Parity Breaking Hydrodynamics in 2+1 Dimensions and Axions in AdS

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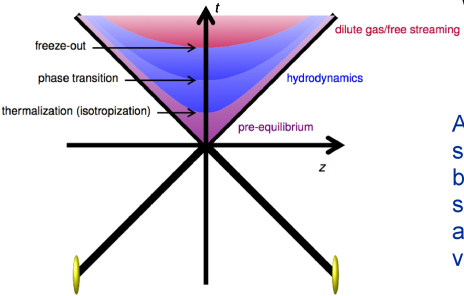
K. Jensen, M. Kaminski, P. Kovtun, R.M., A. Ritz, A. Yarom, arXiv:1112.4498 [hep-th] , to appear in JHEP

Outline

Strong (hydro)dynamics

Hydrodynamics is a universal low energy effective theory for scales $L \gg \ell_{mfp}$ over which an interacting system can achieve local thermal equilibrium

But its application to strongly-interacting theories is nontrivial
➠ requires **thermodynamic functions** & ***transport coefficients***.



AdS/CFT has provided strong-coupling "data" for both, e.g. for models of the sQGP which appears to be a (nearly) perfect fluid with very low η/s.

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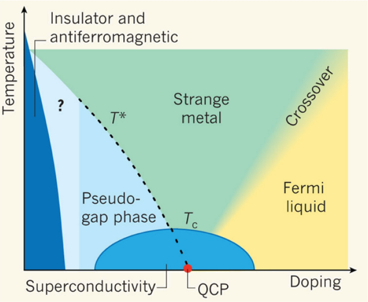

We can ask related questions about the hydrodynamic regime of strongly-correlated systems in 2+1D (e.g. strange metal phase).

➠ Focus of this talk is 2+1D hydrodynamics, emphasizing the role of *parity*[(#)], with applications to (anomalous) Hall transport.

(#) In 2+1D, P: $x^1 \rightarrow -x^1$, analogous to T, and pseudoscalars like magnetic field or vorticity can play an important role.

Strong (hydro)dynamics

- Hall Viscosity from Holography Son-Saremi '11

Break parity through a gravitational axion

$$S = S_{grav}[g,\theta] - \frac{\lambda}{4}\int d^4x\sqrt{-g}\theta(r)\epsilon^{\lambda\rho\alpha\beta}R^{\mu}{}_{\nu\alpha\beta}R^{\nu}{}_{\mu\lambda\rho}$$

With a black brane Ansatz of the form

$$ds^2 = 2H(r)dvdr - r^2f(r)dv^2 + r^2dx_m dx^m$$

the Hall viscosity is

$$\eta_H = -\frac{\lambda}{8\pi G_N}\left.\frac{r^4 f'(r)\theta'(r)}{4H^2(r)}\right|_{r=r_H}$$

Strong (hydro)dynamics

- Another parity breaking term: Gauge axion

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- New dissipationless transport coefficients

Outline

Ideal hydrodynamics

- Conservation laws:

$$\partial_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho \qquad \partial_\mu J^\mu = 0$$

- Constitutive relations (*assuming* local equilibrium):

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p P^{\mu\nu} \qquad J^\mu = \rho u^\mu$$

$$\text{NB}: \quad u^\mu u_\mu = -1, \qquad P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$$

Systematic Corrections

Near equilibrium *iff* $t_{\text{therm}} \ll \Delta t, \Delta l/v$ derivative expansion

$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq}} + \pi^{\mu\nu} \qquad J^{\mu} = J^{\mu}_{\text{eq}} + \nu^{\mu}$$

$$\text{Landau frame}: u_\mu \pi^{\mu\nu} = u_\mu \nu^\mu = 0$$

Dissipative corrections (*assuming P invariance!*)

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \zeta P^{\mu\nu}\partial\cdot u \qquad \sigma^{\mu\nu} \equiv \partial^{<\mu}u^{\nu>}$$

$$\nu^\mu = \sigma P^{\mu\nu}\left(E_\mu - T\partial_\mu \frac{\mu}{T}\right)$$

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Transport coefficients,
positive for $\partial \cdot S \geq 0$

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- Decomposition of $T^{\mu\nu}$ and J^μ with respect to u^μ

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P} P^{\mu\nu} + (q^\mu u^\nu + q^\nu u^\mu) + \pi^{\mu\nu}$$
$$J^\mu = \mathcal{N} u^\mu + \nu^\mu$$

• q^μ and ν^μ are transverse, $\pi^{\mu\nu}$ transverse symmetric traceless

• $\mathcal{E}$, $\mathcal{P}$, q^μ etc. are local functions of T, μ, u^μ and their derivatives

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$$(T, \mu, u^{\mu}) \mapsto (T + \delta T, \mu + \delta\mu, u^{\mu} + \delta u^{\mu})$$

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- Choice of frame: Landau frame

  • Choose u^{μ} = velocity of energy flow $\Rightarrow$ $\boxed{q^{\mu} = 0}$

  • Choose T s.t. $\mathcal{E} = \epsilon_0$ local thermodynamic energy density

  • Choose μ s.t. $\mathcal{N} = \rho_0$ local thermodynamic charge density

$\Rightarrow$ Unique derivative expansion

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  1. Write down a 1-derivative parity odd tensor basis
  2. Entropy current with positive divergence
  3. Use Onsager relations/susceptibility & positivity constraints to further constrain remaining magnetizations

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- Independent (pseudo)scalars, vectors, tensors

scalars	pseudoscalars	transverse vectors	tensors
$\nabla_\mu u^\mu$	$\Omega = -\epsilon^{\mu\nu\rho} u_\mu \nabla_\nu u_\rho$	$V_1^\mu = P^{\mu\nu}\nabla_\nu T$	$\sigma^{\mu\nu}$
	$B = -\frac{1}{2}\epsilon^{\mu\nu\rho} u_\mu F_{\nu\rho}$	$V_2^\mu = F^{\mu\nu} u_\nu = E^\mu$	
		$V_3^\mu = E^\mu - TP^{\mu\nu}\nabla_\nu \frac{\mu}{T}$	

Pseudovectors and pseudotensors are defined via

$$\tilde{V}_i^\mu = \epsilon^{\mu\nu\rho} u_\nu V_{i,\rho}\,, \quad \tilde{\sigma}^{\mu\nu} = \frac{1}{2}(\epsilon^{\mu\alpha\beta} u_\alpha \sigma_\beta{}^\nu + \epsilon^{\nu\alpha\beta} u_\alpha \sigma_\beta{}^\mu)$$

- This is a minimal set of independent quantities.

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The constitutive relations then are

$$
\begin{aligned}
T^{\mu\nu} &= \epsilon_0 u^\mu u^\nu + (P_0 - \tilde{\chi}_B B - \tilde{\chi}_\Omega \Omega - \zeta \nabla_\mu u^\mu) P^{\mu\nu} + \pi^{\mu\nu} \\
J^\mu &= \rho_0 u^\mu + \nu^\mu \\
\pi^{\mu\nu} &= -\eta \sigma^{\mu\nu} - \eta_H \tilde{\sigma}^{\mu\nu} \\
\nu^\mu &= \sigma V_3^\mu + \chi_E E^\mu + \chi_T V_1^\mu + \left[\tilde{\sigma} \tilde{V}_3^\mu + \tilde{\chi}_E \tilde{E}^\mu + \tilde{\chi}_T \tilde{V}_1^\mu\right] \\
dP_0 &= s_0 dT + \rho_0 d\mu \,, \quad \epsilon_0 + P_0 = s_0 T + \rho_0 \mu \,.
\end{aligned}
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- Positivity of divergence entropy current $\Rightarrow$ Second law of thermodynamics

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- In the presence of parity breaking, the canonical entropy current could be corrected Landau Lifshitz 6, Minwalla-Yarom-etal 1105.3733

$$J^\mu_S = \underbrace{s_0 u^\mu - \frac{\mu}{T}\Upsilon^\mu - \frac{u_\nu}{T}\tau^{\mu\nu}}_{J^\mu_{S,can}} + \left(\substack{\text{all possible} \\ \text{single gradient} \\ \text{3-vectors}}\right)$$

NB: Υ and τ include the derivative corrections π and ν plus (magn. and vortical) magnetization contributions at $\mathcal{O}(\partial^1)$

Remarks

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- More convenient basis for first order pseudovectors :

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\tilde{V}_1^\mu = \epsilon^{\mu\nu\rho} u_\nu \nabla_\rho T = -T \tilde{V}_1^\mu + R_0 T \tilde{V}_3^\mu \\
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\tilde{V}_4^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho} F_{\nu\rho} = \tilde{V}_2^\mu + u^\mu B \\
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- Ansatz for the entropy current

$$J_s^\mu = J_{s,can}^\mu + \nu_0(\mu, T)(\nabla \cdot u) u^\mu + \sum_{i=1}^{3} \nu_i(\mu, T) V_i^\mu + \sum_{i=1}^{5} \tilde{\nu}_i(\mu, T) \tilde{V}_i^\mu$$

Remarks

- Schematic structure of the entropy current divergence

$$\nabla_\mu J_s^\mu = \begin{pmatrix}\text{products of}\\ \text{first order}\\ \text{data}\end{pmatrix} + \begin{pmatrix}\text{second order}\\ \text{scalar data}\end{pmatrix}$$

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- Second order contributions have to vanish separately, severly restricting the form of the entropy current. Coupling to a curved background is important: [Minwalla-Yarom 1105.3733]

$$\begin{aligned}\nabla_\alpha J_s^\alpha &= +\left(\nu_2 - \frac{\nu_3}{T}\right)\nabla_\mu E^\mu + \nu_3 \Delta^{\mu\nu}\nabla_\mu \partial_\nu \frac{\mu}{T}\\ &\quad + (\nu_0 + \nu_1)\, u^\alpha \nabla_\alpha \nabla_\mu u^\mu - \nu_1 u^\alpha u^\mu R_{\alpha\mu}\\ &\quad - \tilde{\nu}_2 u^\alpha \nabla_\alpha B + \left(\begin{smallmatrix}\text{products of}\\ \text{first order data}\end{smallmatrix}\right)\\ \Rightarrow &\quad \nu_0 = \nu_1 = \nu_2 = \nu_3 = \tilde{\nu}_2 = 0\end{aligned}$$

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- The products of first order data have to either vanish or be complete squares.
- Ambiguity : $J_s^\mu \to J_s^\mu + \epsilon^{\mu\nu\rho}\nabla_\nu(\tilde{\alpha} u_\rho)$

  Invariants $\tilde{\nu}_4$, $\partial_T \tilde{\nu}_5 + \tilde{\nu}_1$, $\partial_{\frac{\mu}{T}} \tilde{\nu}_5 + \tilde{\nu}_3$

Results

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- Parametrize two invariants with magnetizations

$$T\tilde{\nu}_4 = \mathcal{M}_B\,, \quad \partial_{\frac{\mu}{T}}\tilde{\nu}_5 + \tilde{\nu}_3 = \frac{1}{T}\partial_{\frac{\mu}{T}}\mathcal{M}_\Omega - \mathcal{M}_B$$

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- Constraint fixes the third invariant up to a free function

$$\frac{\partial}{\partial\mu}\left(T^2(\partial_T\tilde{\nu}_5 + \tilde{\nu}_1) - T\partial_T\mathcal{M}_\Omega + 2\mathcal{M}_\Omega\right) = 0$$

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- The role of $f_\Omega(T)$ remains unclear so far. In the magnetovortical frame it contributes to the vorticity magnetization subtraction of the energy density.

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- Transport Coefficients in Landau Frame:

$$(\eta, \zeta, \sigma) \geq 0\,, \qquad \tilde{\sigma}, \tilde{\eta} \in \mathbb{R}\,, \quad \chi_E = \chi_T = 0$$

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\tilde{\chi}_\Omega &= \frac{\partial P_0}{\partial \epsilon_0}\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu\frac{\partial \mathcal{M}_\Omega}{\partial \mu} + f_\Omega(T) - 2\mathcal{M}_\Omega\right) + \frac{\partial P_0}{\partial \rho_0}\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right) \\
\tilde{\chi}_E &= \frac{\partial \mathcal{M}_B}{\partial \mu} - R_0\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right) \\
T\tilde{\chi}_T &= \left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu\frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B\right) - R_0\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu\frac{\partial \mathcal{M}_\Omega}{\partial \mu} + f_\Omega(T) - 2\mathcal{M}_\Omega\right)
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- $\eta, \zeta, \sigma, \tilde{\eta}, \tilde{\sigma}$ are transport coefficients (encode dynamical information)

Results

- Transport Coefficients in Landau Frame:

$$(\eta, \zeta, \sigma) \geq 0\,, \qquad \tilde{\sigma}, \tilde{\eta} \in \mathbb{R}\,, \quad \chi_E = \chi_T = 0$$

$$\begin{aligned}
\tilde{\chi}_B &= \frac{\partial P_0}{\partial \epsilon_0}\left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu\frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B\right) + \frac{\partial P_0}{\partial \rho_0}\frac{\partial \mathcal{M}_B}{\partial \mu} \\
\tilde{\chi}_\Omega &= \frac{\partial P_0}{\partial \epsilon_0}\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu\frac{\partial \mathcal{M}_\Omega}{\partial \mu} + f_\Omega(T) - 2\mathcal{M}_\Omega\right) + \frac{\partial P_0}{\partial \rho_0}\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right) \\
\tilde{\chi}_E &= \frac{\partial \mathcal{M}_B}{\partial \mu} - R_0\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right) \\
T\tilde{\chi}_T &= \left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu\frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B\right) - R_0\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu\frac{\partial \mathcal{M}_\Omega}{\partial \mu} + f_\Omega(T) - 2\mathcal{M}_\Omega\right)
\end{aligned}$$

- Incidentally also frame-invariant!
- $\eta, \zeta, \sigma, \tilde{\eta}, \tilde{\sigma}$ are transport coefficients (encode dynamical information)
- $\tilde{\chi}_B, \tilde{\chi}_E, \tilde{\chi}_T, \tilde{\chi}_\Omega$ are thermodynamic response parameters (calculable from Euclidean Greens functions)

Outline

Linearized Hydrodynamics

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- In the presence of small sources $(A_\mu, h_{\mu\nu})$ the hydrodynamic equations of motion can be solved in linear response theory.

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  Positivity of spectral function, Onsager relations, Reproduce thermodynamic susceptibilities
- Can those reproduce the entropy current results? Can we further determine $\mathcal{M}_B, \mathcal{M}_\Omega, f_\Omega$, such as e.g. $\mathcal{M}_B = \frac{dP}{dB}$?

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Constraints from Linearized Hydro

- The Parity even sector :

$$
\begin{aligned}
T^{\mu\nu} &= \epsilon_0 u^\mu u^\nu + (P_0 - \zeta \nabla_\rho u^\rho) P^{\mu\nu} - \eta \sigma^{\mu\nu} \\
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- Direct computation yields

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G_R^{0,0}(0,0) = \left(\frac{\partial \rho}{\partial \mu}\right)_T + \frac{T\left(s\frac{\partial \rho}{\partial \mu} - \rho \frac{\partial s}{\partial \mu}\right)}{(\epsilon+P)\sigma + T\rho\chi_T} \chi_E
$$

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$$
\begin{aligned}
G_R^{12,12}(\omega, \mathbf{k} = 0) &= -P + i\eta\omega + \mathcal{O}(\omega^2) \\
G_R^{1,1}(\omega, \mathbf{k} = 0) &= -\tfrac{\rho^2}{\epsilon + P} + i\sigma\omega + \mathcal{O}(\omega^2) \\
G_R^{11,11}(\omega, \mathbf{k} = 0) &= C + i(\eta + \zeta)\omega + \mathcal{O}(\omega^2)
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- Possible contributions from higher order terms prevent safe application of many retarded correlators in the full theory without knowledge of second-order hydrodynamics.
- Some Kubo Formulas are safe:

$$\mathcal{C}^0 = \left(\frac{\partial P_0}{\partial \rho_0}\right)_{\epsilon_0} \mathcal{J}^0 + \left(\frac{\partial P_0}{\partial \epsilon_0}\right)_{\rho_0} \mathcal{T}^{00}, \qquad \mathcal{C}^i = \mathcal{J}^i - R_0 \mathcal{T}^{0i}$$

$$\begin{aligned}
\tilde{\chi}_B &= \lim_{k\to 0} \frac{1}{ik} \langle \mathcal{C}^0 \mathcal{J}^2 \rangle_R(0,k) = \lim_{k\to 0} \frac{1}{ik} \left(\frac{\partial P_0}{\partial \rho_0} G_R^{0,2}(0,k) + \frac{\partial P_0}{\partial \epsilon_0} G_R^{00,2}(0,k) \right) \\
\tilde{\chi}_\Omega &= \lim_{k\to 0} \frac{1}{ik} \langle \mathcal{C}^0 \mathcal{T}^{02} \rangle_R(0,k) = \lim_{k\to 0} \frac{1}{ik} \left(\frac{\partial P_0}{\partial \rho_0} G_R^{0,02}(0,k) + \frac{\partial P_0}{\partial \epsilon_0} G_R^{00,02}(0,k) \right) \\
\tilde{\chi}_E &= -\lim_{k\to 0} \frac{1}{ik} \langle \mathcal{C}^2 \mathcal{J}^0 \rangle_R(0,k) = \lim_{k\to 0} \frac{1}{ik} \left(G_R^{0,2}(0,k) - R_0 G_R^{0,02}(0,k) \right) \\
T\tilde{\chi}_T &= -\lim_{k\to 0} \frac{1}{ik} \langle \mathcal{C}^2 \mathcal{T}^{00} \rangle_R(0,k) = \lim_{k\to 0} \frac{1}{ik} \left(G_R^{00,2}(0,k) - R_0 G_R^{00,02}(0,k) \right)
\end{aligned}$$

The Parity Even and Odd Sectors

- Kubo Formulas for parity-odd transport coefficients

$$\tilde{\eta} = \lim_{\omega \to 0} \frac{1}{4\omega} \delta_{ik} \epsilon_{jl} \, \mathrm{Im}\, G_R^{ij,kl}(\omega, 0)$$

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$$\boxed{G_R^{ij}(\omega,\mathbf{k};b_a) = n_i n_j G_R^{ji}(\omega,-\mathbf{k};-b_a) \quad (\Theta\mathcal{O}_i\Theta^{-1} = n_i\mathcal{O}_i)}$$

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- Relating $\langle\mathcal{C}^0\mathcal{C}^2\rangle$ to $\langle\mathcal{C}^2\mathcal{C}^0\rangle$ in this way yields

$$\tilde{\chi}_B - \frac{\rho_0}{\varepsilon_0+P_0}\tilde{\chi}_\Omega = \frac{\partial P_0}{\partial\rho_0}\tilde{\chi}_E + \frac{\partial P_0}{\partial\varepsilon_0}T\chi_T$$

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- Satisfied by the results from the entropy current

$\rightarrow$ Consistency check

Outline

The Magnetovortical Frame

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- Equilibrium constitutive relations are unknown, but can be parametrized by functions of μ, T

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- Magnetic Subtractions: $x_B = r_\Omega = \frac{\partial P}{\partial B}$

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- Used Kubo formulas for $\tilde{\chi}_\Omega, \tilde{\chi}_B, \tilde{\chi}_T, \tilde{\chi}_E$, which are frame independent , to match the expressions for $\tilde{\chi}_\Omega, \tilde{\chi}_B, \tilde{\chi}_T, \tilde{\chi}_E$ in terms of $\mathcal{M}_B, \mathcal{M}_\Omega, f_\Omega$ to this parametrization:

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\begin{aligned}
x_B &= \frac{\partial P}{\partial B}\,, \quad x_\Omega = \frac{\partial P}{\partial \Omega} \\
\mathcal{M}_B &= \frac{\partial P}{\partial B} + h_B(\mu)\,, \quad \mathcal{M}_\Omega = \frac{\partial P}{\partial \Omega} + h_\Omega(\mu, T) \\
e_B &= h_B - \mu h_B'\,, \quad r_B = -h_B'\,, \quad e_\Omega\,,\ r_\Omega \text{ undetermined}
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- If we match to the literature and set $h_B = 0$, and furthermore conjecture the same for vorticity,

$$r_\Omega = \mathcal{M}_B\,, \quad e_\Omega = \mathcal{M}_\Omega - f_\Omega(T)\,, \quad h_\Omega = 0$$

we fully define a different hydrodynamic frame, the magnetovortical frame.

The Magnetovortical Frame

- The constitutive relations in the magnetovortical frame read

$$
\begin{aligned}
T^{\mu\nu} &= \left(\epsilon - \left(\mathcal{M}_\Omega - f_\Omega\right)\Omega\right) u^\mu u^\nu + \left(P - \left(\mathcal{M}_B B + \mathcal{M}_\Omega \Omega\right)\right) \Delta^{\mu\nu} \\
J^\mu &= \left(\rho - \mathcal{M}_B \Omega\right) u^\mu \\
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\end{aligned}
$$

- Consistency Check: Starting from these relations, the susceptibility results

$$
\begin{aligned}
\tilde{\chi}_B &= \frac{\partial P_0}{\partial \epsilon_0}\left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu\frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B\right) + \frac{\partial P_0}{\partial \rho_0}\frac{\partial \mathcal{M}_B}{\partial \mu} \\
\tilde{\chi}_\Omega &= \frac{\partial P_0}{\partial \epsilon_0}\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu\frac{\partial \mathcal{M}_\Omega}{\partial \mu} + f_\Omega(T) - 2\mathcal{M}_\Omega\right) + \frac{\partial P_0}{\partial \rho_0}\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right) \\
\tilde{\chi}_E &= \frac{\partial \mathcal{M}_B}{\partial \mu} - R_0\left(\frac{\partial \mathcal{M}_\Omega}{\partial \mu} - \mathcal{M}_B\right) \\
T\tilde{\chi}_T &= \left(T\frac{\partial \mathcal{M}_B}{\partial T} + \mu\frac{\partial \mathcal{M}_B}{\partial \mu} - \mathcal{M}_B\right) - R_0\left(T\frac{\partial \mathcal{M}_\Omega}{\partial T} + \mu\frac{\partial \mathcal{M}_\Omega}{\partial \mu} + f_\Omega(T) - 2\mathcal{M}_\Omega\right)
\end{aligned}
$$

could have been reproduced from linearized hydro alone.

Outline

A Strongly Coupled Example

- A strongly interacting matter example:

$$S = S_{grav} - \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,\frac{F^2}{4} - \frac{1}{64\pi^2}\int \theta(\phi) F \wedge F$$

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- Fluid-Gravity Correspondence $\Rightarrow$ Constitutive Relations

$$\tilde{\sigma} = \frac{\theta(\phi(r_h))}{8\pi^2} - \frac{\partial\rho}{\partial B} + \mathcal{O}(\mu^2, J_\phi^2)\,, \quad \tilde{\chi}_E = \frac{\partial\rho}{\partial B} + \mathcal{O}(\mu^2, J_\phi^2)$$

$$\sigma = \frac{1}{16\pi G_N} + \mathcal{O}(\mu^2, J_\phi^2)\,, \quad T\tilde{\chi}_T = \frac{\partial\epsilon}{\partial B} + \mathcal{O}(\mu^3, J_\phi^2)$$

$\Rightarrow$ Anomalous Hall Conductivity: $\tilde{\sigma} + \tilde{\chi}_E = \frac{\theta(\phi(r_h))}{8\pi^2}$

Outline

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