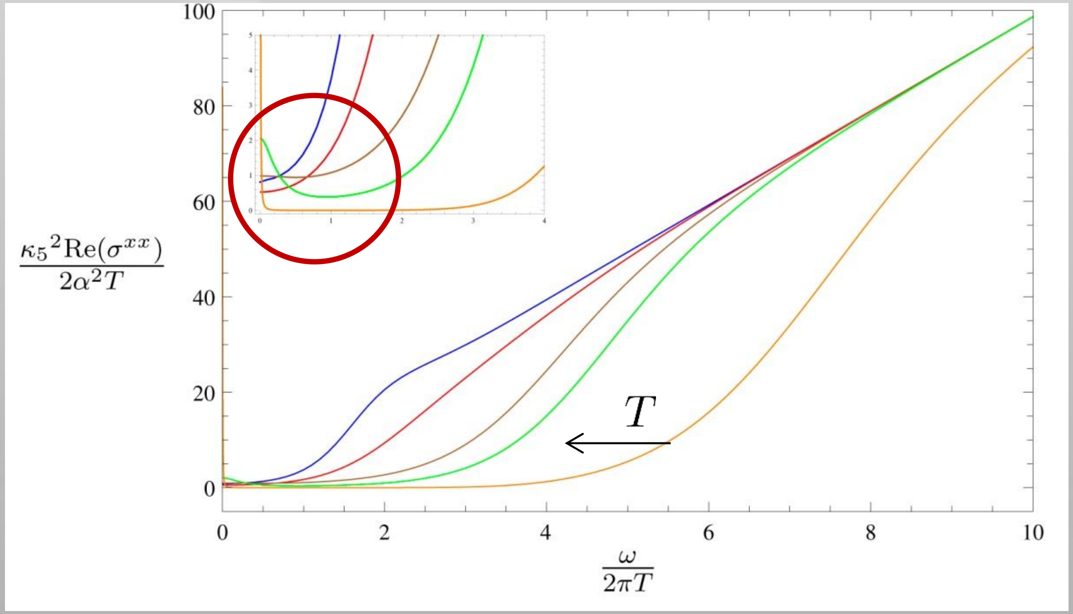
[Erdmenger, DF, Zeller]

$$\begin{pmatrix} \langle J^x \rangle \\ \langle Q^x \rangle \end{pmatrix} = \begin{pmatrix} \sigma^{xx} & T\alpha^{xx} \\ T\alpha^{xx} & T\bar{\kappa}^{xx} \end{pmatrix} \begin{pmatrix} E_x \\ -(\nabla_x T)/T \end{pmatrix}$$

Additional couplings:

$$\begin{pmatrix} \langle J^t_1 \rangle \\ \langle J^t_2 \rangle \\ \langle J^x \rangle \\ \langle Q^x \rangle \end{pmatrix} = \begin{pmatrix} \sigma^{t,t}_{1,1} & \sigma^{t,t}_{1,2} & \sigma^{t,x}_{1,3} & -\mu\sigma^{t,x}_{1,3} \\ \sigma^{t,t}_{2,1} & \sigma^{t,t}_{2,2} & \sigma^{t,x}_{2,3} & -\mu\sigma^{t,x}_{2,3} \\ \sigma^{x,t}_{3,1} & \sigma^{x,t}_{3,2} & \sigma^{xx} & T\alpha^{xx} \\ -\mu\sigma^{x,t}_{3,1} & -\mu\sigma^{x,t}_{3,2} & T\alpha^{xx} & T\bar{\kappa}^{xx} \end{pmatrix} \begin{pmatrix} i\omega a^1_t \\ i\omega a^2_t \\ E_x \\ -\frac{\nabla_x T}{T} \end{pmatrix}$$

Interpretation:
a^1_t, a^2_t rotate charge density into directions 1, 2 without changing its total amount.



<u>Differences:</u>

- Decrease starts at larger ω.
- σ does not vanish for any frequency.
- In fact, it increases again.

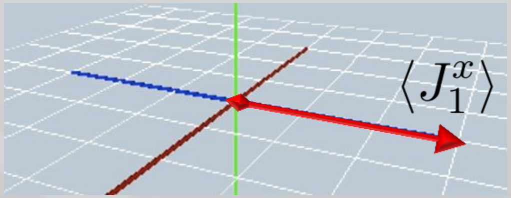
Quasinormal mode

Piezoelectric effect

$$\begin{pmatrix} \langle J^x_\pm \rangle \\ \langle T^{xx}, T^{yy}, T^{tt} \rangle \end{pmatrix} \longleftrightarrow \begin{pmatrix} a^\pm_x \\ h_{xx}, h_{yy}, h_{tt} \end{pmatrix}$$

- Generation of electric current due to elongation/squeezing.
- Generation of mechanical strain due to an external electric field.

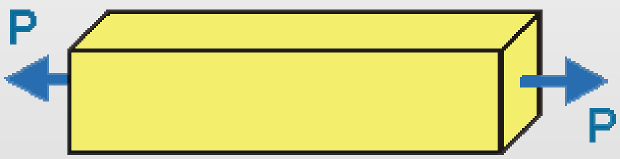
Intuitive picture:



$$\langle J^x_1 \rangle \sim A^1_x$$

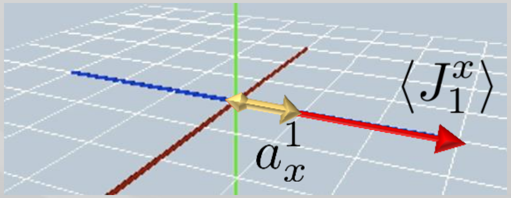
Piezoelectric effect

$$\begin{pmatrix} \langle J^x_\pm \rangle \\ \langle T^{xx}, T^{yy}, T^{tt} \rangle \end{pmatrix} \longleftrightarrow \begin{pmatrix} a^\pm_x \\ h_{xx}, h_{yy}, h_{tt} \end{pmatrix}$$



- Generation of electric current due to elongation/squeezing.
- Generation of mechanical strain due to an external electric field.

Intuitive picture:

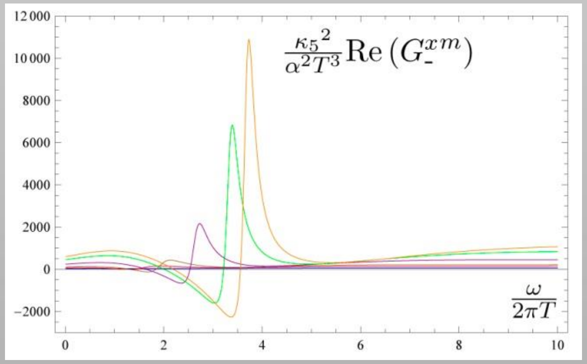


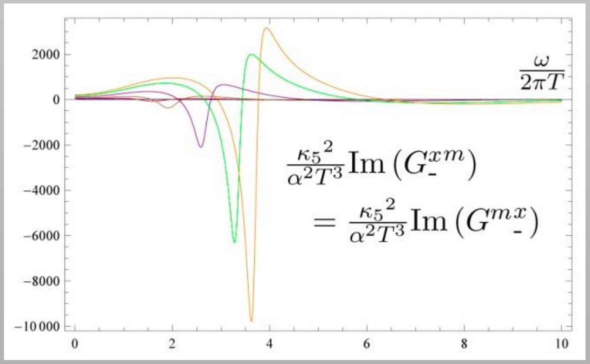
$$\langle J^x_1 \rangle \sim A^1_x + a^1_x$$

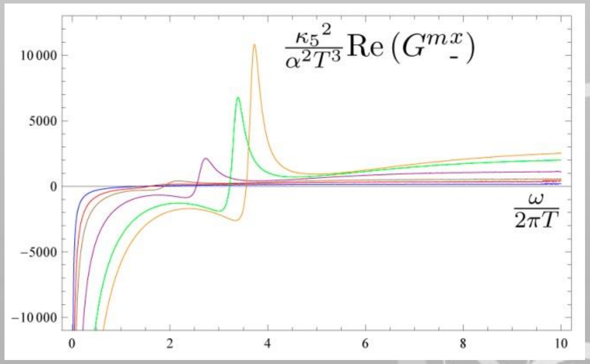
<u>Background</u>

$$\langle J^x_1 \rangle$$
$$\langle T^{xx}, T^{yy}, T^{tt} \rangle$$

[Erdmenger, DF, Zeller]

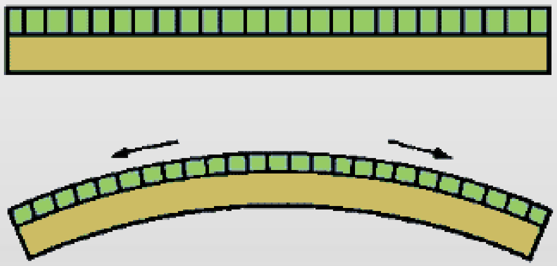






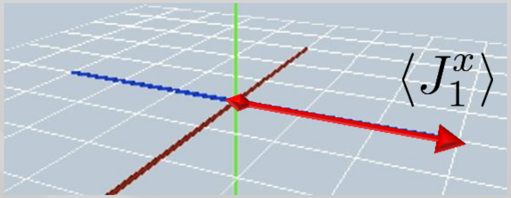
Flexoelectric effect

$$\begin{pmatrix} \langle J^y_\pm \rangle \\ \langle T^{xy} \rangle \end{pmatrix} \longleftrightarrow \begin{pmatrix} a^\pm_y \\ h_{xy} \end{pmatrix}$$



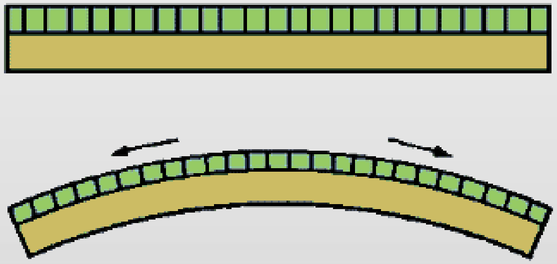
- Generation of electric current due to shear stress.
- Generation of shear deformation due to an external electric field.

Intuitive picture:



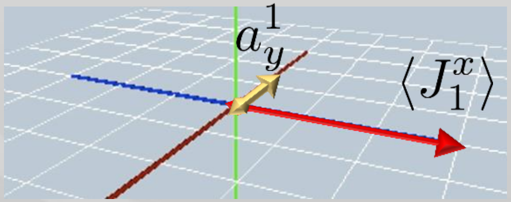
Flexoelectric effect

$$\begin{pmatrix} \langle J^y_\pm \rangle \\ \langle T^{xy} \rangle \end{pmatrix} \longleftrightarrow \begin{pmatrix} a^\pm_y \\ h_{xy} \end{pmatrix}$$



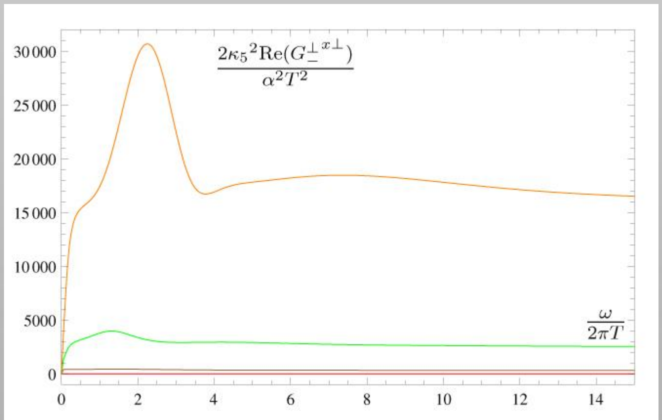
- Generation of electric current due to shear stress.
- Generation of shear deformation due to an external electric field.

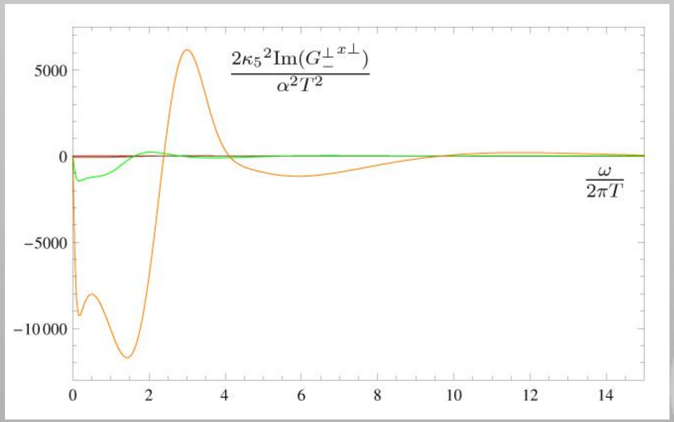
Intuitive picture:



The system tries to cancel the new contribution.

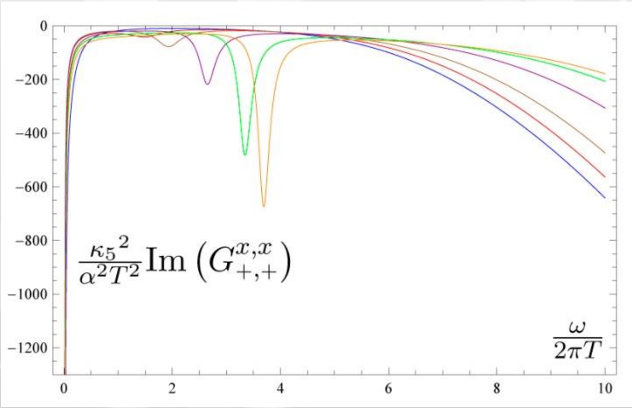
[Erdmenger, Kerner, Zeller]





Goldstone fields and Quasinormal modes

Condensate selects preferred direction $\Longrightarrow$ a_x^2 becomes Goldstone mode.



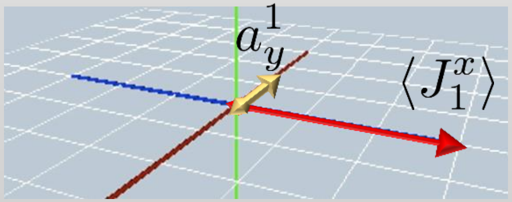
$$A_x^1 = w + \epsilon\, a_x^1,\;\; A_x^2 = \epsilon\, a_x^2$$

$$A_x^{\pm} = (w + \epsilon\,\rho)e^{\pm \mathrm{i}\theta/w}$$

$$a_x^1 \sim \rho,\;\; a_x^2 \sim \theta$$

Other GS modes:

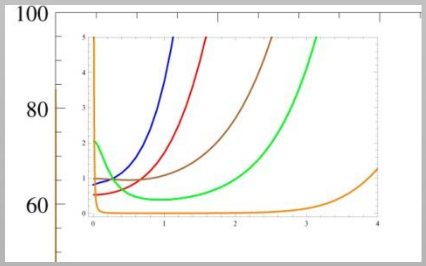
a_y^1, a_z^1

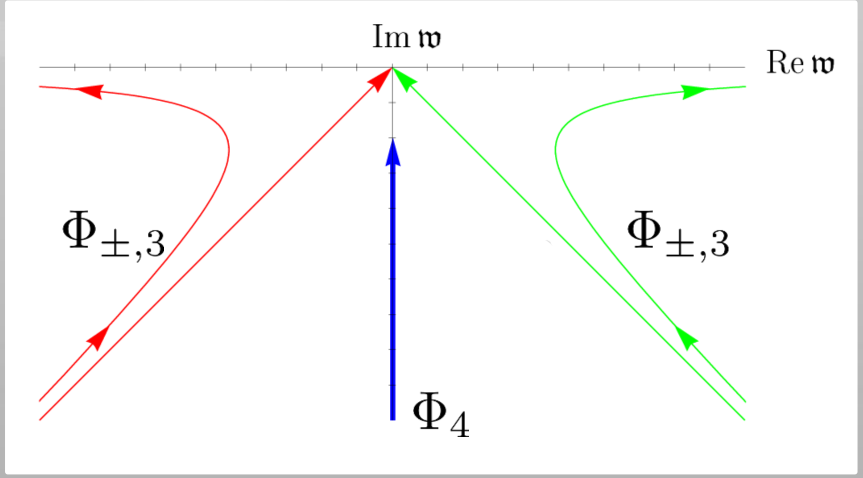


The poles at ω=0 reflect the formation of this massless mode.

Quasinormal modes behavior:

The quasinormal mode of the thermoelectric effect goes up the imaginary axis (ω=0)





The viscosity tensor

[Landau, Lifshitz]

- Internal motion of a system causes dissipation of energy.
- Postulate dissipation function. Its velocity derivatives are frictional forces, linear in u_μ.
- Translation/rotation → No dissipation, so actually linear in $u_{\mu\nu} = \frac{1}{2}\left(\nabla_\mu u_\nu + \nabla_\nu u_\mu\right)$.

$$\Xi = \frac{1}{2}\eta^{\mu\nu\lambda\rho}u_{\mu\nu}u_{\lambda\rho} \quad \Rightarrow \quad T^{\mu\nu}_{\text{diss}} = -\frac{\partial \Xi}{\partial u_{\mu\nu}}$$

- For a transversely isotropic fluid,

$$\begin{aligned}
\eta^{xxxx} &= \zeta_x + \frac{4}{3}\lambda\,, & \eta^{yyyy} &= \eta^{zzzz} = \zeta_y + \frac{\lambda}{3} + \eta_{yz}\,,\\
\eta^{xxyy} &= \eta^{xxzz} = -\frac{2}{3}\lambda\,, & \eta^{yyzz} &= \zeta_y + \frac{\lambda}{3} - \eta_{yz}\,,\\
\eta^{yzyz} &= \eta_{yz}\,, & \eta^{xyxy} &= \eta^{xzxz} = \eta_{xy}\,.
\end{aligned}$$

The viscosity tensor

[Landau, Lifshitz]

- Internal motion of a system causes dissipation of energy.
- Postulate dissipation function. Its velocity derivatives are frictional forces, linear in u_μ.
- Translation/rotation → No dissipation, so actually linear in $u_{\mu\nu} = \frac{1}{2}\left(\nabla_\mu u_\nu + \nabla_\nu u_\mu\right)$.

$$\Xi = \frac{1}{2}\eta^{\mu\nu\lambda\rho} u_{\mu\nu} u_{\lambda\rho} \quad \Rightarrow \quad T^{\mu\nu}_{\text{diss}} = -\frac{\partial \Xi}{\partial u_{\mu\nu}}$$

- For a transversely isotropic conformal fluid,

$$\begin{aligned}
\eta^{xxxx} &= \cancel{\zeta_x} + \frac{4}{3}\lambda\,, & \eta^{yyyy} &= \eta^{zzzz} = \cancel{\zeta_y} + \frac{\lambda}{3} + \eta_{yz}\,,\\
\eta^{xxyy} &= \eta^{xxzz} = -\frac{2}{3}\lambda\,, & \eta^{yyzz} &= \cancel{\zeta_y} + \frac{\lambda}{3} - \eta_{yz}\,,\\
\eta^{yzyz} &= \eta_{yz}\,, & \eta^{xyxy} &= \eta^{xzxz} = \eta_{xy}\,.
\end{aligned}$$

The viscosity tensor

[Landau, Lifshitz]

- Internal motion of a system causes dissipation of energy.
- Postulate dissipation function. Its velocity derivatives are frictional forces, linear in u_μ.
- Translation/rotation → No dissipation, so actually linear in $u_{\mu\nu} = \frac{1}{2}\left(\nabla_\mu u_\nu + \nabla_\nu u_\mu\right)$.

$$\Xi = \frac{1}{2}\eta^{\mu\nu\lambda\rho}u_{\mu\nu}u_{\lambda\rho} \Rightarrow T^{\mu\nu}_{\text{diss}} = -\frac{\partial \Xi}{\partial u_{\mu\nu}}$$

- For a transversely isotropic conformal fluid,

$$\eta^{xxxx} = \cancel{\zeta_x} + \frac{4}{3}\lambda\,, \qquad \eta^{yyyy} = \eta^{zzzz} = \cancel{\zeta_y} + \frac{\lambda}{3} + \eta_{yz}\,,$$

$$\eta^{xxyy} = \eta^{xxzz} = -\frac{2}{3}\lambda\,, \quad \eta^{yyzz} = \cancel{\zeta_y} + \frac{\lambda}{3} - \eta_{yz}\,,$$

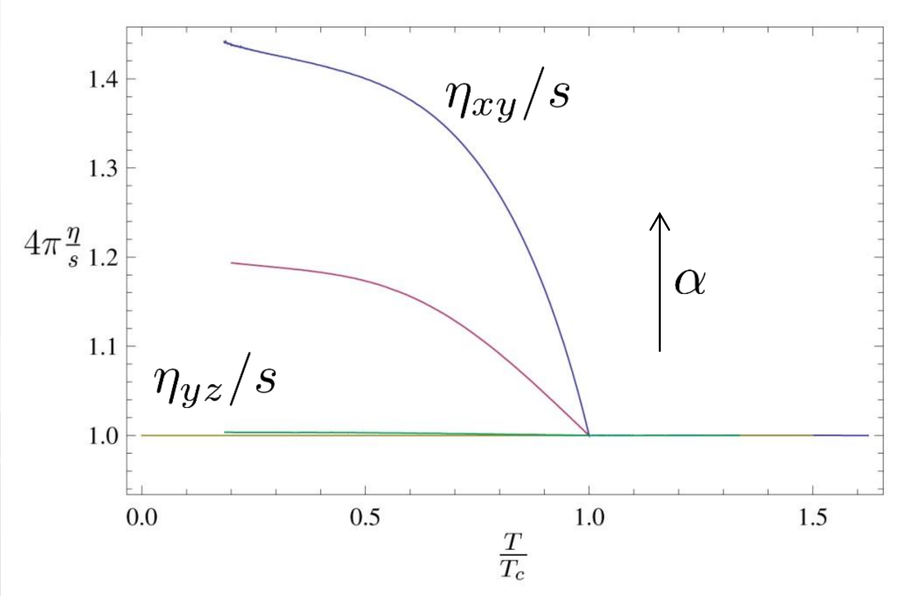
$$\boxed{\eta^{yzyz} = \eta_{yz}\,,} \qquad \boxed{\eta^{xyxy} = \eta^{xzxz} = \eta_{xy}\,.}$$

Shear viscosities

$$\eta_{yz} = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,\langle T_{yz}T_{yz}\rangle \qquad \eta_{xy} = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,\langle T_{xy}T_{xy}\rangle$$

Shear viscosities

[Erdmenger, Kerner, Zeller]
[Kovtun, Son, Starinets, Buchel, Liu, Iqbal]



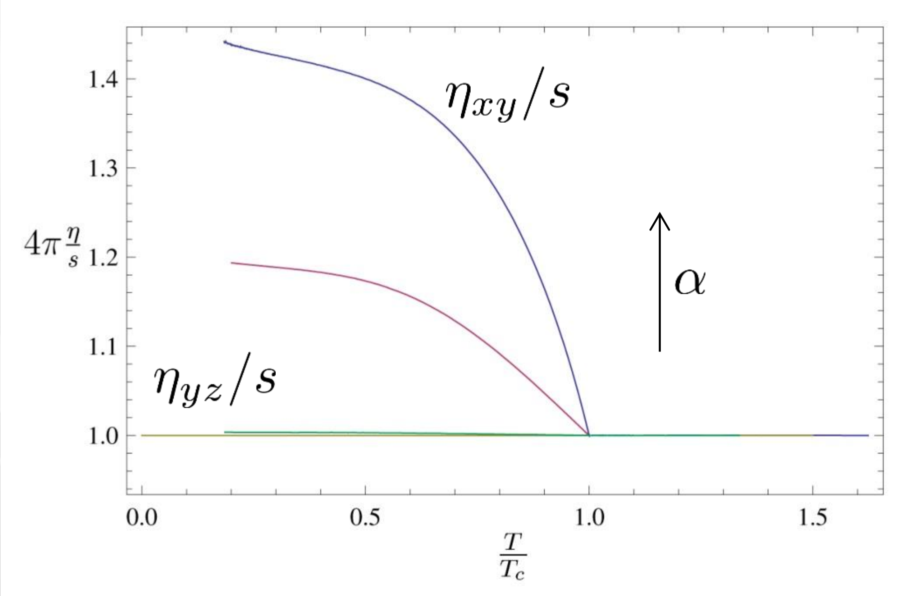
$$\frac{\eta_{yz}}{s} = \frac{1}{4\pi}$$

$$\frac{\eta_{xy}}{s} \geq \frac{1}{4\pi}$$

- In the normal phase, they coincide with the universal value of an isotropic fluid.
- In the superfluid phase, they deviate but the viscosity bound is satisfied.

Shear viscosities

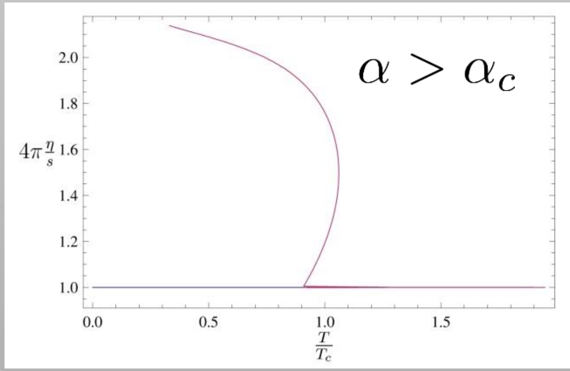
[Erdmenger, Kerner, Zeller]
[Kovtun, Son, Starinets, Buchel, Liu, Iqbal]



$$\frac{\eta_{yz}}{s} = \frac{1}{4\pi}$$

$$\frac{\eta_{xy}}{s} \geq \frac{1}{4\pi}$$

- In the normal phase, they coincide with the universal value of an isotropic fluid.
- In the superfluid phase, they deviate but the viscosity bound is satisfied.



- In the *1st* order phase transition, it is multivalued.
- The presence of anisotropy makes it deviate.

The first normal stress difference

If we assume a conformal fluid,

$$\eta^{xxxx} = \frac{4}{3}\lambda\,, \qquad \eta^{yyyy} = \eta^{zzzz} = \frac{\lambda}{3} + \eta_{yz}\,,$$

$$\eta^{xxyy} = \eta^{xxzz} = -\frac{2}{3}\lambda\,, \quad \eta^{yyzz} = \frac{\lambda}{3} - \eta_{yz}\,.$$

So that the dissipative part of the normal stress difference is:

$$\boxed{T^{xx}_{\text{diss}} - (T^{yy}_{\text{diss}} + T^{zz}_{\text{diss}}) = -\frac{4}{3}\lambda\left(\nabla_x u_x - \frac{1}{2}(\nabla_y u_y + \nabla_z u_z)\right)}$$

Among the physical fields there is

$$\Phi_3(\omega, r) \longrightarrow (\xi_x)^b_0 - (\xi_y)^b_0\,,$$ so its Green's function is identified with

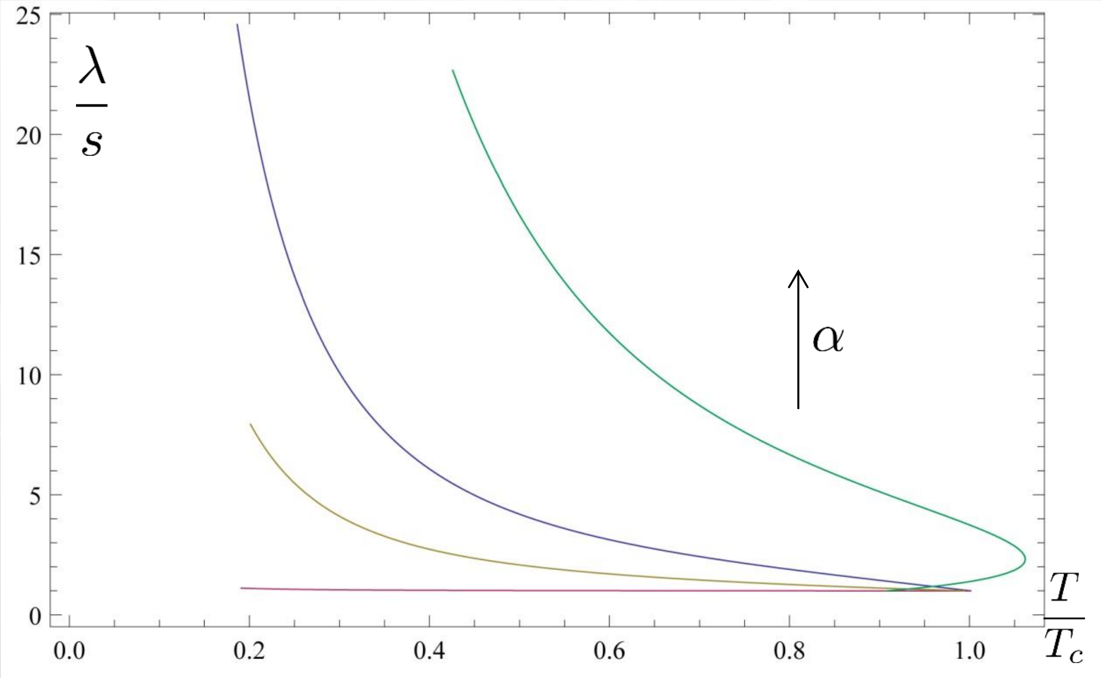
$$G^{m,m}(\omega) = \lim_{|\vec{k}|\to 0}\int \mathrm{d}t\,\mathrm{d}^3x\, e^{-\mathrm{i}k_\mu x^\mu}\theta(t)\left\langle\left[\frac{1}{2}\left(T^x{}_x(t,\vec{x}) - T^y{}_y(t,\vec{x}) - T^z{}_z(t,\vec{x})\right),\right.\right.$$
$$\left.\left.\frac{1}{2}\left(T^x{}_x(0,0) - T^y{}_y(0,0) - T^z{}_z(0,0)\right)\right]\right\rangle$$

Kubo formula:

$$\boxed{\lambda = \lim_{\omega\to 0}\frac{3}{2\,\omega}\,\mathrm{Im}\,G^{m,m}(\omega)}$$

Stress difference viscosity

[Erdmenger, DF, Zeller]

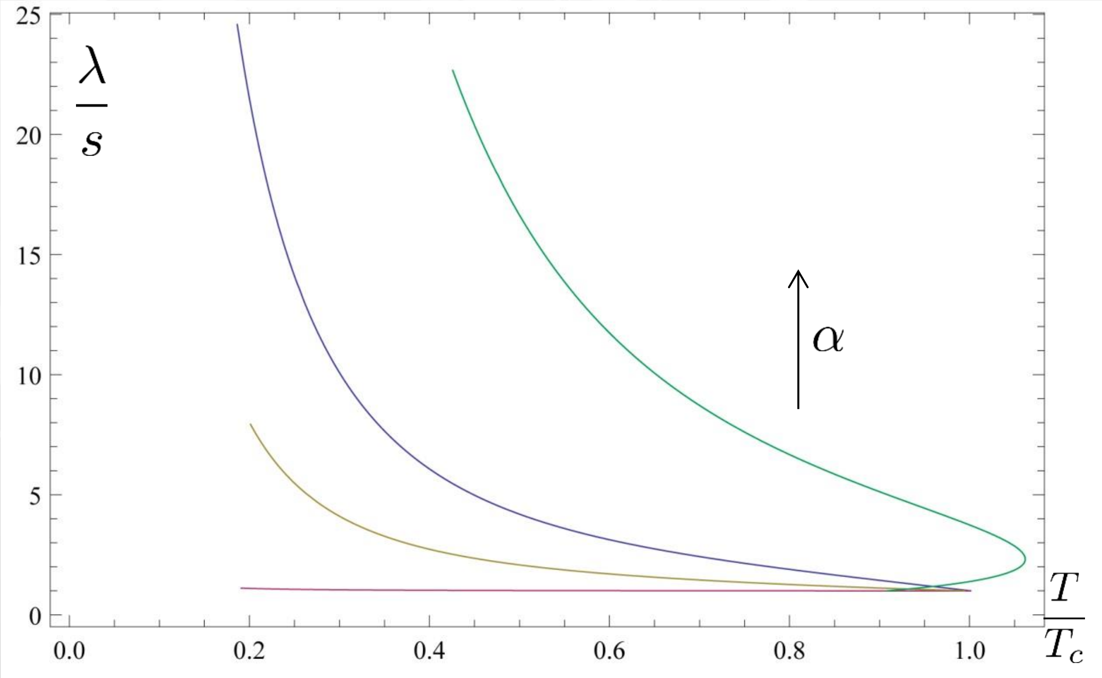


$$\frac{\lambda}{s} \geq \frac{1}{4\pi}$$

Similar to the $1/4\pi$ derivation: $h_{xx} - h_{yy}$ behaves as a minimally coupled scalar in gravity.

Stress difference viscosity

[Erdmenger, DF, Zeller]



$$\frac{\lambda}{s} \geq \frac{1}{4\pi}$$

Similar to the $1/4\pi$ derivation: $h_{xx} - h_{yy}$ behaves as a minimally coupled scalar in gravity.

- Coventionally, normal stresses pull apart compressing surfaces.
- Spinning rod in material $\Longrightarrow$ Fluid is expelled outwards: $\lambda > 0$.
- Effect more pronounced for lower T.

Conclusions

- ✓ Check of the **universal bound** for the ratio η/s.
- ✓ Found **thermoelectric effect** favored by the condensate:
  - Enhancement of conductivity for low T (high ω), suppression above T_c .
  - Sudden increase due to a pole near ω=0, due to quasinormal mode.
- ✓ New phenomena: **Flexoelectric** and **Piezoelectric** effects.
  - Bumps in correlators, related to possible bound states.
- ✓ In the ω=0 limit, found new component of **viscosity tensor**.

- ➢ Results valid as effective macroscopic description of transport properties near T_c .

Outlook:

- Covariant hydrodynamic description of anisotropic superfluids.
- Analysis at finite k: Dispersion relations and new instabilities.

Thank you!