Anomalies and Transport: From the QGP to Weyl-Semimetals on a Superstring

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CTP Crete, 08 Oct 2013

Outline

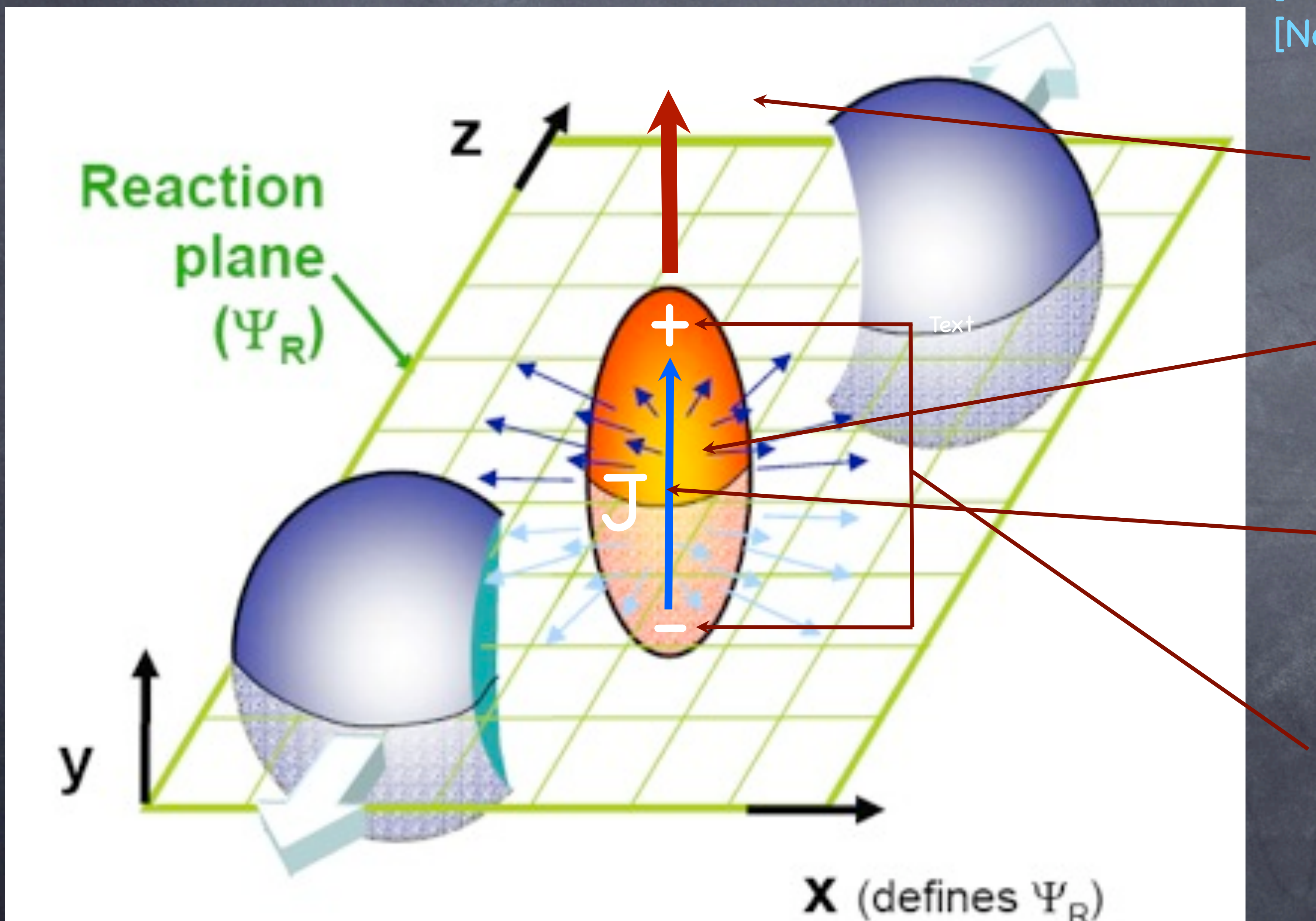
- MOVIE!!
- Chiral Magnetic Effect and Chiral Vortical Effect
- Anomalies and Kubo formulae
- Holography - String Theory
- Weyl Semi-metals
- Conclusions

MOVIE

http://www.youtube.com/watch?v=kXy5EvYu3fw

The CME

[Kharzeev, McLarren, Warringa],
[Fukushima, Kharzeev, Warringa]
[Newman, Son]



Magnetic Field

Net chirality

Electric current

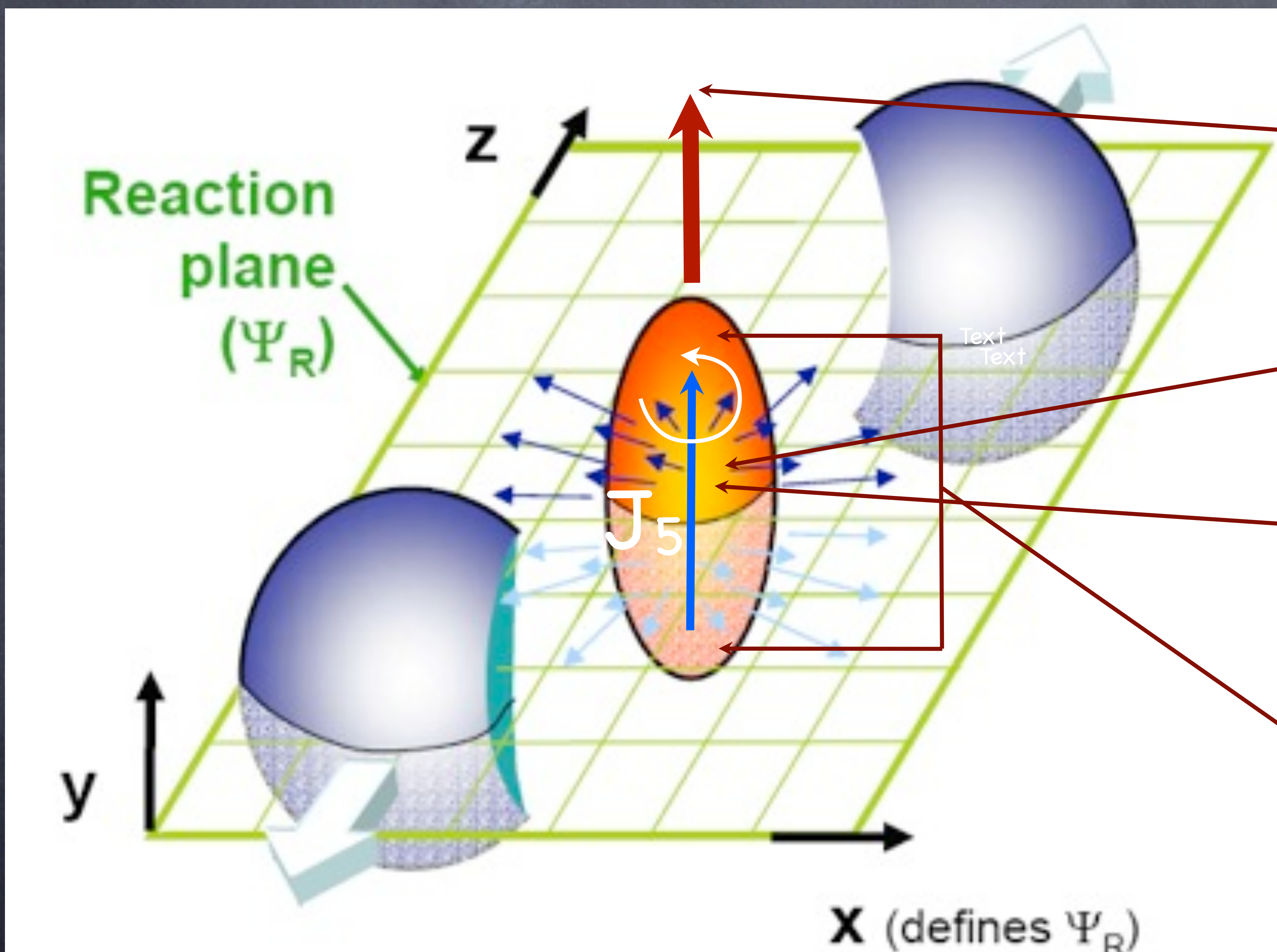
P-odd

T-even

[parity violating currents: Vilenkin '80, Giovannini, Shaposhnikov '98, Alekseev, Chaianov, Fröhlich '98]

The CVE

[Banerjee et al, Erdmenger et al.]



Angular Momentum

Temperature

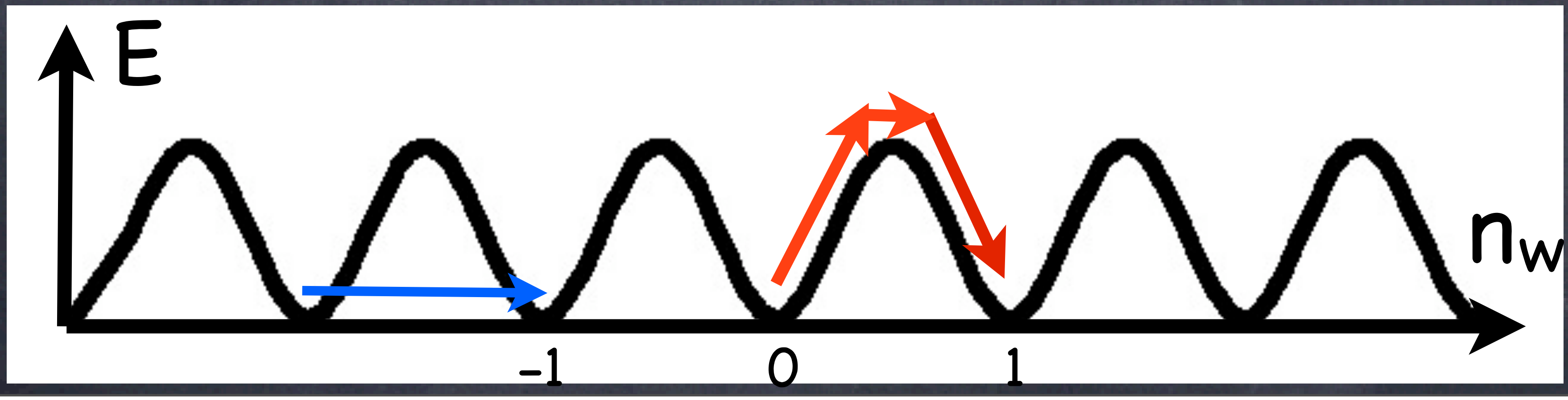
Axial Current

P-even
T-even

[Vilenkin '80, CVE Kharzeev&Son '10, Keren-Zur&Oz '11]

Net chirality

- topological charge $Q_w = \frac{g^2}{32\pi^2}\int d^4x\, F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$
- axial anomaly (QCD) $\partial_\mu j^\mu_5 = 2m_f\langle\bar{\psi}_f i\gamma_5\psi_f\rangle - \frac{N_f g^2}{16\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$
- topologically non trivial gauge field
- effective: axial chemical potential $\mu_5 \longleftrightarrow \Delta Q_5 = 2N_f Q_w$



Kubo formulae

- Anomaly related conductivities

$$\vec{J} = \sigma \vec{B}$$

- Kubo formula, general symmetry group

$$[T^A, T^B] = i f_C^{AB} T^C$$

$$\sigma^{AB} = \lim_{p_j \to 0} \sum_{i,k} \frac{i}{2p_j} \epsilon_{ijk} \langle J_i^A J_k^B \rangle \Big|_{\omega=0}$$

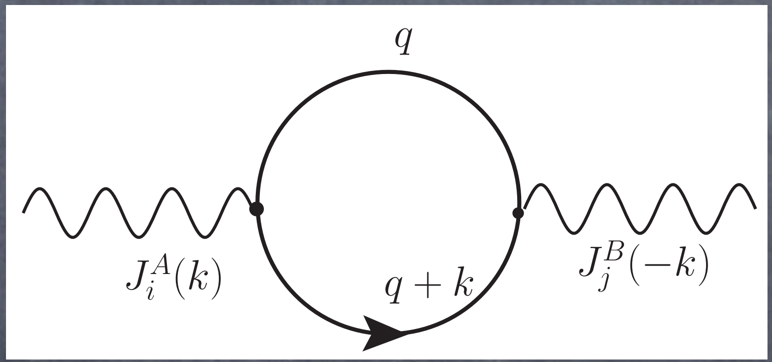
Kubo formulae

- chiral fermions $$J_i^A = \sum_{f,g=1}^{N} (T^A)^g{}_f \bar{\Psi}_g \gamma_i P_+ \Psi^f$$

- chemical potentials and Cartan generators

$$H_A = q_A^f \delta^f{}_g \qquad \mu^f = \sum_A q_A^f \mu_A$$

- 1-loop graph



$$\sigma_{AB} = \frac{1}{8\pi^2} \sum_C \mathrm{tr}\left(T^A\{T^B, H^C\}\right) \mu_C = \frac{1}{4\pi^2} d^{ABC} \mu_C$$

Kubo formulae

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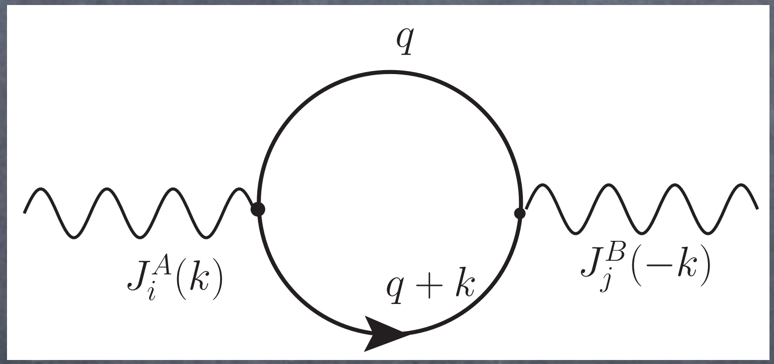

Anomalycoeff

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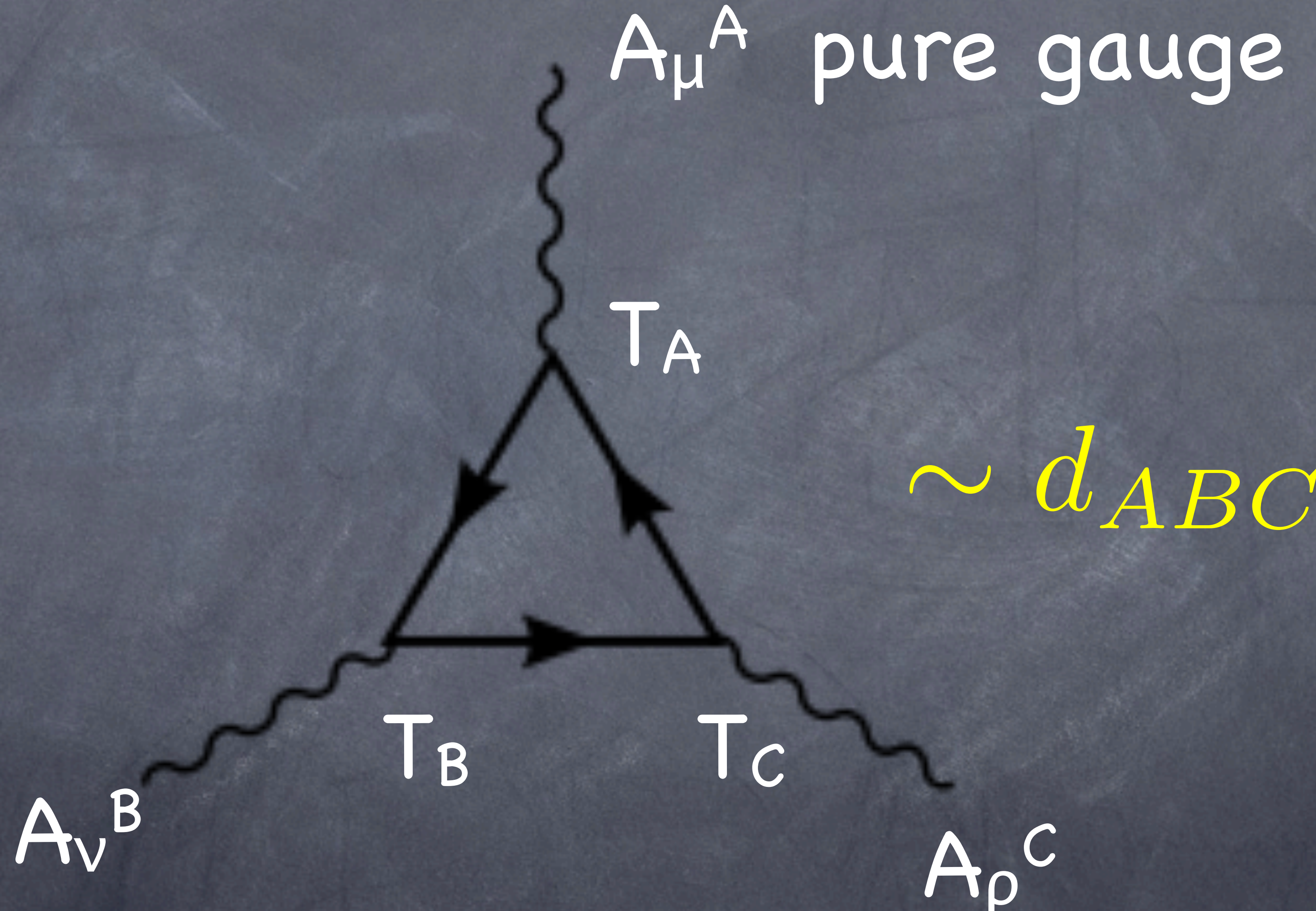


Kubo formulae

- $T_{\mu\nu}$ sourced by metric

$$ds^2 = -(1-2\Phi)dt^2 + 2\vec{A}_g dt d\vec{x} + (1+2\Phi)d\vec{x}^2$$

- A_g "gravitomagnetic field" -> chiral gravitomagnetic effect

$$\vec{J} = \sigma_V \vec{B}_g$$

- chiral vortical effect: fluid velocities

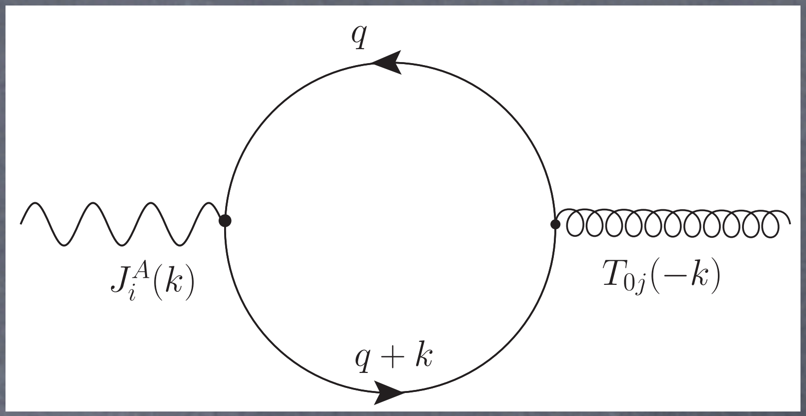
$$u^\mu = (1,0,0,0) \qquad u_\mu = (-1, \vec{A}_g) \qquad J^i = \sigma_V \epsilon^{ijk} \partial_j u_k$$

$$\sigma_V^A = \lim_{p_j \to 0} \sum_{i,k} \frac{i}{2p_j} \epsilon_{ijk} \langle J_i^A T_{k0} \rangle$$

Kubo formulae

- as before: general symmetry group

$$T^{0i} = \frac{i}{2}\sum_{f=1}^{N} \bar{\Psi}_f(\gamma^0\partial^i + \gamma^i\partial^0)P_+\Psi^f$$



$$\sigma_V^A = \frac{1}{8\pi^2}\sum_{f=1}^{N}(T^A)^f{}_f\left[(\mu^f)^2 + \frac{\pi^2}{3}T^2\right]$$

$$= \frac{1}{8\pi^2}\sum_{B,C} d^{ABC}\mu_B\,\mu_C + \frac{T^2}{24}\mathrm{tr}\left(T^A\right)$$

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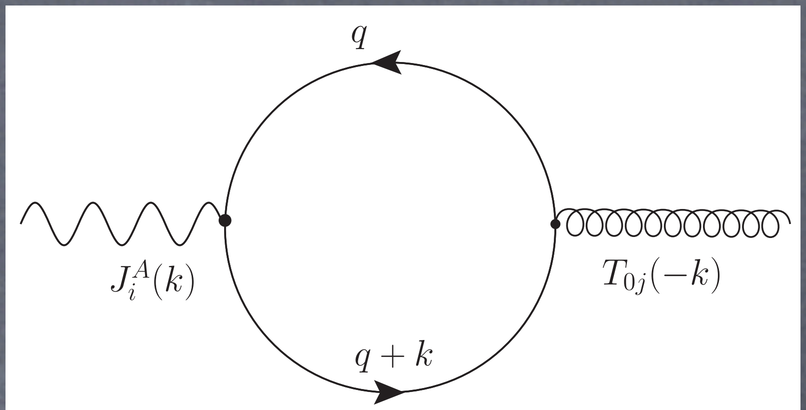

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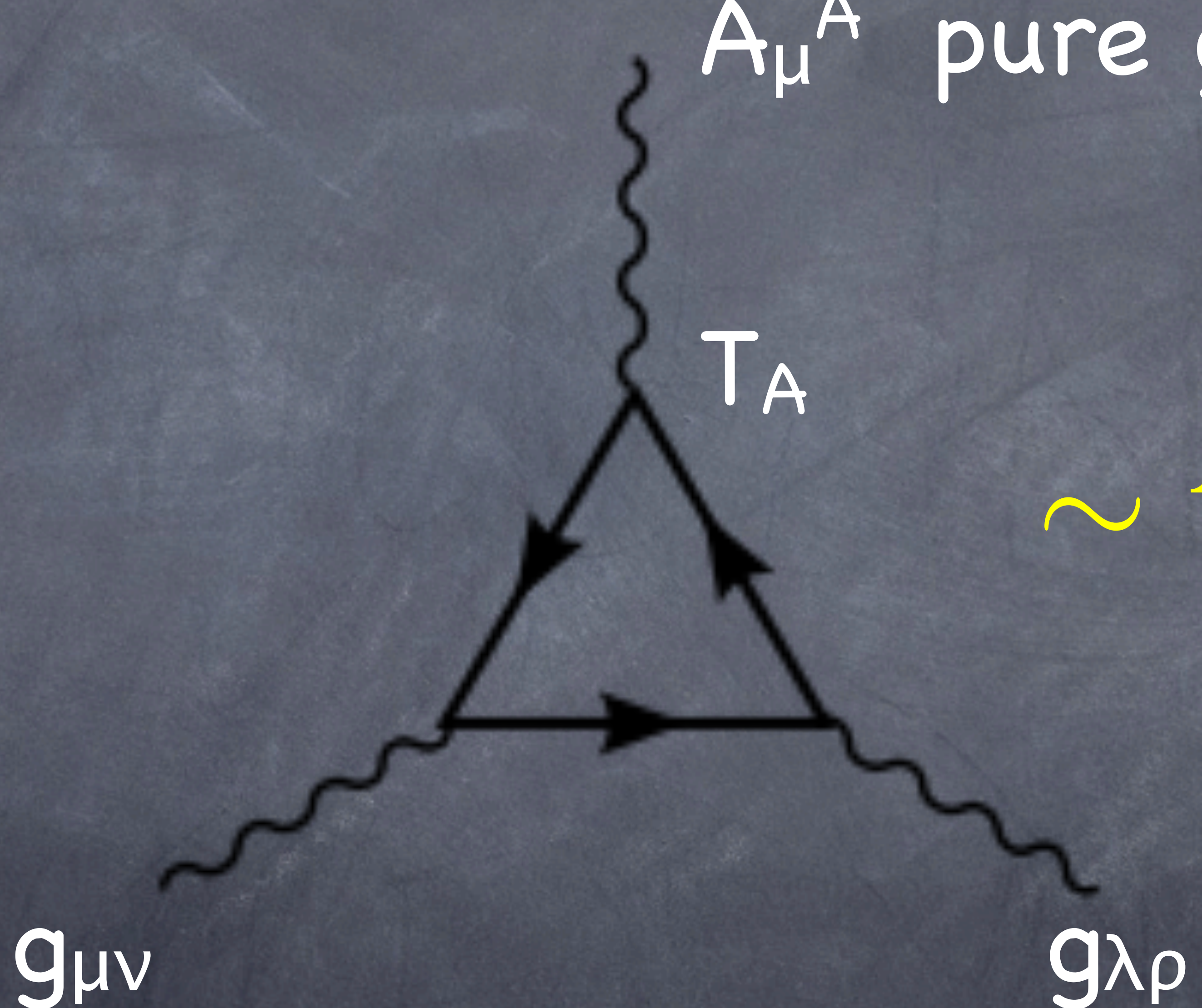
Integration constant -> gravitational anomaly!

Kubo formulae



$$\sim \mathrm{tr}\,(T_A)$$

"Mixed gauge gravitational anomaly"

Anomalous Transport

$$\vec{J}^A = \sigma_{\mathcal{B}}^{AB}\vec{B}^B + \sigma_{\mathcal{V}}^{A}\vec{\omega}$$

$$\vec{J}_\epsilon^A = \sigma_{\mathcal{B}}^{\epsilon,A}\vec{B}^A + \sigma_{\mathcal{V}}^{\epsilon}\vec{\omega}$$

$$\sigma_{\mathcal{B}}^{AB} = \frac{1}{4\pi^2} d^{ABC}\mu_C$$

$$\sigma_{\mathcal{V}}^{A} = \sigma_{\mathcal{B}}^{\epsilon,A} = \frac{1}{8\pi^2} d^{ABC}\mu_B\mu_C + b^A\frac{T^2}{24}$$

$$\sigma_{\mathcal{V}}^{\epsilon} = \frac{1}{12\pi^2} d^{ABC}\mu_A\mu_B\mu_C + b^A\mu_A\frac{T^2}{12}$$

Chiral Anomalies

- Hydrodynamics: fixed by "entropic principle" (except T^2 terms)

[Son,Surowka] [Neimann, Oz] [Loganayagam]

$$\nabla_\mu J^\mu_A = \epsilon^{\mu\nu\rho\lambda}\left(\frac{d_{ABC}}{32\pi^2}F^B_{\mu\nu}F^C_{\rho\lambda} + \frac{b_A}{768\pi^2}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\lambda}\right)$$

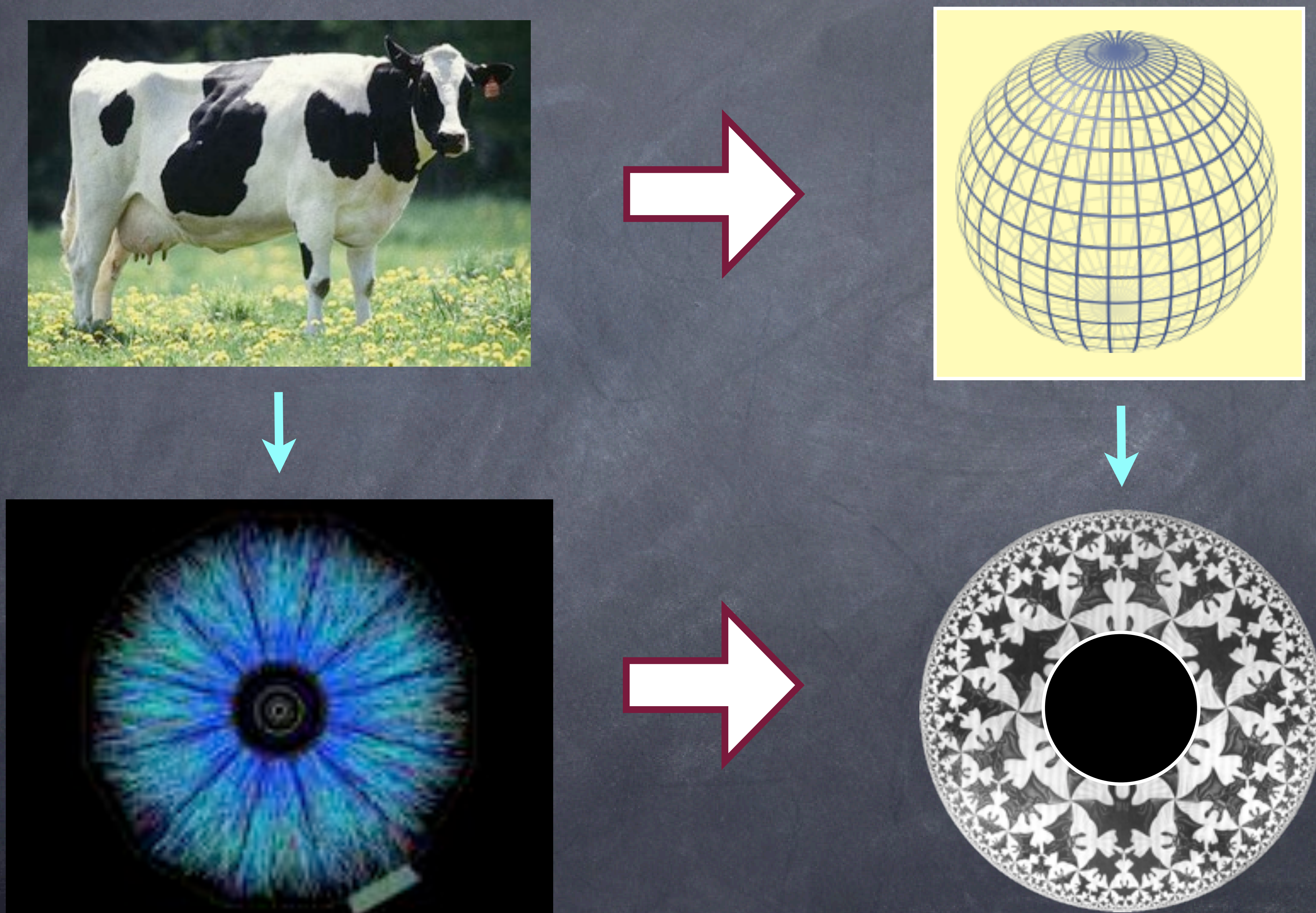
↓ O(2) in derivatives!

↓ O(4) in derivatives!

Mismatch in derivative counting for gravitational anomaly ! ??

[Jensen, Loganayagam, Yarom]

String Theory as spherical cow of sQGP



String Theory as spherical cow of sQGP

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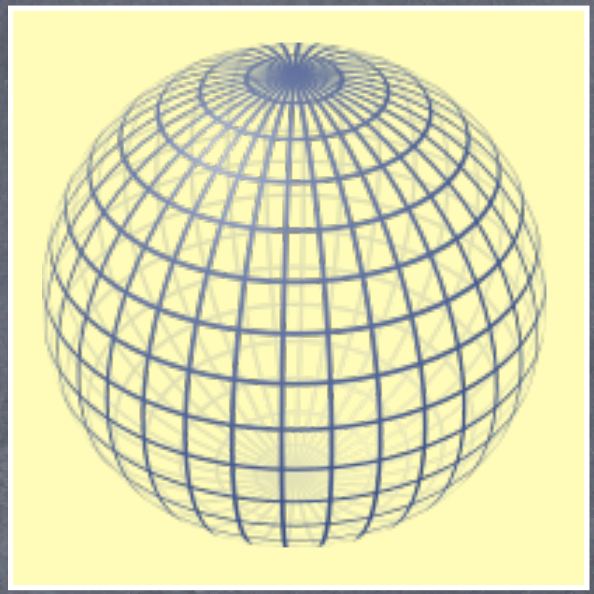

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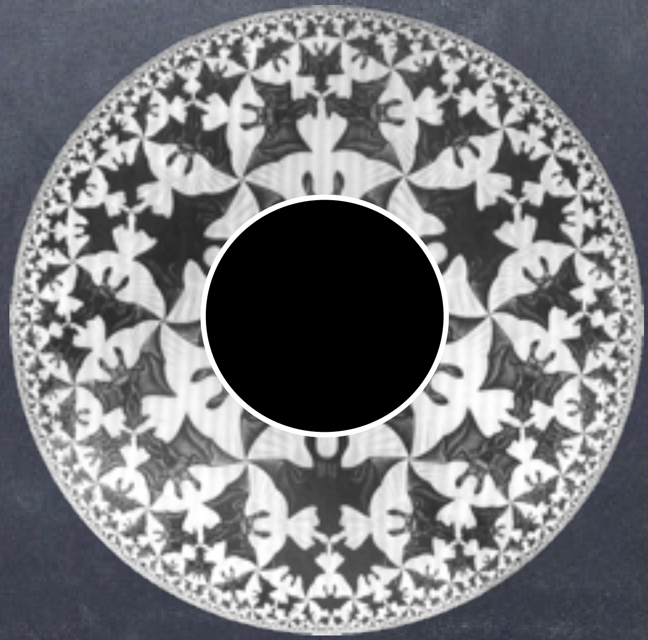

Holography

[Newman], [Banerjee et al.], [Erdmenger et al.] [Yee] [Rebhan, Schmitt, Stricker] [Khalaydzyan, Kirsch], [Hoyos, Nishioka, OBannon]

- mixed gauge gravitational Chern Simons term

$$S = S_{ME} + S_{CS} + S_{GH} + S_{CSK}$$

$$S_{EM} = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + 2\Lambda - \frac{1}{4}F_{MN}F^{MN}\right]$$

$$S_{CS} = \frac{1}{16\pi G}\int d^5x\,\epsilon^{MNPQR}A_M\left(\frac{\kappa}{3}F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)$$

$$S_{GH} = \frac{1}{8\pi G}\int_\partial d^4x\sqrt{-h}\,K$$

$$S_{CSK} = -\frac{1}{2\pi G}\int_\partial d^4x\sqrt{-h}\,\lambda n_M\epsilon^{MNPQR}A_N K_{PL}D_Q K^L_R$$

Holography

- Holography (String Theory):
  5 dim gravity (Anti de Sitter) dual to strongly coupled quantum field theory

- background: charged AdS black hole

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f(r)}dr^2 \qquad A_{(0)} = \left(\beta - \frac{\mu\, r_{\rm H}^2}{r^2}\right)$$

- correlators are

same as weak coupling!
non-renormalization

$$\langle JJ\rangle = -ik_z\left(\frac{\mu}{4\pi^2} - \frac{\beta}{12\pi^2}\right)$$

$$\langle JT\rangle = -ik_z\left(\frac{\mu^2}{8\pi^2} + \frac{T^2}{24}\right)$$

$$\langle TT\rangle = -ik_z\left(\frac{\mu^3}{12\pi^2} + \frac{\mu T^2}{12}\right)$$

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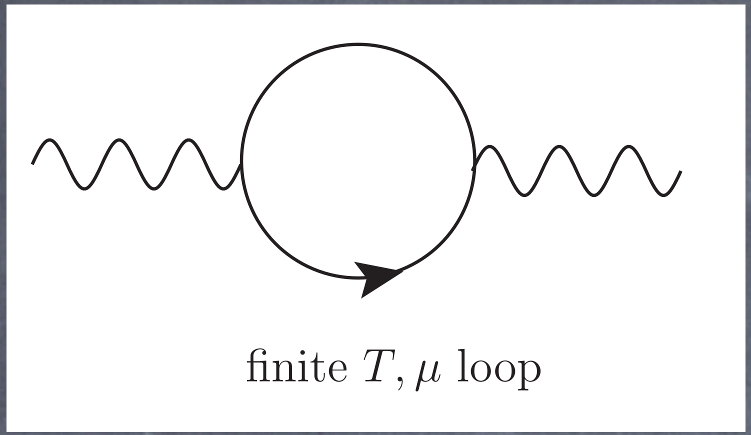
what's this?

Chemical Potentials



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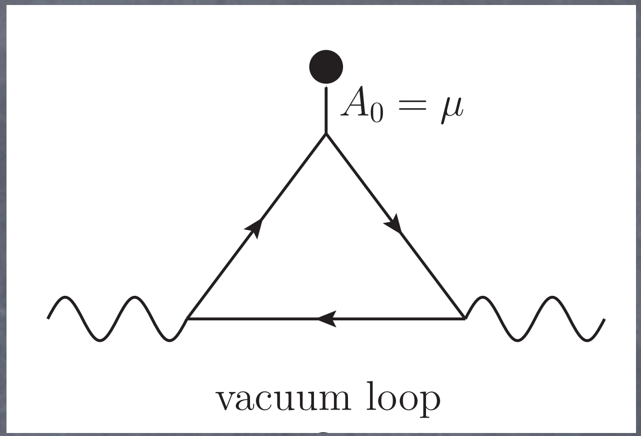

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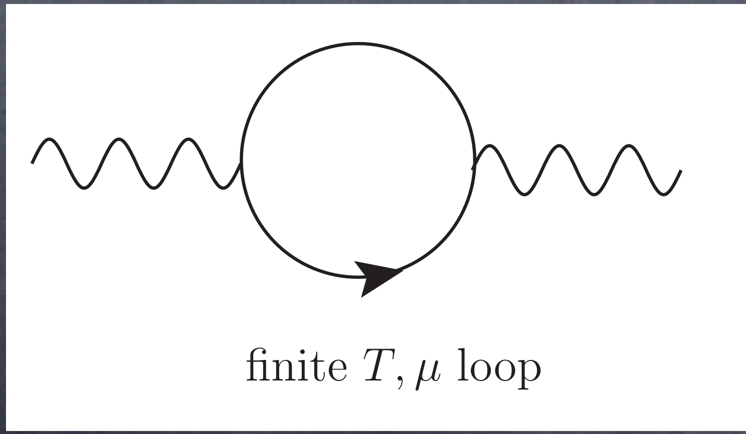


One Dirac Fermion:

- Vector and axial currents

$$J^\mu = \bar{\psi}\gamma^\mu\psi \qquad J_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$$

$$\partial_\mu J^\mu = 0 \qquad \partial_\mu J_5^\mu = \frac{1}{48\pi^2}\epsilon^{\mu\nu\rho\lambda}\left(3F_{\mu\nu}F_{\rho\lambda} + F^5_{\mu\nu}F^5_{\rho\lambda}\right)$$

$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B} + \frac{\mu\mu_5}{2\pi^2}\vec{\omega} - \frac{A_0^5}{2\pi^2}\vec{B}$$

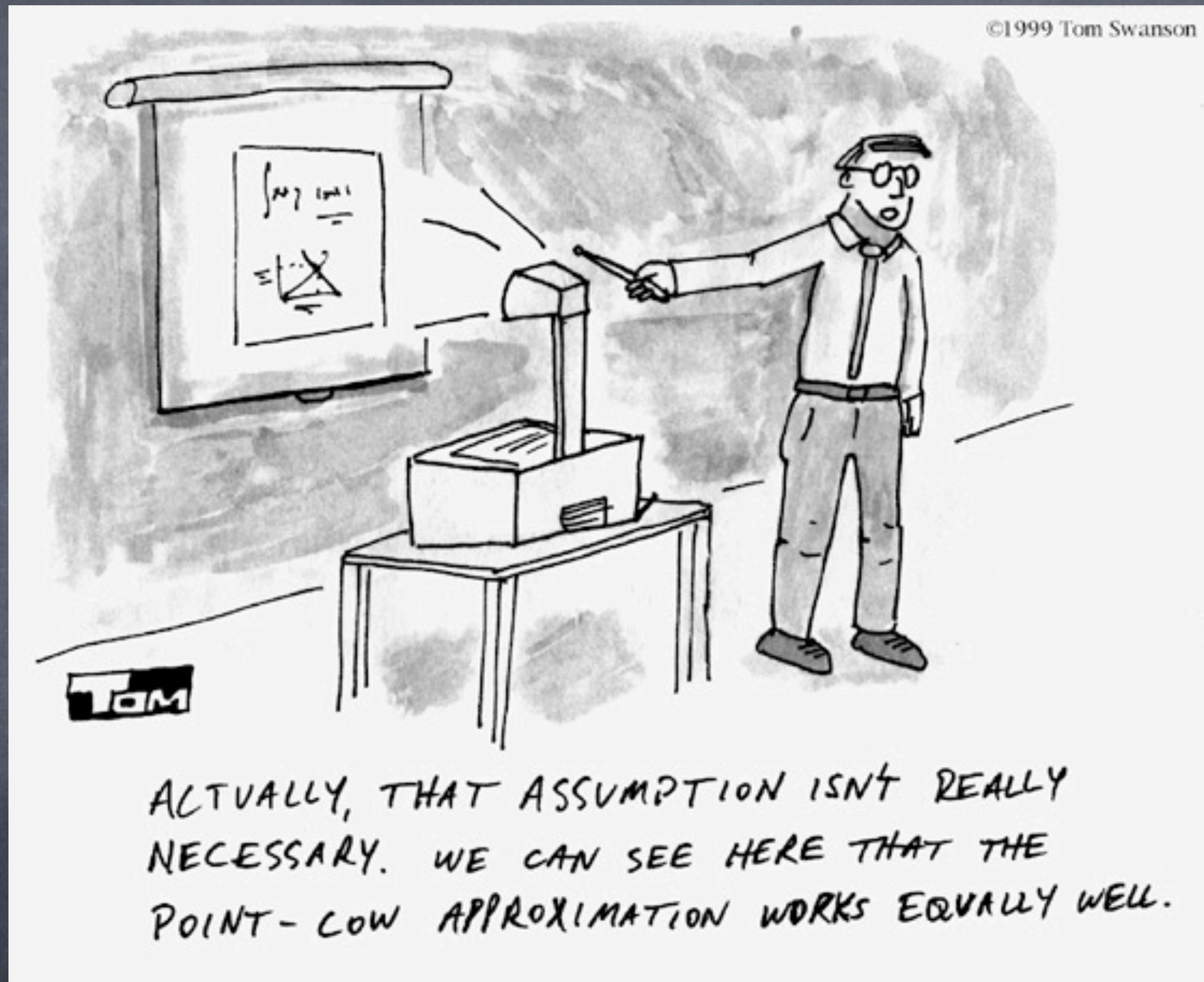
$$\vec{J}_5 = \frac{\mu}{2\pi^2}\vec{B} + \left(\frac{\mu^2+\mu_5^2}{4\pi^2} + \frac{T^2}{12}\right)\vec{\omega}$$

$$\vec{J}_\epsilon = \frac{\mu\mu_5}{2\pi^2}\vec{B} + \left(\frac{3\mu^2\mu_5+\mu\mu_5^2}{6\pi^2} + \frac{\mu_5 T^2}{6}\right)\vec{\omega}$$

Holography


©1999 Tom Swanson
TOM
ACTUALLY, THAT ASSUMPTION ISN'T REALLY NECESSARY. WE CAN SEE HERE THAT THE POINT-COW APPROXIMATION WORKS EQUALLY WELL.

Non-renormalization



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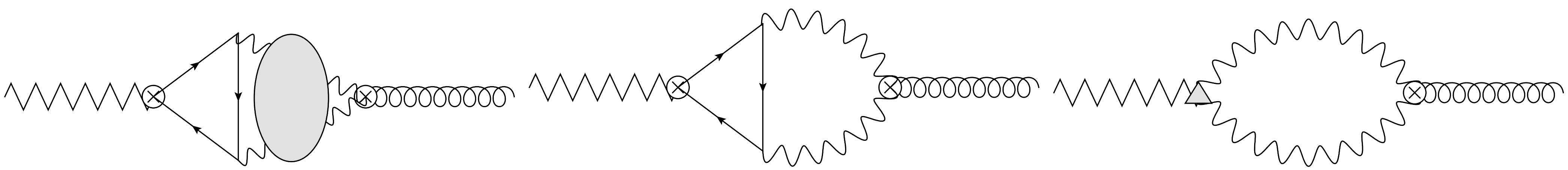
or is it?

Non-renormalization?

- [Golkar, Son] [Hou, Liu, Ren]
- **Dynamical Gauge Fields** $\partial J = F \wedge F$



(a) Anomalous diagrams. (b) Leading order diagram. (c) Effective leading diagram.

- 2-loop correction to CVC of axial current:

$$\sigma_{2-loop} = \frac{g^2 C(R) d(G)}{48\pi^2} T^2$$

- [Gorbar, Miransky, Shovkovy, Wang] CSE in QED @ 2 loop
- [Jensen, Kovtun, Ritz] MHD corrections

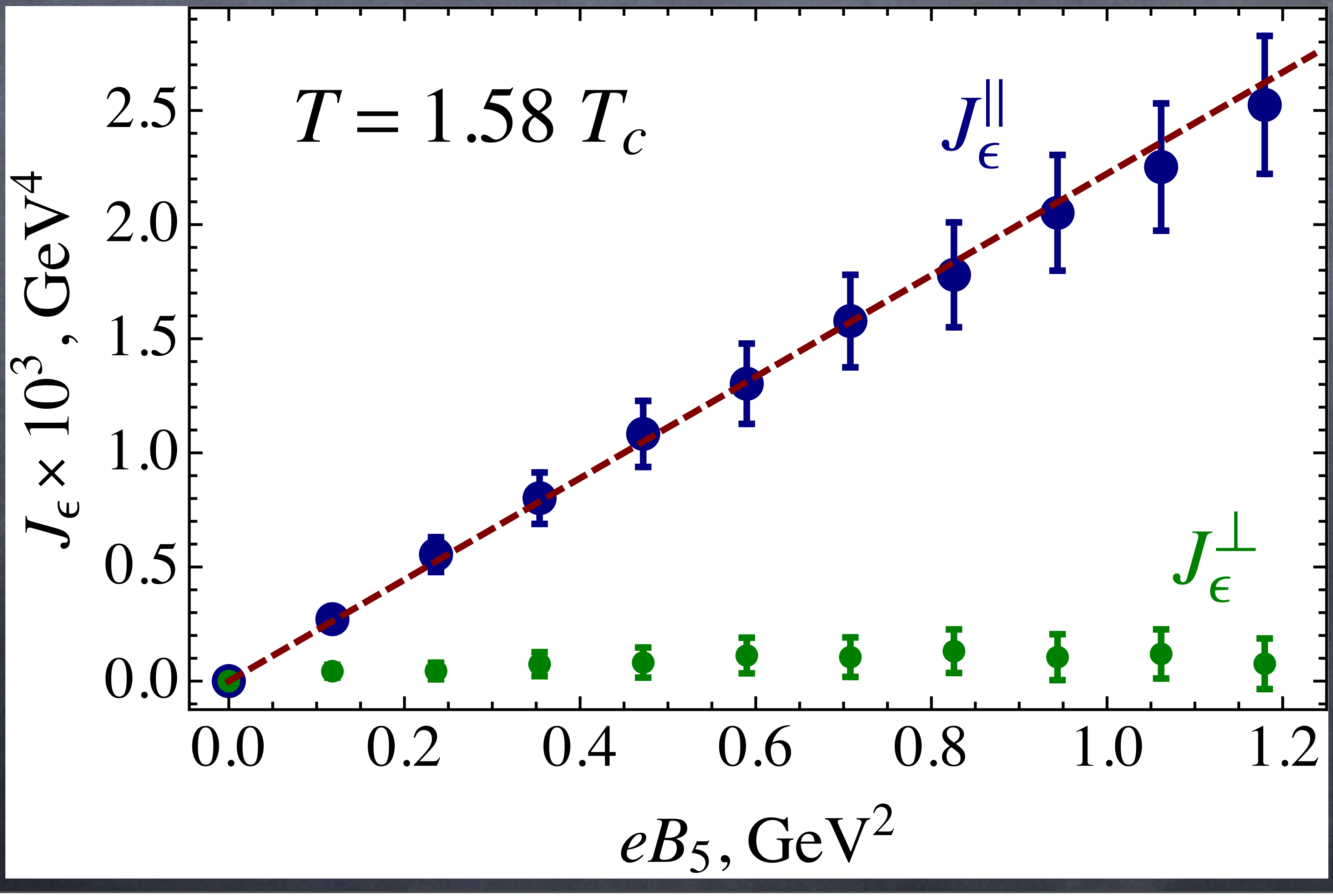
Lattice

[V. Braguta, M.N. Chernodub, K.L., M.I. Polikarpov, M. V. Ulybyshev, arXiv:1303.6266]

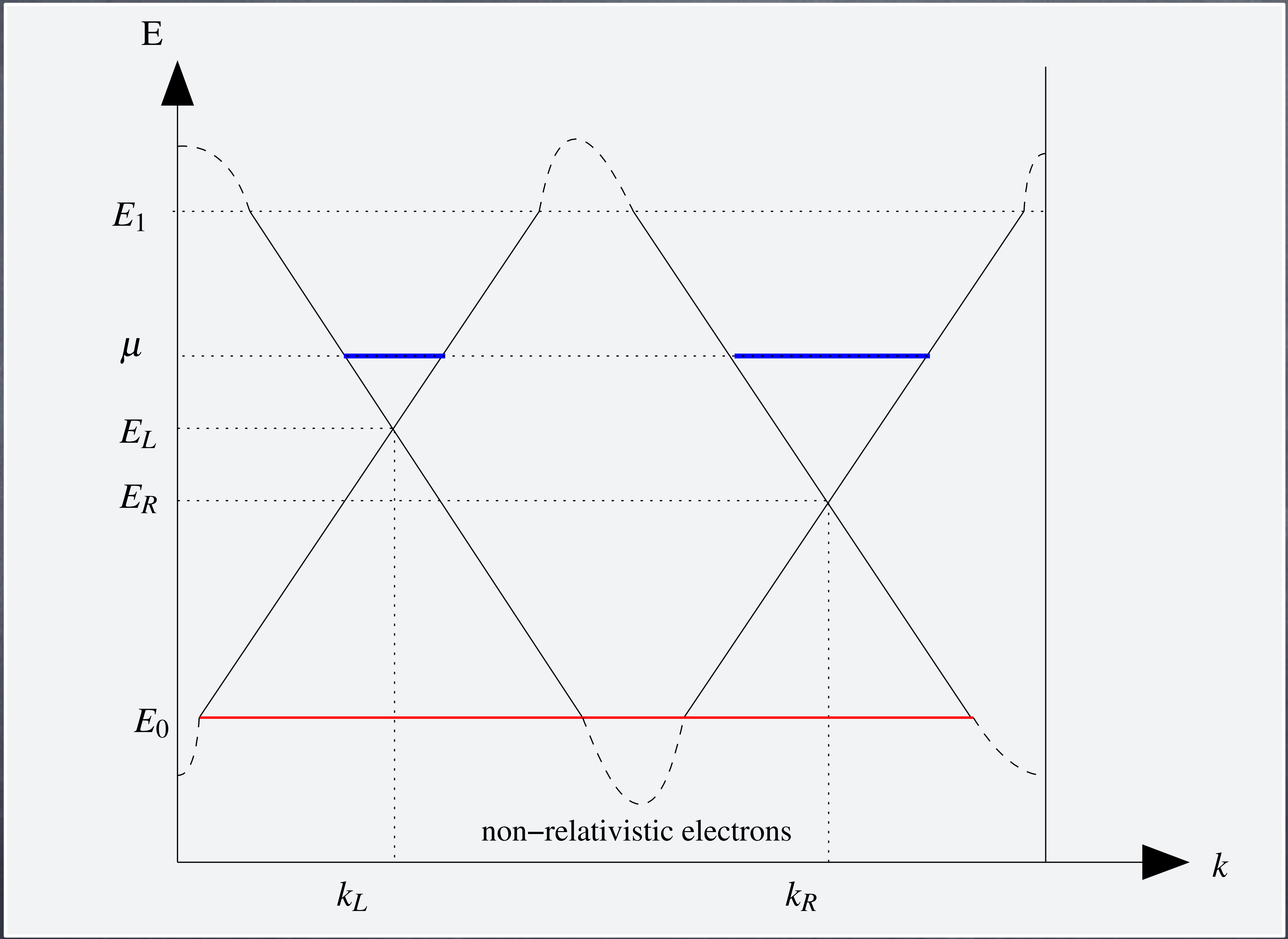
- Axial Magnetic Effect (AME)

$$\vec{J}_\epsilon = c T^2 \vec{B}_5$$

$$c \sim c_{\text{free}}/17$$



Weyl Semi-metals



Weyl Semi-metals

$$\sigma_{WSM} = \sigma(\mu - E_i, T) - \sigma(-(E_i - E_0), 0)$$

- CME $$\vec{J}_{L,R} = \pm \frac{\mu - E_0}{4\pi^2} \vec{B} \qquad J_L + J_R = 0$$

- CVE $$J_{L,R} = \pm \left(\frac{(\mu - E_0)(\mu + E_0 - 2E_{R,L})}{4\pi^2} + \frac{T^2}{12} \right) \vec{\omega}$$

$$\vec{J} = \frac{(E_L - E_R)(\mu - E_0)}{2\pi^2} \vec{\omega}$$

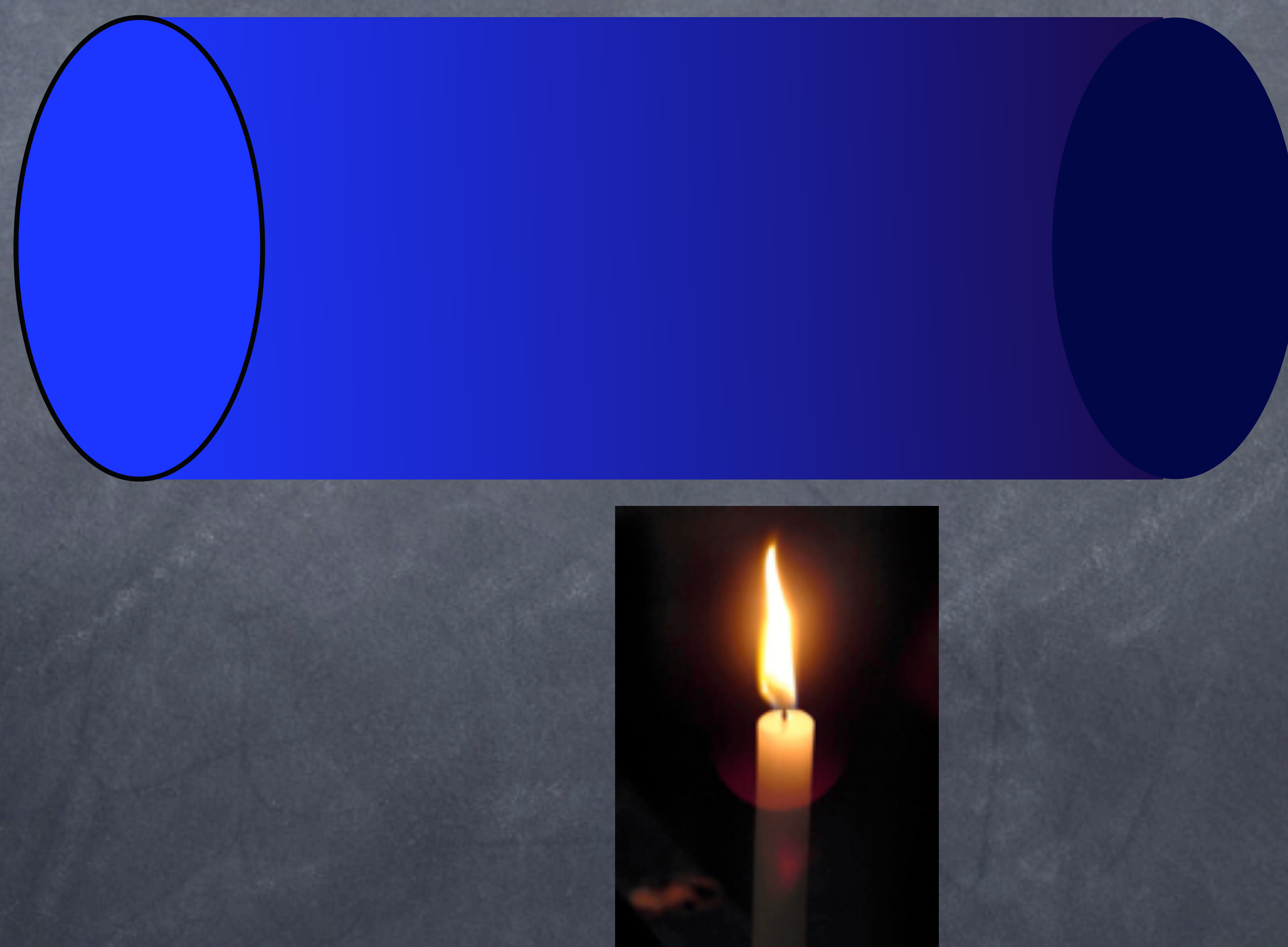
- agrees with kinetic theory

[G. Basar, D. Kharzeev, H.U. Yee, arXiv:1305.6338]

[D.T. Son, Yamamoto, arXiv:1203.2697] [M. Stephanov, arXiv:1207.0747] [I.Zahed, arXiv:1204.1955]

Weyl Semi-metals

➡ T^2 term (gravitational anomaly !)



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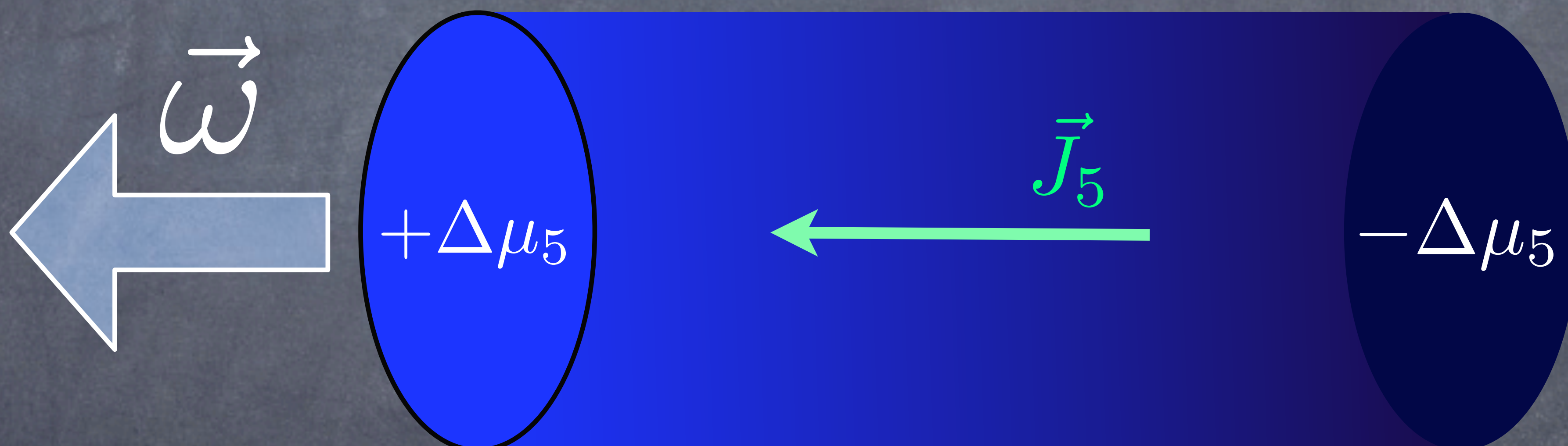

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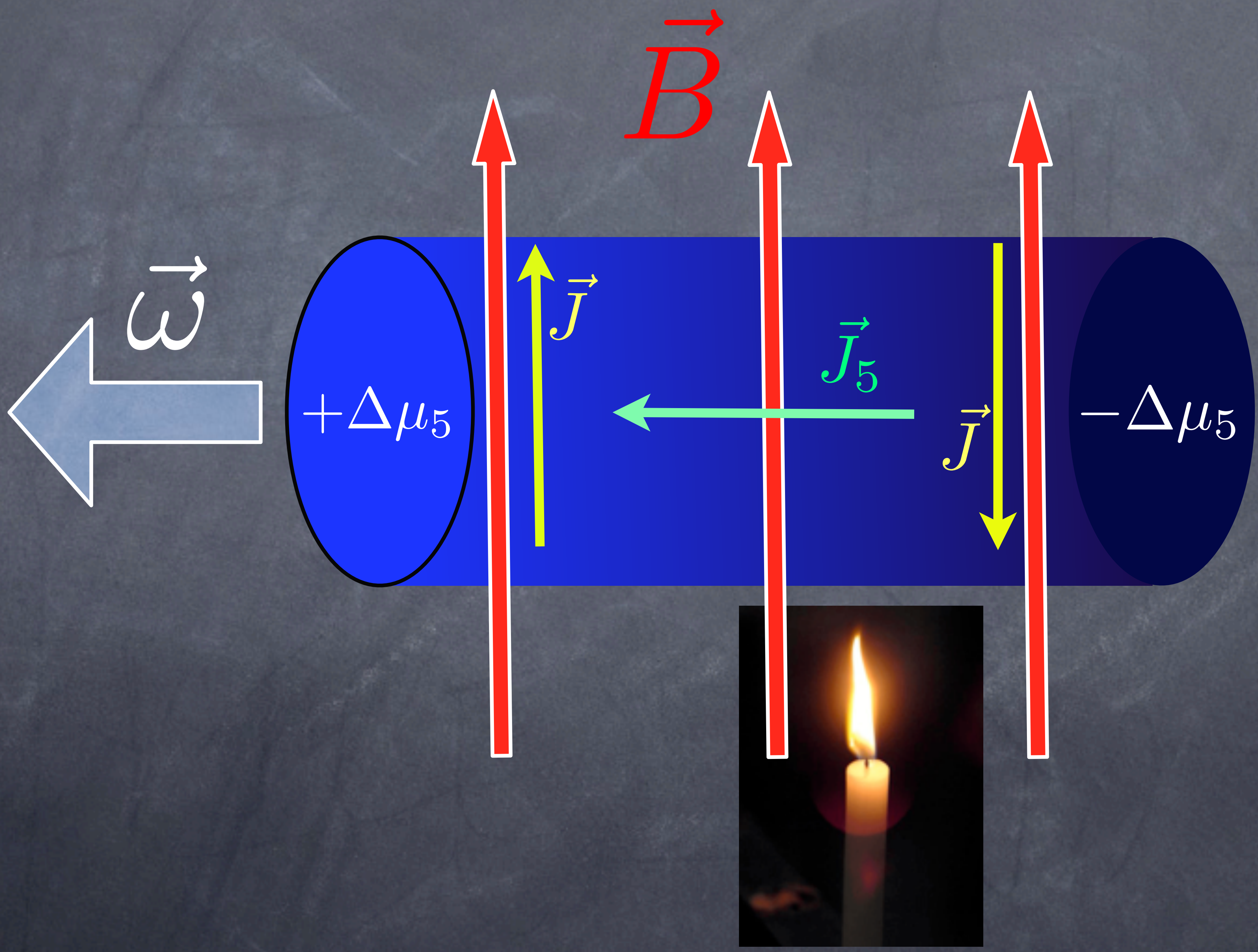

Wrap Up

- Anomalies -> dissipationless transport
- Full 4D classification of anomalous transport
- (non-)renormalization
- QGP
- Weyl Semi-metals promise table top laboratory demonstration of anomalous transport

Thank You!

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