# A Holographic model of the Kondo effect

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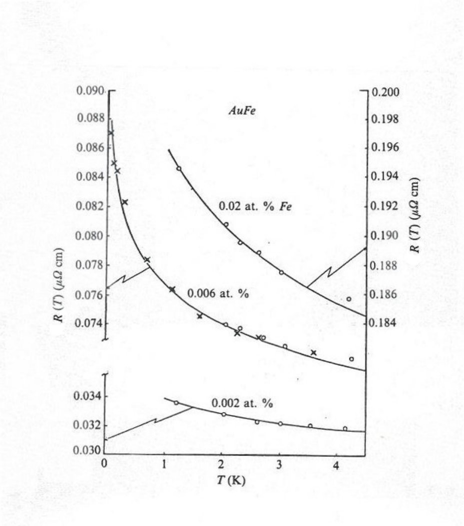
# Outline

- Introduction
- Stringy model
- Bottom-up model
- Future directions

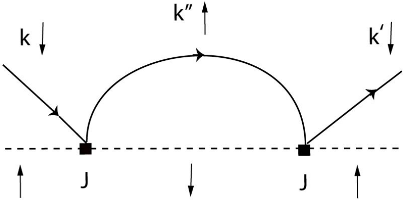
# Kondo effect

Metals: Fermi liquid+phonons+impurities: $\rho \sim \rho_0 + T^2$
But in some metals at low temperatures $\rho \sim -\log(T)$



# Kondo effect

Scattering with magnetic impurities

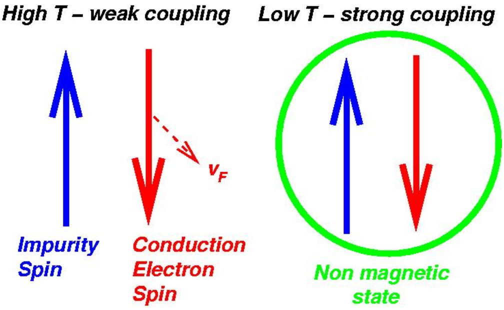



$$\rho \sim \rho_0 \left( 1 + \kappa \log \frac{T}{|\epsilon - \epsilon_F|} \right)$$

Antiferromagnetic coupling $\kappa < 0$

# Kondo effect

Perturbation theory breaks down at $T_K = |\epsilon - \epsilon_F| e^{1/\kappa}$



# Kondo effect

- Single impurity problem solved
  Wilson's RG, Nozières' Fermi liquid description, the Bethe Ansatz, large-$N$ limits, conformal field theory...
- Multiple impurities:
  Heavy fermion compounds with strange metal behaviour

$$\rho \sim T$$

  may be described by a Kondo lattice
- Holography: non-perturbative, large-$N$

Goal today: construct a holographic model for a single impurity

# CFT approach

Kondo model [Kondo' 66; Affleck & Ludwig '90s]:

$$H = \frac{v_F}{2\pi}\,\psi_L^\dagger i\partial_x \psi_L + v_F\, \lambda_K\, \delta(x)\, \vec{S} \cdot \psi_L^\dagger\, \frac{1}{2}\vec{\tau}\, \psi_L,$$

- s-wave reduction: 1+1 dimensions
- Kondo coupling marginal classically (CFT)
- Asymptotic freedom: UV fixed point

## UV CFT

- Symmetries: Spin $SU(N)$, $k$ channels $SU(k)$, Charge $U(1)$
- Kac-Moody algebra: $SU(N)_k \times SU(k)_N \times U(1)$

$$[J_n^a, J_m^b] = i f^{abc} J_{n+m}^c + k\,\frac{n}{2}\,\delta^{ab}\,\delta_{n,-m}$$

- Finite number of highest weight states. $SU(2)_k$: spin $\leq k/2$
- Sugawara construction:

$$H = \frac{1}{2\pi(N+k)} J^a J^a + \frac{1}{2\pi(k+N)} J^A J^A + \frac{1}{4\pi N k} J^2 + \lambda_K\,\delta(x)\,\vec{S}\cdot\vec{J}$$

# IR CFT

- Redefinition of spin current:

$$\mathcal{J}^a \equiv J^a + \pi(N+k)\lambda_K \delta(x) S^a$$

- Critical coupling

$$\lambda_K = \frac{2}{N+k}$$

- Hamiltonian:

$$H = \frac{1}{2\pi(N+k)}\mathcal{J}^a\mathcal{J}^a + \frac{1}{2\pi(k+N)} J^A J^A + \frac{1}{4\pi Nk} J^2$$

No impurity!

[Affleck & Ludwig '95]

# CFT approach

- IR CFT = UV CFT with shifted spectrum
- IR spin representations = UV spin representations + impurity spin

Example: one channel, spin $N = 2$, $SU(2)_1 \times U(1)$, $s_{\rm imp} = 1/2$

- UV: Neveu-Schwarz boundary conditions
  (spin, charge)$=$ $(0, 0)$, $(\pm 1/2, 1)$
- IR: Ramond boundary condition
  (spin, charge)$=$ $(\pm 1/2, 2)$, $(0, 1)$
- Phase of electron wavefunction on a circle changes by $\pi/2$

# CFT approach

Possible phases in $SU(2)_k$:

- Underscreening: $2s_{\rm imp} > k$
  Fermi liquid + impurity of spin $|s_{\rm imp} - k/2|$
- Critical screening: $2s_{\rm imp} = k$ IR fixed point: $k$ free left-movers
- Overscreening: $2s_{\rm imp} < k$ Non-trivial IR fixed point:
  non-Fermi liquid behavior

Qualitatively similar for higher spin

# Large-$N$

- $N \to \infty$, $\lambda_K \to 0$, $\lambda_K N$ fixed
- Spin of impurity: Young tableaux with $Q$ boxes
- Totally antisymmetric representation:

$$S^a = \chi^\dagger T^a \chi$$

  Slave fermions, dimension $[\chi] = 0$

$$\chi^\dagger \chi = Q$$

  Additional $U(N_f)$ symmetry
- Critical ($k = 1$) or overscreening ($k \geq 2$)

Kondo coupling as double-trace deformation:

$$\lambda_K\,\delta(x)\,J^aS^a = \lambda_K\,\delta(x)\,\left(\psi_L^\dagger T^a\psi_L\right)\,\left(\chi^\dagger T^a\chi\right)$$
$$= \frac{1}{2}\lambda_K\,\delta(x)\left(\psi_L^\dagger\chi\right)\left(\chi^\dagger\psi_L\right)$$
$$= \frac{1}{2}\lambda_K\,\delta(x)\,\mathcal{O}\mathcal{O}^\dagger$$

- $\mathcal{O}$ $SU(N)$ singlet, charged under $U(N_f)\times SU(k)\times U(1)$
- Dimensions: $[\psi_L]=[\mathcal{O}]=1/2$
- Mean field transition:

$$T>T_K,\quad \langle\mathcal{O}\rangle=0,\quad SU(k)\times U(N_f)\times U(1)$$

$$T<T_K,\quad \langle\mathcal{O}\rangle\neq 0,\quad SU(k)\times U(N_f)\times U(1)\to U(1)_D$$

[Senthil, Sachdev, Vojta]

# Large-$N$

Summary of Kondo effect at large $N$

- s-wave reduction: 1+1 chiral CFT + impurity
- double-trace coupling
- 0+1 superconductor

These will be the main ingredients to construct a holographic model

# Impurities in stringy models

Supersymmetric defects with localized fermions

- D5/D3 $AdS_2 \subset AdS_5$
  [Kachru, Karch, Yaida] [Harrison, Kachru, Torroba]
- M2/D2 in ABJM $AdS_2 \subset AdS_4$
  [Jensen, Kachru, Karch, Polchinski, Silverstein]
- D6 in ABJM $AdS_2 \subset AdS_4$, [Benincasa, Ramallo]
  with backreaction [Itsios, Sfetsos, Zoakos]
- $D(8-p)$ in $Dp$ background $S^{7-p} \subset S^{8-p}$ [Benincasa, Ramallo]
  other sphere wrappings [Karaiskos, Sfetsos, Tsatis]
- Spectrum of Wilson loops
  [Mueck] [Faraggi, Pando Zayas] [Faraggi, Mueck, Pando Zayas]

# Stringy model

| | $x^0$ | $x^1$ | $x^2$ | $x^3$ | $x^4$ | $x^5$ | $x^6$ | $x^7$ | $x^8$ | $x^9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $N_c$ D3 | • | • | • | • | – | – | – | – | – | – |
| $N_7$ D7 | • | • | – | – | • | • | • | • | • | • |
| $N_5$ D5 | • | – | – | – | • | • | • | • | • | – |

- 3-7 strings = chiral fermions (current algebra)
- 3-5 strings = slave fermions
- 5-7 strings = bifundamental scalar (tachyon)

D5 branes become magnetic flux on D7 branes

$$S_{D7} \subset \int_{D7} P[C_6] \wedge F$$

# Stringy model

- $N_c \to \infty$ : $AdS_5 \times S^5$ background

$$\int_{S^5} F_5 = g_s (2\pi)^2 (2\pi\alpha')^2 \, N_c$$

- 7-7 strings: gauge field $A_\mu$ dual to $J_\mu$

$$S_{D7} \supset -\frac{N_c}{4\pi} \int_{AdS_3} \mathrm{tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$

Global symmetry:

$$U(N_7)_{N_c}$$

# Stringy model

- 5-5 strings: gauge field $a_m$ dual to $Q$

$$S_{D5} \supset N_5 \, T_{D5} \int P[C_4] \wedge f$$

- Dual to completely antisymmetric Wilson loop [Yamaguchi '06; Gomis, Passerini '06]
- Charge $Q =$ number of fundamental strings
- Size of D5 on $S^5$ [Camino,Paredes, Ramallo '01]

$$ds^2_{S^5} = d\theta^2 + \sin^2\theta \, ds^2_{S^4} , \quad \theta_Q = \frac{Q}{N_c}\pi$$

Maximal charge $Q = N_c - 1$

# Stringy model

- 5-7 strings: bifundamental scalar $\Phi$ dual to $\mathcal{O}$
- Double-trace coupling for $\mathcal{O}$ analogous to Kondo coupling
- Double-trace coupling can lead to condensation [Pomoni, Rastelli '08,'10]
- Holographic dual: boundary condition for $\Phi$ [Witten '01]

# Bottom-up model

- s-wave reduction: 1+1 CFT $\longrightarrow AdS_3$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{z^2}\left(\frac{dz^2}{h(z)} - h(z)\, dt^2 + dx^2\right), \quad h(z) = 1 - z^2/z_H^2,$$

Temperature: $T = 1/(2\pi z_H)$

- $SU(N)$-spin, $k = 1$ channel of chiral fermions, $U(1)$ charge

$$S_{CS} = -\frac{N}{4\pi}\int A \wedge dA$$

- Impurity $AdS_2$: $U(1)$ symmetry, operator $\mathcal{O}$

$$S_{AdS_2} = -\int d^3x\, \delta(x)\sqrt{-g}\left[\frac{1}{4} f^{mn} f_{mn} + g^{mn}\left(D_m\Phi\right)^\dagger D_n\Phi + M^2\Phi^\dagger\Phi\right]$$

$$D_m\Phi = \partial_m\Phi + iA_m\Phi - ia_m\Phi$$

# Asymptotics

- Gauge field: charge $Q$ determines the spin representation of impurity

$$a_t(z) = \frac{Q}{z} + \mu$$

Charge is an irrelevant operator: UV behavior is modified
Scalar field effective mass fixed at the BF bound

$$M_{\rm eff}^2 = M^2 - Q^2 = -\frac{1}{4}$$

- Scalar field at the BF bound:

$$\Phi = z^{1/2}(\alpha \log(z) + \beta)$$

UV conformal dimensions $\Delta = \frac{1}{2}$
We set $M^2 = 0$, $Q = -1/2$

# Kondo coupling

- Double-trace deformation = boundary condition [Witten '01]

$$\alpha = \kappa\beta$$

- Renormalization:

$$\Phi = z^{1/2}\beta_0(\kappa_0 \log(\Lambda z) + 1) = z^{1/2}\beta(\kappa \log(\mu z) + 1)$$

- Running coupling

$$\kappa = \frac{\kappa_0}{1 + \kappa_0 \ln\left(\frac{\Lambda}{\mu}\right)}$$

Dynamical scale: $\Lambda_K = \Lambda e^{1/\kappa_0}$

- $\kappa < 0$ "antiferromagnetic": UV asymptotic freedom

# Phases

Normal phase ($\Phi = 0$):

- Background charge $Q = -1/2$:

$$a_t(z) = \frac{Q}{z} + \mu, \quad \mu = -\frac{Q}{z_H}$$

Broken phase ($\Phi \neq 0$):

- Background charge $Q = -1/2$:

$$a_t(z) \simeq \frac{Q}{z} + \mu_T + O((\log z)^3)$$

- Background scalar field:

$$\Phi \simeq (z/z_H)^{1/2} \beta_T (\kappa_T \log(z/z_H) + 1)$$

- Values of $\mu_T$, $\kappa_T$ and $\beta_T$ determined numerically

# Kondo coupling

- Finite temperature solution:

$$\Phi = (z/z_H)^{1/2}\beta_T(\kappa_T \log(z/z_H) + 1) = \beta_0(\kappa_0 \log(\Lambda z) + 1)$$

- Temperature-dependent coupling

$$\kappa_T = \frac{\kappa_0}{1 + \kappa_0 \ln\left(\frac{\Lambda}{2\pi T}\right)}$$

- High-temperatures $T \gg \Lambda_K$:

$$\kappa_T \simeq \frac{1}{\ln\left(\frac{\Lambda}{2\pi T}\right)} < 0$$

- Low temperatures $T \ll \Lambda_K$

$$\kappa_T \simeq \frac{1}{\ln\left(\frac{\Lambda}{2\pi T}\right)} > 0$$

Scalar field @ BF bound $\Phi = e^{-i\omega t}\phi$:

$$h\partial_z(h\partial_z\phi) + \omega^2\phi + \frac{1}{4z^2}\phi = 0$$

Tachyonic modes $\omega = -i\Omega$ exist when

$$\kappa = \frac{\alpha}{\beta} = \frac{1}{H_{\frac{1}{2}\left(\sqrt{4\Omega^2-1}-1\right)} - \log 4}$$

This is possible $\forall\kappa$ except $\kappa_c < \kappa \leq 0$,

$$\kappa_c = \frac{1}{H_{-\frac{1}{2}} - \log 4} \simeq -0.360674$$

Stable at high temperatures, unstable at low temperatures

# Free energy

- Free energy = Euclidean action = - Action
- Counterterms scalar field:

$$S_\Phi = -\int dt\,\sqrt{-\gamma}\left(\frac{1}{2}+\frac{1}{\log\varepsilon}\right)\Phi^\dagger\Phi + \kappa\int dt\,\beta^2$$

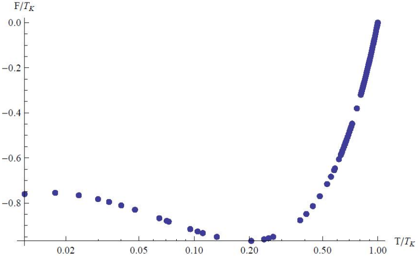
- Counterterms gauge field:

$$S_{a_t} = +\frac{1}{2}\int dt\,\sqrt{-\gamma}\,\gamma^{tt}\,a_t^2$$

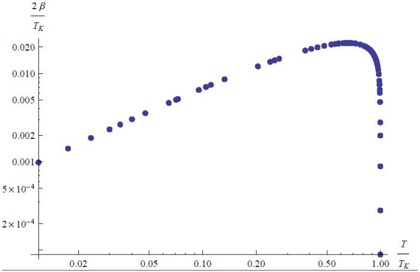
$$F - F_0 = -\alpha\beta - \frac{1}{2}Q(\mu+Q) - 2Q^2\alpha^2 + 2Q^2\alpha\beta - Q^2\beta^2 - \kappa\beta^2$$

+ finite bulk integral

# Thermodynamics of the broken phase




Free energy vs $T/T_K$




Condensate vs $T/T_K$

# Chern-Simons field

$$S = \int dz \int \frac{d\omega d\omega' dq dq'}{(2\pi)^2} \left[ A_\mu(\omega', q') D^{\mu\nu}(\omega, q) \delta(\omega - \omega') \delta(q - q') A_\nu(\omega', q') \right.$$
$$\left. + A_\mu(\omega', q') B^{\mu\nu}(\omega) \delta(\omega - \omega') A_\nu(\omega', q') + j^m(\omega) \delta(\omega - \omega') \delta(q - q') A_m(\omega, q') \right].$$

$$D^{\mu\nu}(\omega, q) = \frac{k}{2\pi} \left[ \epsilon^{\mu z \nu} \partial_z - i\omega \epsilon^{\mu t \nu} + iq \epsilon^{\mu x \nu} \right],$$

$$B^{\mu\nu}(\omega) = \frac{1}{2\pi} \sqrt{-g} g^{mn} \delta^\mu_m \delta^\nu_n \Phi^\dagger \Phi = \frac{1}{2\pi} \phi^2 \sqrt{-g} g^{mn} \delta^\mu_m \delta^\nu_n.$$

$$j^m(\omega) = \sqrt{g} g^{mn} a_n \Phi^\dagger \Phi = -\frac{a_t \phi^2}{h(z)} \delta^m_t$$

# Background fields

- Solutions:

$$F_{tx} = -\frac{2\pi}{k} j^z(\omega) + \delta F_{tx}, \quad F_{zx} = \frac{2\pi}{k} j^t(\omega) + \delta F_{zx}$$

- Normal phase:

$$F_{tx} = F_{zx} = 0$$

- Broken phase:

$$F_{tx} = 0, \quad F_{zx} = \frac{2\pi}{k} \frac{a_t \phi^2}{h(z)} \delta(x)$$

$F_{zx}$ conjugate to $A_t$ : charge localized at the defect

# Fluctuations

$$0 = \frac{k}{2\pi}\delta F_{tx}(\omega, q) + \phi^2 h(z) \int \frac{dq'}{2\pi}\delta A_z(\omega, q'),$$
$$0 = \frac{k}{2\pi}\delta F_{zx}(\omega, q) + \frac{\phi^2}{h(z)} \int \frac{dq'}{2\pi}\delta A_t(\omega, q'),$$
$$0 = \frac{k}{2\pi}\delta F_{zt}(\omega, q).$$

Zero-momentum fluctuations in the broken phase:

$$\delta A_m = \partial_m \lambda, \quad \partial_x \lambda = 0, \quad \lambda(\omega, q) = 2\pi\delta(q)\tilde{\lambda}(\omega)$$

Fluctuations transverse to $AdS_2$ defect $\delta A_x$ decouple from $\delta A_m$
$\Delta = 1$ scalar operator localized at the impurity

# Fluctuations

- Equation of zero-momentum fluctuations

$$\partial_z(h(z)\phi^2\partial_z\tilde{\lambda}(\omega)) + \frac{\omega^2\phi^2}{h(z)}\tilde{\lambda}(\omega) = 0$$

- Boundary:

$$\tilde{\lambda}(\omega) \sim \frac{\sqrt{z}}{\phi h(z)}\left(A Y_0(\omega z) + B J_0(\omega z)\right)$$

- Horizon:

$$\tilde{\lambda}(\omega) \sim \frac{\sqrt{z}}{\phi h(z)}\left(C_{\rm out}(1-z)^{1+i\omega/2} + C_{\rm in}(1-z)^{1-i\omega/2}\right)$$

- Quasinormal modes: $A = 0$, $C_{\rm out} = 0$

# Correlators

- Action of Chern-Simons field fluctuations

$$S = -\lim_{z\to 0}\int \frac{d\omega}{2\pi}\left[\frac{ik\omega}{\pi}\tilde{\lambda}(\omega)\delta A_x(-\omega) + \omega^2\phi^2\tilde{\lambda}(\omega)\partial_z\tilde{\lambda}(-\omega)\right].$$

- Counterterms

$$S_{ct} = -\lim_{z\to 0}\frac{1}{2}\int dtdx\delta(x)\sqrt{-\gamma}\gamma^{tt}A_t^2 = -\lim_{z\to 0}\frac{1}{2}\int\frac{d\omega}{2\pi}\frac{\omega^2}{z}\tilde{\lambda}(\omega)\tilde{\lambda}(-\omega).$$

- Correlation function

$$\langle J^x J^x\rangle(\omega, q, q') \simeq \frac{\delta^2}{\delta A^2}(S + S_{ct})$$

# Correlators

- Two-point function:

$$\langle J^x J^x \rangle(\omega, q, q') \simeq \left[ \frac{8}{\pi^2 \kappa(\omega)} - \frac{4}{\pi} G(\omega) \right] \delta(q)\delta(q'),$$

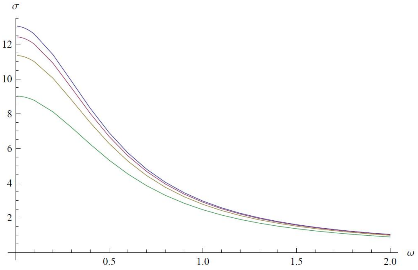
- Frequency-dependent coupling:

$$\kappa(\omega) = \frac{\kappa_T}{1 - \kappa_T \log\left(\frac{\omega e^{\gamma_E}}{2}\right)}$$

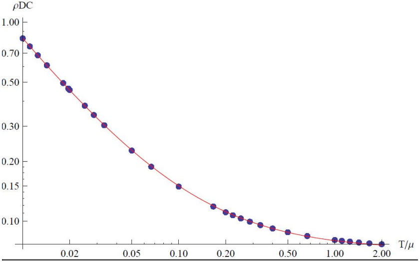
- Conductivity:

$$\sigma(\omega) \propto \frac{1}{\omega} \mathrm{Im}\, G(\omega).$$

# Conductivity & resistivity




conductivity vs $\omega/(2\pi T)$




DC resistivity vs $T/\mu$

Lowest quasinormal mode $\sim$ Kondo resonance $\frac{\omega_0}{2\pi T} \simeq -1.4i$

Resistivity grows at low temperatures!

# Summary

Holographic toy model implements the large-$N$ and CFT approach:

- Current algebra in 1+1 from s-wave reduction
- Wilson line as slave fermions on defect
- Kondo coupling = double-trace coupling

and captures main physical properties:

- Dynamical scale generation and asymptotic freedom
- Raise in the resistivity at low temperatures

# Future directions

- Compute other properties: entropy, heat capacity, magnetic susceptibility, Wilson's ratio, spectrum of operators
- Multi-channel Kondo model
- Impurities in different representations of spin
- Several impurities with interactions
- Models with small spin?