Holographic Wilson Loops and Topological Insulators

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Credits

Work in progress with:

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Outline:

- Motivation: Topological Insulators
- Holographic Conformal Interfaces
- Holographic Wilson Loops
- Static Quark Potential
- Future Directions

Topological Insulators

U(1) symmetry

mass gap in charged sector

topological quantum number distinct from vacuum

Topological Insulators

topological quantum number

A number invariant under continuous deformations that:

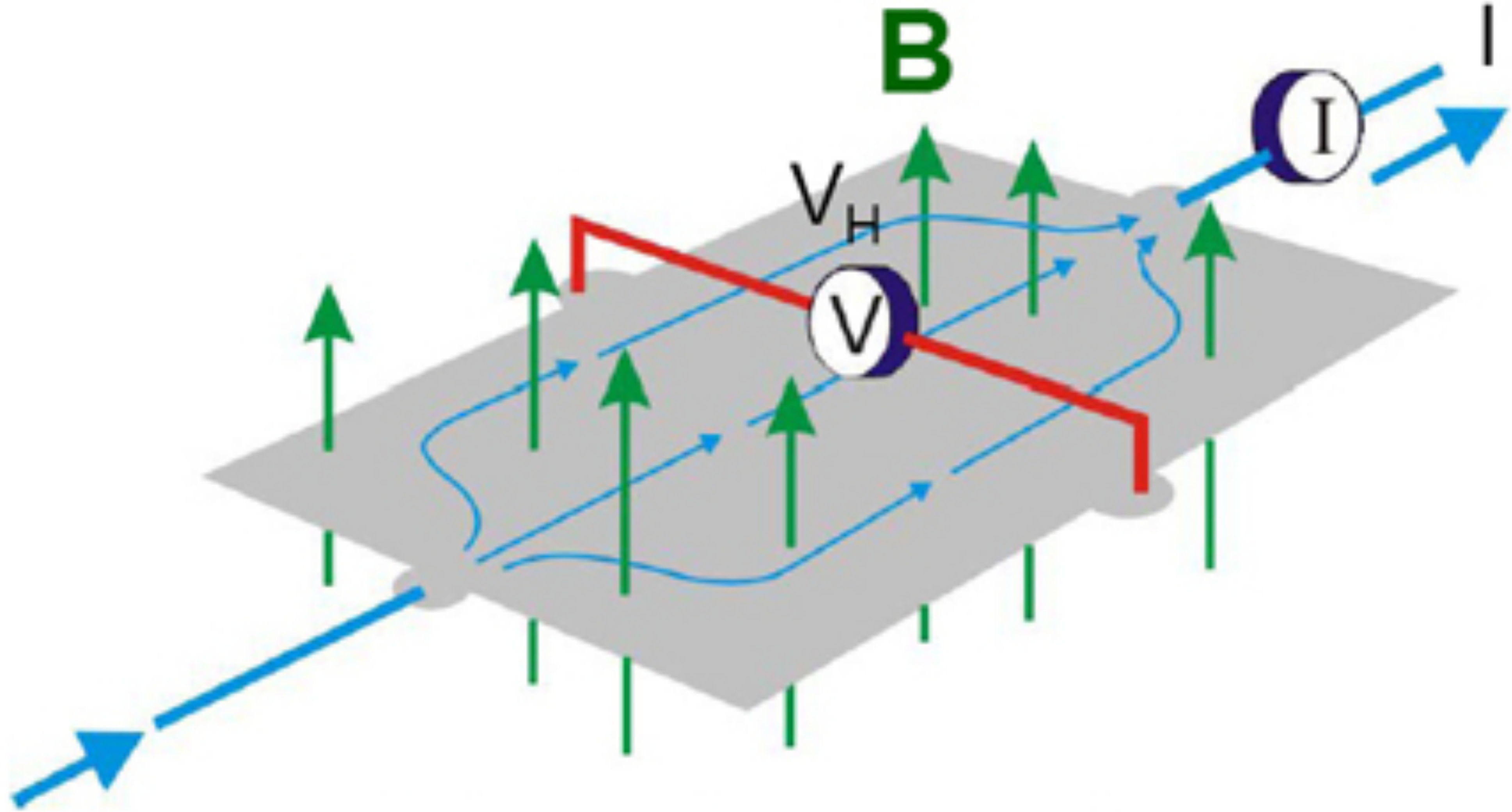
- Preserve all symmetries
- Preserve the mass gap

Topological Insulators

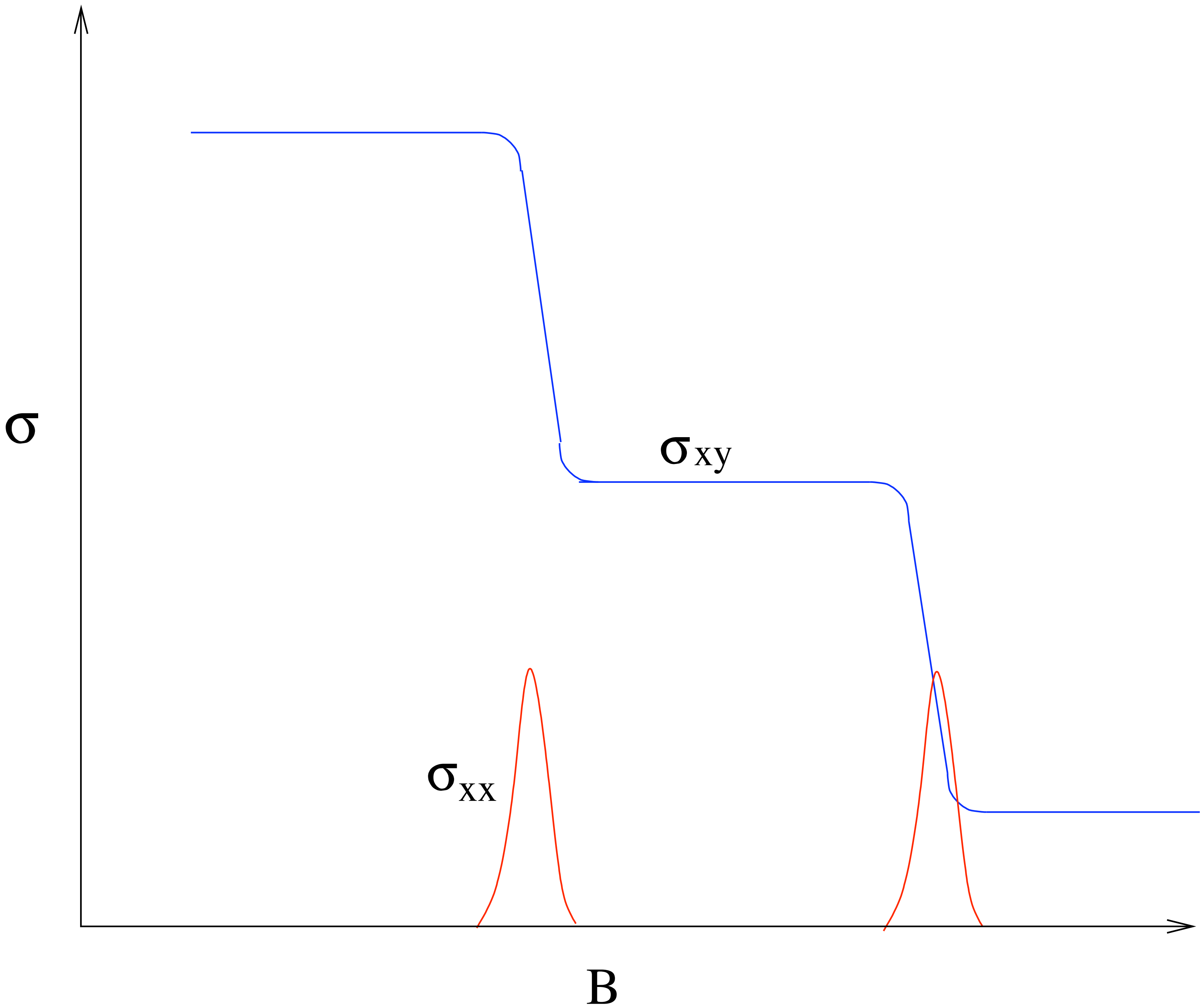
topological quantum number

Cannot be classified (just) by local order parameter

Example: Integer QHE



Example: Integer QHE



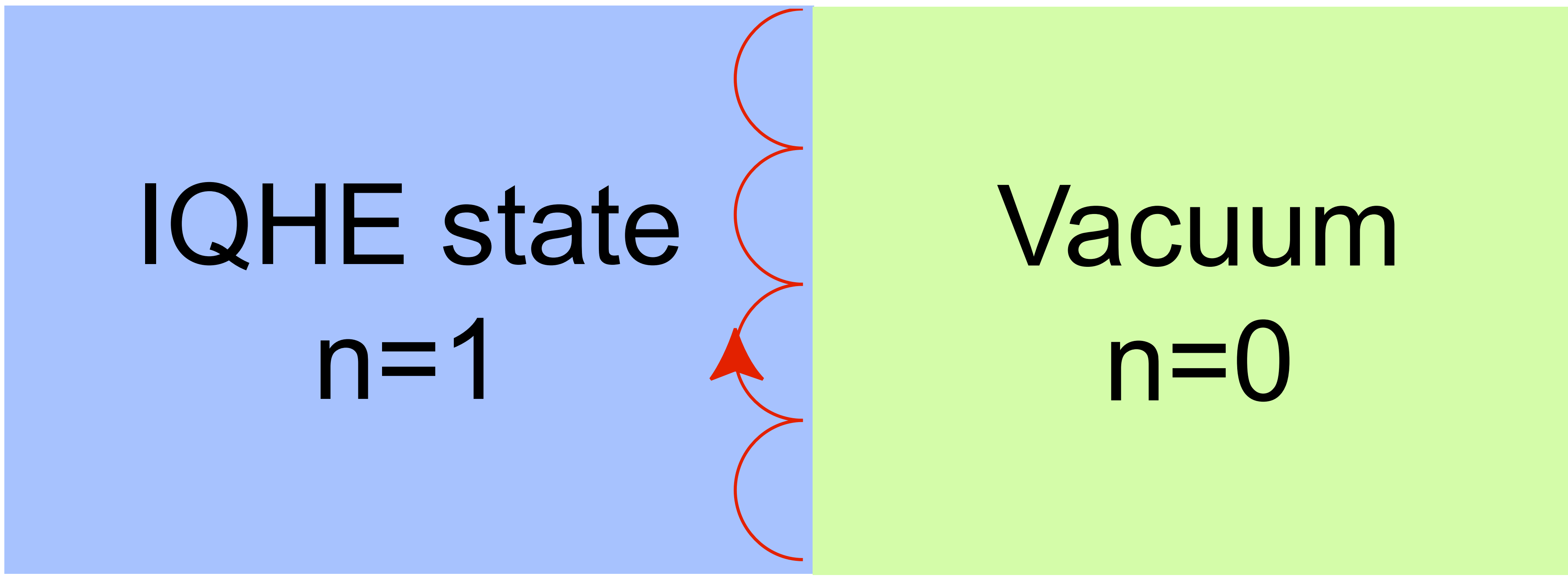
On plateaux:

$$\sigma_{xx} = 0$$

$$\sigma_{xy} = n\frac{e^2}{h}$$

$\mathbb{Z}$ topological quantum number

Edge modes: chiral fermions



$$n = \#\,\text{of edge modes}$$

Observed via:

Transport

Directly (ARPES)

Example: Integer QHE

Effective Description

$$\boxed{S = \frac{k}{4\pi} \int d^3x \, \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho}$$

$$J^i = \frac{\delta S}{\delta A_i} = \frac{k}{2\pi} \epsilon^{ij} E^j$$

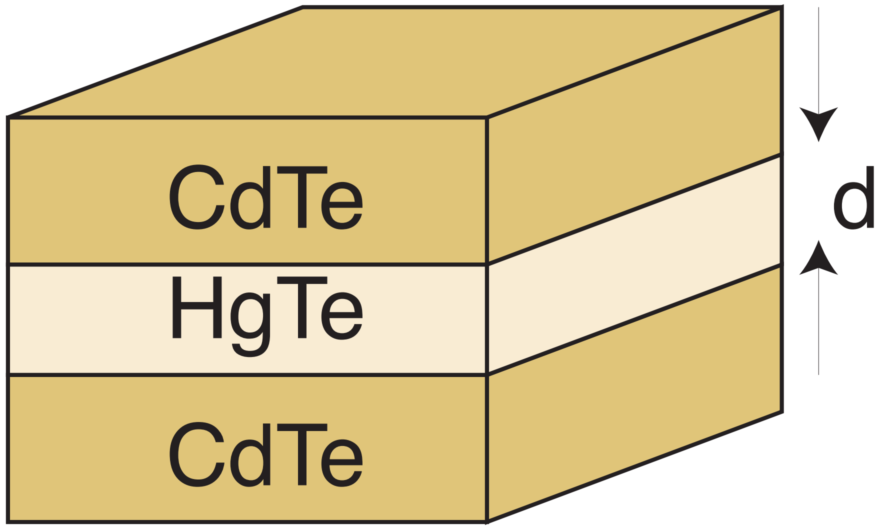
quantization of σ_{xy} $=$ quantization of k

T-invariant Topological Insulators

(2+1)d

Bernevig, Hughes, Zhang 2006

König et al. 2007



(3+1)d

Fu and Kane 2007

Hsieh et al. 2008

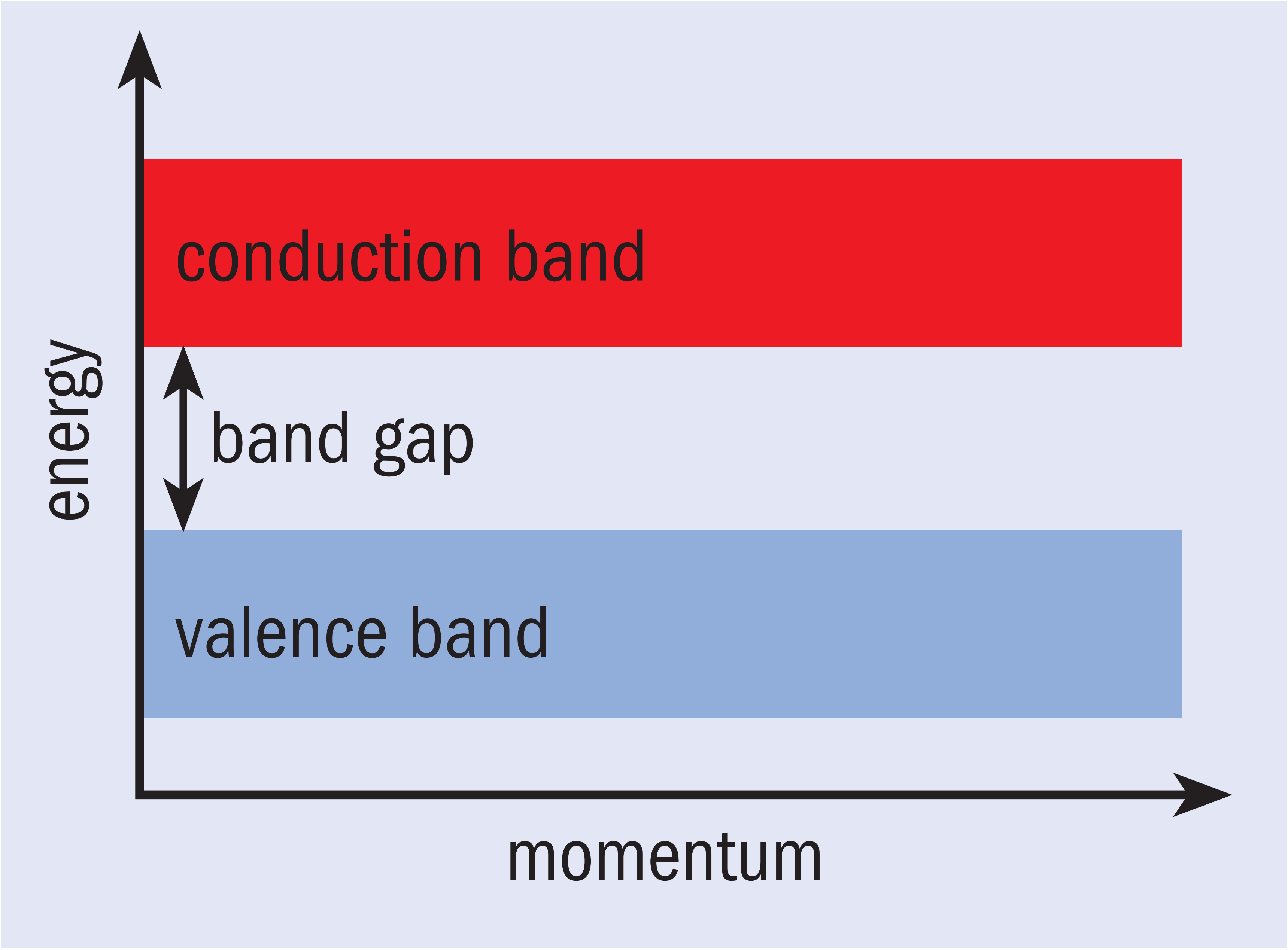
Bi_2Se_3

Bi_2Te_3

Sb_2Te_3

(3+1)d T-invariant Topological Insulators

UV: some band structure



(3+1)d T-invariant Topological Insulators

IR: non-interacting electrons

$$S = \int d^4x\, \bar{\psi}\,(i\partial\!\!\!/ - A\!\!\!/ - M)\,\psi$$

$$T : M \to M^* \qquad M \in \mathbb{R}$$

$\boxed{M > 0}$ or $\boxed{M < 0}$

(3+1)d T-invariant Topological Insulators

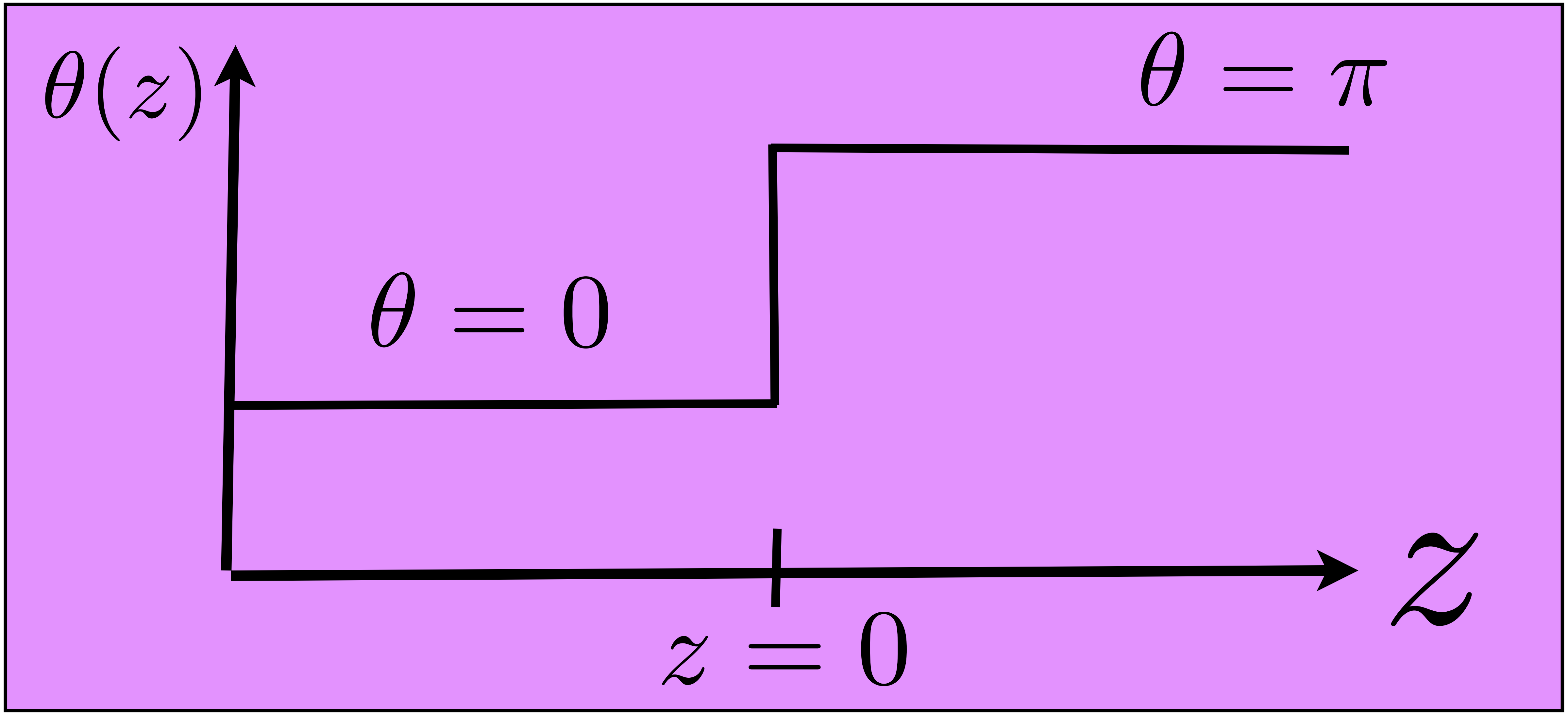
Extreme IR: axion electrodynamics

$$\mathcal{L} = \frac{1}{8\pi}\left[\varepsilon \vec{E}^2 - \frac{1}{\mu}\vec{B}^2\right] + \frac{\theta}{4\pi^2}\vec{E}\cdot\vec{B}$$

$$\theta = \begin{cases} 0 & M > 0 \\ \pi & M < 0 \end{cases} \mod 2\pi$$

$\mathbb{Z}_2$ topological quantum number

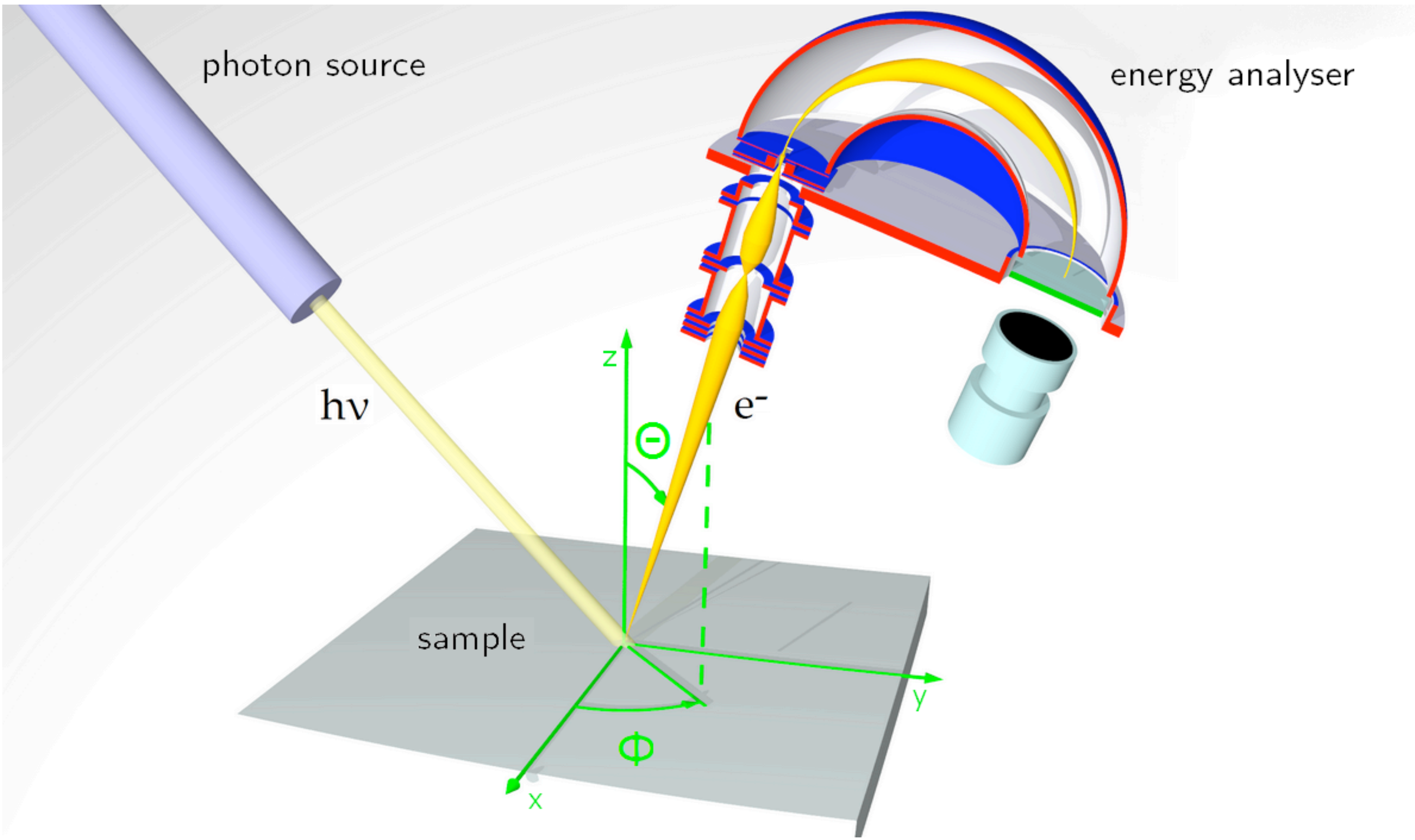
(3+1)d T-invariant Topological Insulators



Edge modes: (2+1)d Dirac fermions

ARPES

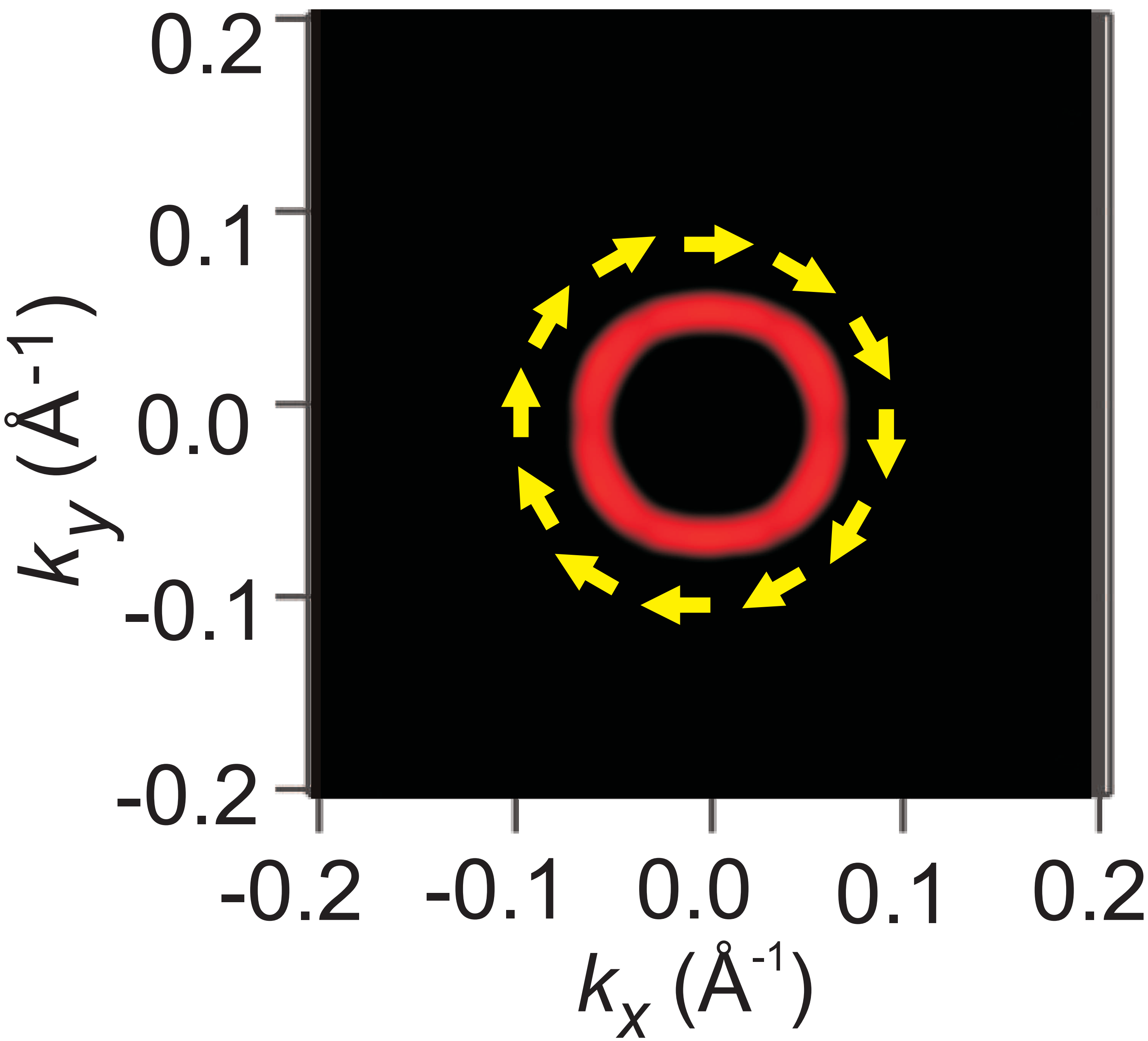
Angle Resolved Photo-Emission Spectroscopy



$$I(\omega, \vec{k}) \propto f(\omega)\mathcal{R}(\omega, \vec{k})$$

(3+1)d T-invariant Topological Insulators

Edge modes: (2+1)d Dirac fermions

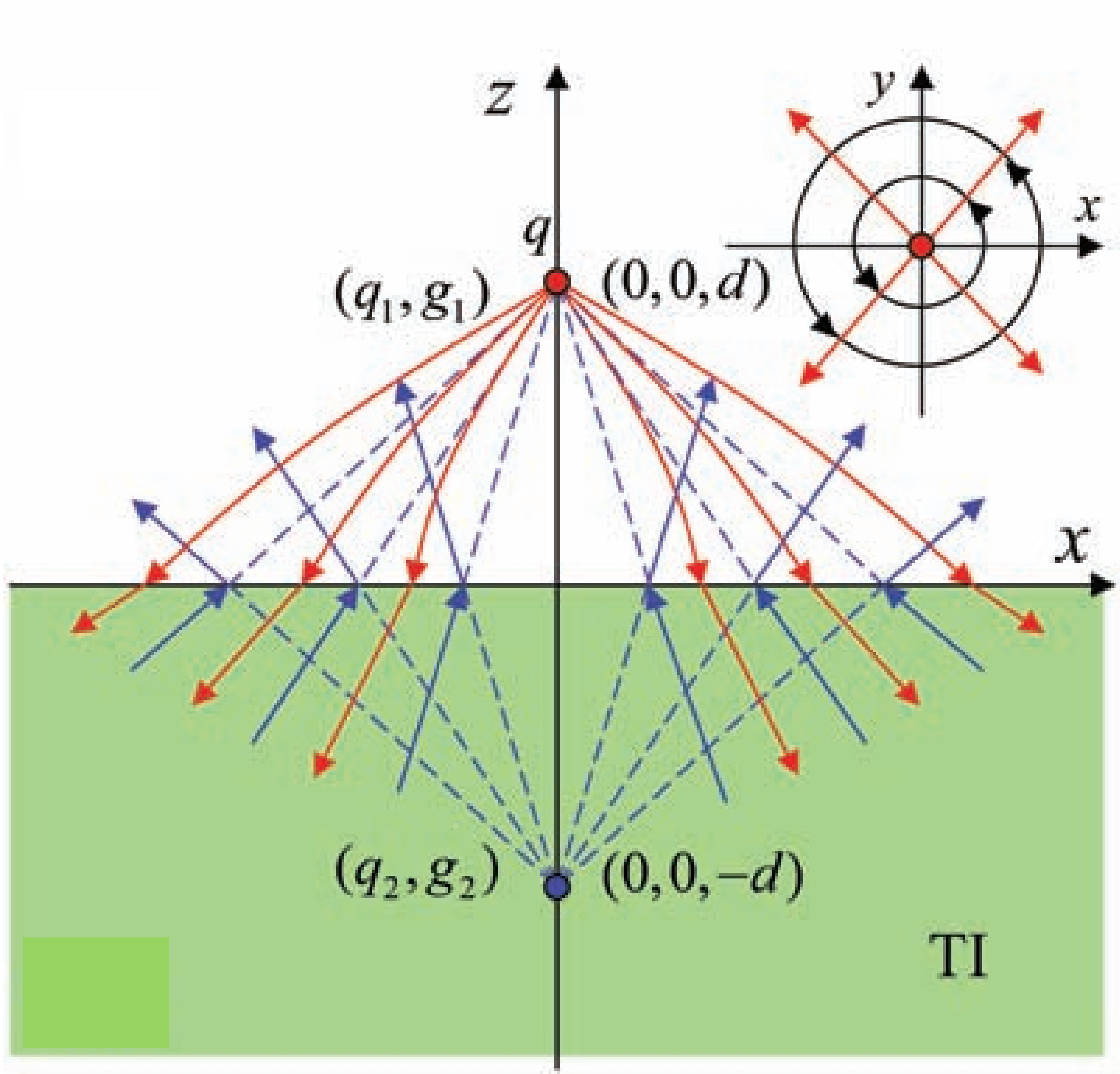


Bi_2Se_3

Xia *et al.* 0812.2078

Hsieh *et al.*
Nature 460, 1101

Jumping θ-angle: image dyons



CLASSIFICATION

at level of free Dirac Hamiltonians

Choices:

- Spatial dimension d
- Number of species
- Mass matrix

Periodic Table of TI's

class\d	0	1	2	3	4	5	6	7	T	C	S
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	0	0	0
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	0	1
AI	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$+$	0	0
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$+$	$+$	1
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	0	$+$	0
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	$-$	$+$	1
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$-$	0	0
CII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$-$	$-$	1
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$-$	0
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$+$	$-$	1

Schnyder, Ryu, Furusaki, Ludwig 2008 Kitaev 2009

Periodic Table of TI's

class\d	0	1	2	3	4	5	6	7	T	C	S
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	0	0	0
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BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$+$	$+$	1
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	0	$+$	0
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	$-$	$+$	1
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$-$	0	0
CII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$-$	$-$	1
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$-$	0
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$+$	$-$	1

Integer QHE

Periodic Table of TI's

class\d	0	1	2	3	4	5	6	7	T	C	S
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AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	0	1
AI	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$+$	0	0
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$+$	$+$	1
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	0	$+$	0
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AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$-$	0	0
CII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$-$	$-$	1
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$-$	0
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	$+$	$-$	1

Time-reversal invariant

What can happen with

INTERACTING

electrons?

Example: Fractional QHE

$$\sigma_{xy} = \frac{p}{q}\frac{e^2}{h}$$

No description in terms of non-interacting electrons

Integer QHE

T-Invariant TI's

Non-interacting electrons

Fractional QHE

Interacting electrons

Integer QHE

T-Invariant TI's

Non-interacting electrons

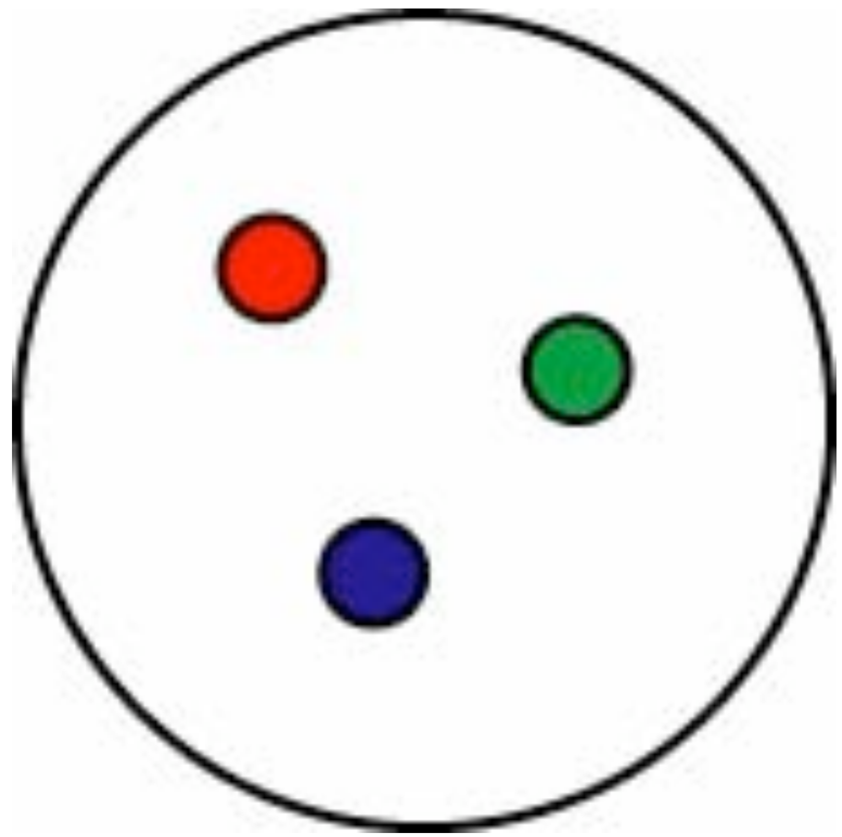
Fractional QHE

Fractional T-Invariant TI's?

Interacting electrons

Fractionalization

Similar to QCD?



Wen 1999

Maciejko, Qi, Karch, Zhang 2010, 2011

Swingle, Barkeshli, McGreevy, Senthil 2010

Fractionalization

"Statistical Gauge Fields"

$$U(1)_{EM} \to U(1)_{EM} \times G$$

Electron

Charge -1 under	Singlet under
$U(1)_{EM}$	G

Fractionalization

Similar to QCD?

$$U(1)_{EM} \times G \simeq U(1)_B \times SU(N_c)$$

Electron	$=$	Baryon
Fractional Electron	$=$	Quark
Statistical Gauge Fields	$=$	Gluons
Fractionalized phase	$=$	Non-confining phase

$\mathcal{N} = 4$ supersymmetric $SU(N_c)$ Yang-Mills

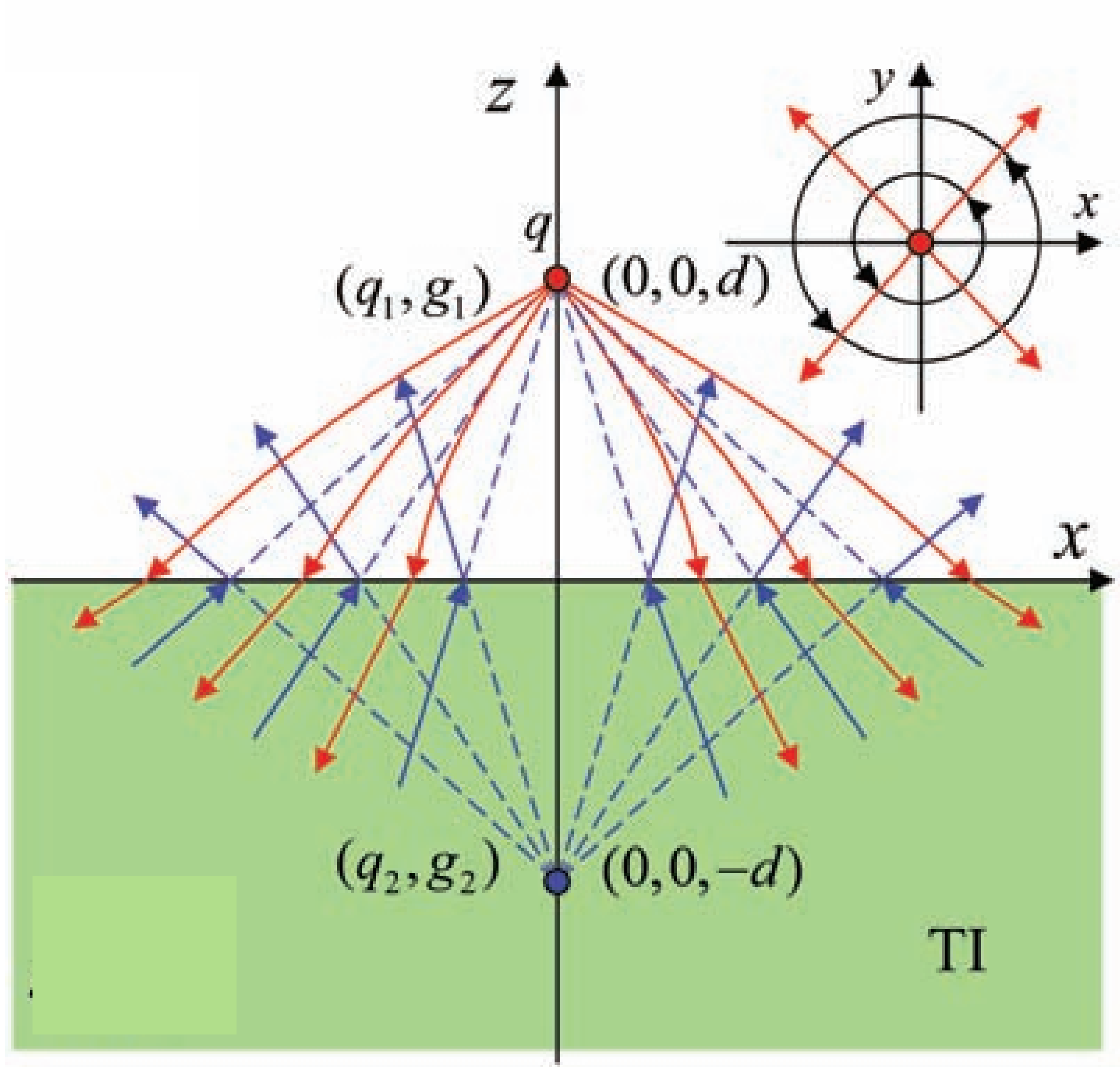
with jumping θ-angle

(3+1)d Fractional T-invariant TI

Karch 2009

GOAL

Compute the potential between test charges
in
$\mathcal{N}=4$ SYM with jumping θ-angle



Outline:

- Motivation: Topological Insulators
- Holographic Conformal Interfaces
- Holographic Wilson Loops
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(3+1)d $\mathcal{N} = 4$ SUSY $SU(N_c)$ YM

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}F_{\mu\nu}F_{\rho\sigma}$$

$$-\frac{1}{2g^2}\mathrm{tr}(D^\mu\Phi^i D_\mu\Phi^i) + \frac{1}{4g^2}\mathrm{tr}([\Phi^i,\Phi^j][\Phi^i,\Phi^j])$$

$$\boxed{\begin{gathered}\lambda = g^2 N_c \\ SO(4,2)\times SO(6) \\ \beta_\lambda = 0\end{gathered}}$$

(3+1)d $\mathcal{N}=4$ SUSY $SU(N_c)$ YM

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}F_{\mu\nu}F_{\rho\sigma}$$

$$-\frac{1}{2g^2}\mathrm{tr}(D^\mu\Phi^i D_\mu\Phi^i) + \frac{1}{4g^2}\mathrm{tr}([\Phi^i,\Phi^j][\Phi^i,\Phi^j])$$

$$N_c \to \infty$$

$$\lambda = g^2 N_c \quad \text{fixed}$$

$\mathcal{N}=4$ SYM

$N_c, \lambda \gg 1$

$=$

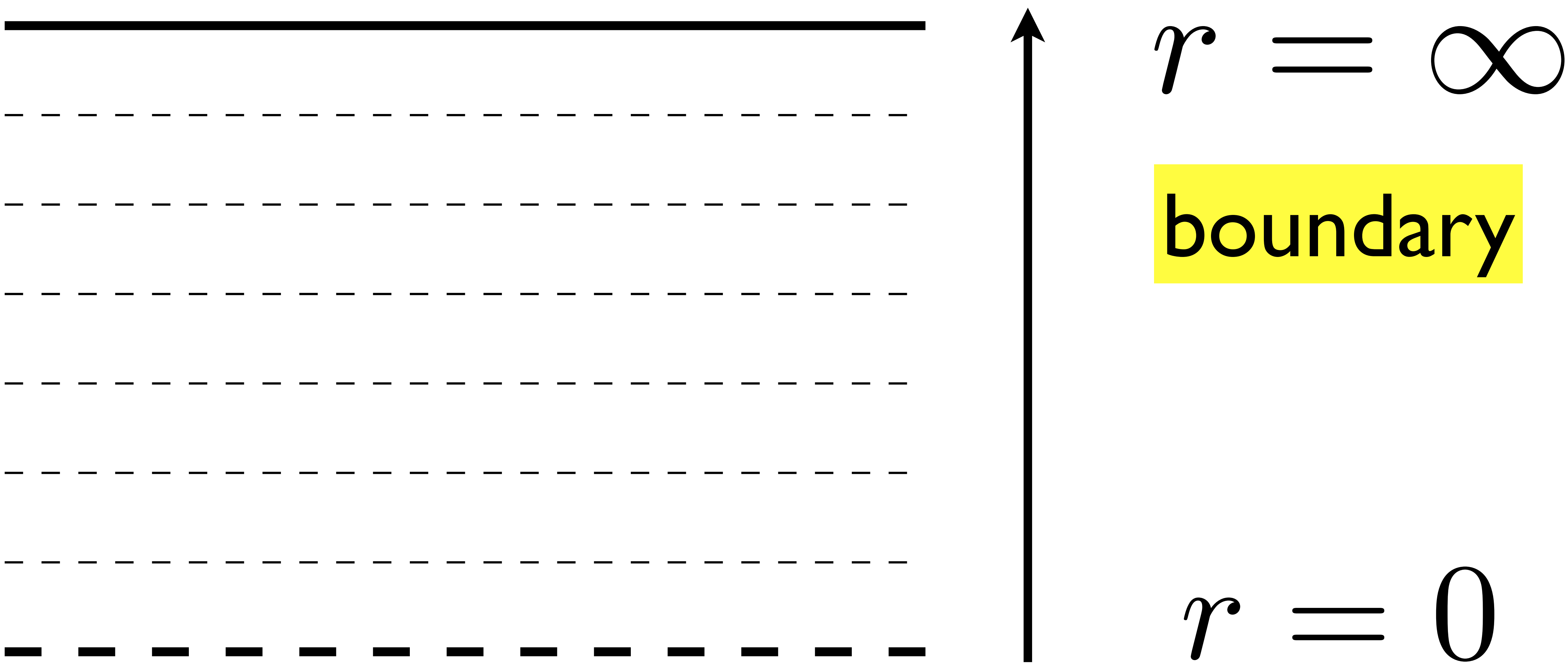
IIB SUGRA

$AdS_5 \times S^5$

global symmetry $=$ isometry

$$SO(4,2) \times SO(6)$$

$$ds^2 = \frac{dr^2}{r^2} + r^2\left(-dt^2 + dx^2 + dy^2 + dz^2\right)$$



“holographic”

The Janus Solution

Bak, Gutperle, Hirano 2003

Solution of type IIB SUGRA

$$ds^2 = R^2\left(\gamma^{-1}h(x)^2dx^2 + h(x)ds^2_{AdS_4} + ds^2_{S^5}\right)$$

$$h(x) = \gamma\left(1 + \frac{4\gamma - 3}{\wp(x) + 1 - 2\gamma}\right)$$

$$\phi(x) = \phi_0 + \sqrt{6(1-\gamma)}\left(x + \frac{4\gamma - 3}{\wp'(\chi)}\left(\ln\frac{\sigma(x+\chi)}{\sigma(x-\chi)} - 2\zeta(\chi)x\right)\right)$$

(and the five-form)

The Janus Solution

One-parameter dilatonic deformation of $AdS_5 \times S^5$

$$ds^2 = R^2 \left(\gamma^{-1} h(x)^2 dx^2 + h(x) ds^2_{AdS_4} + ds^2_{S^5}\right)$$

$$h(x) = \gamma \left(1 + \frac{4\gamma - 3}{\wp(x) + 1 - 2\gamma}\right)$$

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(and the five-form)

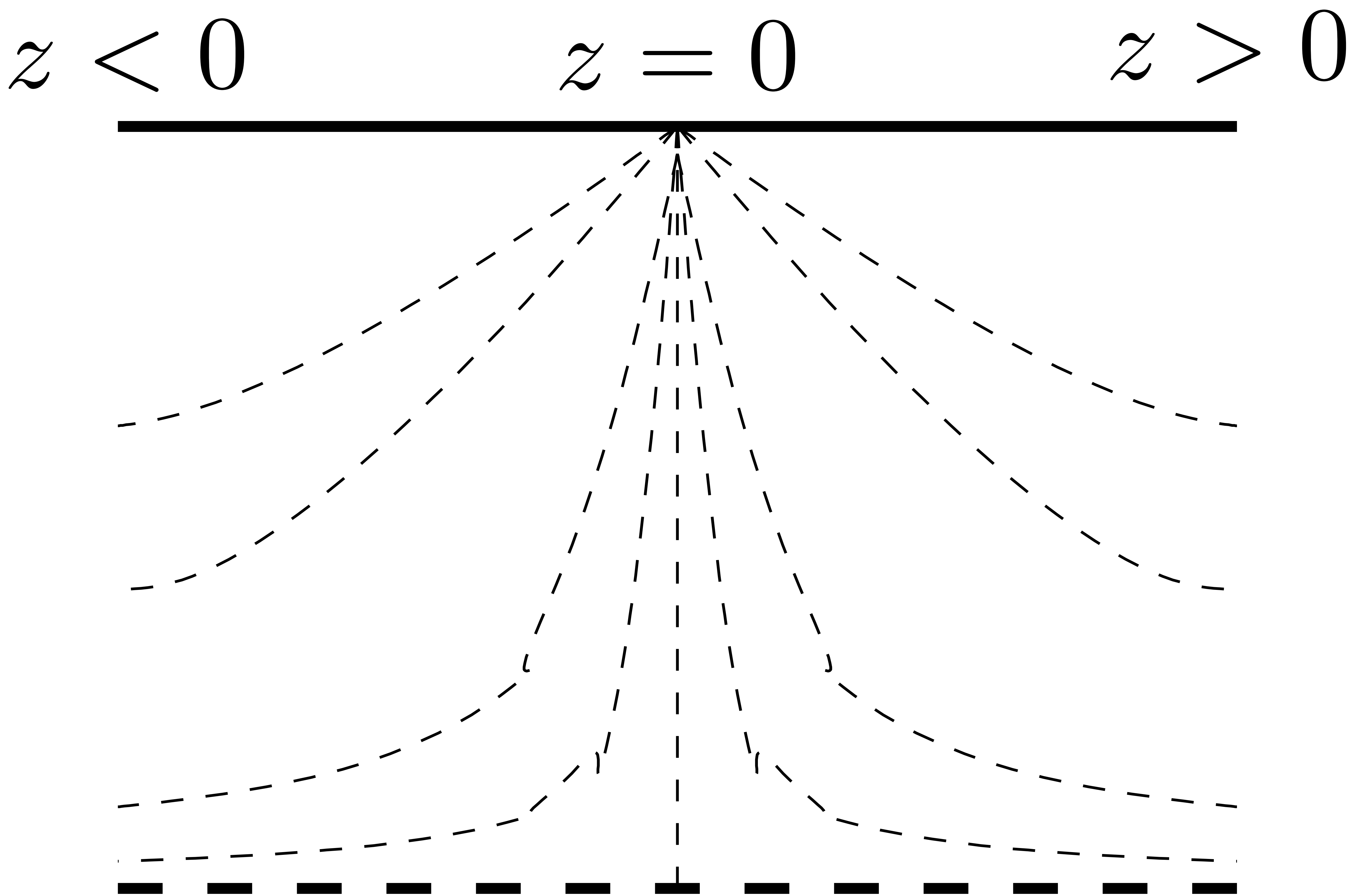
AdS_4 slicing of AdS_5

$$ds^2 = R^2 \left(h(x)^2 dx^2 + h(x) ds^2_{AdS_4} + ds^2_{S^5} \right)$$

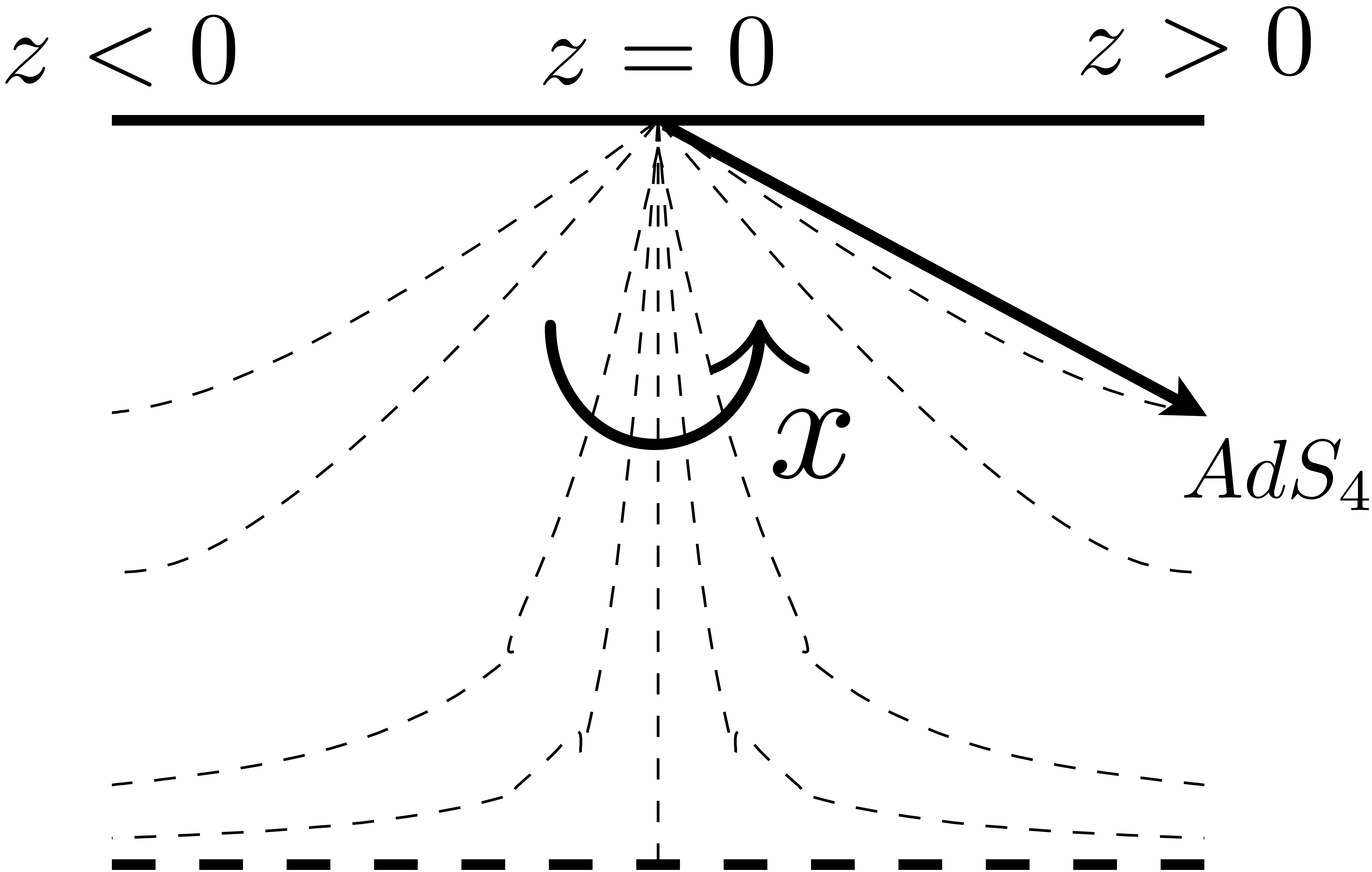
$$h(x) = \frac{1}{1-x^2} \qquad x \in (-1,1)$$

$SO(3,2) \times SO(6)$ isometry manifest

AdS_4 slicing of AdS_5



AdS_4 slicing of AdS_5



The Janus Solution

One-parameter dilatonic deformation of $AdS_5 \times S^5$

$$ds^2 = R^2 \left(\gamma^{-1} h(x)^2 dx^2 + h(x) ds^2_{AdS_4} + ds^2_{S^5}\right)$$

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(and the five-form)

The Janus Solution

Isometry $SO(3,2) \times SO(6)$

$$ds^2 = R^2 \left(\gamma^{-1} h(x)^2 dx^2 + h(x) ds^2_{AdS_4} + ds^2_{S^5} \right)$$

$$h(x) = \gamma \left(1 + \frac{4\gamma - 3}{\wp(x) + 1 - 2\gamma} \right)$$

$$\phi(x) = \phi_0 + \sqrt{6(1-\gamma)} \left(x + \frac{4\gamma - 3}{\wp'(\chi)} \left(\ln \frac{\sigma(x+\chi)}{\sigma(x-\chi)} - 2\zeta(\chi) x \right) \right)$$

(and the five-form)

The Janus Solution

One free parameter

$\gamma \in (3/4, 1)$

$$ds^2 = R^2\left(\gamma^{-1} h(x)^2 dx^2 + h(x) ds^2_{AdS_4} + ds^2_{S^5}\right)$$

$$h(x) = \gamma\left(1 + \frac{4\gamma - 3}{\wp(x) + 1 - 2\gamma}\right)$$

$$\phi(x) = \phi_0 + \sqrt{6(1-\gamma)}\left(x + \frac{4\gamma - 3}{\wp'(\chi)}\left(\ln\frac{\sigma(x+\chi)}{\sigma(x-\chi)} - 2\zeta(\chi)x\right)\right)$$

(and the five-form)

The Janus Solution

Breaks ALL SUSY

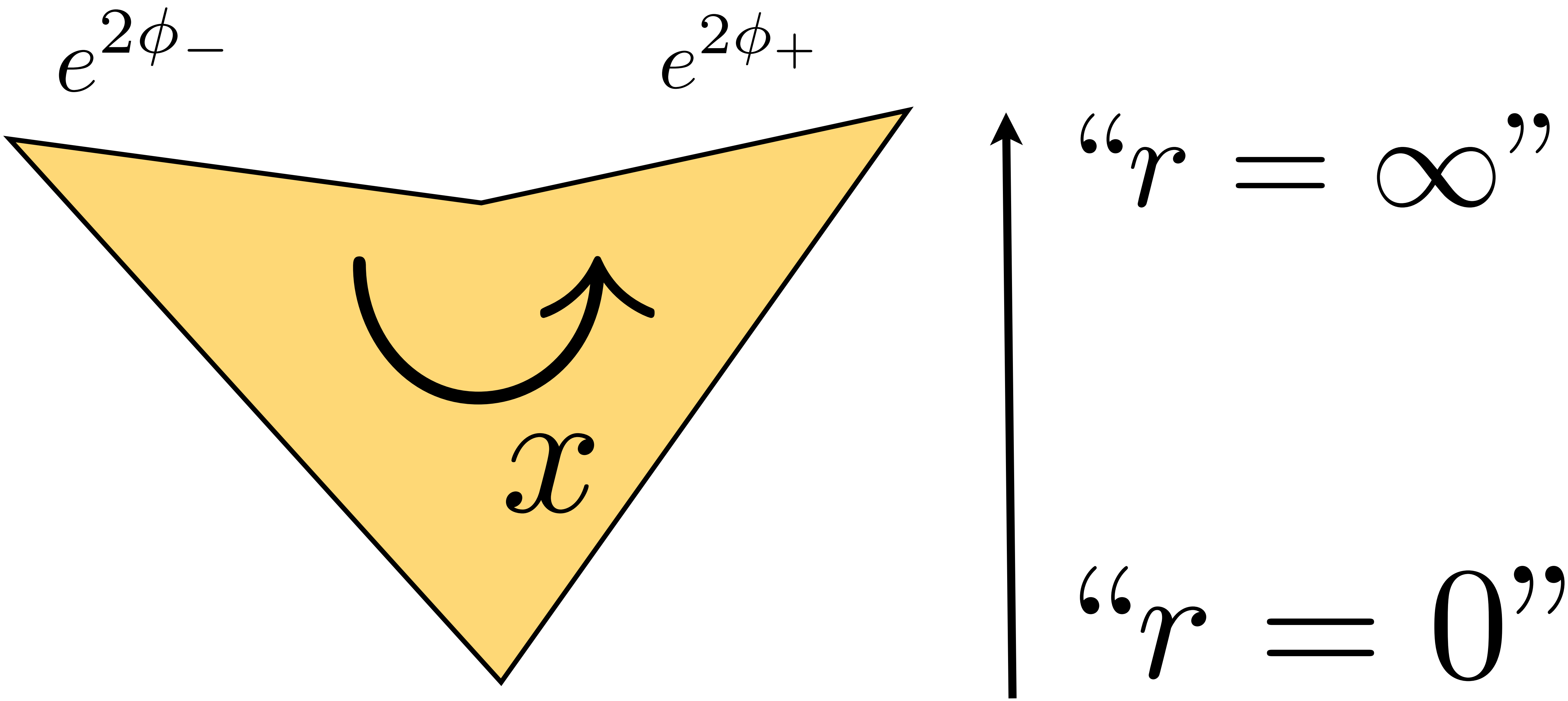
$$ds^2 = R^2\left(\gamma^{-1}h(x)^2 dx^2 + h(x) ds^2_{AdS_4} + ds^2_{S^5}\right)$$

$$h(x) = \gamma\left(1 + \frac{4\gamma - 3}{\wp(x) + 1 - 2\gamma}\right)$$

$$\phi(x) = \phi_0 + \sqrt{6(1-\gamma)}\left(x + \frac{4\gamma - 3}{\wp'(\chi)}\left(\ln\frac{\sigma(x+\chi)}{\sigma(x-\chi)} - 2\zeta(\chi)x\right)\right)$$

(and the five-form)

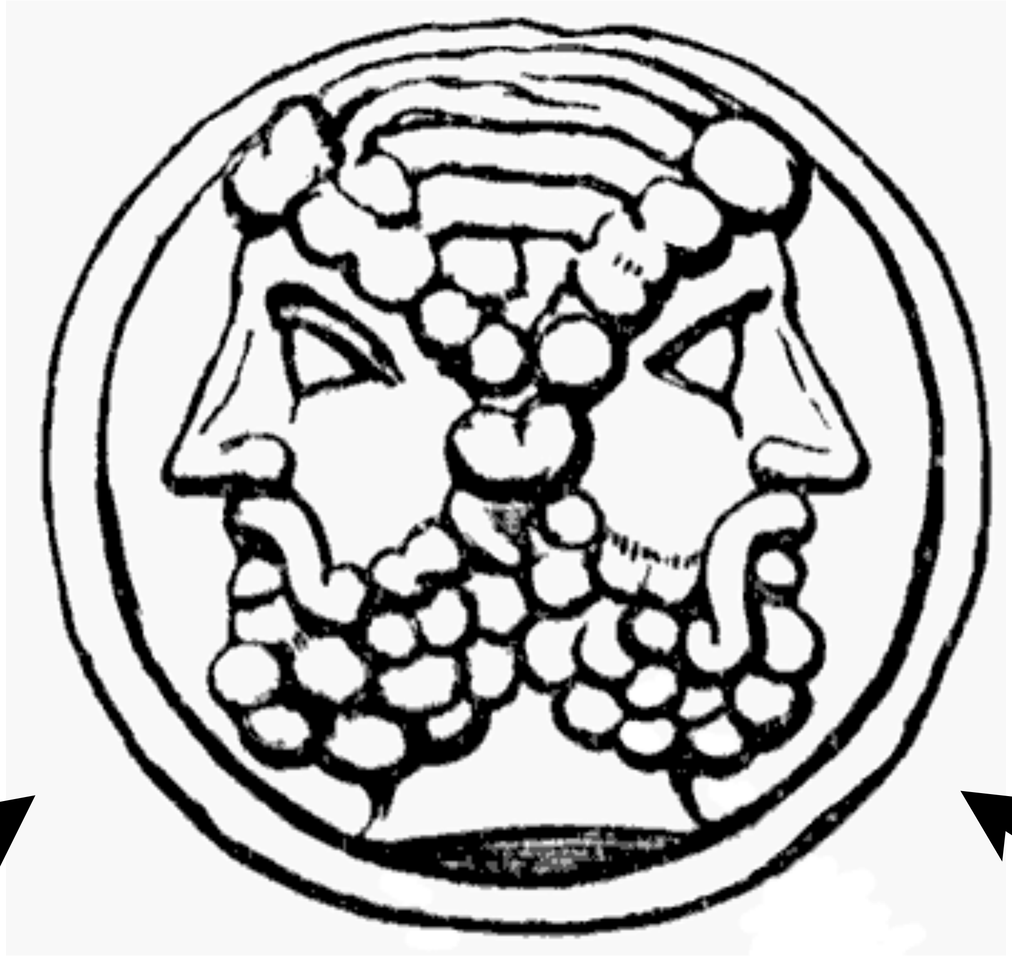
The Janus Solution



Free parameter: jump in dilaton

Janus

Roman god of beginnings, transitions, gates, and doorways

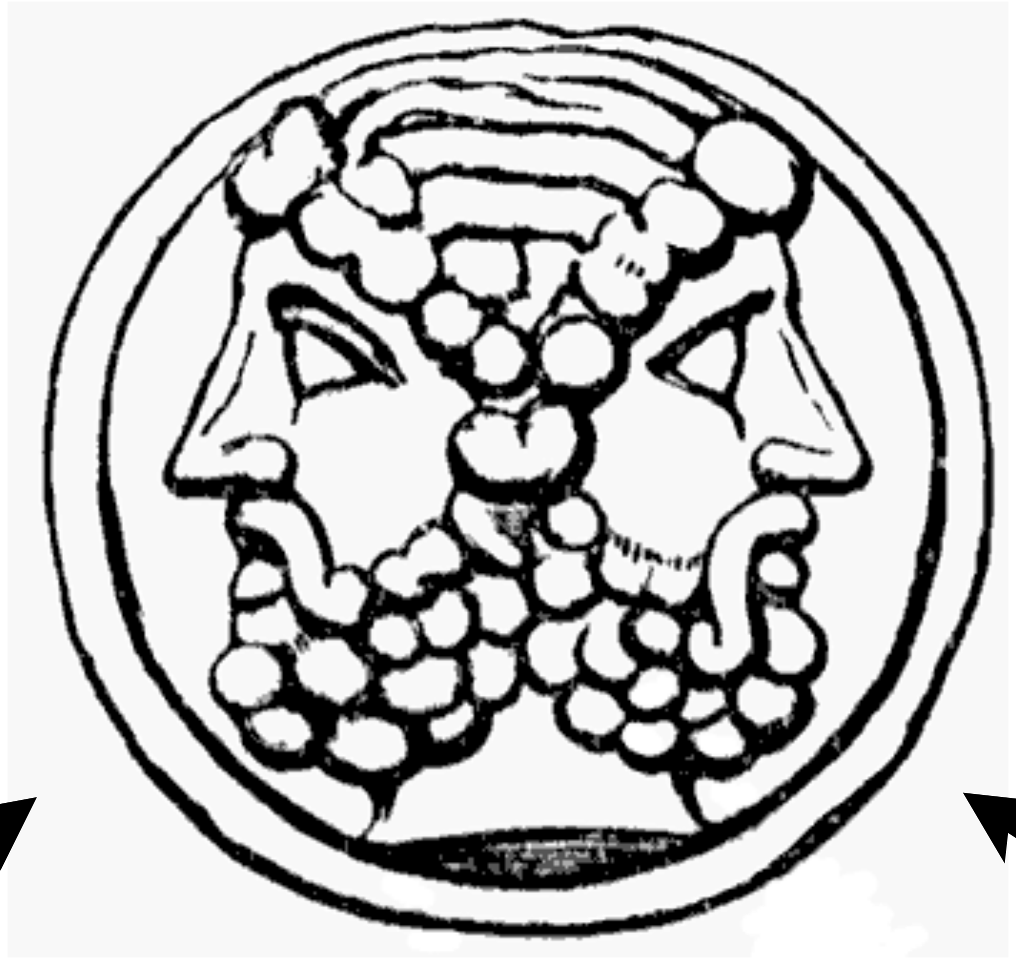


Towards Past

Towards Future

Janus

Root of "January" and "Janitor"



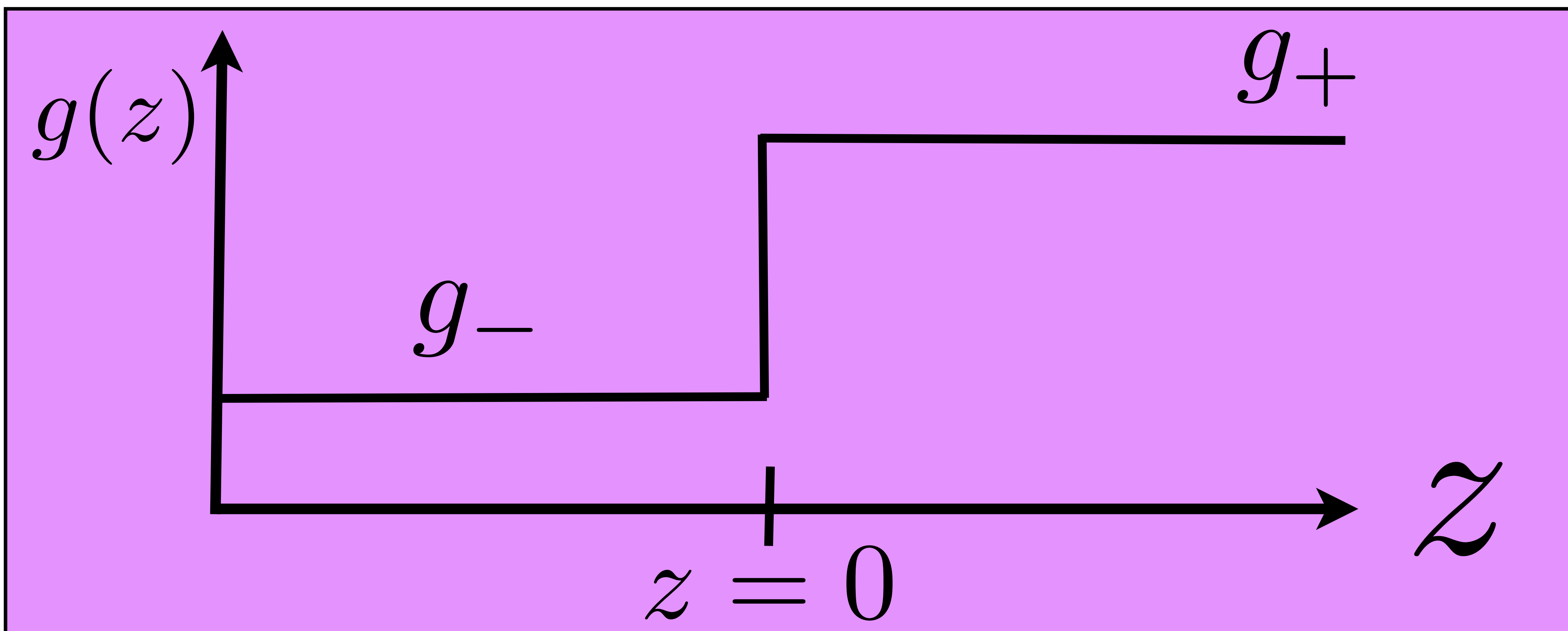
Towards Past

Towards Future

$$e^{2\phi} = g^2/2\pi$$

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}F_{\mu\nu}F_{\rho\sigma}$$

$$-\frac{1}{2g^2}\mathrm{tr}(D^\mu\Phi^i D_\mu\Phi^i) + \frac{1}{4g^2}\mathrm{tr}([\Phi^i,\Phi^j][\Phi^i,\Phi^j])$$



Jumping coupling

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}F_{\mu\nu}F_{\rho\sigma}$$

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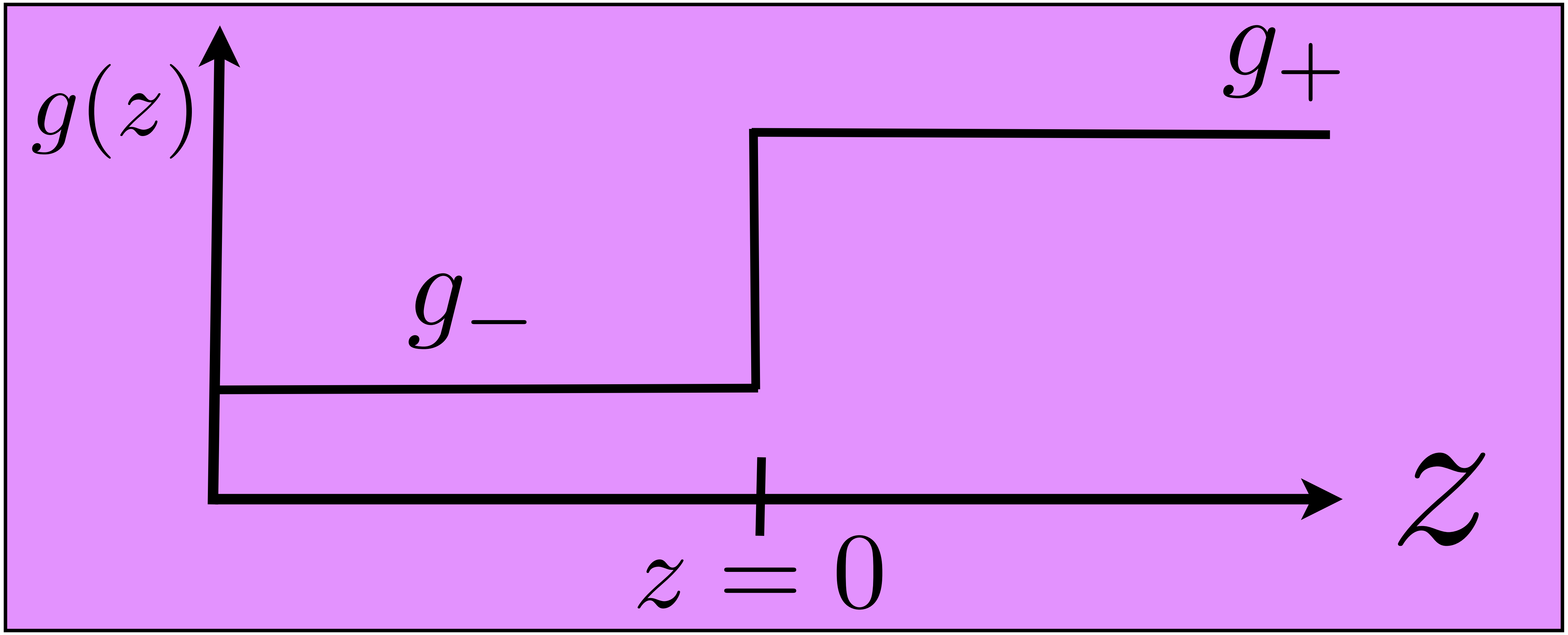

Jumping coupling

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}F_{\mu\nu}F_{\rho\sigma}$$

$$-\frac{1}{2g^2}\mathrm{tr}(D^\mu\Phi^i D_\mu\Phi^i) + \frac{1}{4g^2}\mathrm{tr}([\Phi^i,\Phi^j][\Phi^i,\Phi^j])$$

Breaks all SUSY

Preserves SO(3,2) x SO(6)

“Conformal Interface”

Dielectric Interfaces

$$\mathcal{L} = \frac{1}{8\pi}\left[\varepsilon \vec{E}^2 - \frac{1}{\mu}\vec{B}^2\right]$$

$$\varepsilon \propto 1/g^2$$

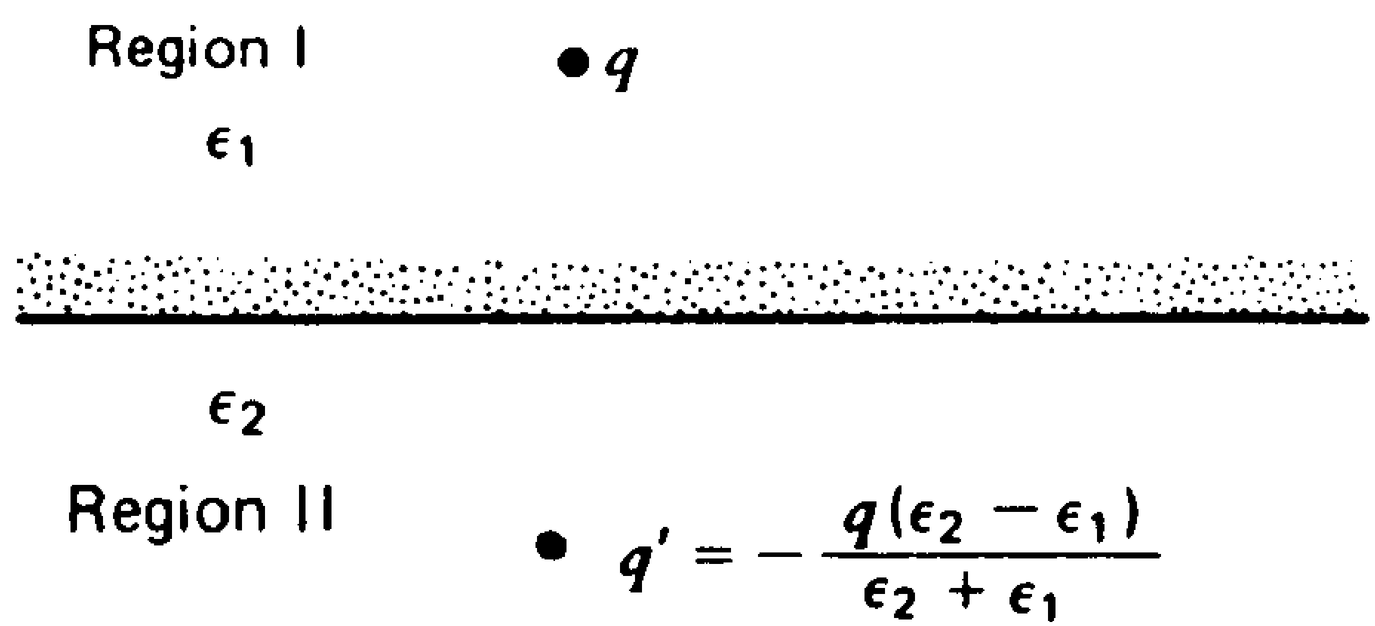

Image charges!

$$\boxed{SL(2,\mathbb{R})}$$

$$\boxed{\tau = C_0 + ie^{-2\phi}}$$

$$\tau \to \frac{a\tau + b}{c\tau + d} \qquad a, b, c, d \in \mathbb{R}$$

$$ab - cd = 1$$

Jumping dilaton $\longrightarrow$ Jumping axion

$$\boxed{SL(2,\mathbb{R})}$$

$$\boxed{\tau = \frac{\theta}{2\pi} + i\frac{2\pi}{g^2}}$$

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \qquad a, b, c, d \in \mathbb{R}$$

$$ab - cd = 1$$

Jumping coupling $\longrightarrow$ Jumping θ-angle

Topological Insulator

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Wilson Loops in $\mathcal{N} = 4$ SYM

$$W_R[C] = \frac{1}{N_c} \mathrm{tr}_R P \exp\left[\oint_C ds \left(i A_\mu \dot{x}^\mu + \Phi_i \theta^i |\dot{x}|\right)\right]$$

$R = N_c$ of $SU(N_c)$

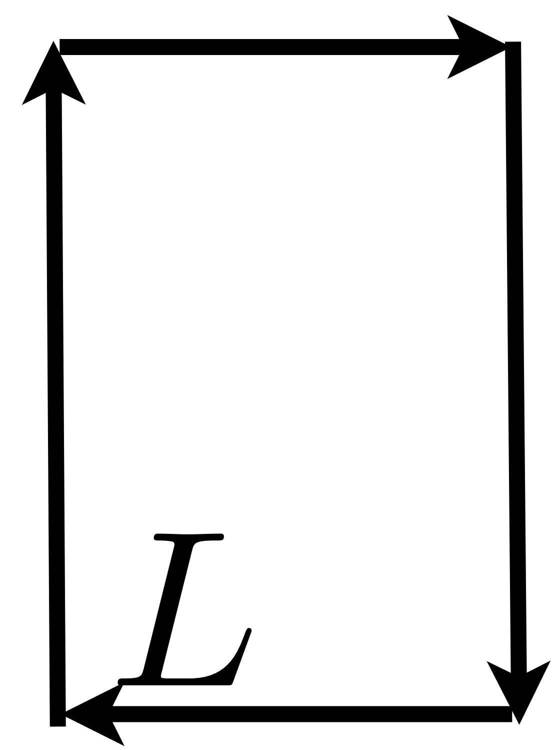
$$C = \Big\uparrow T$$

Wilson Loops in $\mathcal{N} = 4$ SYM

$$W_R[C] = \frac{1}{N_c} \mathrm{tr}_R P \exp\left[\oint_C ds \left(iA_\mu \dot{x}^\mu + \Phi_i \theta^i |\dot{x}|\right)\right]$$

$R = N_c$ of $SU(N_c)$

$C =$



T

Wilson Loops in $\mathcal{N} = 4$ SYM

$$W_R[C] = \frac{1}{N_c} \mathrm{tr}_R P \exp\left[\oint_C ds\left(iA_\mu \dot{x}^\mu + \Phi_i \theta^i |\dot{x}|\right)\right]$$

$$\boxed{V(L) = -\lim_{T\to\infty} \frac{1}{T} \ln\langle W[C]\rangle}$$

$$\boxed{V(L) = \frac{f(\lambda)}{L}}$$

Wilson Loops in $\mathcal{N}=4$ SYM

$$\lambda \ll 1$$

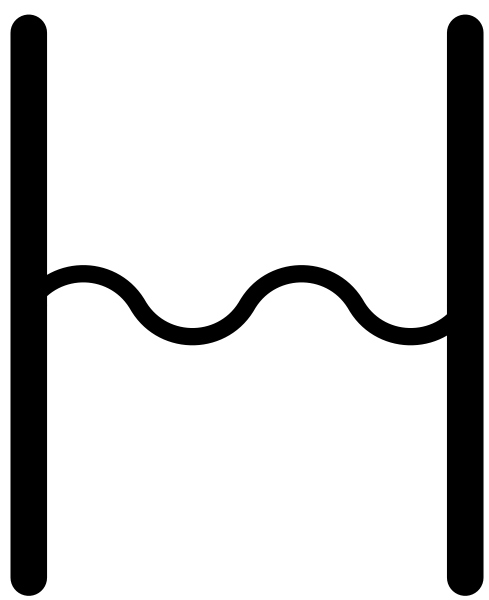
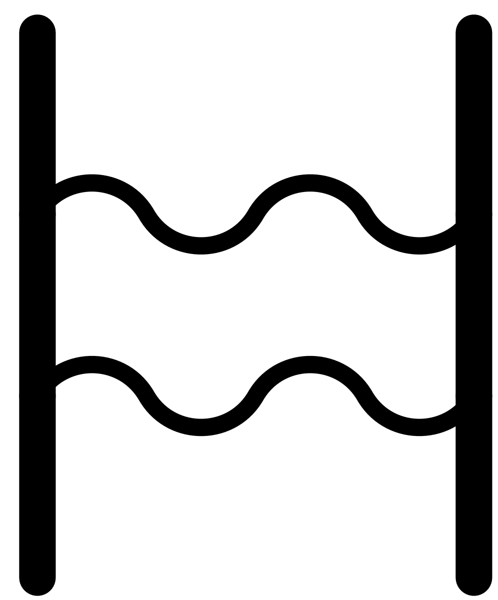
Perturbatively

$$\langle W[C]\rangle = 1 - \frac{N_c}{2}\oint_C ds \oint_C d\tilde{s}\,\Big(\dot{x}^{\mu}(s)\dot{x}^{\nu}(\tilde{s})\langle P A_{\mu}(x(s))A_{\nu}(x(s))\rangle$$

$$- |\dot{x}(s)||\dot{x}(\tilde{s})|\theta^{i}\theta^{j}\langle P\Phi^{i}(x(s))\Phi^{j}(x(\tilde{s}))\rangle\Big) + \ldots$$

Wilson Loops in $\mathcal{N} = 4$ SYM

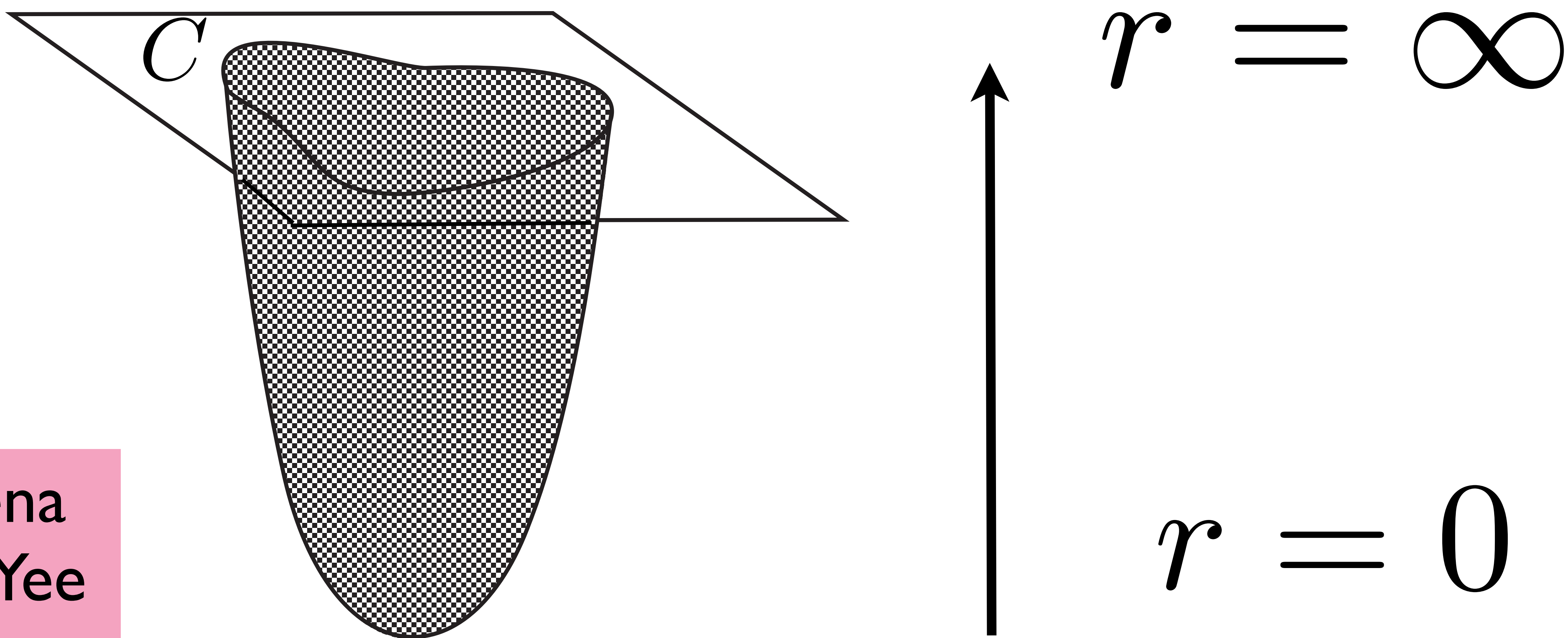
Sum "ladder" diagrams

$1 +$  $+$  $+ \cdots$

Wilson Loops in $\mathcal{N} = 4$ SYM

$$\lambda \gg 1$$

Holographically



Maldacena
Rey and Yee
1998

Wilson Loops in $\mathcal{N}=4$ SYM

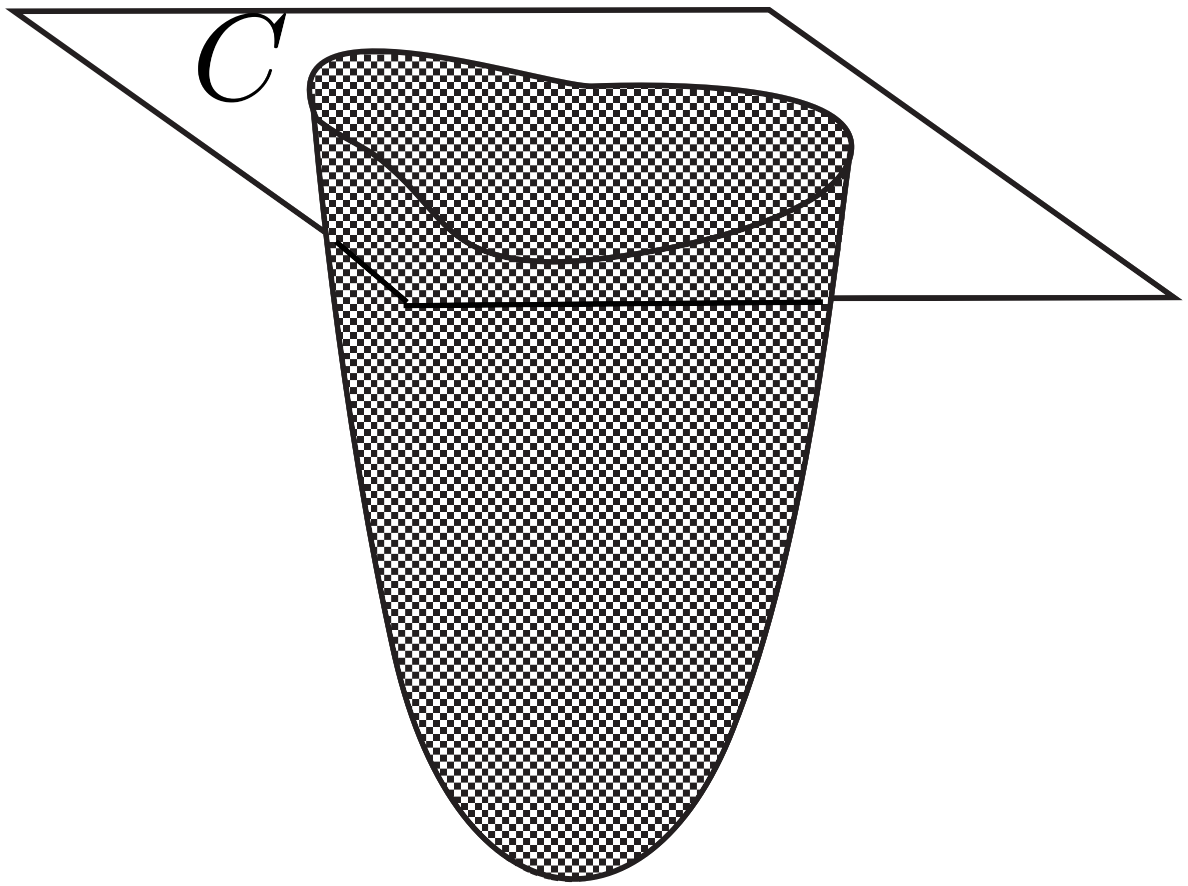
$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d^2\sigma\,\sqrt{-\det g}$$

$$S_{NG}\big|_{\text{solution}} = A$$

$$\langle W[C]\rangle \propto e^{-A}$$

$$V(L) = -\lim_{T\to\infty}\frac{1}{T}\ln\langle W[C]\rangle = \lim_{T\to\infty}\frac{A}{T}$$

Wilson Loops in $\mathcal{N} = 4$ SYM



$S_{NG}|_{\text{solution}}$

diverges at

$r = \infty$

Infinite self-energy

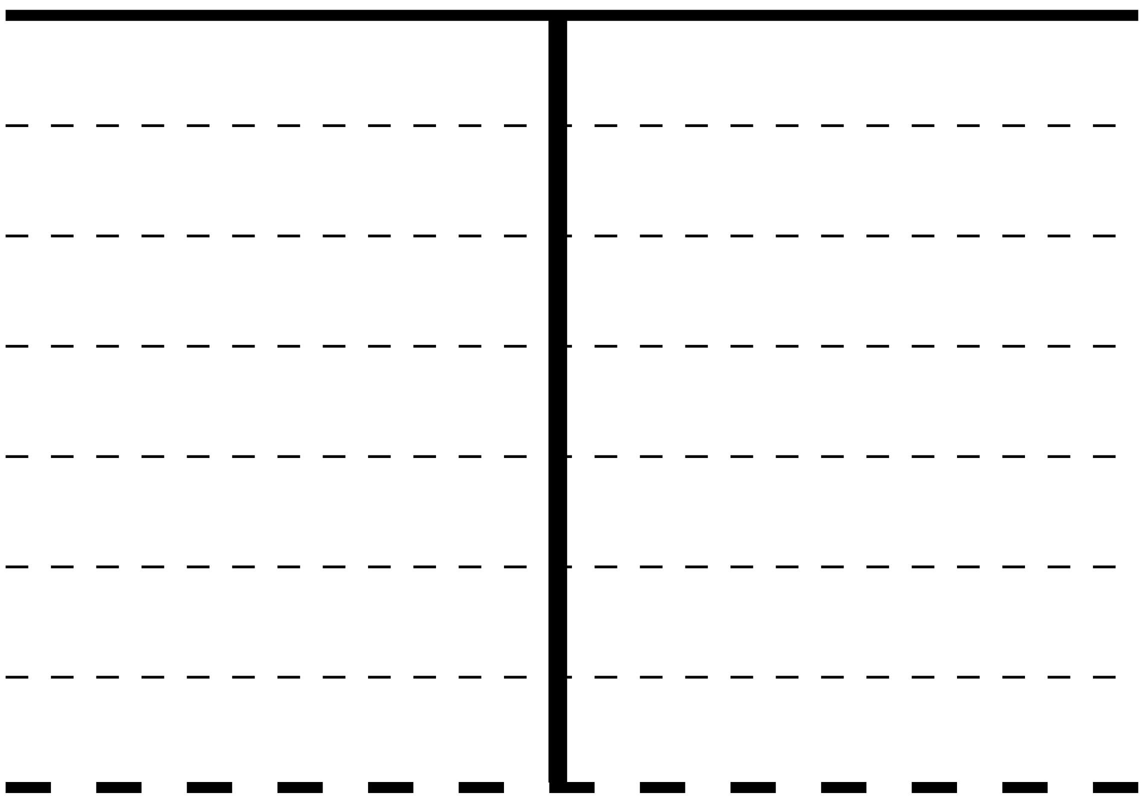
Wilson Loops in $\mathcal{N} = 4$ SYM

Drukker, Gross, Ooguri 1999

Legendre transform

$$A = S_{NG} - \int d\sigma\, P_r\, r \Big|_{\partial \mathrm{AdS}}$$

Wilson Loops in $\mathcal{N} = 4$ SYM



"Straight string"

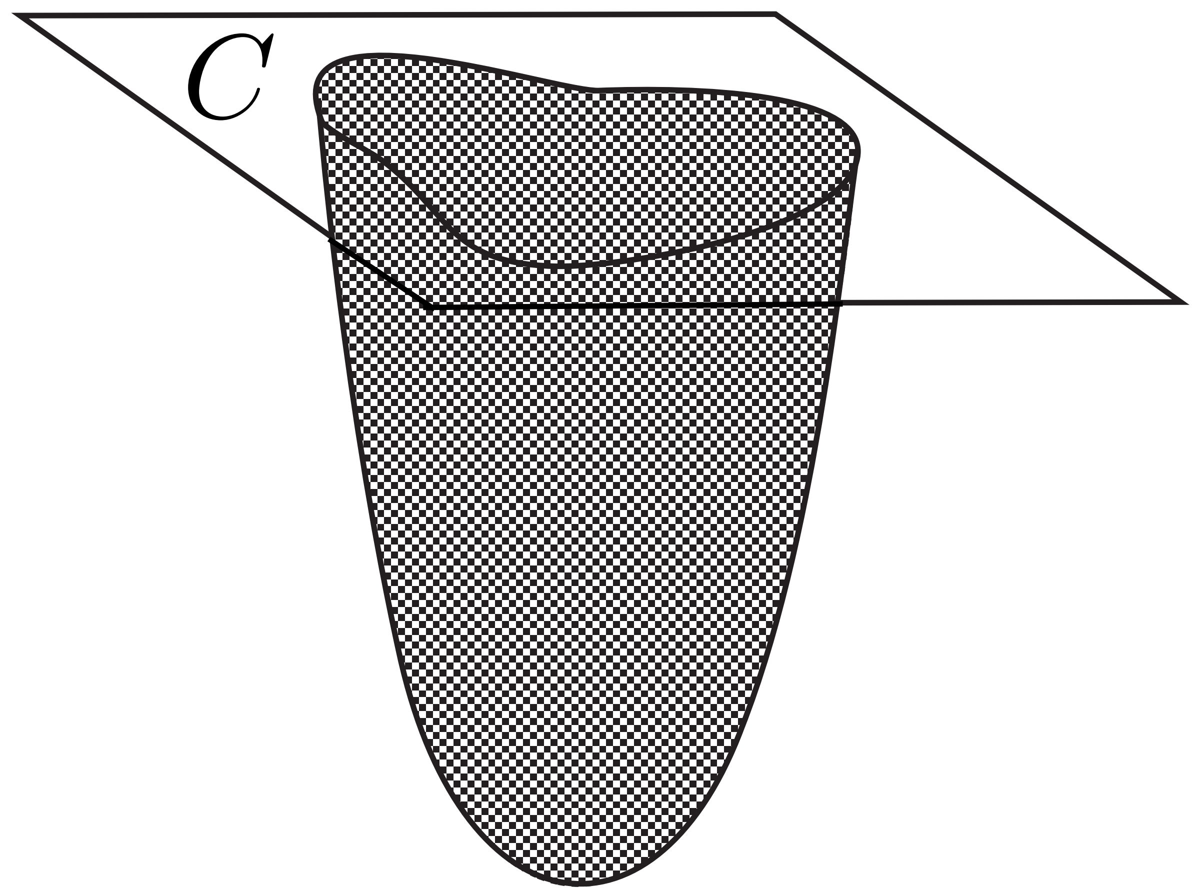
Legendre transform cancels the divergence!

$$A = 0 \quad \Rightarrow \quad \langle W[C] \rangle = 1$$

due to SUSY

$\mathcal{N}=4$ SYM jumping g

Wilson Loops from AdS_5



Legendre transform = Subtracting straight string = Subtracting self-energy

Maldacena
Rey and Yee
1998

Holography

$$V(L) = -\frac{4\pi^2}{\Gamma(1/4)^4}\frac{\sqrt{2\lambda}}{L} \qquad \lambda \gg 1$$

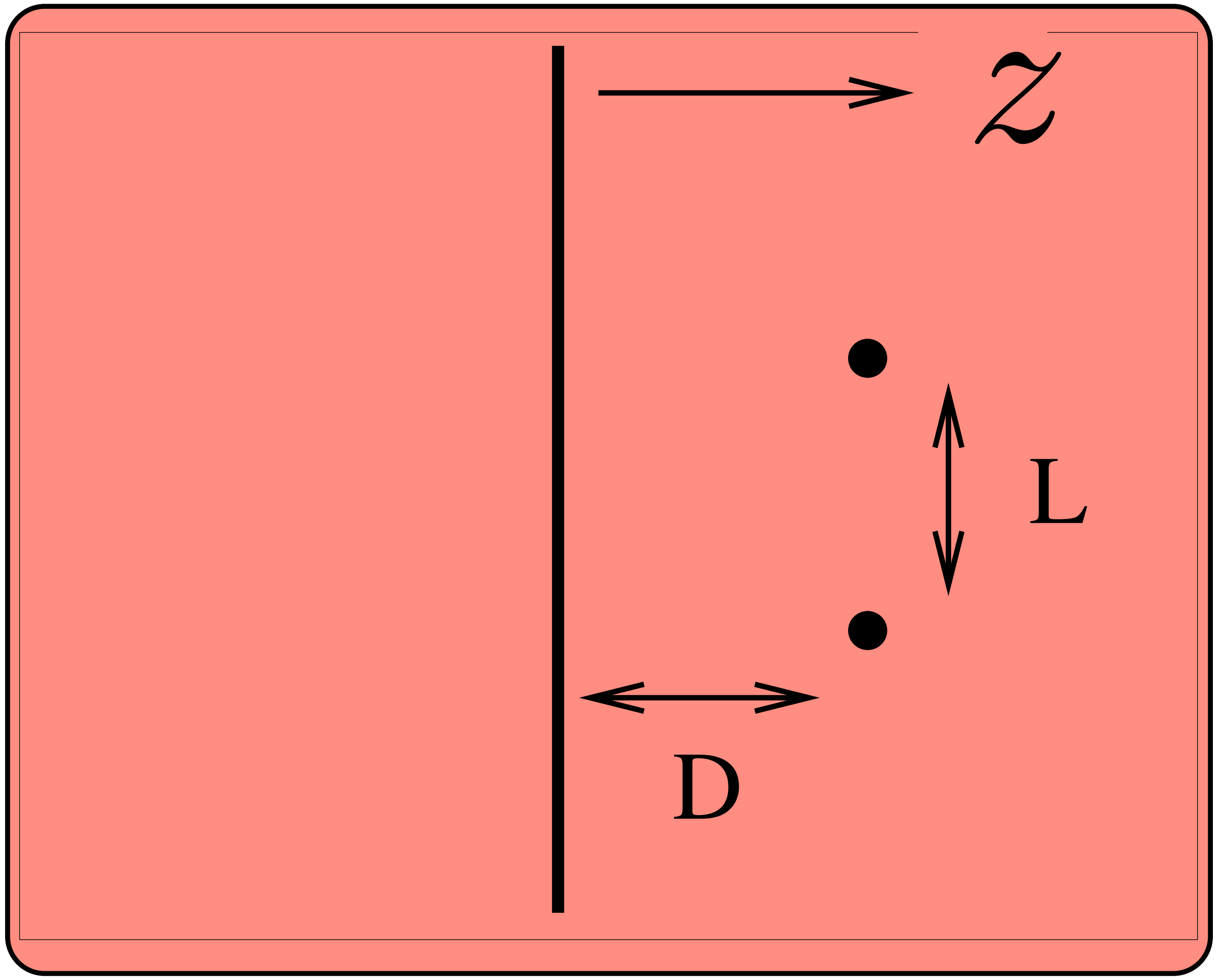
Erickson
Semenoff
Zarembo
2000

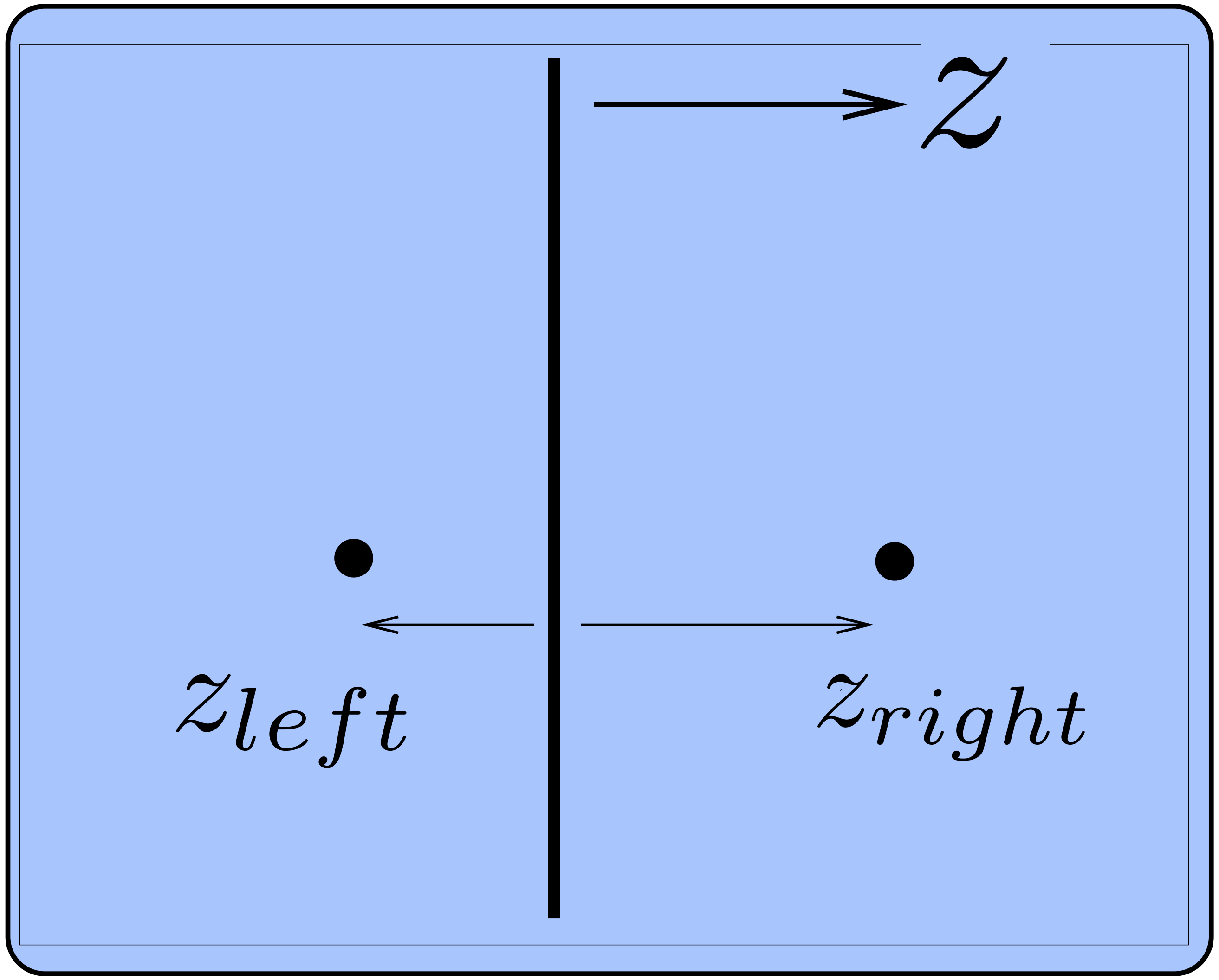
Ladder Diagrams

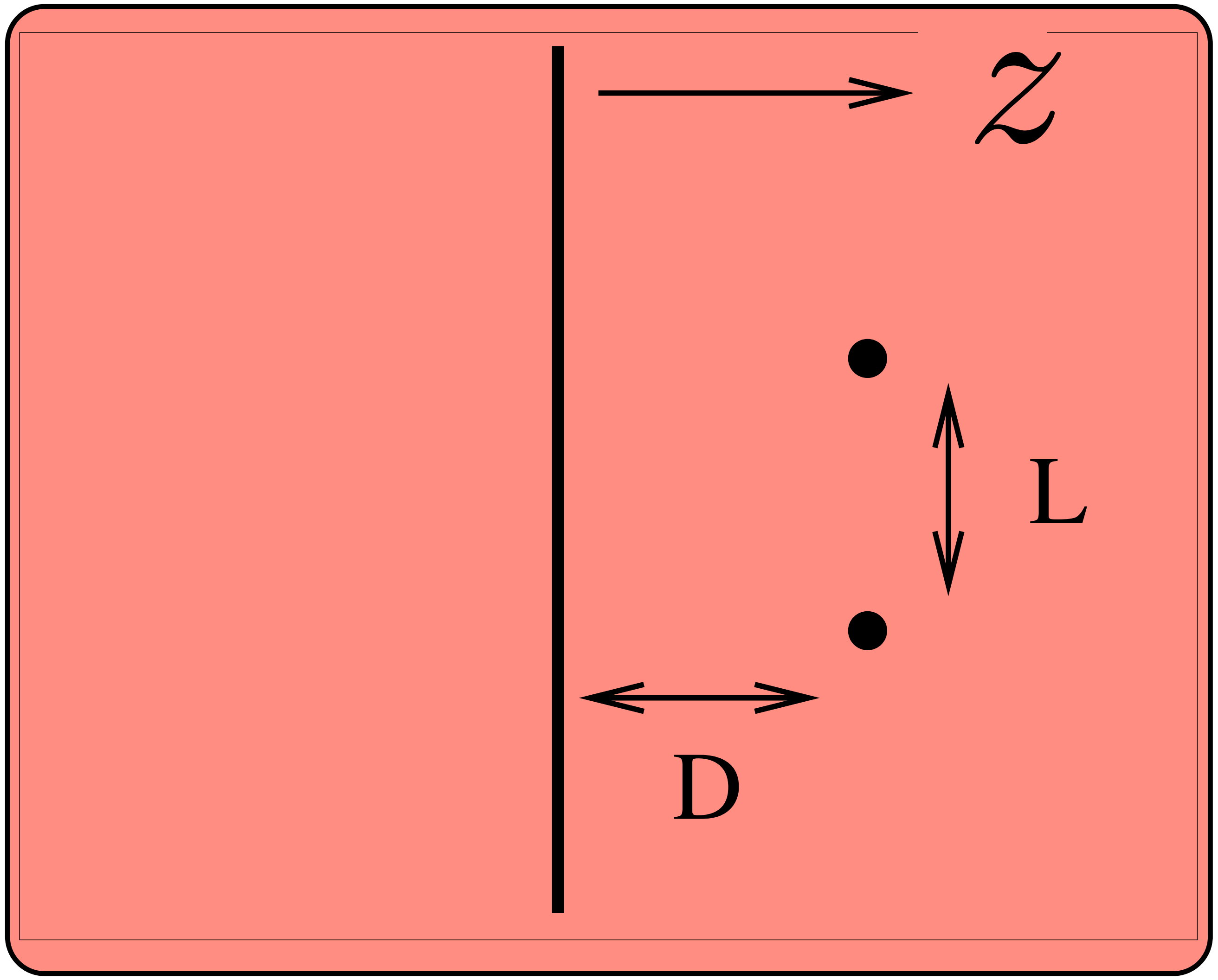
$$V(L) = \begin{cases} -\dfrac{1}{4\pi}\dfrac{2\lambda}{L}, & \lambda \ll 1 \\ \\ -\dfrac{1}{\pi}\dfrac{\sqrt{2\lambda}}{L}, & \lambda \gg 1. \end{cases}$$

Wilson Loops in $\mathcal{N} = 4$ SYM

Conformal Interface







$$V(L,D) = \frac{f(\lambda, D/L)}{L}$$

Wilson Loops in $\mathcal{N} = 4$ SYM

Conformal Interface

Perturbatively

$\langle PA_\mu(x(s))A_\nu(x(s))\rangle$ acquires image terms

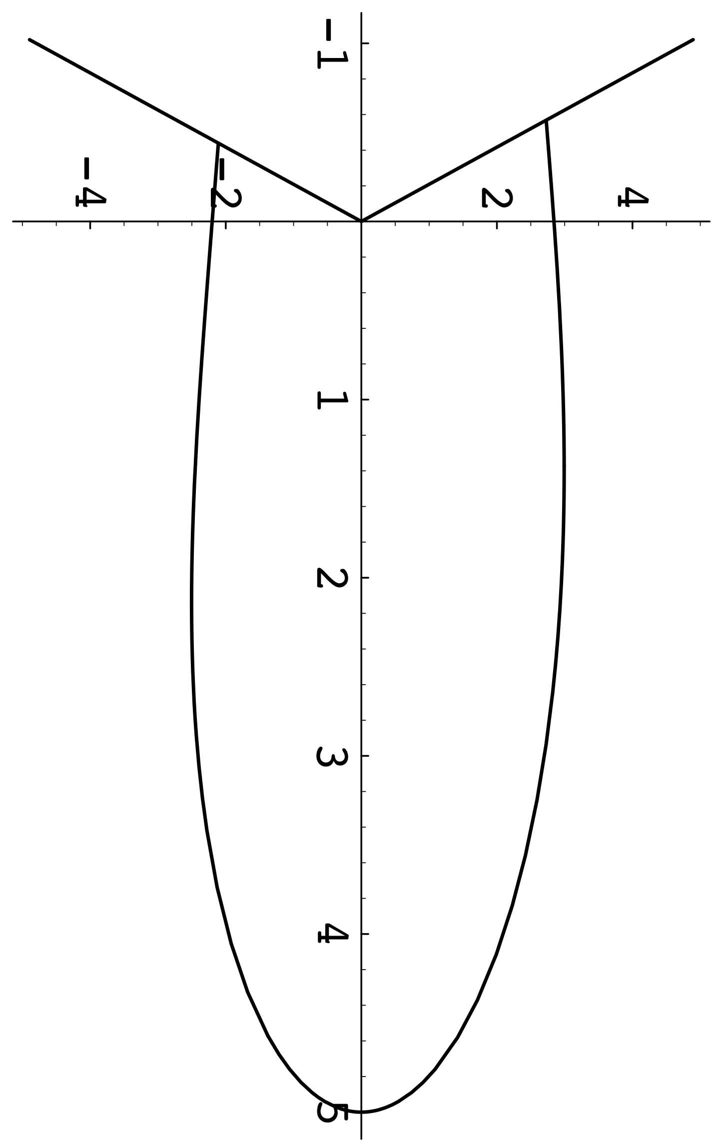
$\langle P\Phi^i(x(s))\Phi^j(x(\tilde{s}))\rangle$ unchanged

Clark, Freedman, Karch, Schnabl 2004

Wilson Loops in $\mathcal{N} = 4$ SYM

Conformal Interface

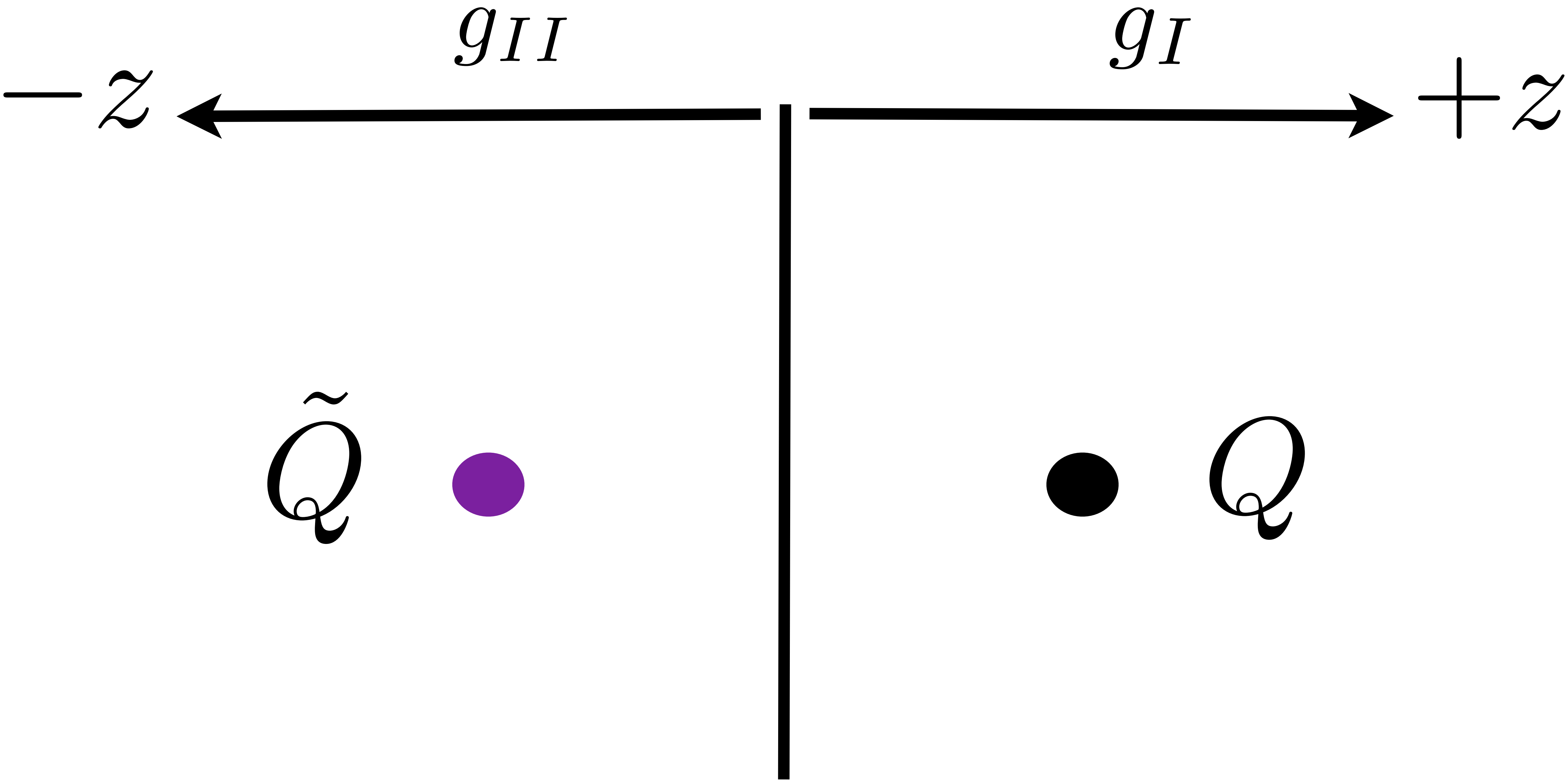
Holographically



Outline:

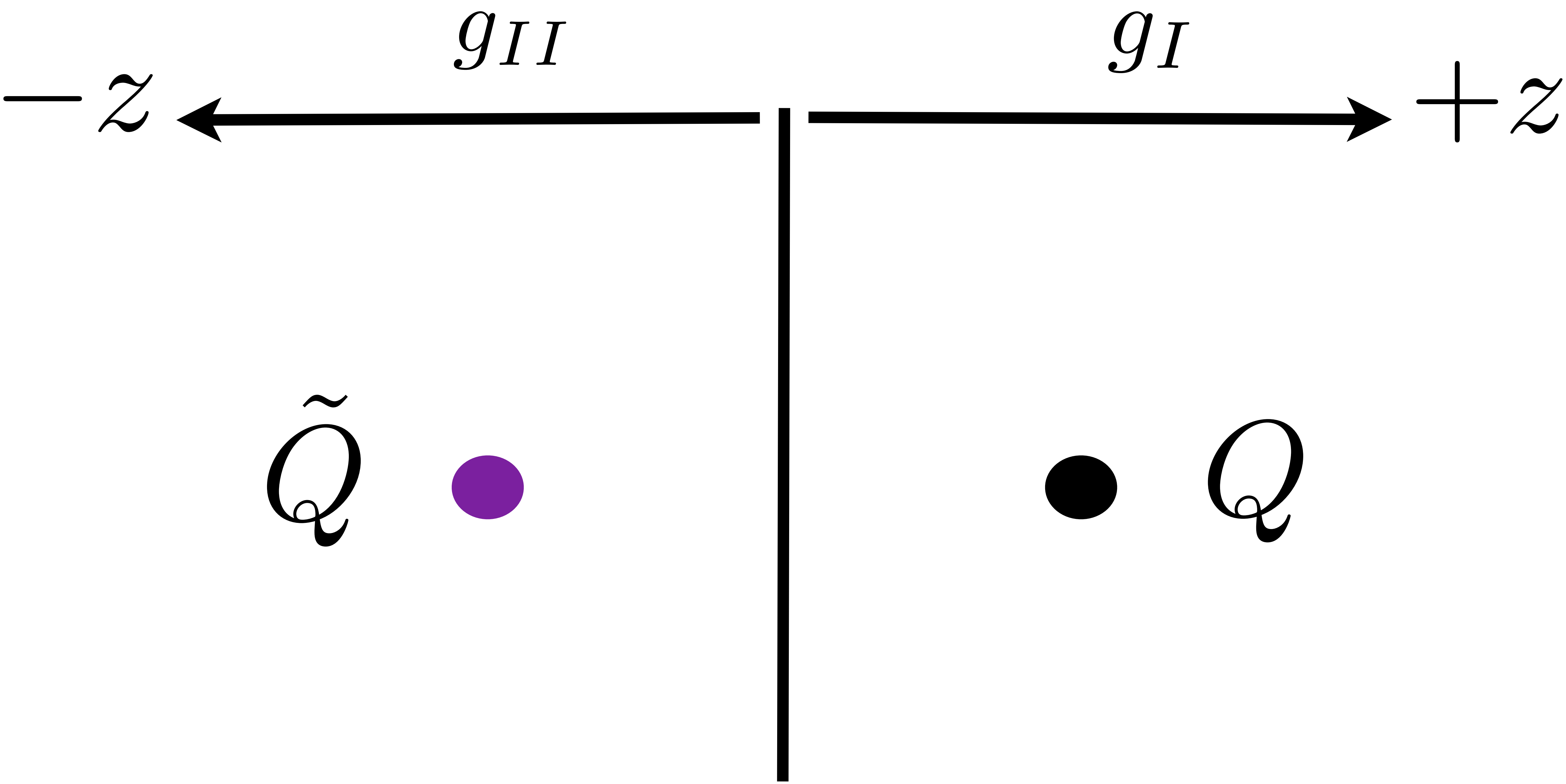
- Motivation: Topological Insulators
- Holographic Conformal Interfaces
- Holographic Wilson Loops
- Static Quark Potential
- Future Directions

Electromagnetism



$$\tilde{Q} = Q\,\frac{g_{II}^2 - g_I^2}{g_{II}^2 + g_I^2}$$

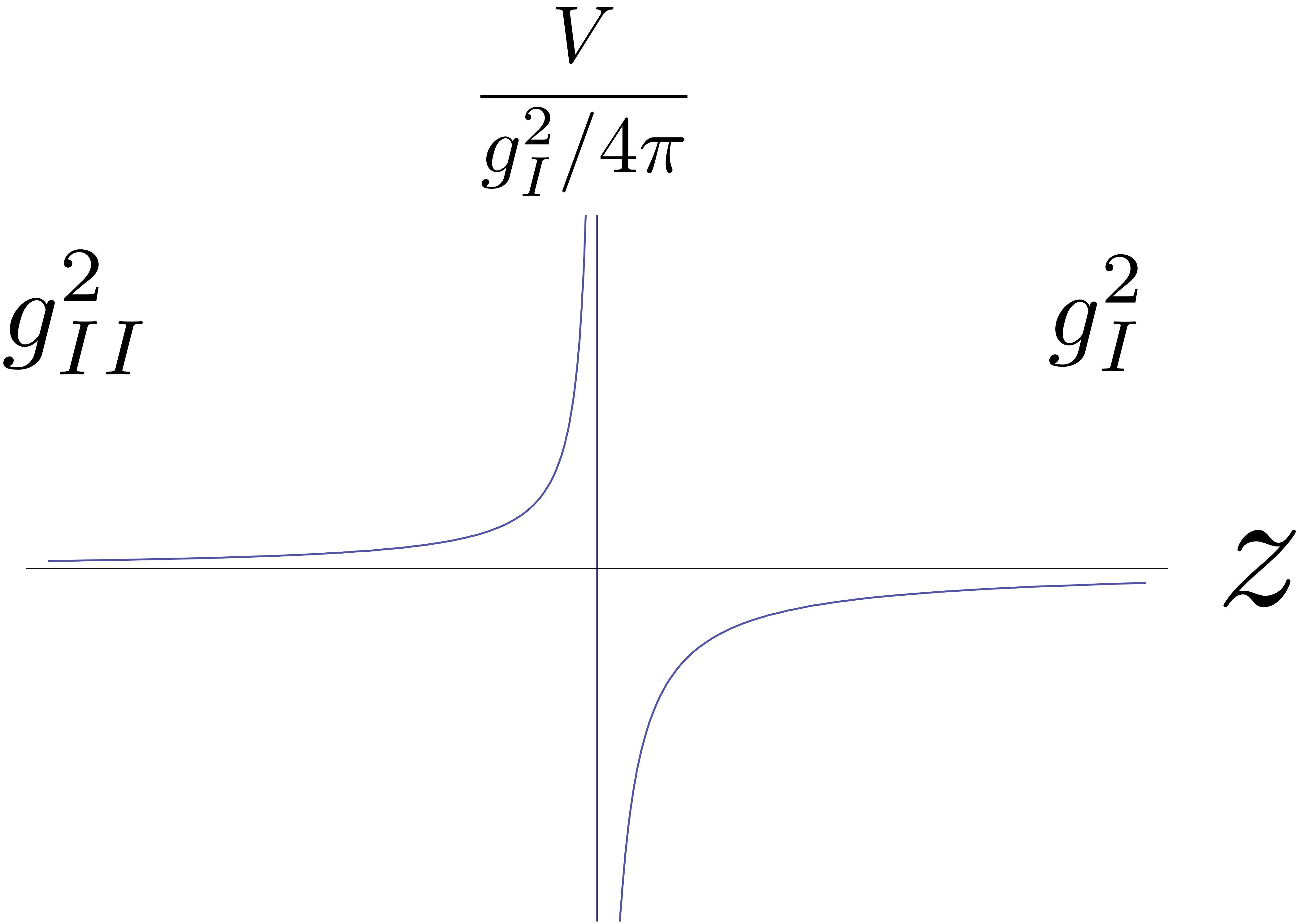
Electromagnetism



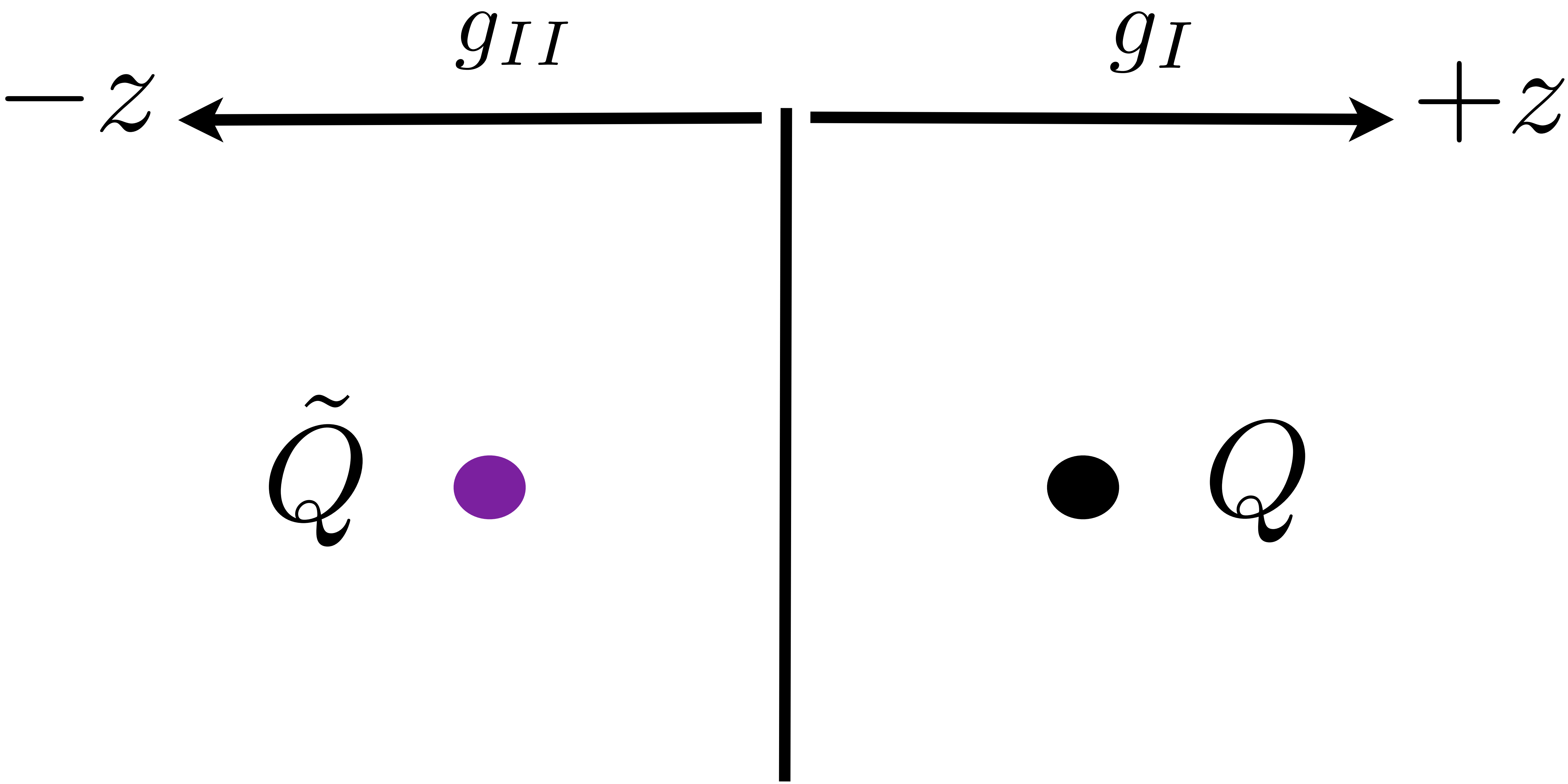
Q attracted to side with SMALLER coupling

Electromagnetism

$$Q = +1 \qquad g_I^2 > g_{II}^2$$



$\mathcal{N}=4$ SYM jumping g



$\langle W[C] \rangle$ with straight-line C

$\mathcal{N} = 4$ SYM jumping g

Straight string in Janus

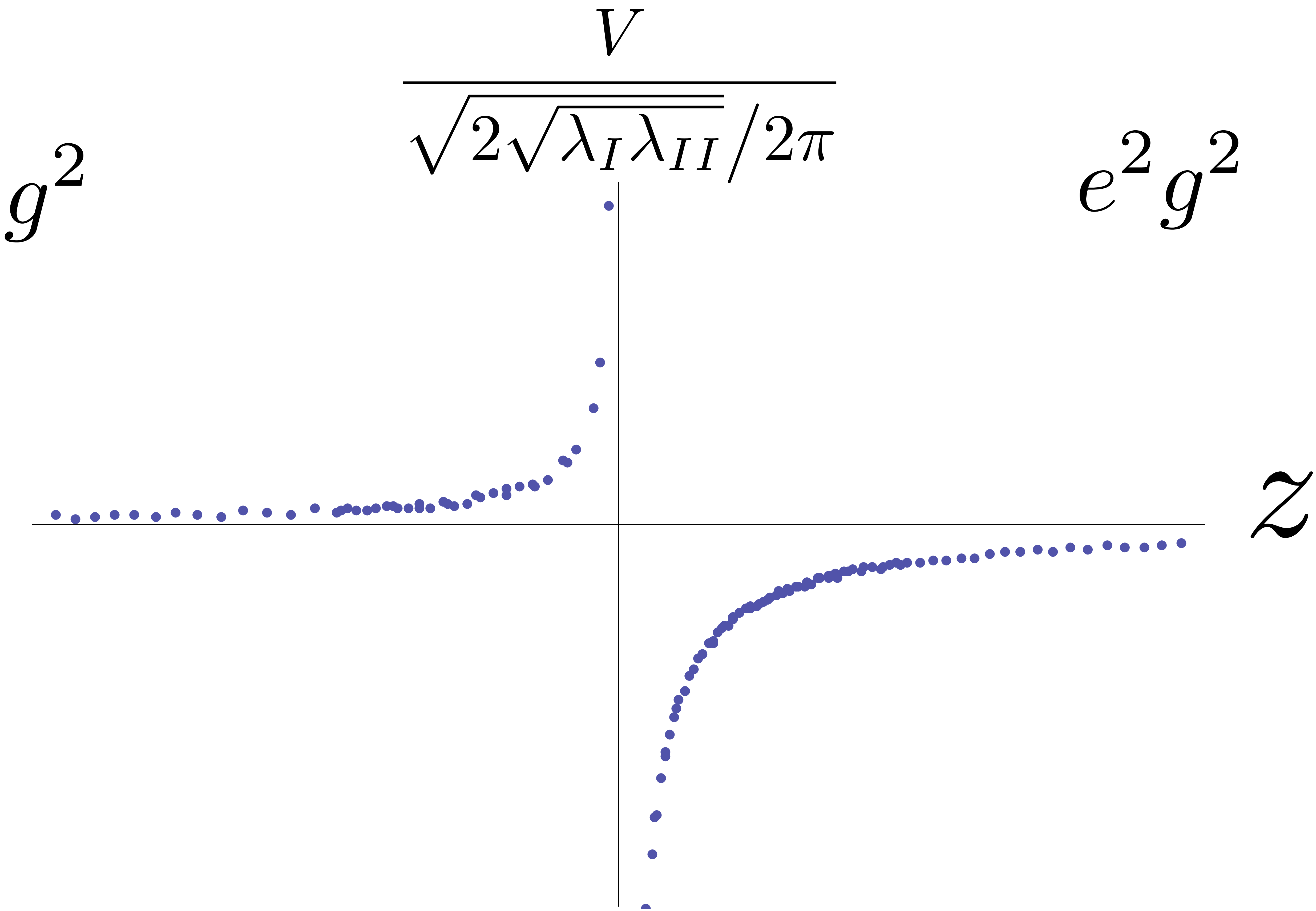
$$A = S_{NG} - \int d\sigma \, P_r \, r \Big|_{\partial \mathrm{AdS}}$$

$$A \neq 0 \quad (!)$$

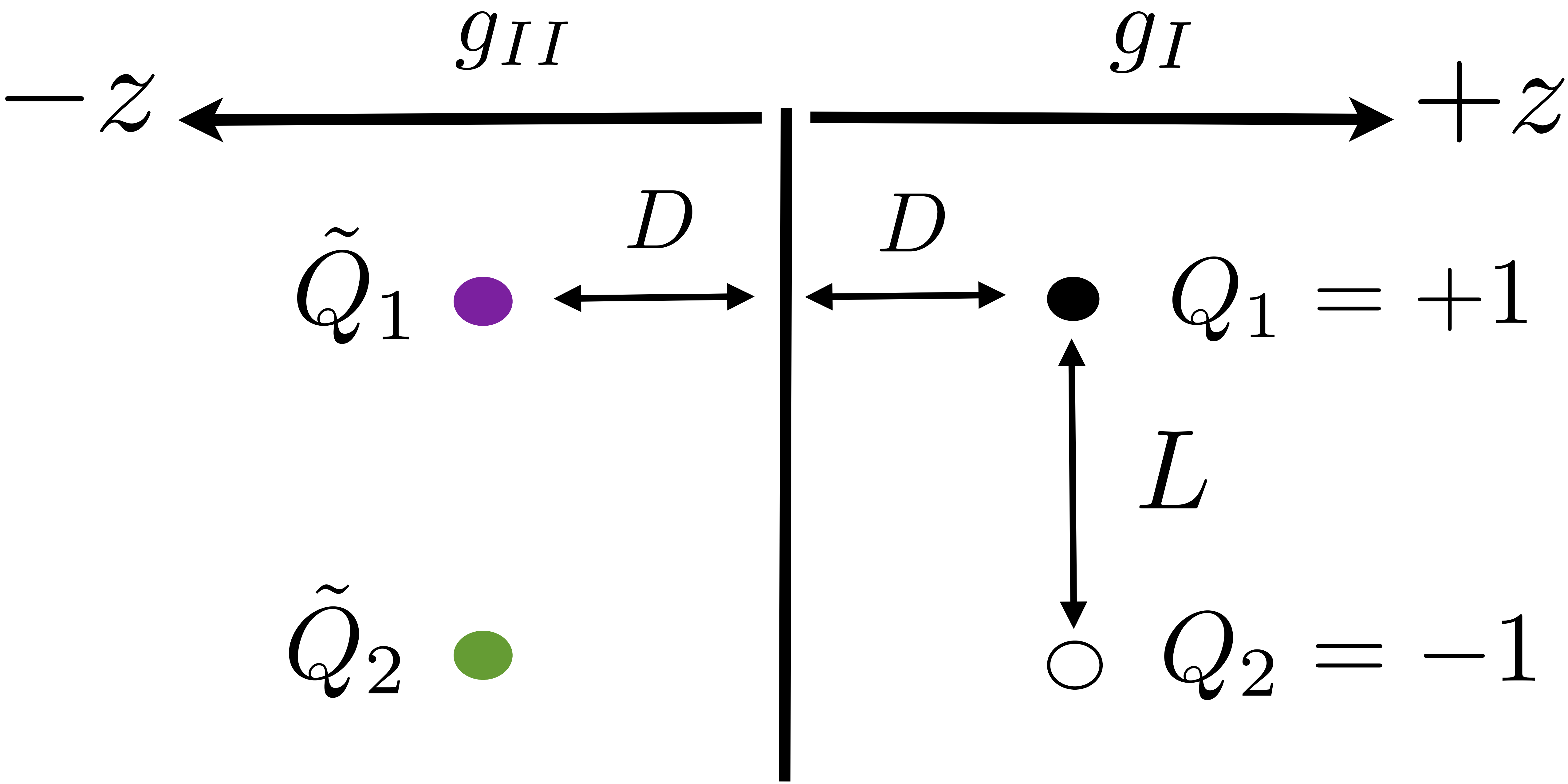
$$V = \lim_{T \to \infty} \frac{A}{T} \neq 0$$

Interaction energy with image charge

$\mathcal{N} = 4$ SYM jumping g

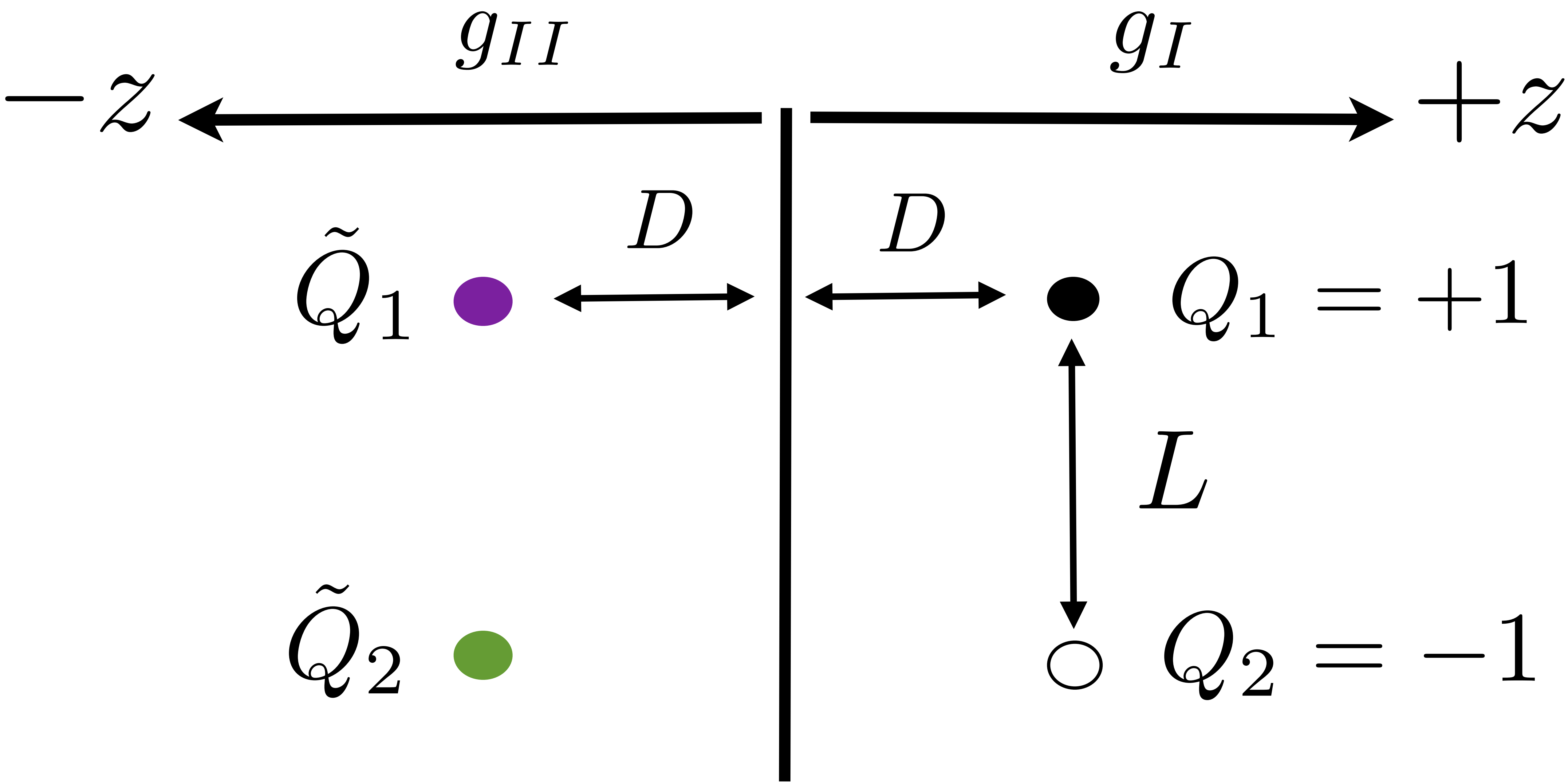


Electromagnetism



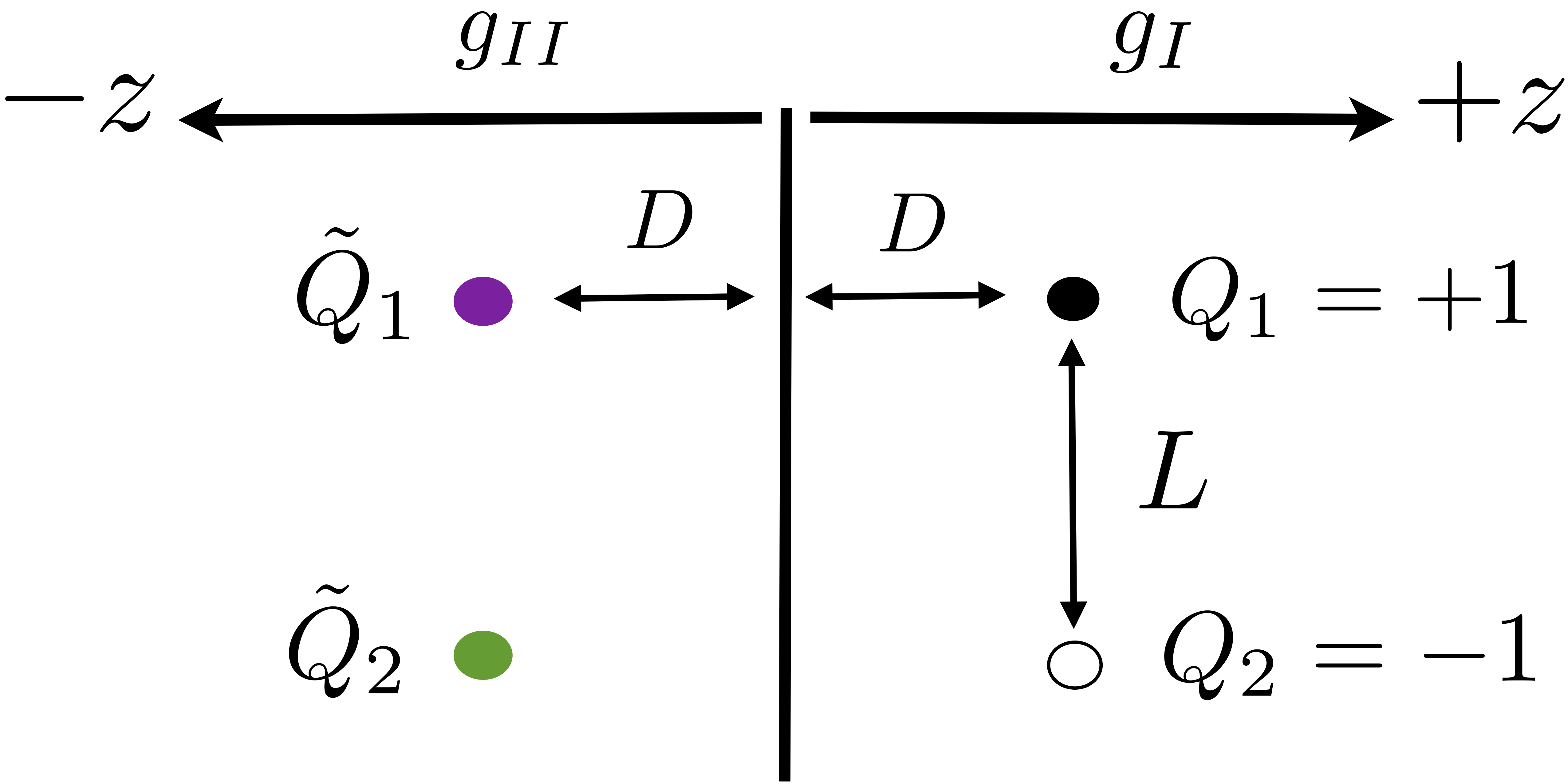
$$\tilde{Q} = Q\frac{g_{II}^2 - g_I^2}{g_{II}^2 + g_I^2}$$

Electromagnetism



$$V = \frac{g_I^2}{4\pi}\left[\frac{Q_1 Q_2}{L} + \frac{Q_1\tilde{Q}_1}{2D} + \frac{Q_2\tilde{Q}_2}{2D} + \frac{Q_1\tilde{Q}_2}{\sqrt{L^2+4D^2}}\right]$$

Electromagnetism



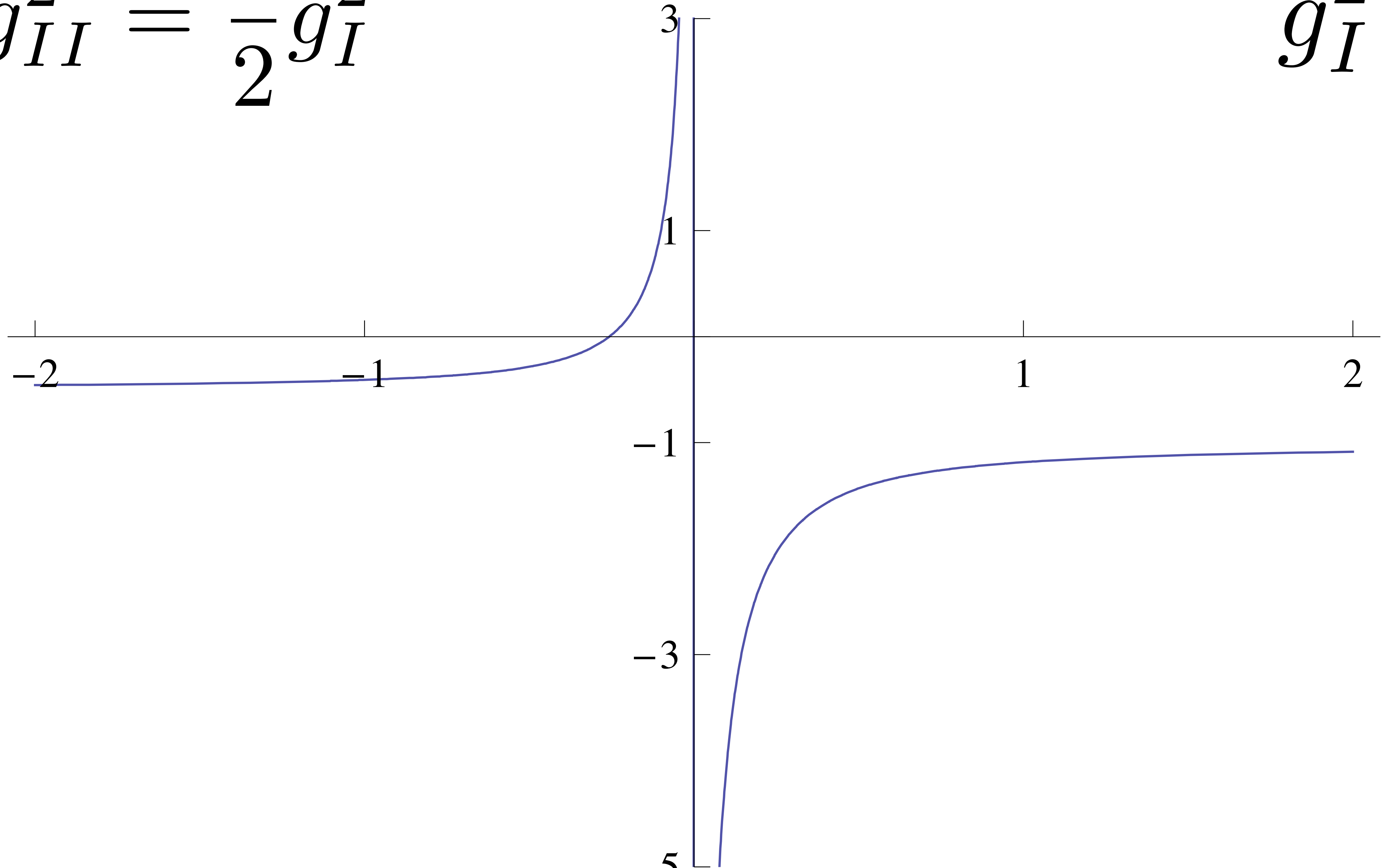
Again attracted to side with SMALLER coupling

Electromagnetism

$$\frac{VL}{g_I^2/4\pi}$$

$$g_{II}^2 = \frac{1}{2} g_I^2$$

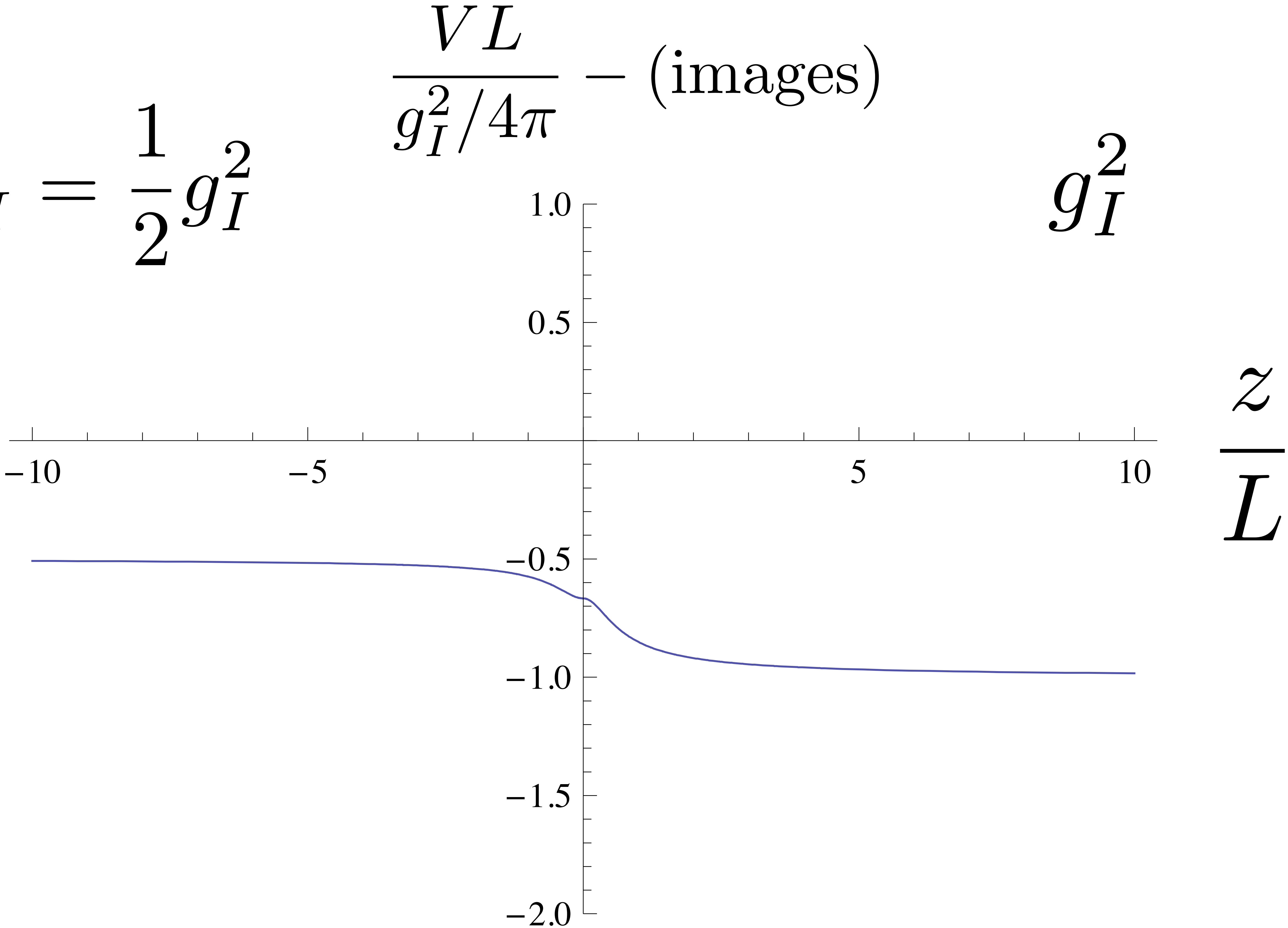
$$g_I^2$$



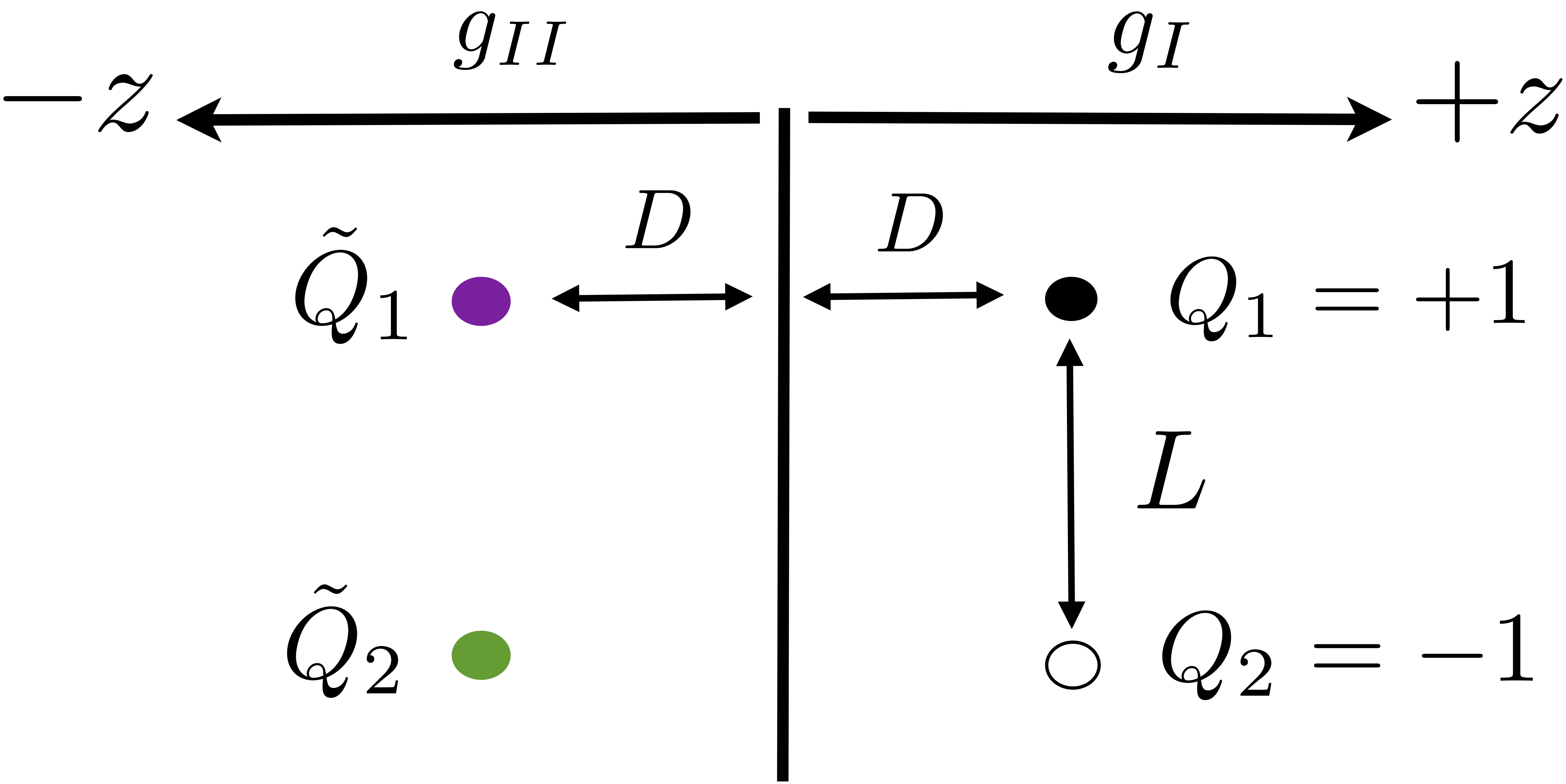
$$\frac{z}{L}$$

Electromagnetism

$$g_{II}^2 = \frac{1}{2} g_I^2$$

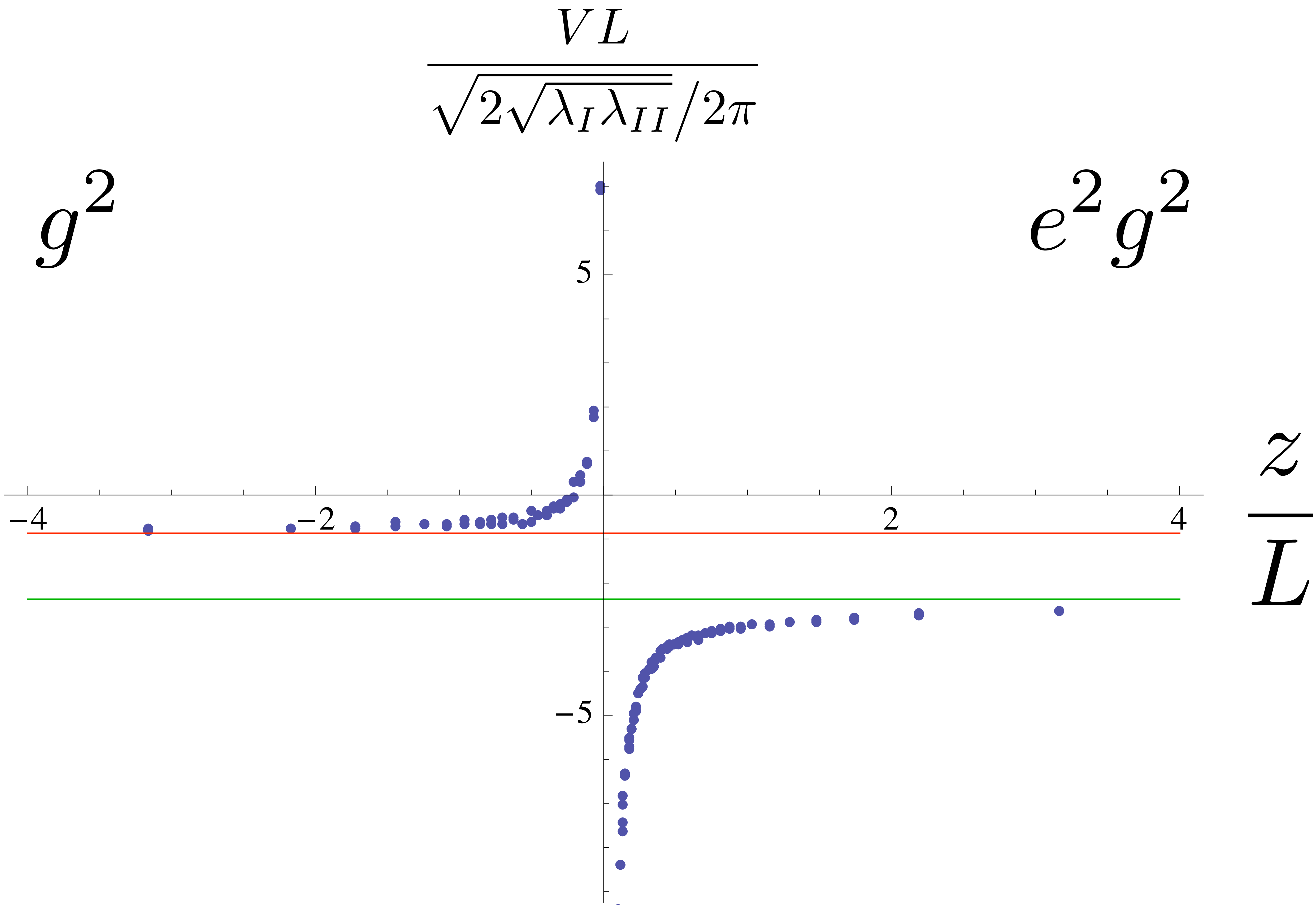


$\mathcal{N} = 4$ SYM jumping g



$\langle W[C] \rangle$ with rectangular C

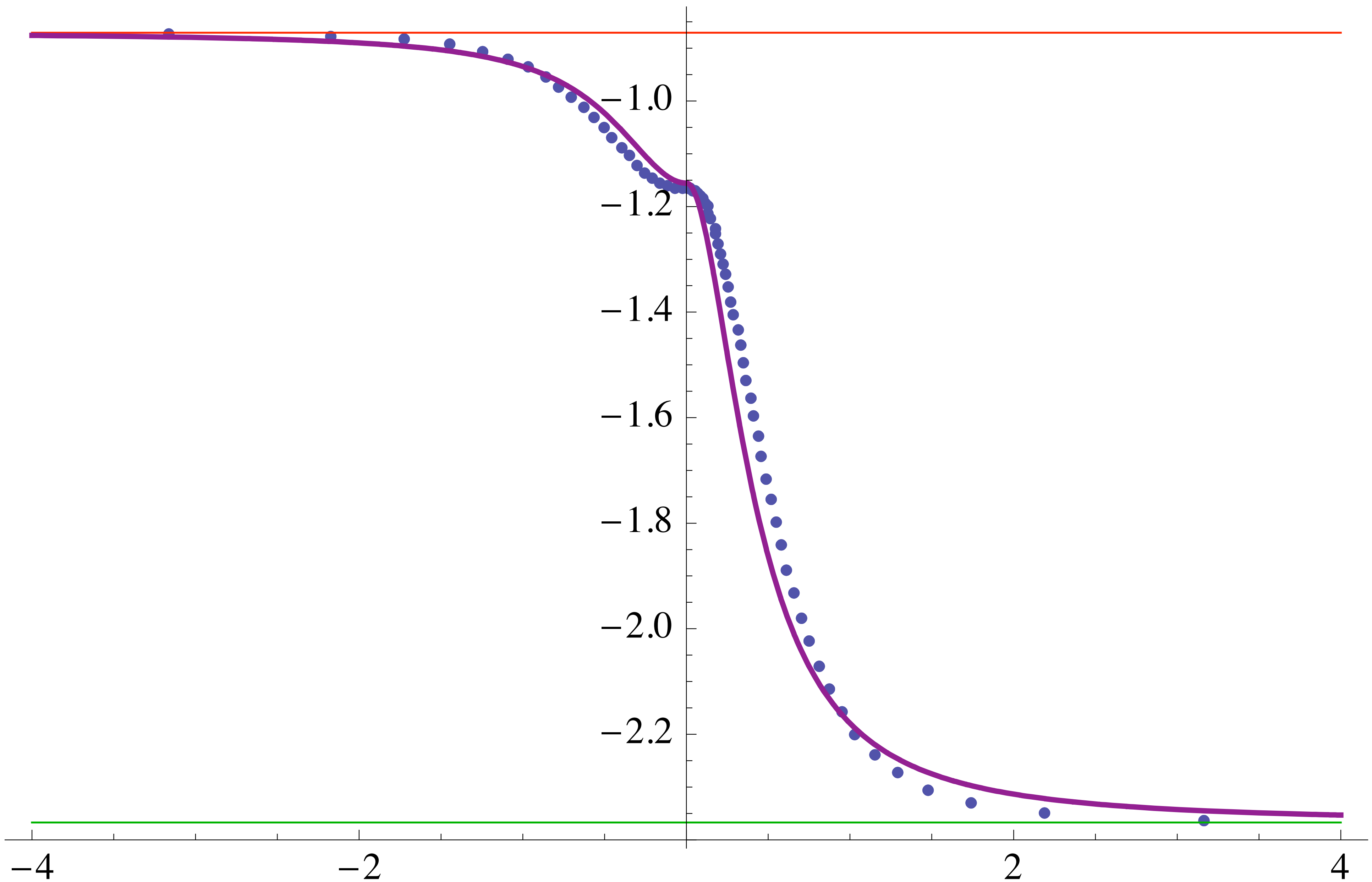
$\mathcal{N} = 4$ SYM jumping g



$\mathcal{N}=4$ SYM jumping g

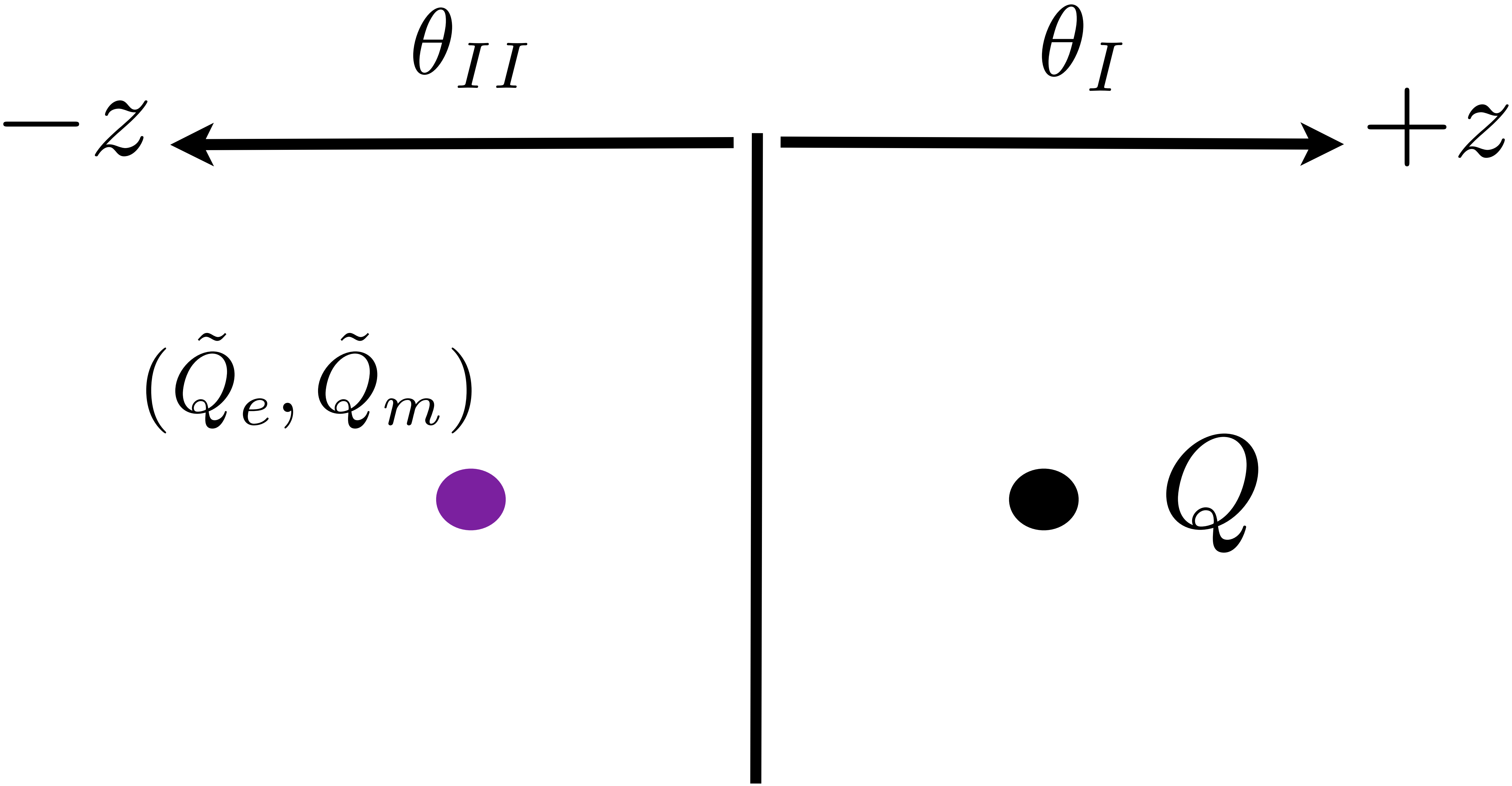
$$\frac{VL}{\sqrt{2\sqrt{\lambda_I\lambda_{II}}/2\pi}} - (\text{images})$$

g^2 $e^2 g^2$



$\dfrac{z}{L}$

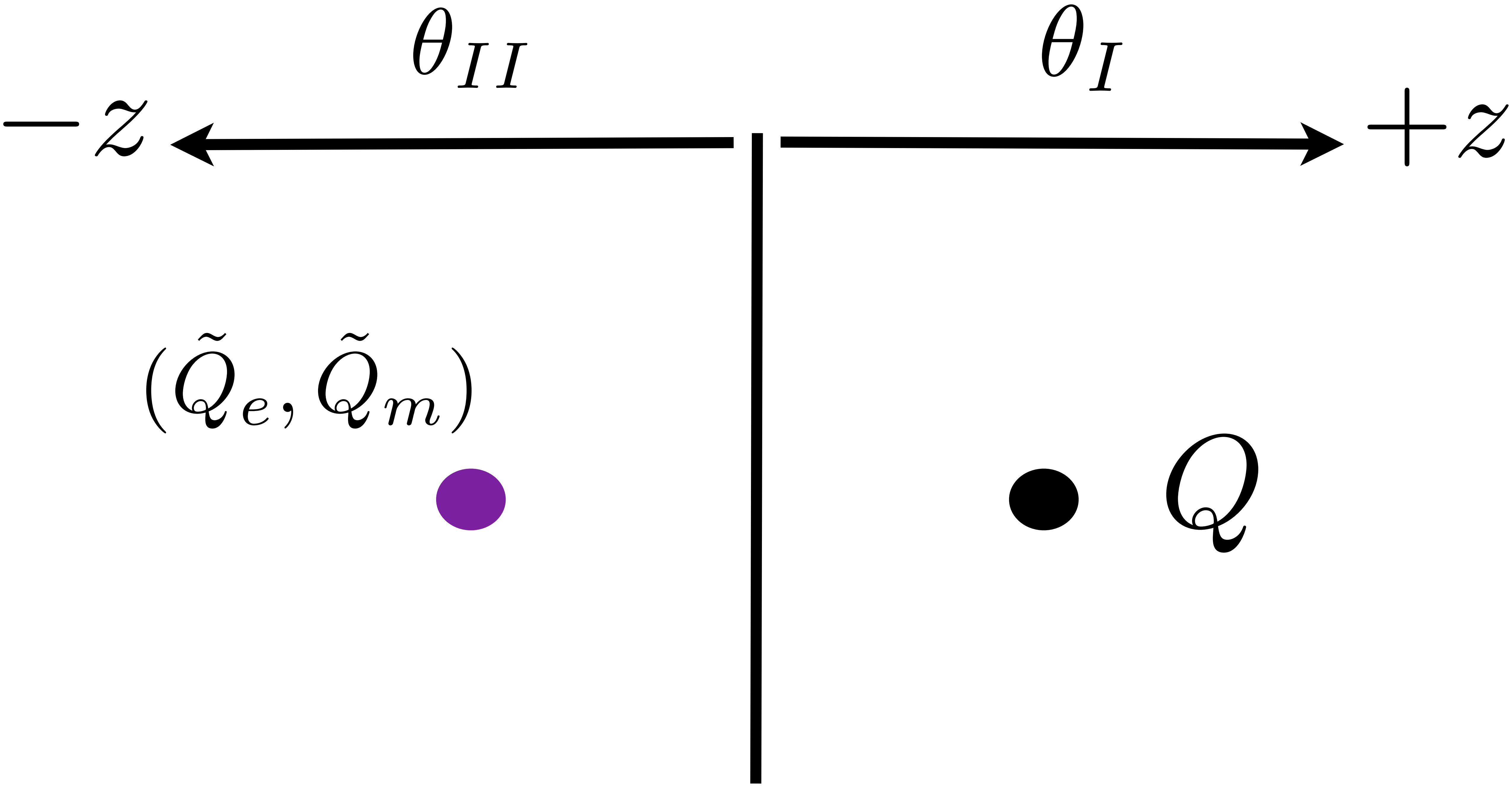
Electromagnetism



$$\tilde{Q}_e = \frac{-Q\, g^4\, (\theta_{II} - \theta_I)^2}{16\pi^2 + g^4\, (\theta_{II} - \theta_I)^2}$$

$$\tilde{Q}_m = \frac{-4\pi Q\, g^2 (\theta_{II} - \theta_I)}{16\pi^2 + g^4\, (\theta_{II} - \theta_I)^2}$$

Electromagnetism



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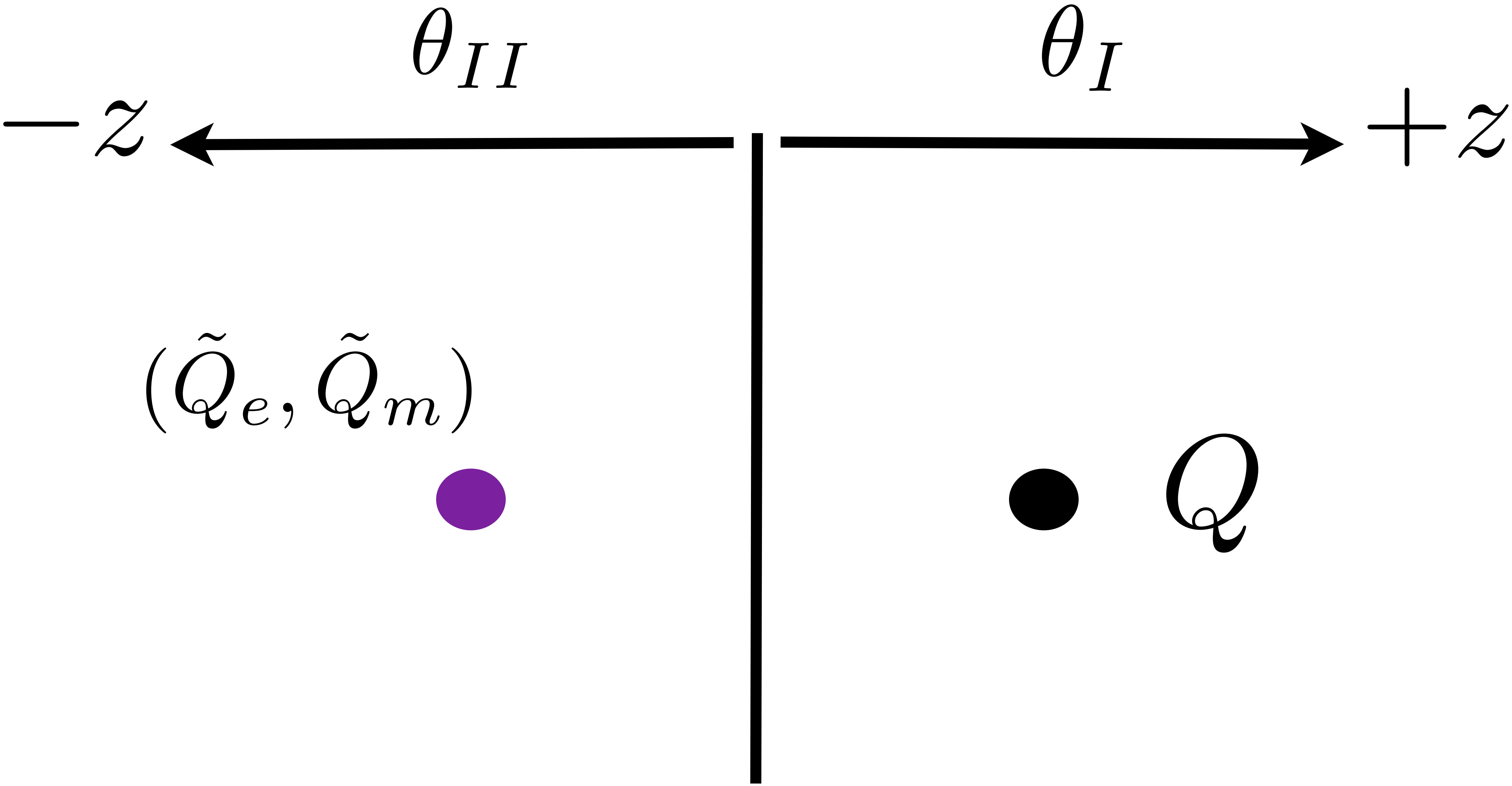
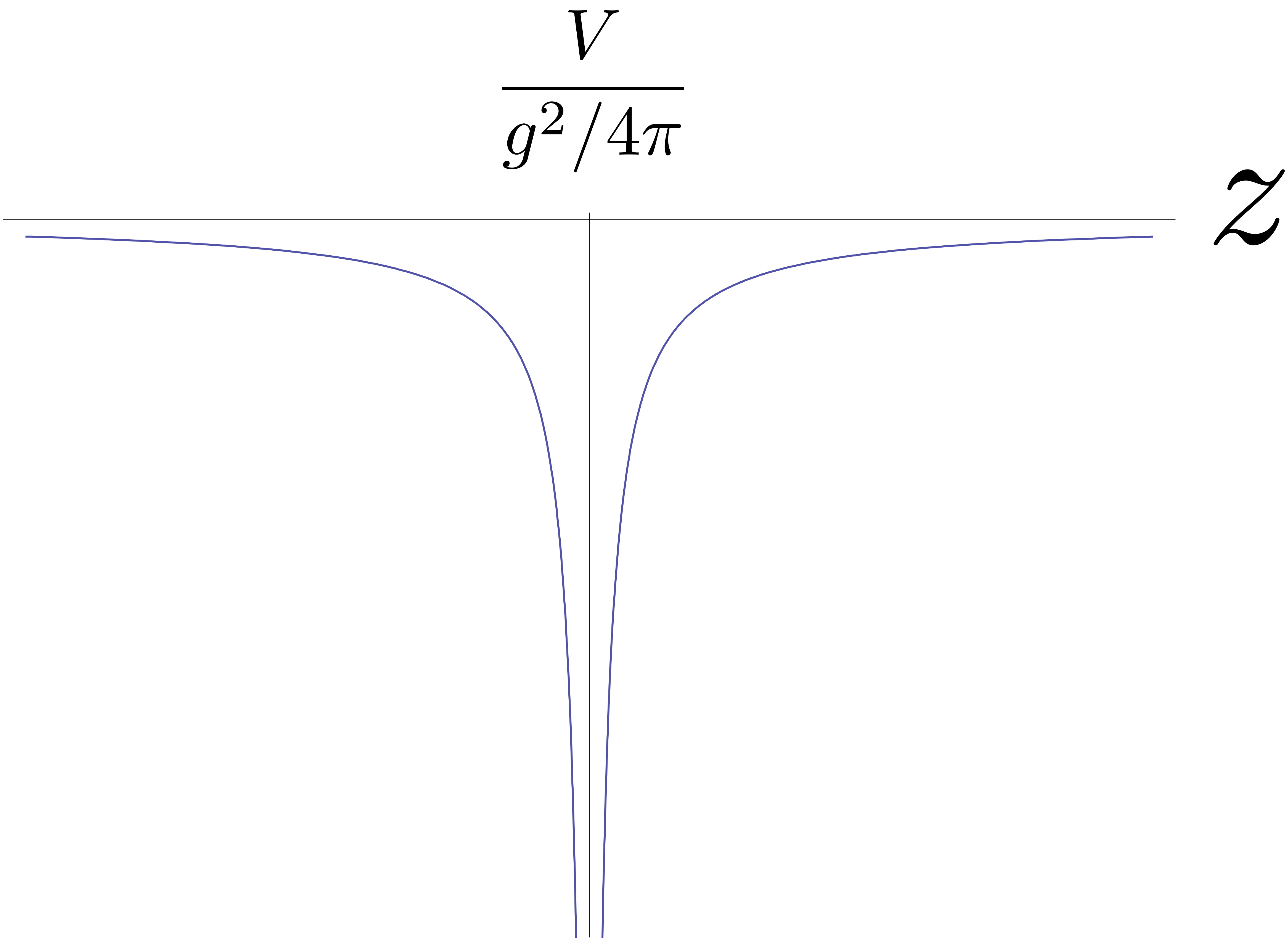
Electromagnetism

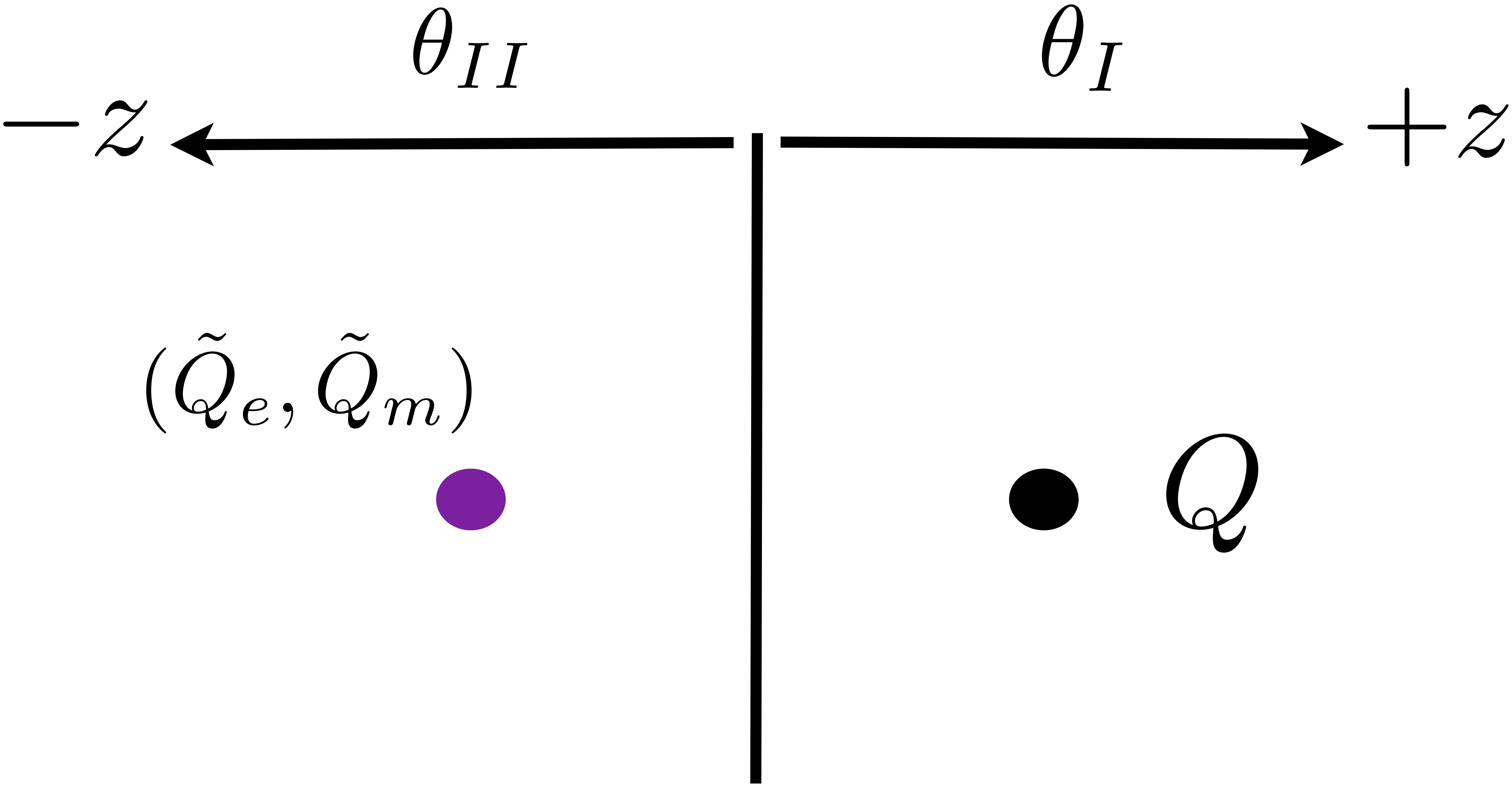


Interface always attractive!

Electromagnetism



$\mathcal{N}=4$ SYM jumping θ



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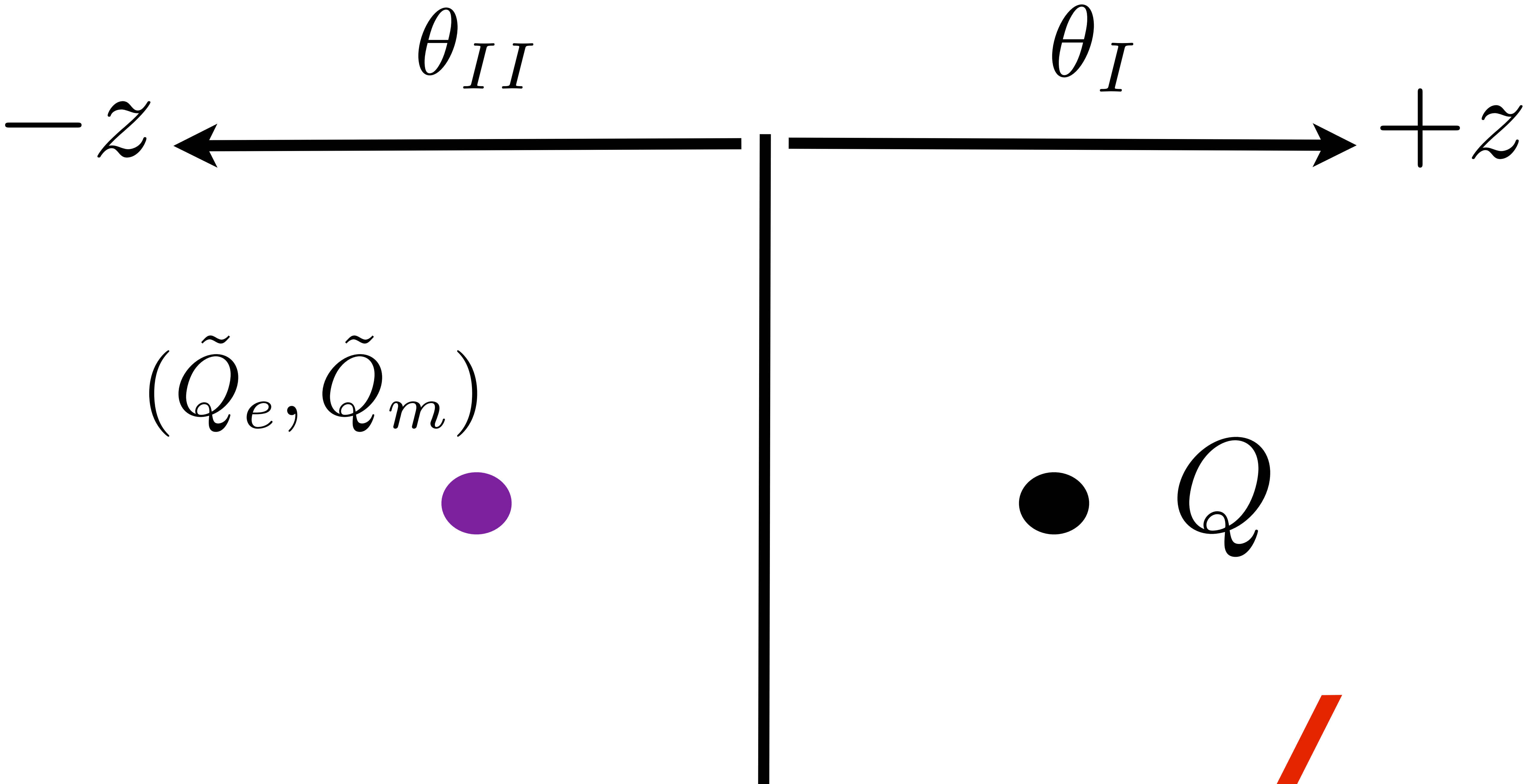
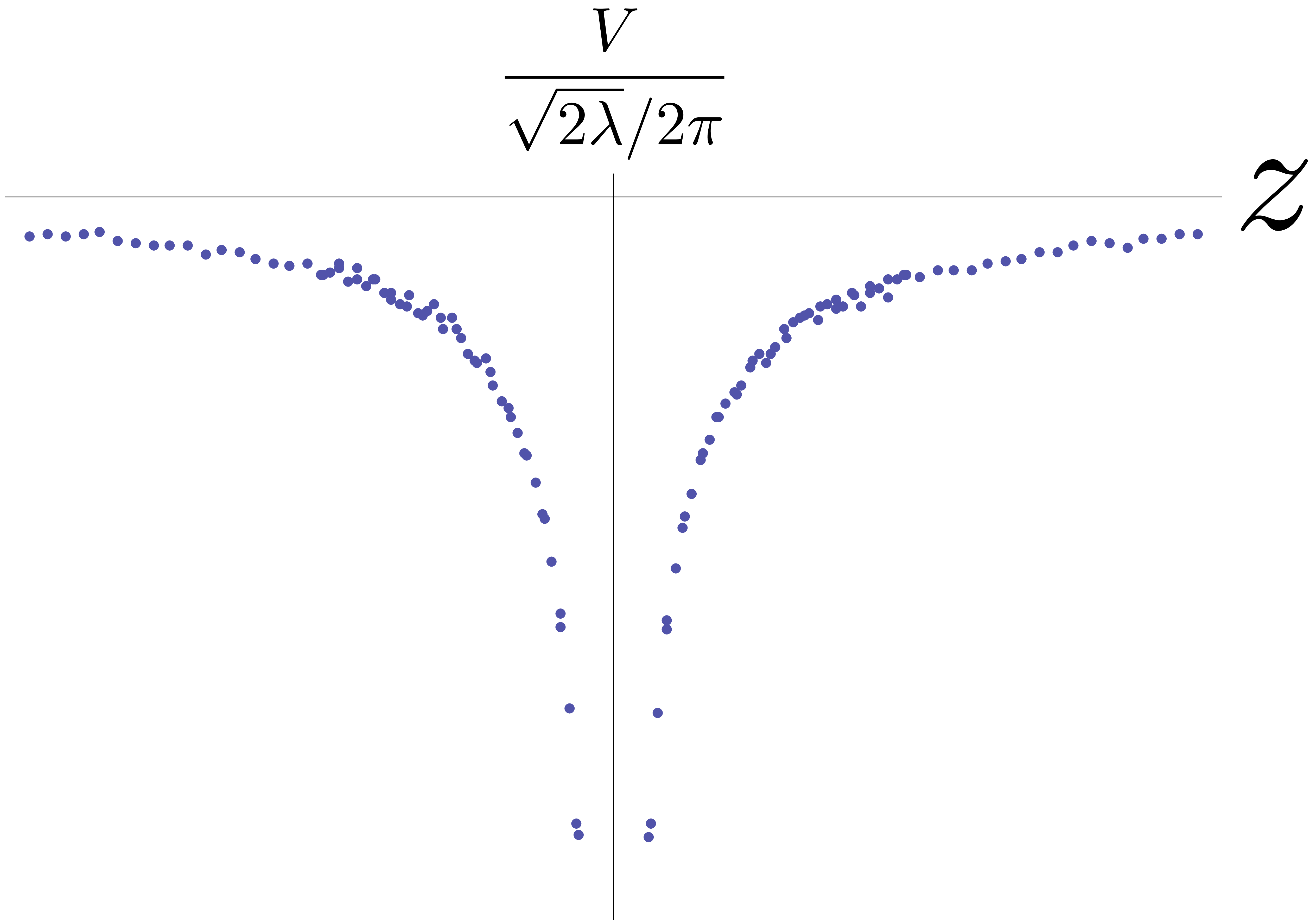
$\mathcal{N}=4$ SYM jumping θ



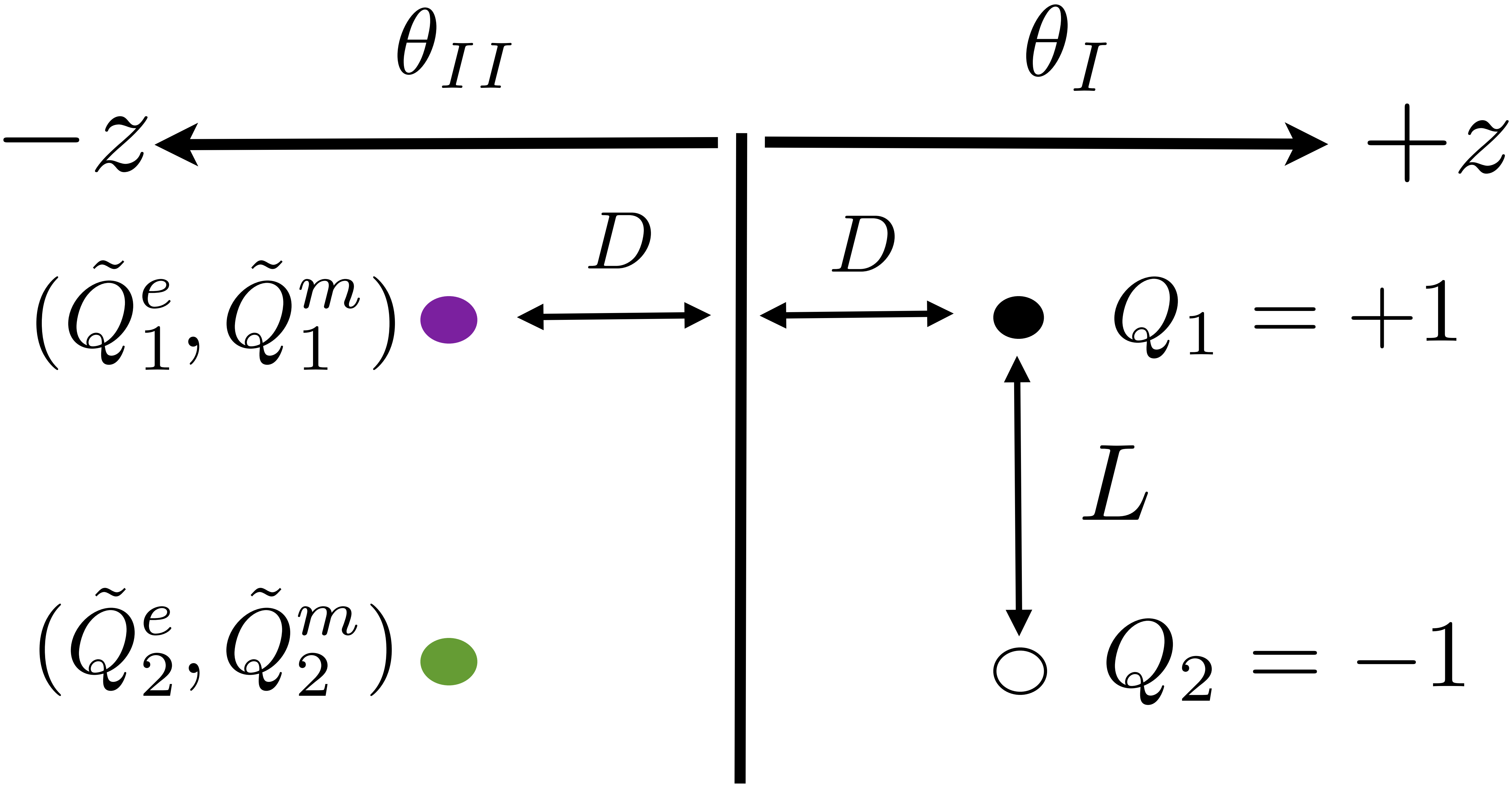
$$\tilde{Q}_e = \frac{-Q\, g^4\, (\theta_{II}-\theta_I)^2}{16\pi^2 + g^4\, (\theta_{II}-\theta_I)^2}$$

$$\tilde{Q}_m = \frac{-4\pi\, Q\, g^2\, (\theta_{II}-\theta_I)}{16\pi^2 + g^4\, (\theta_{II}-\theta_I)^2}$$

$\mathcal{N}=4$ SYM jumping θ

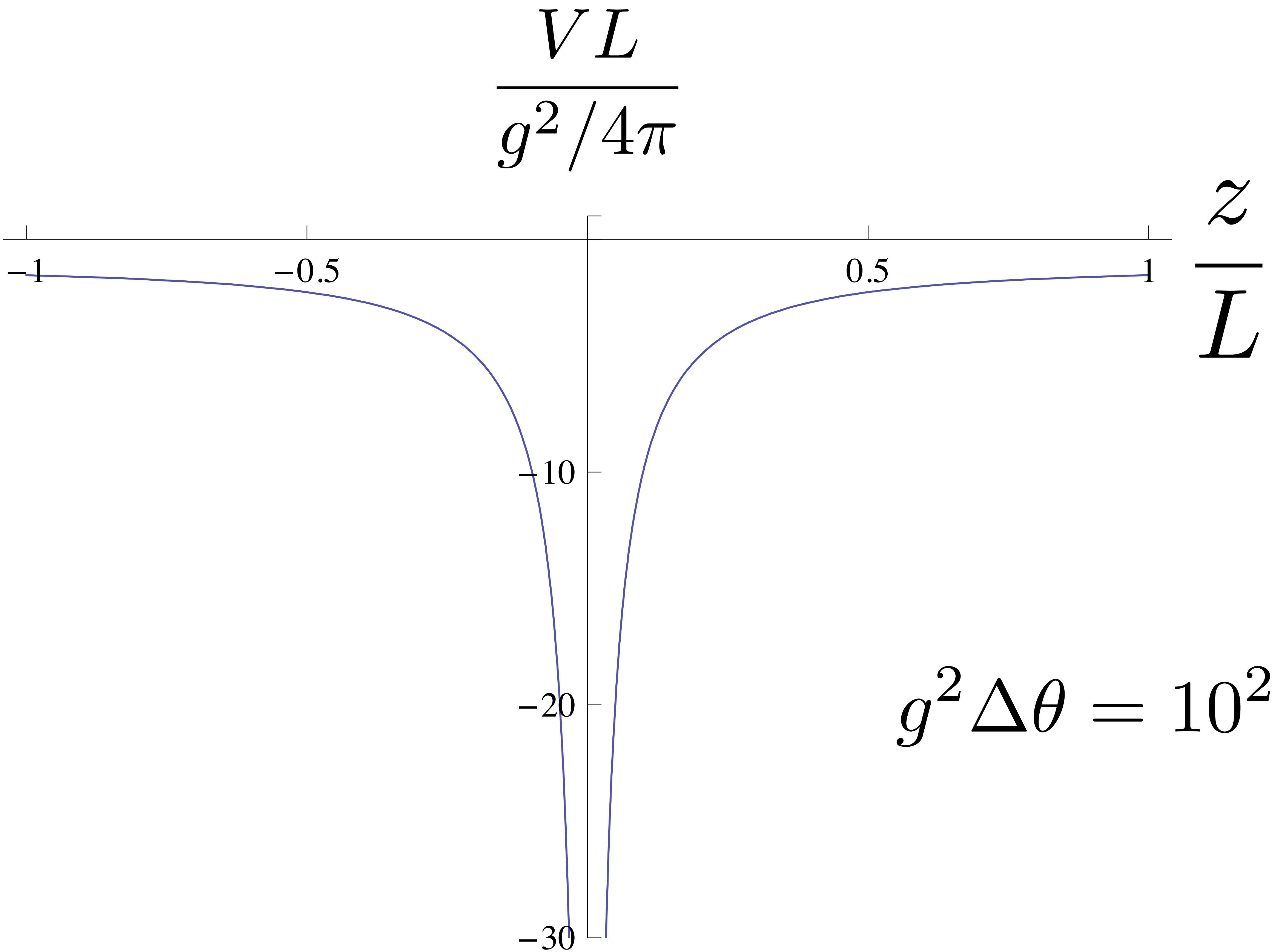


Electromagnetism



Interface always attractive!

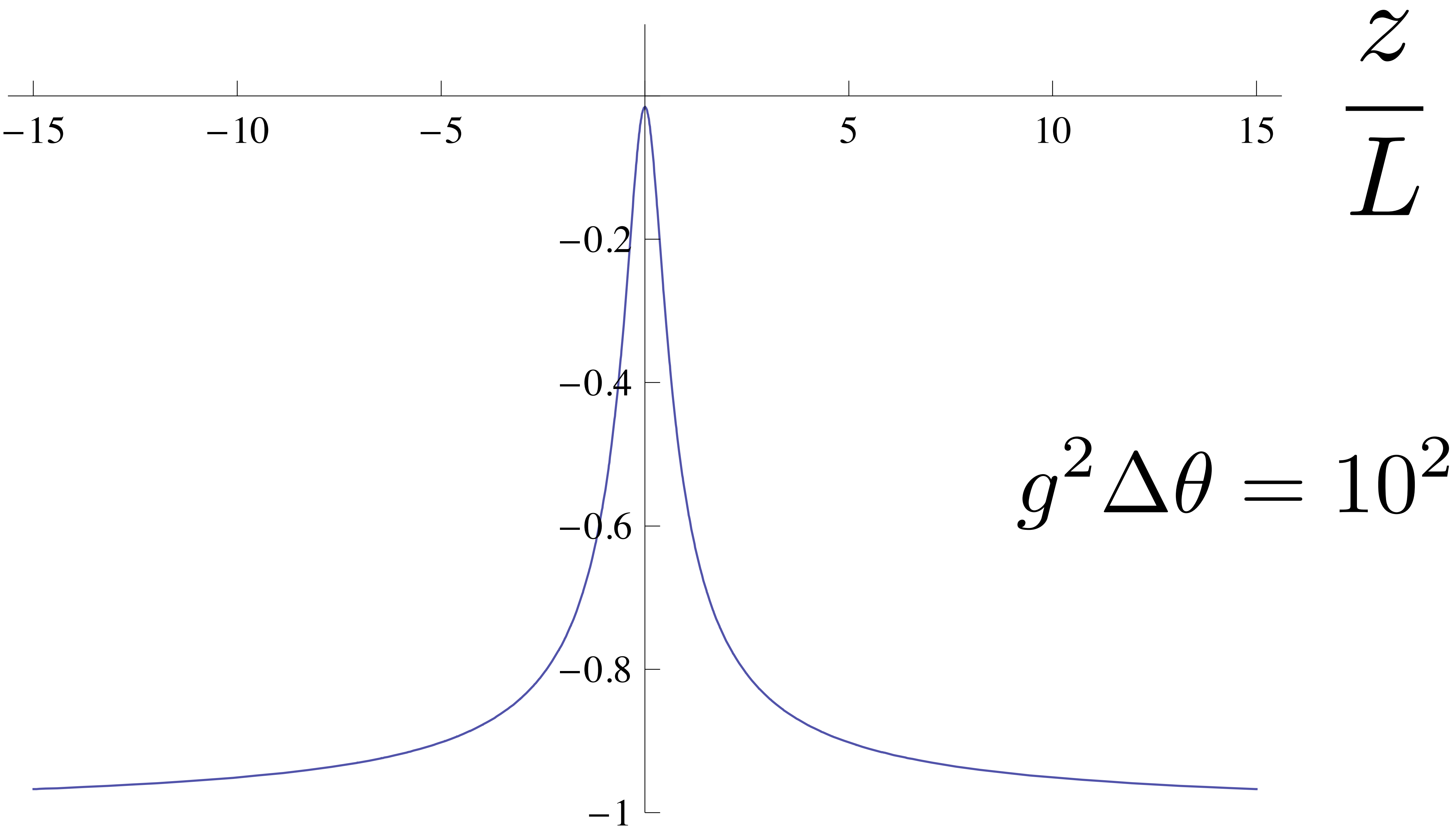
Electromagnetism



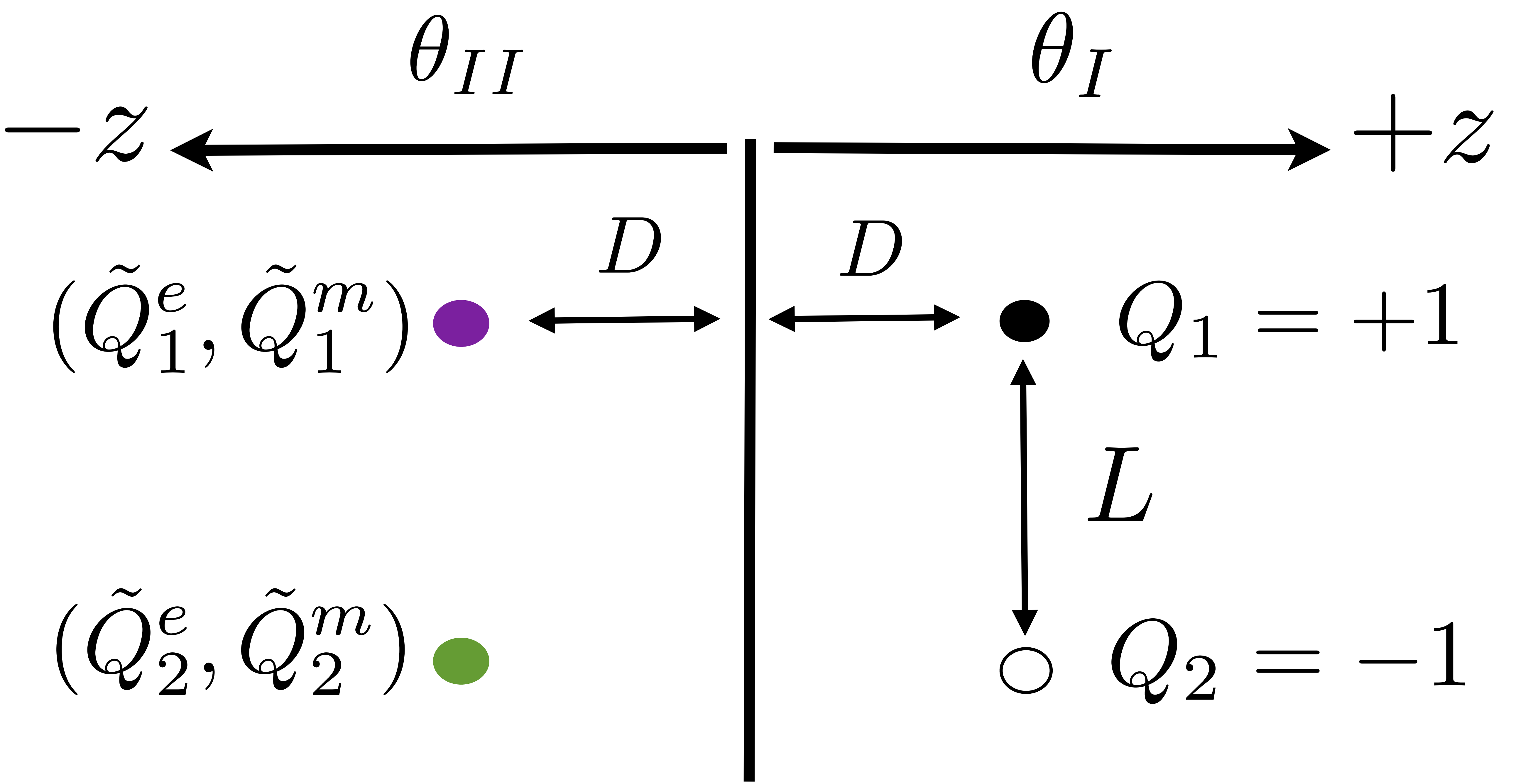
$$g^2\Delta\theta = 10^2$$

Electromagnetism

$$\frac{VL}{g^2/4\pi} - (\text{images})$$



$\mathcal{N} = 4$ SYM jumping θ



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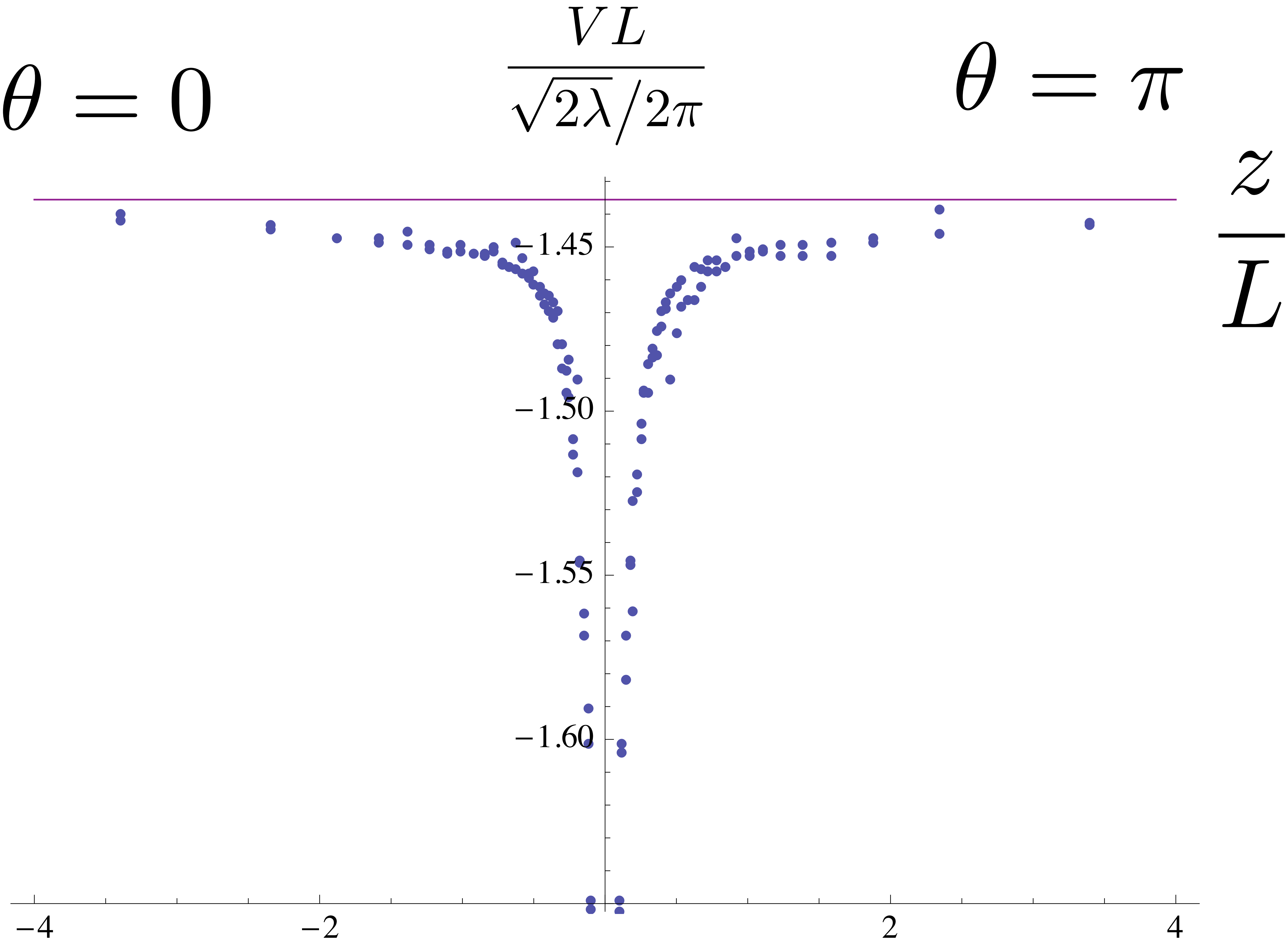
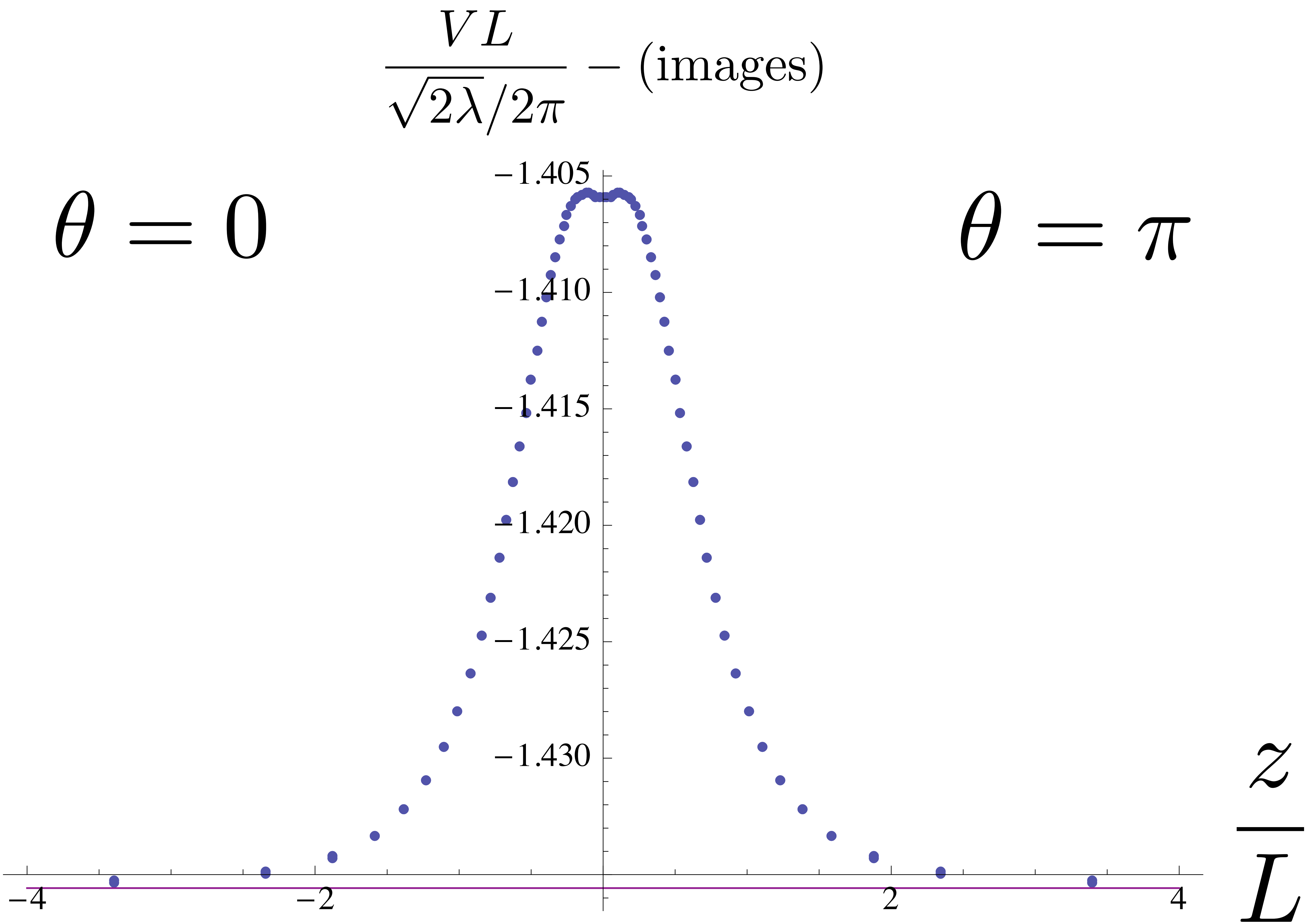
$\mathcal{N}=4$ SYM jumping θ

$\theta = 0$

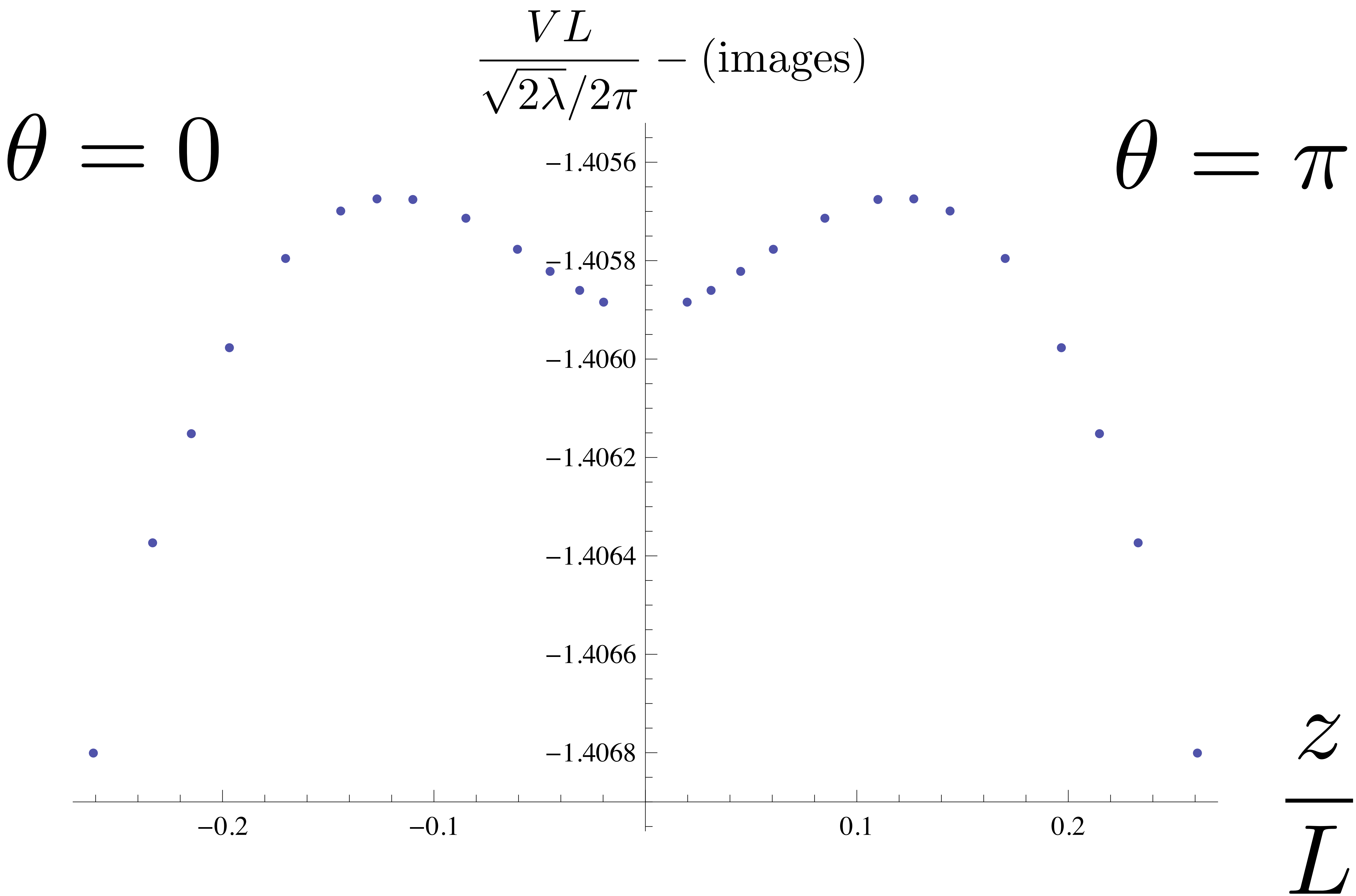
$\theta = \pi$



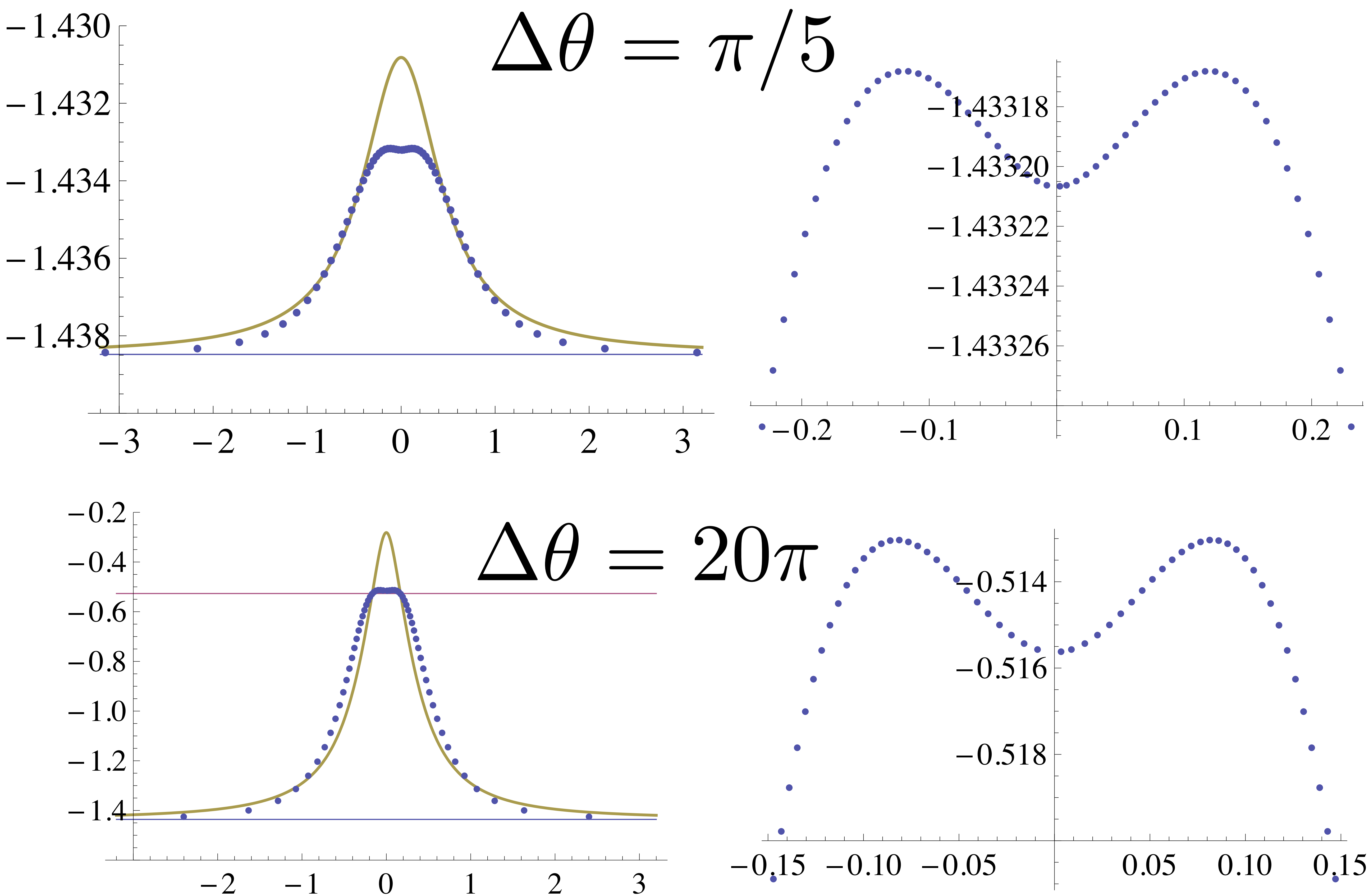
$\mathcal{N}=4$ SYM jumping θ



$\mathcal{N} = 4$ SYM jumping θ



$\mathcal{N}=4$ SYM jumping θ



Outline:

- Motivation: Topological Insulators
- Holographic Conformal Interfaces
- Holographic Wilson Loops
- Static Quark Potential
- Future Directions

Future Directions

- Circular Wilson loops?
- Other representations?
- Image strings?
- Accelerating charges?

Thank You.