From AdS to Ricci flat: holography and the Gregory-Laflamme instability

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2013.05.15
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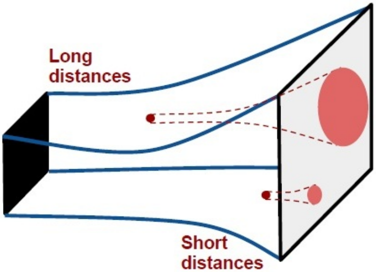
Based on arXiv:1211.2815 (published in PRD RC) and ongoing work with M. Caldarelli, J. Camps and K. Skenderis

Holography in AdS

[MALDACENA'97]: Type IIB SUGRA on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ sYM.

$N \to +\infty$ with $\lambda = g_{YM}^2 N$ fixed
$\Rightarrow$ Gravity is **classical**

$\lambda \gg 1 \Rightarrow$ String Theory reduces to **Einstein gravity** $+ O(1)$ matter fields



$$\text{Fields in AdS } \Psi(\rho, z^\mu) \quad \longleftrightarrow \quad \text{Local operators in CFT } \mathcal{O}(z^\mu)$$

$$Z_{CFT}[\Phi_0] = \langle e^{-\int g[\Phi_0]\mathcal{O}_\Phi} \rangle \sim e^{-S_\Lambda\left[\Phi(\rho,z^\mu) \underset{\rho\to 0}{\sim} \Phi_0(z^\mu)\rho^\Delta\right]}\Bigg|_{on-shell}$$

$$\Phi(\rho, z^\mu) \underset{\rho\to 0}{\sim} \Phi_0(z^\mu)\, \rho^\Delta + \Phi_1(z^\mu)\, \rho^{d-\Delta} + \dots, \quad m^2 L^2 = \Delta(d-\Delta)$$

Φ_0 is the source (non-normalisable), Φ_1 vev of $\mathcal{O}_\Phi$ (normalisable)

Holography for non-AdS spacetimes

- The arguments for holography are not directly dependent upon asymptotics: scaling of the black hole entropy with the area of the event horizon, geometrisation of the RG flow.
- The best-grounded case of the duality is also the simplest symmetry-wise: **scale invariant**, no running of the beta function. Conformal symmetry (no running of the coupling) basically fixes (the asymptopia of) the gravity dual: (Poincaré) AdS. Other scaling symmetries? Different asymptotics?
- Other cases: nonconformal branes, [Kanitscheider&al'08, Wiseman&Withers'08], related to AdS by generalised dimensional reduction [Kanitscheider&Skenderis'09]. Turn on KK vectors in the reduction [Gouteraux&al'11]
- Non-relativistic holography: Schrödinger [Guica&al'10, Chemissany&al'12], Horava gravity [Janiszewski&Karch'12, Horava&al'12]

Flat space holography

- **Holographic counterterms** for asymptotically flat spacetimes, [Kraus&al'99,Mann&Marolf'05]
- Defining feature of AdS holography: the Fefferman-Graham expansion (in $d+1$ dimensions)

$$\frac{\mathrm{d}s^2}{\ell^2} = \frac{\mathrm{d}\rho^2}{4\rho^2} + \frac{1}{\rho}\mathfrak{g}_{\mu\nu}(\rho, z^\lambda)\mathrm{d}z^\mu \mathrm{d}z^\nu \,,$$

$$\mathfrak{g}(\rho, z) = g_{(0)}(z) + \rho g_{(2)}(z) + \ldots + \rho^{\frac{d}{2}}\left(g_{(d)}(z) + h_{(d)}(z)\log\rho\right) + \ldots,$$

- Algebraic equations for the coefficients organised in terms of powers of the radial coordinates
- Flat spacetimes: differential equations $\Rightarrow$ nonlocal.
- BMS group, [Barnich&al]; 3D gravity, [Barnich&al, Bagchi&al].
- Define the field theory on the lightcone boundary: time as an extra holographic coordinate, [de Boer&Solodukhin'03, Caldeira Costa'12]
- Microscopic theory?

Aim of this talk

- Prove the following gravitational statement:

  Asymptotically locally AdS spacetimes with a transverse planar subspace can be mapped to Ricci-flat spacetimes with a transverse sphere.

- Take advantage of this to take a (small) step towards holography in Ricci-flat spacetimes.

- In the hydrodynamic limit, derive the hydro stress-tensor for flat p-branes and the dispersion relation of the Gregory-Laflamme instability from AdS quantities.

- **What I will not do**: set up a holographic dictionary for asymptotically flat spacetimes.

Some intuition from nonconformal branes

$$S=\int \mathrm{d}^{p+2}x\,\sqrt{-g}\left[\mathcal{R}-\frac{1}{2}\partial\phi^2+V_0 e^{-\delta\phi}\right]$$

The above action has the following planar black holes:

$$\mathrm{d}s^2=r^{\frac{2\theta}{p}}\left[\frac{L^2\mathrm{d}r^2}{r^2 f(r)}+\frac{-f(r)\mathrm{d}t^2+\mathrm{d}R^2_{(p)}}{r^2}\right],\quad f(r)=1-\frac{r^{p+1-\theta}}{r_0^{p+1-\theta}}$$

$$e^{\phi}=r^{\frac{2\theta}{p\delta}},\quad \theta=\frac{p^2\delta^2}{p\delta^2-2},\quad V_0=\frac{(p-\theta)(p+1-\theta)}{L^2},$$

- Violate hyperscaling $S\sim T^{p-\theta}$, near-extremal limit of hairy AdS black holes [Gouteraux&Kiritsis'11] (irrelevant gauge field).
- $\theta<0$ ($\delta^2<2/\mathrm{p}$): thermodynamics similar to the AdS planar black hole.
- $\theta>0$ ($\delta^2>2/\mathrm{p}$): thermodynamics similar to the Schwarzschild black hole.

From AdS to nonconformal branes

$$S = \int \mathrm{d}^{d+1}x \sqrt{-g}\,[\mathcal{R} - 2\Lambda]$$

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$$+$$

$$\mathrm{d}s^2_{(d+1)} = e^{-\delta\phi}\mathrm{d}s^2_{(p+2)} + e^{\frac{2\phi}{p\delta}\left(1-\frac{p}{2}\delta^2\right)}\mathrm{d}X^2_{(d-p-1)}\,, \qquad \mathcal{R}[X]_{ij} = \frac{\mathcal{R}[X]h_{ij}}{d-p-1}$$

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$$\Downarrow$$

$$S = \int \mathrm{d}^{p+2}x\,\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}\partial\phi^2 - 2\Lambda e^{-\delta\phi} + \mathcal{R}[X]e^{-\frac{2\phi}{p\delta}}\right]$$

X is constrained to be an Einstein space by the eoms. The low-d equations read

$$G_{\alpha\beta} = \frac{1}{2}\partial_\alpha\phi\partial_\beta\phi + \frac{g_{\alpha\beta}}{2}\left[-2\Lambda e^{-\delta\phi} + \mathcal{R}[X]e^{-\frac{2\phi}{p\delta}}\right]$$

$$\Box\phi = -2\delta\Lambda e^{-\delta\phi} + \frac{2}{p\delta}\mathcal{R}[X]e^{-\frac{2\phi}{p\delta}}$$

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$$\Box\phi=-2\delta\Lambda e^{-\delta\phi}+\frac{2}{p\delta}\mathcal{R}[X]e^{-\frac{2\phi}{p\delta}}$$

The low-d action and field equations are invariant under

$$-2\Lambda\longleftrightarrow\mathcal{R}[X]\,,\qquad \delta\longleftrightarrow\frac{2}{p\delta}$$ [Goutéraux&al'11]

Pointed out by [Charmousis&Gregory'03] for Weyl metrics.

From AdS to Ricci-flat spacetimes

Let us take advantage of this invariance and derive our previous statement.
Classical solutions of the **action**

$$S = \int \mathrm{d}^{d+1}x \sqrt{-g}\,[\mathcal{R} - 2\Lambda]$$

which respect the following symmetries

$$\mathrm{d}s^2_{(d+1)} = e^{-\delta\phi}\mathrm{d}s^2_{(p+2)} + e^{\frac{2\phi}{p\delta}\left(1-\frac{p}{2}\delta^2\right)}\mathrm{d}R^2_{(d-p-1)}\,, \qquad \mathcal{R}[R] = 0$$

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can be mapped to classical solutions of the **action**

$$S = \int \mathrm{d}^{p+n+3}x \sqrt{-g}\, \mathcal{R}$$

which respect the symmetries

$$\mathrm{d}s^2_{(p+n+3)} = e^{-\frac{2\phi}{p\delta}}\mathrm{d}s^2_{(p+2)} + e^{\delta\phi\left(1-\frac{2}{p\delta^2}\right)}\mathrm{d}X^2_{(n+1)}\,, \qquad \mathcal{R}[X]_{ij} = \frac{\mathcal{R}[X]h_{ij}}{n+1}$$

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Mapping the AdS planar black hole to a Ricci flat p-brane

Start from the nonconformal black brane

$$\mathrm{d}s^2 = r^{\frac{2\theta}{p}} \left[\frac{L^2 \mathrm{d}r^2}{r^2 f(r)} + \frac{-f(r)\mathrm{d}t^2 + \mathrm{d}R^2_{(p)}}{r^2} \right], \quad f(r) = 1 - \frac{r^{p+1-\theta}}{r_0^{p+1-\theta}}$$

$$e^{\phi} = r^{\frac{2\theta}{p\delta}} = r^{\frac{2p\delta}{p\delta^2-2}}, \quad V_0 = \frac{(p-\theta)(p+1-\theta)}{L^2},$$

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If $\delta^2 \le \frac{2}{p}$, lift using

$$\frac{p\delta^2}{2} = \frac{d-p-1}{d-1}, \qquad V_0 = -2\Lambda$$

and find

$$\mathrm{d}s^2 = \frac{L^2 \mathrm{d}r^2}{r^2 f(r)} + \frac{-f(r)\mathrm{d}t^2 + \mathrm{d}R^2_{(d+p-1)}}{r^2}$$

$$f(r) = 1 - \frac{r^d}{r_0^d}$$

The AdS planar black hole

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The AdS planar black hole

If $\delta^2 \geq \frac{2}{p}$, lift using

$$\frac{p\delta^2}{2} = \frac{n+p+1}{n+1}, \qquad V_0 = \mathcal{R}[X]$$

and find

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \mathrm{d}R^2_{(p)} + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}X_{(n+1)}$$

$$f(r) = 1 - \frac{r_0^n}{r^n}$$

The Ricci-flat black p-brane

Analytic continuation

Going from one side (AdS) to the other (Ricci-flat) seems to imply

$$\left(\boxed{\frac{p\delta^2}{2} = \frac{d-p-1}{d-1}} \qquad \boxed{\frac{p\delta^2}{2} = \frac{n+p+1}{n+1}} \right)$$

$$d = -n\,!$$

This is useful to derive Ricci-flat solutions from AdS ones without going through the nonconformal branes. It does not mean Ricci flat theories are AdS theories in negative dimensions...

The analytic continuation always occur at the level of the low-d theory, where d and n are nolonger spacetime dimensions: generalised dimensional reduction [Kanitscheider&Skenderis'09].

Families of AdS solutions are mapped to **families** of Ricci-flat solutions (d and n must not be fixed)

Holographic dictionary for nonconformal branes,

[KANITSCHEIDER&AL'08,WITHERS&WISEMAN'08,KANITSCHEIDER&SKENDERIS'09]

Can be obtained in the dual frame from the AdS dictionary using the dimensional reduction

1. FG-expansion (specialising to planar boundaries)

$$\frac{\mathrm{d}s^2_{(p+2)}}{\ell^2} = \frac{\mathrm{d}\rho^2}{4\rho^2} + \frac{1}{\rho}\mathfrak{g}_{\alpha\beta}(\rho, z^\gamma)\mathrm{d}z^\alpha \mathrm{d}z^\beta \,,$$

$$\mathfrak{g}_{\alpha\beta} = \delta^\mu_\alpha \delta^\nu_\beta \mathfrak{g}_{\mu\nu} \Rightarrow \mathfrak{g}(\rho, z) = g_{(0)}(z) + \rho^{\frac{d}{2}} g_{(d)}(z) + \ldots \,,$$

2. The lower-dimensional scalar operator derives from the breathing mode

$$g_{ab} = \delta^\mu_a \delta^\nu_b g_{\mu\nu} = \rho e^{\frac{2\phi}{n+1}} \delta_{ab} = e^\psi \delta_{ab}$$

$$\psi(\rho, z) = \psi_{(0)} + \rho^{\frac{d}{2}} \psi_{(d)}(z) + \ldots \,.$$

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- It is now straightforward to derive the 1-point functions of the lower-dimensional dual operators:

$$T_{\alpha\beta} = \frac{\ell^{d-1} d e^{\psi_{(0)}}}{16\pi G_N} g_{(d)\alpha\beta}\,, \qquad \mathcal{O}_\phi = -\frac{\ell^{d-1} d e^{\psi_{(0)}} \psi_{(d)}}{16\pi G_N}\,.$$

- The reduced Ward identity shows that the scalar operator parametrises deviation from conformality:

$$T_\alpha{}^\alpha = -(n+p+1)\mathcal{O}_\phi\,.$$

- The scalar operator acts as a source in the stress-tensor conservation equation:

$$\partial^\alpha T_{\alpha\beta} - \partial_\alpha \psi_{(0)} \hat{\mathcal{O}}_\phi = 0\,.$$

Towards a holographic dictionary for Ricci-flat p-branes?

Lift up the nonconformal FG expansion

$$\mathrm{d}s^2 = \left(1 - \frac{\ell^{n+1}}{n\,r^n}\mathcal{O}_\phi - \frac{\ell^{n+1}\Box_z\mathcal{O}_\phi}{2n(n-2)r^{n-2}}\right)\left(\mathrm{d}r^2 + \eta_{\alpha\beta}\mathrm{d}z^\alpha\mathrm{d}z^\beta + r^2\mathrm{d}\Omega^2_{n+1}\right)$$
$$- \left(\frac{\ell^{n+1}}{n\,r^n}T_{\alpha\beta} + \frac{\ell^{n+1}\Box_z T_{\alpha\beta}}{2n(n-2)r^{n-2}}\right)\mathrm{d}z^\alpha\mathrm{d}z^\beta + O\left(\frac{T^2}{r^{2n}}\right) + O\left(\frac{\partial^4 T}{r^{n-4}}\right).$$

Regulate potential terms spoiling the asymptotics? Derivative expansion?
Can these terms be resummed?

Towards a holographic dictionary for Ricci-flat p-branes? (2)

What is the meaning of $T_{\alpha\beta}$? Look at the linear perturbation at large r

$$\mathrm{d}s^2 = (\eta_{AB} + h_{AB} + \dots)\,\mathrm{d}x^A \mathrm{d}x^B$$

$$\Downarrow$$

$$\Box_{r,z}\left(h_{AB} - \frac{h^C_C}{2}\eta_{AB}\right) = \delta^\alpha_A \delta^\beta_B T_{\alpha\beta}\delta(r)$$

(Minus) The holographic stress-tensor acts as a source for the faraway gravitational field

Still true at linear order and higher derivatives

Note: Other methods [MANN&MAROLF'05, KRAUS&AL'99, BROWN&YORK'92] do not yield a finite stress-tensor.

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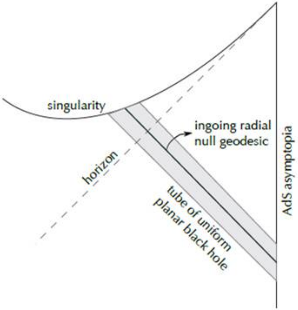
Fluid metrics in AdS

Generically, field theories are expected to equilibrate locally at high enough density. Thus, they should be amenable to a hydrodynamic description in a suitable long wavelength limit.
AdS/CFT: hydrodynamic limit on both sides. In this limit, Einstein equations equivalent to Navier-Stokes equations.



[HUBENY&AL'11]

Patch-wise construction: build a perturbative black hole with slowly-varying temperature and fluid velocity.

[BATTHACHARYYA&AL'07,'08]

Correct the solution order by order in a **derivative expansion** to account for viscous corrections.

Fluid metric and stress-tensor in AdS

Original construction, [BATTHACHARYYA&AL'07,'08]: in EF coordinates.

To derive the holographic stress-tensor, go to FG coordinates and select the $d/2$ mode in the FG expansion, [CALDARELLI&AL'12].

$$
\begin{aligned}
&\nabla^\mu T_{\mu\nu} = 0 \\
T_{\mu\nu} &= P\left(g_{\mu\nu} + d u_\mu u_\nu\right) - 2\eta\sigma_{\mu\nu} - 2\eta\tau_\omega \left[u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + \omega_\mu{}^\lambda \sigma_{\lambda\nu} + \omega_\nu{}^\lambda \sigma_{\mu\lambda}\right] \\
&+ 2\eta b \left[u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + \sigma_\mu{}^\lambda \sigma_{\lambda\nu} - \frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{d-1} P_{\mu\nu}\right] \\
b &\equiv \frac{d}{4\pi T} \quad P = \frac{1}{b^d} \quad \eta = \frac{s}{4\pi} = \frac{1}{b^{d-1}} \quad \tau_\omega = b \int_1^\infty \frac{\xi^{d-2} - 1}{\xi(\xi^d - 1)} \mathrm{d}\xi\,.
\end{aligned}
$$

Conformal symmetry organises the allowed conformal structure [BAIER&AL'07].

Sound modes $(\delta b,\, \delta u_\mu)$ are **stable** [BHATTACHARYYA&AL'07,BAIER&AL'07]

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Blackfolds in Ricci flat spacetimes

Idea: describe the dynamics of long wavelength deformations of black branes by an effective worldvolume theory, [EMPARAN&AL'09].

Possible whenever there is a **separation of scales**: Horizon length scale much smaller than the background curvature radius: $r_0 << L$.

The dynamics of black branes is captured by two sets of equations, [CAMPS&EMPARAN'12]:

extrinsic (bending the brane)

intrinsic (internal fluctuations of the brane)

Our map recovers the (1st order) blackfold metric from the (1st order) fluid/gravity metric [CALDARELLI&AL'13].

Blackfold hydrodynamic stress tensor from AdS

Get the Ricci-flat second order stress tensor: $\nabla^\alpha T_{\alpha\beta} = 0$

$$
\begin{aligned}
T_{\alpha\beta} = {} & P\left(\eta_{\alpha\beta} - n u_\alpha u_\beta\right) - 2\eta\sigma_{\alpha\beta} - \zeta\theta P_{\alpha\beta} \\
& + 2\eta\tau_\omega \left[P_\alpha{}^\gamma P_\beta{}^\delta \dot{\sigma}_{\gamma\delta} - \frac{\theta\sigma_{\alpha\beta}}{n+1} + 2\omega_{(\alpha}{}^\gamma \sigma_{\beta)\gamma}\right] + \zeta\tau_\omega \left[P_{\alpha\beta}\dot{\theta} - \frac{1}{n+1}\theta^2 P_{\alpha\beta}\right] \\
& - 2\eta r_0 \left[P_\alpha{}^\gamma P_\beta{}^\delta \dot{\sigma}_{\gamma\delta} + \left(\frac{2}{p} + \frac{1}{n+1}\right)\theta\sigma_{\alpha\beta} + \sigma_\alpha{}^\gamma \sigma_{\gamma\beta} + \frac{\sigma^2}{n+1} P_{\alpha\beta}\right] \\
& - \zeta r_0 \left[P_{\alpha\beta}\dot{\theta} + \left(\frac{1}{p} + \frac{1}{n+1}\right)\theta^2 P_{\alpha\beta}\right]
\end{aligned}
$$

$$
P = -\Omega_{(n+1)} r_0^n\,, \qquad \epsilon = -(n+1)P\,, \qquad c_s^2 = -\frac{1}{n+1}\,,
$$

$$
\eta = \frac{s}{4\pi} = \Omega_{(n+1)} r_0^{n+1}\,, \qquad \zeta = 2\eta\left(\frac{1}{p} - c_s^2\right), \quad \tau_\omega = \frac{r_0}{n}\mathrm{Harmonic}\left(-\frac{2}{n} - 1\right).
$$

"Hidden" conformal symmetry still organises its coefficients and tensor structure

Coincides at 1st order with previous results, [Camps&al'10]

Divergent for $n = 1$ and $n = 2$.

Black strings $n = 1$

Trouble: subleading terms in the Fefferman-Graham expansion now become of the same order as the boundary metric and the stress-tensor. More explicitly

$$\mathfrak{g} = \eta + \rho^{d/2} g_{(d/2)} + \rho^{1+d/2} g_{(1+d/2)} + \rho^{d+1} g_{(d+1)} + \rho^{3d/2} g_{(3d/2)} + \rho^{1+3d/2} g_{(1+3d/2)} + \dots$$

For $n = 1 \leftrightarrow d = -1$, terms are mixing and logarithms appear:

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For $n = 1 \leftrightarrow d = -1$, terms are mixing and logarithms appear:

$d + 1 = 0$: changes the boundary metric

$1 + 3d/2 = -1/2$: contributes to the stress-tensor

This can be cured by a second-order change of boundary coordinates.

(Need also to change boundary conditions to avoid spurious divergences in metric functions)

Black strings $n = 1$: "conformal anomalies"

The equation for the stress-tensor is modified by non-linear pieces acting like conformal anomalies

$$
\begin{aligned}
&\partial_\mu T^\mu{}_\nu + \frac{1}{4} T^{\mu\rho} T_\mu{}^\sigma \partial_\rho \Box T_{\nu\sigma} + \frac{1}{2} T^{\mu\rho} T^{\sigma\kappa} \partial_{\mu\rho\sigma} T_{\nu\kappa} - \frac{37}{48} T^{\mu\rho} T^{\sigma\kappa} \partial_{\nu\mu\rho} T_{\sigma\kappa} + \frac{1}{3} T^{\mu\rho} T^{\sigma\kappa} \partial_{\nu\mu\sigma} T_{\rho\kappa} \\
&- \frac{1}{2} T^{\mu\rho} \partial_\mu T_\nu{}^\sigma \Box T_{\rho\sigma} + \frac{1}{4} T^{\mu\rho} \partial_\mu T_\rho{}^\sigma \Box T_{\nu\sigma} + \frac{11}{24} T^{\mu\rho} \partial_\mu T^{\sigma\kappa} \partial_{\nu\rho} T_{\sigma\kappa} - \frac{4}{3} T^{\mu\rho} \partial_\mu T^{\sigma\kappa} \partial_{\nu\sigma} T_{\rho\kappa} \\
&+ T^{\mu\rho} \partial_\mu T^{\sigma\kappa} \partial_{\sigma\kappa} T_{\nu\rho} + \frac{1}{4} T^{\mu\rho} \partial_\nu T_\mu{}^\sigma \Box T_{\rho\sigma} - \frac{13}{48} T^{\mu\rho} \partial_\nu T^{\sigma\kappa} \partial_{\mu\rho} T_{\sigma\kappa} + \frac{7}{6} T^{\mu\rho} \partial_\nu T^{\sigma\kappa} \partial_{\mu\sigma} T_{\rho\kappa} \\
&- \frac{37}{48} T^{\mu\rho} \partial_\nu T^{\sigma\kappa} \partial_{\sigma\kappa} T_{\mu\rho} + \frac{7}{6} T^{\mu\rho} \partial^\sigma T_\mu{}^\kappa \partial_{\nu\kappa} T_{\rho\sigma} - \frac{1}{3} T^{\mu\rho} \partial^\sigma T_\mu{}^\kappa \partial_{\nu\rho} T_{\sigma\kappa} - T^{\mu\rho} \partial^\sigma T_\mu{}^\kappa \partial_{\rho\kappa} T_{\nu\sigma} \\
&+ \frac{1}{32} T^{\mu\rho} \partial^\sigma T_{\mu\rho} \Box T_{\nu\sigma} - \frac{1}{2} T^{\mu\rho} \partial^\sigma T_\nu{}^\kappa \partial_{\mu\kappa} T_{\rho\sigma} - \frac{1}{2} T^{\mu\rho} \partial^\sigma T_\nu{}^\kappa \partial_{\mu\rho} T_{\sigma\kappa} + \frac{1}{2} T^{\mu\rho} \partial^\sigma T_\nu{}^\kappa \partial_{\mu\sigma} T_{\rho\kappa} \\
&- \frac{3}{8} T^{\mu\rho} \partial^\sigma T_\nu{}^\kappa \partial_{\sigma\kappa} T_{\mu\rho} + \frac{1}{2} T_\nu{}^\mu \partial_\mu T^{\rho\sigma} \Box T_{\rho\sigma} - \frac{1}{2} T_\nu{}^\mu \partial^\rho T_\mu{}^\sigma \Box T_{\rho\sigma} - \frac{1}{4} T_\nu{}^\mu \partial^\rho T^{\sigma\kappa} \partial_{\mu\rho} T_{\sigma\kappa} \\
&+ \frac{1}{8} \partial^\mu T_\nu{}^\rho \partial_\mu T^{\sigma\kappa} \partial_\rho T_{\sigma\kappa} - \frac{1}{2} \partial^\mu T_\nu{}^\rho \partial_\mu T^{\sigma\kappa} \partial_\sigma T_{\rho\kappa} - \frac{1}{2} \partial^\mu T_\nu{}^\rho \partial_\rho T^{\sigma\kappa} \partial_\sigma T_{\mu\kappa} + \partial^\mu T_\nu{}^\rho \partial^\sigma T_\mu{}^\kappa \partial_\kappa T_{\rho} \\
&- \frac{1}{48} \partial^\mu T^{\rho\sigma} \partial^\kappa T_{\rho\sigma} \partial_\nu T_{\mu\kappa} + \frac{1}{6} \partial^\mu T^{\rho\sigma} \partial_\nu T_\mu{}^\kappa \partial_\rho T_{\kappa\sigma} - \frac{5}{12} \partial_\nu T^{\mu\rho} \partial^\sigma T_\mu{}^\kappa \partial_\kappa T_{\rho\sigma} = 0\,,
\end{aligned}
$$

(Cadabra!)

Black strings $n = 1$: Renormalised stress-tensor

Eventually, one finds a divergence-free, Landau frame stress-tensor:

$$
\begin{aligned}
T_{\alpha\beta} \;=\;& P\left(\eta_{\alpha\beta} - u_\alpha u_\beta\right) - 2\eta\sigma_{\alpha\beta} - \zeta\theta P_{\alpha\beta} \\
& + b\eta\left[\frac{13}{8}\sigma_{\gamma(\alpha}\omega_{\beta)}{}^{\gamma} + \frac{15}{16}\sigma_{\alpha\gamma}\sigma_{\beta}{}^{\gamma} + \frac{9}{16}\omega_{\alpha\gamma}\omega^{\gamma}{}_{\beta} + \frac{7}{4}u^{\gamma}\partial_\gamma\sigma_{\alpha\beta} + \frac{9}{16}\theta\sigma_{\alpha\beta}\right. \\
& \left. + \frac{9}{16}a_\alpha a_\beta - \frac{7}{2}a^{\gamma}u_{(\alpha}\sigma_{\beta)\gamma} + P_{\alpha\beta}\left(-\frac{5}{32}\omega^2 - \frac{69}{32}\sigma^2 + \frac{15}{16}\theta^2 - \frac{15}{8}\dot{\theta} - \frac{3}{32}a^2\right)\right] \\
& + b\zeta\left[\frac{15}{16}\theta\sigma_{\alpha\beta} + P_{\alpha\beta}\left(\frac{7}{8}\dot{\theta} - \frac{15}{8}\theta^2 + \frac{15(p+2)}{64p}\theta^2\right)\right]
\end{aligned}
$$

$$
P = b\,, \qquad \eta = b^2\,, \qquad \zeta = \eta\frac{p+2}{p}
$$

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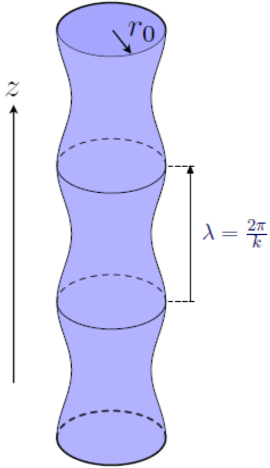
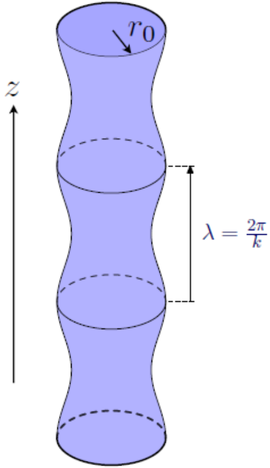
GL instability



Sufficiently long black strings are unstable to long wavelength, linear, spherically symmetric perturbations

[Gregory&Laflamme'94]

GL instability



Sufficiently long black strings are unstable to long wavelength, linear, spherically symmetric perturbations

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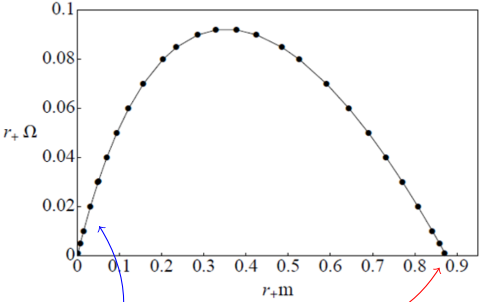

Threshold mode at finite k_c

The hydro approximation should capture the small k behaviour

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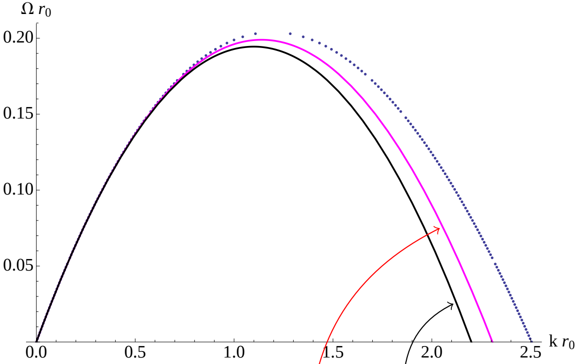
Dispersion relation $n = 7$ (numerics P. Figueras)

$$\Omega = \frac{1}{\sqrt{n+1}} k - \frac{2+n}{n(1+n)} k^2 + \frac{(2+n)[2+n(2\tau_\omega - 1)]}{2n^2(1+n)^{3/2}} k^3 + O(k^4)$$



Correct shape captured already at quadratic order [CAMPS&AL'10].

The low n fit is improved at cubic order, but is not so good for finite k: expected

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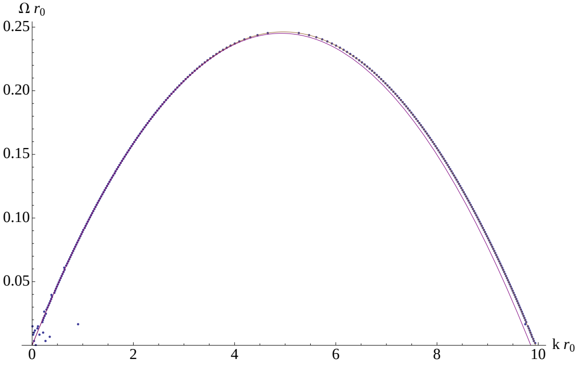
Dispersion relation $n = 100$ (numerics P. Figueras)

$$\Omega = \frac{1}{\sqrt{n+1}} k - \boxed{\frac{2+n}{n(1+n)} k^2} + \boxed{\frac{(2+n)[2+n(2\tau_\omega - 1)]}{2n^2(1+n)^{3/2}} k^3} + O(k^4)$$



Impressive agreement!

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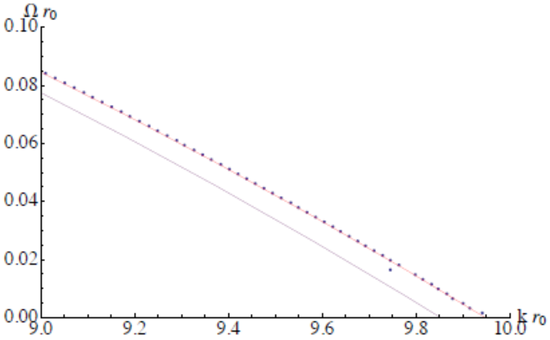
Dispersion relation $n = 100$ (numerics P. Figueras)

$$\Omega = \frac{1}{\sqrt{n+1}}k - \boxed{\frac{2+n}{n(1+n)}k^2} + \boxed{\frac{(2+n)[2+n(2\tau_\omega - 1)]}{2n^2(1+n)^{3/2}}k^3} + O(k^4)$$



Does the large n limit capture the threshold mode exactly? [Emparan&al'13]

$$k = \frac{4\pi T}{\sqrt{n}}\left(\boxed{1} - \boxed{\frac{1}{2n}} + O\left(\frac{1}{2n^2}\right)\right)$$ agrees with [Kol&al'07, Emparan&al'13]

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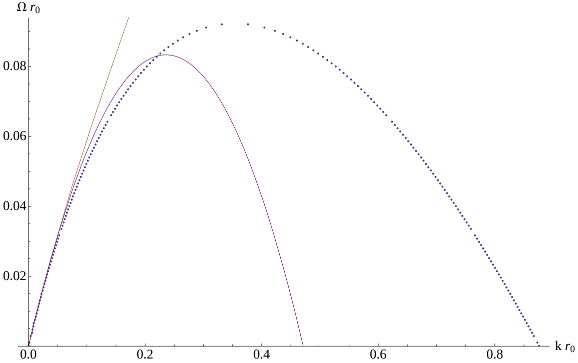
Dispersion relation for $n = 1$ (numerics P. Figueras)

$$\Omega = \frac{1}{\sqrt{3}}k - \frac{2}{3}k^2 \boxed{+\frac{5}{6\sqrt{3}}k^3} + O(k^4)\,.$$



The $\boxed{k^3 \text{ term}}$ spoils the capture of the threshold mode: but then we had not right to expect it in low n. Asymptotic expansion?

Summary

- Asymptotically locally AdS solutions with a planar subspace can be mapped to Ricci-flat solutions with a transverse sphere using generalised dimensional reduction.
- Hydrodynamic metrics in AdS and in Ricci-flat are equivalent. The holographic stress-tensor becomes the source of the p-brane effective stress tensor. "Hidden" conformal symmetry organises its tensor structure.
- We used this to obtain the cubic dispersion relation of the GL instability. At large n, the curve lies on top of the numerics. For low n, the cubic term improves over the quadratic one, but the finite k results are expectedly less impressive.
- Asymptotic expansion for $n = 1, 2$?

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Outlook

- Deformations of the sphere? (extrinsic blackfold perturbations, [Camps&Emparan'12])
- Curved boundary metrics (Schwarzschild black hole)?
- The limit $n = -1$ (vanishing transverse sphere) recovers known results about the Rindler fluid, [Strominger&al'11, Skenderis&al'11, Skenderis&al'12 ,Oz&al'12]. Holography?
- Connection with literature on "hidden" conformal symmetry in general asymptotically flat black holes? [Castro&al'10,Cvetic&Larsen'11]

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