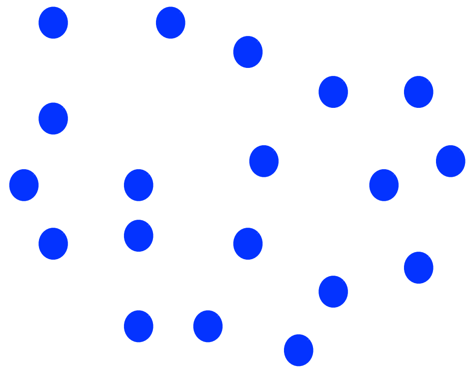
Hydrodynamics and the entropy current

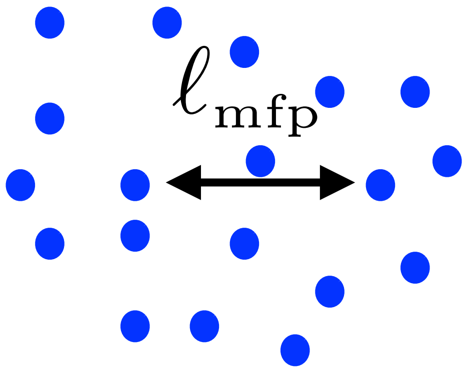
Amos Yarom

Together with: J. Bhattacharya, S. Bhattacharyya, J. Erdmenger, M. Haack, C. Herzog, K. Jensen, M. Kaminski, P. Kovtun, N. Lisker, R. Meyer, S. Minwalla, A. Ritz, P. Surowka

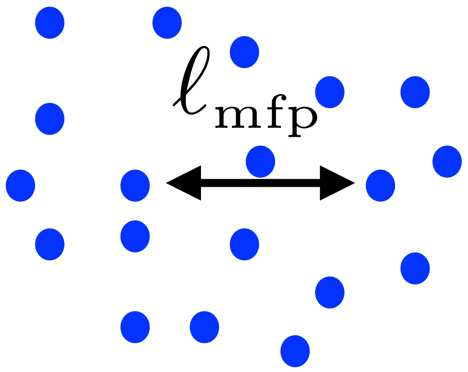
Hydrodynamics



Hydrodynamics

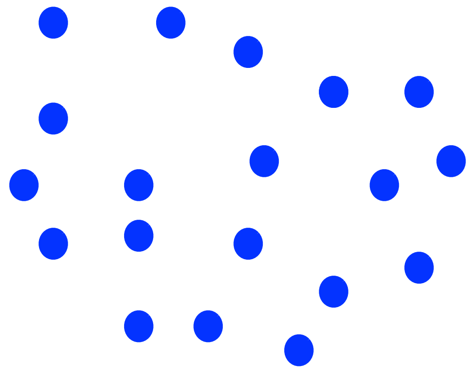


Hydrodynamics



$$L \gg \ell_{\rm mfp}$$

Hydrodynamics



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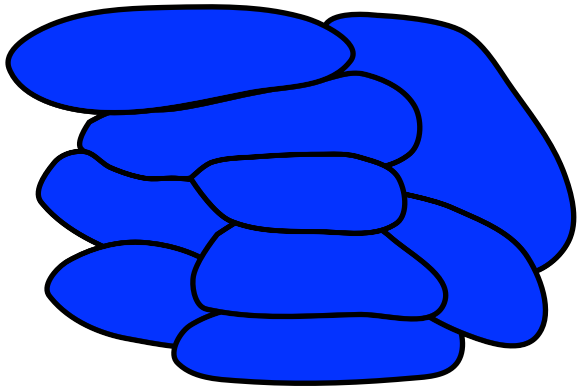
Hydrodynamics



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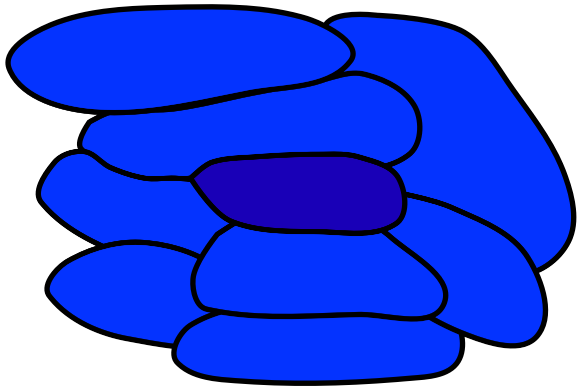
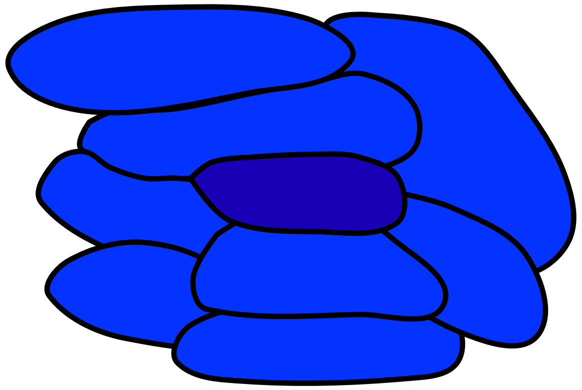
Hydrodynamics



$$L \gg \ell_{\mathrm{mfp}}$$

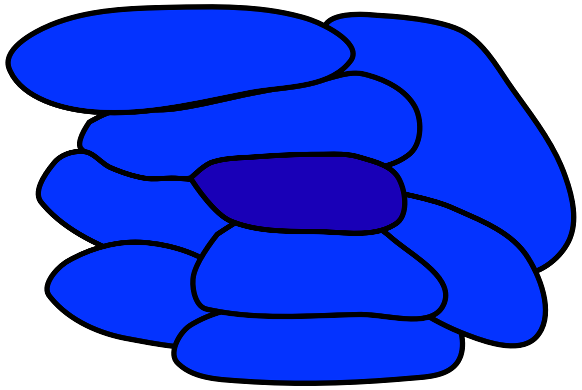
Hydrodynamics

$\epsilon(x^\mu)$ Energy density



$L \gg \ell_{\rm mfp}$

Hydrodynamics



$L \gg \ell_{\mathrm{mfp}}$

$\epsilon(x^\mu)$ Energy density

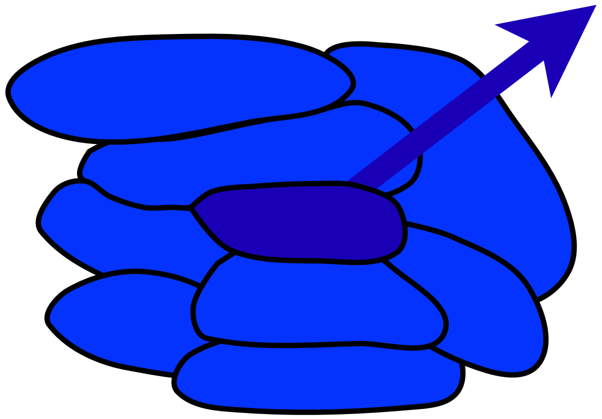
$\rho(x^\mu)$ Charge density

Hydrodynamics

$\epsilon(x^\mu)$ Energy density

$\rho(x^\mu)$ Charge density

$u^\nu(x^\mu)$ Velocity field



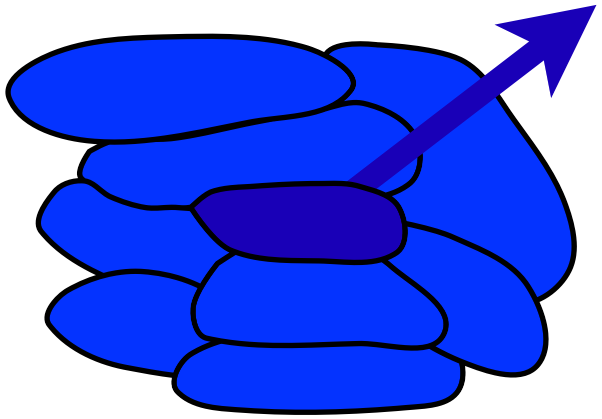
$L \gg \ell_{\rm mfp}$

Hydrodynamics

$\epsilon(x^\mu)$ **Energy density**

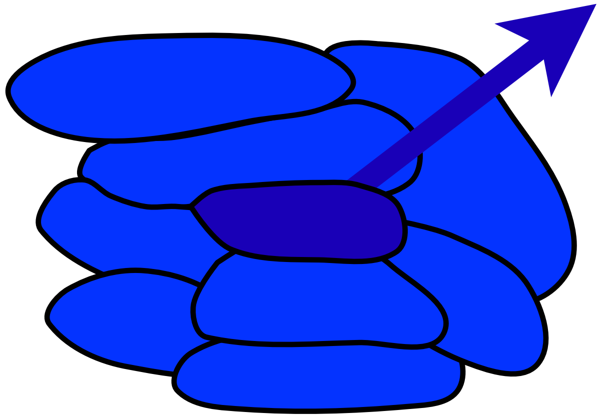
$\rho(x^\mu)$ **Charge density**

$u^\nu(x^\mu)$ **Velocity field** $(u_\mu u^\mu = -1)$



$L \gg \ell_{\rm mfp}$

Hydrodynamics



$L \gg \ell_{\rm mfp}$

$\epsilon(x^\mu)$ **Energy density**

$\rho(x^\mu)$ **Charge density**

$u^\nu(x^\mu)$ **Velocity field** $(u_\mu u^\mu = -1)$

$$\partial_\mu T^{\mu\nu} = 0$$

Hydrodynamics

$\epsilon(x^\mu)$ Energy density

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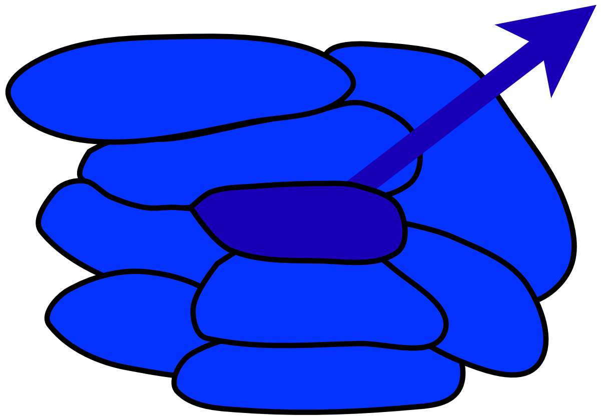
$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



$L \gg \ell_{\rm mfp}$

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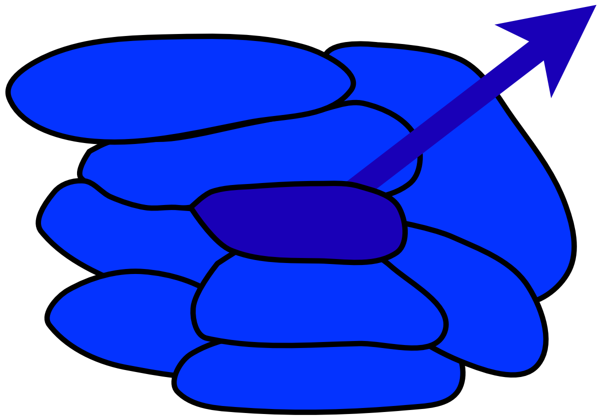
Hydrodynamics



$\epsilon(x^\mu)$ Energy density

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$L \gg \ell_{\mathrm{mfp}}$

$$\partial_\mu T^{\mu\nu} = 0$$

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$$T^{\mu\nu} = ?$$

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Hydrodynamics

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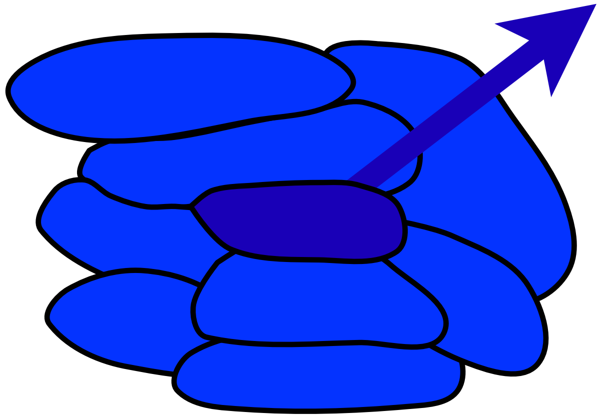

$$L \gg \ell_{\mathrm{mfp}}$$

To leading order the fields are uniform:

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$$T^{\mu\nu} = ?$$

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Hydrodynamics

$\epsilon(x^\mu)$ Energy density

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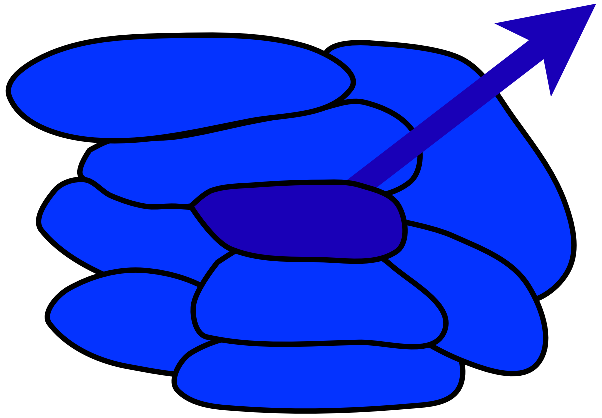

$$L \gg \ell_{\rm mfp}$$

To leading order the fields are uniform:

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Lorentz covariant tensors:	Lorentz covariant vectors:

$$T^{\mu\nu} = ?$$

$$J^\mu = ?$$

Hydrodynamics

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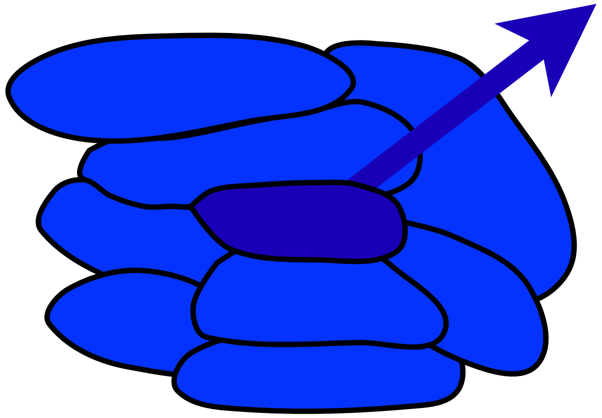

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Hydrodynamics

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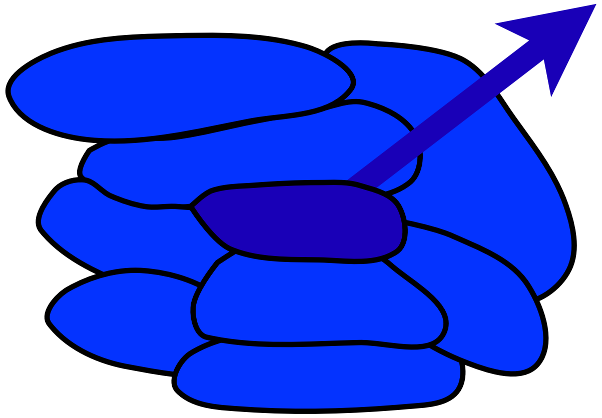

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Lorentz covariant tensors:	Lorentz covariant vectors:
$u^\mu u^\nu$ $\eta^{\mu\nu}$	

$$T^{\mu\nu} = ?$$

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Hydrodynamics

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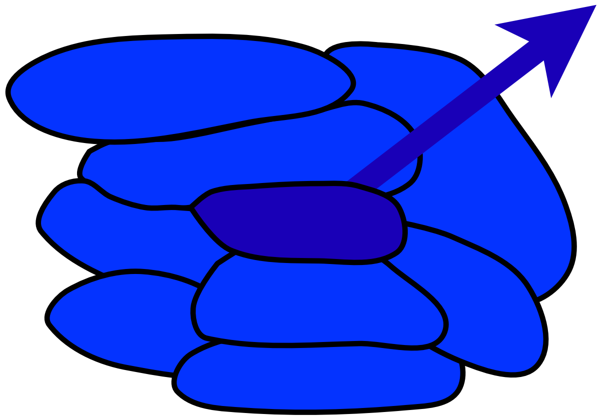

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Lorentz covariant tensors:	Lorentz covariant vectors:
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$$T^{\mu\nu} = ?$$

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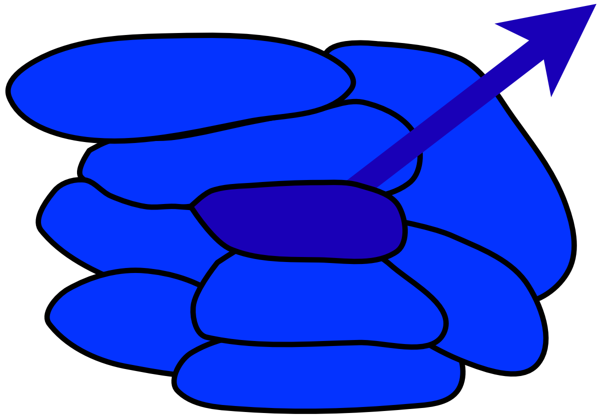
Hydrodynamics



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$$T^{\mu\nu} = T_1 u^\mu u^\nu + T_2 P^{\mu\nu}$$

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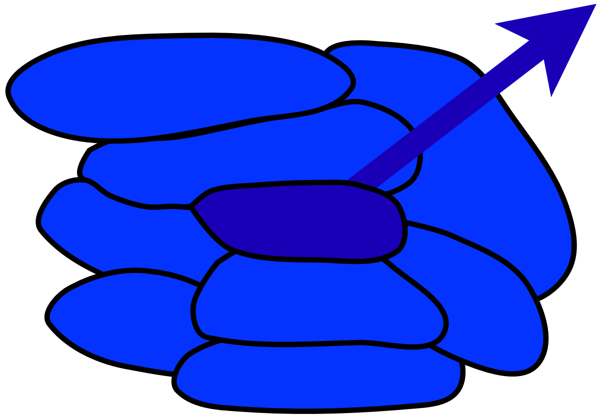

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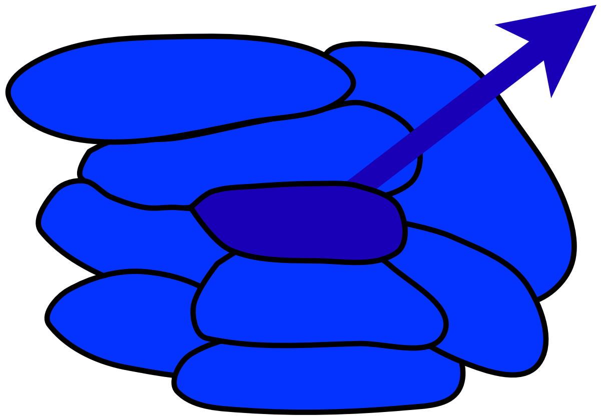
Hydrodynamics



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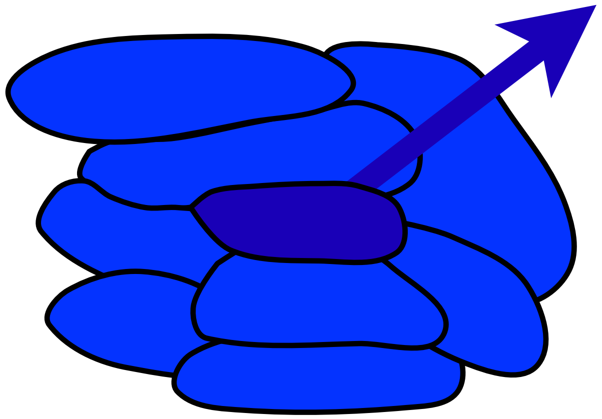

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Hydrodynamics

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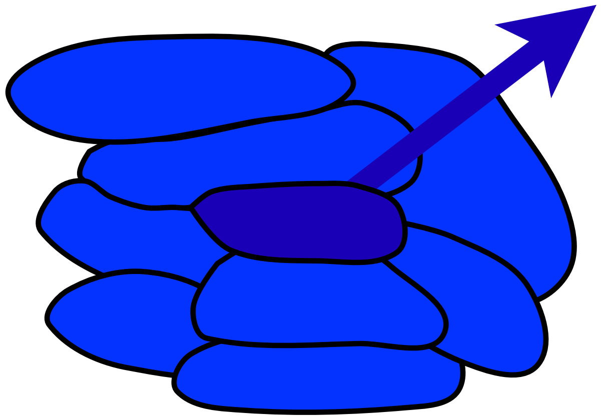


$$L \gg \ell_{\rm mfp}$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

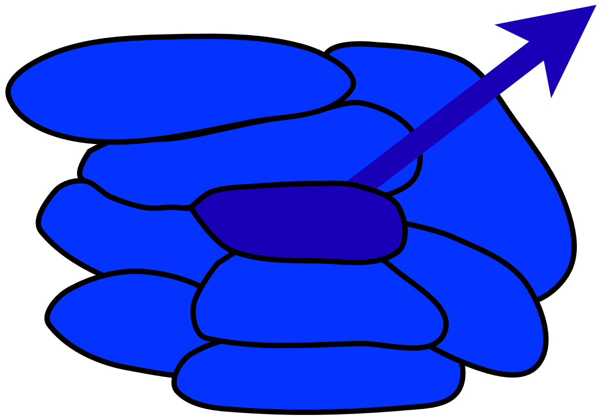
To leading order the fields are uniform. So we can always boost to a configuration where:

$$u^\mu = (1,0,0,0)$$

$$T^{\mu\nu} = T_1 u^\mu u^\nu + T_2 P^{\mu\nu}$$

$$J^\mu = T_3 u^\mu$$

Hydrodynamics



$\epsilon(x^\mu)$ Energy density

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$$L \gg \ell_{\rm mfp}$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$$T^{\mu\nu} = \begin{pmatrix} T_1 & 0 & 0 & 0 \\ 0 & T_2 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_2 \end{pmatrix}$$

$$J^\mu = T_3 u^\mu$$

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Hydrodynamics

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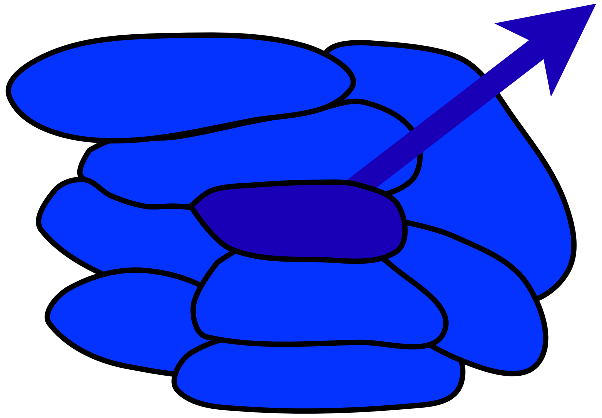

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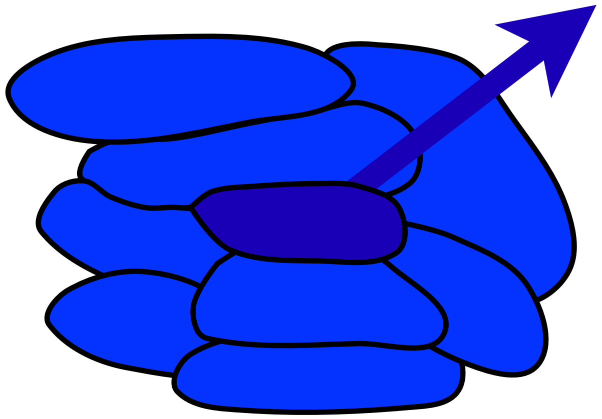
$$J^\mu = (T_3\, 0\, 0\, 0)$$

Hydrodynamics



$\epsilon(x^\mu)$ Energy density

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$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$

$$L \gg \ell_{\rm mfp}$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$T_1 = \epsilon$$

To leading order the fields are uniform. So we can always boost to a configuration where:

$$u^\mu = (1,0,0,0)$$

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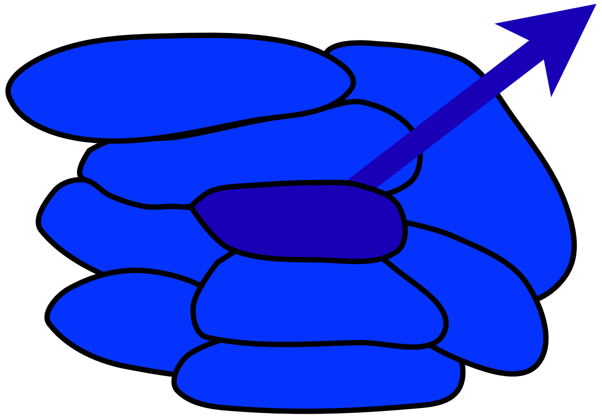
$$T^{\mu\nu} = \begin{pmatrix} T_1 & 0 & 0 & 0 \\ 0 & T_2 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_2 \end{pmatrix}$$

$$J^\mu = (T_3\,0\,0\,0)$$

Hydrodynamics



$\epsilon(x^\mu)$ Energy density

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$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$

$$L \gg \ell_{\rm mfp}$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$T_1 = \epsilon$$

$$T_2 = P$$

To leading order the fields are uniform. So we can always boost to a configuration where:

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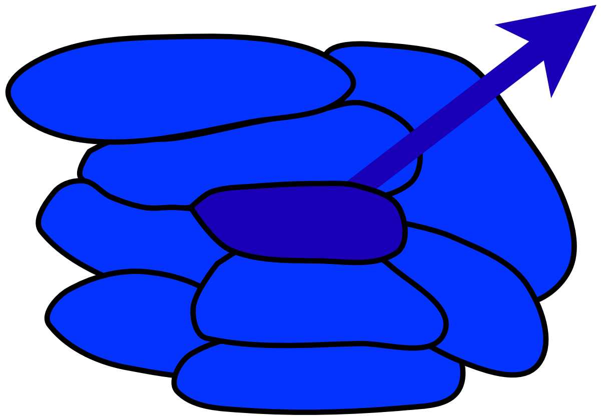
$$J^\mu = (T_3\, 0\, 0\, 0)$$

Hydrodynamics



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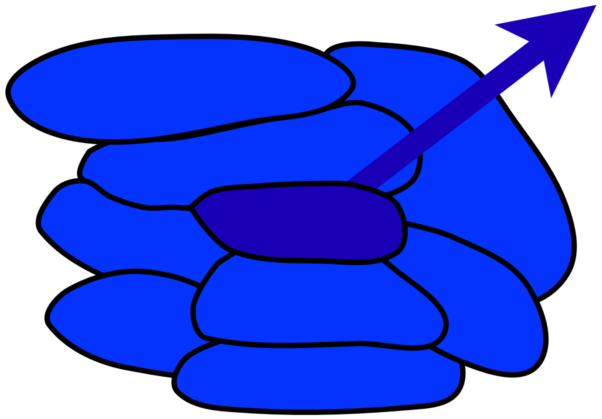
Hydrodynamics



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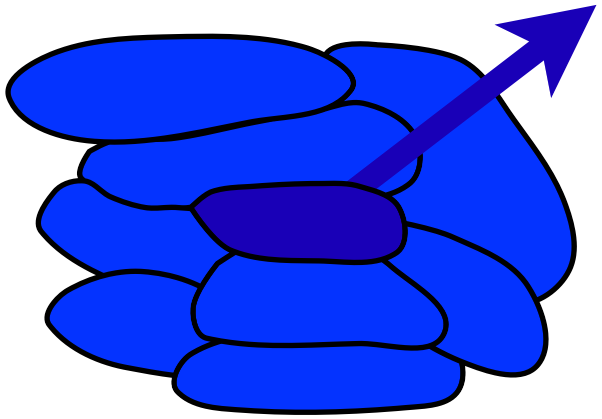

$L \gg \ell_{\rm mfp}$

To leading order the fields are uniform.

$$\partial_\mu T^{\mu\nu} = 0$$

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$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu}$$

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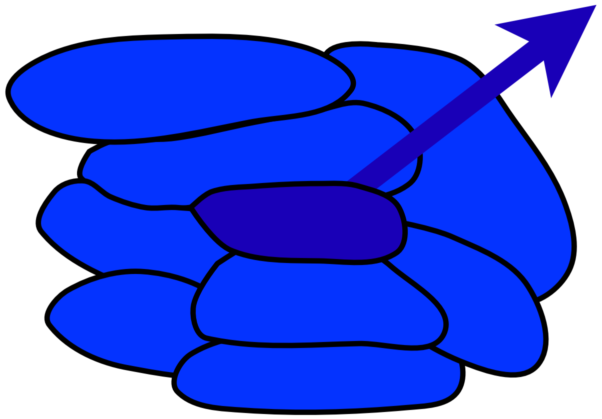
Thermodynamics



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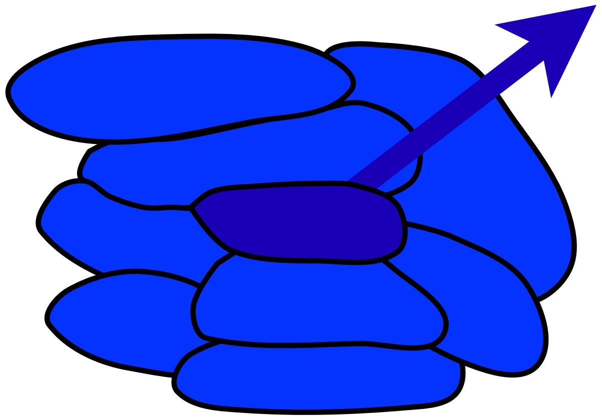

$$L \gg \ell_{\rm mfp}$$

To leading order the fields are uniform. At subleading order we allow slowly varying fields

$$\partial_\mu T^{\mu\nu} = 0$$

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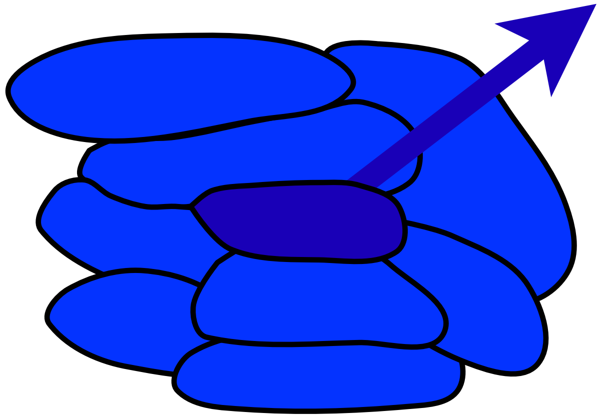

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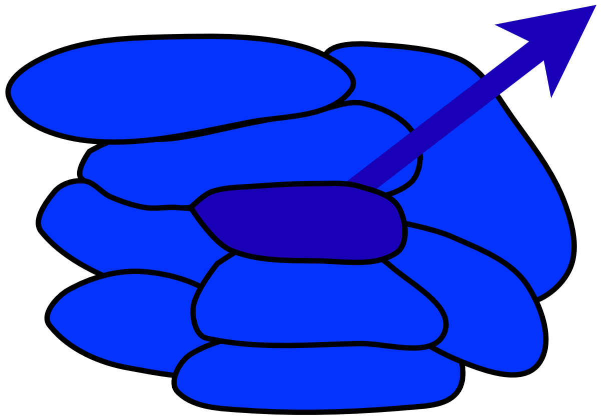

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Scalars	
Vectors	
Tensors	

Hydrodynamics

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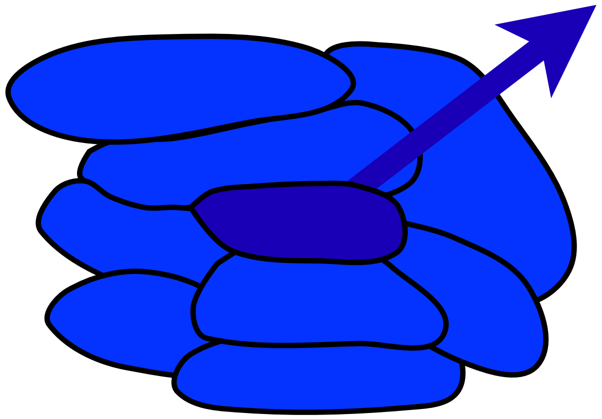

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$$\partial_\mu J^\mu = 0$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} + \tilde{T}^{\mu\nu}$$

$$J^\mu = \rho u^\mu + \tilde{J}^\mu$$

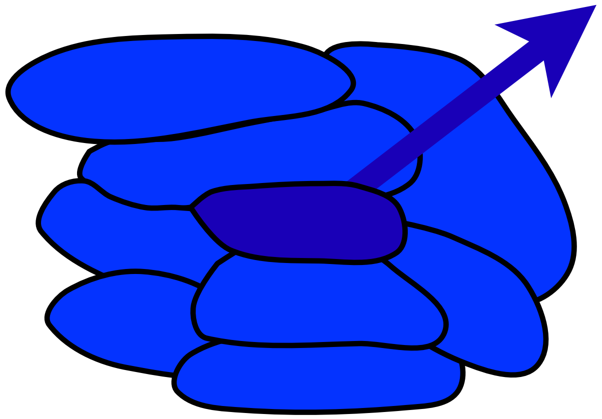
Scalars	$\partial_\mu u^\mu$ $u^\mu \partial_\mu \rho$ $u^\mu \partial_\mu \epsilon$
Vectors	
Tensors	

Hydrodynamics

$\epsilon(x^\mu)$ Energy density

$\rho(x^\mu)$ Charge density

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



$$L \gg \ell_{\mathrm{mfp}}$$

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Scalars	$\partial_\mu u^\mu$ $\quad u^\mu \partial_\mu \rho$ $\quad u^\mu \partial_\mu \epsilon$
Vectors	$P^{\mu\nu}\partial_\nu \epsilon$ $\quad P^{\mu\nu}\partial_\nu \rho$ $\quad u^\nu \partial_\nu u^\mu$
Tensors	

Hydrodynamics

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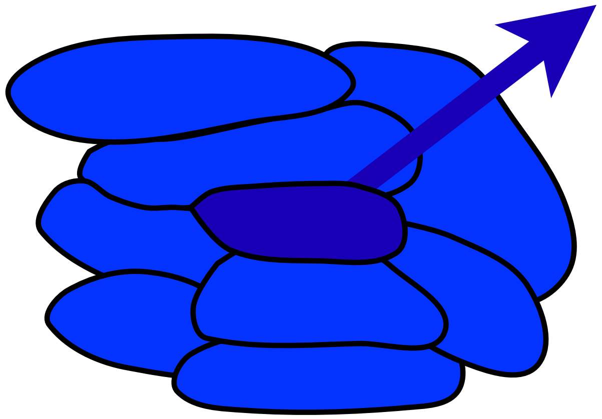

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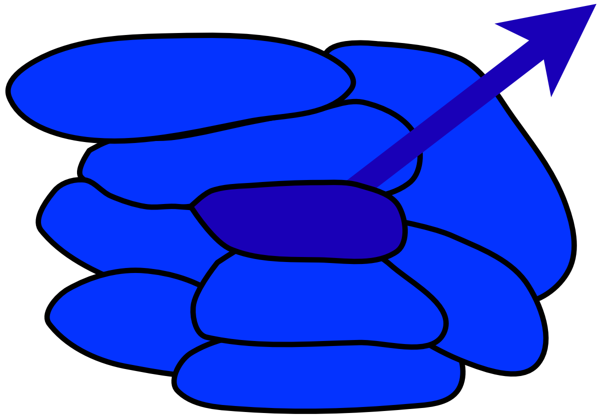
Scalars	$\partial_\mu u^\mu$ $u^\mu \partial_\mu \rho$ $u^\mu \partial_\mu \epsilon$
Vectors	$P^{\mu\nu}\partial_\nu \epsilon$ $P^{\mu\nu}\partial_\nu \rho$ $u^\nu \partial_\nu u^\mu$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

$\epsilon(x^\mu)$ Energy density $\leftrightarrow$ $T(x^\mu)$

$\rho(x^\mu)$ Charge density

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



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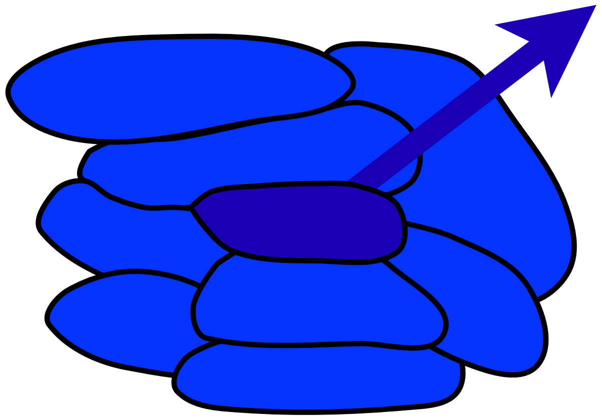
Scalars	$\partial_\mu u^\mu$ $\quad u^\mu \partial_\mu \rho$ $\quad u^\mu \partial_\mu \epsilon$
Vectors	$P^{\mu\nu}\partial_\nu \epsilon$ $\quad P^{\mu\nu}\partial_\nu \rho$ $\quad u^\nu \partial_\nu u^\mu$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

$\epsilon(x^\mu)$ **Energy density** $\leftrightarrow \ T(x^\mu)$

$\rho(x^\mu)$ **Charge density** $\leftrightarrow \ \mu(x^\mu)$

$u^\nu(x^\mu)$ **Velocity field** $(u_\mu u^\mu = -1)$



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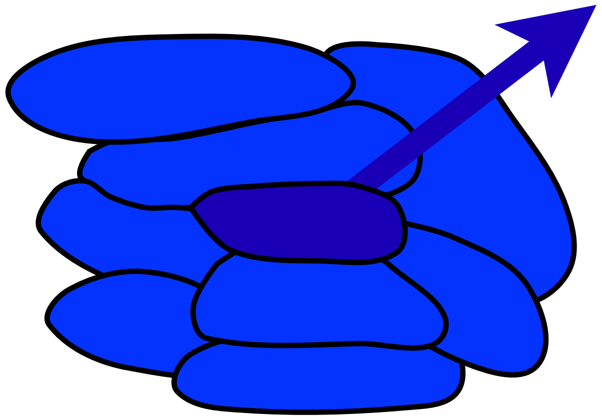
Scalars	$\partial_\mu u^\mu \quad u^\mu \partial_\mu \rho \quad u^\mu \partial_\mu \epsilon$
Vectors	$P^{\mu\nu}\partial_\nu \epsilon \quad P^{\mu\nu}\partial_\nu \rho \quad u^\nu \partial_\nu u^\mu$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics



$$T(x^\mu)$$

$$\mu(x^\mu)$$

$u^\nu(x^\mu)$ **Velocity field** $(u_\mu u^\mu = -1)$

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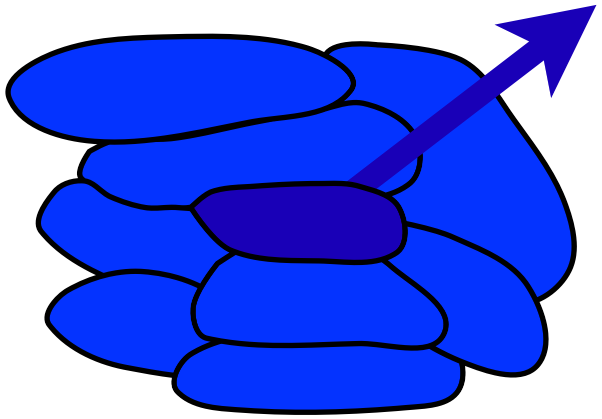
Scalars	$\partial_\mu u^\mu$ $u^\mu \partial_\mu \rho$ $u^\mu \partial_\mu \epsilon$
Vectors	$P^{\mu\nu}\partial_\nu \epsilon$ $P^{\mu\nu}\partial_\nu \rho$ $u^\nu \partial_\nu u^\mu$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

$T(x^\mu)$ Temperature

$\mu(x^\mu)$ Chemical potential

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



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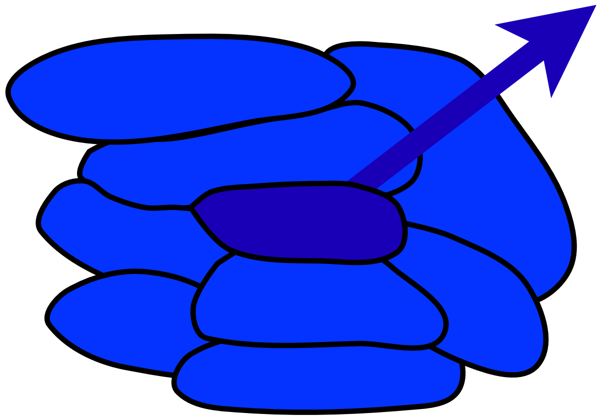
Scalars	$\partial_\mu u^\mu$ $u^\mu \partial_\mu \rho$ $u^\mu \partial_\mu \epsilon$
Vectors	$P^{\mu\nu}\partial_\nu \epsilon$ $P^{\mu\nu}\partial_\nu \rho$ $u^\nu \partial_\nu u^\mu$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

$T(x^\mu)$ Temperature

$\mu(x^\mu)$ Chemical potential

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



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Scalars	$\partial_\mu u^\mu \qquad u^\mu \partial_\mu \rho \qquad u^\mu \partial_\mu \epsilon$
Vectors	$P^{\mu\nu}\partial_\nu \epsilon \qquad P^{\mu\nu}\partial_\nu \rho \qquad u^\nu \partial_\nu u^\mu$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

$T(x^\mu)$ Temperature

$\mu(x^\mu)$ Chemical potential

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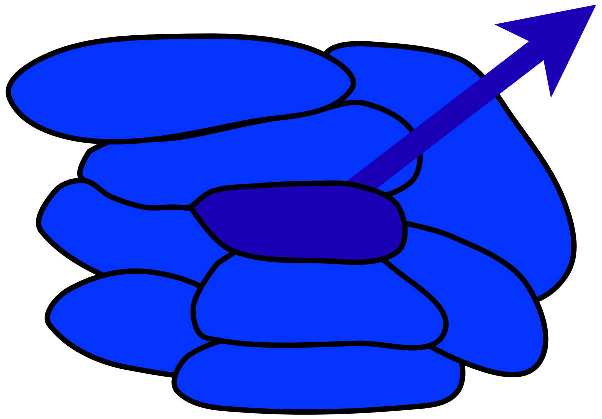

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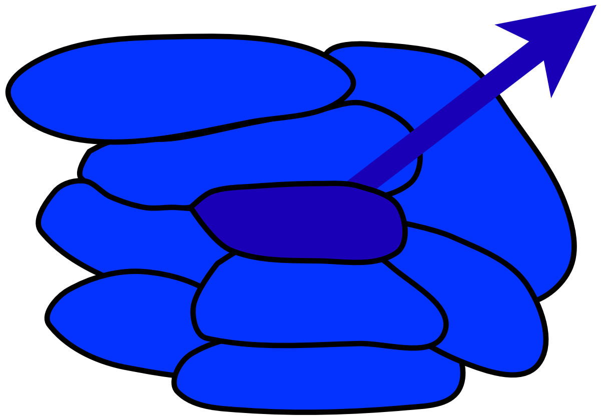
Scalars	$\partial_\mu u^\mu$ $\quad u^\mu \partial_\mu \mu$ $\quad u^\mu \partial_\mu T$
Vectors	$P^{\mu\nu}\partial_\nu T$ $\quad P^{\mu\nu}\partial_\nu \frac{\mu}{T}$ $\quad u^\nu \partial_\nu u^\mu$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

$T(x^\mu)$ Temperature

$\mu(x^\mu)$ Chemical potential

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



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$$\partial_\mu J^\mu = 0$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} + \tilde{T}^{\mu\nu}$$

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Scalars	$\partial_\mu u^\mu$ $u^\mu \partial_\mu \mu$ $u^\mu \partial_\mu T$
Vectors	$P^{\mu\nu} \partial_\nu T$ $P^{\mu\nu} \partial_\nu \frac{\mu}{T}$ $u^\nu \partial_\nu u^\mu$
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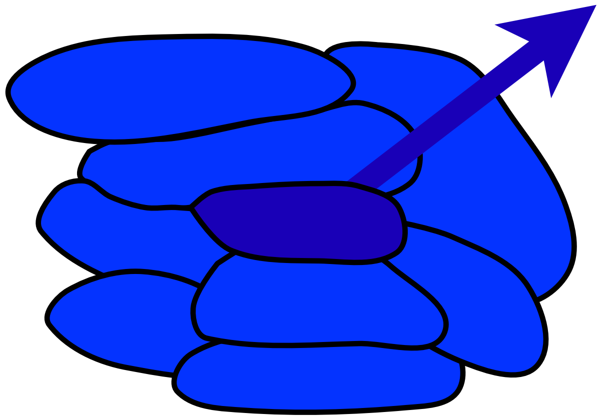

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Hydrodynamics

$T(x^\mu)$ Temperature

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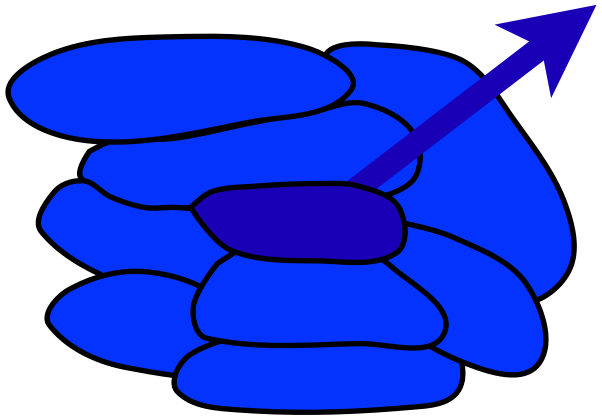

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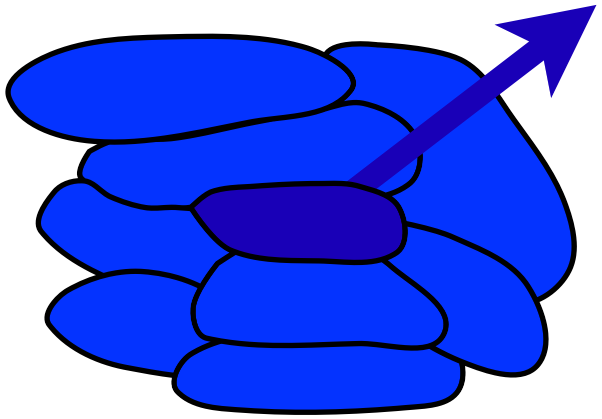
Vectors	$P^{\mu\nu}\partial_\nu T$ $P^{\mu\nu}\partial_\nu \frac{\mu}{T}$ $u^\nu \partial_\nu u^\mu$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

$T(x^\mu)$ Temperature

$\mu(x^\mu)$ Chemical potential

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



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Hydrodynamics

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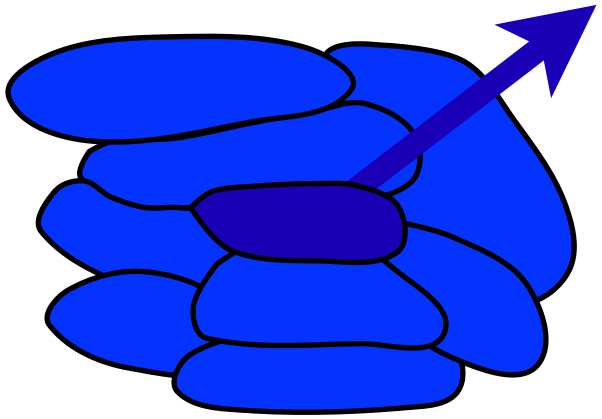

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Vectors	$P^{\mu\nu}\partial_\nu T$ $\quad$ $P^{\mu\nu}\partial_\nu \frac{\mu}{T}$ $\quad$ $u^\nu \partial_\nu u^\mu$
Tensors	$\partial_{\langle\mu} u_{\nu\rangle}$

Hydrodynamics

$T(x^\mu)$ Temperature

$\mu(x^\mu)$ Chemical potential

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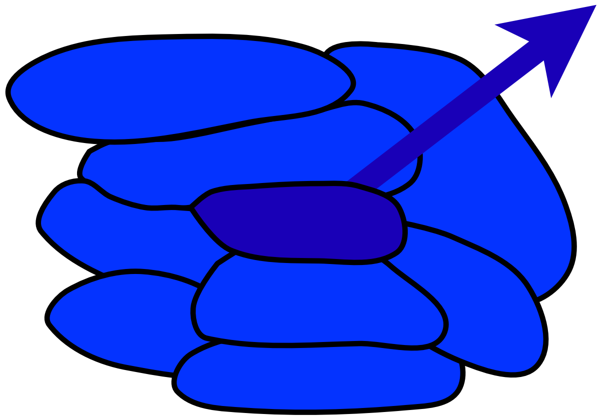

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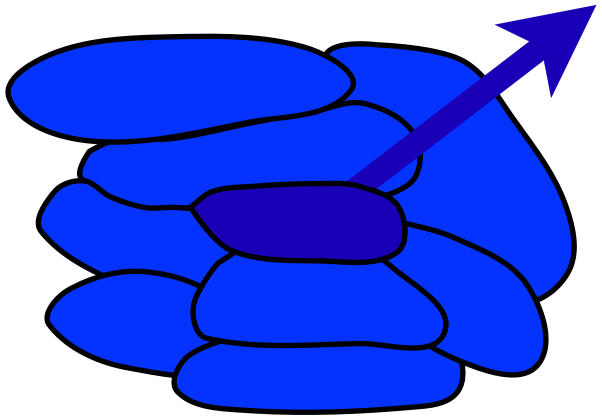

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Hydrodynamics

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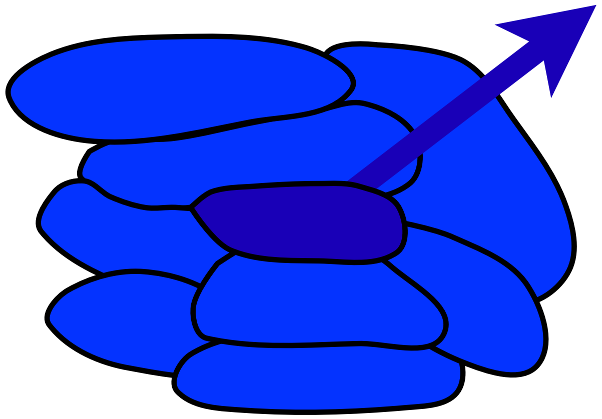

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Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

$T(x^\mu)$ Temperature

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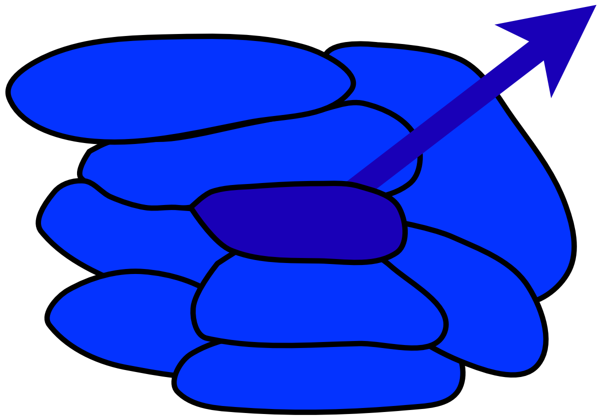

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To leading order the fields are uniform. At subleading order we allow slowly varying fields

In a local rest frame:

$$T^{00} = \epsilon + \tilde{T}^{00}$$

Scalars	$\partial_\mu u^\mu$ $\cancel{u^\mu \partial_\mu \mu}$ $\cancel{u^\mu \partial_\mu T}$
Vectors	$P^{\mu\nu}\partial_\nu T$ $P^{\mu\nu}\partial_\nu \frac{\mu}{T}$ $\cancel{u^\nu \partial_\nu u^\mu}$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

$T(x^\mu)$ Temperature

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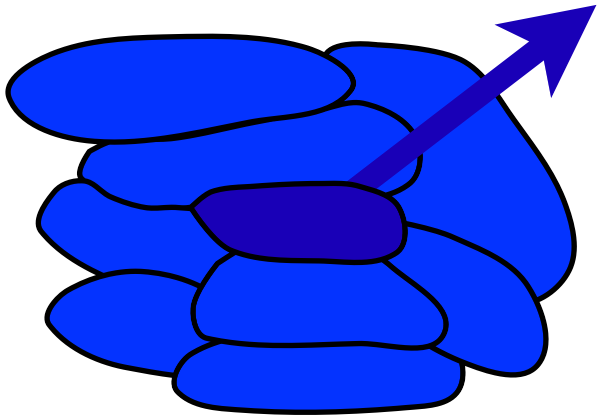

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In a local rest frame:

$$T^{00} = \epsilon + \tilde{T}^{00}$$

=out of equilibrium local energy density

Scalars	$\partial_\mu u^\mu$ $\cancel{u^\mu \partial_\mu \mu}$ $\cancel{u^\mu \partial_\mu T}$
Vectors	$P^{\mu\nu}\partial_\nu T$ $P^{\mu\nu}\partial_\nu \frac{\mu}{T}$ $\cancel{u^\nu \partial_\nu u^\mu}$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

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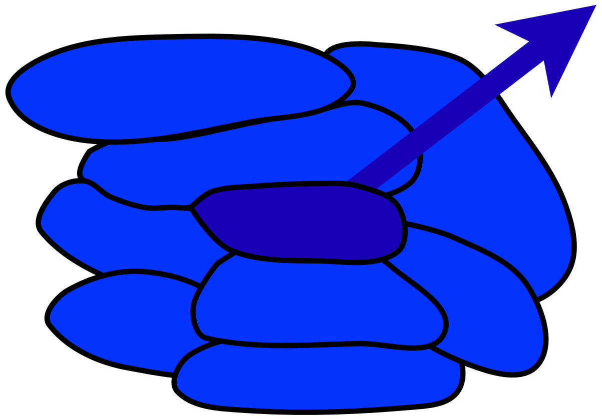

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In a local rest frame:

$$T^{00} = \epsilon + \tilde{T}^{00}$$

=out of equilibrium local energy density

$$J^0 = \rho + \tilde{J}^0$$

Scalars	$\partial_\mu u^\mu$ $\cancel{u^\mu \partial_\mu \mu}$ $\cancel{u^\mu \partial_\mu T}$
Vectors	$P^{\mu\nu}\partial_\nu T$ $P^{\mu\nu}\partial_\nu \frac{\mu}{T}$ $\cancel{u^\nu \partial_\nu u^\mu}$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

$T(x^\mu)$ Temperature

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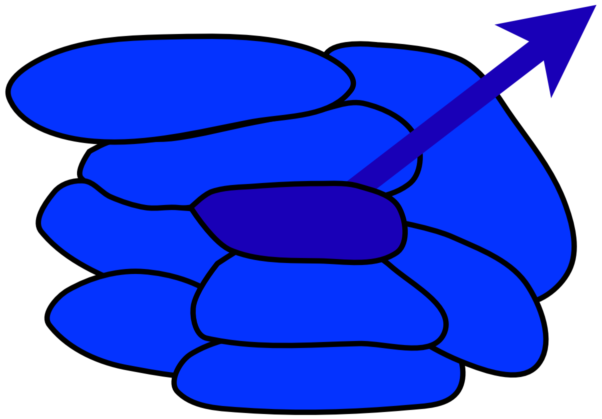
$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



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In a local rest frame:

$$T^{00} = \epsilon + \tilde{T}^{00}$$ =out of equilibrium local energy density

$$J^0 = \rho + \tilde{J}^0$$ =out of equilibrium local charge density

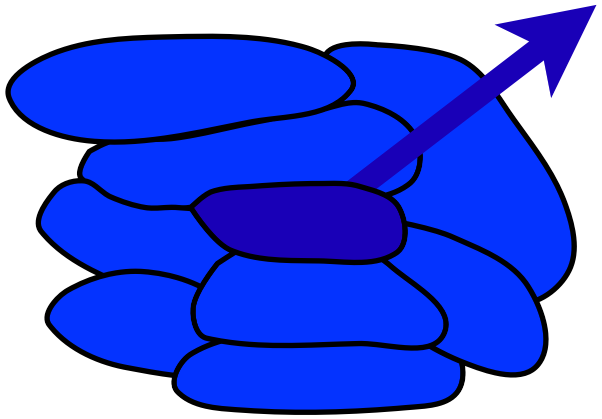
Scalars	$\partial_\mu u^\mu$ $\cancel{u^\mu \partial_\mu \mu}$ $\cancel{u^\mu \partial_\mu T}$
Vectors	$P^{\mu\nu}\partial_\nu T$ $P^{\mu\nu}\partial_\nu \frac{\mu}{T}$ $\cancel{u^\nu \partial_\nu u^\mu}$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

$T(x^\mu)$ Temperature $\to T' = T + \delta T$

$\mu(x^\mu)$ Chemical potential

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



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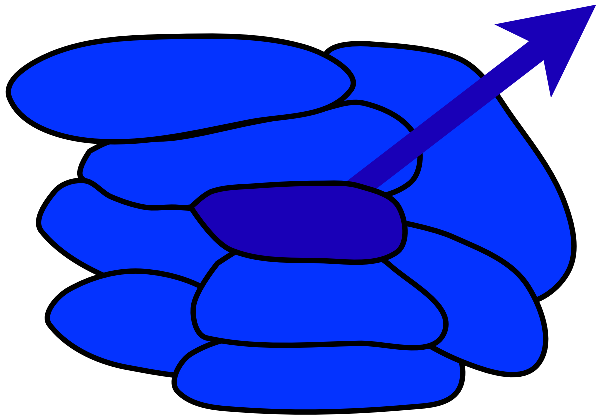
$$J^0 = \rho + \tilde{J}^0$$ =out of equilibrium local charge density

Scalars	$\partial_\mu u^\mu$ $\cancel{u^\mu \partial_\mu \mu}$ $\cancel{u^\mu \partial_\mu T}$
Vectors	$P^{\mu\nu}\partial_\nu T$ $P^{\mu\nu}\partial_\nu \frac{\mu}{T}$ $\cancel{u^\nu \partial_\nu u^\mu}$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics



$T(x^\mu)$ **Temperature** $\rightarrow T' = T + \delta T$

$\mu(x^\mu)$ **Chemical potential** $\rightarrow \mu' = \mu + \delta\mu$

$u^\nu(x^\mu)$ **Velocity field** $(u_\mu u^\mu = -1)$

$$L \gg \ell_{\mathrm{mfp}}$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} + \tilde{T}^{\mu\nu}$$

$$J^\mu = \rho u^\mu + \tilde{J}^\mu$$

To leading order the fields are uniform. At subleading order we allow slowly varying fields

In a local rest frame:

$$T^{00} = \epsilon + \tilde{T}^{00}$$

=out of equilibrium local energy density

$$J^0 = \rho + \tilde{J}^0$$

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Scalars	$\partial_\mu u^\mu \qquad \cancel{u^\mu\partial_\mu\mu} \qquad \cancel{u^\mu\partial_\mu T}$
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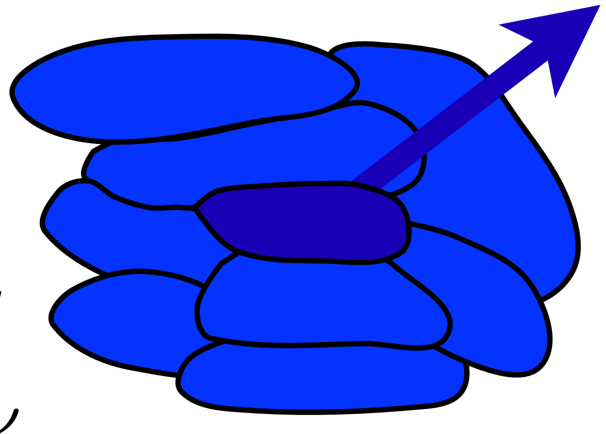
Tensors	$\partial_{\langle\mu} u_{\nu\rangle}$

Hydrodynamics



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Hydrodynamics

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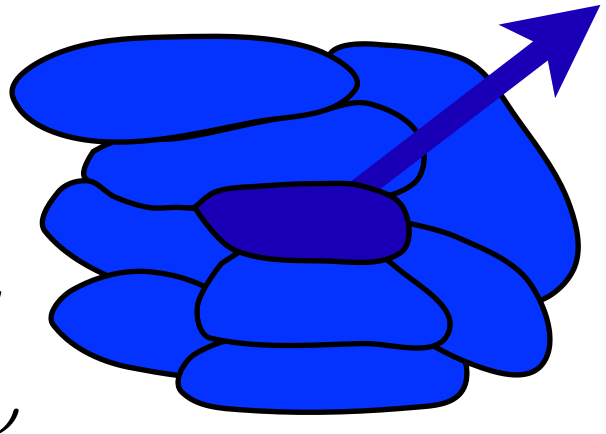

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$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} + \tilde{T}^{\mu\nu}$$

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To leading order the fields are uniform. At subleading order we allow slowly varying fields

In a local rest frame:

$$T^{00} = \epsilon + \tilde{T}^{00} = \epsilon$$

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Scalars	$\partial_\mu u^\mu$ $\cancel{u^\mu \partial_\mu \mu}$ $\cancel{u^\mu \partial_\mu T}$
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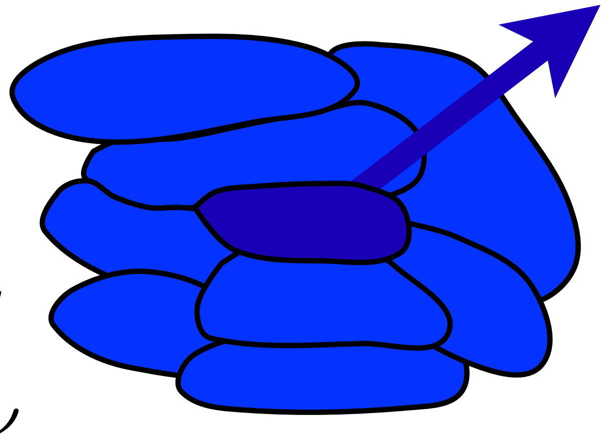

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To leading order the fields are uniform. At subleading order we allow slowly varying fields

In a local rest frame:

$$T^{00} = \epsilon + \tilde{T}^{00} \;\; = \epsilon$$

$$J^0 = \rho + \tilde{J}^0 \quad = \rho$$

Scalars	$\partial_\mu u^\mu$ $\cancel{u^\mu \partial_\mu \mu}$ $\cancel{u^\mu \partial_\mu T}$
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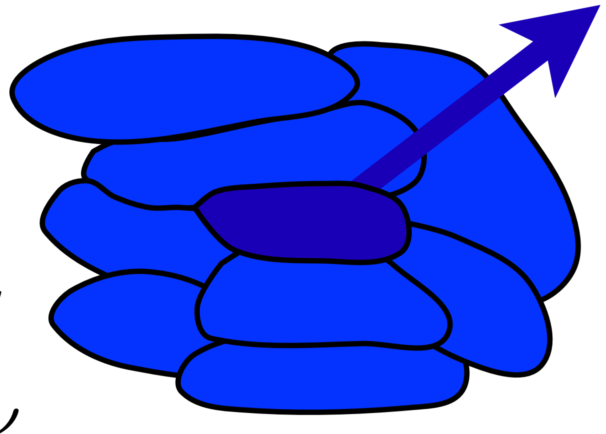
Hydrodynamics



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$$T^{0i} = 0$$

Scalars	$\partial_\mu u^\mu$ $\cancel{u^\mu \partial_\mu \mu}$ $\cancel{u^\mu \partial_\mu T}$
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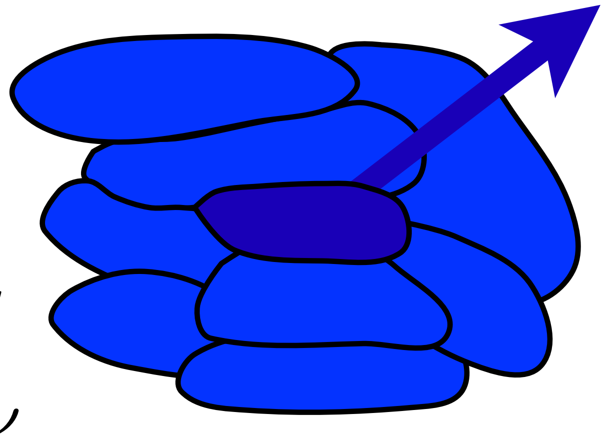
Hydrodynamics



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Landau frame:

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$$T^{0i} = 0$$

Scalars	$\partial_\mu u^\mu$ $\cancel{u^\mu \partial_\mu \mu}$ $\cancel{u^\mu \partial_\mu T}$
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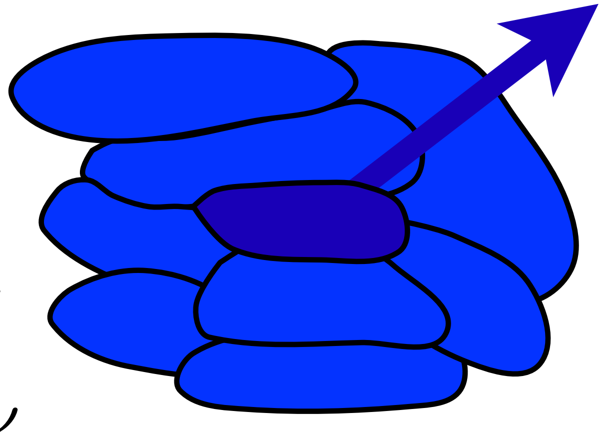
Hydrodynamics



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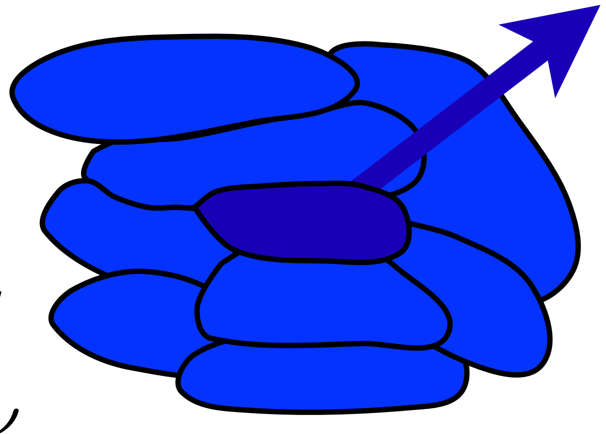
$$J^i = 0$$

Hydrodynamics



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Landau frame:

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$$T^{0i} = 0$$

⇒

$$u_\mu \tilde{T}^{\mu\nu} = 0$$

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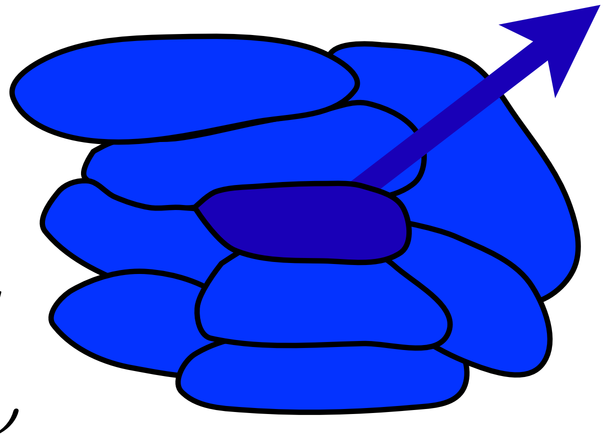
Hydrodynamics



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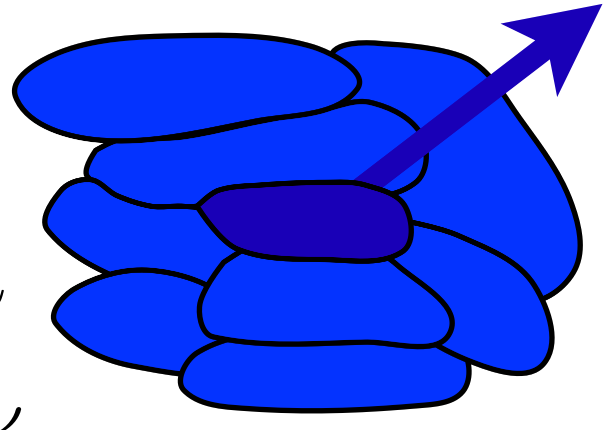
Hydrodynamics



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Scalars	$\partial_\mu u^\mu$	
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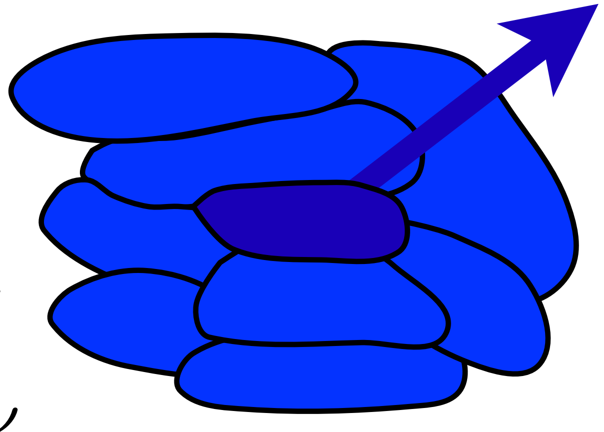
Hydrodynamics



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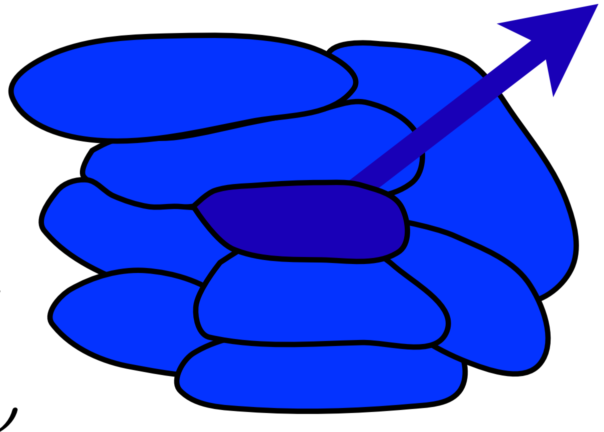
Hydrodynamics



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Landau frame:

$$u_\mu \tilde{T}^{\mu\nu} = 0$$

$$u_\mu \tilde{J}^\mu = 0$$

Scalars	$\partial_\mu u^\mu$
Vectors	$P^{\mu\nu}\partial_\nu T$ $\quad P^{\mu\nu}\partial_\nu \frac{\mu}{T}$
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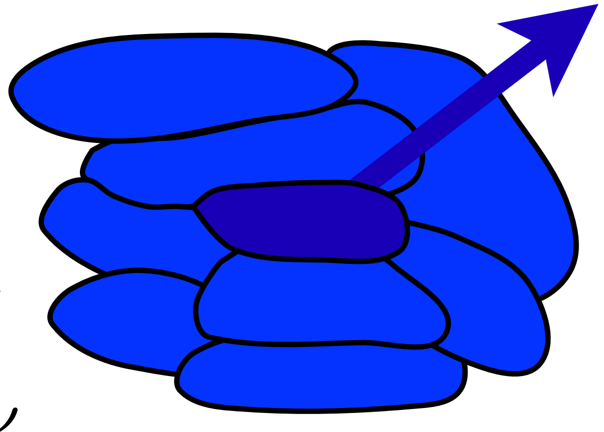
Hydrodynamics



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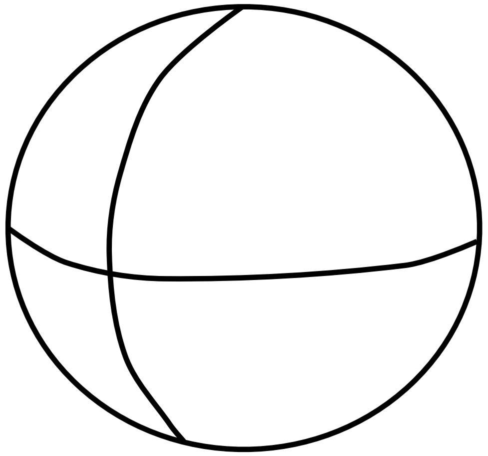
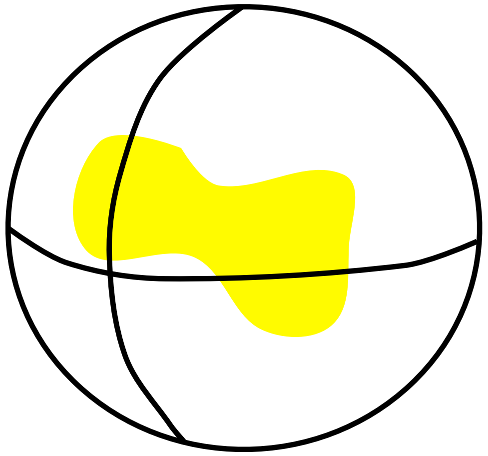
An entropy current

An entropy current



An entropy current

$$\frac{dS}{dt}$$



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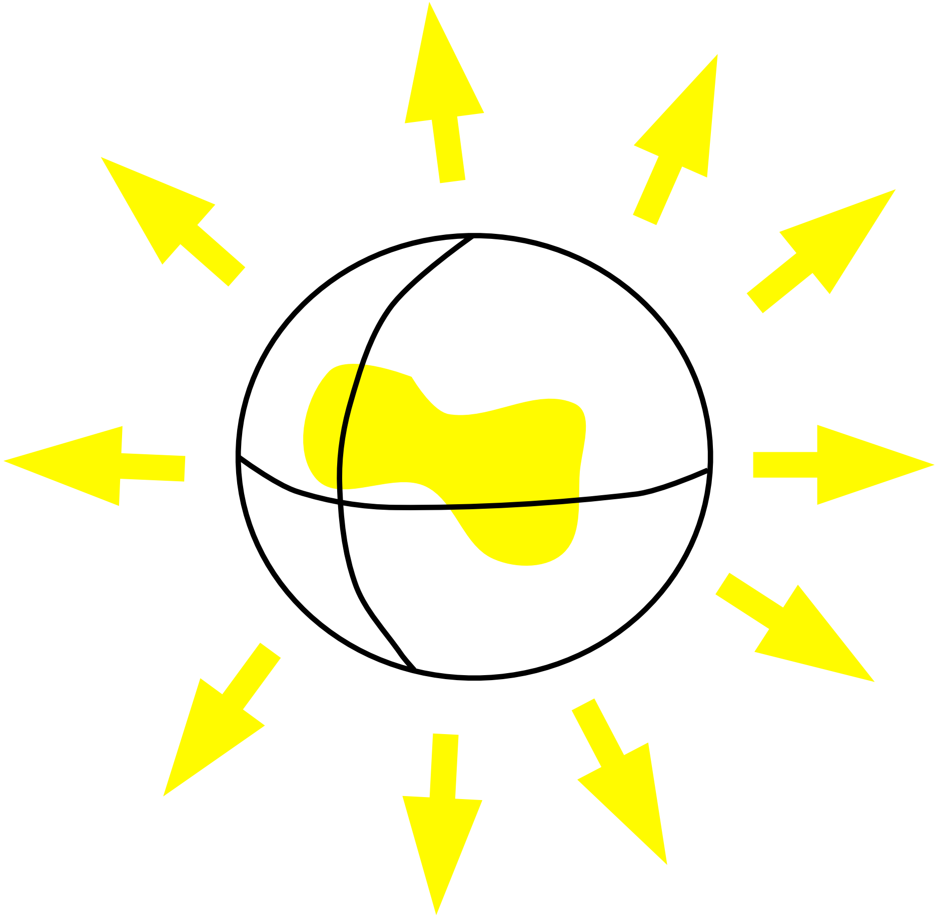
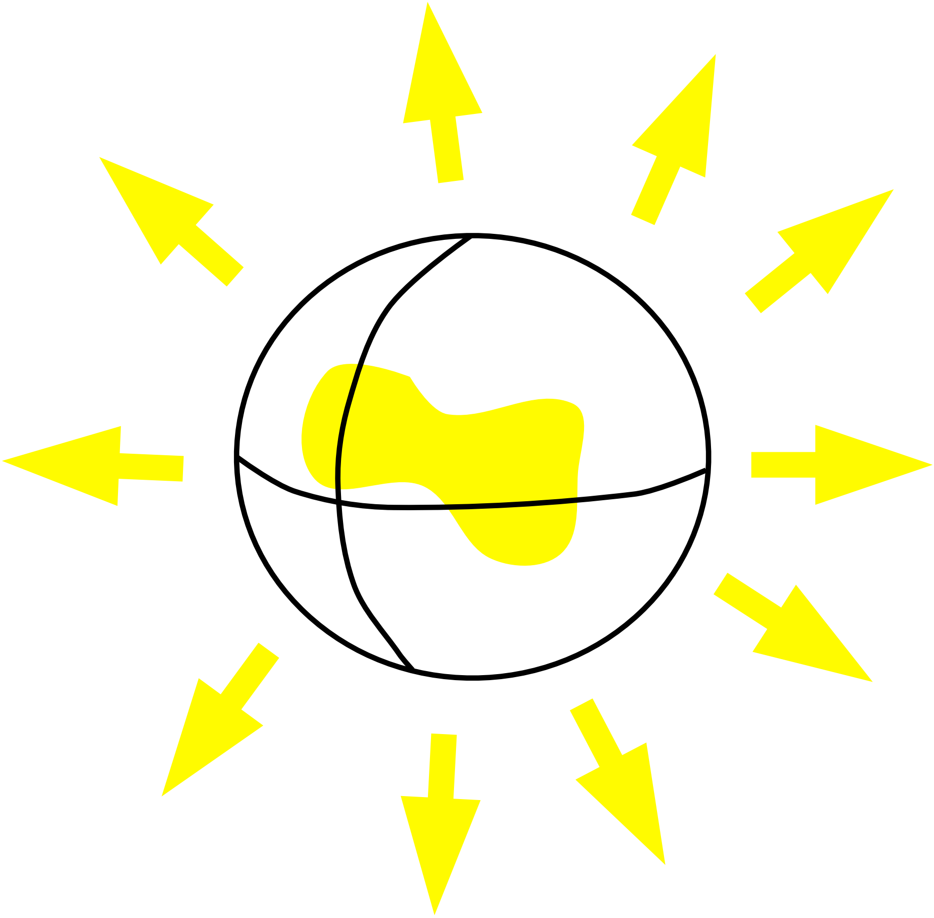


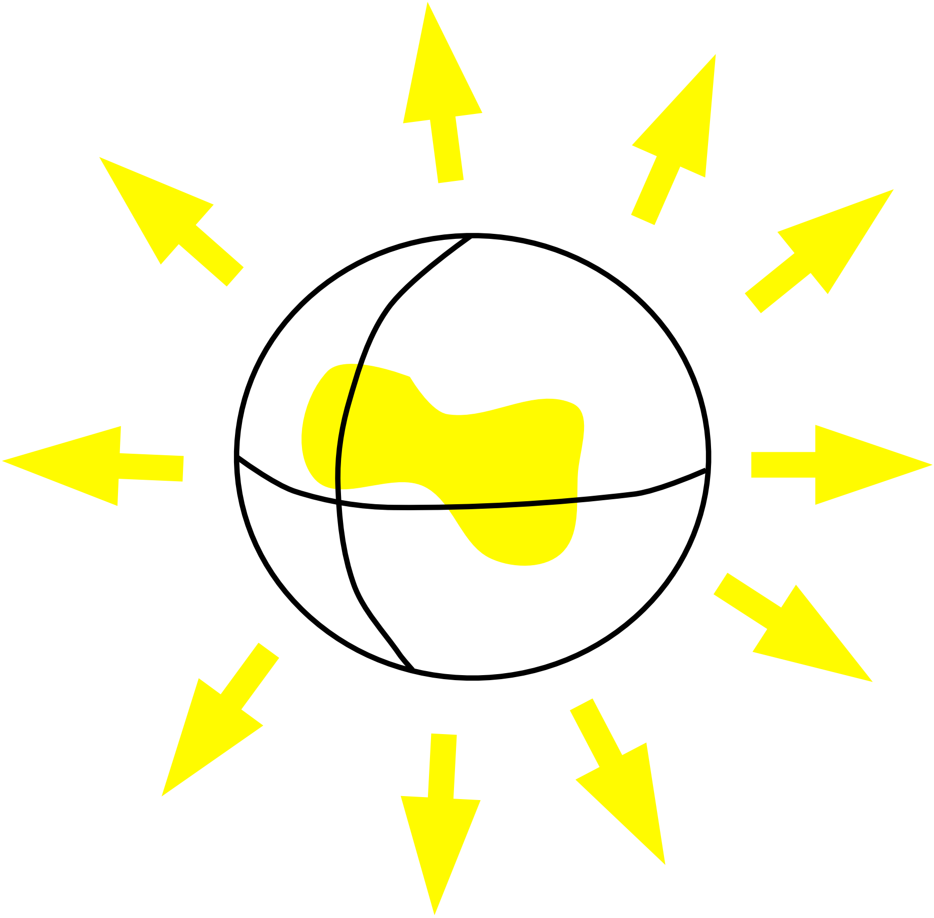
$$\frac{dS}{dt} + \int \vec{S} \cdot d\vec{a}$$

An entropy current



$$\frac{dS}{dt} + \int \vec{S} \cdot d\vec{a} \quad \geq 0$$

An entropy current

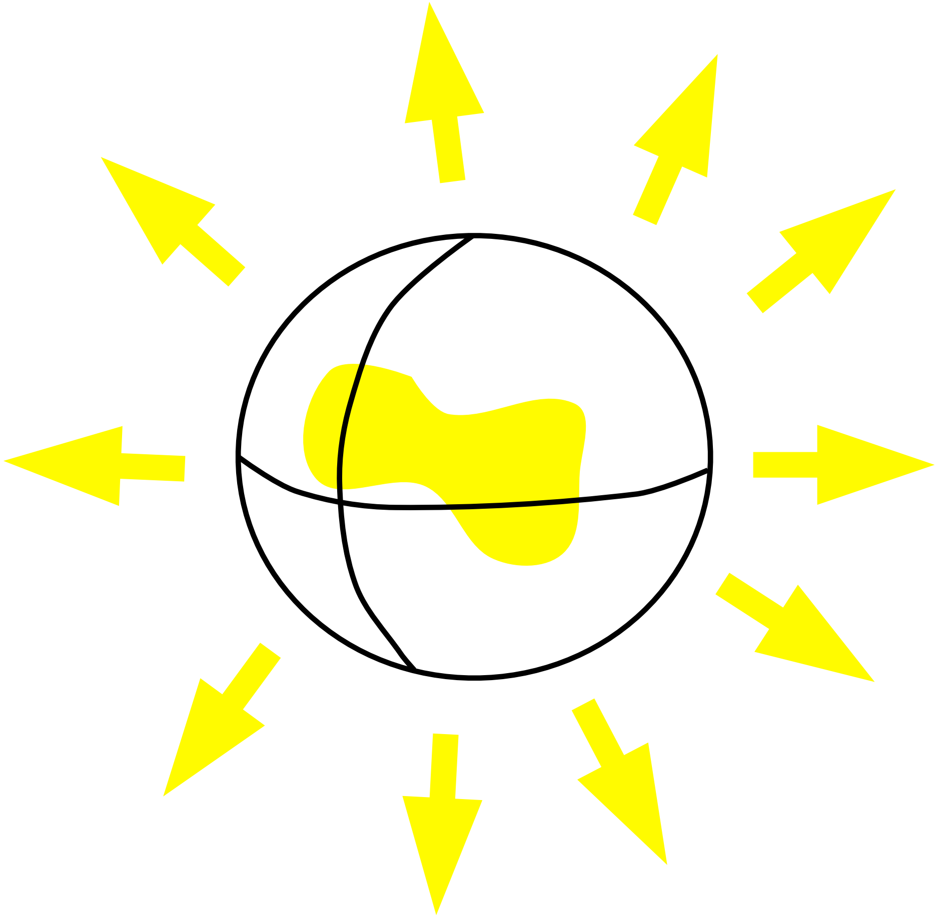


$$\frac{dS}{dt} + \int \vec{S} \cdot d\vec{a} \quad \geq 0$$

Local version of the 2nd law:

1) $\partial_\mu J^\mu_s \geq 0$

An entropy current



$$\frac{dS}{dt} + \int \vec{S} \cdot d\vec{a} \quad \geq 0$$

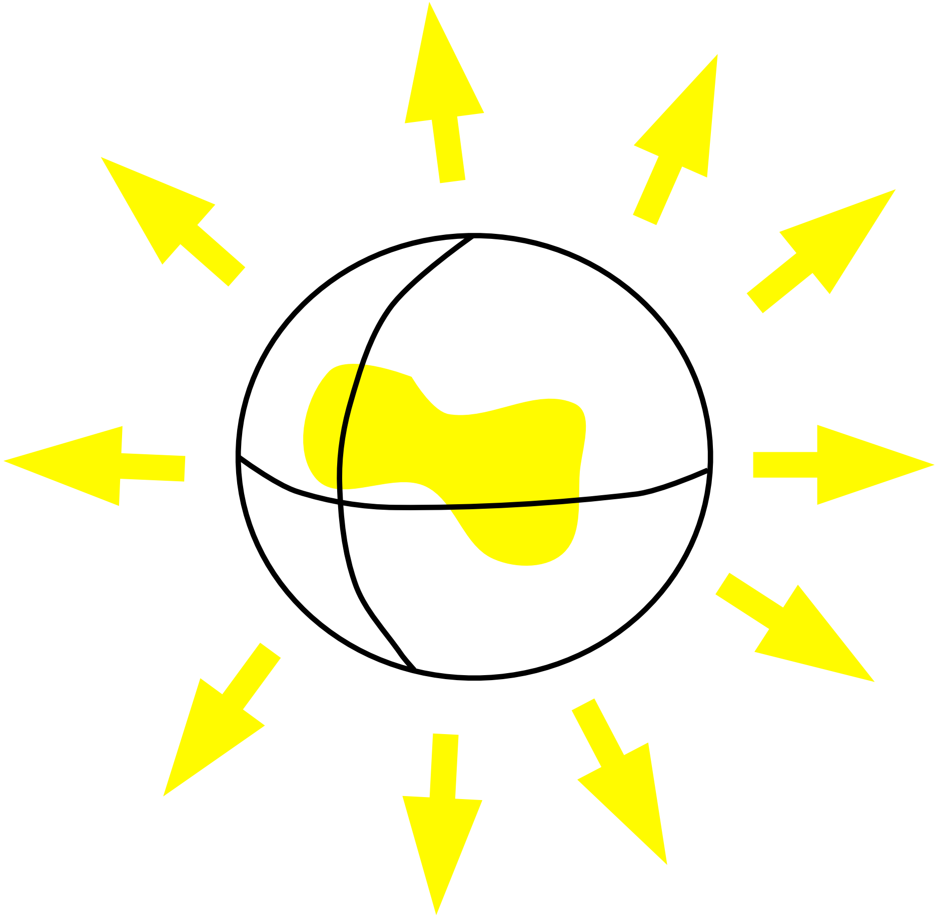
Local version of the 2nd law:

1) $\partial_\mu J_s^\mu \geq 0$

2) In equilibrium: $u^\mu = (1,0,0,0)$

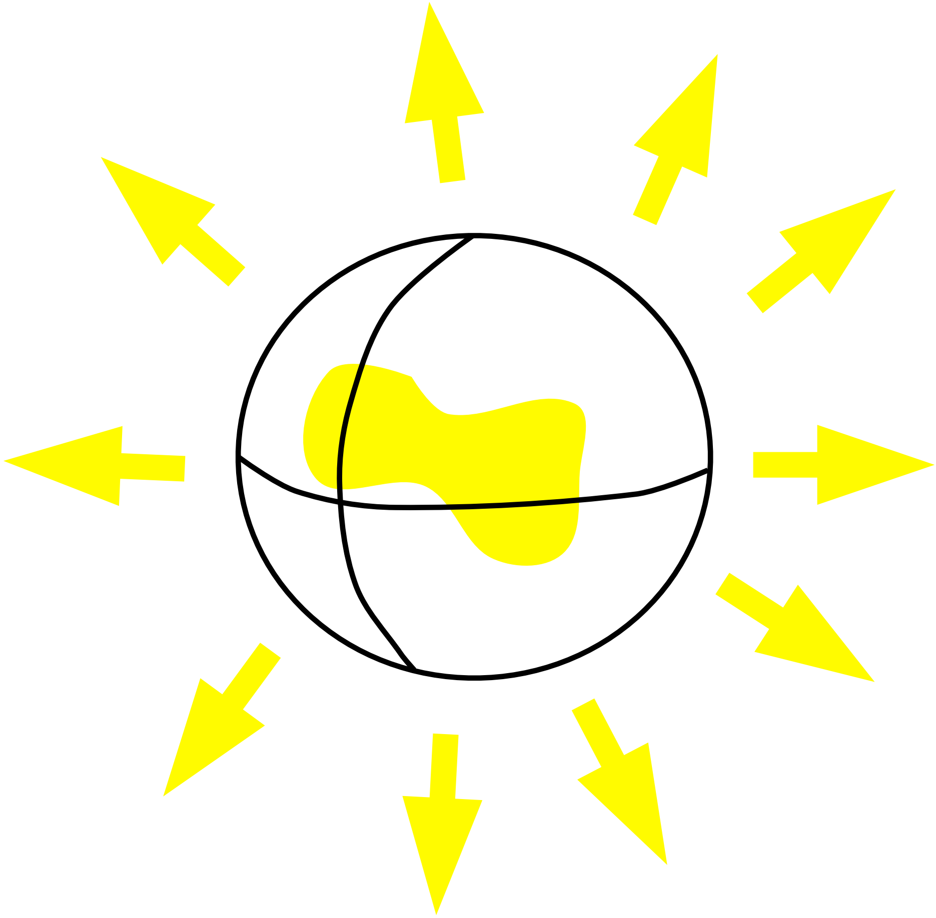
$$J_s^\mu = (s,0,0,0)$$

An entropy current



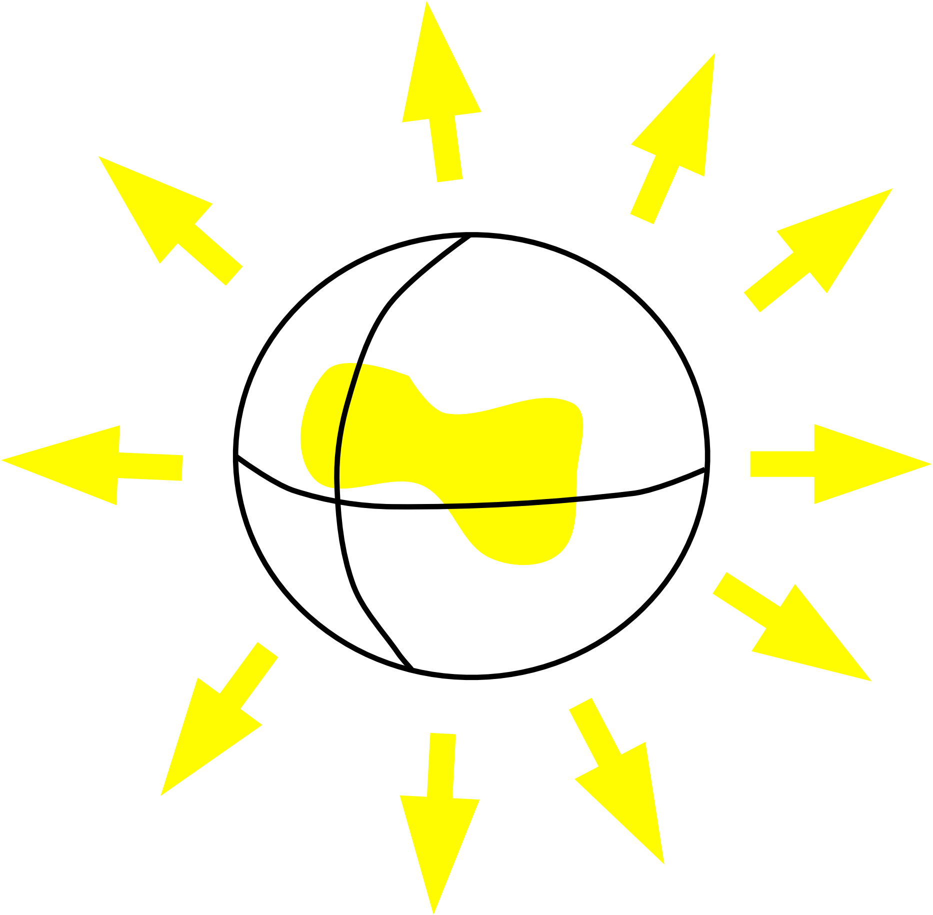
1) Locality?

An entropy current



1) Locality?
2) Properties?

An entropy current



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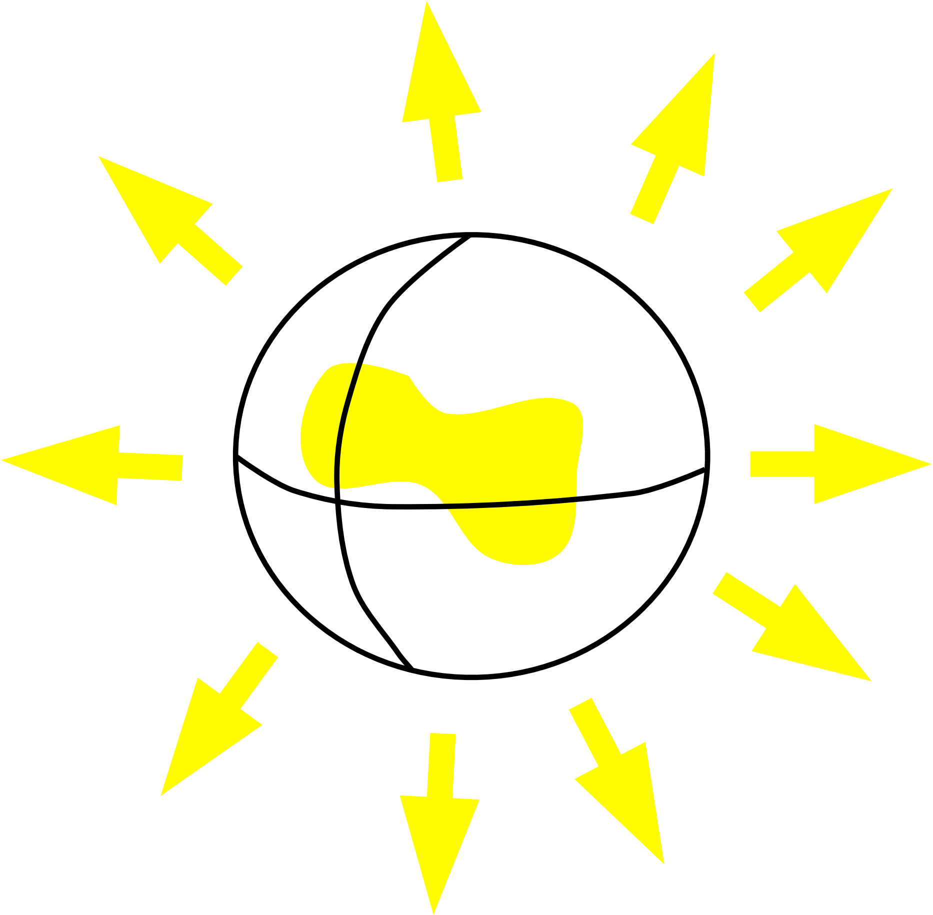

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$$J_s^\mu = s u^\mu + \tilde{J}_s^\mu$$

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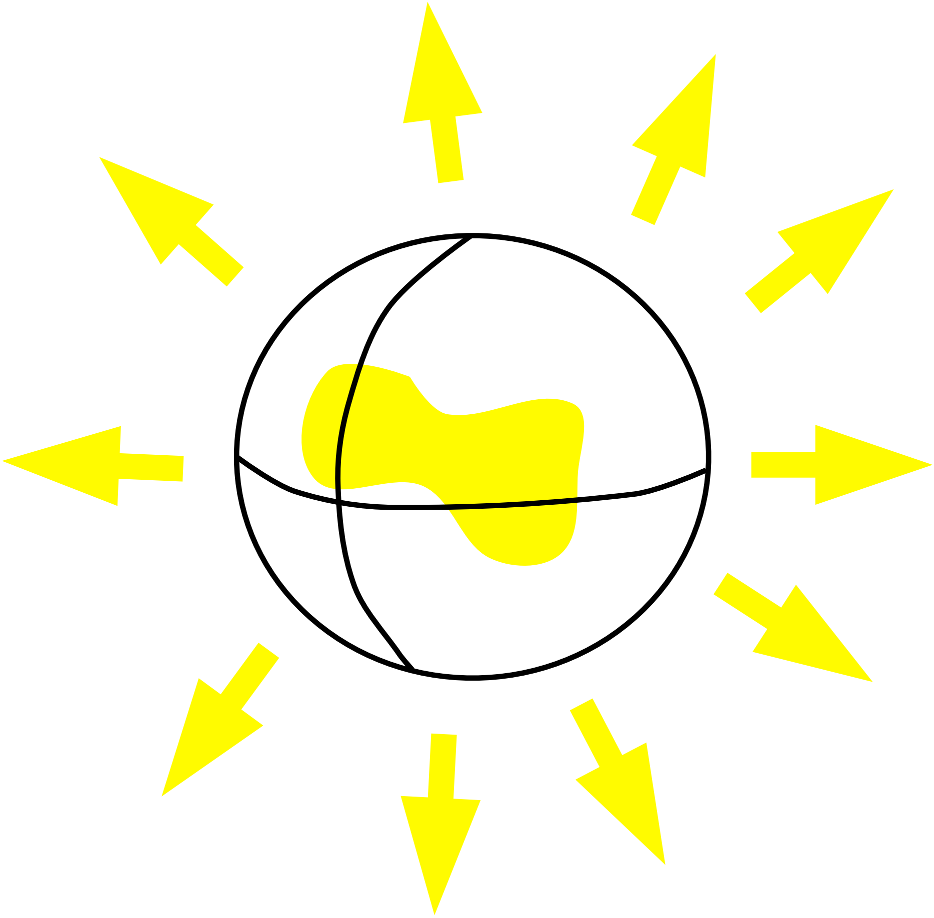

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Scalars	$\partial_\mu u^\mu$	
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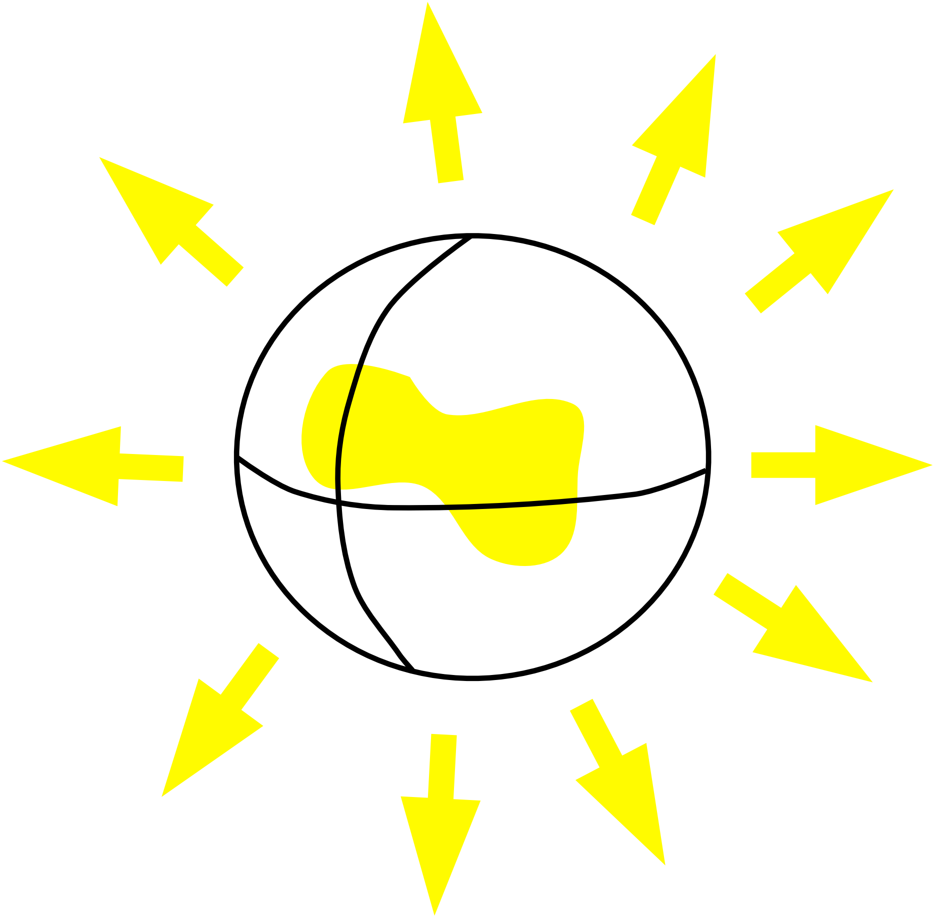
An entropy current



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Local version of the 2nd law:

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$$J^\mu_s = s u^\mu + \theta \partial_\alpha u^\alpha u^\mu$$

Scalars	$\partial_\mu u^\mu$	
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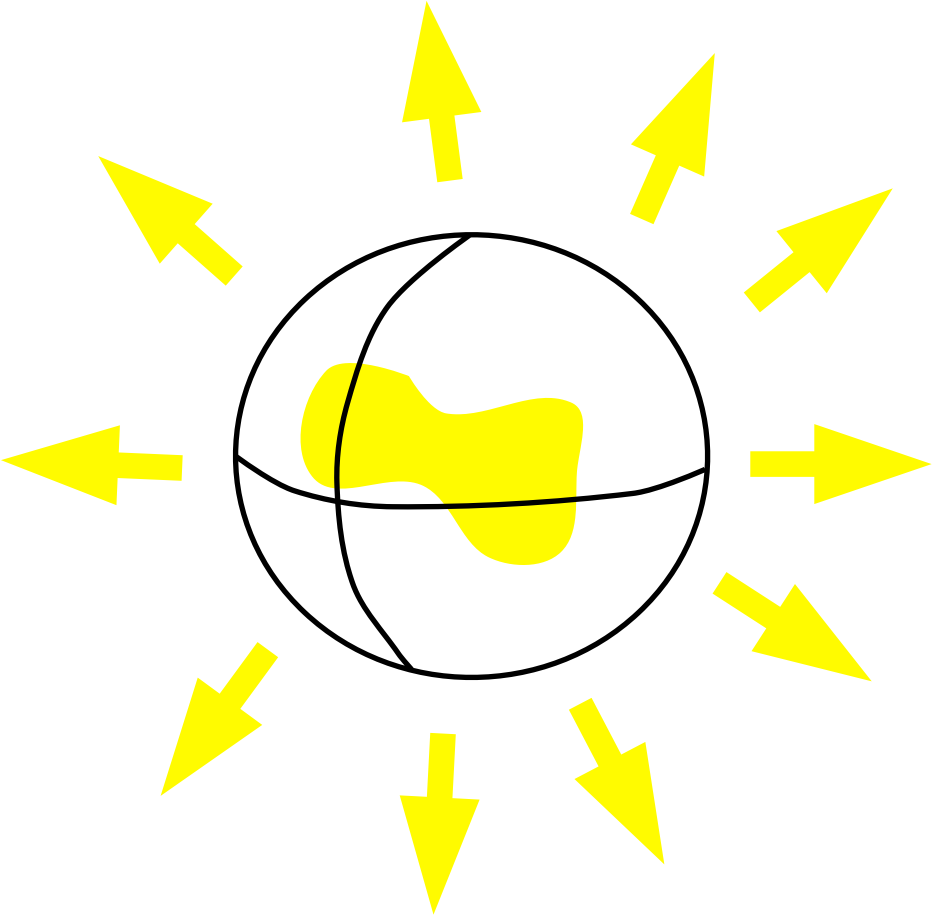

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$$J^\mu_s = su^\mu + \theta \partial_\alpha u^\alpha u^\mu$$
$$+ (\sigma + \frac{\mu}{T}\kappa) P^{\mu\nu} \partial_\nu \frac{\mu}{T}$$
$$+ (\sigma_T + \frac{\mu}{T}\kappa) P^{\mu\nu} \partial_\nu T$$

Scalars	$\partial_\mu u^\mu$
Vectors	$P^{\mu\nu}\partial_\nu T$ $\quad P^{\mu\nu}\partial_\nu \frac{\mu}{T}$
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Hydrodynamics

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$$\partial_\mu T^{\mu\nu} = 0$$

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Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

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$$J_s^\mu = s u^\mu + \theta\partial_\alpha u^\alpha u^\mu + (\sigma + \frac{\mu}{T}\kappa)P^{\mu\nu}\partial_\nu \frac{\mu}{T} + (\sigma_T + \frac{\mu}{T}\kappa_T)P^{\mu\nu}\partial_\nu T$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J_s^\mu \geq 0$$

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

$$J^\mu = \rho u^\mu - \kappa P^{\mu\nu}\partial_\nu \frac{\mu}{T} - \kappa_T P^{\mu\nu}\partial_\nu T$$

$$J_s^\mu = s u^\mu + \theta \partial_\alpha u^\alpha u^\mu + (\sigma + \frac{\mu}{T}\kappa) P^{\mu\nu}\partial_\nu \frac{\mu}{T} + (\sigma_T + \frac{\mu}{T}\kappa_T) P^{\mu\nu}\partial_\nu T$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$\partial_\mu J_s^\mu =$ one-derivative expressions **+** two-derivative expressions

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

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Hydrodynamics

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$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$\partial_\mu J_s^\mu =$ two-derivative expressions

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

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$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$\partial_\mu J_s^\mu =$ two-derivative terms

[illegible] expressions

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

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$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$\partial_\mu J^\mu_s =$ product of two one-derivative terms **+** genuine two-derivative terms

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

$$J^\mu = \rho u^\mu - \kappa P^{\mu\nu}\partial_\nu \frac{\mu}{T} - \kappa_T P^{\mu\nu}\partial_\nu T$$

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$$\partial_\mu T^{\mu\nu} = 0 \qquad (\partial_\alpha u^\alpha)^2$$

$$\partial_\mu J^\mu = 0$$

$\partial_\mu J_s^\mu =$ product of two one-derivative terms **+** genuine two-derivative terms

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + PP^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

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$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$(\partial_\alpha u^\alpha)^2$

$\partial_\alpha \partial^\alpha \mu$

$\partial_\mu J_s^\mu =$ product of two one-derivative terms + genuine two-derivative terms

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

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$$\partial_\mu T^{\mu\nu} = 0$$

$$(\partial_\alpha u^\alpha)^2 \qquad \partial_\alpha \partial^\alpha \mu$$

$$\partial_\mu J^\mu = 0$$

$\partial_\mu J^\mu_s =$ product of two one-derivative terms **+** genuine two-derivative terms ≥ 0

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

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$$\partial_\mu T^{\mu\nu} = 0$$

$$(\partial_\alpha u^\alpha)^2$$

$$\partial_\alpha \partial^\alpha \mu$$

genuine two-derivative terms **=0**

$$\partial_\mu J^\mu = 0$$

$\partial_\mu J^\mu_s =$ product of two one-derivative terms **+** genuine two-derivative terms ≥ 0

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

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$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$$(\partial_\alpha u^\alpha)^2 \qquad \partial_\alpha \partial^\alpha \mu$$

$\partial_\mu J_s^\mu =$ product of two one-derivative terms **+** genuine two-derivative terms ≥ 0

genuine two-derivative terms **=0**

$$\theta = 0$$

$$\sigma = 0$$

$$\sigma_T = 0$$

$$\kappa_T = 0$$

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

$$J^\mu = \rho u^\mu - \kappa P^{\mu\nu}\partial_\nu \frac{\mu}{T}$$

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$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$$(\partial_\alpha u^\alpha)^2 \qquad \partial_\alpha \partial^\alpha \mu$$

$\partial_\mu J_s^\mu =$ product of two one-derivative terms **+** genuine two-derivative terms ≥ 0

genuine two-derivative terms **=0**

$$\theta = 0$$

$$\sigma = 0$$

$$\sigma_T = 0$$

$$\kappa_T = 0$$

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

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$$\partial_\mu T^{\mu\nu} = 0$$

$$\boxed{(\partial_\alpha u^\alpha)^2}$$

$$\partial_\mu J^\mu = 0$$

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Hydrodynamics

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$$\partial_\mu T^{\mu\nu} = 0$$

$$(\partial_\alpha u^\alpha)^2$$

$$\partial_\mu J^\mu = 0$$

$\partial_\mu J^\mu_s =$ product of two one-derivative terms ≥ 0

product of two one-derivative terms ≥ 0

$$\kappa > 0$$

$$\eta > 0$$

$$\zeta > 0$$

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^{\mu} u^{\nu} + P P^{\mu\nu} - 2\eta_{\langle} \partial_{\mu} u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu} \partial_{\alpha} u^{\alpha}$$

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$$\partial_{\mu} T^{\mu\nu} = 0$$

$$\partial_{\mu} J^{\mu} = 0$$

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

$$J^\mu = \rho u^\mu - \kappa P^{\mu\nu}\partial_\nu \frac{\mu}{T}$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

Thermodynamic input:

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

$$J^\mu = \rho u^\mu - \kappa P^{\mu\nu}\partial_\nu \frac{\mu}{T}$$

$$\partial_\mu T^{\mu\nu} = 0$$

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Thermodynamic input: $P(\mu, T)$

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^{\mu} u^{\nu} + P P^{\mu\nu} - 2\eta_{\langle} \partial_{\mu} u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu} \partial_{\alpha} u^{\alpha}$$

$$J^{\mu} = \rho u^{\mu} - \kappa P^{\mu\nu} \partial_{\nu} \frac{\mu}{T}$$

$$\partial_{\mu} T^{\mu\nu} = 0$$

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Thermodynamic input: $P(\mu, T)$

Transport:

Hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

$$J^\mu = \rho u^\mu - \kappa P^{\mu\nu}\partial_\nu \frac{\mu}{T}$$

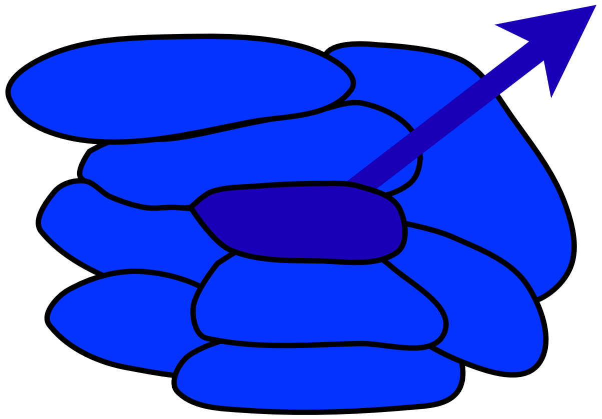
$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

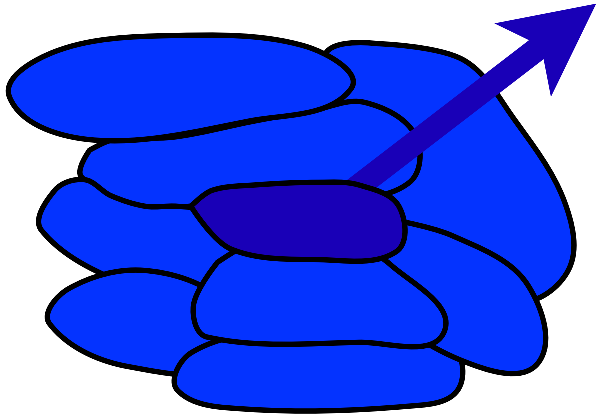
Thermodynamic input: $P(\mu, T)$

Transport: κ, η, ζ

Hydrodynamics



Hydrodynamics

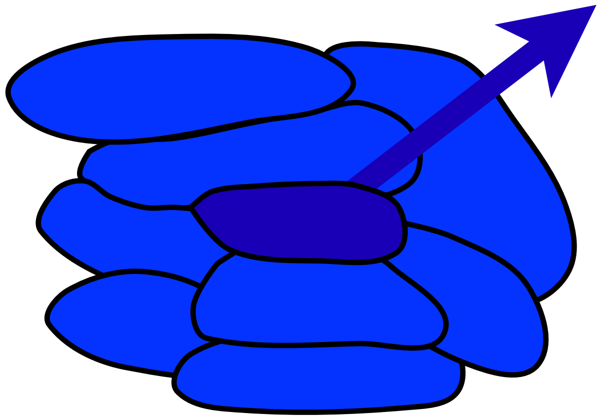


$T(x^\mu)$ Temperature

$\mu(x^\mu)$ Chemical potential

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$

Hydrodynamics



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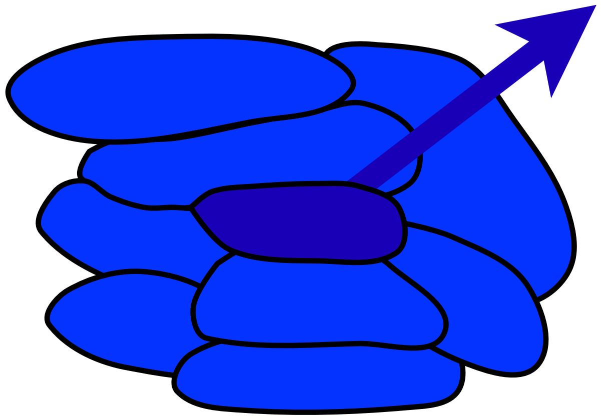
Hydrodynamics



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$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu}$$

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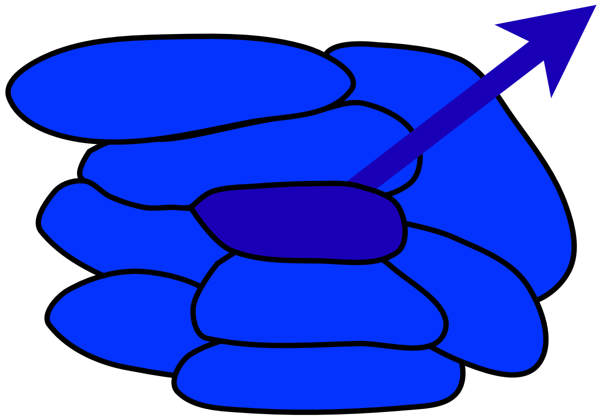
Hydrodynamics



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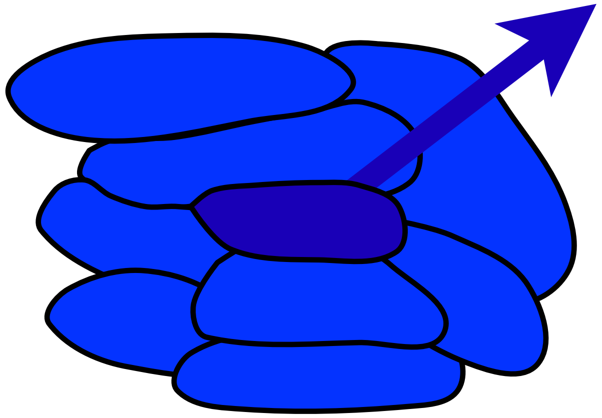
Scalars	$\partial_\mu u^\mu$
Vectors	$P^{\mu\nu}\partial_\nu T \quad P^{\mu\nu}\partial_\nu \frac{\mu}{T}$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics



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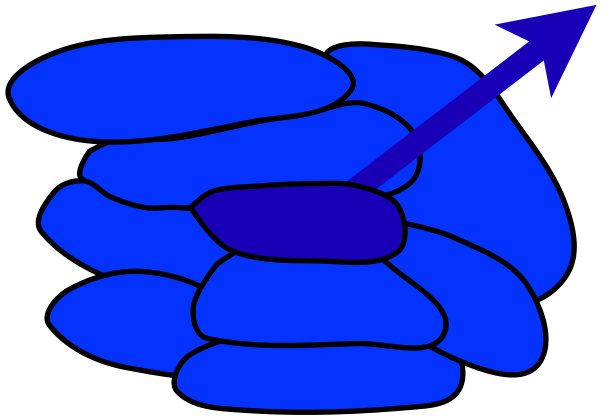
Hydrodynamics



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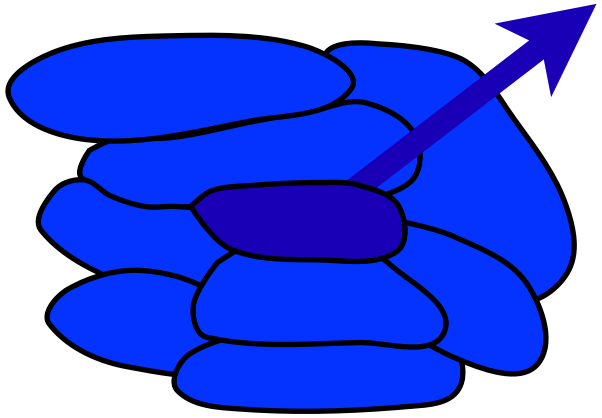
Scalars	$\partial_\mu u^\mu$
Vectors	$P^{\mu\nu}\partial_\nu T$ $\quad P^{\mu\nu}\partial_\nu \frac{\mu}{T}$
Tensors	$\langle\partial_\mu u_\nu\rangle$

Hydrodynamics



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Scalars	$\partial_\mu u^\mu$
Vectors	$P^{\mu\nu}\partial_\nu T$ $\quad P^{\mu\nu}\partial_\nu \frac{\mu}{T}$
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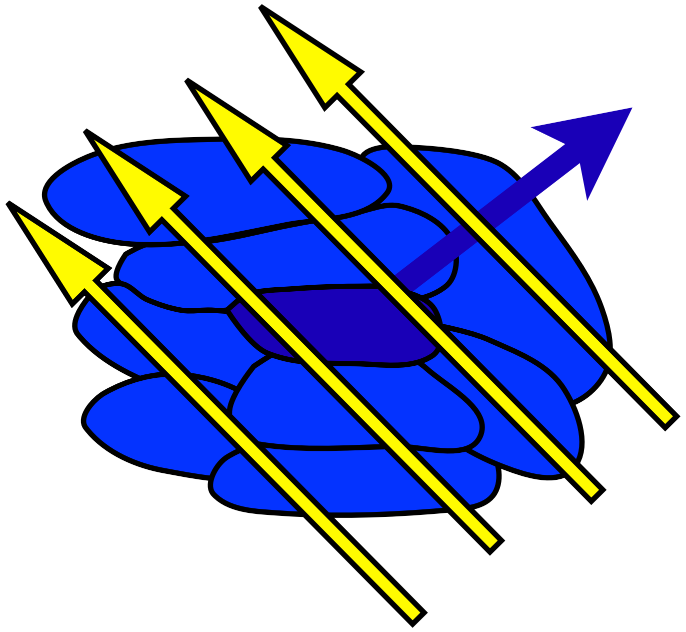
Hydrodynamics



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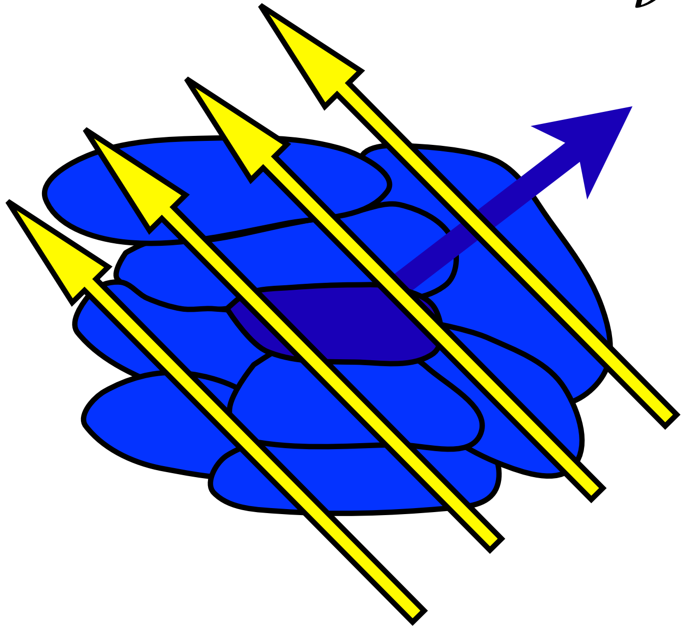
Scalars	$\partial_\mu u^\mu$
Vectors	$P^{\mu\nu}\partial_\nu T$ $\quad P^{\mu\nu}\partial_\nu \frac{\mu}{T}$
Tensors	$\langle\partial_\mu u_\nu\rangle$

Hydrodynamics

$$E^\mu = F^{\mu\nu} u_\nu$$



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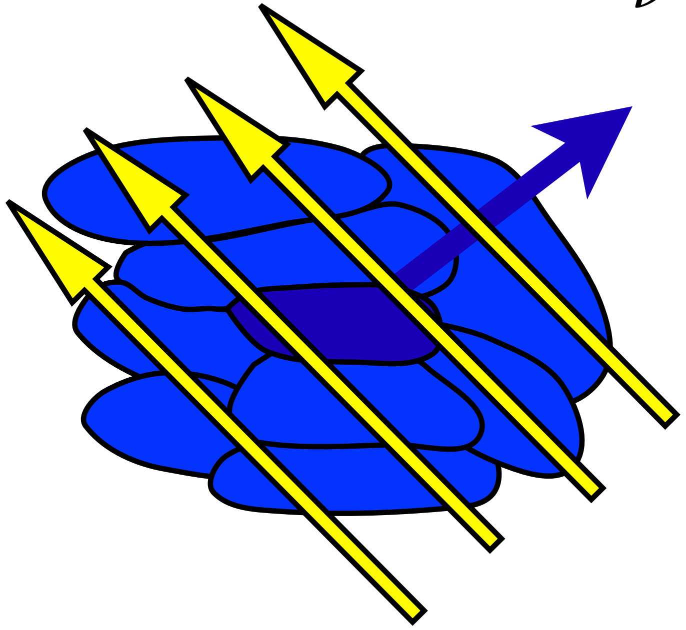
Scalars	$\partial_\mu u^\mu$
Vectors	$P^{\mu\nu}\partial_\nu T$ $\quad P^{\mu\nu}\partial_\nu \frac{\mu}{T}$
Tensors	$\langle \partial_\mu u_\nu \rangle$

Hydrodynamics

$E^{\mu} = F^{\mu\nu} u_{\nu}$



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$$J^{\mu} = \rho u^{\mu} - \kappa P^{\mu\nu} \partial_{\nu} \frac{\mu}{T}$$

$$\partial_{\mu} T^{\mu\nu} = 0$$

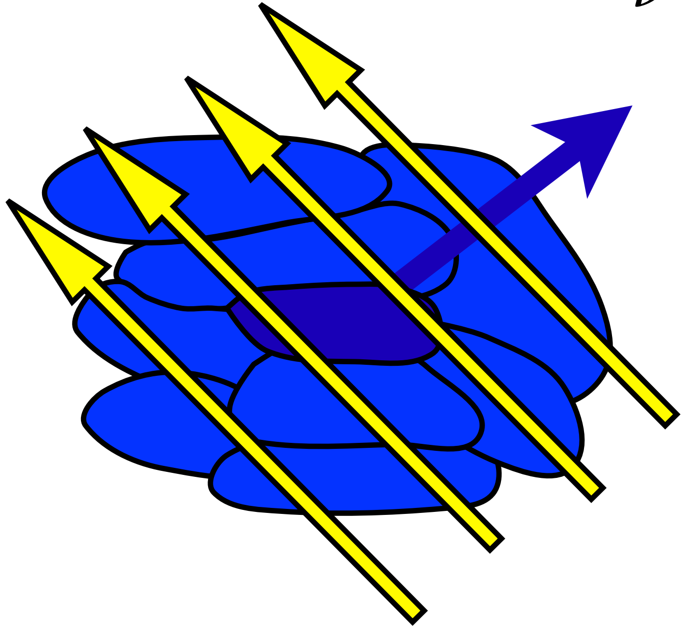
$$\partial_{\mu} J^{\mu} = 0$$

$$\partial_{\mu} J^{\mu}_{s} \geq 0$$

Scalars	$\partial_{\mu} u^{\mu}$
Vectors	$P^{\mu\nu} \partial_{\nu} T$ $\quad P^{\mu\nu} \partial_{\nu} \frac{\mu}{T}$ $\quad E^{\mu}$
Tensors	$\langle \partial_{\mu} u_{\nu} \rangle$

Hydrodynamics

$E^\mu = F^{\mu\nu} u_\nu$



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$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

$$J^\mu = \rho u^\mu - \kappa P^{\mu\nu}\partial_\nu \frac{\mu}{T} + \sigma E^\mu$$

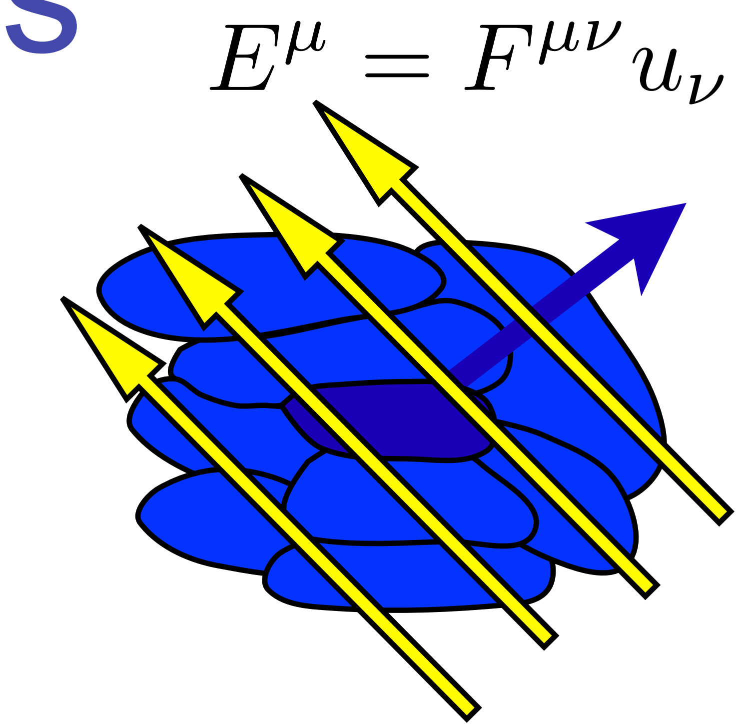
$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J_s^\mu \geq 0$$

Scalars	$\partial_\mu u^\mu$
Vectors	$P^{\mu\nu}\partial_\nu T$ $\quad P^{\mu\nu}\partial_\nu \frac{\mu}{T}$ $\quad E^\mu$
Tensors	$\langle\partial_\mu u_\nu\rangle$

Hydrodynamics



$$E^\mu = F^{\mu\nu} u_\nu$$

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$$J^\mu = \rho u^\mu - \kappa \left(P^{\mu\nu} \partial_\nu \frac{\mu}{T} - \frac{E^\mu}{T} \right)$$

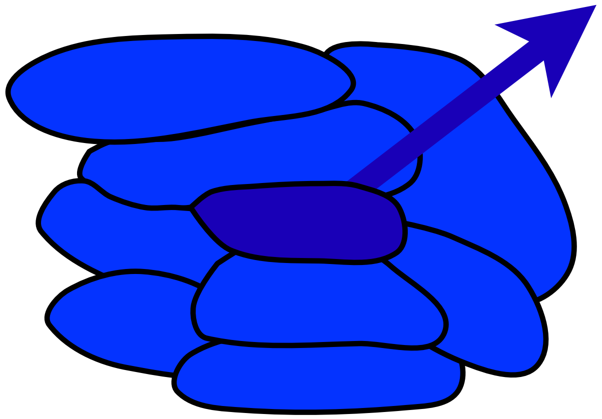
$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J^\mu_s \geq 0$$

Scalars	$\partial_\mu u^\mu$
Vectors	$P^{\mu\nu}\partial_\nu T$ $\quad P^{\mu\nu}\partial_\nu \frac{\mu}{T}$ $\quad E^\mu$
Tensors	$\langle \partial_\mu u_\nu \rangle$

$\not{P}$ Hydrodynamics



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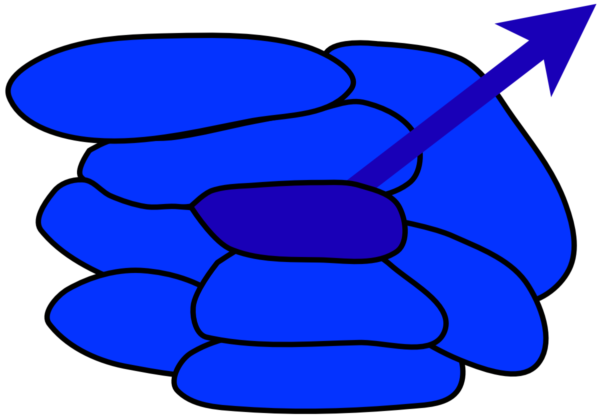
$\not{P}$ Hydrodynamics 2+1

Avron, Seller, Zograf (1995)



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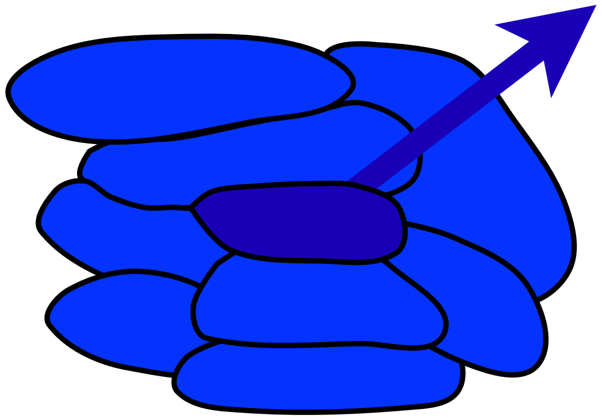

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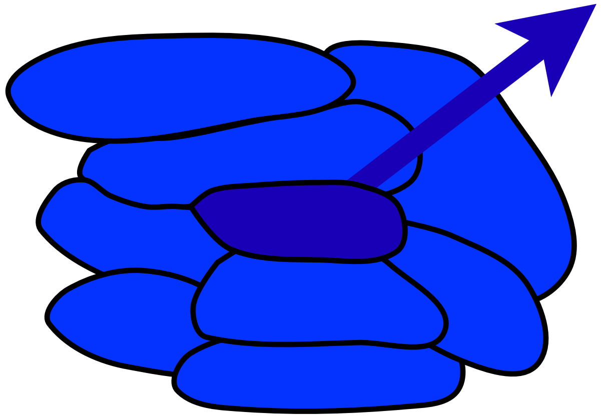
$\not{P}$ Hydrodynamics 2+1

Jensen, Kaminski, Kovtun, Meyer, Ritz, AY (2011)



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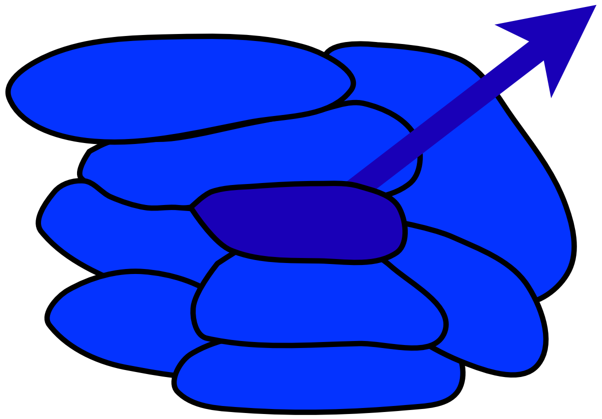

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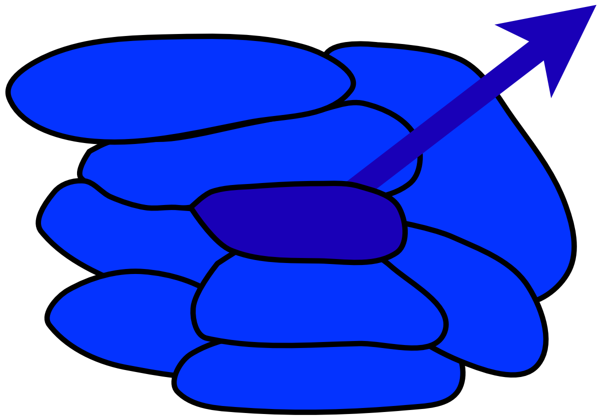

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$\not{P}$ Hydrodynamics

2+1

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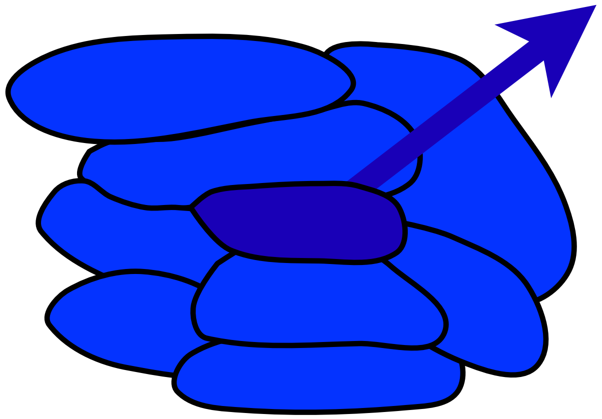

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$\not{P}$ Hydrodynamics 2+1

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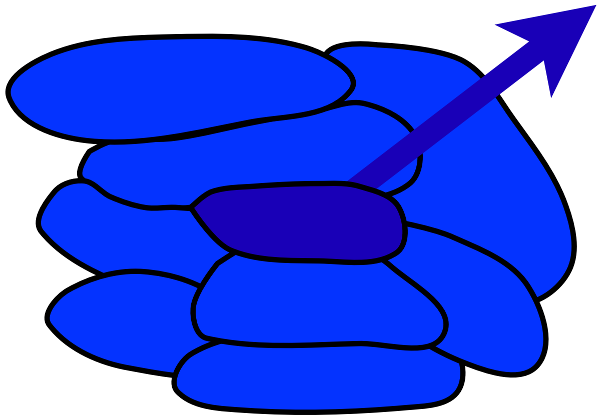

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$$T\tilde{\chi}_T = \left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu\frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B\right) - R_0\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu\frac{\partial \mathcal{M}_\Omega}{\partial \mu} + f_\Omega(T) - 2\mathcal{M}_\Omega\right),$$

$$\tilde{\chi}_E = \frac{\partial \mathcal{M}_B}{\partial \mu} - R_0\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right)$$

$$\tilde{\chi}_\Omega = \frac{\partial P_0}{\partial \epsilon_0}\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu\frac{\partial \mathcal{M}_\Omega}{\partial \mu} + f_\Omega(T) - 2\mathcal{M}_\Omega\right) + \frac{\partial P_0}{\partial \rho_0}\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right)$$

$$\tilde{\chi}_B = \frac{\partial P_0}{\partial \epsilon_0}\left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu\frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B\right) + \frac{\partial P_0}{\partial \rho_0}\frac{\partial \mathcal{M}_{\mathcal{B}}}{\partial \mu}$$

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3+1

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B^μ

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ω^μ B^μ

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$\not{P}$ Hydrodynamics 3+1

Son, Surowka (2009)

$$\tilde{\kappa}_\omega = C\mu^2\left(1 - \frac{2}{3}\frac{\mu\rho}{\epsilon + P}\right)$$

$$\tilde{\kappa}_B = C\mu^2\left(1 - \frac{1}{2}\frac{\mu\rho}{\epsilon + P}\right)$$

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$\not{P}$ Hydrodynamics 3+1

Erdmenger, Haack, Kaminski, AY (2008)
Banerjee, Bhattacharya, Bhattacharyya, Dutta, Loganyagam, Surowka, (2008)
Son, Surowka (2009)

$T(x^\mu)$ Temperature

$\mu(x^\mu)$ Chemical potential

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$

$$\tilde{\kappa}_\omega = C\mu^2 \left(1 - \frac{2}{3}\frac{\mu\rho}{\epsilon + P}\right)$$

$$\tilde{\kappa}_B = C\mu^2 \left(1 - \frac{1}{2}\frac{\mu\rho}{\epsilon + P}\right)$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P P^{\mu\nu} - 2\eta_{\langle}\partial_\mu u_{\nu\rangle} - \frac{\zeta}{3} P^{\mu\nu}\partial_\alpha u^\alpha$$

$$J^\mu = \rho u^\mu - \kappa\left(P^{\mu\nu}\partial_\nu \frac{\mu}{T} - \frac{E^\mu}{T}\right) + \tilde{\kappa}_\omega \omega^\mu + \tilde{\kappa}_B B^\mu$$

$$\partial_\mu T^{\mu\nu} = 0$$

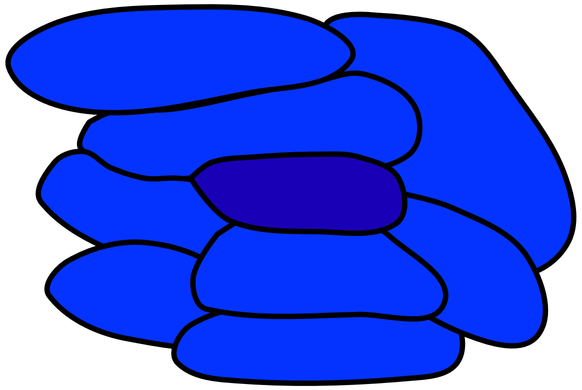
$$\partial_\mu J^\mu = C E^\mu B_\mu$$

$$\partial_\mu J^\mu_s \geq 0$$

ω^μ B^μ

Scalars	$\partial_\mu u^\mu$
Vectors	$P^{\mu\nu}\partial_\nu T$ $P^{\mu\nu}\partial_\nu \frac{\mu}{T}$ E^μ $\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$ $\epsilon^{\mu\nu\rho\sigma} F_{\nu\rho} u_\sigma$
Tensors	$\langle\partial_\mu u_\nu\rangle$

Superfluid hydrodynamics

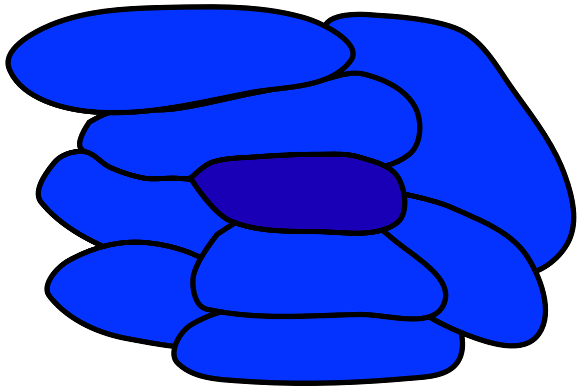


$\epsilon(x^\mu)$ Energy density

$\rho(x^\mu)$ Charge density

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$

Superfluid hydrodynamics



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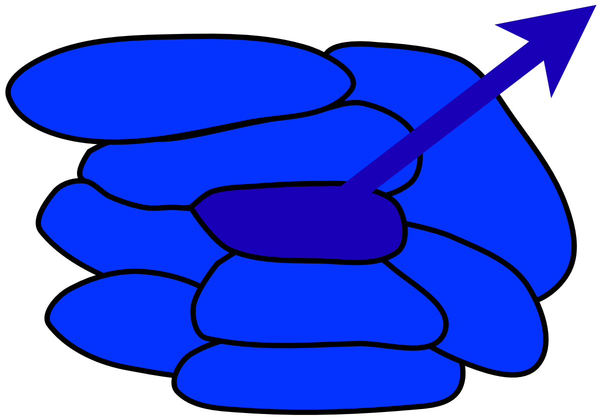
Superfluid hydrodynamics



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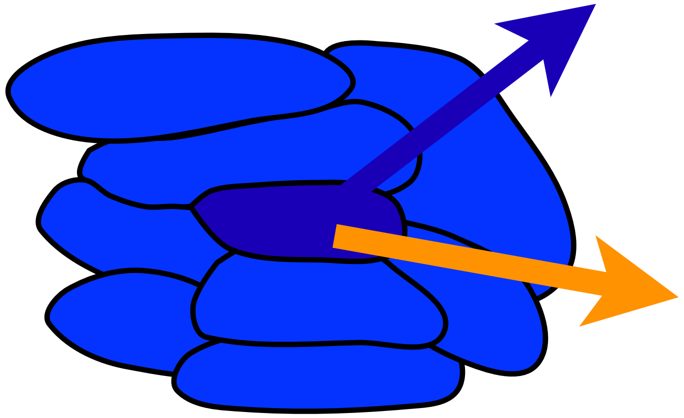

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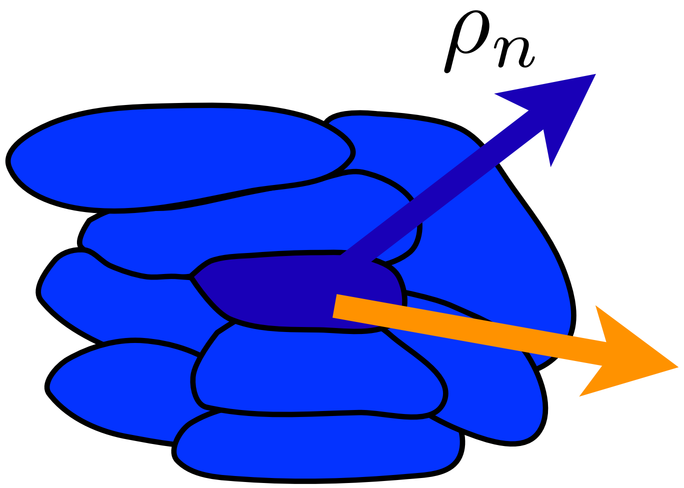

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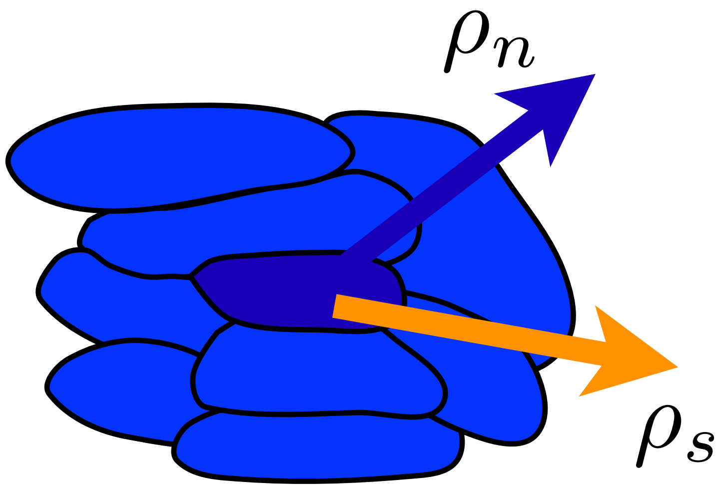

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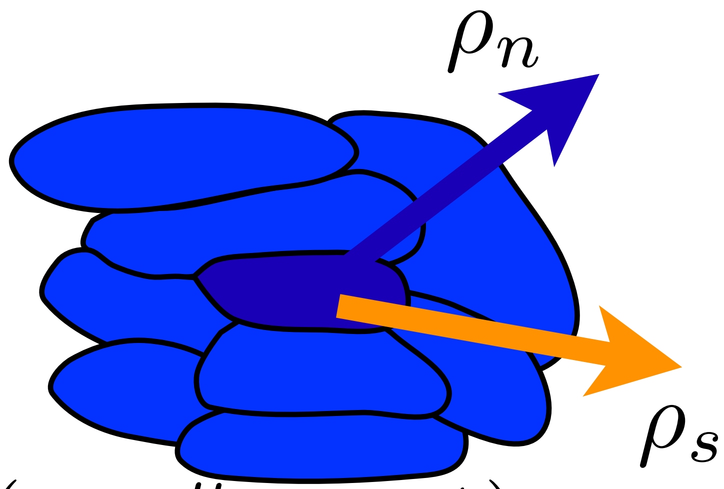
Superfluid hydrodynamics



$\epsilon(x^\mu)$	Energy density
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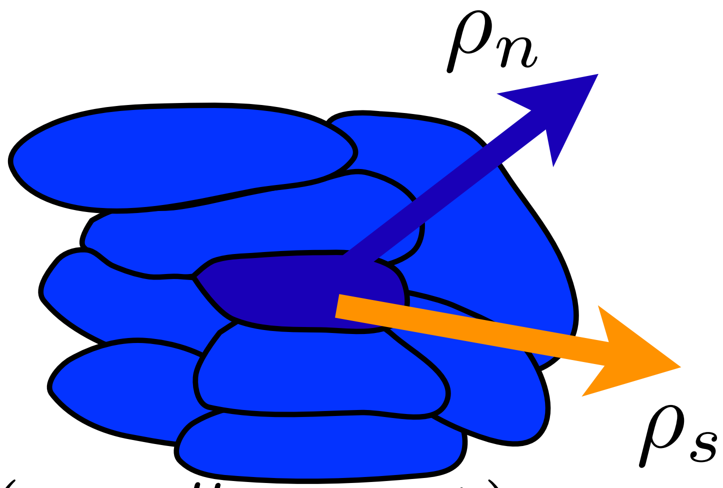
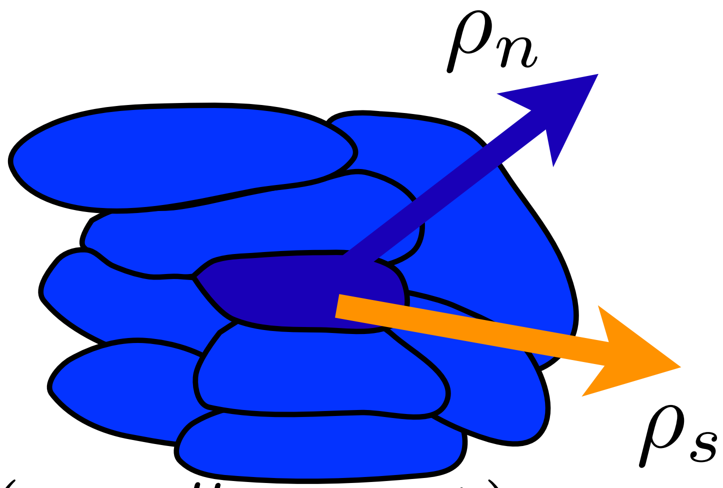
Superfluid hydrodynamics



$\epsilon(x^\mu)$	Energy density
$\rho(x^\mu)$	Charge density
$u^\nu(x^\mu)$	Velocity of uncondensed phase $(u_\mu u^\mu = -1)$
$\partial^\nu \phi(x^\mu)$	Velocity of condensate

Superfluid hydrodynamics



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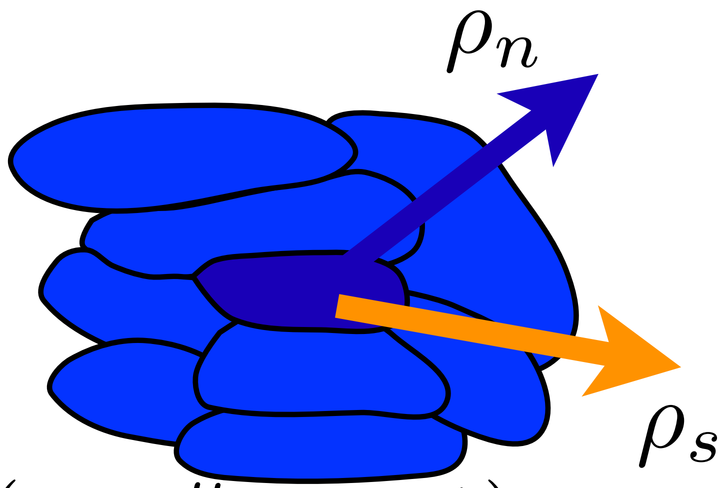
Superfluid hydrodynamics



$\epsilon(x^\mu)$	Energy density
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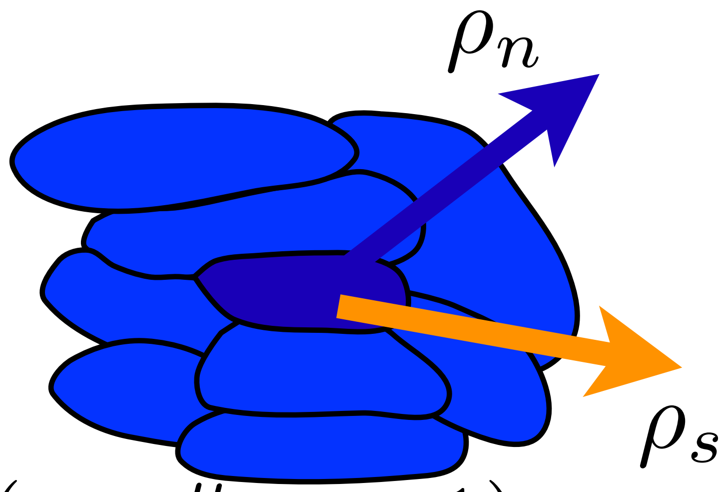

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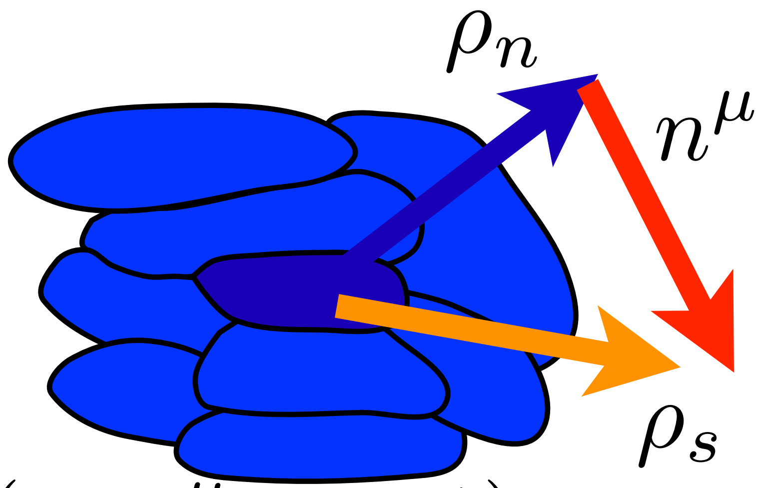

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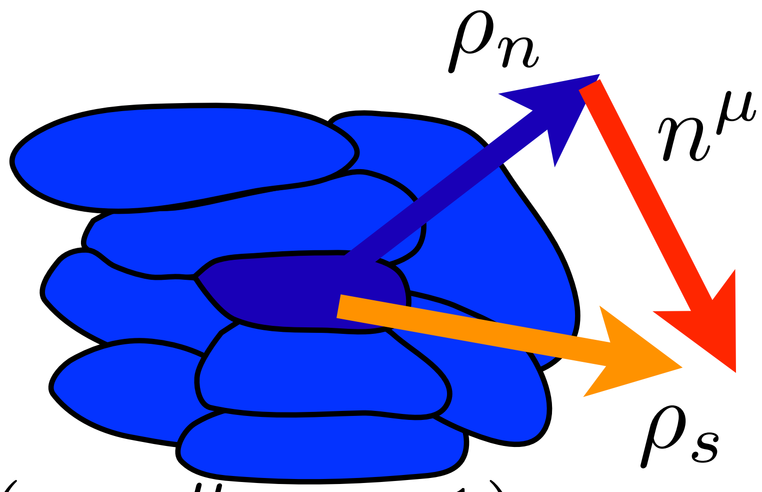
$$n^\mu = P^{\mu\nu} \partial_\nu \phi$$

Superfluid hydrodynamics



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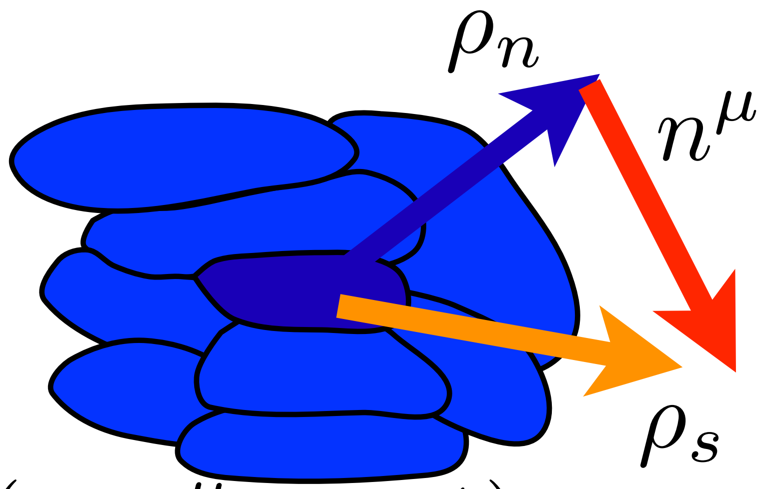
$$\mu_T = u^\nu \partial_\nu \phi$$

Superfluid hydrodynamics



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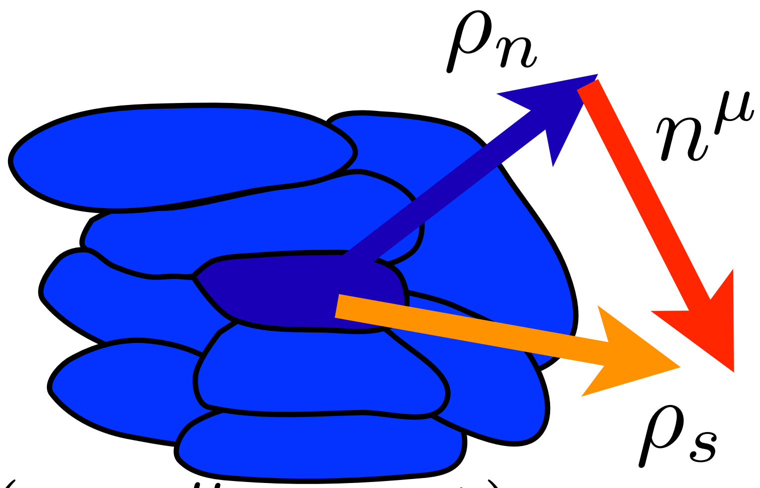
$$T^{\mu\nu} = T_1 u^\mu u^\nu + T_2 \eta^{\mu\nu} + 2T_3 u^{(\mu} n^{\nu)} + T_4 n^\mu n^\nu + \mathcal{O}(\partial)$$

$$J^\mu = T_5 u^\mu + T_6 n^\mu + \mathcal{O}(\partial)$$

Superfluid hydrodynamics



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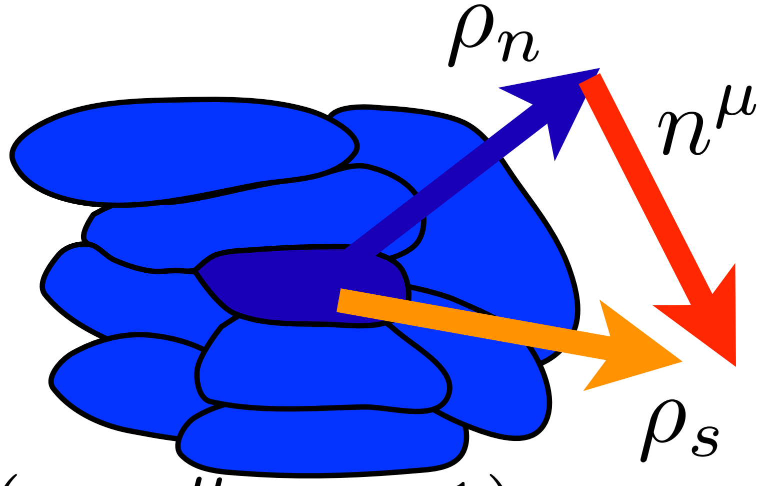
Superfluid hydrodynamics



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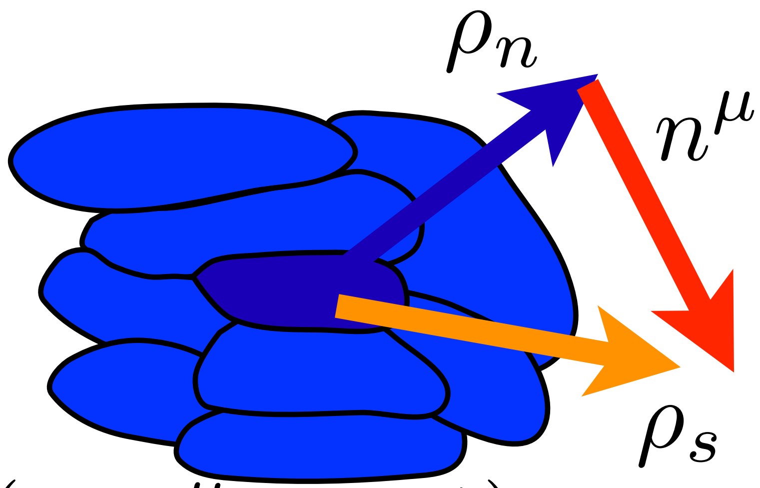
Superfluid hydrodynamics



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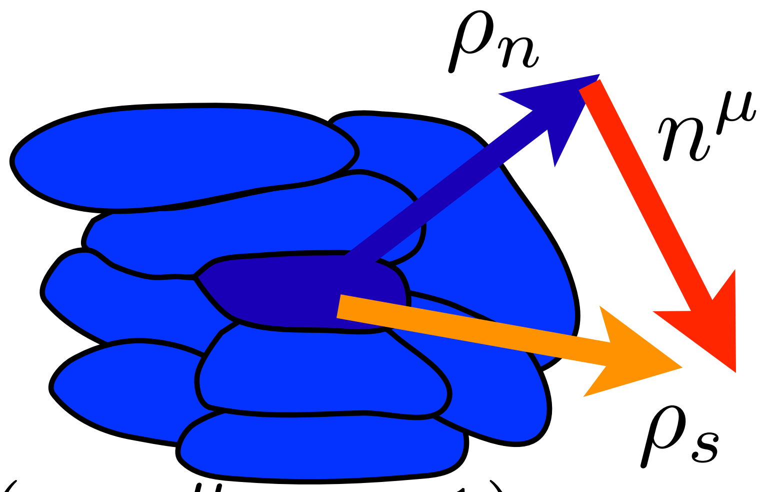

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$\mathcal{O}(\partial)$ corrections:

Superfluid hydrodynamics

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$\mathcal{O}(\partial)$ corrections:

Scalars	7
Pseudo scalars	2
Vectors	7
Pseudo vectors	7
Tensors	2
Pseudo tensors	2

Superfluid hydrodynamics

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$\mathcal{O}(\partial)$ corrections:

Scalar sector: 10 parameters which sit in a 4 X 4 positive semi-definite symmetric matrix

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Pseudo scalars	2
Vectors	7
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Superfluid hydrodynamics

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$\mathcal{O}(\partial)$ **corrections:**

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Pseudo scalar sector: 2 parameters

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Superfluid hydrodynamics

$$\mu + \mathcal{O}(\partial) = u^\nu \partial_\nu \phi$$

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Vector + pseudo vector sector: 6 parameters (+2 from pseudoscalar sector)

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Scalar sector: 10 parameters which sit in a 4 X 4 positive semi-definite symmetric matrix

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Vector + pseudo vector sector: 6 parameters (+2 from pseudoscalar sector)

Tensor sector: 1 parameter

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Pseudo-tensor sector: 1 parameter

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20 parameters:

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Pseudo-tensor sector: 1 parameter

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20 parameters: 14 parity even + 6 parity odd

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Holographic hydrodynamics

$$S = \int \sqrt{-g} \left(R + \frac{12}{L^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) d^5 x$$

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$g_{\mu\nu}$

A_μ

Holographic hydrodynamics

$$S = \int \sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) d^5x$$

$$g_{\mu\nu} \longrightarrow \langle T_{\mu\nu} \rangle$$

$$A_\mu$$

Holographic hydrodynamics

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$$A_\mu \Longrightarrow \langle J_\mu \rangle$$

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$$g_{\mu\nu} \Rightarrow \langle T_{\mu\nu} \rangle$$

$$A_\mu \Rightarrow \langle J_\mu \rangle$$

$$ds^2 = -r^2 f(r) dt^2 + r^2 dx^2 + \sigma(r) dt dr$$

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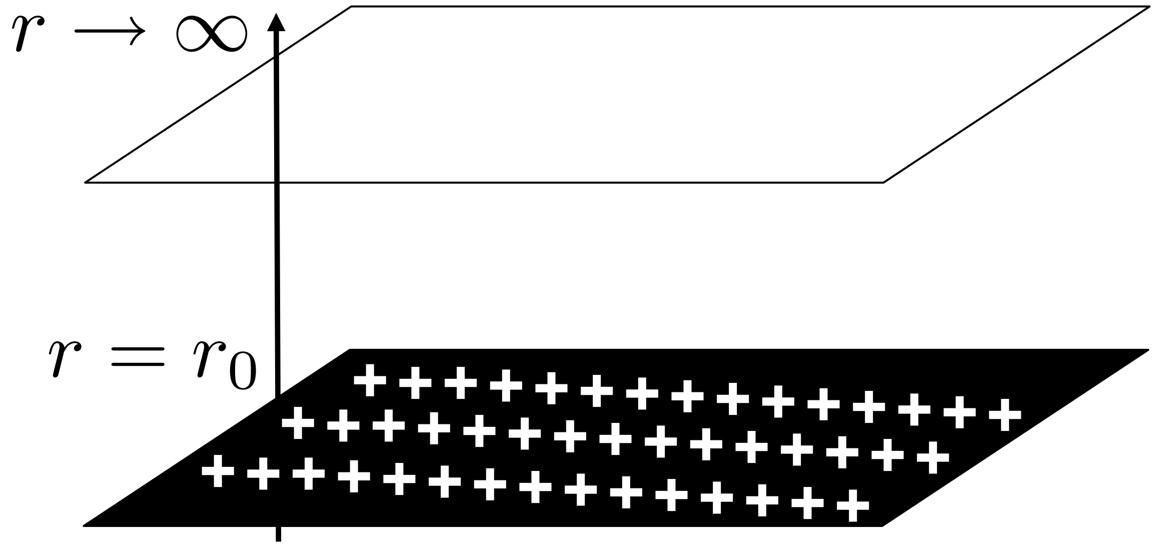

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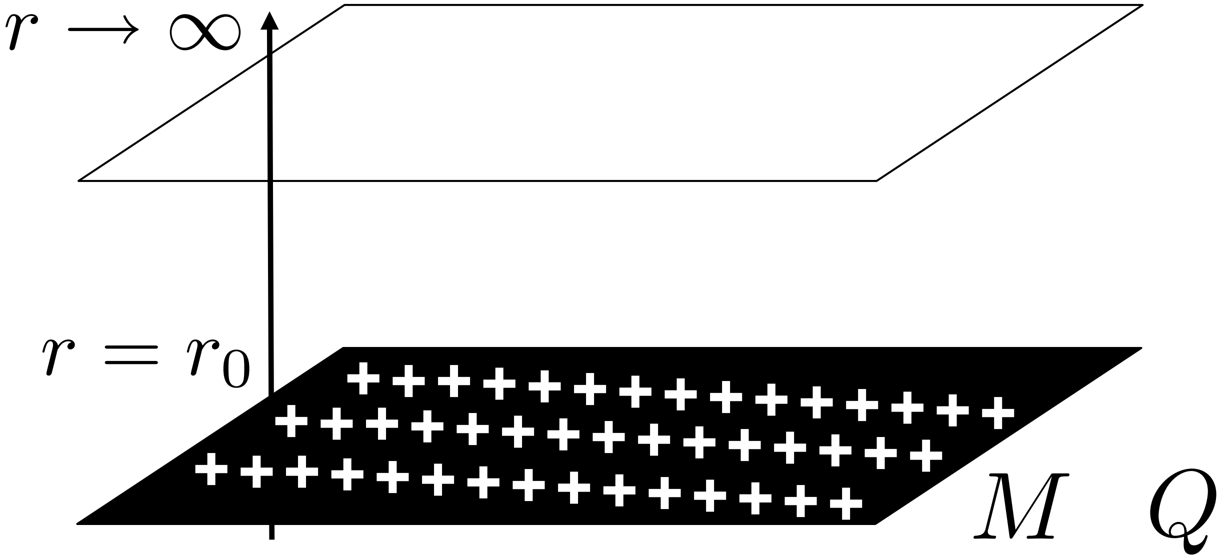

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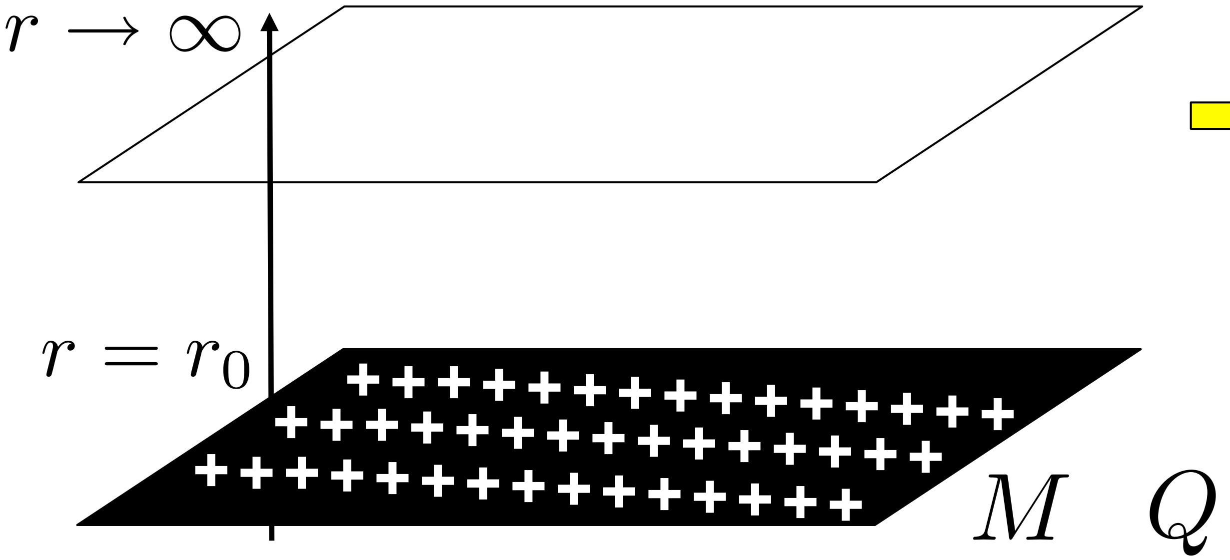


$$\Rightarrow \quad \langle T^{\mu\nu} \rangle = \begin{pmatrix} 3P & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

$$J^\mu = \begin{pmatrix} \rho & 0 & 0 & 0 \end{pmatrix}$$

Holographic hydrodynamics

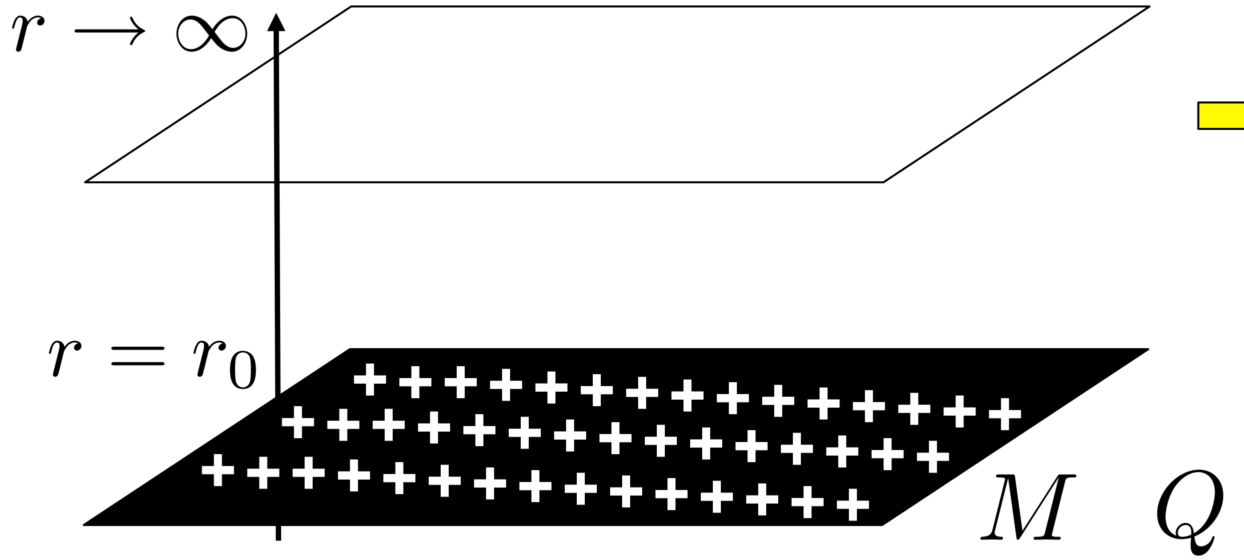
$$S = \int \sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) d^5x$$

$$g_{\mu\nu} \Longrightarrow \langle T_{\mu\nu}\rangle$$

$$A_\mu \Longrightarrow \langle J_\mu\rangle$$

$$ds^2 = -r^2 f(r)dt^2 + r^2 dx^2 + \sigma(r)dtdr$$

$$A = A_0 dt + A_5 dr$$



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μ $\quad$ T

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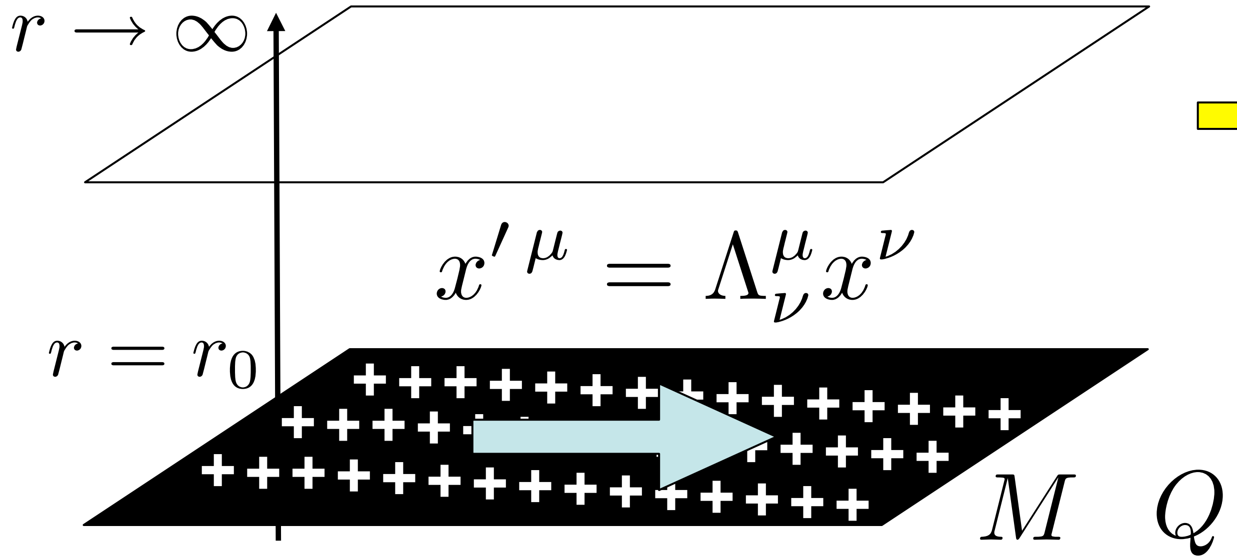

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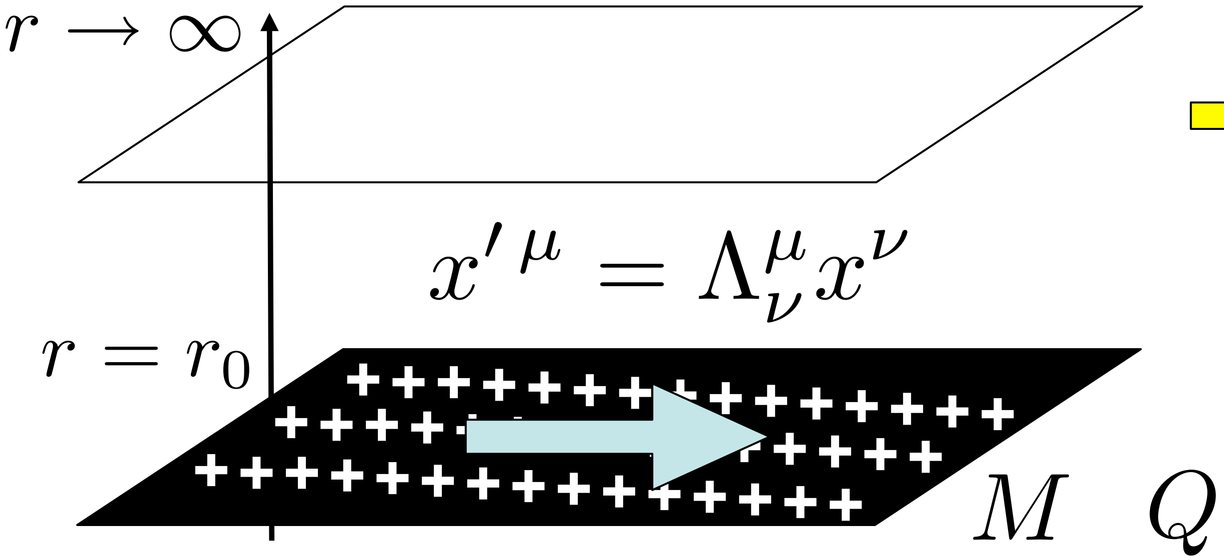

$$x'^{\,\mu} = \Lambda^\mu_\nu x^\nu$$

$$\Rightarrow$$

$$\langle T^{\mu\nu} \rangle = \begin{pmatrix} 3P & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

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Holographic hydrodynamics

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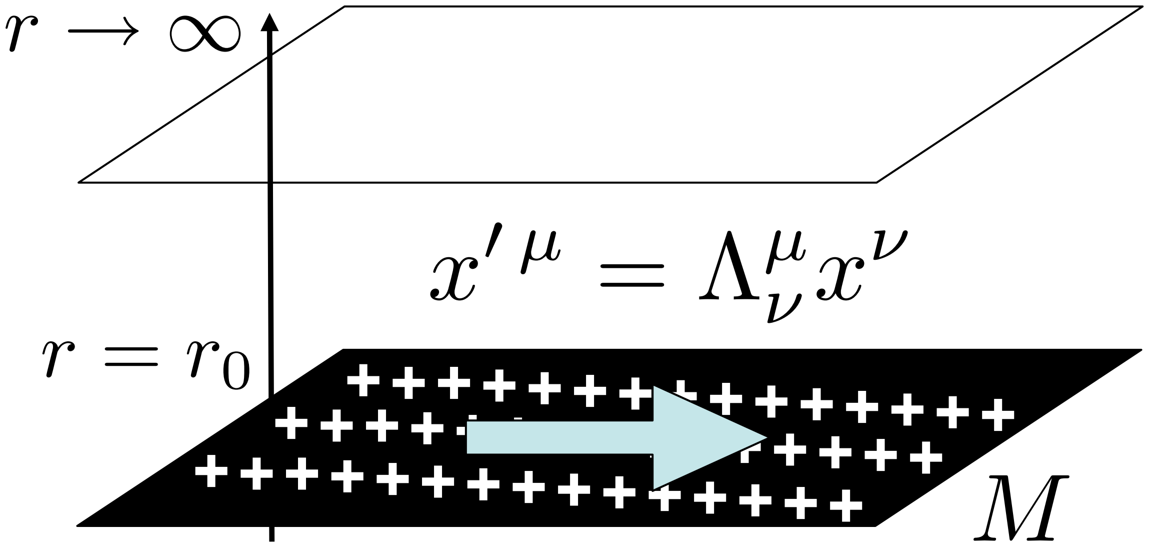

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$M \quad Q \quad \beta^i \qquad \mu \qquad T$

Holographic hydrodynamics

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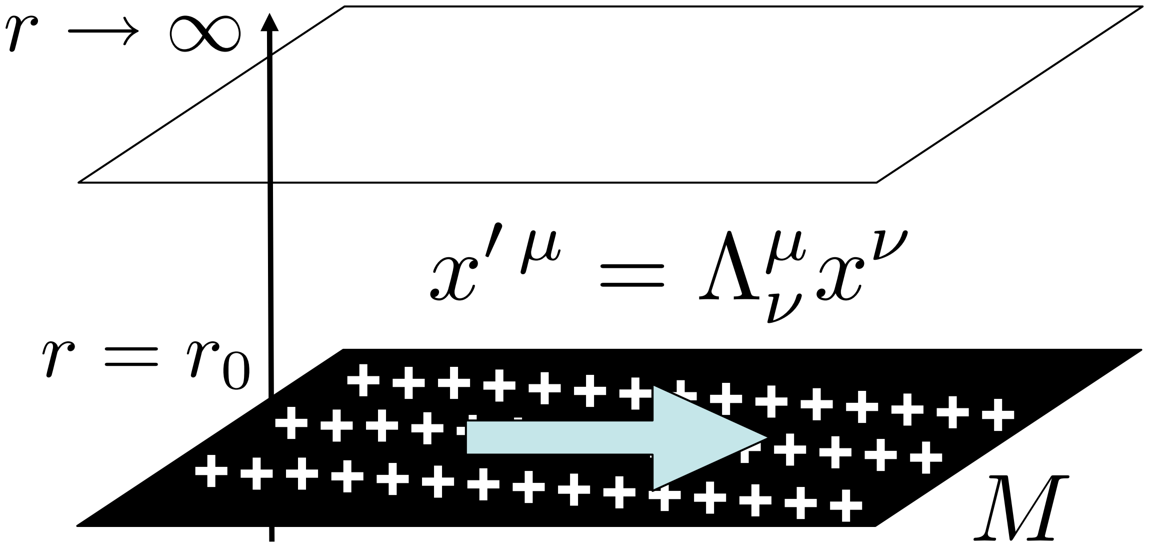

$$\Rightarrow \quad \langle T^{\mu\nu}\rangle = (\epsilon + P)u^\mu u^\nu + P\eta^{\mu\nu}$$

$$\langle J^\mu\rangle = \rho u^\mu$$

$M \quad Q \quad \beta^i \qquad \mu \qquad T$

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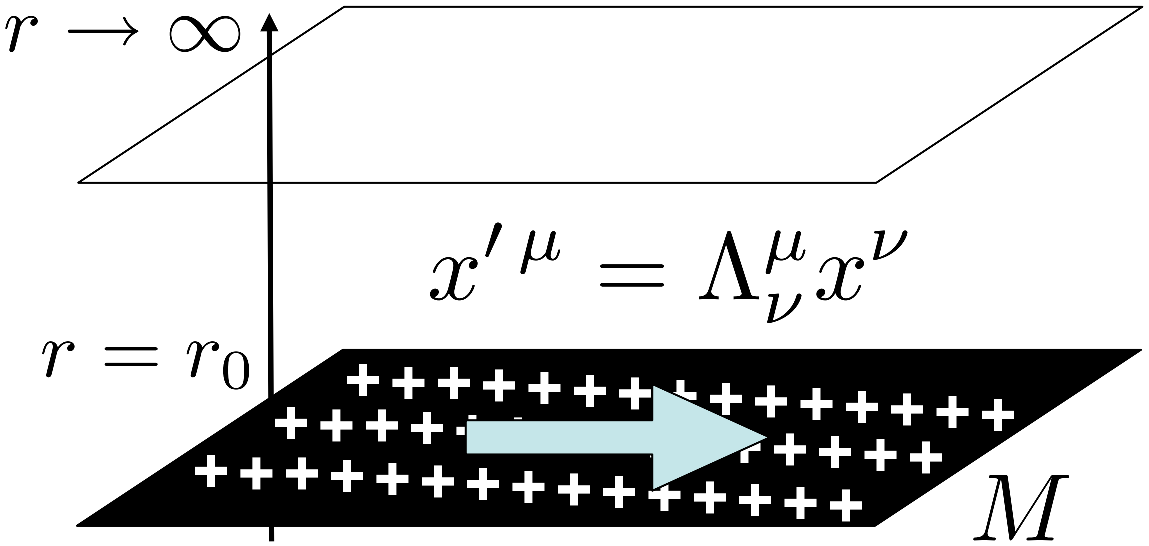

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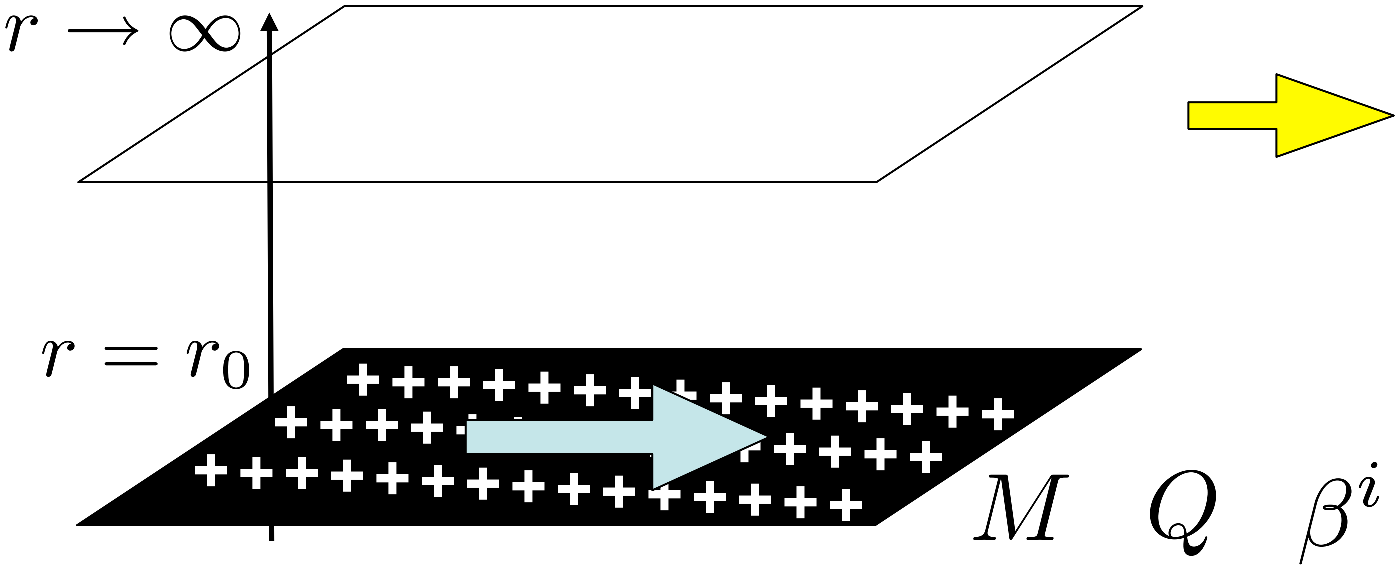

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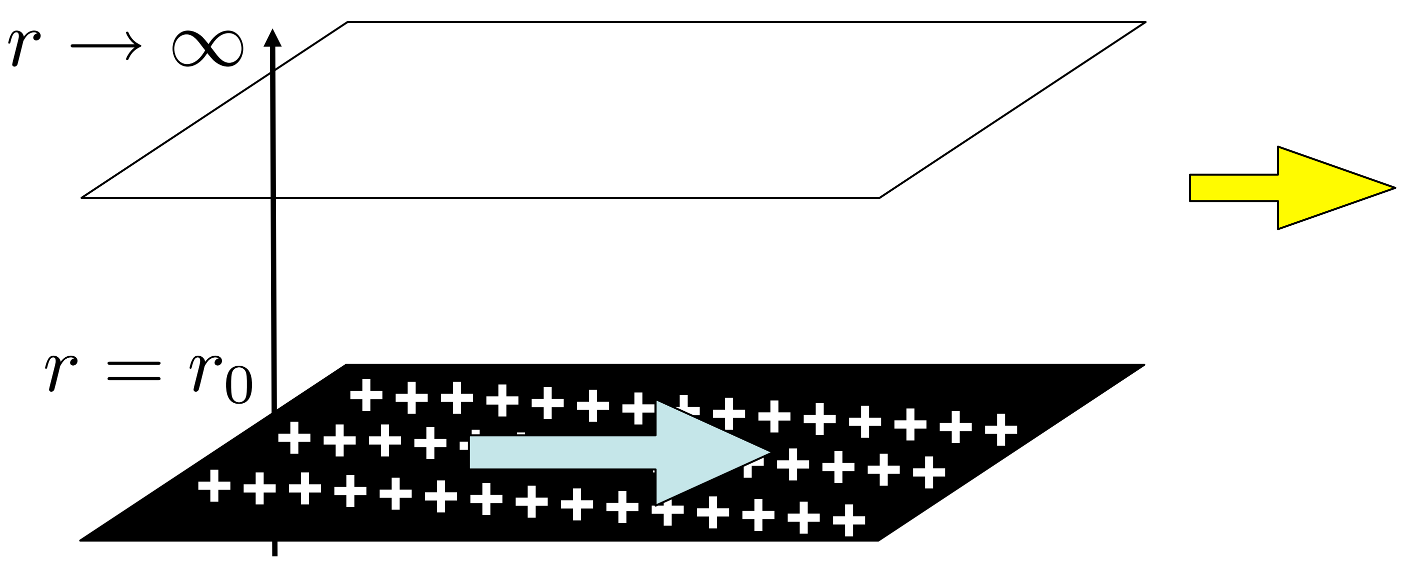

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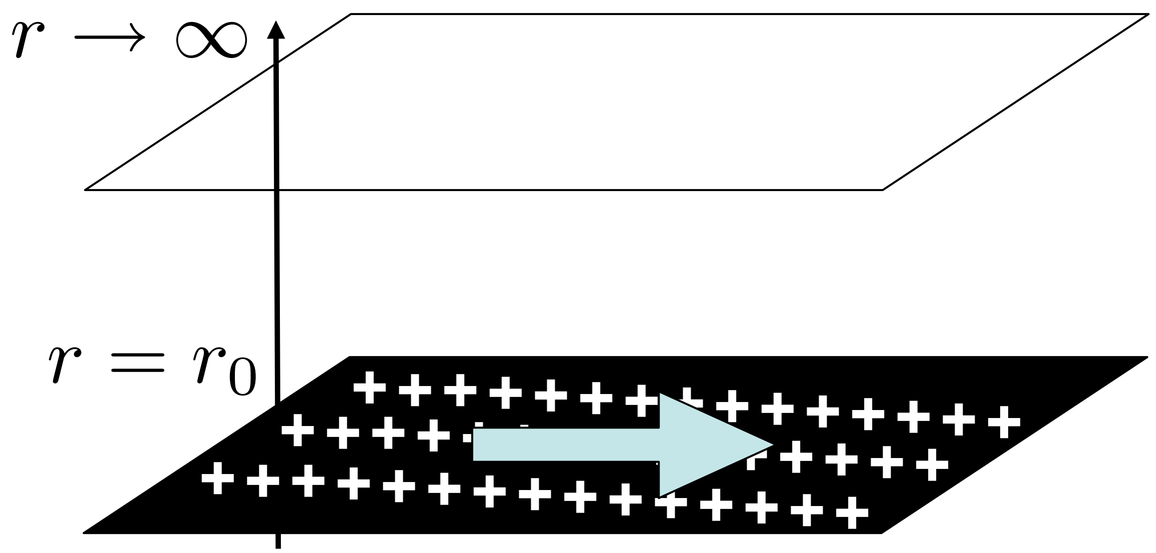

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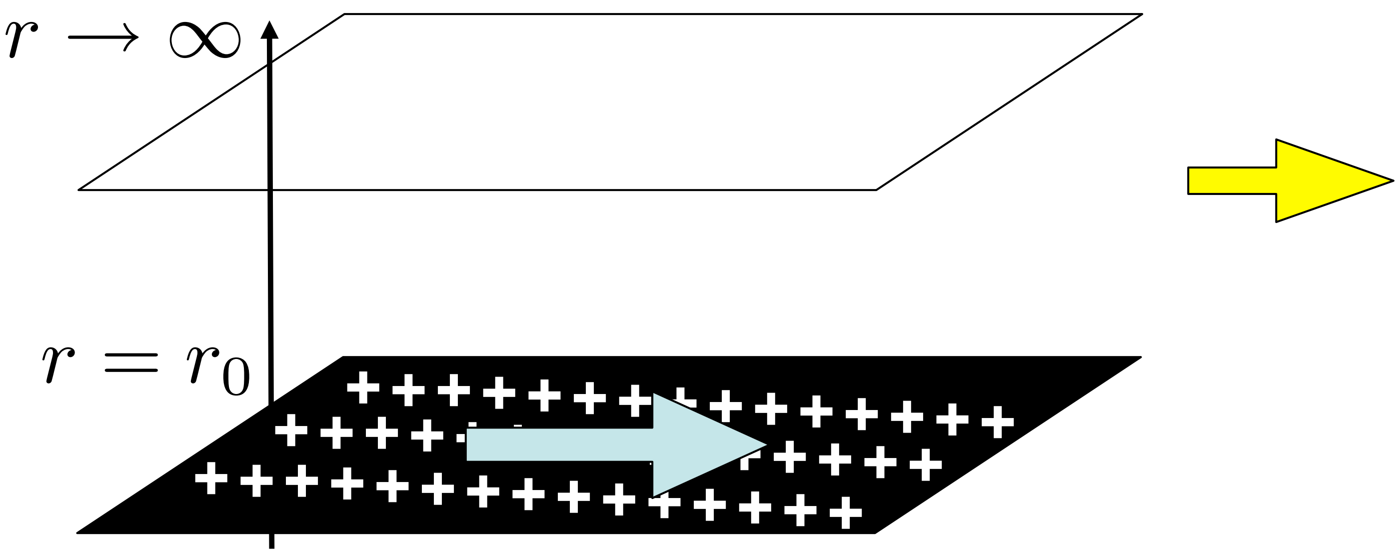

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$M(x)$ $Q(x)$ $\beta^i(x)$ μ T u^μ

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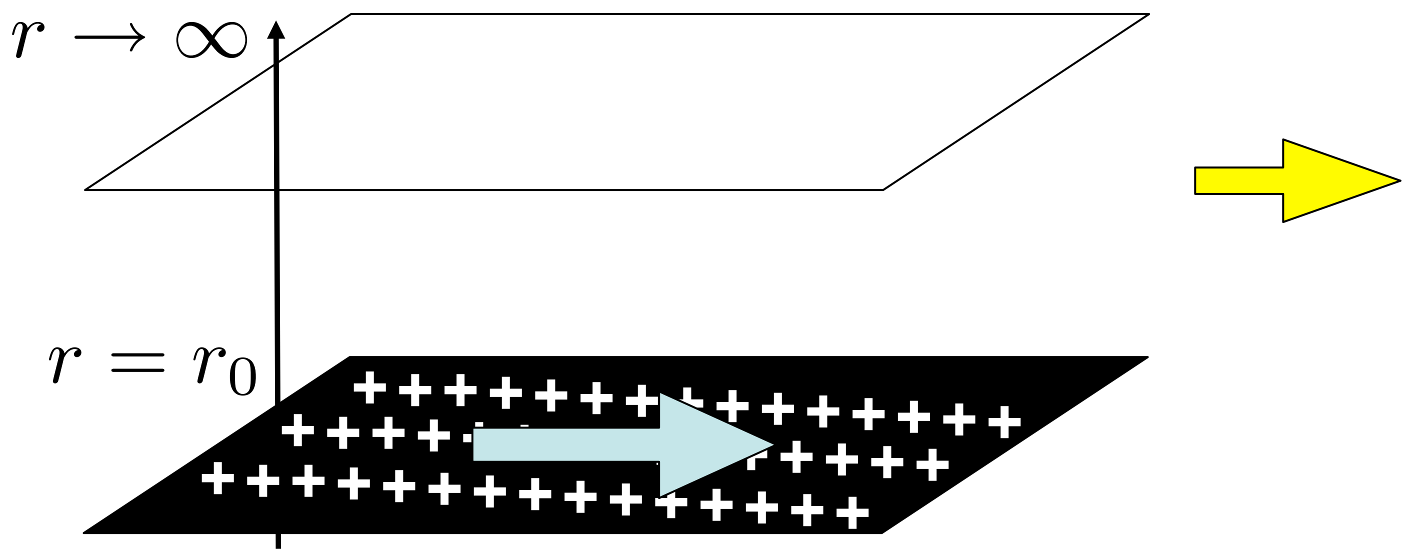

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$M(x) \quad Q(x) \quad \beta^i(x) \qquad \mu \quad T \quad u^\mu$

1.Promote the thermodynamic fields to be spacetime dependent.

Holographic hydrodynamics

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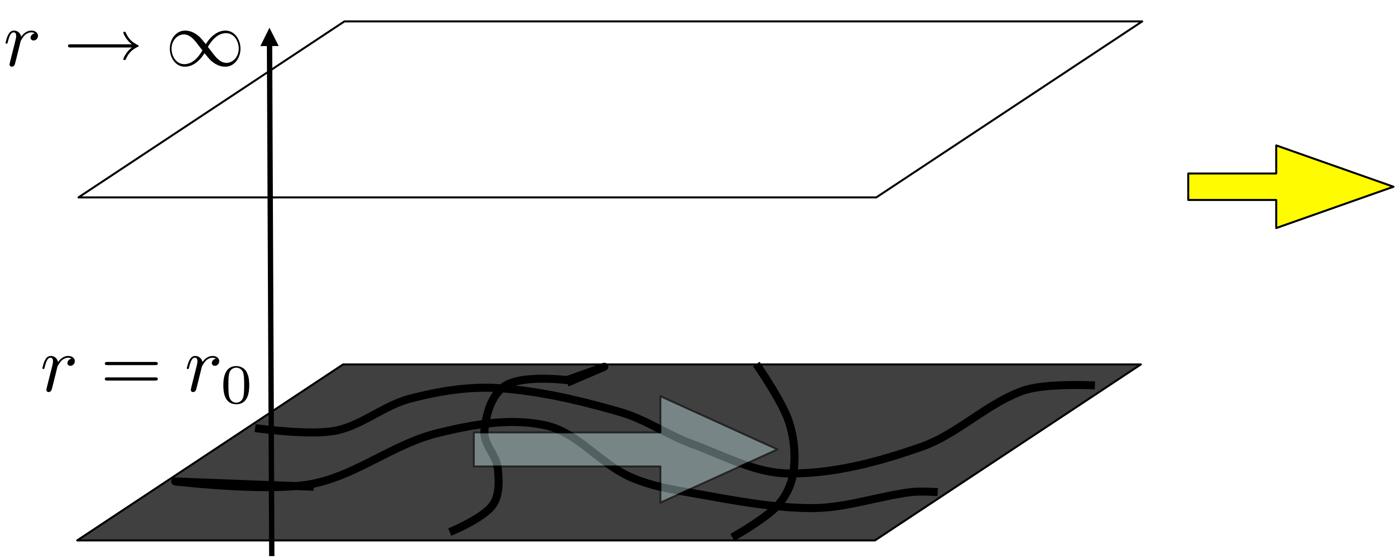

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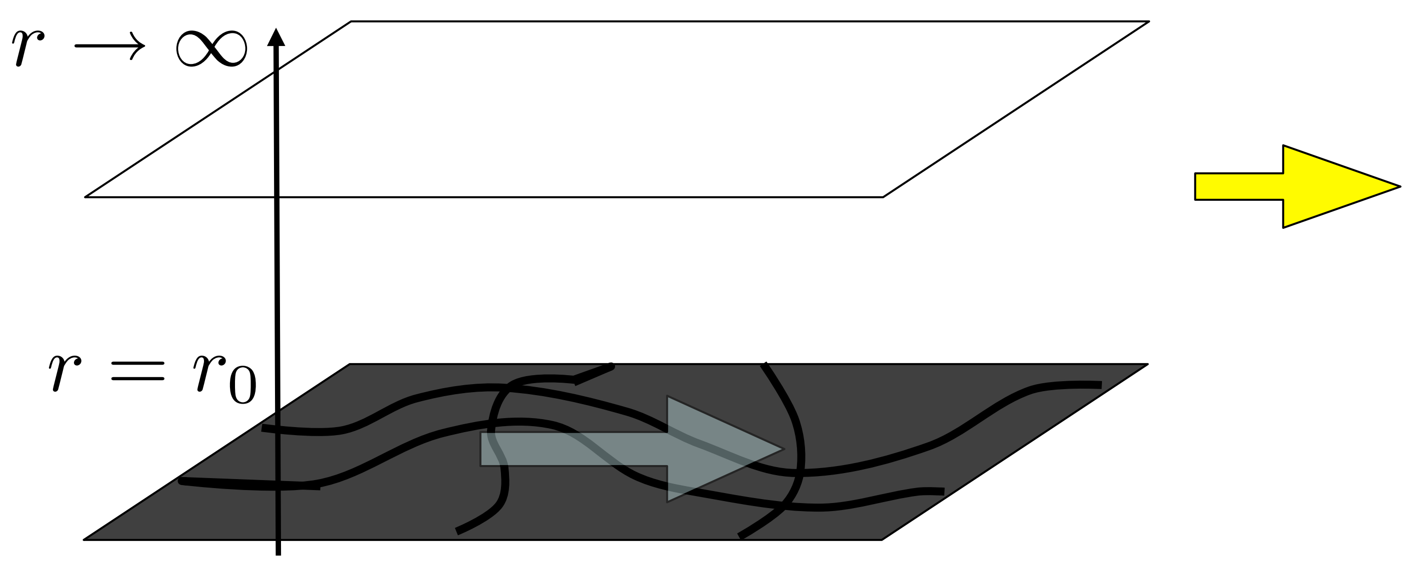

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$M(x) \quad Q(x) \quad \beta^i(x) \qquad \mu \quad T \quad u^\mu$

1.Promote the thermodynamic fields to be spacetime dependent.

2. Allow for corrections to the metric and gauge field.

Holographic hydrodynamics

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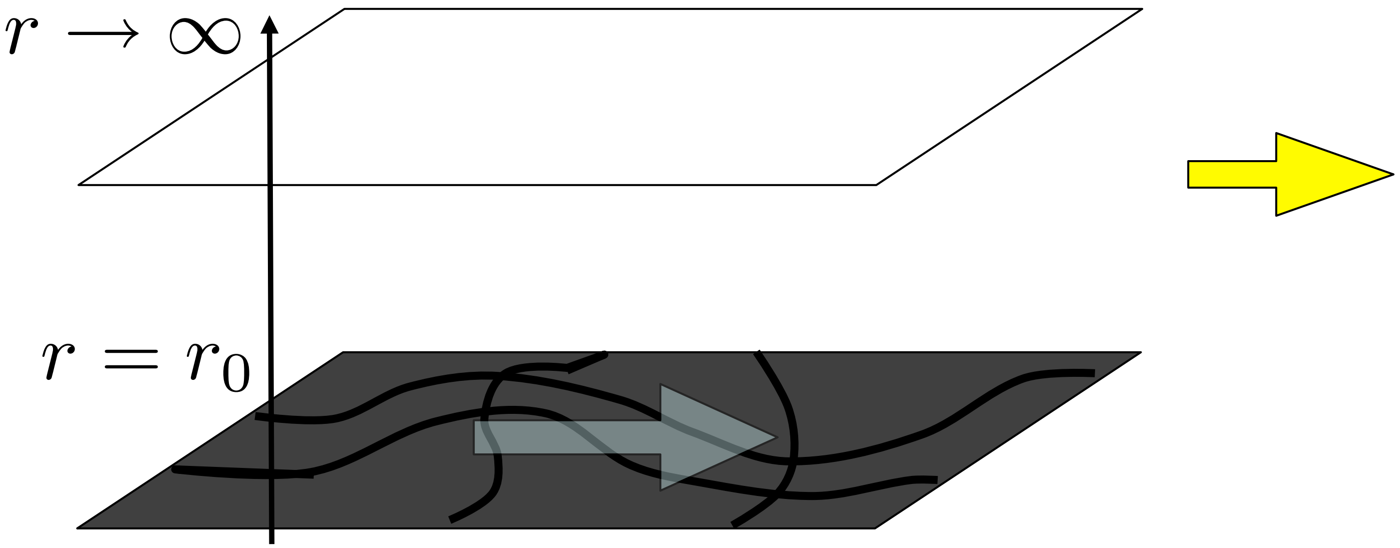

$$\langle T^{\mu\nu} \rangle = (\epsilon + P) u^\mu u^\nu + P \eta^{\mu\nu}$$
$$- \eta \sigma^{\mu\nu}$$

$$\langle J^\mu \rangle = \rho u^\mu - \kappa P^{\mu\nu} \partial_\nu \frac{\mu}{T}$$

$$M(x) \quad Q(x) \quad \beta^i(x) \qquad \mu \quad T \quad u^\mu$$

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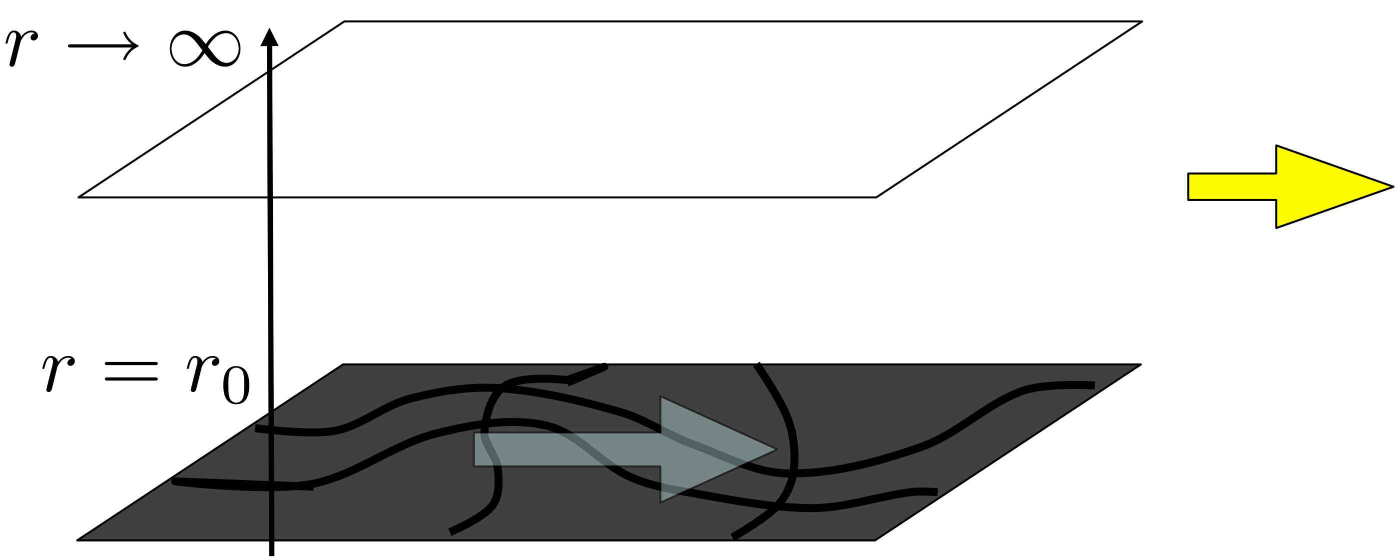

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Holographic hydrodynamics

Bhattacharyya, Hubeny, Minwalla, Rangamani (2007)

$$S = \int \sqrt{-g} \left(R + \frac{12}{L^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) d^5 x$$

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Erdmenger, Haack, Kaminski, AY (2008)

Banerjee, Bhattacharya, Bhattacharyya, Dutta, Loganyagam, Surowka, (2008)

$$S = \int \sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) - C\epsilon^{\mu\nu\rho\sigma\tau}F_{\mu\nu}F_{\rho\sigma}A_\tau d^5x$$

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Jensen, Kaminski, Kovtun, Meyer, Ritz, AY (2011)

$$S = \int \sqrt{-g}\left(R + \frac{12}{L^2} - \partial_\mu\phi\partial^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) - \theta(\phi)\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}d^4x$$

Holographic hydrodynamics

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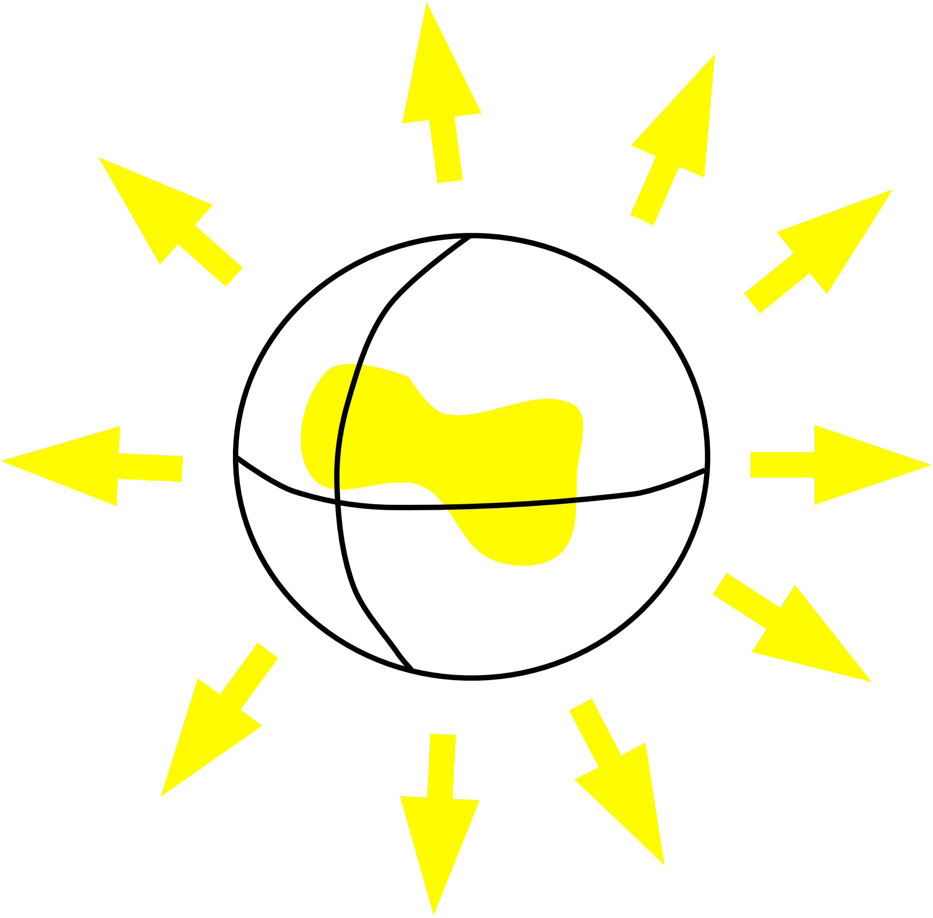
Bhattacharyya, Bhattacharya, Minwalla, AY (2011)

Bhattacharyya, Bhattacharya, Minwalla, (2011)

Herzog, Lisker, Surowka, AY, (2011)

$$S = \int \sqrt{-g}\left(R + \frac{12}{L^2} - V_\phi(\phi)|D_\mu\phi - iqA_\mu\phi|^2 - V(\phi) - V_F(\phi)F_{\mu\nu}F^{\mu\nu}\right) - C\epsilon^{\mu\nu\rho\sigma\tau}A_\mu F_{\nu\rho}F_{\sigma\tau}d^5x$$

An entropy current



$$\frac{dS}{dt} + \int \vec{S} \cdot d\vec{a} \quad \geq 0$$

Local version of the 2nd law:

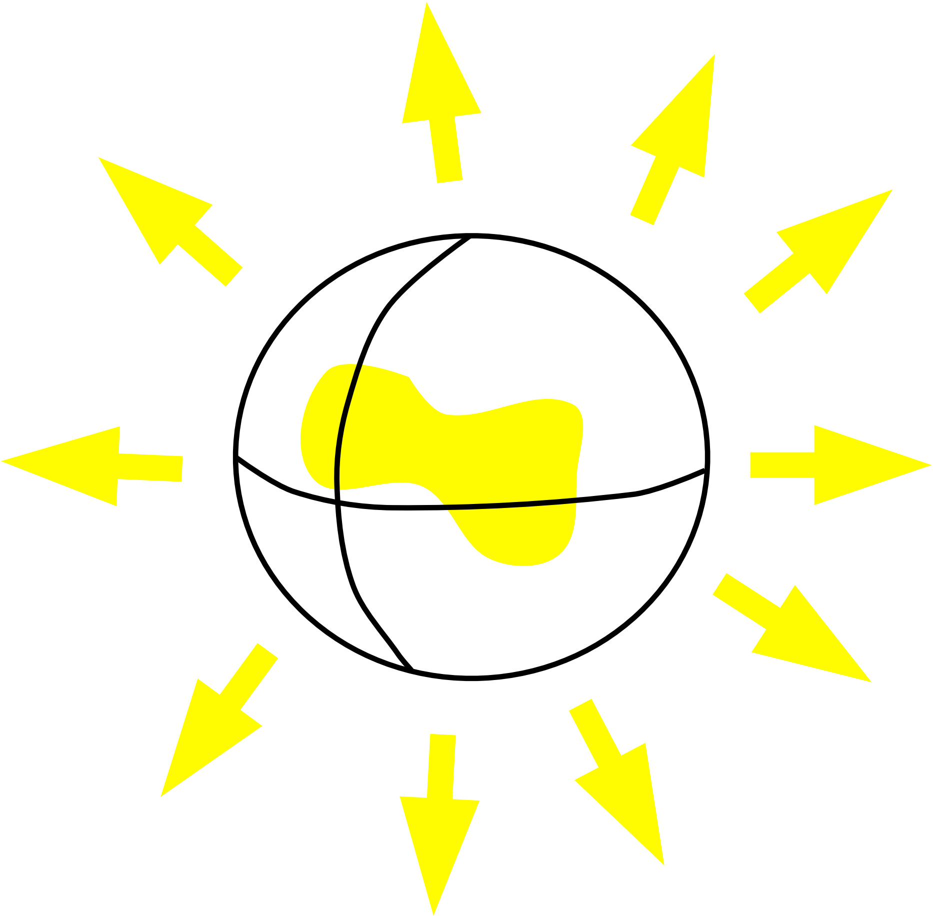
1) $\partial_\mu J_s^\mu \geq 0$

2) In equilibrium: $u^\mu = (1,0,0,0)$

$$J_s^\mu = (s,0,0,0)$$

$\partial_\mu J_s^\mu =$ product of two one-derivative terms **+** genuine two-derivative terms ≥ 0

An entropy current



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$$J_s^\mu = (s,0,0,0)$$

1) Locality?
2) Properties?

$\partial_\mu J_s^\mu =$ product of two one-derivative terms **+** genuine two-derivative terms ≥ 0

An alternative to the entropy current

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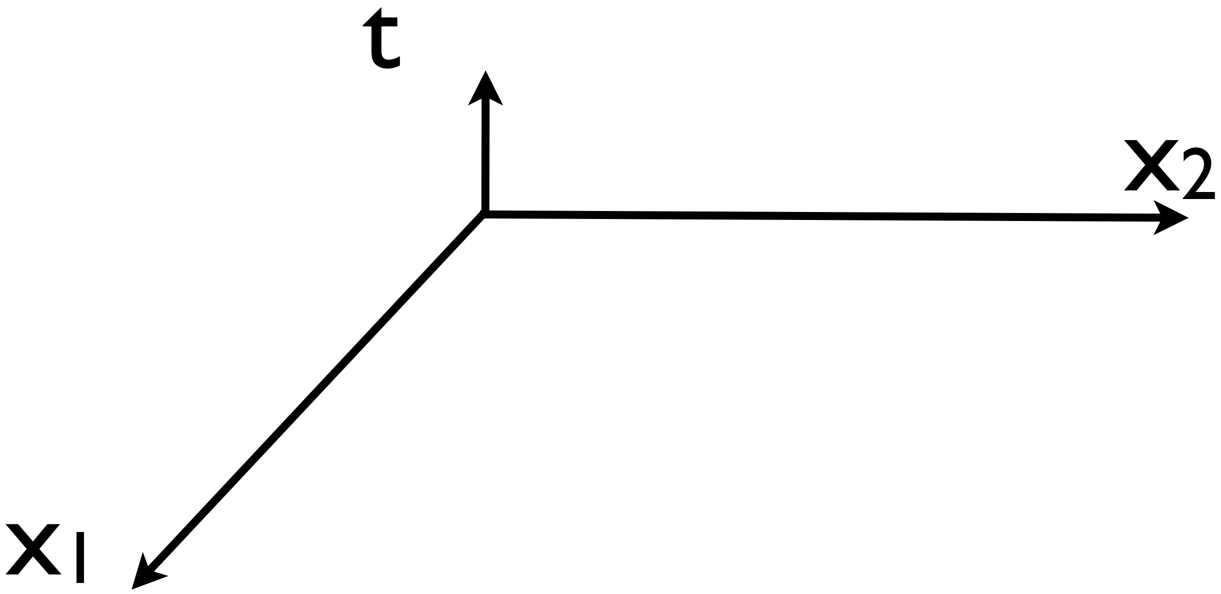
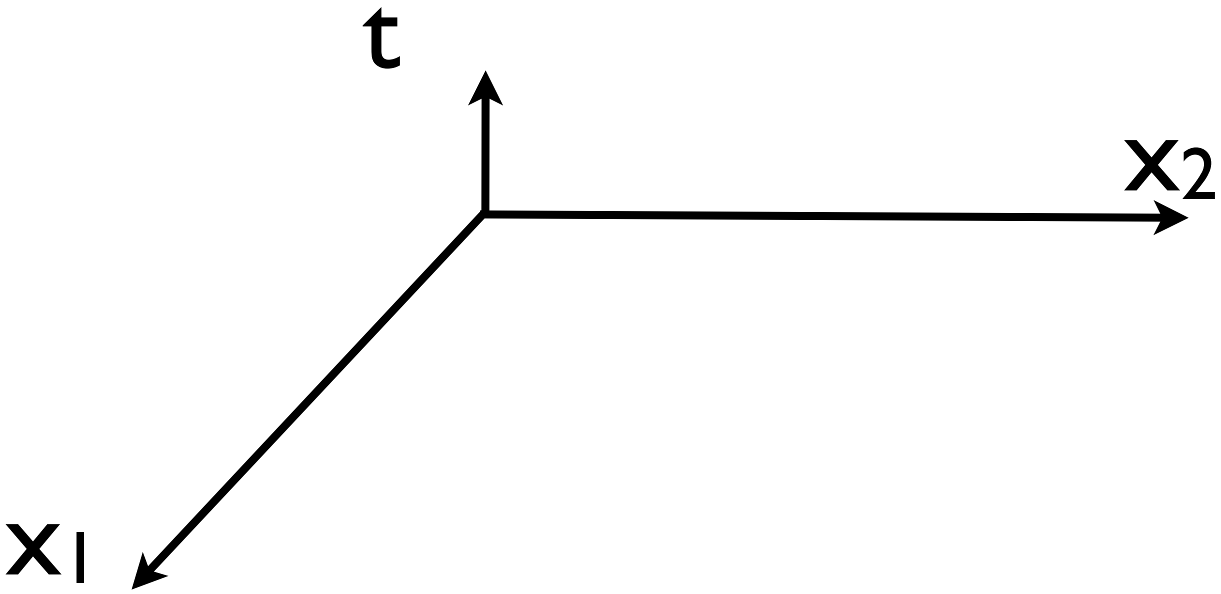


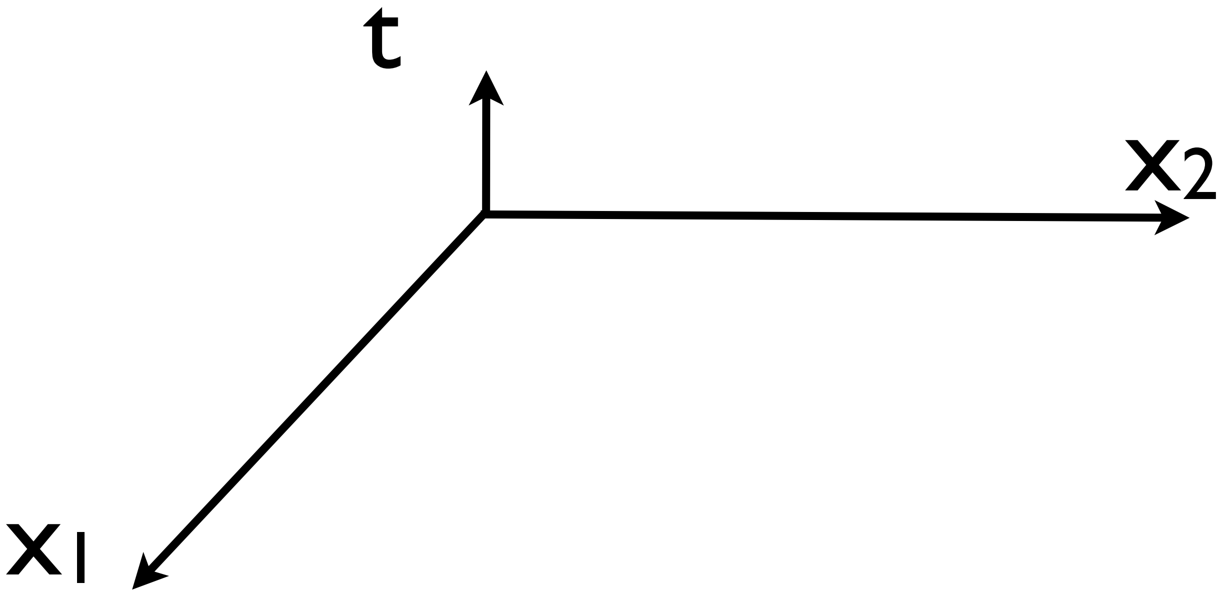
An alternative to the entropy current

V^μ



An alternative to the entropy current

$$V^{\mu} = (1, 0)$$



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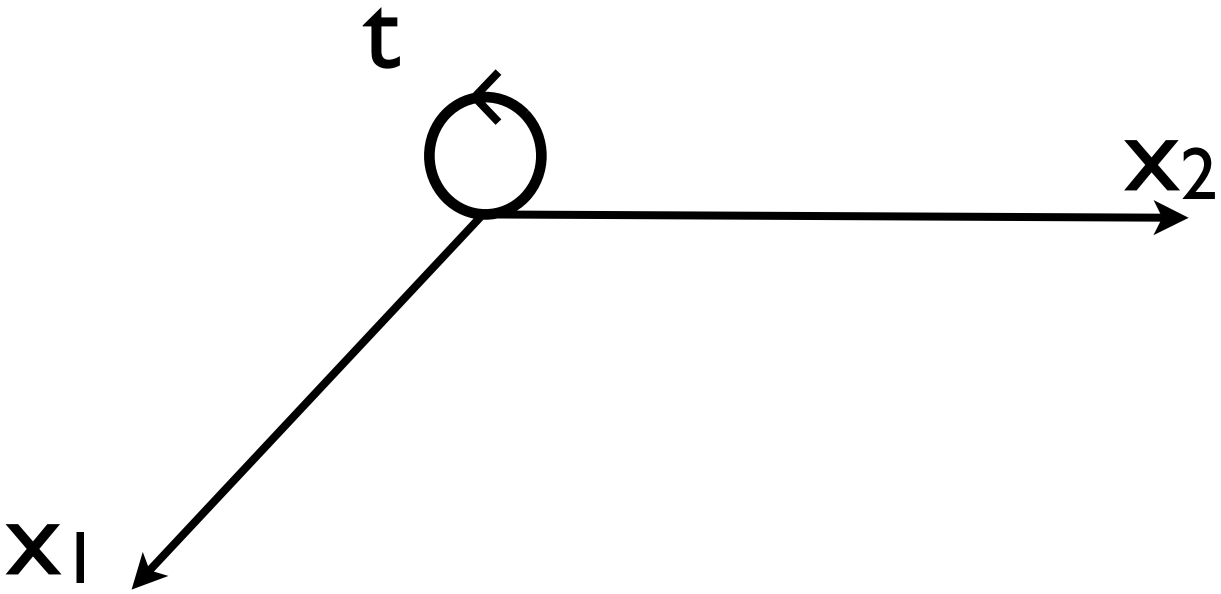
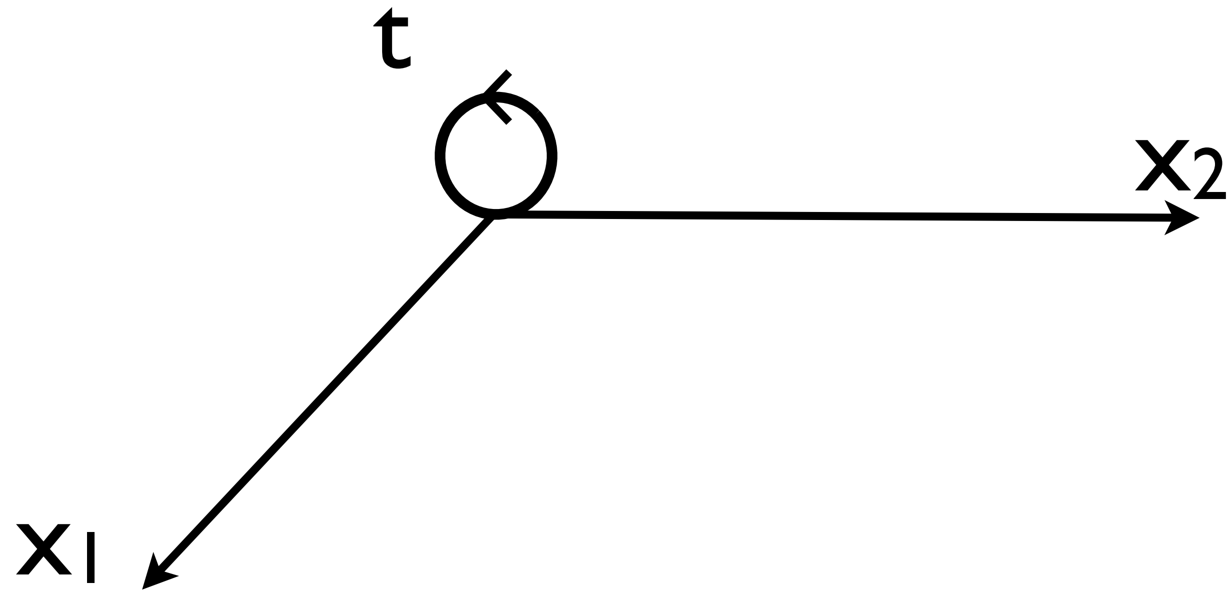


An alternative to the entropy current

$W[g, A]$

$V^\mu = (1, 0)$



An alternative to the entropy current

$$W[g, A]$$

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$$\mathcal{L}_V g = 0$$

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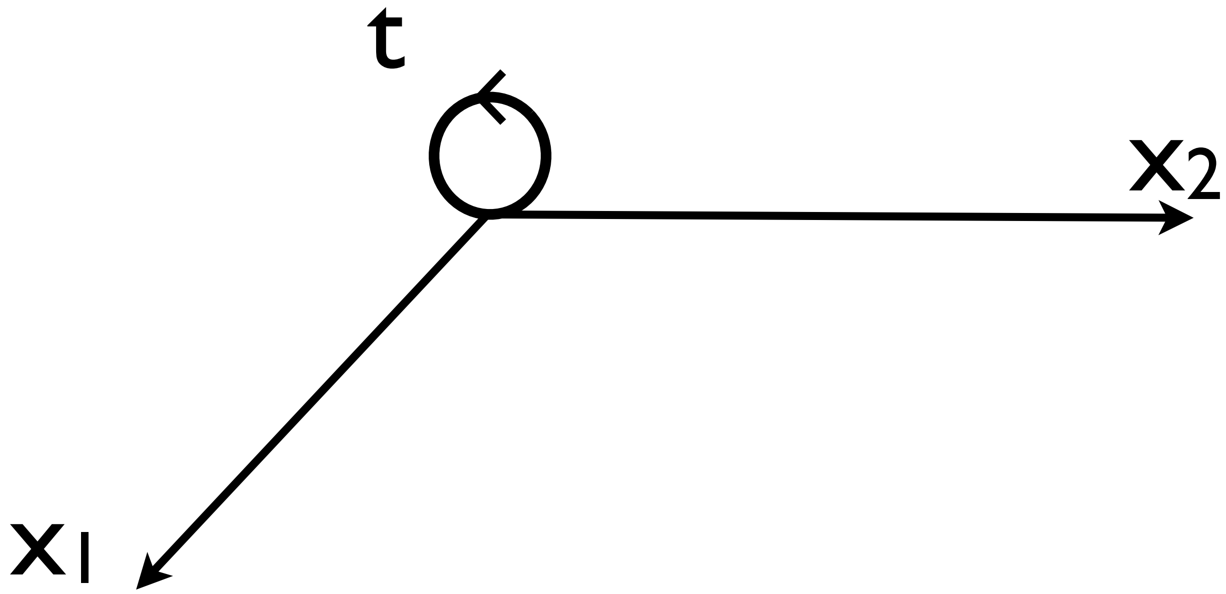


An alternative to the entropy current

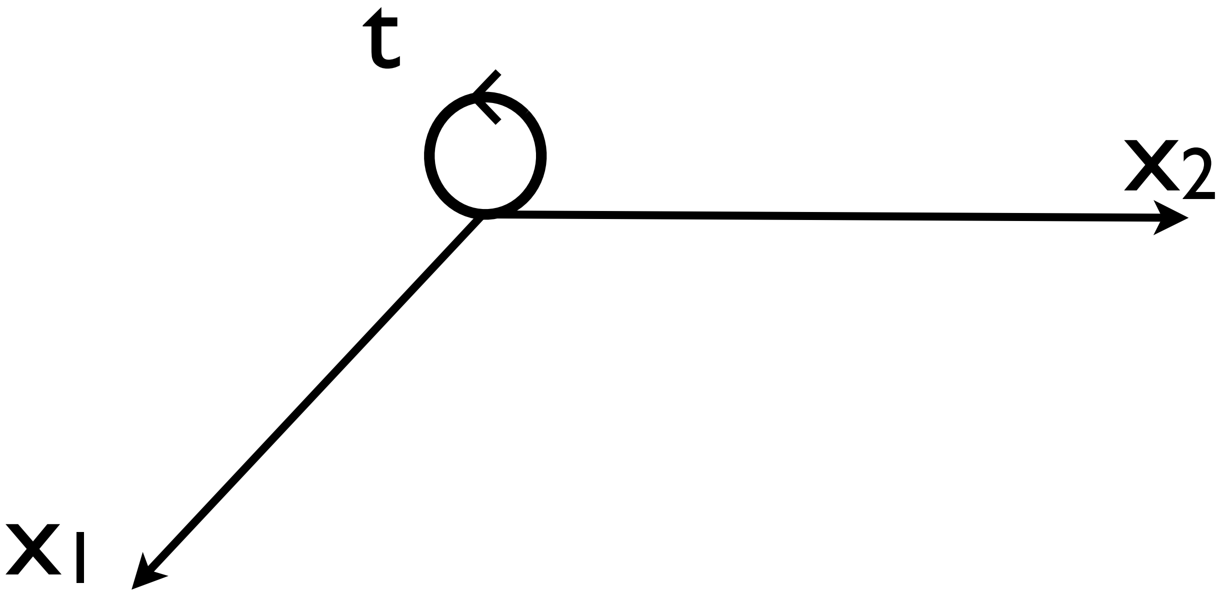
$$W[g, A]$$

$$\mathcal{L}_V g = 0$$

$$\mathcal{L}_V A = 0$$

$$T = \int_0^\beta \sqrt{g_{00}}$$

$$V^\mu = (1, 0)$$



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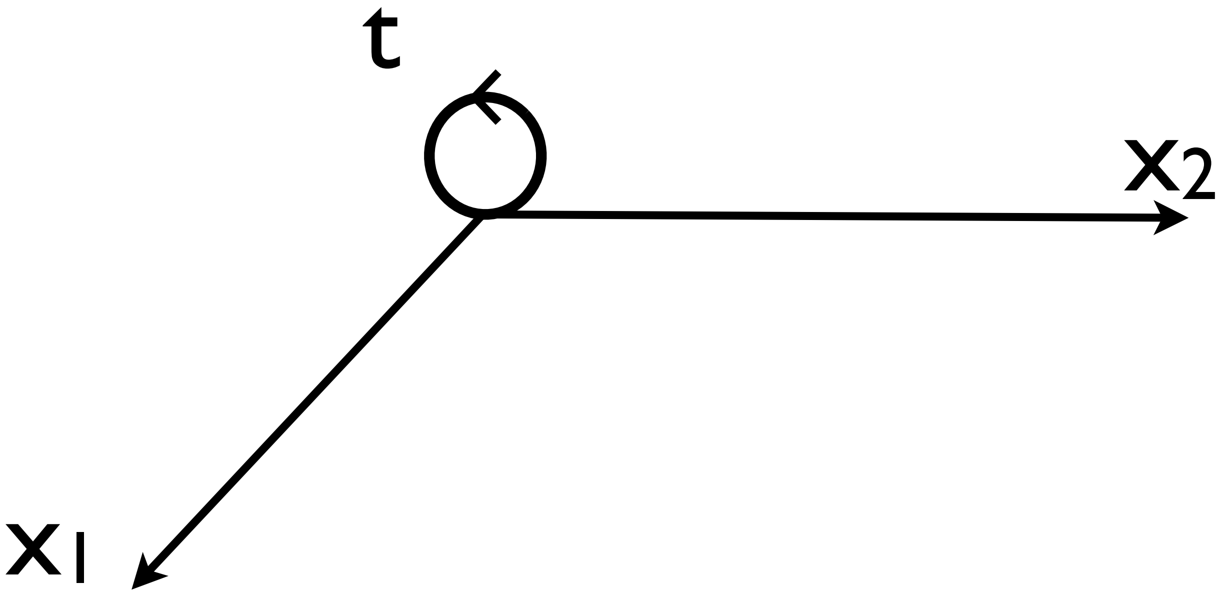

$$T = \int_0^{\beta} \sqrt{g_{00}}$$

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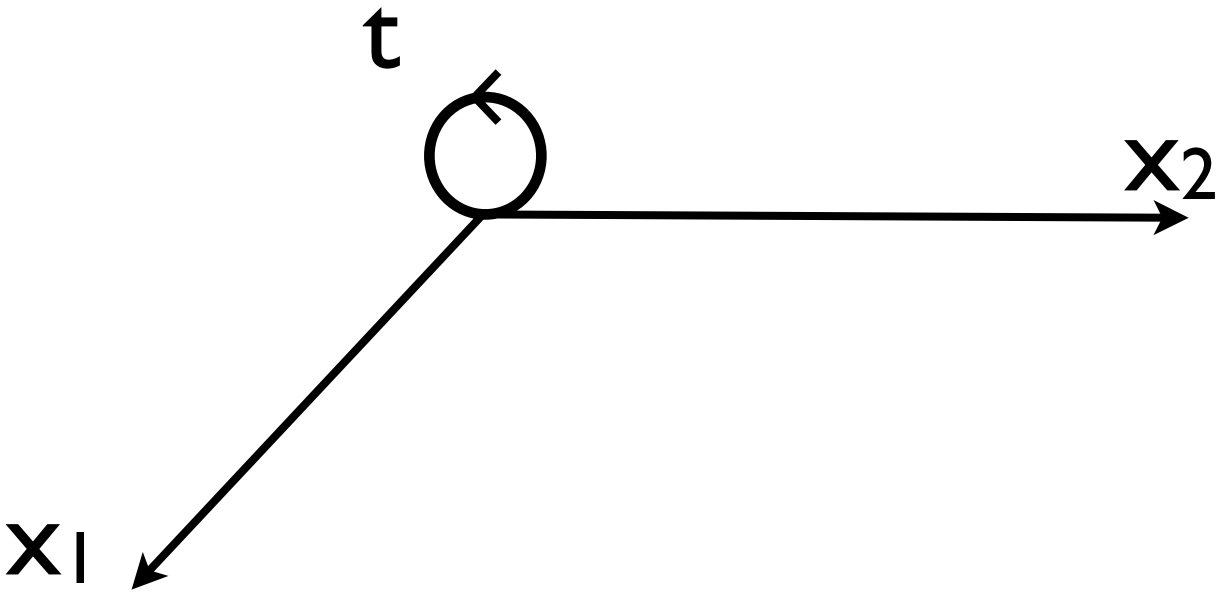
$$V^{\mu}$$



An alternative to the entropy current

$$W[g,A] = \int \sqrt{-g} P(T,\mu) d^d x$$

$$V^\mu = (1,0)$$

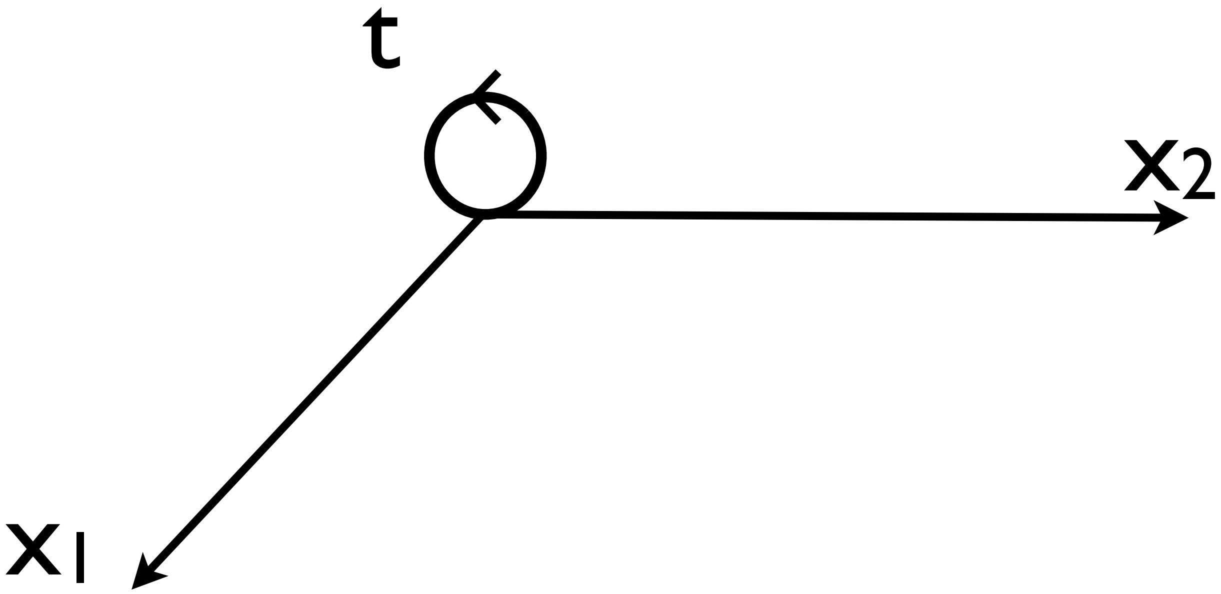
$$\mathcal{L}_V g = 0$$

$$\mathcal{L}_V A = 0$$

$$T = \int_0^\beta \sqrt{g_{00}}$$

$$\mu = \ln P_A / T$$

$$V^\mu$$



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$$W[g,A] = \int \sqrt{-g} P(T,\mu) d^d x$$

$$J^\mu = \frac{1}{\sqrt{-g}} \frac{\delta W}{\delta A_\mu}$$

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$$J^\mu = \frac{1}{\sqrt{-g}}\frac{\delta W}{\delta A_\mu} = \frac{\partial P}{\partial \mu}\frac{V^\mu}{\sqrt{-V^2}}$$

An alternative to the entropy current

$$W[g,A] = \int \sqrt{-g}P(T,\mu)d^dx$$

$$u^\mu = V^\mu/\sqrt{-V^2}$$

$$J^\mu = \frac{1}{\sqrt{-g}}\frac{\delta W}{\delta A_\mu} = \frac{\partial P}{\partial \mu}\frac{V^\mu}{\sqrt{-V^2}}$$

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$$W[g,A] = \int \sqrt{-g} P(T,\mu) d^d x$$

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$$\kappa_T = 0$$

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$$B = \epsilon^{\mu\nu\rho} u_\mu F_{\nu\rho}$$

$$\Omega = \epsilon^{\mu\nu\rho} u_\mu \partial_\nu u_\rho$$

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An alternative to the entropy current

$$W[g,A] = \int \sqrt{-g}P(T,\mu) + \mathcal{M}_B(T,\mu)B + \mathcal{M}_\Omega(T,\mu)\Omega d^3x$$

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$$\kappa_T = 0$$

$$\tilde{\chi}_E = \frac{\partial \mathcal{M}_B}{\partial \mu} - R_0 \left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B \right)$$

$$\tilde{\chi}_B = \frac{\partial P_0}{\partial \epsilon_0} \left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu \frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B \right) + \frac{\partial P_0}{\partial \rho_0}\frac{\partial \mathcal{M}_\mathcal{B}}{\partial \mu}$$

$$T\tilde{\chi}_T = \left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu \frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B \right) - R_0 \left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu \frac{\partial \mathcal{M}_\Omega}{\partial \mu} - 2\mathcal{M}_\Omega \right),$$

$$\tilde{\chi}_\Omega = \frac{\partial P_0}{\partial \epsilon_0} \left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu \frac{\partial \mathcal{M}_\Omega}{\partial \mu} - 2\mathcal{M}_\Omega \right) + \frac{\partial P_0}{\partial \rho_0} \left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B \right)$$

An alternative to the entropy current

$$W[g,A] = \int \sqrt{-g}P(T,\mu) + \mathcal{M}_B(T,\mu)B + \mathcal{M}_\Omega(T,\mu)\Omega d^3x$$

An alternative to the entropy current

$$W[g,A] \; = \int \sqrt{-g} P(T,\mu) + \alpha_1(T,\mu) R + \alpha_2(T,\mu) u^\alpha u^\beta R_{\alpha\beta} + \ldots d^3x$$

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$$\begin{aligned}
\Pi_{\mu\nu} = \;& -\eta\sigma_{\mu\nu} - \zeta P_{\mu\nu}\Theta \\
& T\Big[\tau\ (u.\nabla)\sigma_{\langle\mu\nu\rangle} + \kappa_1 R_{\langle\mu\nu\rangle} + \kappa_2 F_{\langle\mu\nu\rangle} + \lambda_0\ \Theta\sigma_{\mu\nu} \\
& + \lambda_1\ \sigma_{\langle\mu}{}^{a}\sigma_{a\nu\rangle} + \lambda_2\ \sigma_{\langle\mu}{}^{a}\omega_{a\nu\rangle} + \lambda_3\ \omega_{\langle\mu}{}^{a}\omega_{a\nu\rangle} + \lambda_4\ \mathfrak{a}_{\langle\mu}\mathfrak{a}_{\nu\rangle}\Big] \\
& + TP_{\mu\nu}\Big[\zeta_1(u.\nabla)\Theta + \zeta_2 R + \zeta_3 R_{00} + \xi_1\Theta^2 + \xi_2\sigma^2 + \xi_3\omega^2 + \xi_4\mathfrak{a}^2\Big]
\end{aligned}$$

An alternative to the entropy current

$$W[g,A] \;= \int \sqrt{-g}P(T,\mu) + \alpha_1(T,\mu)R + \alpha_2(T,\mu)u^\alpha u^\beta R_{\alpha\beta} + \ldots d^3x$$

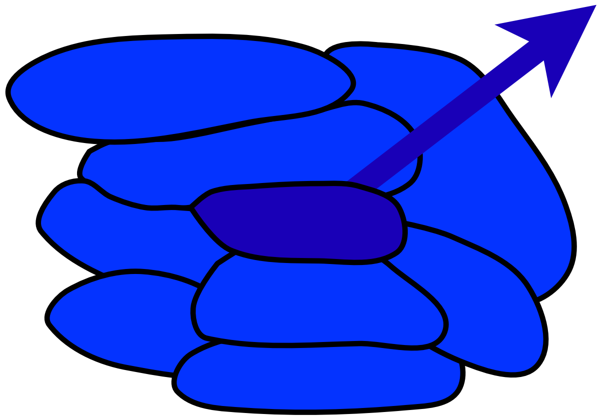
$$\kappa_2 = \kappa_1 + T\frac{d\kappa_1}{dT}$$

$$\zeta_2 = \frac{1}{2}\left[s\frac{d\kappa_1}{ds} - \frac{\kappa_1}{3}\right]$$

$$\zeta_3 = \left(s\frac{d\kappa_1}{ds} + \frac{\kappa_1}{3}\right) + \left(s\frac{d\kappa_2}{ds} - \frac{2\kappa_2}{3}\right) + \frac{s}{T}\left(\frac{dT}{ds}\right)\lambda_4$$

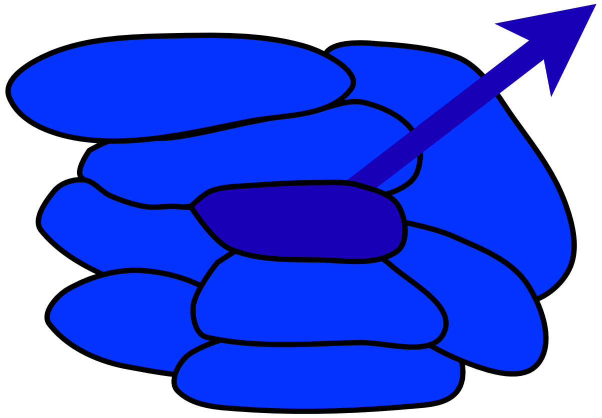
$$\begin{aligned}\xi_3 = {} & \frac{3}{4}\left(\frac{s}{T}\right)\left(\frac{dT}{ds}\right)\left(T\frac{d\kappa_2}{dT} + 2\kappa_2\right) - \frac{3\kappa_2}{4} + \left(\frac{s}{T}\right)\left(\frac{dT}{ds}\right)\lambda_4 \\ & + \frac{1}{4}\left[s\frac{d\lambda_3}{ds} + \frac{\lambda_3}{3} - 2\left(\frac{s}{T}\right)\left(\frac{dT}{ds}\right)\lambda_3\right] \\ \xi_4 = {} & -\frac{\lambda_4}{6} - \frac{s}{T}\left(\frac{dT}{ds}\right)\left(\lambda_4 + \frac{T}{2}\frac{d\lambda_4}{dT}\right) - T\left(\frac{d\kappa_2}{dT}\right)\left(\frac{3s}{2T}\frac{dT}{ds} - \frac{1}{2}\right) \\ & - \frac{Ts}{2}\left(\frac{dT}{ds}\right)\left(\frac{d^2\kappa_2}{dT^2}\right)\end{aligned}$$

Summary



$\epsilon(x^\mu)$	Energy density
$\rho(x^\mu)$	Charge density
$u^\nu(x^\mu)$	Velocity field $(u_\mu u^\mu = -1)$

Summary



$\epsilon(x^\mu)$ Energy density

$\rho(x^\mu)$ Charge density

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J_s^\mu \geq 0$$

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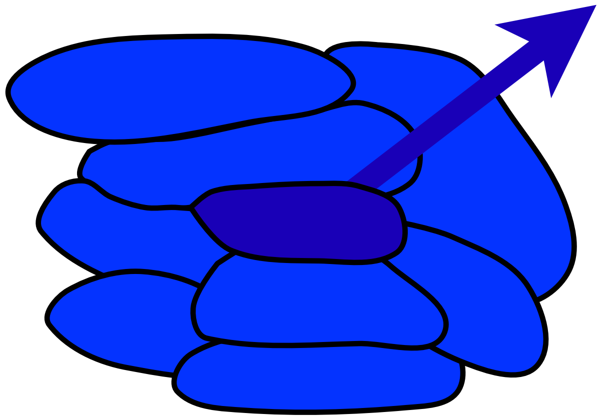

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$\not{P}$ hydrodynamics 2+1

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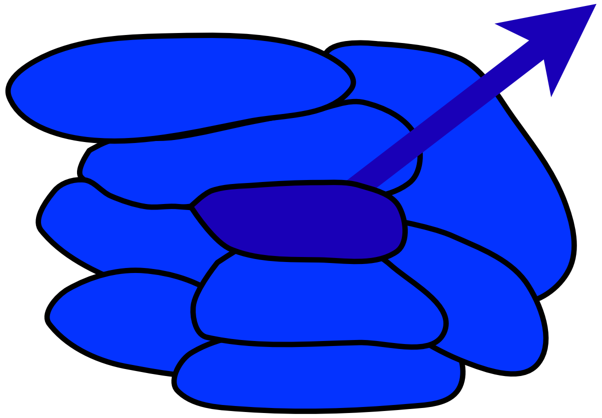

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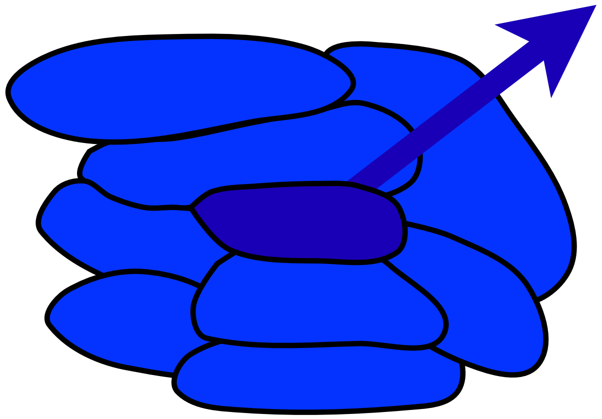

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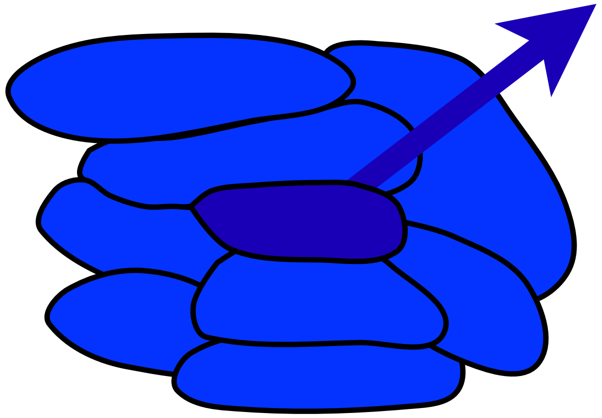
$\not{P}$ superfluids 3+1

Summary



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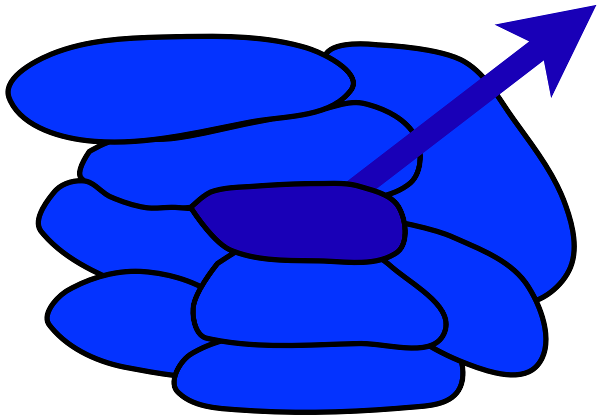

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$$W[g, A]$$

$$J^\mu = \frac{1}{\sqrt{-g}} \frac{\delta W}{\delta A_\mu}$$

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta W}{\delta g_{\mu\nu}}$$

Thank you

Superfluid hydrodynamics

$$\mu + \mathcal{O}(\partial) = u^\nu \partial_\nu \phi$$

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$\mathcal{O}(\partial)$ corrections:

20 parameters: 14 parity even + 6 parity odd

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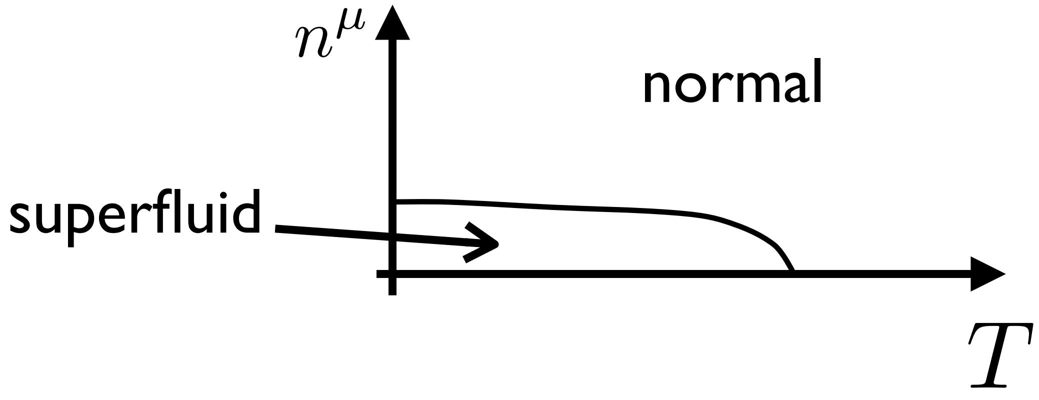

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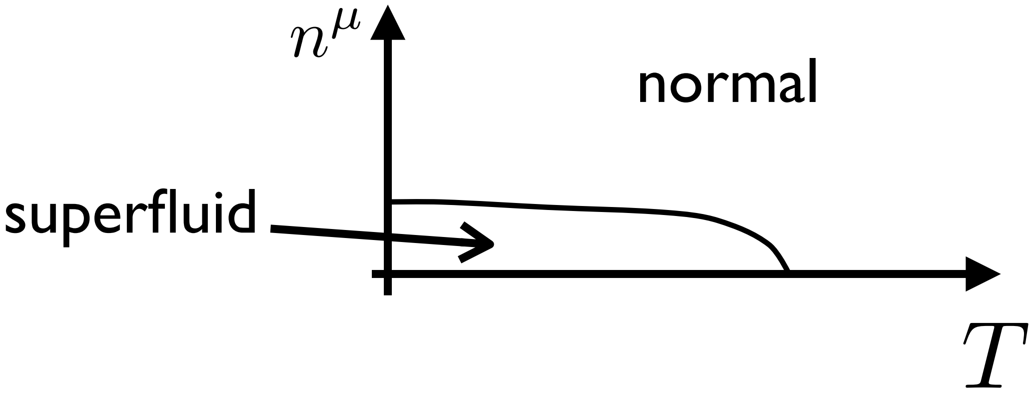

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$\mathcal{O}(\partial)$ corrections:

7 parameters: 5 parity even + 2 parity odd

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5 parameters: 3 parity even + 2 parity odd

Superfluid hydrodynamics

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Superfluid phase

$$\tilde{\kappa}_\omega(\mu, T) \in \mathbf{R}$$

$$\tilde{\kappa}_B(\mu, T) \in \mathbf{R}$$

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Normal phase

$$\tilde{\kappa}_\omega = c\mu^2 \left(1 - \frac{2}{3} \frac{\mu\rho}{\epsilon + P} \right)$$

$$\tilde{\kappa}_B = c\mu^2 \left(1 - \frac{1}{2} \frac{\mu\rho}{\epsilon + P} \right)$$

Superfluid phase

$$\tilde{\kappa}_\omega(\mu, T) \in \mathbf{R}$$

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