Real-time dynamics of the quark-gluon plasma from the lattice

Marco Panero

Instituto de Física Teórica, Universidad Autónoma de Madrid/CSIC

Heraklion, 3 December 2013

Outline

Outline

Motivation—The five W's and one H

What?	At temperatures T of the order of 200 MeV, hadrons give way to a deconfined state of matter: the quark-gluon plasma (QGP)
When?	Believed to have existed in Nature until a few microseconds after the Big Bang
Where?	Reproduced in heavy-ion collision experiments, in which a *strongly coupled* QGP is *indirectly* observed
Why?	Theoretical understanding of the *dynamical* evolution of the QGP is crucial
Who?	Based on work in collaboration with Kari Rummukainen (Helsinki) and Andreas Schäfer (Regensburg), arXiv:1307.5850
How?	Non-perturbative, first-principle approach via lattice simulations

Motivation—The five W's and one H

What? • At temperatures T of the order of 200 MeV, hadrons give way to a deconfined state of matter: the quark-gluon plasma (QGP)

When? • Believed to have existed in Nature until a few microseconds after the Big Bang

Where? • Reproduced in heavy-ion collision experiments, in which a *strongly coupled* QGP is *indirectly* observed

Why? • Theoretical understanding of the *dynamical* evolution of the QGP is crucial

Who? • Based on work in collaboration with Kari Rummukainen (Helsinki) and Andreas Schäfer (Regensburg), arXiv:1307.5850

How? • Non-perturbative, first-principle approach via lattice simulations

Motivation—The five W's and one H

What? • At temperatures T of the order of 200 MeV, hadrons give way to a deconfined state of matter: the quark-gluon plasma (QGP)

When? • Believed to have existed in Nature until a few microseconds after the Big Bang

Where? • Reproduced in heavy-ion collision experiments, in which a *strongly coupled* QGP is *indirectly* observed

Why? • Theoretical understanding of the *dynamical* evolution of the QGP is crucial

Who? • Based on work in collaboration with Kari Rummukainen (Helsinki) and Andreas Schäfer (Regensburg), arXiv:1307.5850

How? • Non-perturbative, first-principle approach via lattice simulations

Motivation—The five W's and one H

What?	• At temperatures T of the order of 200 MeV, hadrons give way to a deconfined state of matter: the quark-gluon plasma (QGP)
When?	• Believed to have existed in Nature until a few microseconds after the Big Bang
Where?	• Reproduced in heavy-ion collision experiments, in which a *strongly coupled* QGP is *indirectly* observed
Why?	• Theoretical understanding of the *dynamical* evolution of the QGP is crucial
Who?	• Based on work in collaboration with Kari Rummukainen (Helsinki) and Andreas Schäfer (Regensburg), arXiv:1307.5850
How?	• Non-perturbative, first-principle approach via lattice simulations

Motivation—The five W's and one H

What?	At temperatures T of the order of 200 MeV, hadrons give way to a deconfined state of matter: the quark-gluon plasma (QGP)
When?	Believed to have existed in Nature until a few microseconds after the Big Bang
Where?	Reproduced in heavy-ion collision experiments, in which a *strongly coupled* QGP is *indirectly* observed
Why?	Theoretical understanding of the *dynamical* evolution of the QGP is crucial
Who?	Based on work in collaboration with Kari Rummukainen (Helsinki) and Andreas Schäfer (Regensburg), arXiv:1307.5850
How?	Non-perturbative, first-principle approach via lattice simulations

Motivation—The five W's and one H

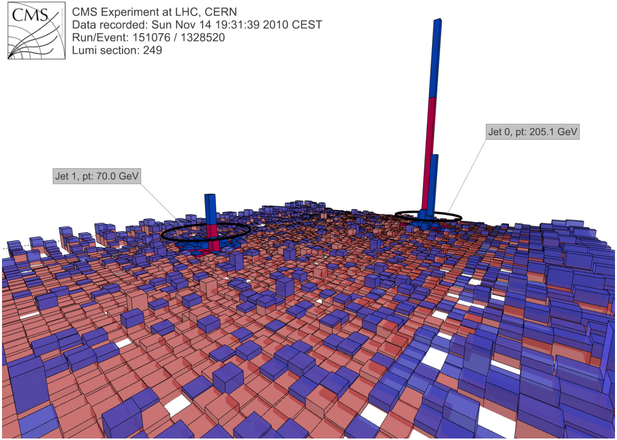
- **What?** At temperatures T of the order of 200 MeV, hadrons give way to a deconfined state of matter: the quark-gluon plasma (QGP)
- **When?** Believed to have existed in Nature until a few microseconds after the Big Bang
- **Where?** Reproduced in heavy-ion collision experiments, in which a *strongly coupled* QGP is *indirectly* observed
- **Why?** Theoretical understanding of the *dynamical* evolution of the QGP is crucial
- **Who?** Based on work in collaboration with Kari Rummukainen (Helsinki) and Andreas Schäfer (Regensburg), arXiv:1307.5850
- **How?** Non-perturbative, first-principle approach via lattice simulations

Motivation—The five W's and one H

- **What?** At temperatures T of the order of 200 MeV, hadrons give way to a deconfined state of matter: the quark-gluon plasma (QGP)
- **When?** Believed to have existed in Nature until a few microseconds after the Big Bang
- **Where?** Reproduced in heavy-ion collision experiments, in which a *strongly coupled* QGP is *indirectly* observed
- **Why?** Theoretical understanding of the *dynamical* evolution of the QGP is crucial
- **Who?** Based on work in collaboration with Kari Rummukainen (Helsinki) and Andreas Schäfer (Regensburg), arXiv:1307.5850
- **How?** Non-perturbative, first-principle approach via lattice simulations

Jet quenching

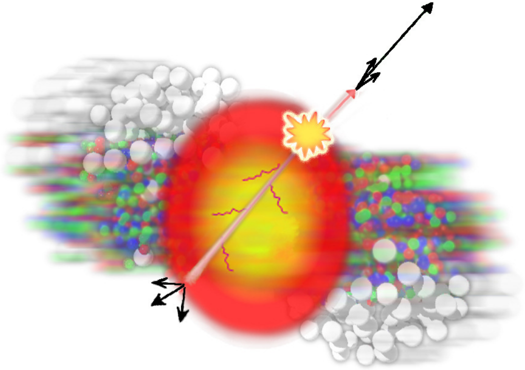
Jet quenching is the suppression of high-p_T particles and back-to-back correlations in nuclear collisions



Jet quenching

Jet quenching is the suppression of high-p_T particles and back-to-back correlations in nuclear collisions

Provides important experimental evidence for the quark-gluon plasma (QGP) existence Bjorken, 1982



Outline

Overview of the theoretical approach

Jet quenching belongs to the class of *hard probes* to heavy-ion collisions, involving a large energy scale Q (see Casalderrey-Solana and Salgado, 2007)

QCD factorization theorems:

$$\sigma_{(M+N\to \text{hadron})} = f_M(x_1, Q^2) \otimes f_N(x_2, Q^2) \otimes \sigma(x_1, x_2, Q^2) \otimes D_{\text{parton}\to\text{hadron}}(z, Q^2)$$

$f_A(x, Q^2)$: parton distribution functions

$\sigma(x, y, Q^2)$: short-distance cross-section

$D_{\text{parton}\to\text{hadron}}(z, Q^2)$: fragmentation function

Overview of the theoretical approach

Jet quenching belongs to the class of *hard probes* to heavy-ion collisions, involving a large energy scale Q (see Casalderrey-Solana and Salgado, 2007)

QCD factorization theorems:

$$\sigma_{(M+N\to\mathrm{hadron})} = f_M(x_1, Q^2) \otimes f_N(x_2, Q^2) \otimes \sigma(x_1, x_2, Q^2) \otimes D_{\mathrm{parton}\to\mathrm{hadron}}(z, Q^2)$$

$f_A(x, Q^2)$: parton distribution functions

$\sigma(x, y, Q^2)$: short-distance cross-section

$D_{\mathrm{parton}\to\mathrm{hadron}}(z, Q^2)$: fragmentation function

Here: Focus on propagation of a high-energy parton in QGP medium

Hard parton propagation in QGP

Multiple soft-scattering description, in the *eikonal approximation* Baier et al., 1997

Leading effect: *transverse momentum broadening*, described by the jet quenching parameter:

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L}$$

Can be evaluated in terms of a *collision kernel* $C(p_\perp)$ (differential parton-plasma constituents collision rate)

$$\hat{q} = \int \frac{\mathrm{d}^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp)$$

$C(p_\perp)$ can be related to a two-point correlator of *light-cone Wilson lines*

Computing the jet quenching parameter

What tools are available?

- Perturbation theory (PT) expansions
  - ✓ Based on first principles
  - ✓ Well established technology
  - ✓ Problems with infrared divergences are well understood
  - ✗ May not be reliable at RHIC or LHC temperatures
- Holographic computations
  - ✓ Mathematically *beautiful*
  - ✓ Ideally suited for strong coupling
  - ✗ Not directly applicable to real-world QCD
- Lattice simulations
  - ✓ Based on first principles
  - ✓ Well established (computer) technology
  - ✓ Do not rely on weak- or strong-coupling assumptions
  - ✗ Euclidean setup, so *generally* unsuitable for real-time phenomena

Outline

Key idea

Energy scale hierarchy in high-temperature, perturbative QCD:

$$g^2T/\pi \text{ (ultrasoft)} \ll gT \text{ (soft)} \ll \pi T \text{ (hard)}$$

IR divergences accounted for by 3D effective theories Braaten and Nieto, 1995 Kajantie et al., 1995:

- electrostatic QCD (3D Yang-Mills + adjoint scalar field) for soft scale
- magnetostatic QCD (3D pure Yang-Mills) for ultrasoft scale

Large NLO corrections hindering PT due to *soft*, essentially *classical* fields

Observation: Soft contributions to physics of light-cone partons *insensitive* to parton velocity $\longrightarrow$ Can turn the problem Euclidean! Caron-Huot, 2008

Proof

Spatially separated ($|t| < |z|$) light-like Wilson lines Ghiglieri *et al.*, 2013

$$G^<(t, x_\perp, z) = \int \mathrm{d}\omega \mathrm{d}^2 p_\perp \mathrm{d}p^z \tilde{G}^<(\omega, p_\perp, p^z) e^{-i(\omega t - x_\perp \cdot p_\perp - z p^z)}$$

$$= \int \mathrm{d}\omega \mathrm{d}^2 p_\perp \mathrm{d}p^z \left[\frac{1}{2} + n_\mathrm{B}(\omega)\right] \left[\tilde{G}_\mathrm{R}(\omega, p_\perp, p^z) - \tilde{G}_\mathrm{A}(\omega, p_\perp, p^z)\right] e^{-i(\omega t - x_\perp \cdot p_\perp - z p^z)}$$

Shift $p'^z = p^z - \omega t/z$, integrate over frequencies by analytical continuation into upper (lower) half-plane for retarded (advanced) contribution $\longrightarrow$ sum over Matsubara frequencies

$$G^<(t, x_\perp, z) = T \sum_{n \in \mathbb{Z}} \int \mathrm{d}^2 p_\perp \mathrm{d}p'^z \tilde{G}_\mathrm{E}(2\pi n T, p_\perp, p'^z + 2\pi i n T t/z) e^{i(x_\perp \cdot p_\perp + z p'^z)}$$

- $n \neq 0$ contributions: exponentially suppressed at large separations
- Soft contribution: from $n = 0$ mode. Time-independent: evaluate in EQCD

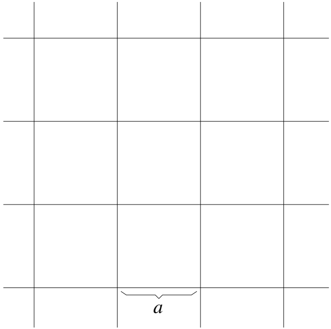
Outline

So, what is this "lettuce cage theory"[1] all about?

[1] Courtesy of Dragon Dictation, as reported by Andreas Kronfeld, 26 November 2013

Basics of lattice QCD – I

- Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing a



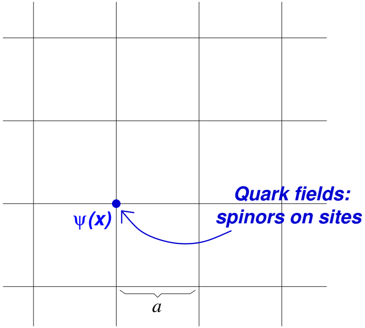
- Transcribe gauge and fermion d.o.f. to lattice elements, build lattice observables
- Isotropic lattice action for Yang-Mills theory Wilson, 1974

$$S = \beta \sum_{\Box} \left(1 - \frac{1}{N} \,\mathrm{Re}\,\mathrm{Tr}\, U_{\Box} \right), \quad \text{with: } \beta = \frac{2N}{g^2} a^{D-4}$$

- A gauge-invariant, non-perturbative regularization
- Exact symmetries at any value of a

Basics of lattice QCD – I

- Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing a
- Transcribe gauge and fermion d.o.f. to lattice elements, build lattice observables



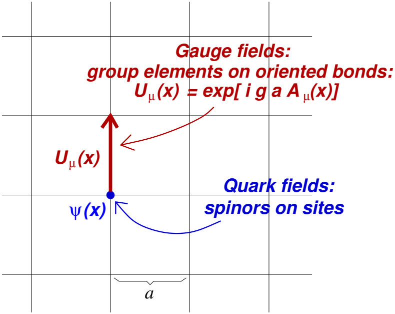
- Isotropic lattice action for Yang-Mills theory Wilson, 1974

$$S = \beta \sum_{\square} \left(1 - \frac{1}{N} \,\mathrm{Re}\,\mathrm{Tr}\, U_{\square} \right), \quad \text{with: } \beta = \frac{2N}{g^2} a^{D-4}$$

- A gauge-invariant, non-perturbative regularization
- Exact symmetries at *any* value of a

Basics of lattice QCD – I

- Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing a
- Transcribe gauge and fermion d.o.f. to lattice elements, build lattice observables



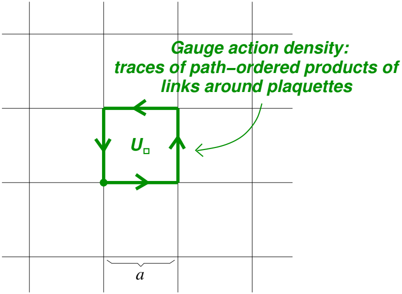
- Isotropic lattice action for Yang-Mills theory Wilson, 1974

$$S = \beta \sum_{\square} \left(1 - \frac{1}{N} \, \mathrm{Re}\, \mathrm{Tr}\, U_{\square} \right), \quad \text{with: } \beta = \frac{2N}{g^2} a^{D-4}$$

- A gauge-invariant, non-perturbative regularization
- Exact symmetries at *any* value of a

Basics of lattice QCD – I

- Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing a
- Transcribe gauge and fermion d.o.f. to lattice elements, build lattice observables



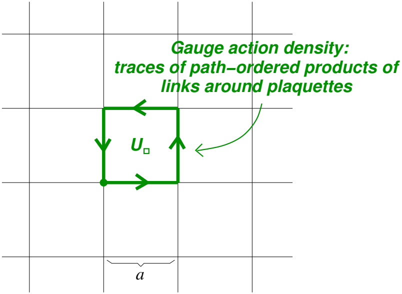
- Isotropic lattice action for Yang-Mills theory Wilson, 1974

$$S = \beta \sum_{\square} \left(1 - \frac{1}{N} \, \mathrm{Re} \, \mathrm{Tr} \, U_\square \right), \quad \text{with: } \beta = \frac{2N}{g^2} a^{D-4}$$

- A gauge-invariant, non-perturbative regularization
- Exact symmetries at *any* value of a
  - Gauge symmetry

Basics of lattice QCD – I

- Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing a
- Transcribe gauge and fermion d.o.f. to lattice elements, build lattice observables



- Isotropic lattice action for Yang-Mills theory Wilson, 1974

$$S = \beta \sum_{\square} \left(1 - \frac{1}{N} \text{ Re Tr } U_\square \right), \quad \text{with: } \beta = \frac{2N}{g^2} a^{D-4}$$

$$S \xrightarrow[a \to 0]{} \frac{1}{2} \int \mathrm{d}^D x \text{ Tr } \left(F_{\mu\nu}^2 \right) \left[1 + O(a^2) \right]$$

- A gauge-invariant, non-perturbative regularization

Basics of lattice QCD – I

- Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing a
- Transcribe gauge and fermion d.o.f. to lattice elements, build lattice observables
- Isotropic lattice action for Yang-Mills theory Wilson, 1974

$$S = \beta \sum_{\square} \left(1 - \frac{1}{N} \, \mathrm{Re} \, \mathrm{Tr} \, U_{\square} \right), \quad \text{with: } \beta = \frac{2N}{g^2} a^{D-4}$$

- A gauge-invariant, non-perturbative regularization
- Exact symmetries at *any* value of a
  - Gauge symmetry
  - Discrete translations
  - Discrete rotations
  - Global discrete symmetries of the continuum ($\mathcal{C}$, $\mathcal{P}$, $\mathcal{T}$, ...)
- Suitable for numerical simulation: Sample configuration space according to a *statistical weight* proportional to $\exp(-S)$
- Expectation values from statistical averages

$$\langle \mathcal{O} \rangle = \frac{\int \prod \mathrm{d}U_{\mu}(x) \mathcal{O} \exp(-S)}{\int \prod \mathrm{d}U_{\mu}(x) \exp(-S)}$$

Basics of lattice QCD – I

- Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing a
- Transcribe gauge and fermion d.o.f. to lattice elements, build lattice observables
- Isotropic lattice action for Yang-Mills theory Wilson, 1974

$$S = \beta \sum_{\square} \left(1 - \frac{1}{N} \text{ Re Tr } U_{\square} \right), \quad \text{with: } \beta = \frac{2N}{g^2} a^{D-4}$$

- A gauge-invariant, non-perturbative regularization
- Exact symmetries at *any* value of a
  - Gauge symmetry
  - Discrete translations
  - Discrete rotations
  - Global discrete symmetries of the continuum ($\mathcal{C}$, $\mathcal{P}$, $\mathcal{T}$, ...)
- Suitable for numerical simulation: Sample configuration space according to a *statistical weight* proportional to $\exp(-S)$
- Expectation values from statistical averages

$$\langle \mathcal{O} \rangle = \frac{\int \prod \mathrm{d}U_\mu(x) \mathcal{O} \exp(-S)}{\int \prod \mathrm{d}U_\mu(x) \exp(-S)}$$

Basics of lattice QCD – I

- Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing a
- Transcribe gauge and fermion d.o.f. to lattice elements, build lattice observables
- Isotropic lattice action for Yang-Mills theory Wilson, 1974

$$S = \beta \sum_{\square} \left(1 - \frac{1}{N} \, \mathrm{Re}\, \mathrm{Tr}\, U_{\square} \right), \quad \text{with: } \beta = \frac{2N}{g^2} a^{D-4}$$

- A gauge-invariant, non-perturbative regularization
- Exact symmetries at *any* value of a
  - Gauge symmetry
  - Discrete translations
  - Discrete rotations
  - Global discrete symmetries of the continuum ($\mathcal{C}$, $\mathcal{P}$, $\mathcal{T}$, ...)
- Suitable for numerical simulation: Sample configuration space according to a *statistical weight* proportional to $\exp(-S)$
- Expectation values from statistical averages

$$\langle \mathcal{O} \rangle = \frac{\int \prod \mathrm{d}U_\mu(x) \mathcal{O} \exp(-S)}{\int \prod \mathrm{d}U_\mu(x) \exp(-S)}$$

Basics of lattice QCD – I

- Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing a
- Transcribe gauge and fermion d.o.f. to lattice elements, build lattice observables
- Isotropic lattice action for Yang-Mills theory Wilson, 1974

$$S = \beta \sum_{\square} \left(1 - \frac{1}{N} \text{ Re Tr } U_{\square} \right), \quad \text{with: } \beta = \frac{2N}{g^2} a^{D-4}$$

- A gauge-invariant, non-perturbative regularization
- Exact symmetries at *any* value of a
  - Gauge symmetry
  - Discrete translations
  - Discrete rotations
  - Global discrete symmetries of the continuum $(\mathcal{C}, \mathcal{P}, \mathcal{T}, \dots)$
- Suitable for numerical simulation: Sample configuration space according to a *statistical weight* proportional to $\exp(-S)$
- Expectation values from statistical averages

$$\langle \mathcal{O} \rangle = \frac{\int \prod \mathrm{d}U_\mu(x) \mathcal{O} \exp(-S)}{\int \prod \mathrm{d}U_\mu(x) \exp(-S)}$$

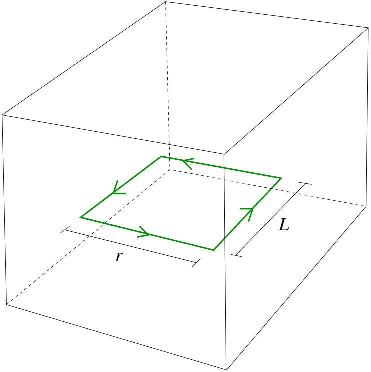
Basics of lattice QCD – II

- Setting the scale (for a choice of lattice parameters): Match a lattice observable to its continuum value
- Extrapolation to the continuum limit $a \to 0$ in the presence of a *continuous* phase transition of the lattice theory
- Simulation of fermionic fields along same principles, but:
  - involves *non-local* determinant of Dirac operator
  - unphysical doubler modes to be removed
  - additional challenges at finite quark chemical potential
- Lattice QCD now in an era of precision calculations; results show striking agreement with experimental data
- Note: *Euclidean* formalism

Basics of lattice QCD – II

- Setting the scale (for a choice of lattice parameters): Match a lattice observable to its continuum value
  - Example: confining static potential from large Wilson loops

$$\langle \mathcal{W}(r, L) \rangle \propto \exp\left(-\sigma a^2 \cdot \frac{r}{a} \cdot \frac{L}{a}\right)$$



  - Fit σa^2 from simulation results
  - Deduce a using $\sigma = (440\text{MeV})^2$ and 197 MeV $\simeq 1$ fm^{-1}
- Extrapolation to the continuum limit $a \to 0$ in the presence of a *continuous*

Basics of lattice QCD – II

- Setting the scale (for a choice of lattice parameters): Match a lattice observable to its continuum value
  - Example: confining static potential from large Wilson loops

$$\langle \mathcal{W}(r, L) \rangle \propto \exp\left(-\sigma a^2 \cdot \frac{r}{a} \cdot \frac{L}{a}\right)$$

  - Fit σa^2 from simulation results
  - Deduce a using $\sigma = (440\text{MeV})^2$ and 197 MeV $\simeq$ 1 fm^{-1}
- Extrapolation to the continuum limit $a \to 0$ in the presence of a *continuous* phase transition of the lattice theory
- Simulation of fermionic fields along same principles, but:
  - involves *non-local* determinant of Dirac operator
  - unphysical doubler modes to be removed
  - additional challenges at finite quark chemical potential
- Lattice QCD now in an era of precision calculations; results show striking agreement with experimental data
- Note: *Euclidean* formalism

Basics of lattice QCD – II

- Setting the scale (for a choice of lattice parameters): Match a lattice observable to its continuum value
  - Example: confining static potential from large Wilson loops

$$\langle \mathcal{W}(r, L) \rangle \propto \exp\left(-\sigma a^2 \cdot \frac{r}{a} \cdot \frac{L}{a}\right)$$

  - Fit σa^2 from simulation results
  - Deduce a using $\sigma = (440\text{MeV})^2$ and 197 MeV $\simeq$ 1 fm^{-1}
- Extrapolation to the continuum limit $a \to 0$ in the presence of a *continuous* phase transition of the lattice theory
- Simulation of fermionic fields along same principles, but:
  - involves *non-local* determinant of Dirac operator
  - unphysical doubler modes to be removed
  - additional challenges at finite quark chemical potential
- Lattice QCD now in an era of precision calculations; results show striking agreement with experimental data
- Note: *Euclidean* formalism

Basics of lattice QCD – II

- Setting the scale (for a choice of lattice parameters): Match a lattice observable to its continuum value
- Extrapolation to the continuum limit $a \to 0$ in the presence of a *continuous* phase transition of the lattice theory
- Simulation of fermionic fields along same principles, but:
  - involves *non-local* determinant of Dirac operator
  - unphysical doubler modes to be removed
  - additional challenges at finite quark chemical potential
- Lattice QCD now in an era of precision calculations; results show striking agreement with experimental data
- Note: *Euclidean* formalism

Basics of lattice QCD – II

- Setting the scale (for a choice of lattice parameters): Match a lattice observable to its continuum value
- Extrapolation to the continuum limit $a \to 0$ in the presence of a *continuous* phase transition of the lattice theory
  - Does it *really* work?
  - Yes: Continuum theory as *a good low-energy effective theory* for lattice theory
- Simulation of fermionic fields along same principles, but:
  - involves *non-local* determinant of Dirac operator
  - unphysical doubler modes to be removed
  - additional challenges at finite quark chemical potential
- Lattice QCD now in an era of precision calculations; results show striking agreement with experimental data
- Note: *Euclidean* formalism

Basics of lattice QCD – II

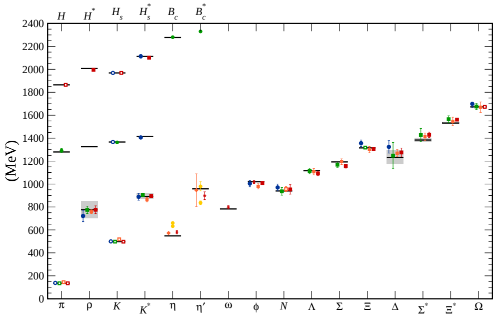
- Setting the scale (for a choice of lattice parameters): Match a lattice observable to its continuum value
- Extrapolation to the continuum limit $a \to 0$ in the presence of a *continuous* phase transition of the lattice theory
  - Does it *really* work?
  - Yes: Continuum theory as *a good low-energy effective theory* for lattice theory
- Simulation of fermionic fields along same principles, but:
  - involves *non-local* determinant of Dirac operator
  - unphysical doubler modes to be removed
  - additional challenges at finite quark chemical potential
- Lattice QCD now in an era of precision calculations; results show striking agreement with experimental data
- Note: *Euclidean* formalism

Basics of lattice QCD – II

- Setting the scale (for a choice of lattice parameters): Match a lattice observable to its continuum value
- Extrapolation to the continuum limit $a \to 0$ in the presence of a *continuous* phase transition of the lattice theory
- Simulation of fermionic fields along same principles, but:
  - involves *non-local* determinant of Dirac operator
  - unphysical doubler modes to be removed
  - additional challenges at finite quark chemical potential
- Lattice QCD now in an era of precision calculations; results show striking agreement with experimental data
- Note: *Euclidean* formalism

Basics of lattice QCD – II

- Setting the scale (for a choice of lattice parameters): Match a lattice observable to its continuum value
- Extrapolation to the continuum limit $a \to 0$ in the presence of a *continuous* phase transition of the lattice theory
- Simulation of fermionic fields along same principles, but:
  - involves *non-local* determinant of Dirac operator
  - unphysical doubler modes to be removed
  - additional challenges at finite quark chemical potential
- Lattice QCD now in an era of precision calculations; results show striking agreement with experimental data



Kronfeld, 2012

- Note: *Euclidean* formalism

Basics of lattice QCD – II

- Setting the scale (for a choice of lattice parameters): Match a lattice observable to its continuum value
- Extrapolation to the continuum limit $a \to 0$ in the presence of a *continuous* phase transition of the lattice theory
- Simulation of fermionic fields along same principles, but:
  - involves *non-local* determinant of Dirac operator
  - unphysical doubler modes to be removed
  - additional challenges at finite quark chemical potential
- Lattice QCD now in an era of precision calculations; results show striking agreement with experimental data
- Note: *Euclidean* formalism

Electrostatic QCD on the lattice

Super-renormalizable EQCD Lagrangian

$$\mathcal{L} = \frac{1}{4} F^a_{ij} F^a_{ij} + \mathrm{Tr}\left((D_i A_0)^2\right) + m_{\mathrm{E}}^2 \mathrm{Tr}\left(A_0^2\right) + \lambda_3 \left(\mathrm{Tr}\left(A_0^2\right)\right)^2$$

Parameters chosen (by matching) to reproduce soft physics of high-T QCD

- 3D gauge coupling: $g_{\mathrm{E}}^2 = g^2 T + \ldots$
- Debye mass parameter: $m_{\mathrm{E}}^2 = \left(1 + \frac{n_f}{6}\right) g^2 T + \ldots$
- 3D quartic coupling: $\lambda_3 = \frac{9 - n_f}{24\pi^2} g^4 T + \ldots$

Standard Wilson lattice regularization Hietanen *et al.*, 2008

Our setup: QCD with $n_f = 2$ light flavors, two temperature ensembles:

- $T \simeq 398$ MeV
- $T \simeq 2$ GeV

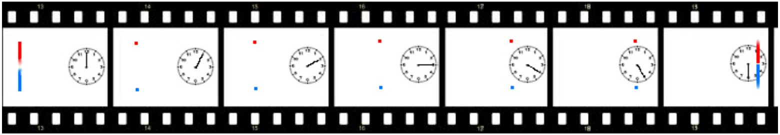
Closely related studies in MQCD Laine, 2012 Benzke *et al.*, 2012

Operator implementation

Effective theory: purely spatial

but

Operator describes *real time* evolution



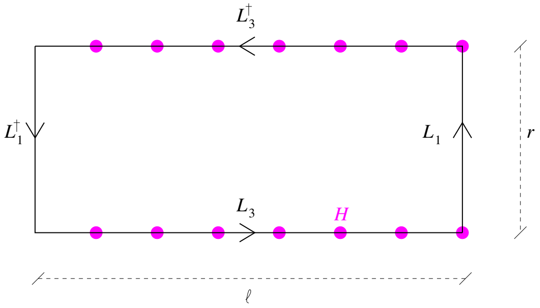
Operator implementation

Light-cone Wilson line correlator

$$\langle W(\ell, r)\rangle = \left\langle \mathrm{Tr}\left(L_3 L_1 L_3^\dagger L_1^\dagger\right)\right\rangle \sim \exp\left[-\ell V(r)\right]$$

with

$$L_3 = \prod U_3 H \qquad L_1 = \prod U_1 \qquad H = \exp(-a g_{\mathrm{E}}^2 A_0)$$



Well-defined renormalization properties D'Onofrio *et al.*, to appear

Outline

$\hat{q}$ *estimate*

Contribution to $\hat{q}$ related to the curvature of $V(r)$ near the origin

$\hat{q}$ *estimate*

Contribution to $\hat{q}$ related to the curvature of $V(r)$ near the origin

Data fitted with a procedure similar to Laine, 2012

$$V/g_{\mathrm{E}}^2 = Arg_{\mathrm{E}}^2 + B(rg_{\mathrm{E}}^2)^2 + C(rg_{\mathrm{E}}^2)^2 \ln(rg_{\mathrm{E}}^2) + \dots$$

$\hat{q}$ estimate

Contribution to $\hat{q}$ related to the curvature of $V(r)$ near the origin

Data fitted with a procedure similar to Laine, 2012

$$V/g_{\mathrm{E}}^2 = Arg_{\mathrm{E}}^2 + B(rg_{\mathrm{E}}^2)^2 + C(rg_{\mathrm{E}}^2)^2 \ln(rg_{\mathrm{E}}^2) + \dots$$

Purely NP soft contribution to $\hat{q}$ is quite large

$$\hat{q}_{\mathrm{EQCD}}^{\mathrm{NP}} \sim 0.5 g_{\mathrm{E}}^6$$

$\hat{q}$ *estimate*

Contribution to $\hat{q}$ related to the curvature of $V(r)$ near the origin

Data fitted with a procedure similar to Laine, 2012

$$V/g_{\rm E}^2 = Arg_{\rm E}^2 + B(rg_{\rm E}^2)^2 + C(rg_{\rm E}^2)^2 \ln(rg_{\rm E}^2) + \ldots$$

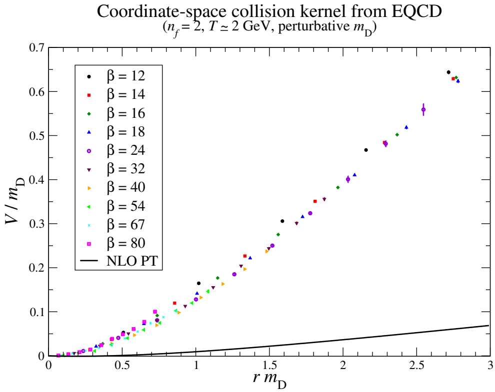
Purely NP soft contribution to $\hat{q}$ is quite large

$$\hat{q}_{\rm EQCD}^{\rm NP} \sim 0.5 g_{\rm E}^6$$

Approximate estimate $\hat{q} \sim 6\ \mathrm{GeV}^2/\mathrm{fm}$ at RHIC temperatures

Lattice versus perturbation theory

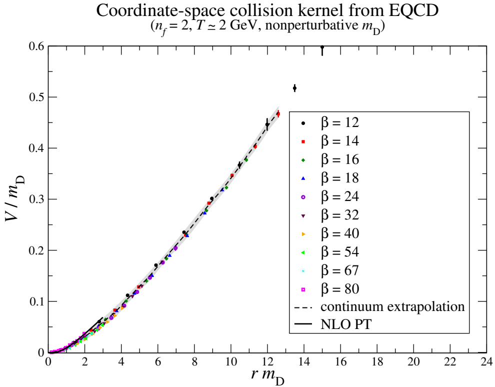
"Naïve" comparison with NLO PT



Lattice versus perturbation theory

Discrepancy reduced if data are plotted in terms of non-perturbative m_D

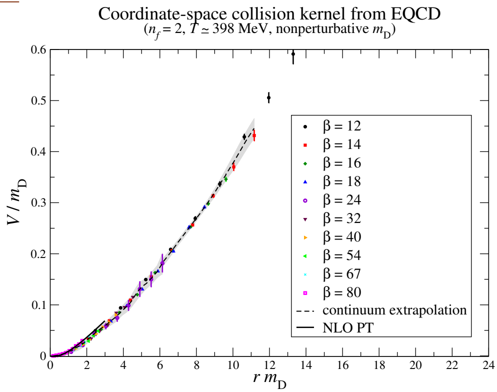
Laine and Philipsen, 2008



Lattice versus perturbation theory

Discrepancy reduced if data are plotted in terms of non-perturbative m_D

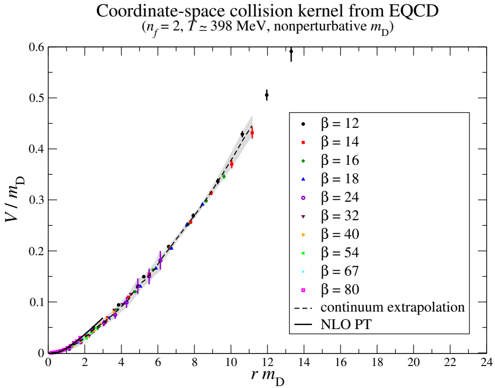
Laine and Philipsen, 2008



Lattice versus perturbation theory

Discrepancy reduced if data are plotted in terms of non-perturbative m_{D}

Laine and Philipsen, 2008



Using NP value for m_{D} in

$$\hat{q}^{\mathrm{NLO}}_{\mathrm{EQCD}} = g^4 T^2 m_{\mathrm{D}} \mathcal{C}_{\mathrm{f}} \mathcal{C}_{\mathrm{a}} \frac{3\pi^2 + 10 - 4\ln 2}{32\pi^2}$$

yields again $\hat{q} \sim 6\ \mathrm{GeV}^2/\mathrm{fm}$ at RHIC temperatures

Outline

Conclusions

- Lattice approach *possible* for certain real-time problems—see also Ji, 2013
- Here: focus on soft physics in thermal QCD Laine and Rothkopf, 2013 Cherednikov *et al.*, 2013
- Outlined approach is *systematic*
- Tentative estimate of jet quenching parameter
- Clear indication for large non-perturbative effects
- Results in ballpark of
  - holographic computations Liu, Rajagopal and Wiedemann, 2006 Armesto, Edelstein and Mas, 2006 Gürsoy, Kiritsis, Michalogiorgakis and Nitti, 2009 ✓
  - estimates from phenomenological models Dainese *et al.*, 2004 Eskola *et al.*, 2004 Bass *et al.*, 2008 ✓