Einstein-Maxwell-Dilaton theories: holographic applications and generalised dimensional reduction

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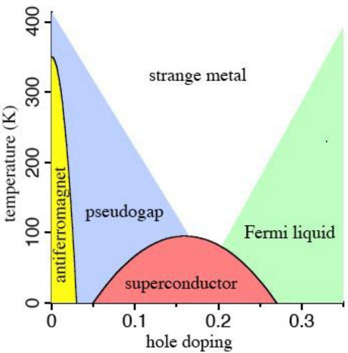
arXiv:1110.2320, arXiv:1107.2116, arXiv:1005.4690 and ongoing

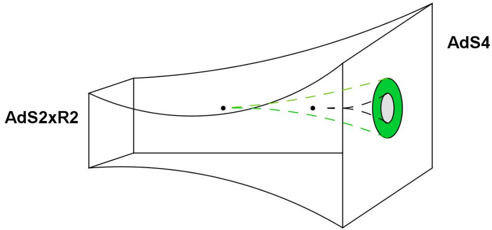
Strong coupling in Condensed Matter Systems



- Some materials (e.g. high T_c or heavy fermion superconductors) display unconventional behaviour in the normal phase: strange metals.
  - non-Fermi Liquid (no weakly-coupled quasiparticles).
  - $C_\mu \sim \sigma_{DC}^{-1} \sim T \quad (\omega << T << \mu)$.
  - $\sigma_{AC} \sim \omega^{-2/3} \quad (T \lesssim \omega << \mu)$.

- Unconventional behaviours at low T ($T << \mu$) in CMS often controlled by Quantum Critical points (T=0).
- QC points cannot be described by a theory of weakly-coupled particles. Strong coupling $\Rightarrow$ Holography?

Holography in AdS spacetimes

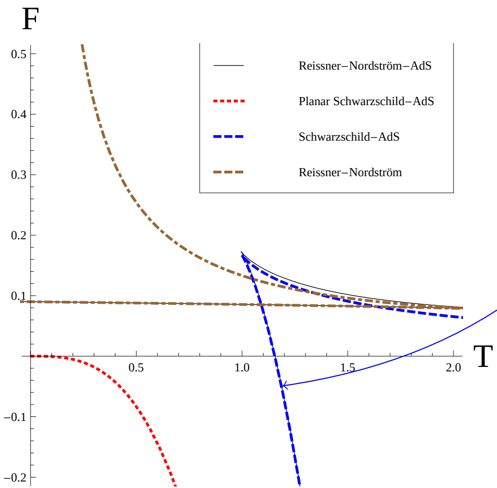


$$S = \int \mathrm{d}^4x\sqrt{-g}\left(R - 2\Lambda - \tfrac{1}{4}F^2\right)$$

$$\mathrm{d}s^2 = \tfrac{1}{r^2}\left(-\mathrm{d}t^2 + \mathrm{d}r^2 + \mathrm{d}x^2 + \mathrm{d}y^2\right)$$

- Canonical duality ([MALDACENA98]): sYM(N=4) in **strong coupling** and supergravity on $AdS_5 \times \mathbf{S}^5$ (weak coupling).
- **Relativistic scale invariance**: $(r, x, y, t) \to \lambda(r, x, y, t) \Rightarrow$ UV-IR relation: the radial bulk coordinate r **geometrises** the boundary RG flow.
- Local U(1) gauge field in the bulk $\Leftrightarrow$ global gauge field on the boundary: finite density of boundary carriers.

Holography in AdS spacetimes



- Above T_c, HP phase transition between thermal AdS and large AdS black holes.
- [Witten98]: formation of a bulk event horizon ⇔ deconfined boundary phase.
- Flat AdS boundary: destroys HP phase transition.
- Need to simulate horizon curvature: can be done with a scalar (relevant) operator and exponential potential: [Gursoy,Kiritsis, Mazzanti&Nitti.08].

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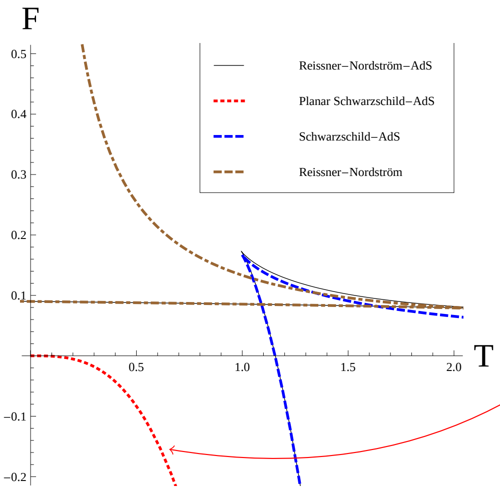

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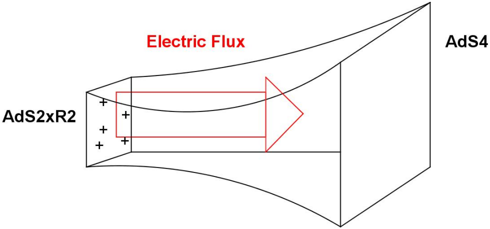
Relativistic IR fixed points



The AdS_4 charged black hole **collapses** to an $AdS_2 \times \mathbf{R}_2$ metric in the IR:
$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2\right) + dx^2 + dy^2$

Emergent IR scaling behaviour:
Non-Fermi liquid [Lee08, Liu&al,Zaanen&al.09]
Charged horizon: $\lim_{r\to 0} \int_{\mathbf{R}_2} \star F = ct$

But

- finite entropy at zero temperature;
- $\rho \sim \langle J^t \rangle^{-1} T^2$, $\sigma_{AC} \sim \omega^2$
- unstable to particle pair production near the horizon: condensation of complex scalar (superfluid, [Hartnoll&al08]); electron stars, [Hartnoll&al10,11].
- Emergent IR Lifshiz scaling, [Horowitz&Roberts09,Gubser&Nellore09], [Tong&al09].

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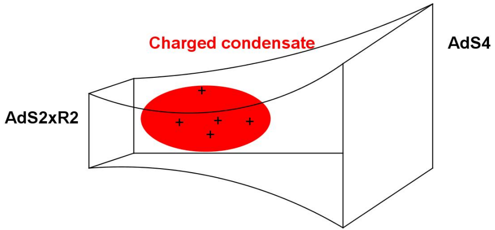

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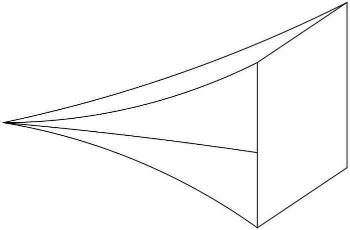
Non-relativistic fixed points [Balasubramanian&McGreevy,Kachru&al,Son08]



$$S=\int\sqrt{-g}\left(R-2\Lambda-\frac{F^2}{4}-m^2A^2\right)$$

$$\mathrm{d}s^2=-\frac{\mathrm{d}t^2}{r^{2z}}+\frac{\mathrm{d}r^2+\mathrm{d}x^2+\mathrm{d}y^2}{r^2}\,,\ A_t\sim r^{-z}$$

Anisotropic scaling
$(x,y)\to\lambda(x,y)$, $t\to\lambda^z t$

Hyperscaling: $S\sim T^{\frac{2}{z}}$

- Zero horizon electric flux: $lim_{r\to\infty}\int_{\mathbf{R}_2}\star F=0$, $S(T=0)=0$.
- $\rho\sim\langle J^t\rangle^{-1}T^{2/z}$, $\sigma_{AC}\sim\omega^2$ [Hartnoll&al08].
- **'Mesonic' phase**, [Hartnoll&Huijse11]: charge carried by composite boundary gauge-invariant operators $\leftrightarrow$ charged matter in the bulk.
- But geodesically incomplete: unstable? [Horowitz&Way11]
- Can one have both $S(T=0)=0$ and charge carried by the horizon?

Hyperscaling violation and fractionalisation

- **Fractionalised phases**: charge carried by boundary gauge-charged operators ⇒ introduce a bulk neutral scalar: drives the IR dynamics (Effective Holographic Theories, [BG,Kiritsis&al10])
- EMD theories:

$$S_{EMD} = \int \mathrm{d}^4x \sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}\mathrm{e}^{\gamma\phi}F^2 - 2\Lambda e^{-\delta\phi}\right]$$

- $\delta = 0$: Lifshitz IR (hyper)scaling, [Taylor08,Kachru&al09].

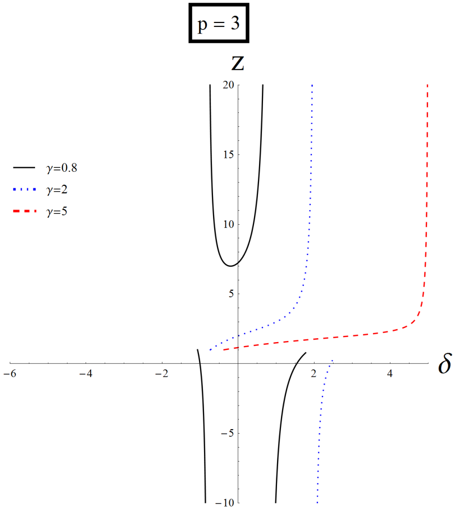
$$z = 1 + \frac{4}{\gamma^2} \geq 1$$

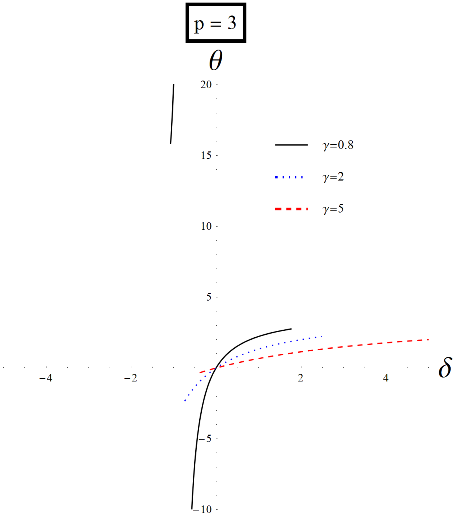
- $\delta \neq 0$: hyperscaling violating IR fixed points, [BG,Kiritsis&al10, BG&Kiritsis, Sachdev&al.11, Kachru&al.12]

$$\mathrm{d}s^2 = r^\theta\left[-\frac{\mathrm{d}t^2}{r^{2z}} + \frac{\mathrm{d}r^2 + \mathrm{d}x^2 + \mathrm{d}y^2}{r^2}\right], \quad e^{\delta\phi} \sim r^\theta$$

- conformal to Lifshitz [Perlmutter10,BG&Kiritsis11]

$T = 0$ IR fixed point: generalised Lifshitz structure



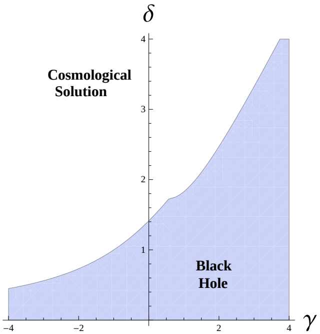


Generalised IR fixed point at finite temperature

$$\mathrm{d}s^2_{(4)} = -V(p)\,\mathrm{e}^{\gamma\phi}\mathrm{d}t^2 + \frac{u\mathrm{e}^{\delta\phi}\mathrm{d}p^2}{w(-\Lambda)V(p)} + \mathrm{e}^{(\delta-\gamma)\frac{\phi}{2}}\mathrm{d}\mathbf{R}^2_{(2)}$$

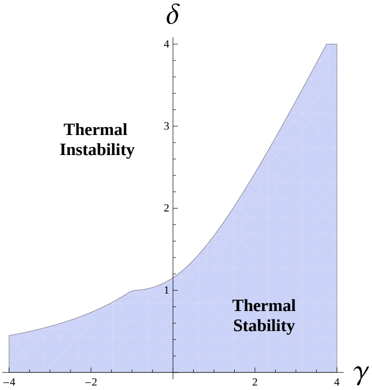
$$V(p) = p(p-2m)\,, \quad \mathrm{e}^{\phi} = p^{\frac{-4(\gamma-\delta)}{w}}\,, \quad A = 2\sqrt{\frac{-v}{w}}p\,\mathrm{d}t$$

$$u = \gamma^2 - \gamma\delta + 2\,, \quad v = \delta^2 - \gamma\delta - 2\,, \quad w = 3\gamma^2 - \delta^2 - 2\gamma\delta + 4$$



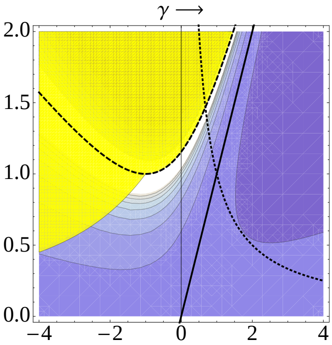
- 1 singularity ($p=0$), 1 event horizon ($p=2m$) if $u>0$, $v<0$, $w>0$ (else, naked cosmological singularity); singular extremal limit $m=0$.
- $\gamma=\delta$: $AdS_2\times\mathbf{R}^2$.
- $\delta=0$: Lifshitz solution, $z = 1+\frac{4}{\gamma^2} > 1$ [Taylor08,Kachru et al.09]
- Arbitrary γ,δ: **unusual** asymptotics.

Thermodynamics



- Single branch of solutions
- $sign[F] \sim -sign[w - 2(v+u)]$
- $sign[C_T] \sim sign[w - 2(v+u)]$
- $w - 2(v+u) > 0$: thermally stable
- $w - 2(v+u) < 0$: thermally unstable
- No phase transitions
- If $w = 4(v+u)$, $C_T \sim T$

AC and DC conductivity



Contour levels of the scaling exponent n_{AC} in the $(\gamma\,,\,\delta)$ plane

- Yellow region: unstable.
- Lighter regions: higher scaling exponent ($n_{AC} > 1.5$).

- Fluctuations of A_i and g_{ti}: sources in the boundary theory. $\sigma_x \sim \omega^{n_{AC}}$
- Strange metals: $n_{AC} < 0\,(\sim -0.66)$.
- Einstein-Maxwell case: NHEL $AdS_2 \times \mathbf{R}^2 \Rightarrow \sigma_x \sim \omega^2$.
- EMD: non-conventional NHE geometry $\Rightarrow$ non-conventional scaling.
- Charged EMD background: Stable region $\Rightarrow n_{AC} > 0$
  $\gamma = \delta\ (AdS_2 \times \mathbf{R}^2) \Rightarrow n_{AC} = 2$
- drag force result: $\rho \sim \sigma_{DC}^{-1} \sim C_\mu \sim T^{\frac{2(v+u)}{w-2(v+u)}}$
  Linear scaling if $w = 4(v+u)$.

Some questions

1. Can we set up holography for such non-trivial, non-conformal spacetimes?
2. Can we explain the scaling properties of the EMD backgrounds ?
3. Can we explain the scaling of their thermodynamics and of their transport coefficients?

Holography for Einstein-Dilaton theories [Kanitscheider&Skenderis09]

- ED theories are by *diagonal reductions* of AdS_{n+p+1} over R^n

$$S = \int \mathrm{d}^{n+p+1}x\,[R - 2\Lambda] \Rightarrow S = \int \mathrm{d}^{p+1}x\left[R - \frac{1}{2}\partial\phi^2 - 2\Lambda e^{-\delta\phi}\right]$$

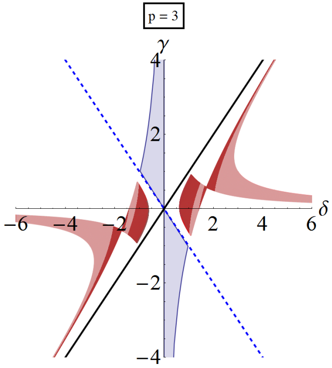
- It is a consistent truncation: all $(p+1)$ solutions are $(n+p+1)$ solutions.
- The eoms depend smoothly on n and can be analytically continued: the reduction is generalised.

$$\delta^2 = \frac{2n}{(p-1)(n+p-1)} \in \mathbb{R}\,, \qquad 0 \leq \delta^2 \leq \frac{2}{p-1}.$$

- In that range, the holographic dictionary (asymptotics, counterterms, 1-point functions, etc.) for ED spacetimes follows straightforwardly from the AdS case.
- The dilatonic planar black hole in $p+1$ uplifts to its AdS counterpart in $n+p+1$.
- Conserved charges and hydrodynamics can also be derived.
- $\delta^2 > 2/(p-1)$: Reduction over a sphere from $\Lambda = 0$ GR.

Application to the EMD IR fixed points

$$\mathcal{L}_{(n+4)} = R - \frac{1}{2}\partial\Phi^2 - e^{\Gamma\Phi}F^2_{[q+2]} - 2\Lambda$$



- Solid black line: $AdS_2 \times R_{n+2}$, $\Phi = 0, \Gamma = 0, q = 0$
- Dotted blue line: $AdS_{n+2} \times R_2$, $\Phi = 0, \Gamma = 0, q = n$
- Blue region: Dilatonic Lifshitz solution $z = 1 + \frac{2(n+2)}{\Gamma^2}$, $q = 0$
- Red region: Dilatonic 2-brane , $q = 0$, $\Lambda = 0$
- All lower-dimensional scalings derive from the usual higher-dimensional ones.
- The theory effectively lives in $4 + n$ dimensions.

Conclusions

- Charged horizons can be associated with fractionalised phases, with non-trivial IR emergent scalings.
- The near-extremal geometry of EMD theories is **universal**: it encompasses all main types of IR fixed points (AdS, Lifshitz, hyperscaling violating).
- Interesting laboratories for the recovery of strongly-coupled Condensed Matter phenomena, such as **strange metallicity**.
- Generalised dimensional reduction can be used to put the non-conformal holographic dictionary on firmer grounds, and explain the lower-dimensional scalings. It provides a fast way of computing interesting quantities (thermodynamics, hydrodynamics).
- **Hidden fermi surfaces**, [Sachdev&al11]: $\theta = 1$. Uplift?