Metal - Insulator transition in holography

Aristomenis Donos

Imperial College London

Talk at CCTP

April, 2013

Based on
arXiv:1212.2998 with S. Hartnoll

Outline

Outline

Holographic Phases

- Analyse the behavior of strongly coupled CFTs when held at finite temperature and charge density and/or in a uniform magnetic field.
  - Construct charged black hole solutions with AdS asymptotics
  - Calculate the free energies and deduce the phase diagram
- What type of thermal phases are possible?
- What kind of zero temperature ground states can we have?
- Do we find interesting new behaviour in the far IR? e.g. Lifshitz, Schrodinger, hyperscaling violating, ..., something new??
- Analyse hydrodynamics and transport

Holographic phases

Superconductivity/Superfluidity:

[Gubser], [Hartnoll, Herzog, Horowitz]

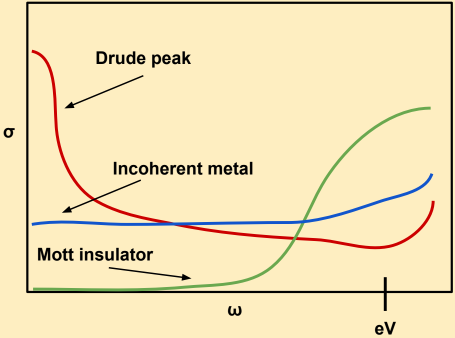
- A bulk charged scalar takes a VEV giving mass to the $U(1)$ gauge field e.g. s-wave superconductors
- Higher rank charged fields take a VEV giving rise to anisotropic superfluidity order e.g. p-wave, d-wave superconductors

Spatially modulated phases:

[Domokos, Harvey], [Nakamura, Oogiru, Park], [AD, Gauntlett, Pantelidou], [Bergman, Jokela, Lifschytz, Lippert], [Iizuka, Kachru, Kundu, Narayan, Sircar, Trivedi]

- A neutral/charged bulk field takes a modulated VEV breaking spatial translations
- Current/Momentum density waves, CDWs, FFLO-like

Metal-Insulator transition



- Materials with charged d.o.f. can be conductors or insulators
- At the transition a bad metallic phase can appear
- Strong coupling dynamics suggested to take place $\rightarrow$ answer in AdS/CFT?

Linear response in holography

Can we carry out such “experiments” in holography?

- Describes field theories at strong coupling
- Allows for the calculation of correlators e.g. $G^R_{\mathcal{O}_1\mathcal{O}_2}(\omega, k)$ by studying small perturbations around a black hole background
- Kubo's formula for linear response allows the direct computation of the optical conductivity

$$\sigma = \frac{1}{\imath\omega} G^R_{J_x J_x}(\omega, k = 0)$$

- Use to study transport of phases of holographic matter

Holographic metals

Consider bulk theory with metric $g_{\mu\nu}$ a cosmological constant and a $U(1)$ gauge field A_μ

- Study CFT $\rightarrow$ asymptote to AdS_4
- Deform by chemical potential $\rightarrow$ $A_\mu \approx \mu\, dt + \cdots$
- Finite temperature $\rightarrow$ regular Killing horizon

In $D = 4$ Einstein-Maxwell theory

$$\mathcal{L}_{EM} = \sqrt{-g}\left(\frac{1}{2}R + 6 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

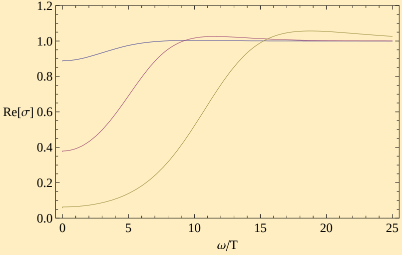
the above translate to the AdS-RN black brane

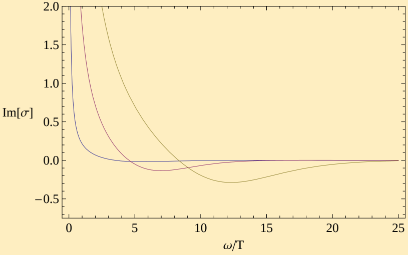
$$ds_4^2 = -g(r)\,dt^2 + g(r)^{-1}\,dr^2 + r^2\,\left(dx_1^2 + dx_2^2\right)$$

$$A = \mu\,\left(1 - \frac{r_+}{r}\right)\,dt, \quad g = 2r^2 - \left(2r_+^2 + \frac{\mu^2}{2}\right)\frac{r_+}{r} + \mu^2\frac{r_+^2}{2r^2}$$

Holographic metals

Calculate conductivity for the resulting charged medium [Hartnoll]



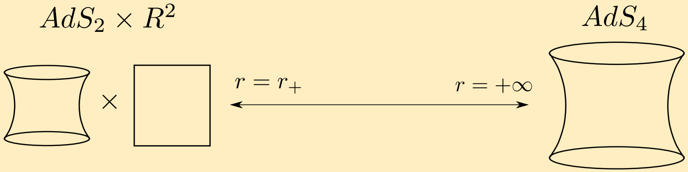


- The chemical potential breaks the $1 + 2$ dim Poincare group down to $T_t \times E(2)$
- The delta function $\mathrm{Re}[\sigma(\omega)] \propto \delta(\omega)$ reflects the translational invariance of the background $\rightarrow$ have to couple the current to heavy degrees of freedom $\rightarrow$ natural by breaking translations

Outline

Holographic metals

At $T = 0$ the RN-AdS black brane becomes an interpolating solution



- ✗ Finite entropy at $T = 0$, OK for field theories at large N (?)
- ✓ Finite spectral weight at $\omega = 0$, $k \neq 0$
- ! Einstein-Maxwell-Scalar theories have solutions conformal to $AdS_2 \times \mathbb{R}^2$ with zero entropy and finite spectral weight at $\omega = 0$, $k \neq 0$ [Charmousis, Gouteraux, Kim, Kiritsis, Meyer], [Hartnoll, Shoughoulian]

Holographic metals

In order to maintain the same low and high energy physics we need to have a UV-IR benign lattice:

- Introduce a UV relevant deformation $\mathcal{O}(x)$
- The IR operator should be irrelevant with respect to AdS_2
- Solve the PDEs and show that the IR remains $AdS_2 \times \mathbb{R}^2$

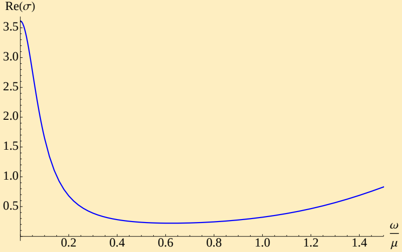
Natural choice for the lattice operators is to have a non-uniform chemical potential in a spatial direction:

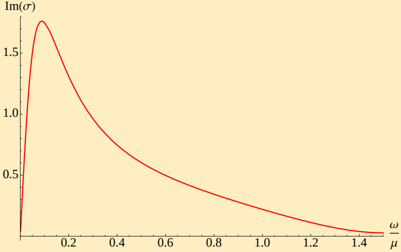
$$\mu(x) = \mu_0 + A_0 \cos(k_L x)$$

Can check that only involves irrelevant AdS_2 operators in Einstein-Maxwell theory with dimensions $\Delta_i(k_L, \mu_0) > 1/2$.

Holographic metals

Calculating the conductivity with respect to the background deformed by the lattice resolves the low ω delta function to a Drude peak

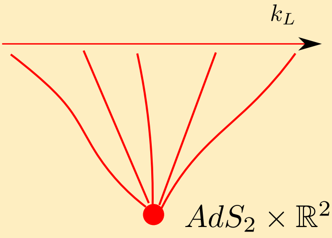


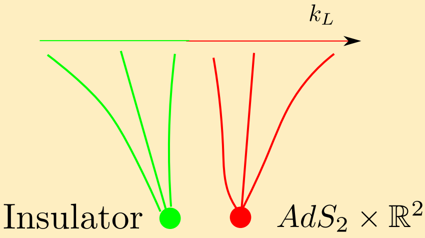


- The resistivity $\rho = \mathrm{Re}[\sigma(0)]^{-1}$ is a power law $\rho \propto T^{\Delta(k_L,\mu)}$ for small T [Hartnoll, Hofman]
- Mid-infrared power law $|\sigma| \propto \omega^{-2/3} + C$ [Horowitz, Santos, Tong]

Metal-Insulator transition

In Einstein-Maxwell the picture inferred from the AdS_2 spectrum $\Delta_i(k_L, \mu_0) > 1/2$ is





- In order to flow to an insulator, localization effects have to be important
- The translation breaking lattice has to break the translations of the $E(2)$ group in the IR. One way is if $\Delta_i(k_L, \mu_0) < 1/2$ for some values of k_L.

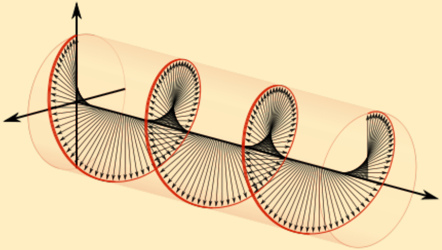
Outline

Technical simplification

We consider theories deformed by chemical potential μ and a lattice. To avoid having to solve PDEs:

- Consider $1+3$ dimensional CFTs $\Rightarrow$ 5 bulk dimensions
- Uniform potential μ breaks the Poincare group to $T_t \times E(3)$, we still need to break translations of $E(3)$
- To reduce the problem to ODEs keep the constant bulk radius slices homogeneous

$\rightarrow$ Keep ∂_{x_2}, ∂_{x_3} and $p^{-1}\,\partial_{x_1} + (x_2\partial_{x_3} - x_3\partial_{x_2})$
Give a helical structure to preserve a Bianchi VII_0 subgroup



The model

Consider model with a metric $g_{\mu\nu}$, a $U(1)$ gauge field A_μ and a (not too massive) 1-form B_μ

$$S = \int d^5x\sqrt{-g}\left(R + 12 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{4}W_{ab}W^{ab} - \frac{m^2}{2}B_aB^a\right) - \frac{\kappa}{2}\int B\wedge F\wedge W$$

- A_μ used to deform by a uniform chemical potential
- B_μ used to introduce helical “lattice”
- CS coupling κ helps to flow to an insulating geometry
- Will consider $m^2 = 0$

The ansatz

Making the consistent ansatz

$$A = a(r)dt\,, \qquad B = w(r)\omega_2\,,$$
$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + e^{2v_1(r)}\omega_1^2 + e^{2v_2(r)}\omega_2^2 + e^{2v_3(r)}\omega_3^2$$

with the left-invariant Killing one-forms

$$\omega_1 = dx_1\,, \qquad \omega_2 + i\omega_3 = e^{ipx_1}(dx_2 + idx_3)$$

yields a non-linear system of ODEs for the radial functions.

Basic ingredients for a metallic state are captured:

- AdS_5 with $a = w = 0$, $U = r^2$ and $v_i = \ln r$
- $AdS_2 \times \mathbb{R}^3$ with $w = 0$, $a = 2\sqrt{6}\,r$, $U = 12\,r^2$ and $v_i = 0$

RG flows

Close to the AdS_5 the boundary conditions are fixed by the deformation parameters μ, λ and p

$$a = \mu + \frac{\nu}{r^2} + \cdots\,, \quad w = \lambda + \frac{\beta - \lambda\, p^2/2 \log r}{r^2} + \cdots\,,$$
$$U = r^2 - \frac{\epsilon/3 + p^2\,\lambda^2/6\,\log r}{r^2} + \cdots$$
$$v_i = \log r + \frac{g_i + s_i\,\lambda^2\,p^2/24\,\log r}{r^4} + \cdots$$

- In the IR of the geometry we impose boundary conditions for the existence of a regular black horizon at temperature T
- What are the possible zero temperature IR behaviors?
- Always $AdS_2 \times \mathbb{R}^3$ with varying λ and p?

RG flows

Instructive to find the spectrum of operators on $AdS_2 \times \mathbb{R}^3$.
Perturb the background as

$$U = 12\, r^2 (1 + \varepsilon\, u_1 r^\delta),\; v_i = v_o (1 + \varepsilon\, v_{i\,1} r^\delta),\; a = 2\sqrt{6}\, r (1 + \varepsilon\, a_1 r^\delta)$$
$$w = \varepsilon\, w_1 r^\delta$$

for small ε and find the values for δ.

- They come in pairs $\delta_\pm^{(i)}(p) = -1/2 \pm \nu^{(i)}(p)$, for stable solutions $\nu^{(i)}(p) \geq 0$
- To "shoot out" from $AdS_2 \times \mathbb{R}^3$ we need to use to modes with $\delta_+^{(i)}(p) \geq 0$
- If one of the modes is relevant $\delta_+^{(j)}(p) < 0$, a flow cannot exist

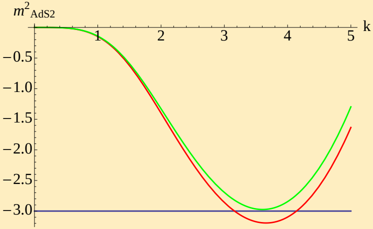
RG flows

This is where the CS coupling κ plays a central role

i) For $\kappa = 0$ we have $\delta^{(i)}(p) \geq 0$ for all p

ii) For $0 < \kappa \leq 1/\sqrt{2}$ we have a mode with $\delta^{(i)}(p) < 0$ for a range of p

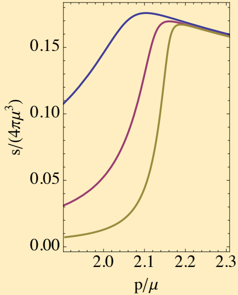
iii) For $\kappa > 1/\sqrt{2}$ the mode can have complex $\delta^{(i)}(p)$ indicating a phase transition



Option iii) leads to spontaneous breaking of translations, we will go with option ii) so that $AdS_2 \times \mathbb{R}^3$ is dynamically stable.

Black holes

- Good indication that for $0 < \kappa \leq 1/\sqrt{2}$ we could flow down to something which is not $AdS_2 \times \mathbb{R}^3$.
- Look at low temperature black holes and vary p



- Plot entropy density s vs p for $T/\mu = 10^{-3}$, 10^{-4}, 10^{-5} and fixed lattice strength
- Jump in the entropy indicates phase transition

Insulating IR

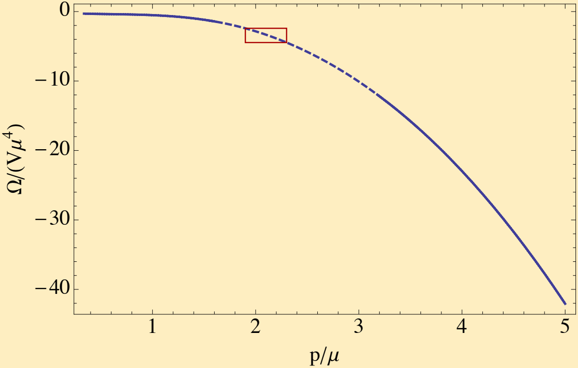
- It turns out there is a symmetry breaking solution. At leading order in small r

$$ds_5^2 = -r^2\, dt^2 + r^{-2}\, dr^2 + r^{-2/3}\, \omega_1^2 + r^{4/3}\, \omega_2^2 + r^{2/3}\, \omega_3^2$$
$$A = 0, \quad B = w_0\, \omega_2$$

- Strong effect from the lattice in the IR leads to a ground state which breaks translations
- It is a solution of Einstein-Maxwell, the CS coupling helps connect to AdS_5 in the UV
- It doesn't have any relevant mode
- At low temperatures it leads to entropy $s \propto T^{2/3}$

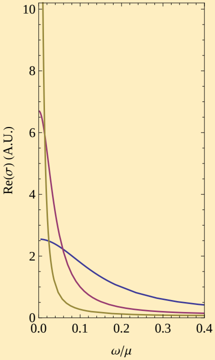
Domain walls

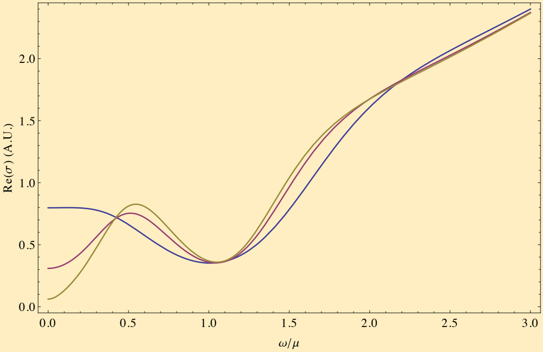
- Construct solutions at strictly zero temperature



- Left solid part: Domain wall with insulating IR
- Right solid part: Domain wall with $AdS_2 \times \mathbb{R}^3$
- Dashed part: Low temperature black hole (numerically hard to get the domain walls)

AC Conductivity

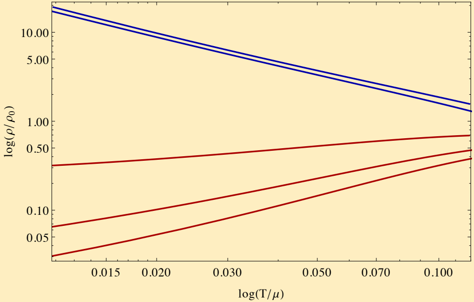




- Left: AC conductivity for black holes with metallic $AdS_2 \times \mathbb{R}^3$ IR. At low ω agreement with Drude peak
- Right: AC conductivity for black holes with insulating IR. At low omega $\mathrm{Re}[\sigma(\omega)] \propto \omega^{4/3}$
- Opposite temperature dependence of DC conductivity $\rho = \mathrm{Re}[\sigma(0)]^{-1}$ on T

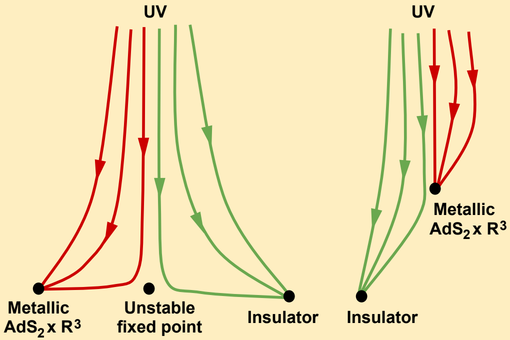
Resistivity

Plotting resistivity vs temperature for different lattice momenta p



- Red: Metallic phase, resistivity increases as a power law $\rho \propto T^{\Delta(p)}$
- Blue: Insulating phase, resistivity decreases as a power law $\rho \propto T^{-4/3}$

Different scenarios for the transition



- For a range of $0 < \kappa < 0.57$ there is a third scaling solution which has relevant or unstable modes and can mediate the transition. It can be either continuous of first order. [Hartnoll, Huijse]
- For $\kappa > 0.57$ this solution doesn't exist in the model and the transition is infinite order

Outline

Summary - Plans

- Constructed black holes with broken translations using ODEs
- AC conductivity with a Drude peak
- Interaction driven metal-insulator transition
- It can be embedded in string theory through $D = 5$ minimal gauged SUGRA
- Explore the phase diagram more thoroughly
- Consider other theories with neutral scalars ($N = 4^+$ Roman's theory)
- Construct insulating geometries with a gap
- Construct Mott insulators? $\rightarrow$ necessarily solve PDEs