Two Applications of Holography

Umut Gürsoy

CERN

University of Crete, Heraklion—May 2nd, 2012

Based on:
arXiv:1111.0580, w/ M. Caselle, L. Castagnini, A. Feo, F. Gliozzi, M. Panero, A. Schafer
arXiv:1112:5074, ongoing, w/ E. Plauschinn, H. Stoof, S. Vandoren

Outline

- Universal thermodynamics in the quark-gluon plasma: Universal features of the interaction measure in finite T gauge theories.
- Holography and ARPES sum-rules: Elementary Weyl fermions in a strongly coupled CFT.

APPLICATION I

The interaction measure

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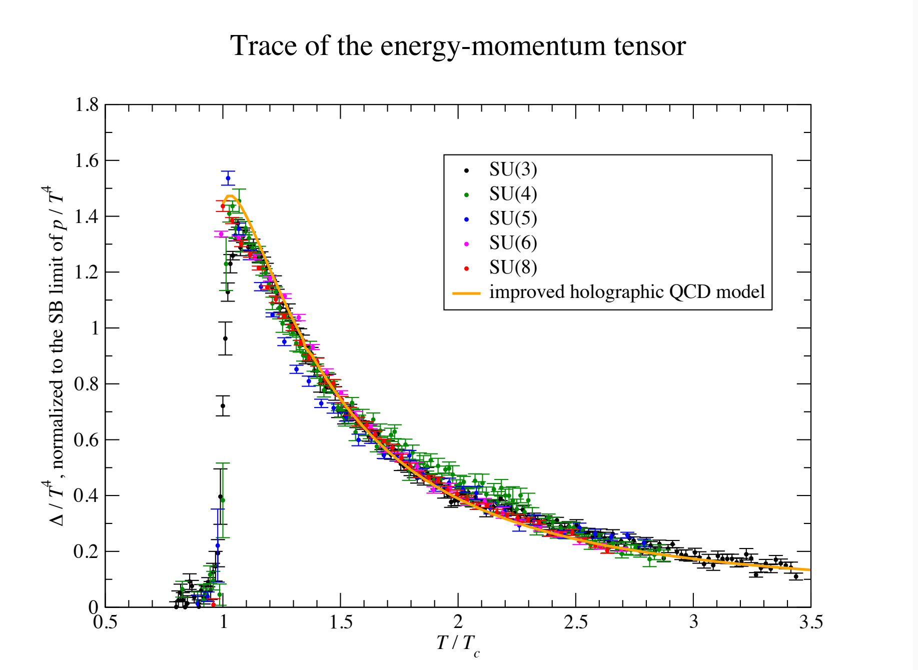

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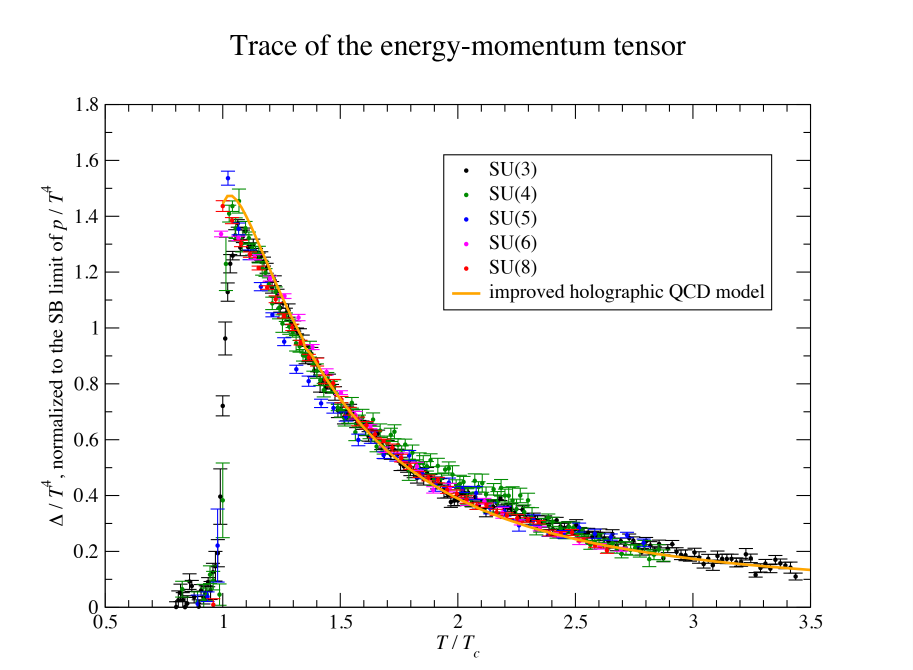

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Observations: (a) Large N is not bad at all,

(b) (improved) holography captures the correct behavior.

An intriguing universal behavior

Pisarski '11, U.G. et al '11

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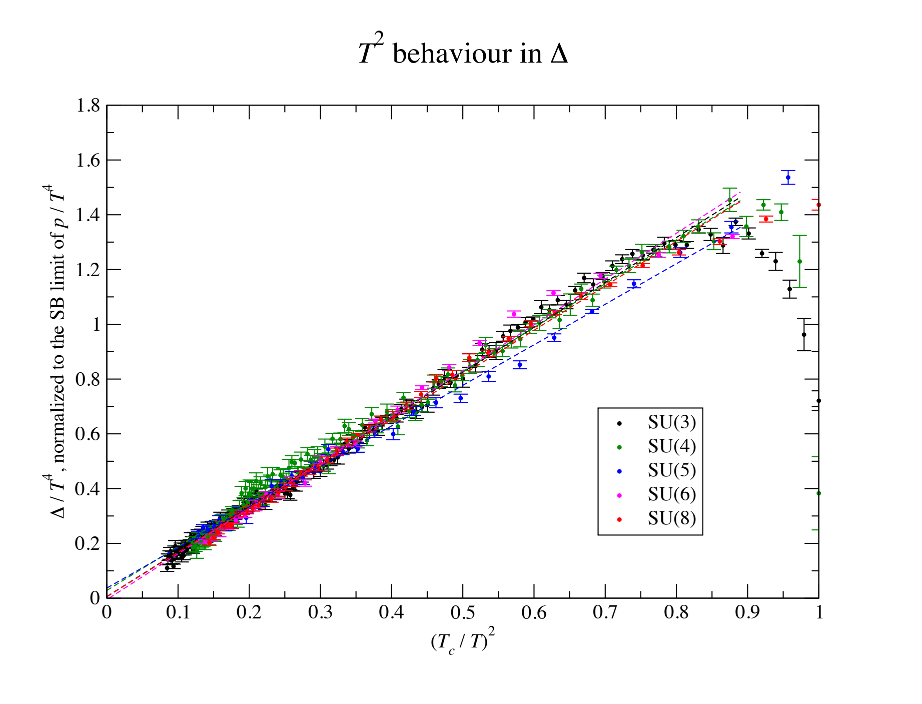

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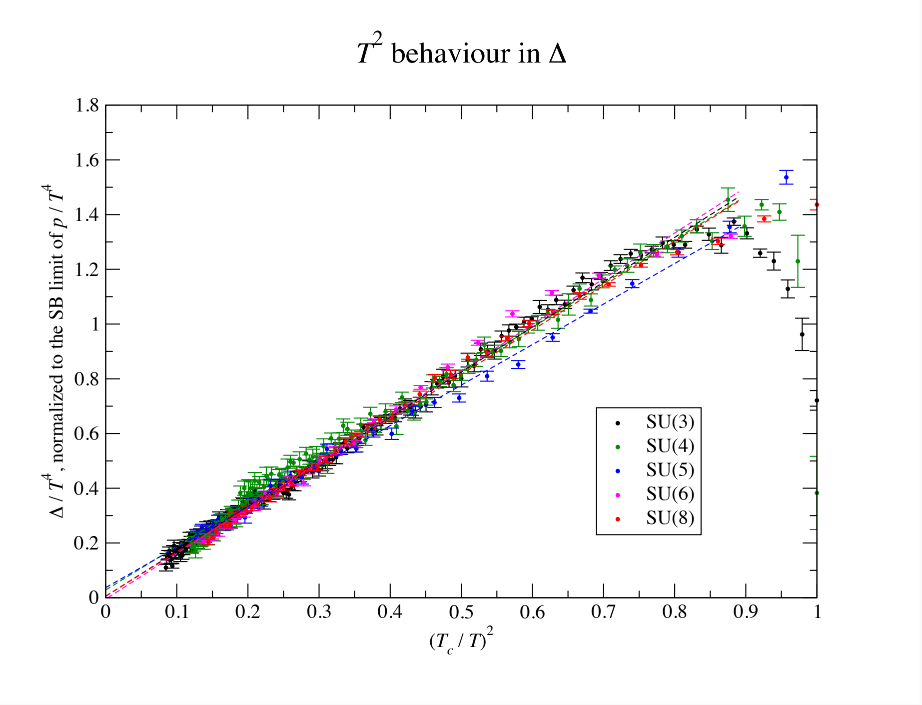

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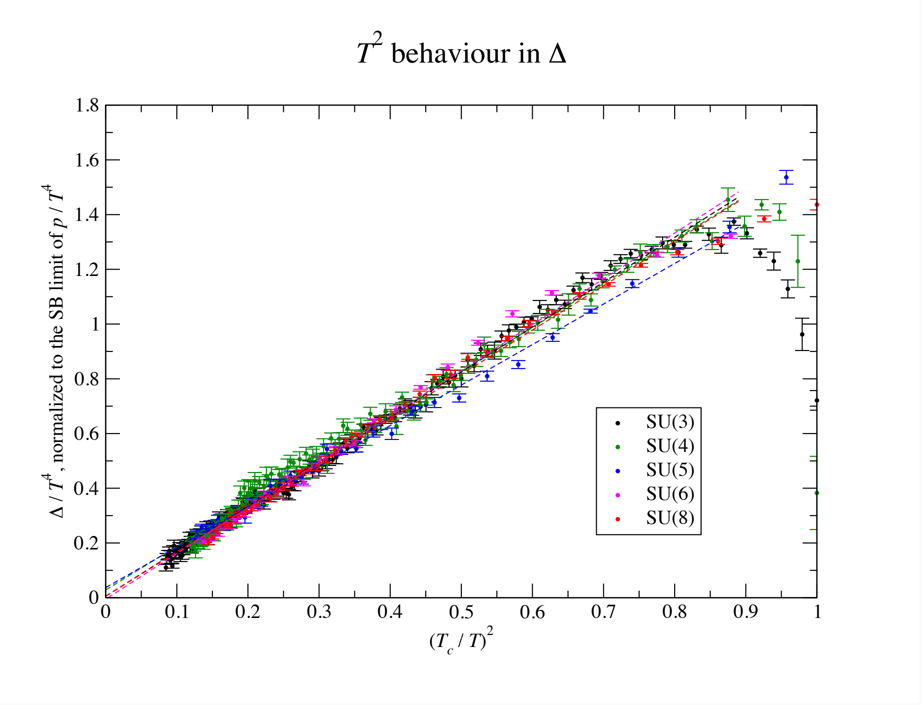

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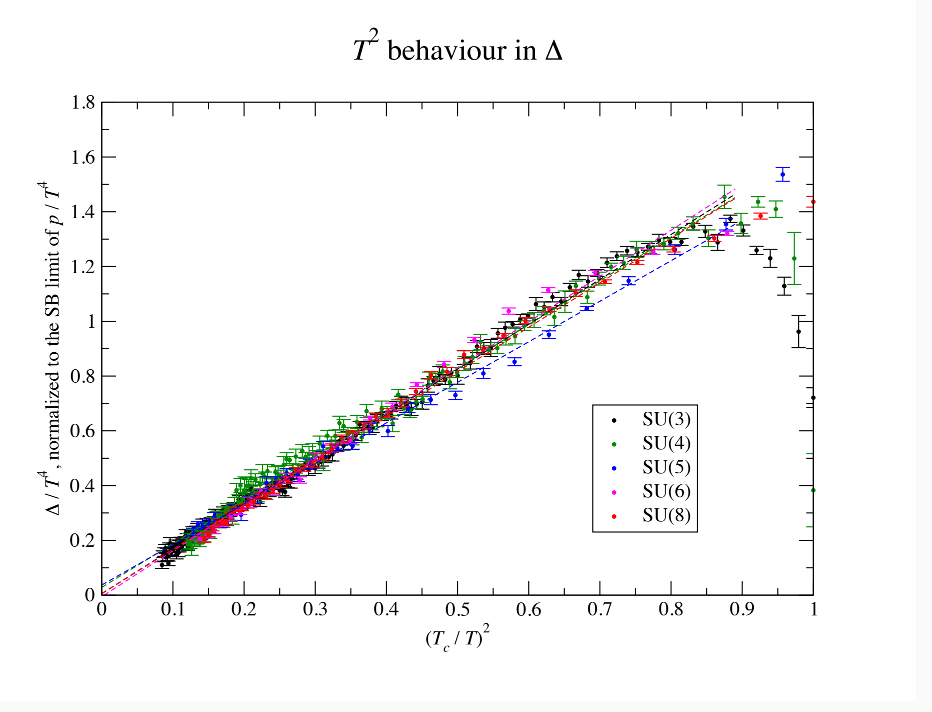

- Data collapses on $1/T^2$ (3+1D)
- No satisfactory explanation in field theory
  Because gauge theory is strongly coupled in that range.

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This approach is most suitable for universal and generic results.

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1. Spectrum of the glue- balls: $m_n^2 \propto n^{\alpha-1}$, for large n;
2. The nature of the deconfining transition: for large α, transition more discontinuous, whereas for $\alpha \to 1$, it is continuous;
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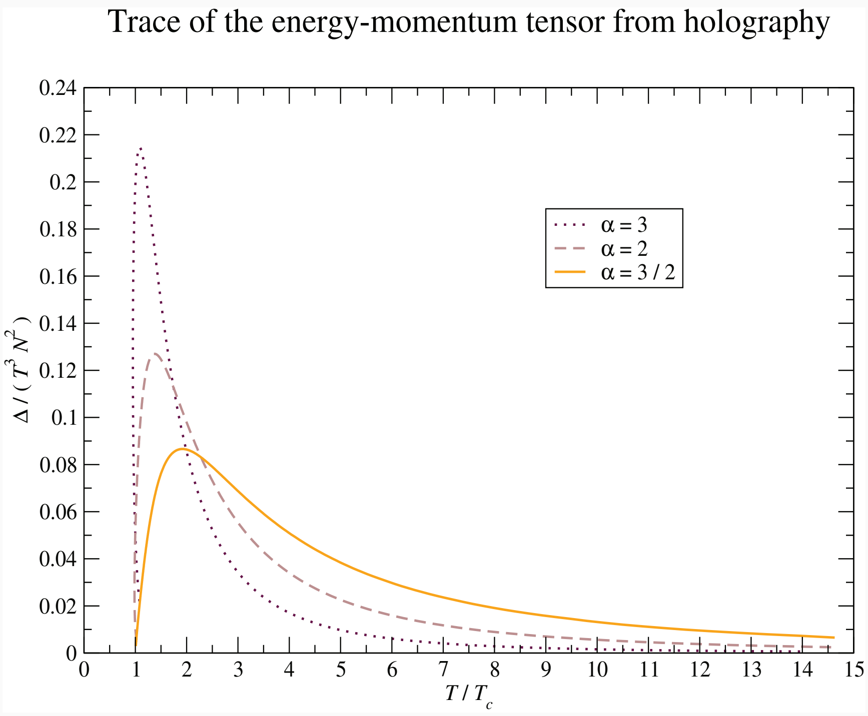

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- Calculate the thermodynamics from the corresponding black-hole solution:
  $ds^2 = b^2(r)\left(dt^2 f(r) + dx^2 + \frac{dr^2}{f(r)}\right)$; Horizon $f(r_h) = 0$.
- Basic thermodynamics: $\frac{\Delta}{N^2 T^4} = \frac{S}{T^3} - \frac{4}{T^4}\int_{T_c}^{T} S(\tilde{T})d\tilde{T}$
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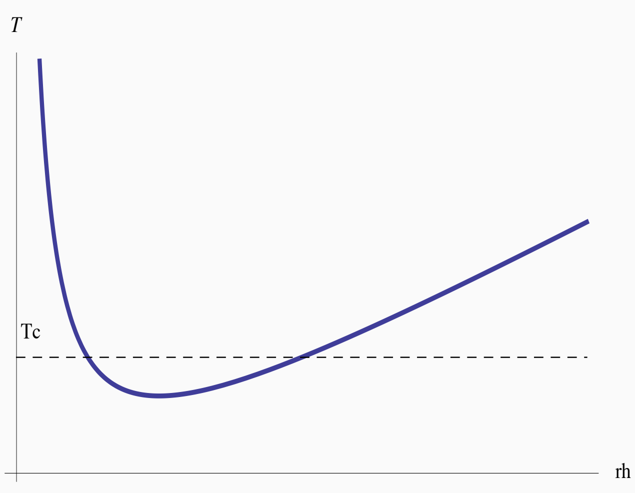

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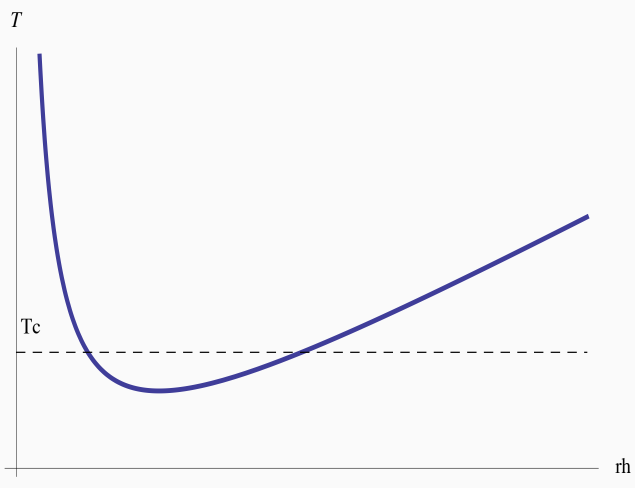

- Indeed $T(r_h)$ is approximately conformal
- We need the first confining correction to $b(r)$:
- IR of V: $\Rightarrow b(r_h) \approx \frac{\ell}{r} e^{-(r\Lambda)^\alpha}$

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Outlook:

- Apparently proportionality factors also universal Pisarski ’11
- Other universal phenomena on the $\mu - T$ plane

APPLICATION II

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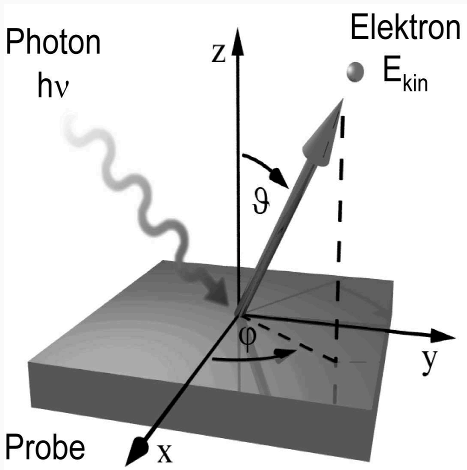

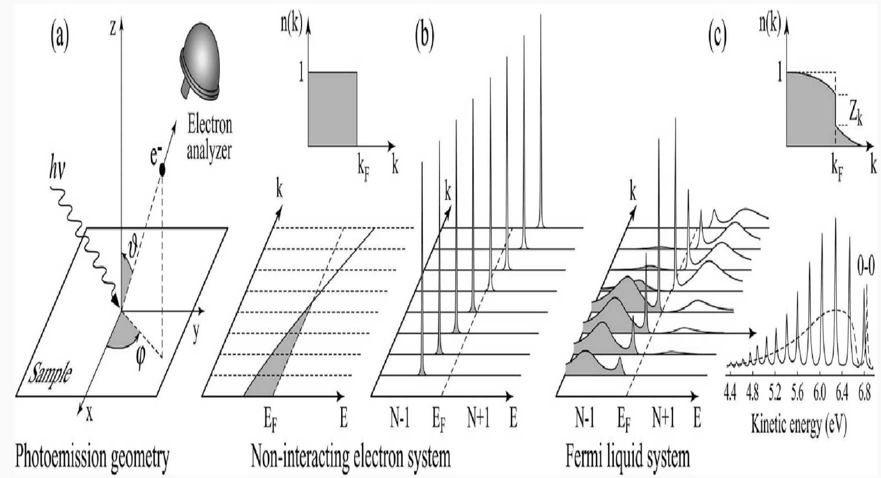

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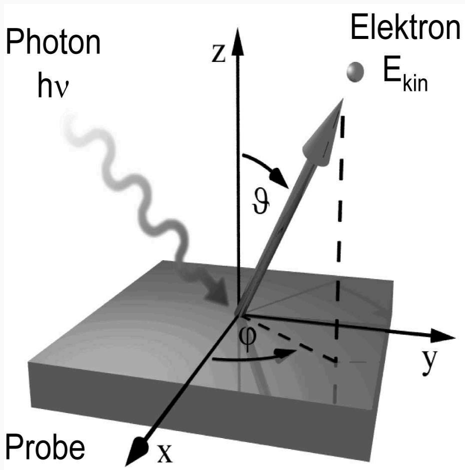

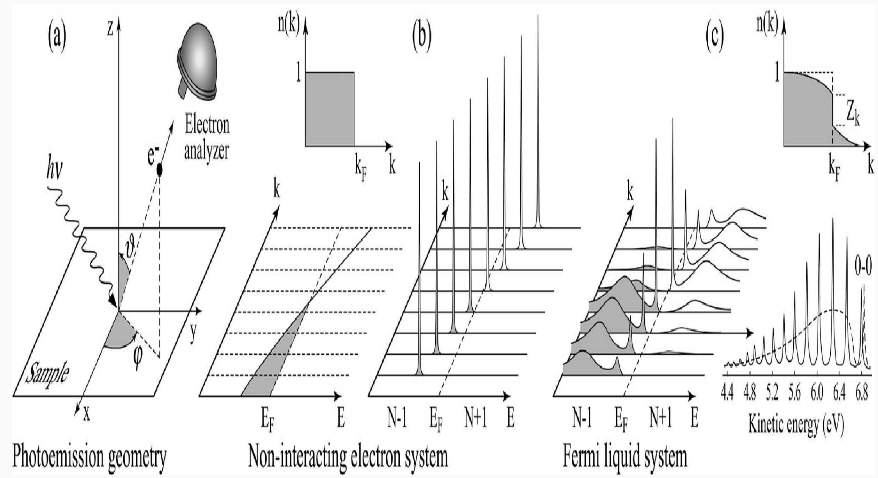

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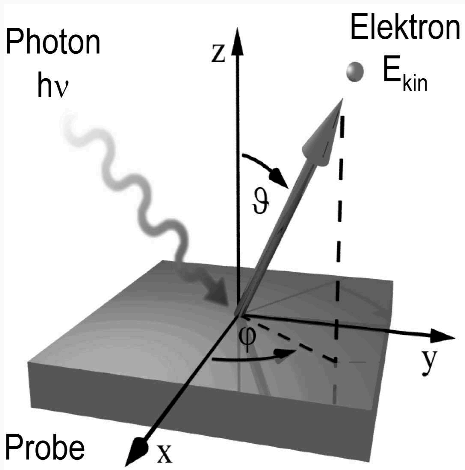

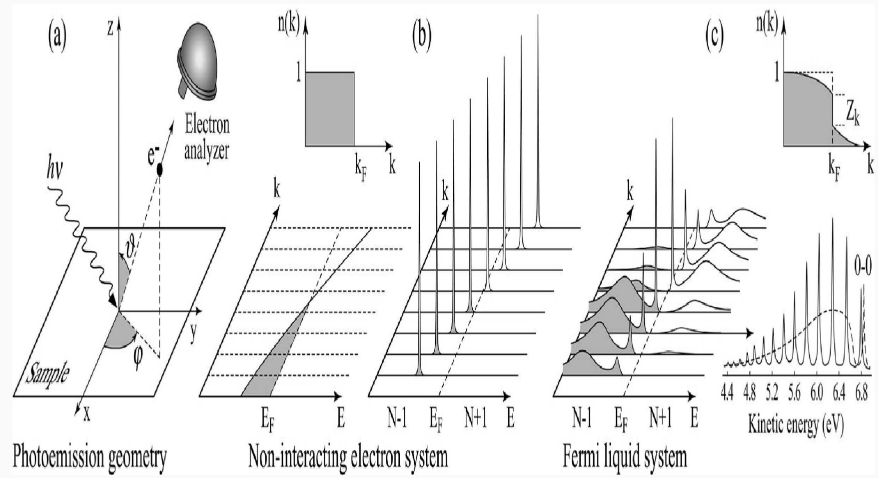

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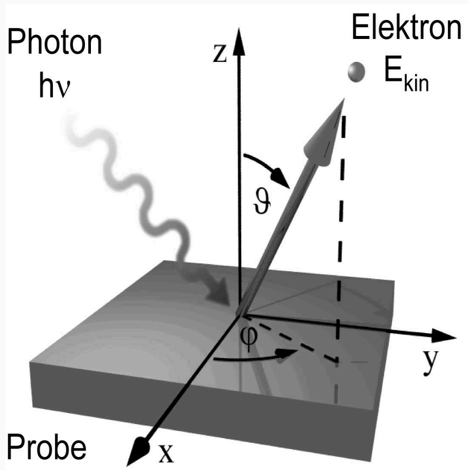

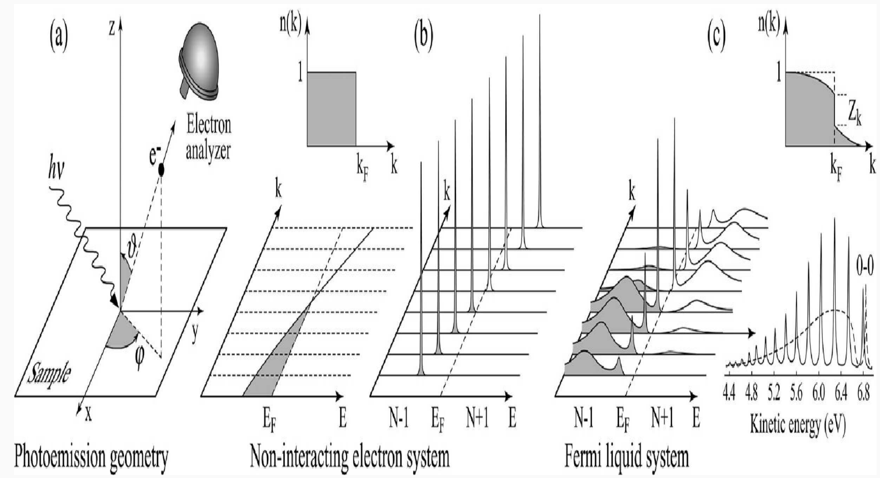

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(*) ARPES sum-rule: $\frac{1}{\pi}\int d\omega \mathrm{Im}[G(\omega, k)] = 1, \qquad \forall k,\ T$

(*) From canonical com. rels of elementary operators

(*) Difficult in AdS/CFT, as $\mathcal{O}$ is composite $\Rightarrow$ UV divergences.

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How to derive this holographically?

(not only semi-holographically as in Faulkner, Polchinski '11.)

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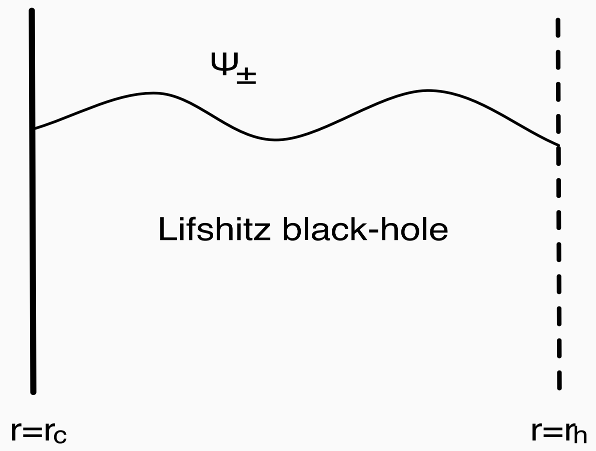

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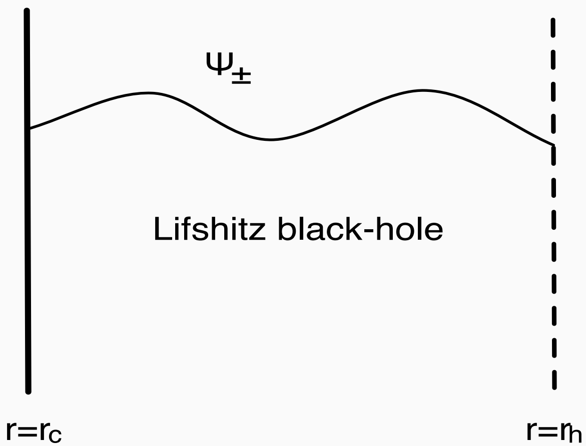

Introduce $\Psi = \Psi_+ + \Psi_-$

with $\Psi_+(r_0) \Leftrightarrow \psi_+$, $\Psi_-(r_0) \Leftrightarrow \langle \mathcal{O}_- \rangle$

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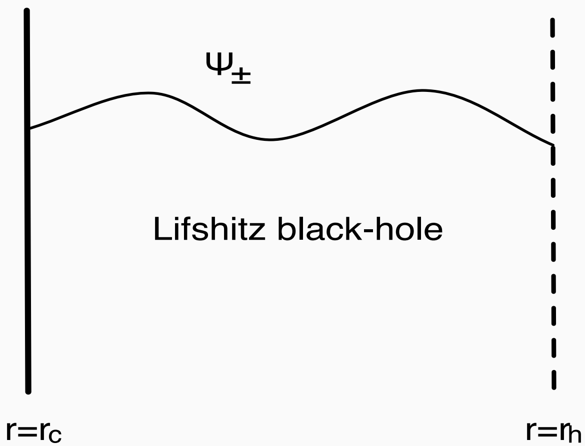

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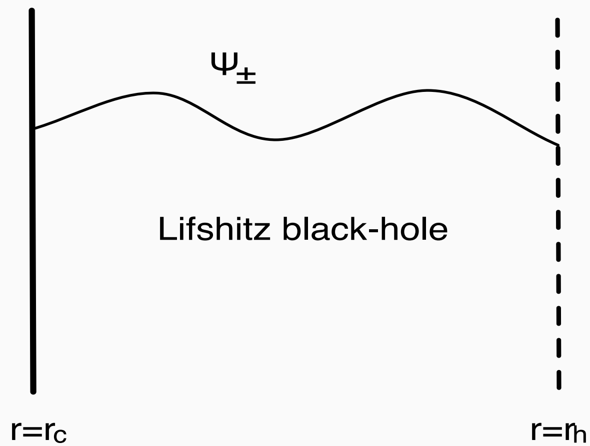

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Results:

$$G_R[\psi_+] = \frac{-1}{\not{p}(1+g\ p^{2M-1})} \qquad \text{(for } z=1\text{)}$$

$$G_R[\psi_+] = \frac{-1}{\omega - \eta\ \vec{\sigma}\cdot\hat{k}\ k^z g\ \omega^{\frac{2M}{z}} \left(f_1(\omega/k^z) + \vec{\sigma}\cdot\hat{k}\ f_2(\omega/k^z)\right)} \qquad \text{(for generic } z\text{)}$$

with $f_2(u,M) = f_1(u,M)\sqrt{1+(f_1(u,M)f_1(u,-M))^{-1}}$.

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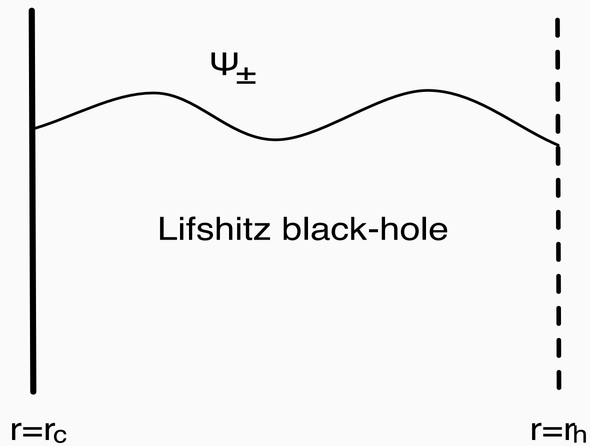

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with $f_2(u, M) = f_1(u, M)\sqrt{1 + (f_1(u, M) f_1(u, -M))^{-1}}$.

Satisfies the sum-rule and Kramers-Kronig for $-z/2 < M < z/2$!

Details

The background:

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{r^2 f(r)} - f(r)\, r^{\,2z}\mathrm{d}t^2 + r^{\,2}\mathrm{d}\vec{x}^{\,2}\,, \qquad f(r) = 1 - \left(\frac{r_h}{r}\right)^{d+z-1}\,.$$

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Compute $G_{\mathcal{O}}$ of a fermonic operator $\mathcal{O}_-$ on Lifshitz:

solve Dirac $(\not{D} - M)\Psi = 0$. Decompose $\Gamma^{\underline{r}}\,\Psi_\pm = \pm\,\Psi_\pm$. UV asymptotics:

$$\psi_\pm = r^{\pm M - \frac{1}{2}(z+d-1)}\Big(1 + \cdots\Big)A_\pm + r^{\mp M - \frac{1}{2}(z+d+1)}\Big(1 + \cdots\Big)B_\pm\ ,$$

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- Discover $M = \Delta + \frac{z+d-1}{2}$
- Find $G_{\mathcal{O}}$ from $A_- \sim G_{\mathcal{O}}\, A_+$.
- Define $\psi_-(r,p) = -i\,\xi(r,p)\, \psi_+(r,p)$
- Then $G_{\mathcal{O}}(\omega) = \lim_{r\to\infty} r^{2M} \xi(r,\omega), \quad -\frac{z}{2} < M < \frac{z}{2},$

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- Parity: $G^+_{\mathcal{O}}(\omega, \vec{k}) = G^-_{\mathcal{O}}(\omega, -\vec{k})$.
- Particle-hole: $\mathrm{Tr}\, G^\dagger_{\mathcal{O}}(\vec{k}, \omega) = -\mathrm{Tr}\, G_{\mathcal{O}}(\vec{k}, -\omega)$.
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Eigenvalues of ξ $\Leftrightarrow$ components of $G_{\mathcal{O}}$.

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- Legendre transform w.r.t. ψ_+
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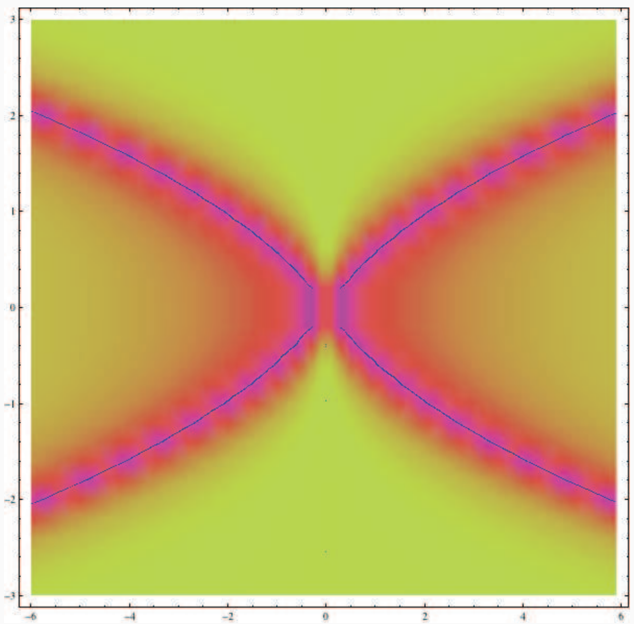

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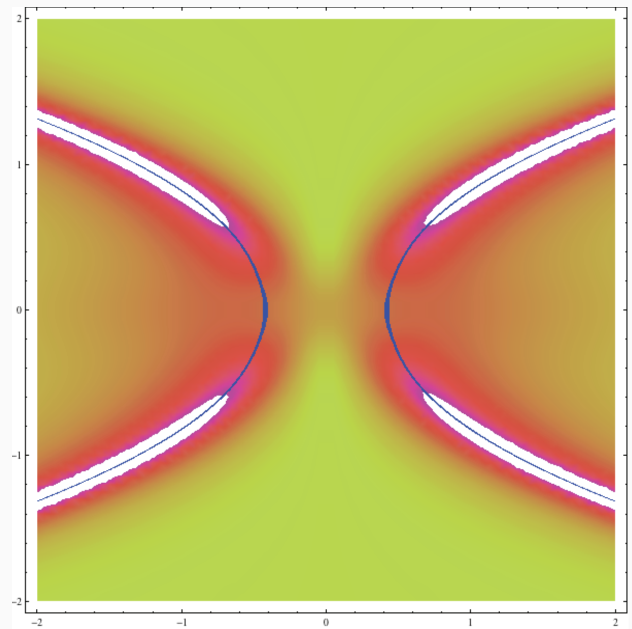

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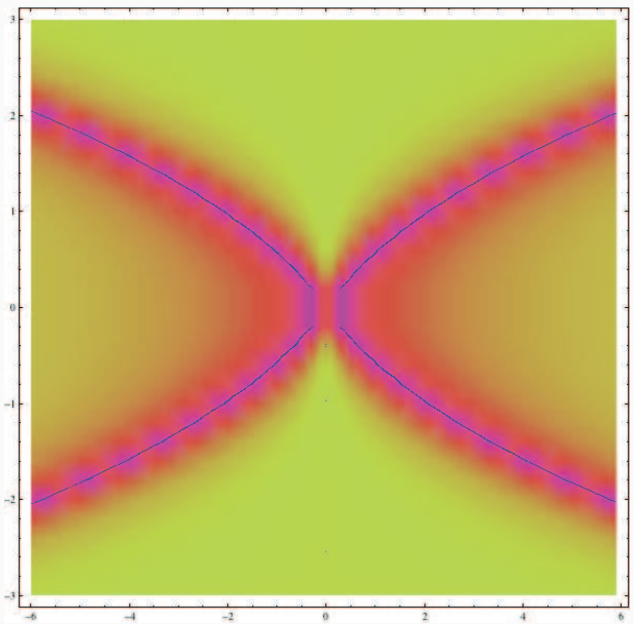

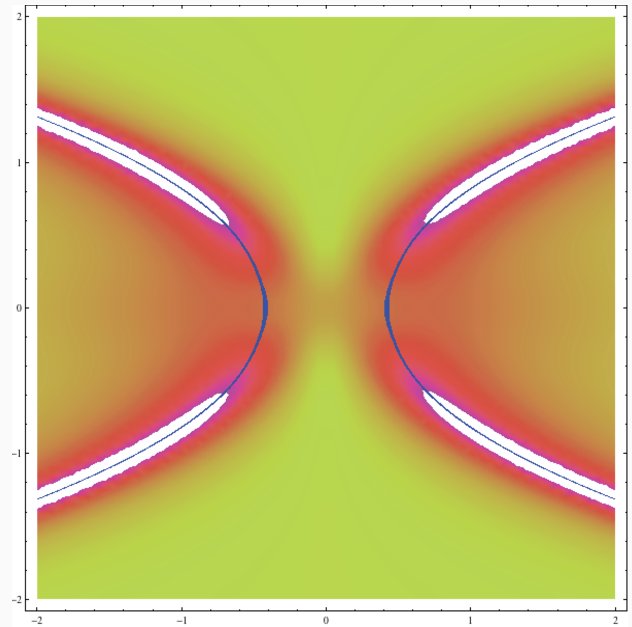

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One proves that, dispersion is gapped for $M < 0 \Rightarrow \Delta < \frac{z+d-1}{2}$:

From $Re[G_R^{-1}] = 0 \Rightarrow \omega(0) = \left(g\ (2z)^{-\frac{2M}{z}} \frac{\Gamma(\frac{1}{2} - \frac{M}{z})}{\Gamma(\frac{1}{2} + \frac{M}{z})} |\cos(\pi(M + \frac{1}{2}))| \right)^{\frac{z}{z-2M}}$

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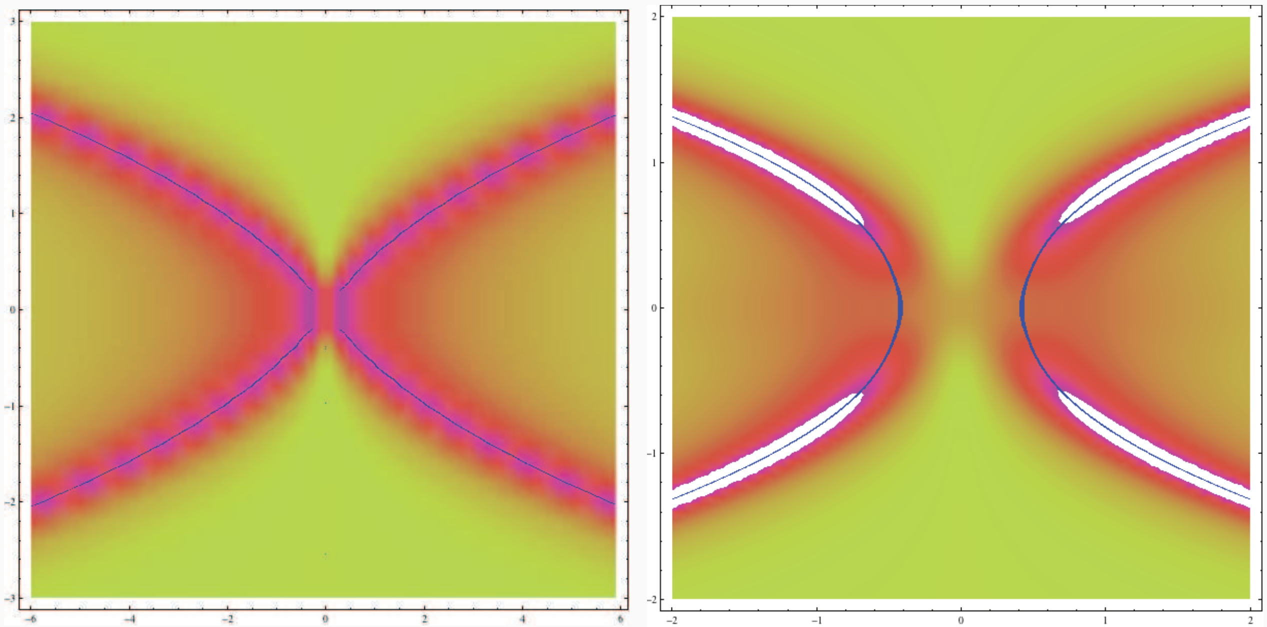

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Spontaneous gap generation (similar to technicolor):

chiral fermion acquires mass when the dominant channel it couples in the CFT has $\Delta < \frac{z+d-1}{2}$.

A topological phase transition:

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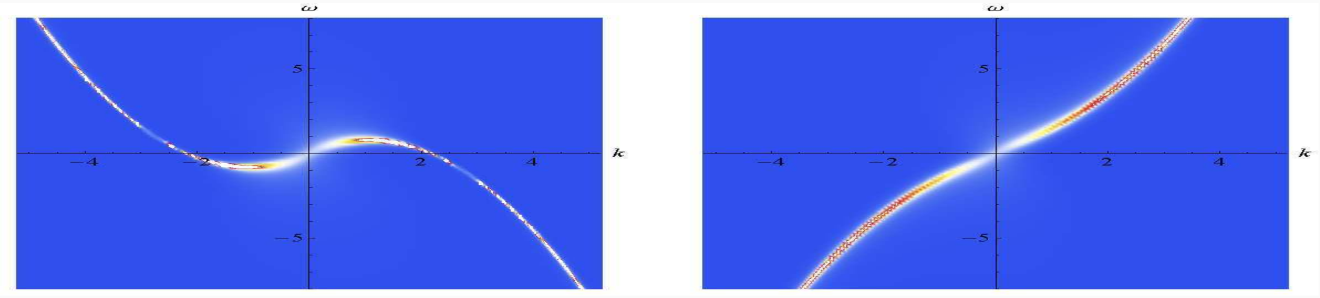
- A change in the number of Fermi surfaces.
- Fermi surface: $k(\omega \to 0) = finite$ in the dispersion.



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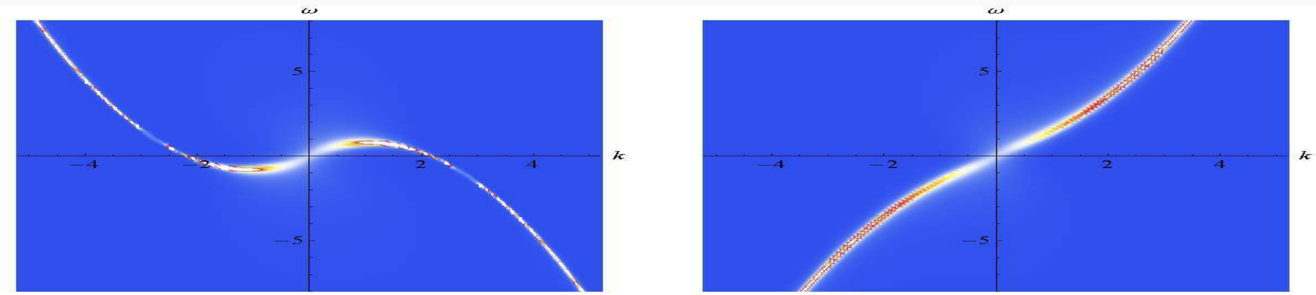

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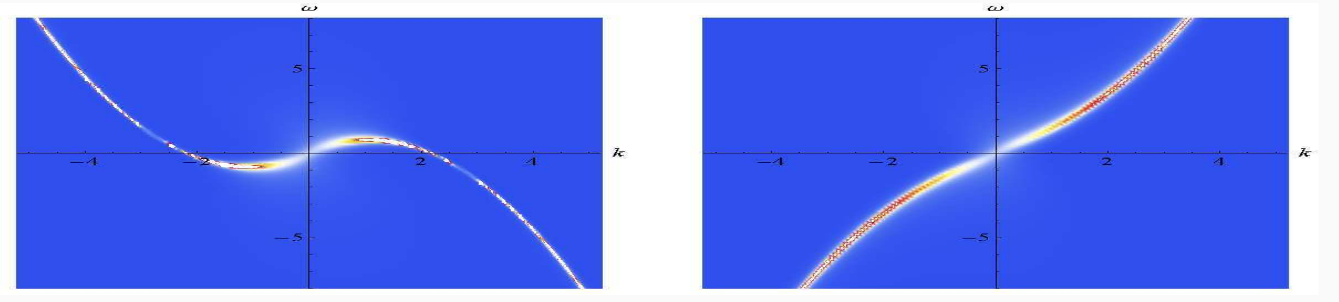

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  For $1/2 < |M| < z/2, \eta > 0$ and $0 < |M| < 1/2, \eta < 0$.
- η is the "spin-orbit coupling"
- Location of the Fermi surface:
  $$k_F = \left(\frac{g}{\eta}|c_1|\right)^{\frac{1}{z-2M}} \qquad \text{with} \qquad c_1 = 2^{-2M}\frac{\Gamma(\frac{1}{2}-M)}{\Gamma(\frac{1}{2}+M)}.$$

Outlook

- Explore these transitions in detail U.G. et al, ongoing
- How to tune the parameters of the UV theory to produce them
- Finite T and μ in detail
- Consider more general situations than Hertz-Millis
- Break particle-hole symmetry and apply to cold-atoms.

THANK YOU !