Holographic Type II Goldstone Bosons

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[arXiv:1302.5641, arXiv:1307.8100]

Crete CTP, 07-10-2013

Outline

- Goldstone theorems
- Field theoretical model
- Holographic model(s)
- Landau criterion
- Summary

Goldstone Theorems

- Spontaneously broken continuous symmetry

$$\boxed{\lim_{k\to 0}\omega(k) = 0}$$

- At least one mode
- No constraint on power: $\omega(k) \propto k^n$
- Lorentz symmetry:
  - $\omega(k) = ck$
  - One mode for every broken generator

Goldstone Theorems

- No Lorentz symmetry
  - State: temperature T, density mu
  - Principally: non-relativistic, Lifshitz, ...
- Classification : [Nielsen-Chadha '74]
  - Type I : $\omega \propto k^{2n+1}$
  - Type II: $\omega \propto k^{2n}$

Goldstone Theorems

- Chadha-Nielsen

$$\boxed{N_I + 2N_{II} \geq N_{BG}}$$

- Brauner-Watanabe-Murayama

$$\boxed{\begin{aligned} &\langle [Q_a, Q_b] \rangle = B_{ab} \\ &N_I + N_{II} = N_{BG} - \frac{1}{2}\text{rank}(B_{ab}) \end{aligned}}$$

- Brauner-Murayama-Watanabe, Nicolis-Piazza, Kapustin ("massive" Goldstone)

$$\boxed{\omega = q\mu}$$

Field Theory Model

T. Schafer, D. T. Son, M. A. Stephanov, D. Toublan and J. J. M. Verbaarschot, [hep-ph/0108210]

V. A. Miransky and I. A. Shovkovy, [hep-ph/0108178]

$$\mathcal{L} = (\partial_0 - i\mu)\Phi^\dagger(\partial_0 + i\mu)\Phi - \vec{\partial}\Phi^\dagger\vec{\partial}\Phi - M^2\Phi^\dagger\Phi - \lambda^4(\Phi^\dagger\Phi)^2$$

Doublet of U(2) $\quad \phi = (\phi_1, \phi_2)^T \quad \phi = (0, v)^T$

μ inside overall U(1)

$$\omega_1^2 = \frac{\mu^2 - M^2}{3\mu^2 - M^2} p^2 + O(p^4)\,, \quad \rightarrow \text{Type I Goldstone}$$

$$\omega_2^2 = 6\mu^2 - 2M^2 + O(p^2)\,,$$

$$\omega_3^2 = p^2 - 2\mu\omega_3\,, \qquad \omega_3 \approx \frac{p^2}{2\mu} \quad \rightarrow \text{Type II Goldstone}$$

$$\omega_4^2 = p^2 + 2\mu\omega_4\,. \qquad \omega_4 = 2\mu \quad \rightarrow \text{“Massive Goldstone”}$$

Holography

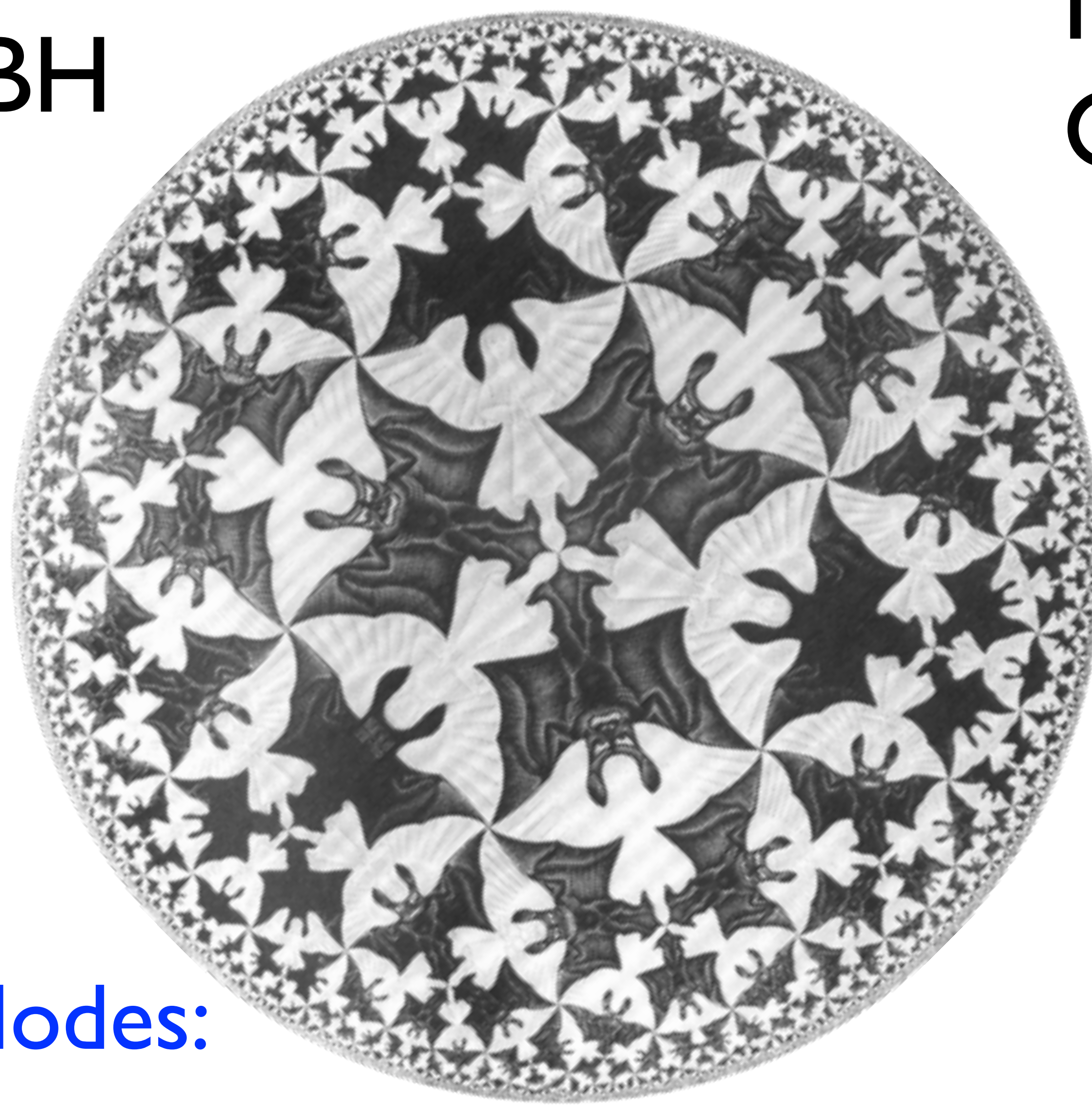
- Global on boundary = local in Bulk
- U(2) gauge fields + scalar in doublet
- gauged model,
- Chemical potential only in U(1)
- Holographic Goldstone modes = Quasinormal Modes
- Simplest: U(2) generalization of HHH

  [Hartnoll-Herzog-Horowitz]
- Decoupling limit

Holography

3D AdS
Schwarzschild BH

Maxwell +
Charged Scalar

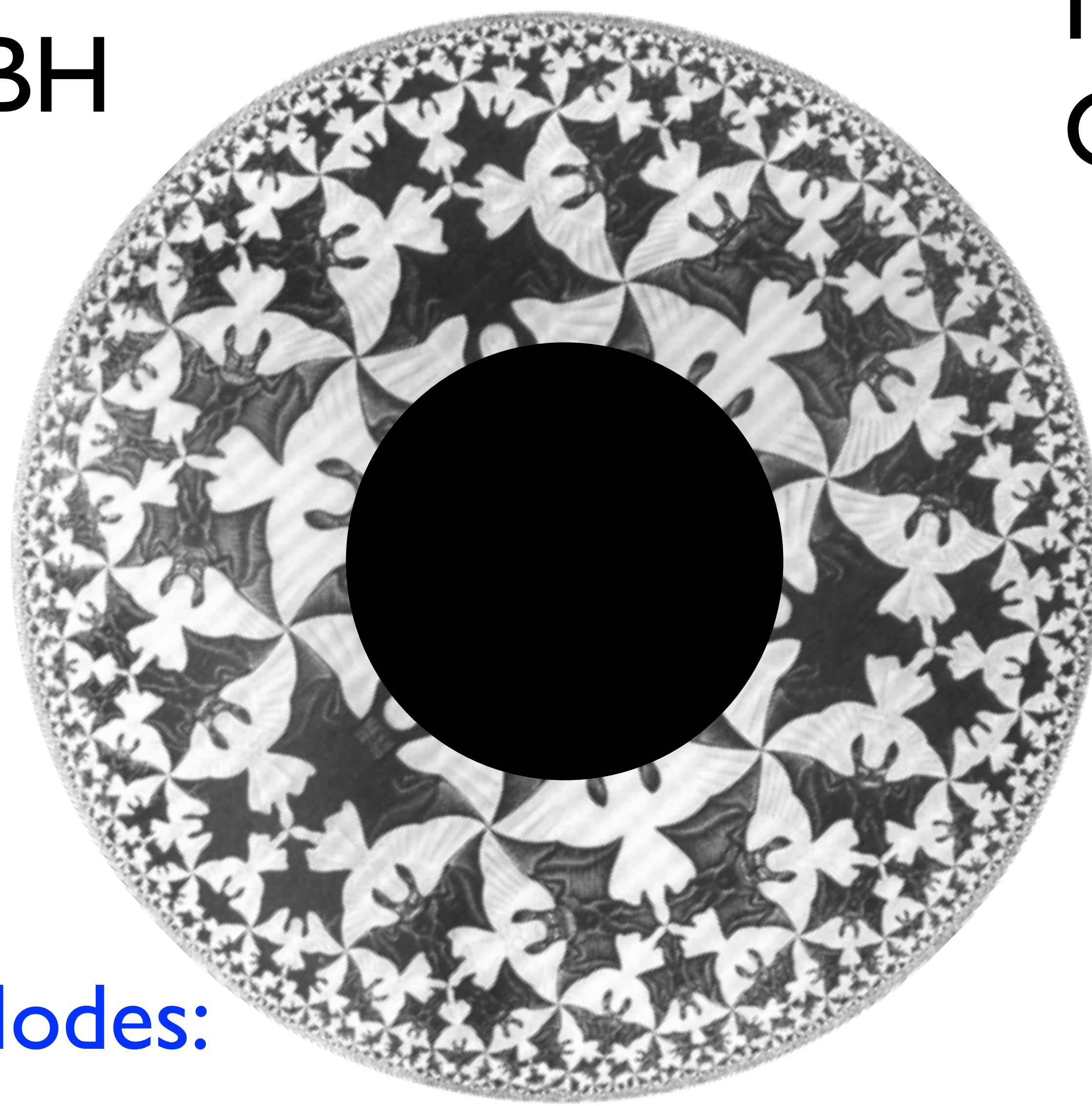


Quasinormal Modes:

Holography

3D AdS
Schwarzschild BH

Maxwell +
Charged Scalar



Quasinormal Modes:

Holography

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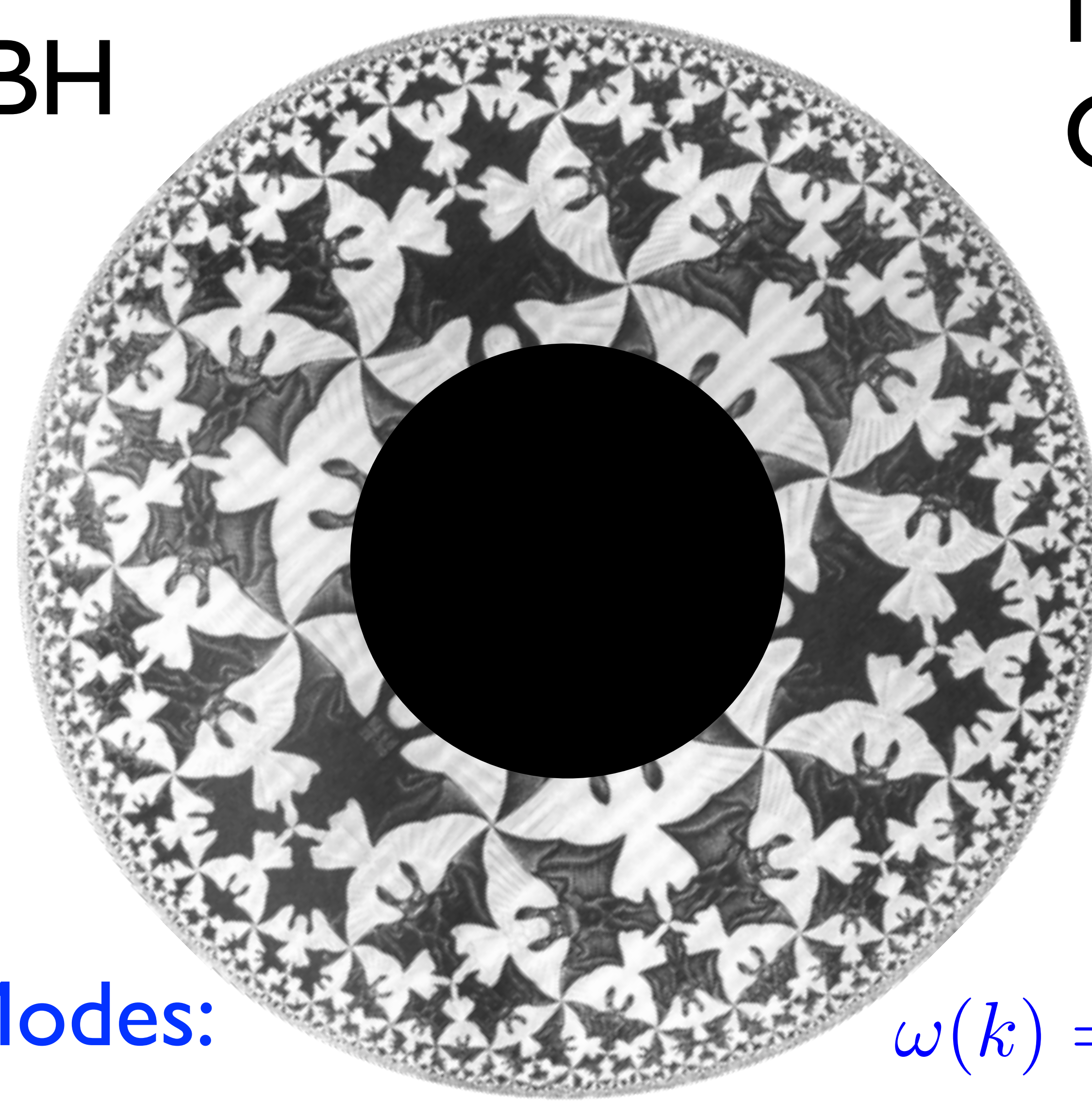
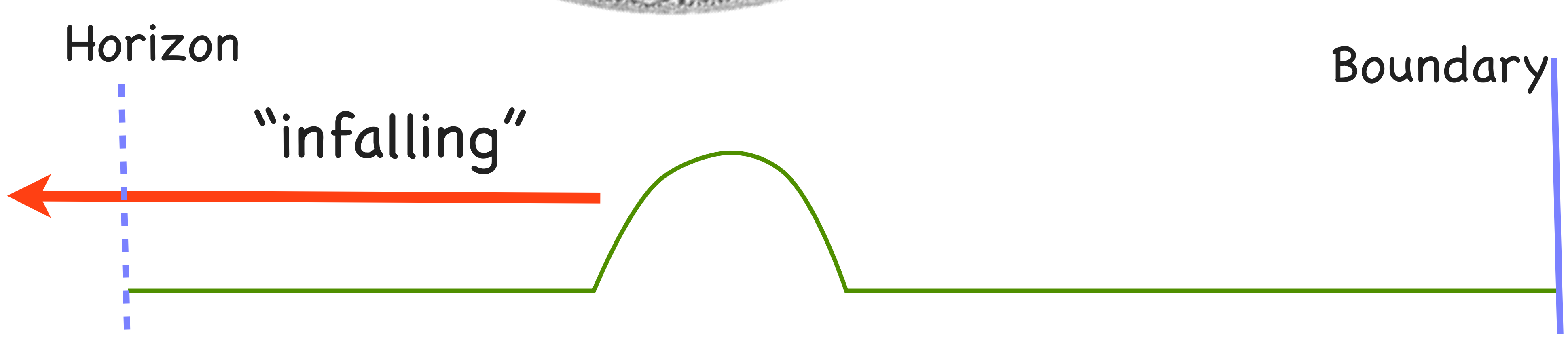


Quasinormal Modes:

$$\omega(k) = \pm\Omega(k) - i\,\Gamma(k)$$



Holography

U(2) decomposition:

$$\sigma_- = \frac{1}{2}(\sigma_0 - \sigma_3) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

broken:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

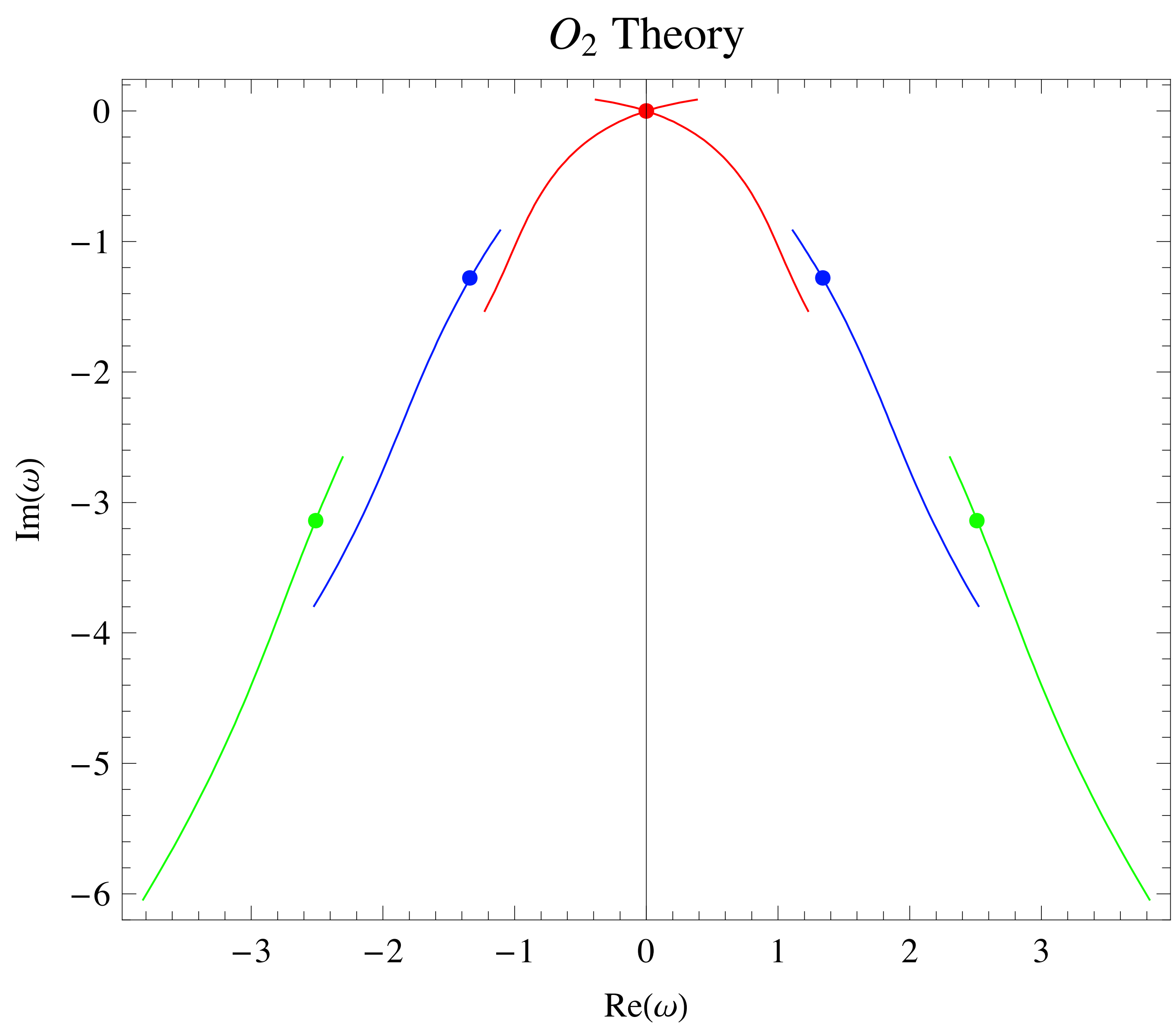
unbroken:

$$\sigma_+ = \frac{1}{2}(\sigma_0 + \sigma_3) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Holography

Normal phase (high T)

I. Amado, M. Kaminski and K. L., [arXiv:0903.2209 [hep-th]]



Scalar QNMs doubly degenerate

There are also 4 Diffusion modes $\omega = -iDk^2$ $D = \dfrac{3}{4\pi T}$

Holography

$$\Psi'' + \left(\frac{f'}{f} + \frac{2}{\rho}\right)\Psi' + \frac{\chi^2}{f^2}\Psi - \frac{m^2}{f}\Psi = 0\,,$$

Scalar

$$\chi'' + \frac{2}{\rho}\chi' - \frac{2\Psi^2}{f}\chi = 0\,,$$

σ_- Sector

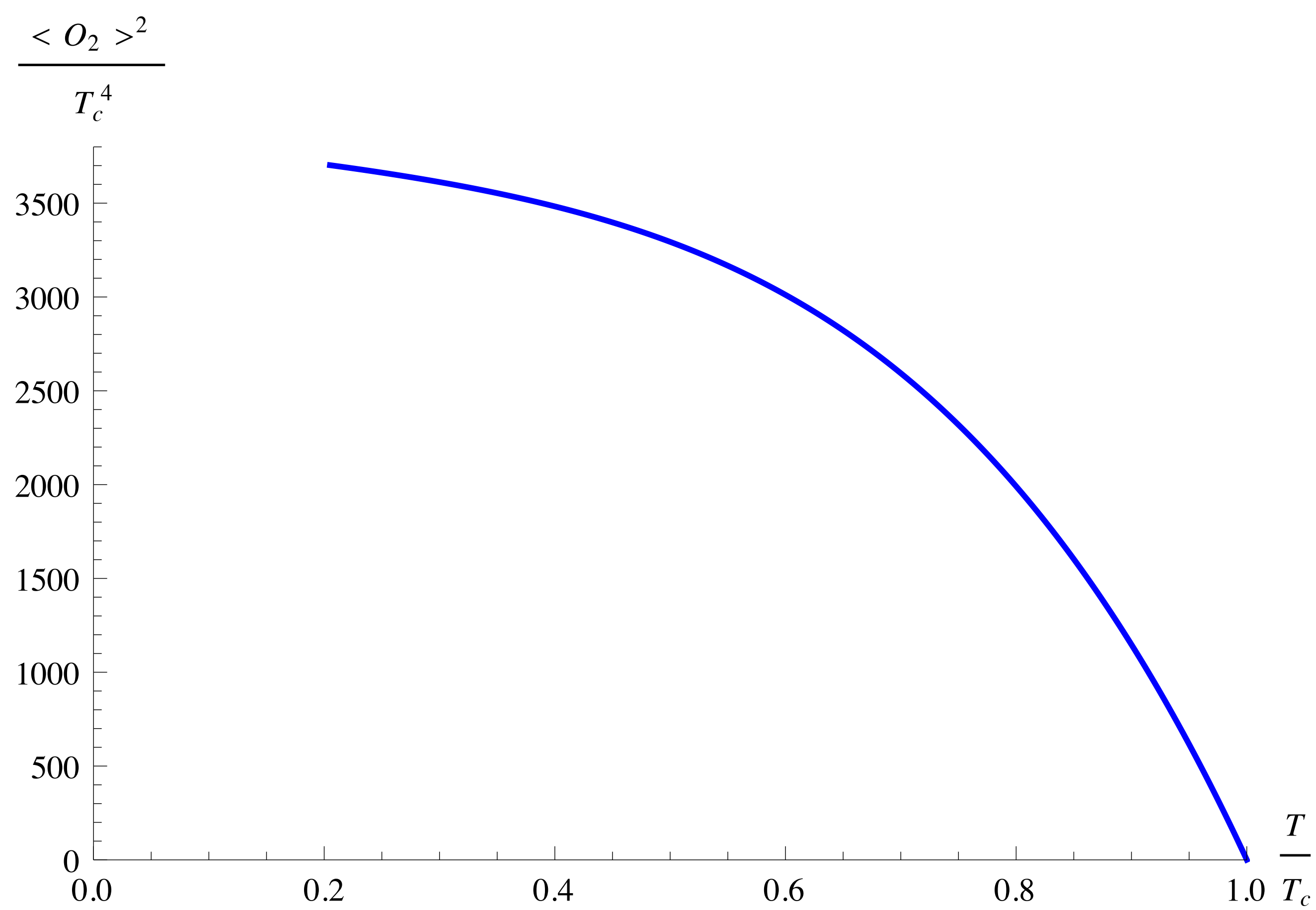
$$\xi'' + \frac{2}{\rho}\xi' = 0\,,$$

σ_+ Sector

$$\chi = \bar{\mu}_\chi - \frac{\bar{n}_\chi}{\rho} + O\left(\frac{1}{\rho^2}\right)$$

$$\xi = \bar{\mu}_\xi - \frac{\bar{n}_\xi}{\rho} + O\left(\frac{1}{\rho^2}\right)$$

$$\Psi = \frac{\psi_1}{\rho} + \frac{\psi_2}{\rho^2} + O\left(\frac{1}{\rho^3}\right)$$



b.c.s: $\bar{\mu}_\chi = \bar{\mu}_\xi = \bar{\mu}.$

$\psi_1 = 0.$

$\psi_2 \propto \langle O \rangle$

Holography

$$\Psi'' + \left(\frac{f'}{f} + \frac{2}{\rho}\right)\Psi' + \frac{\chi^2}{f^2}\Psi - \frac{m^2}{f}\Psi = 0\,,$$

Scalar

$$\chi'' + \frac{2}{\rho}\chi' - \frac{2\Psi^2}{f}\chi = 0\,,$$

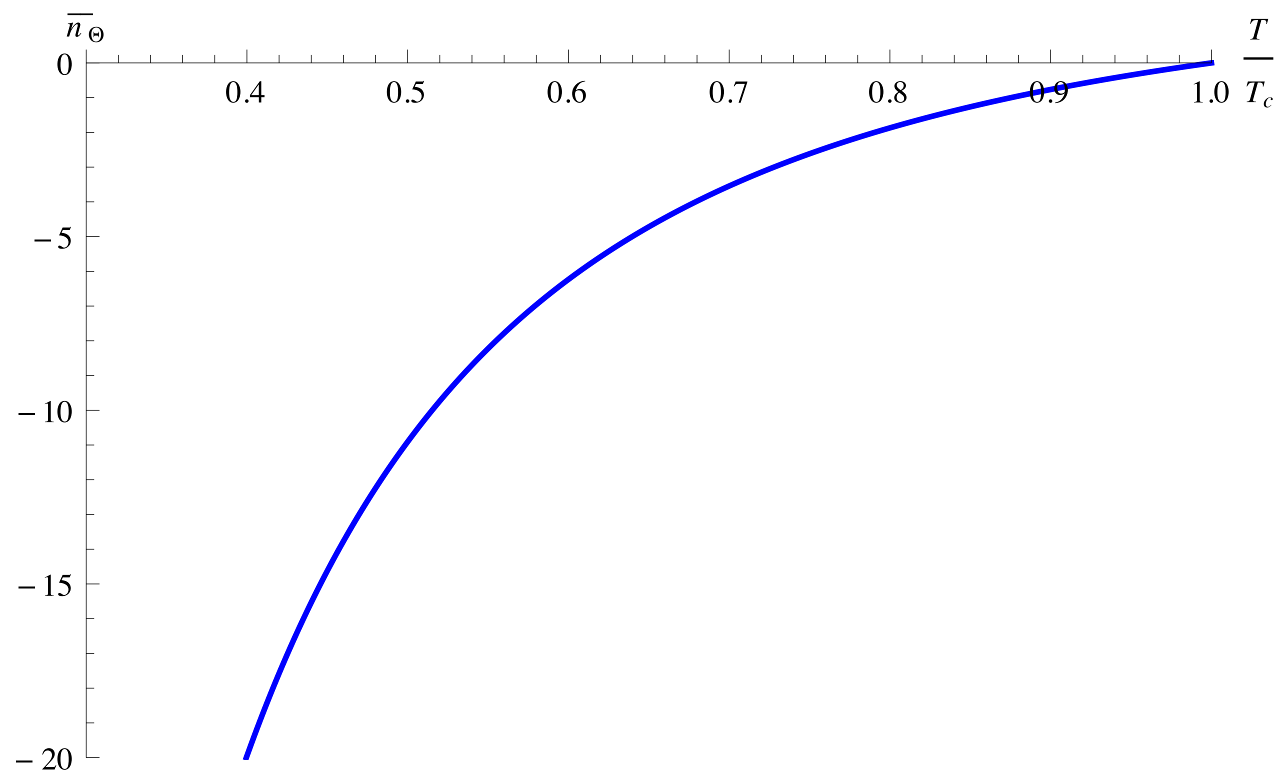
σ_- Sector

$$\xi'' + \frac{2}{\rho}\xi' = 0\,,$$

σ_+ Sector

Charge in σ_3 Sector:

(Charge in σ_0 sector always >0)



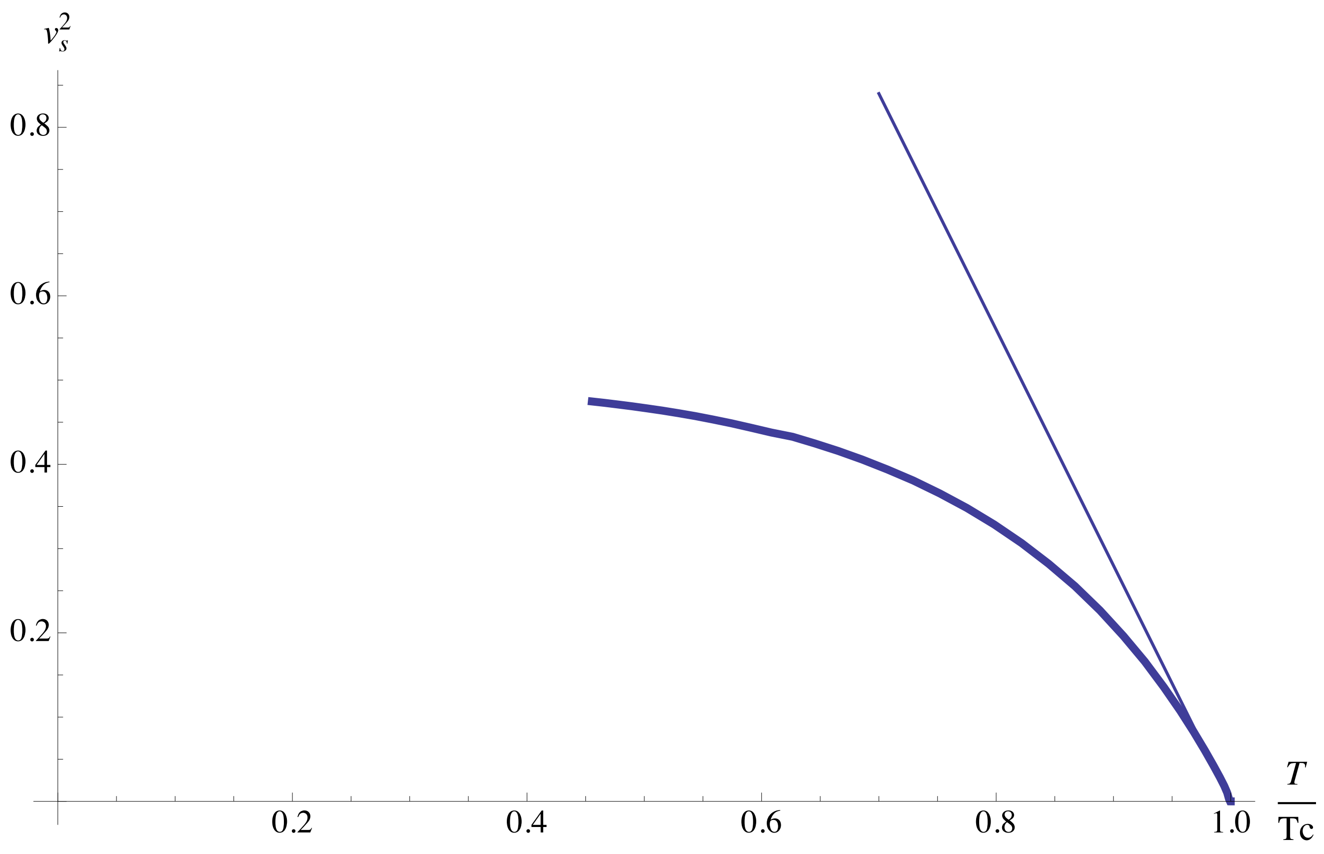
triggers 2nd phase transition to p-wave

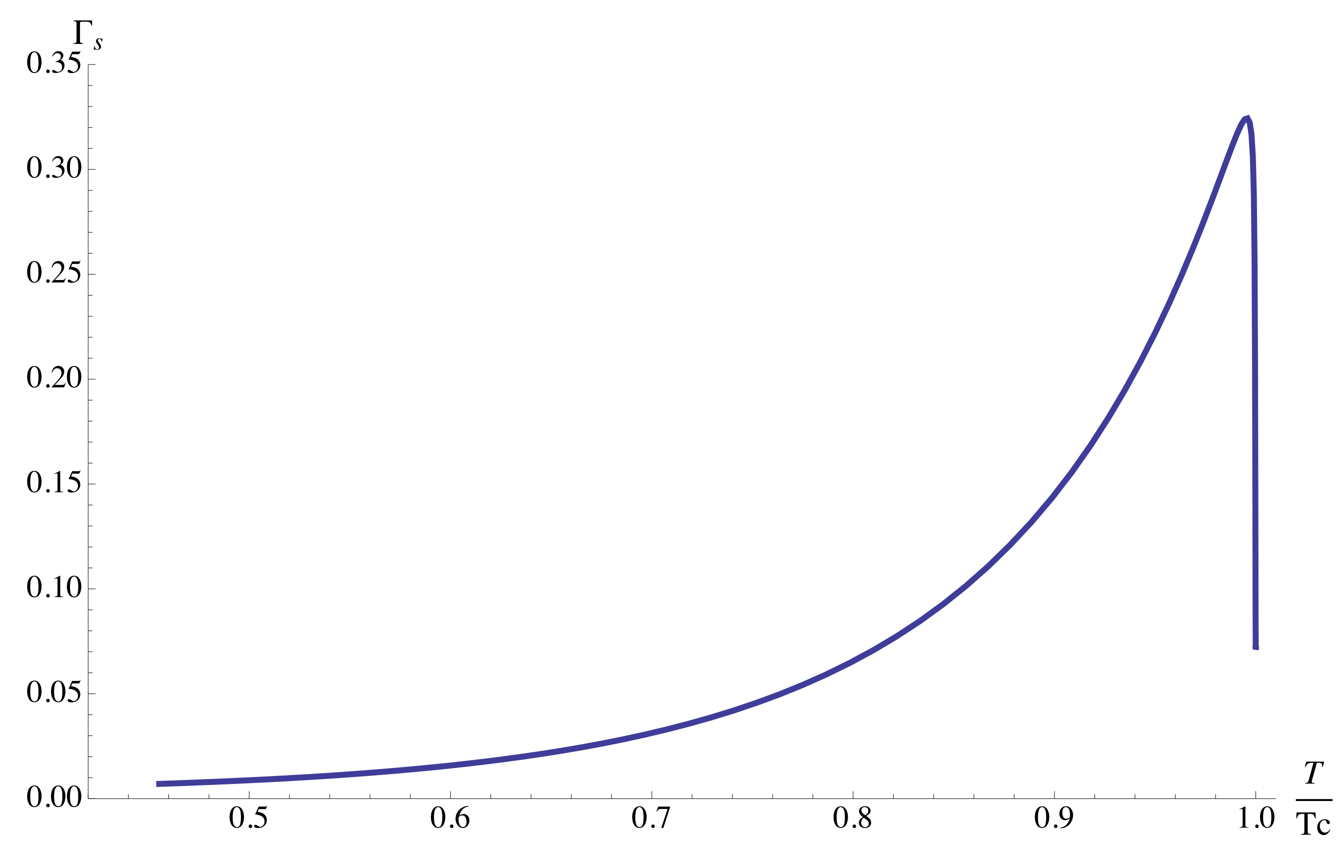
[I. Amado, D. Arean, A. Jimenez-Alba, L. Melgar, I. Salazar-Landea, arXiv:1309.5986]

Holography

“σ-” Sector:

Broken phase, Type I (4th sound) $\omega = v_s k + (b - i\Gamma_s)k^2$

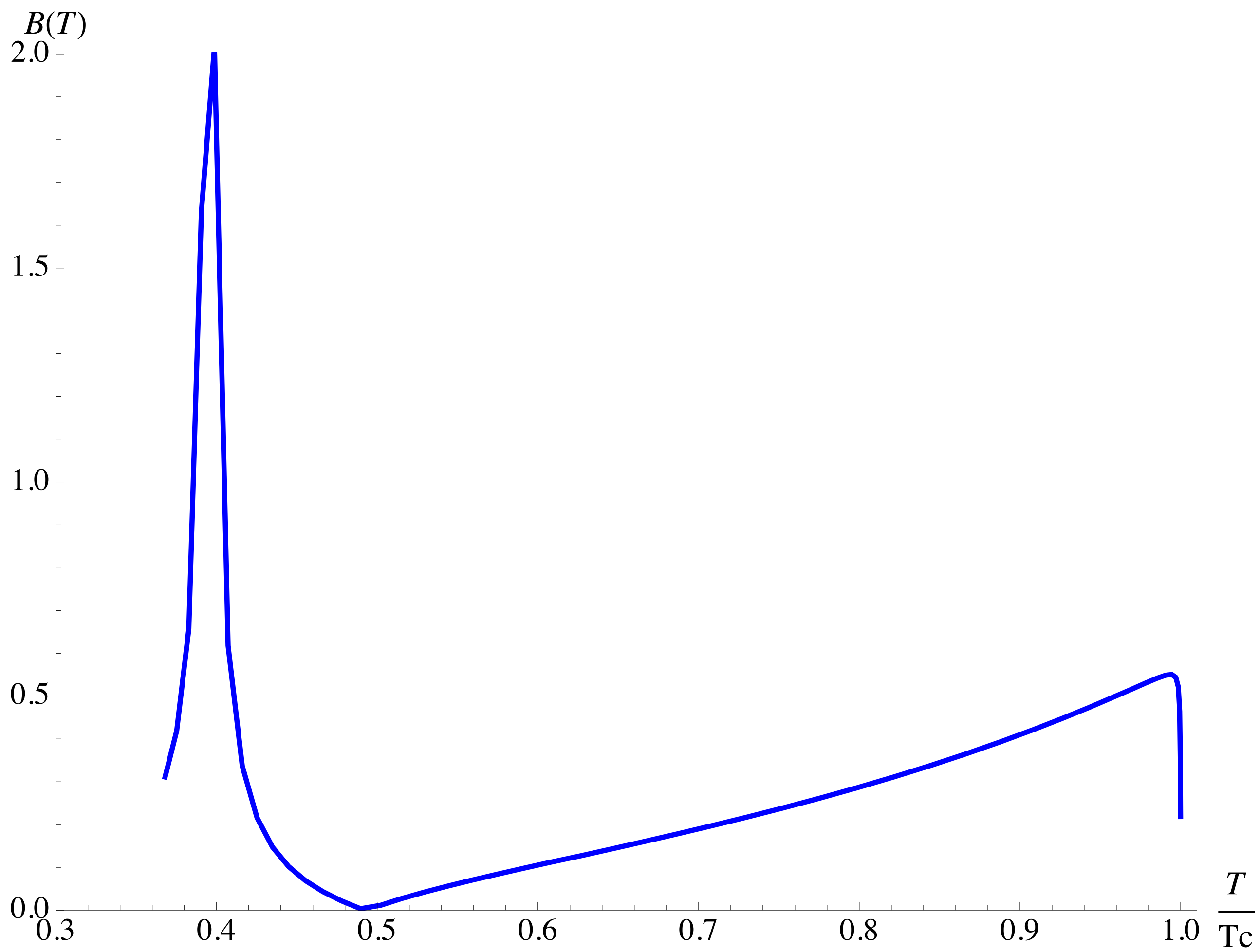


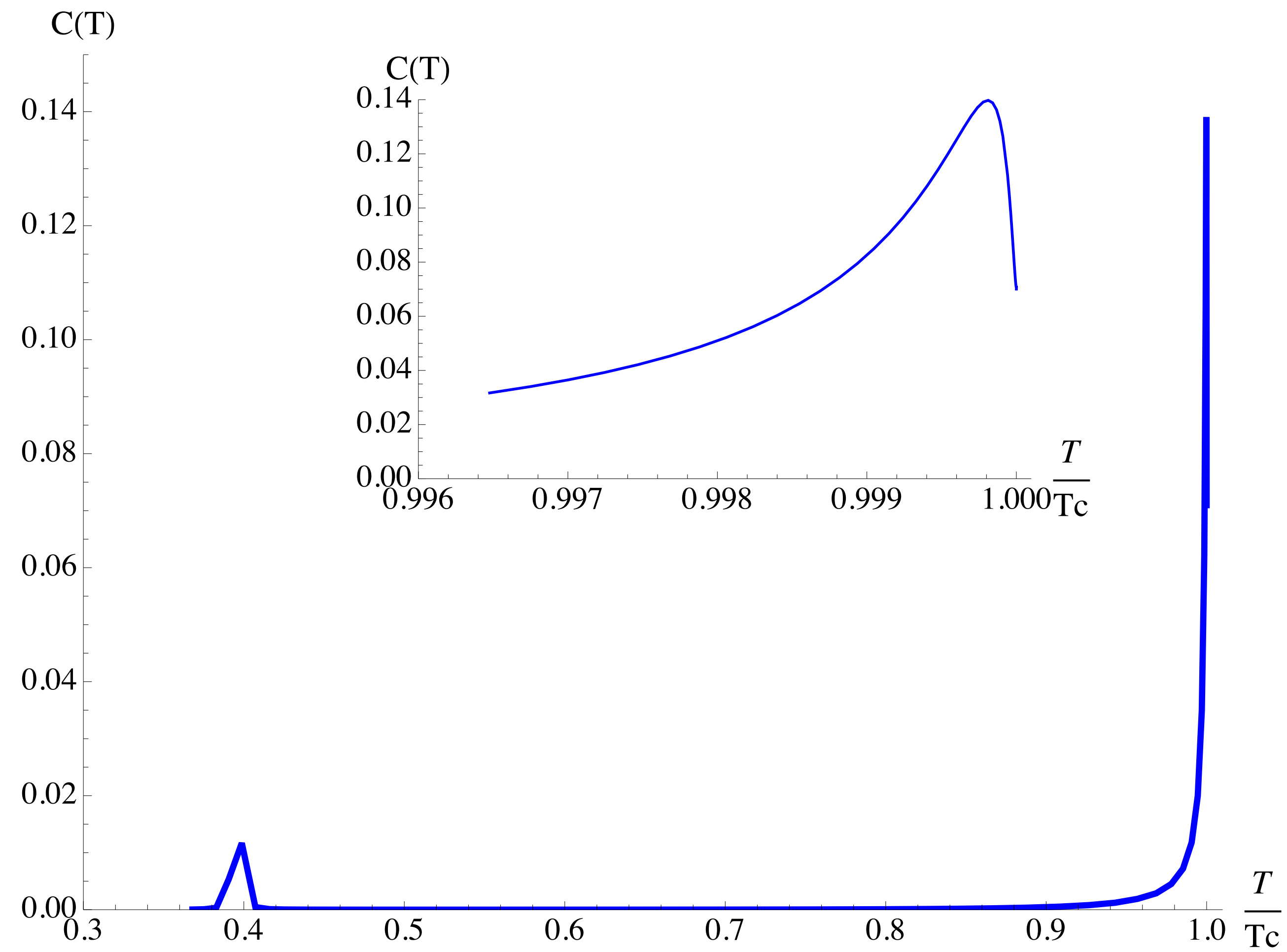


Holography

"$\sigma_{1,2}$" Sector:

Broken phase, Type II $\quad \omega = (B - iC)k^2$





Holography

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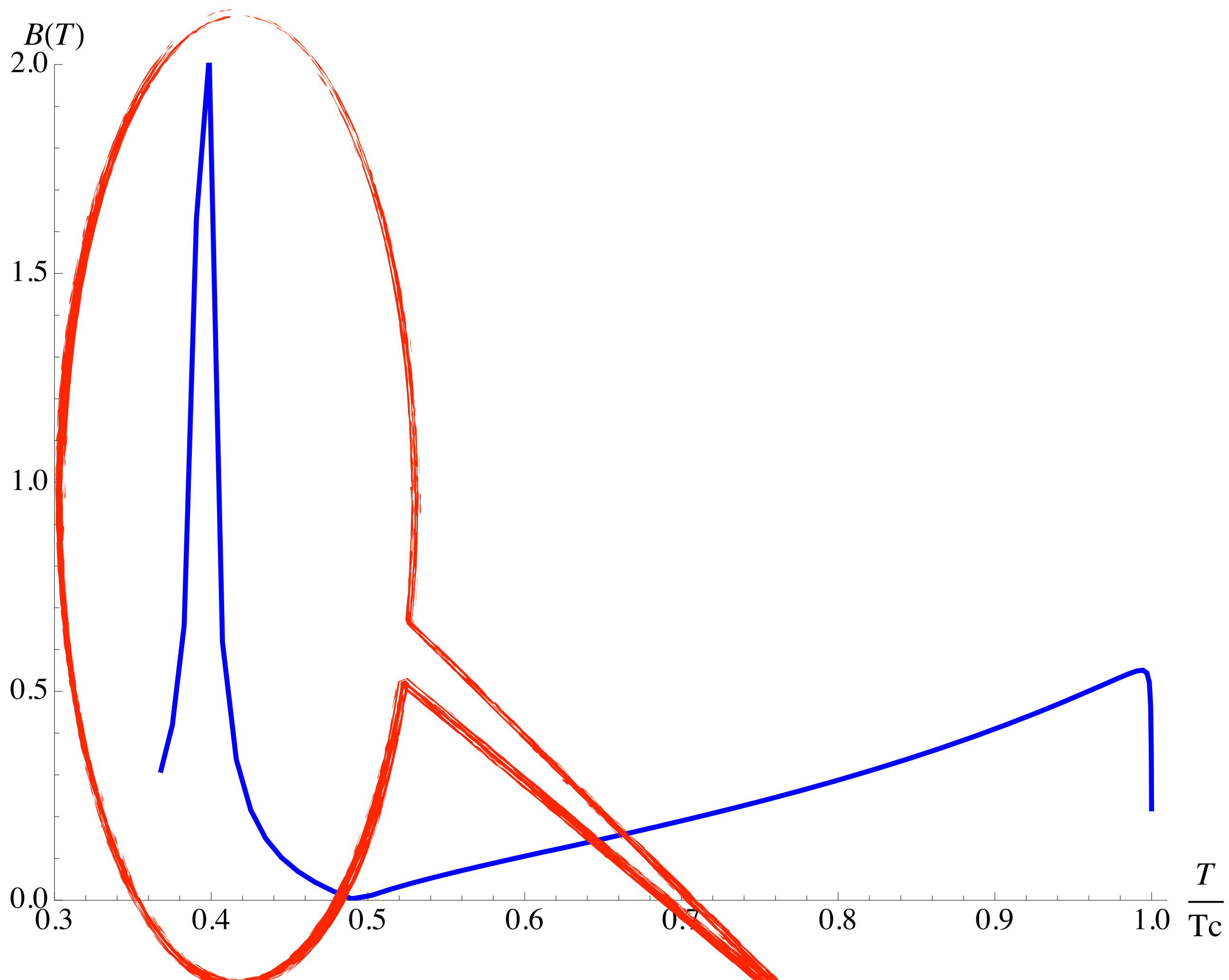
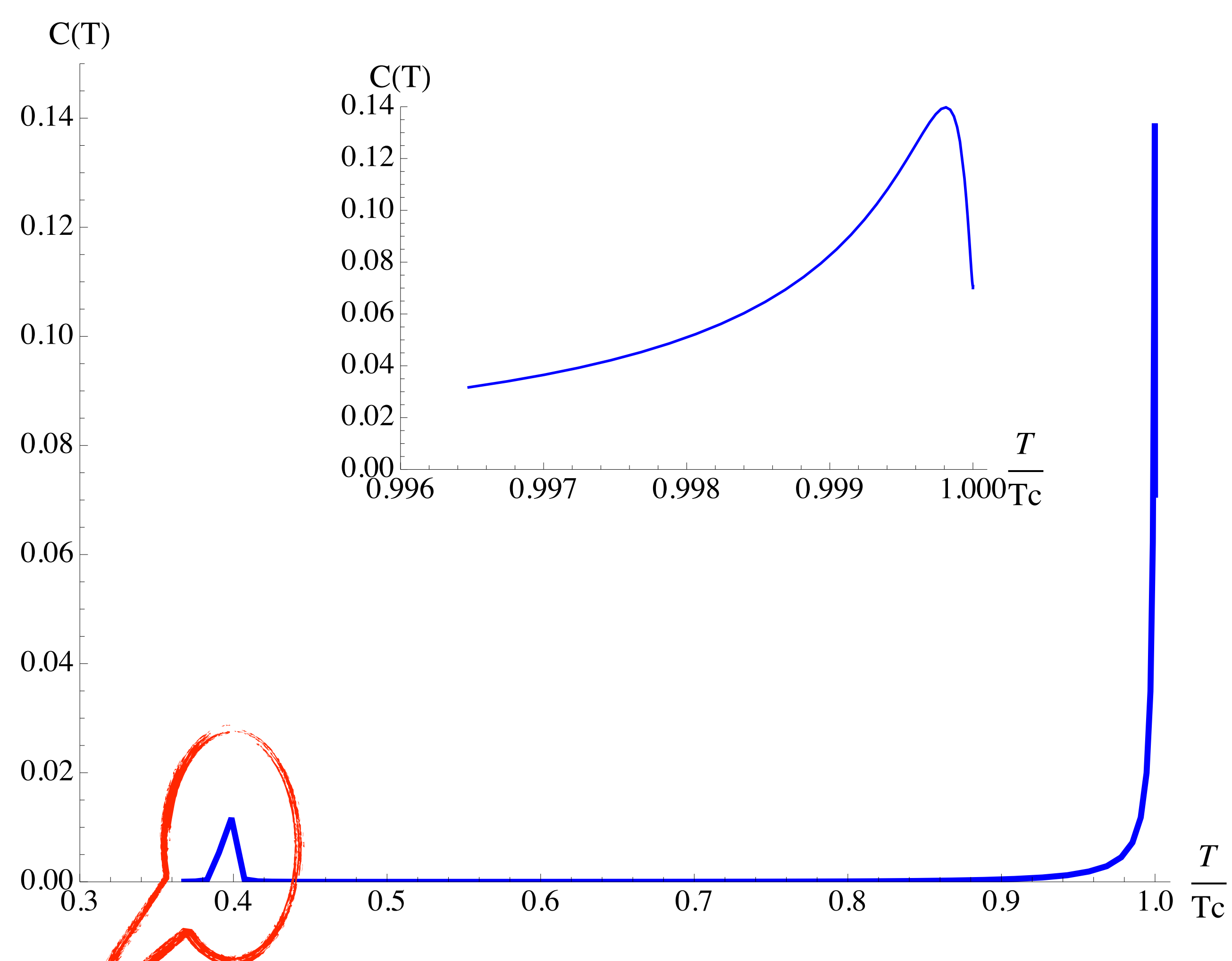


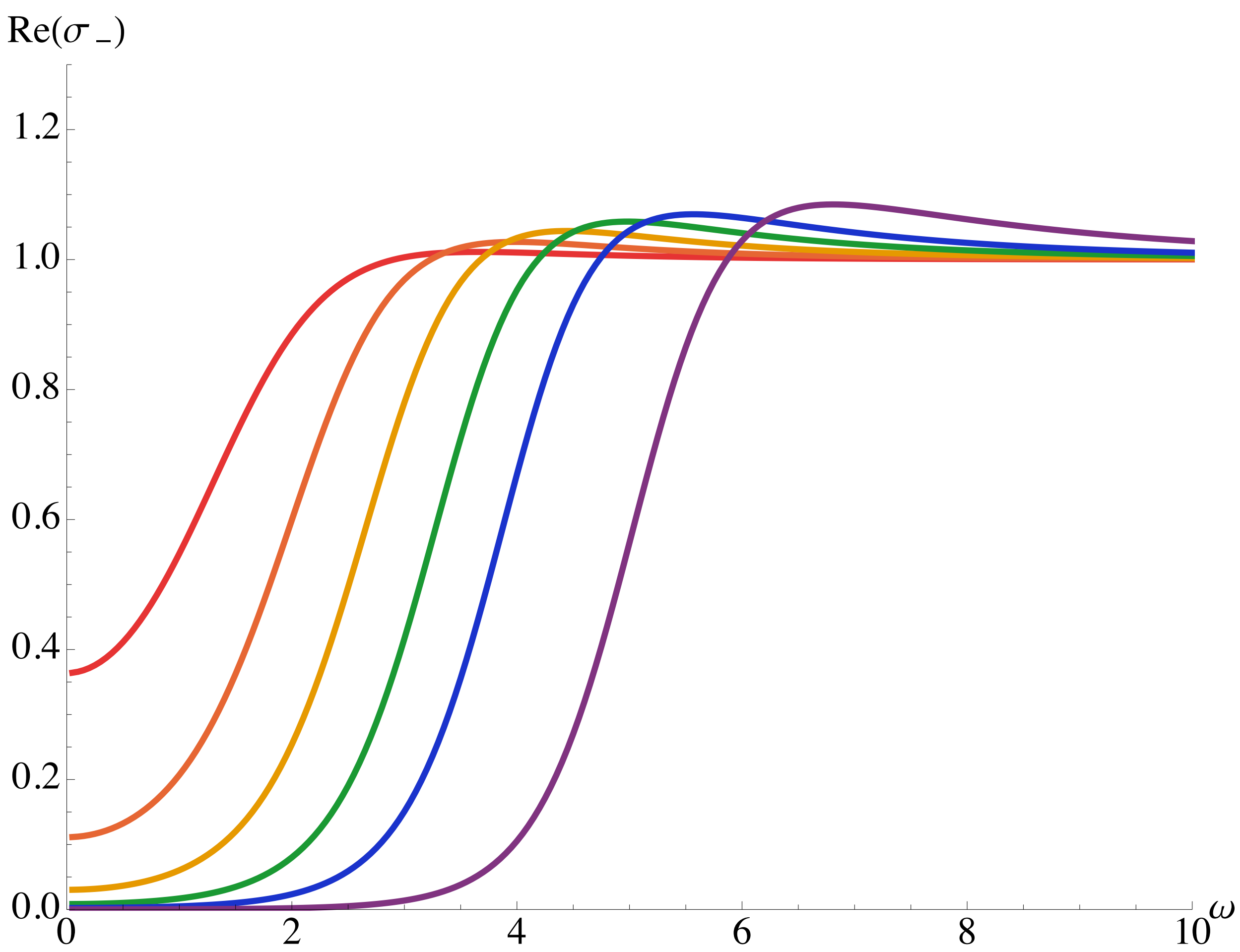


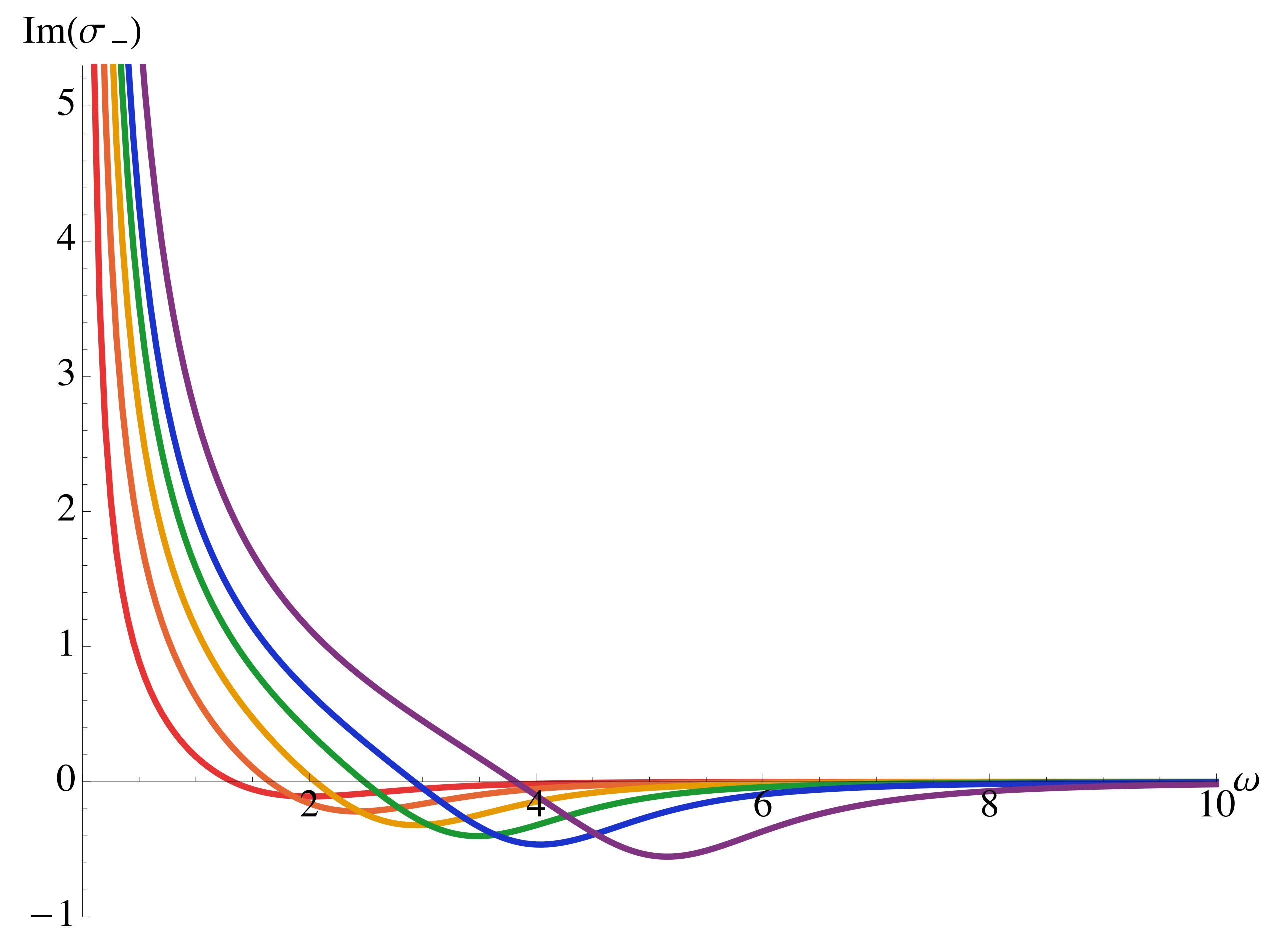
additional unstable mode: p-wave condensate

Holography

"σ₋" Sector:

Conductivities related to type I



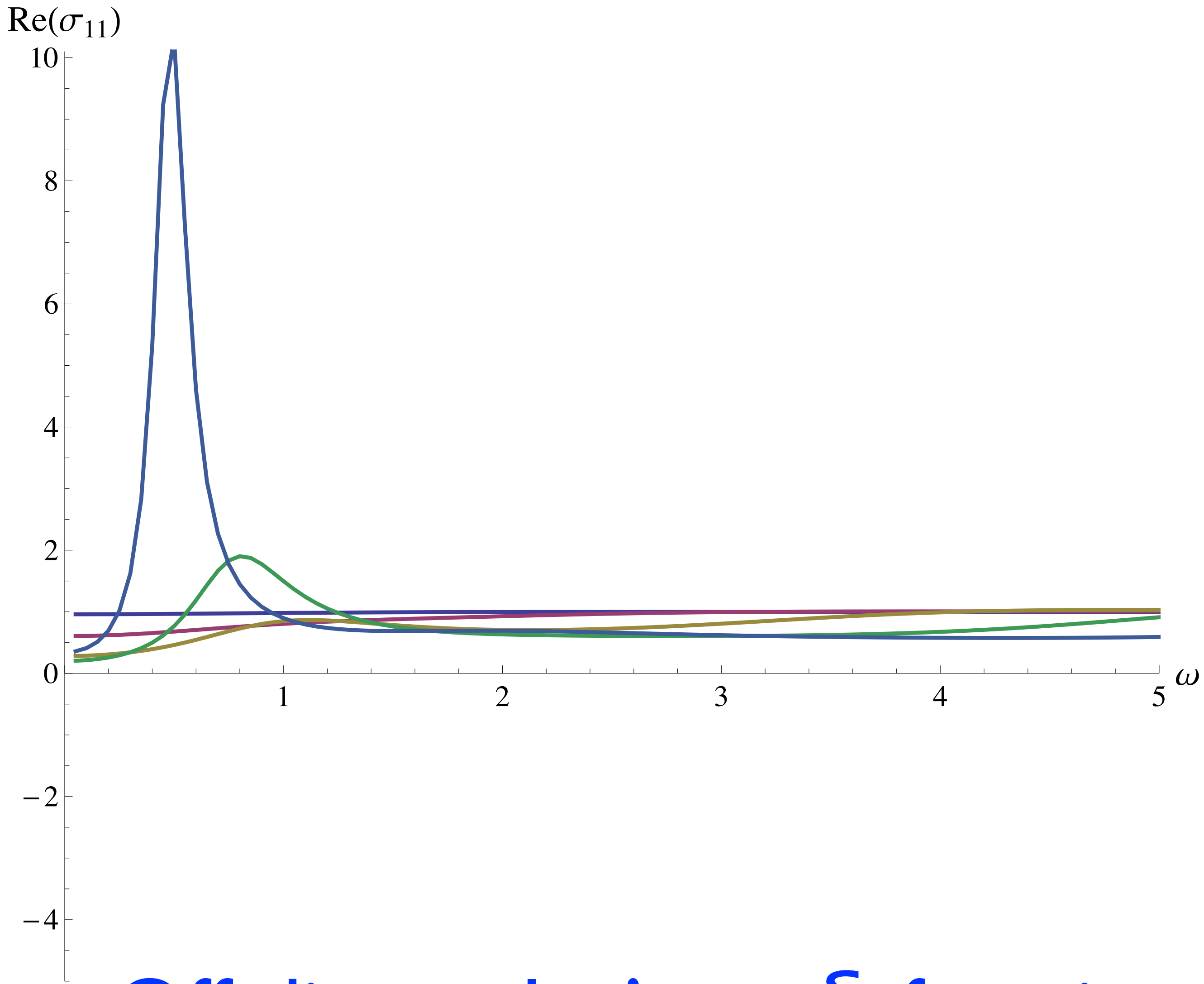


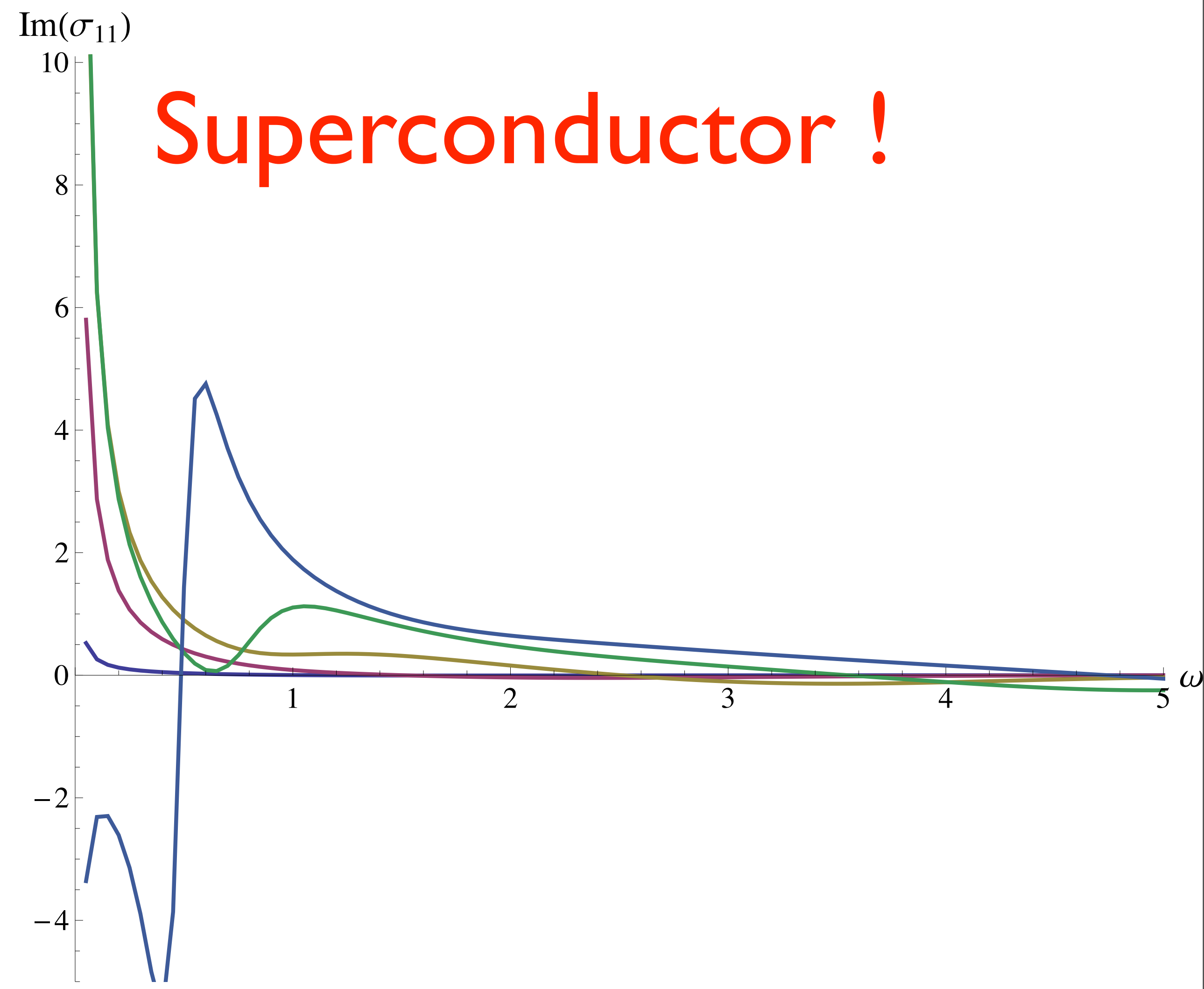
(just HHH model)

Holography

"$\sigma_{1,2}$" Sector:

Conductivities related to type II, diagonal





Superconductor !

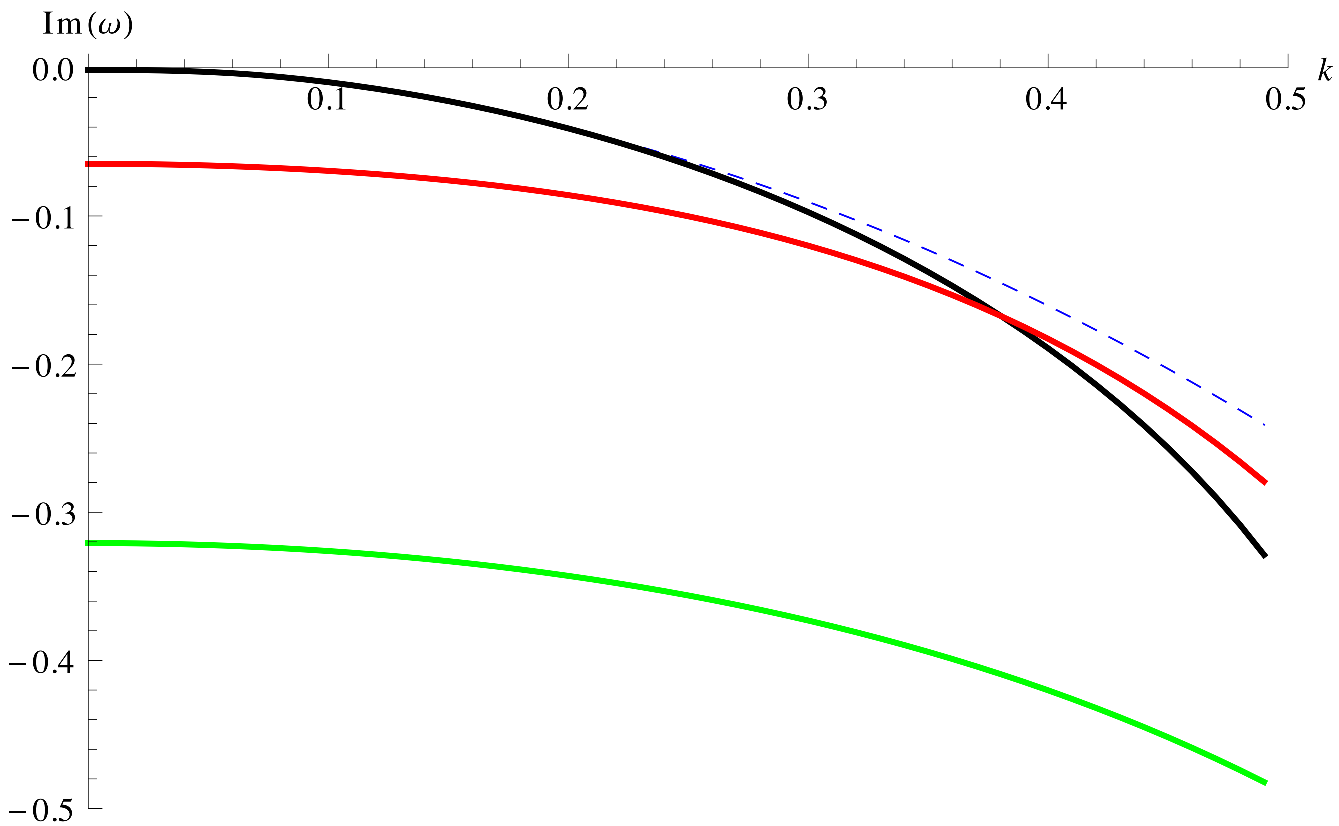
Off-diagonal σ's: no δ-function pole !

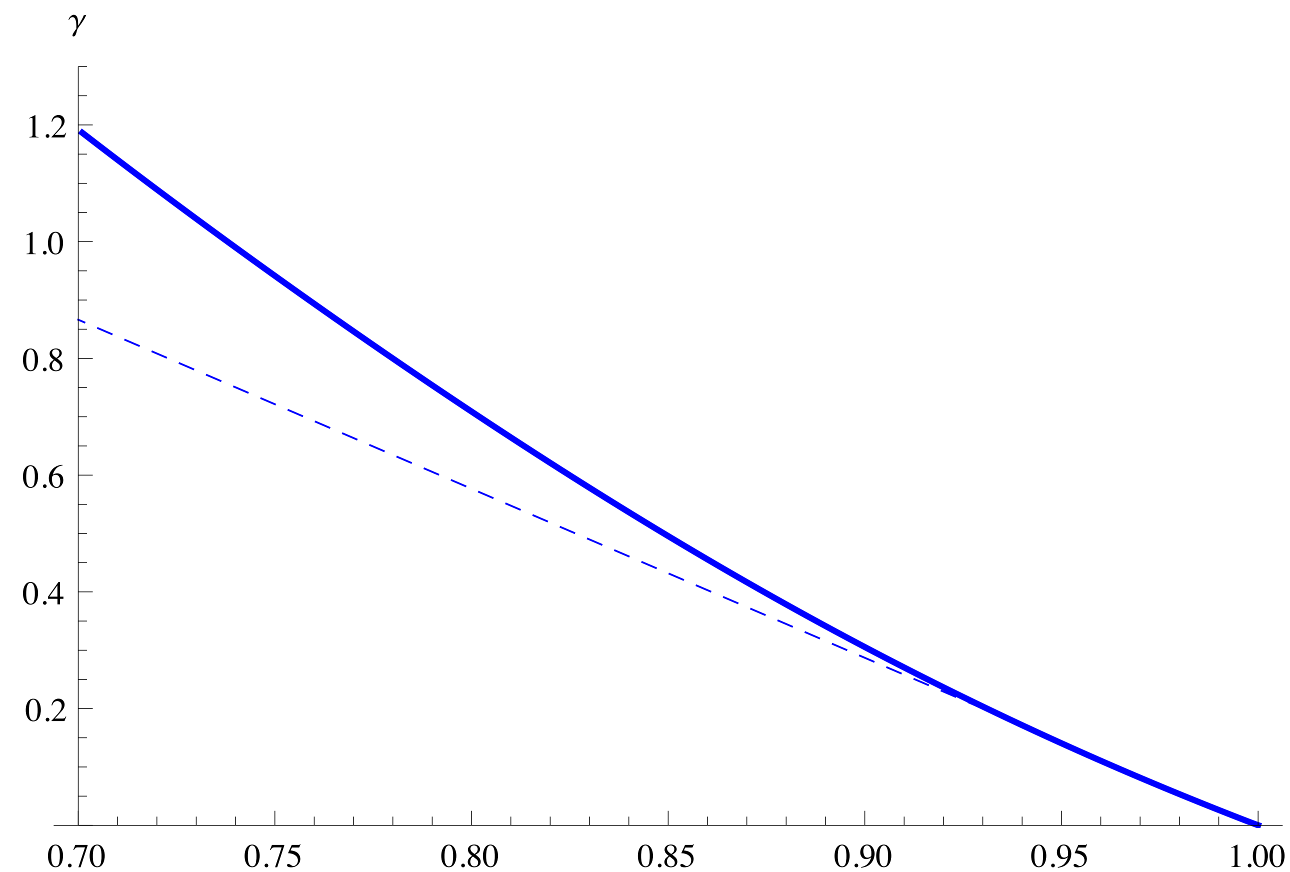
Holography

Fate of the diffusion modes in broken phase

"σ₋" Sector: gapped (pseudo) diffusion

$$\omega = -i\gamma - iDk^2$$





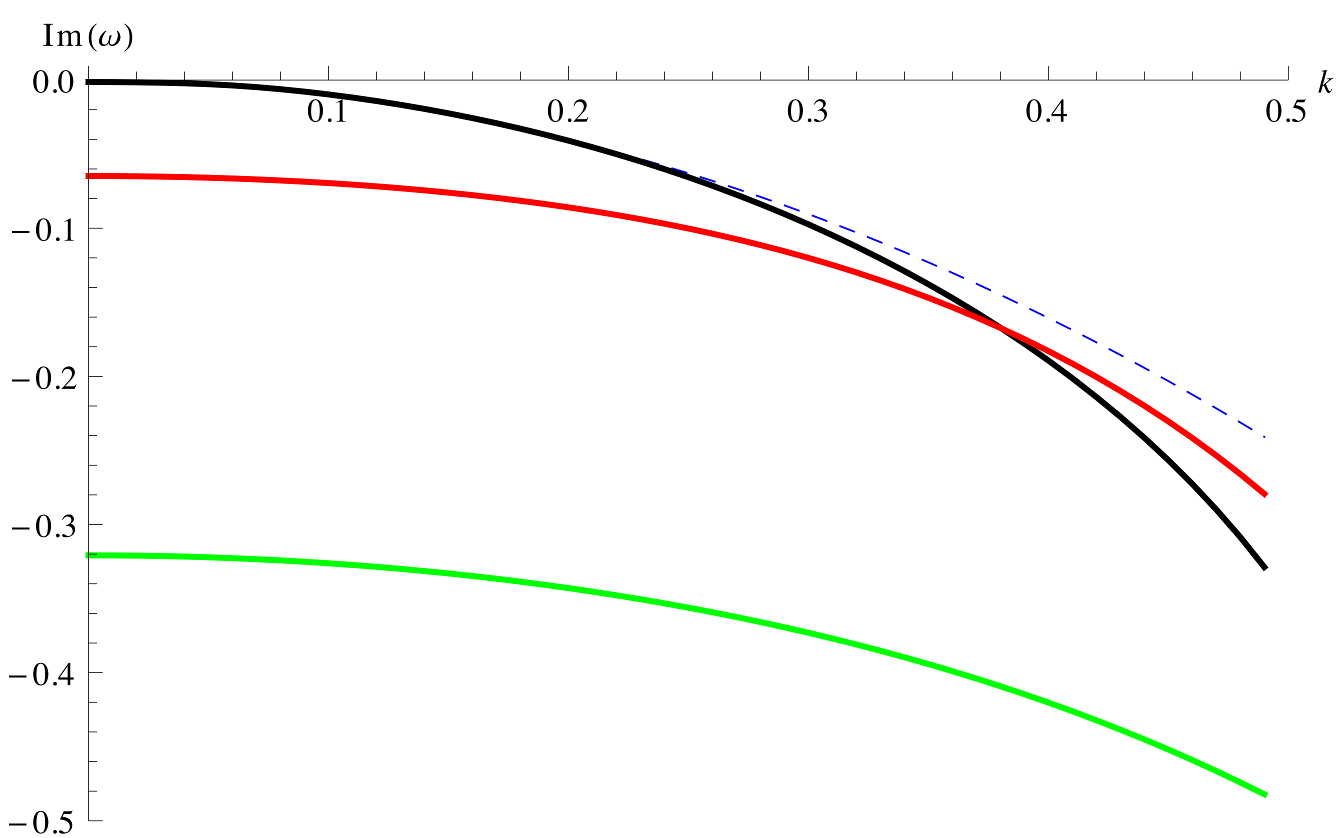
Holography

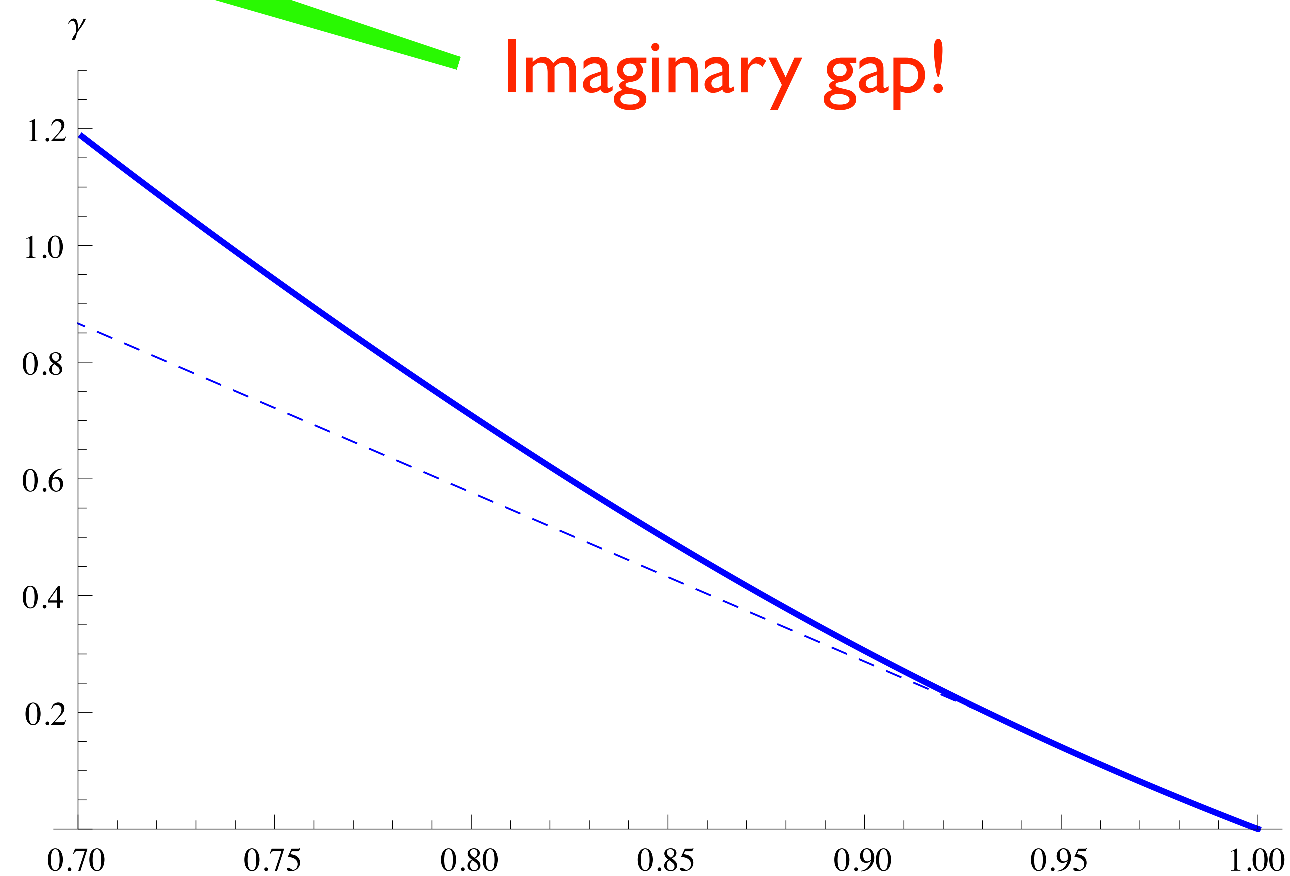
Fate of the diffusion modes in broken phase

"σ₋" Sector: gapped (pseudo) diffusion

$$\omega = -i\gamma - iDk^2$$

Imaginary gap!

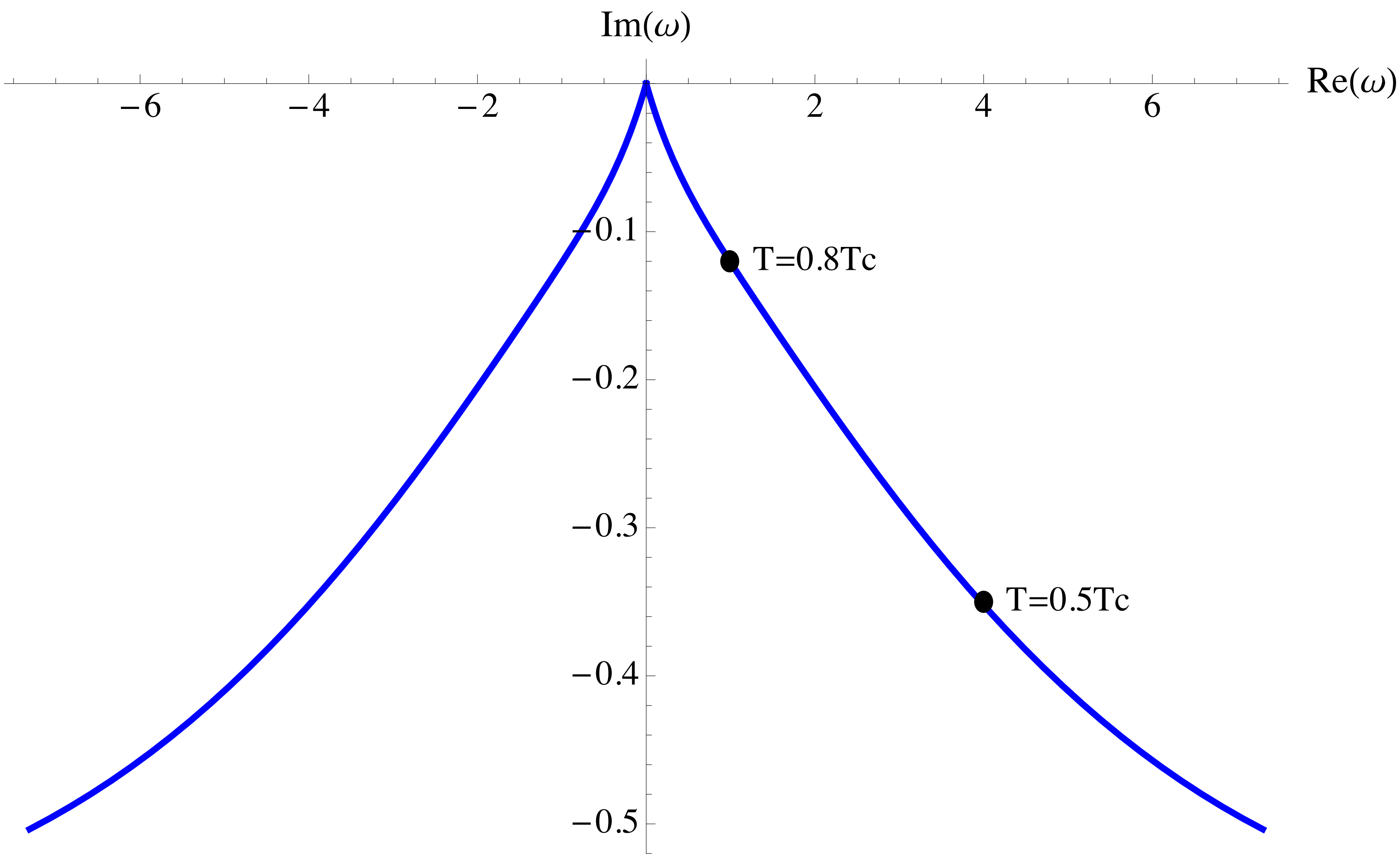




Holography

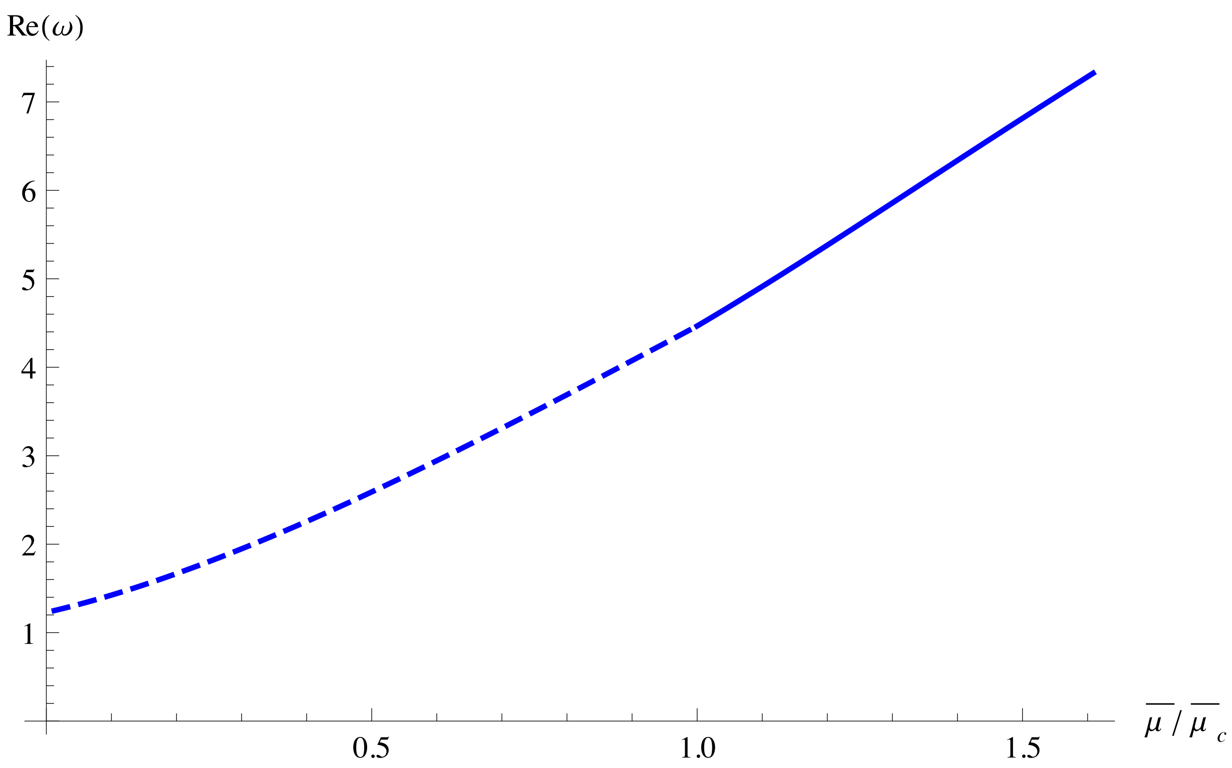
Fate of the diffusion modes in broken phase

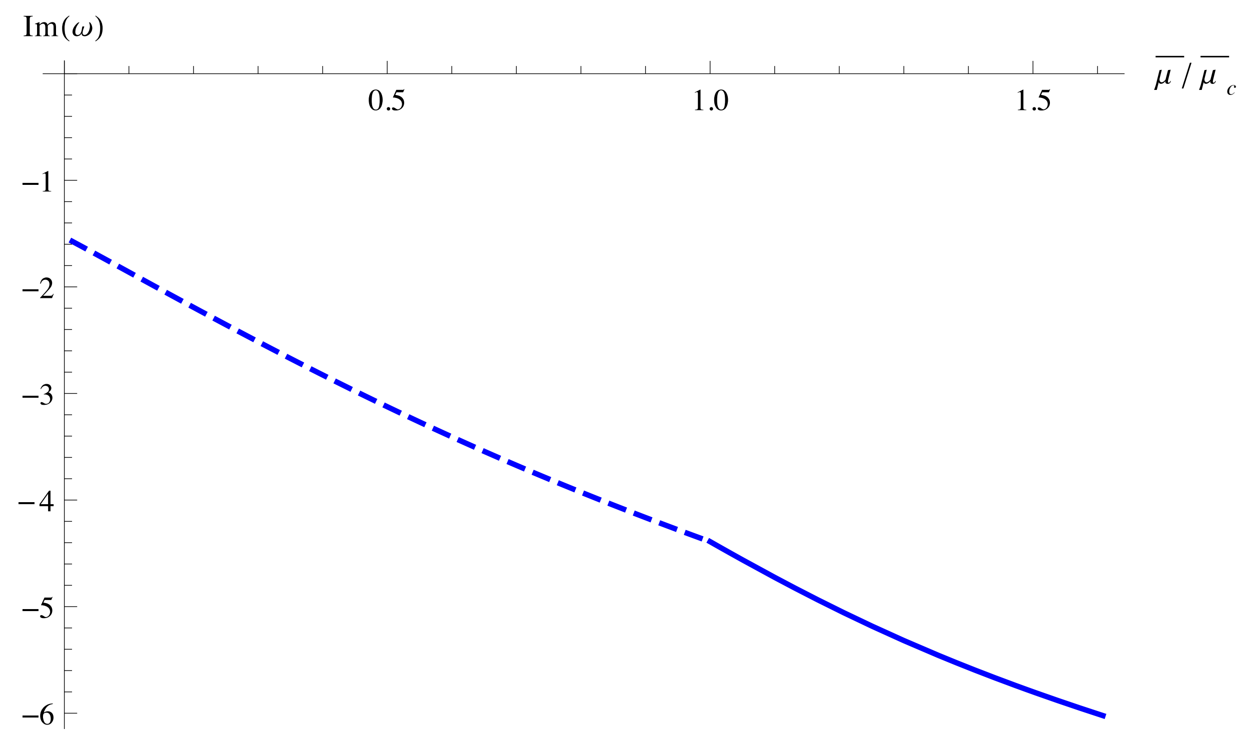
$\sigma_{1,2}$ - Sector: 2 gapped modes $\omega = \pm\Omega - i\Gamma$



Holography

“Massive” Goldstone $Re(\omega) \approx 1.1q\mu$





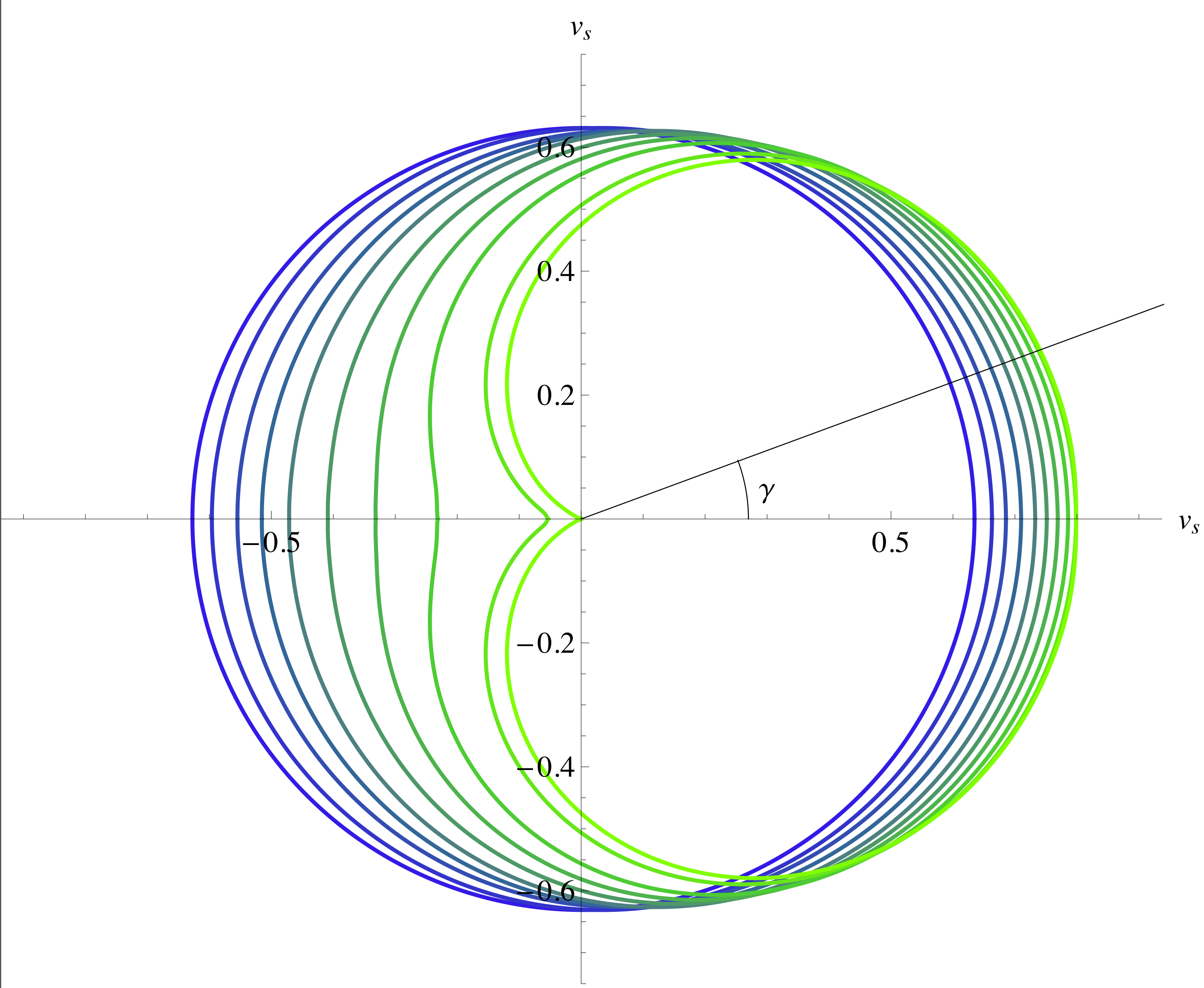
Landau criterion

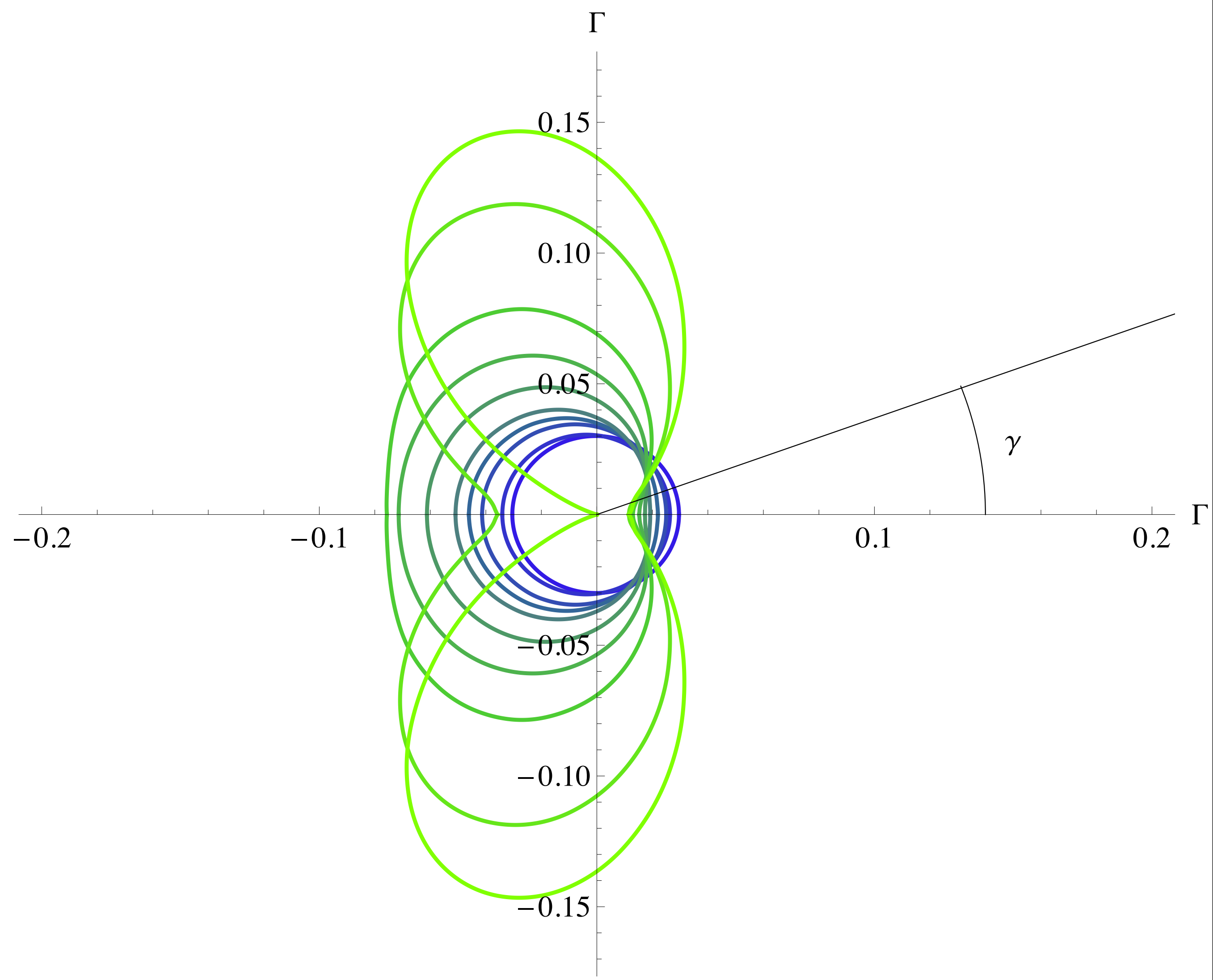
- Superflow = spatial gauge field on boundary
- Critical superfluid velocity
  - T=0 via boosts $\omega(p) + \vec{p}.\vec{S} \leq 0 \;\; \Rightarrow \;\; S_c = \min \frac{\omega(p)}{p}$
  - T>0 more complicated
  - basic idea: negative energy , instability
  - QNMs:

$$\Re(\omega(p,S,T)) \leq 0 \qquad\qquad \Im(\omega(p,S,T)) \geq 0$$

Landau criterion

Type I Goldstone



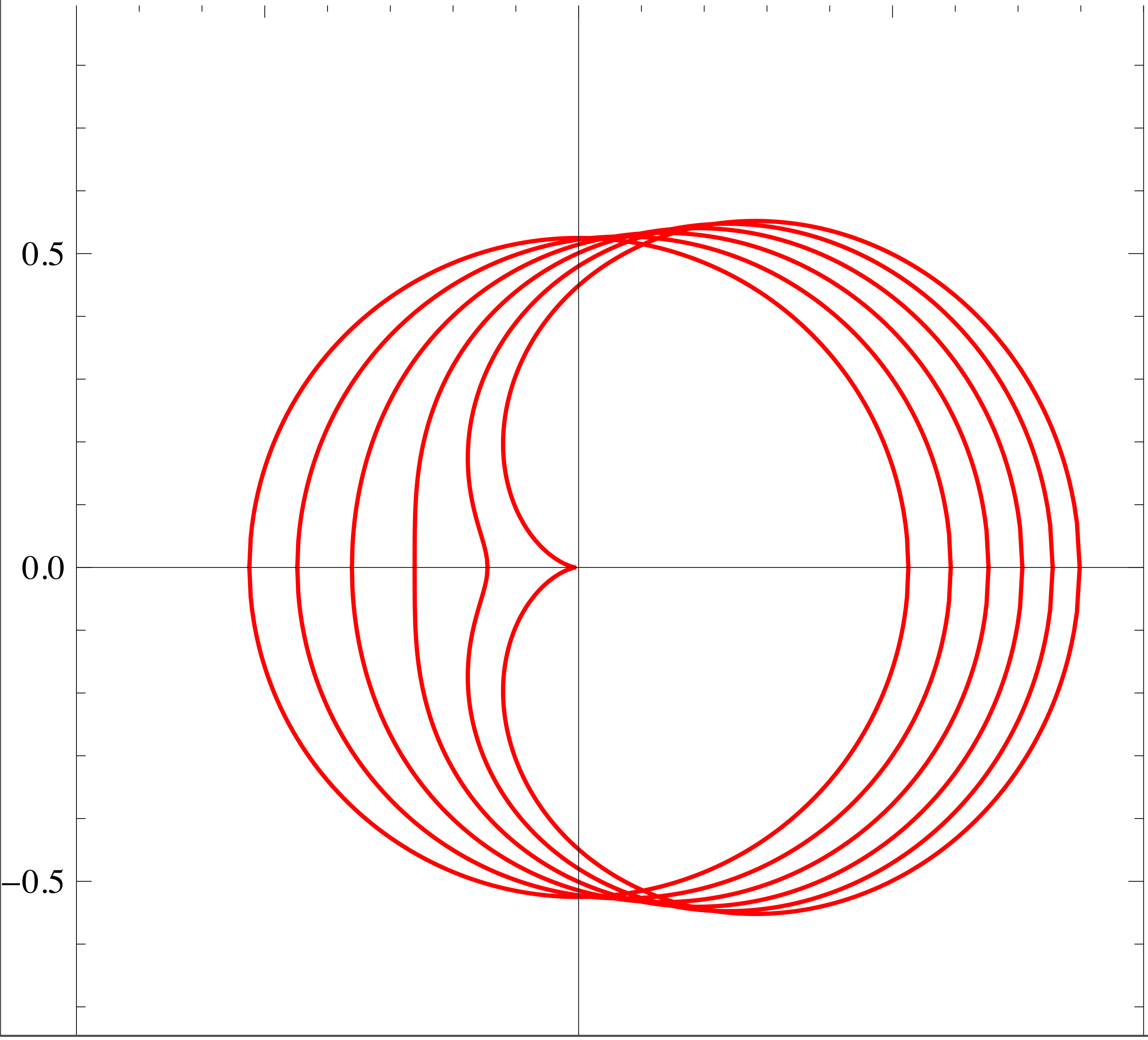


Landau criterion

Weak Coupling

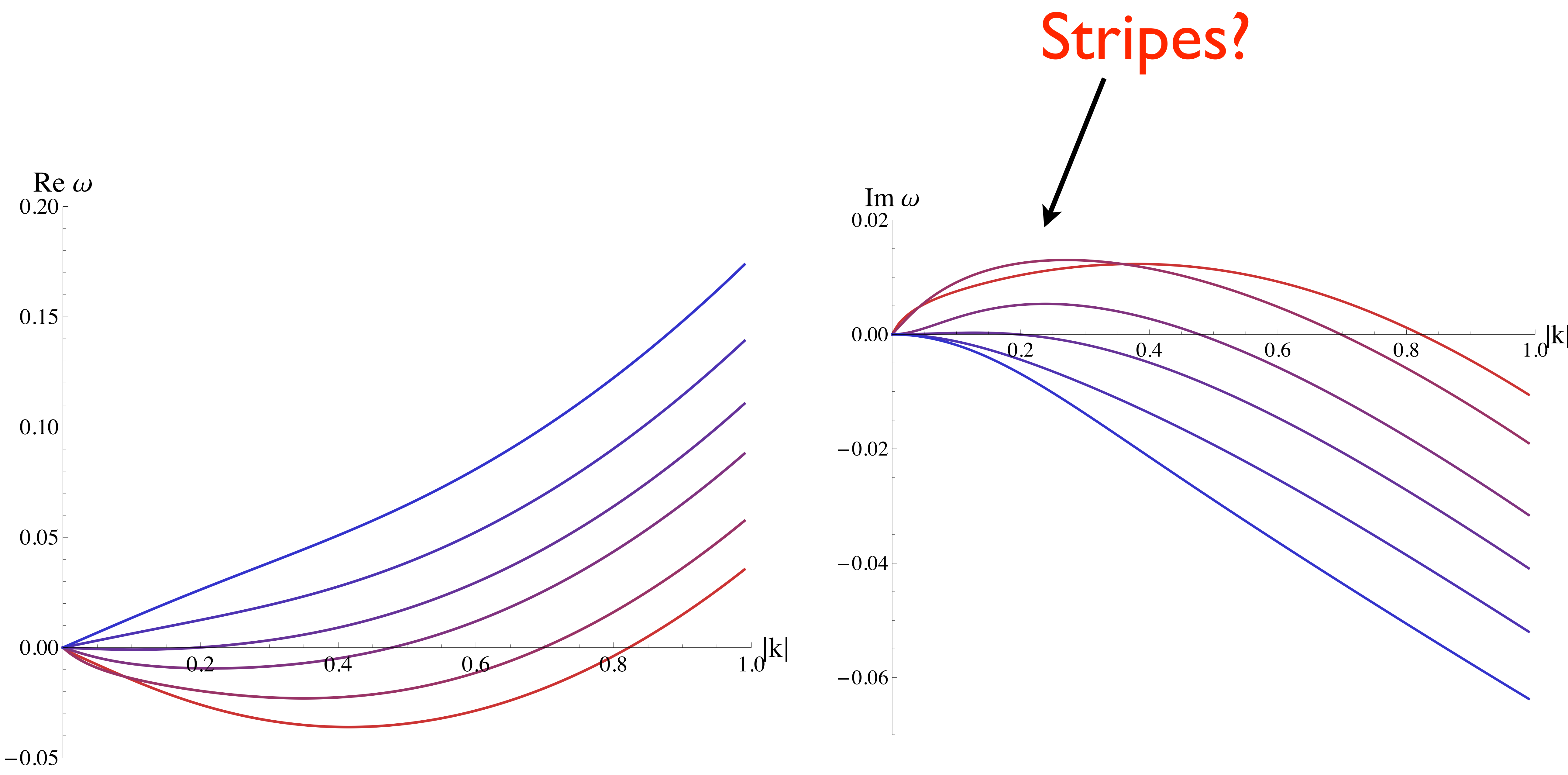
(courtesy: A. Schmitt)

[Alford, Mallavarpu, Schmitt, Stetina] to appear



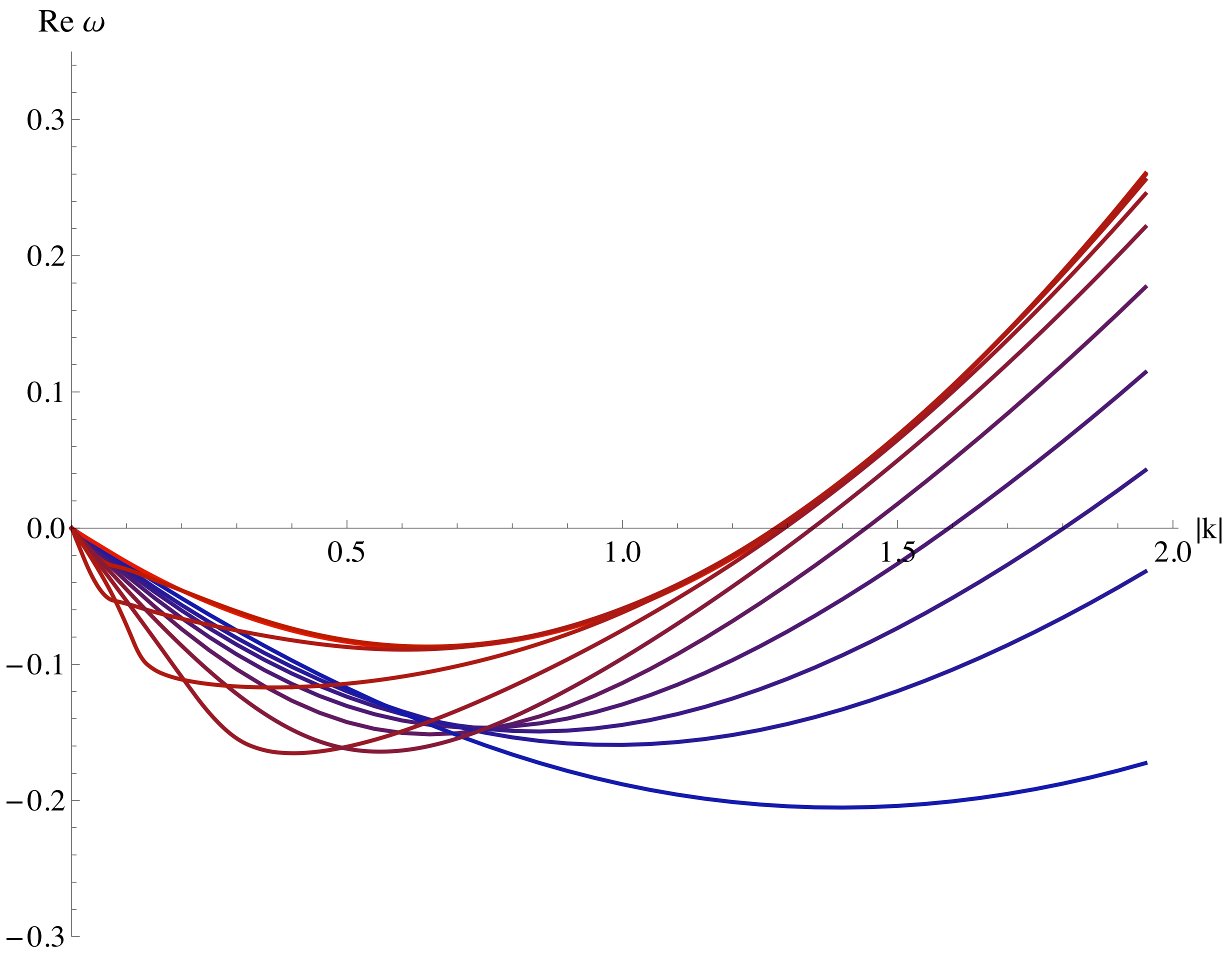
Landau criterion

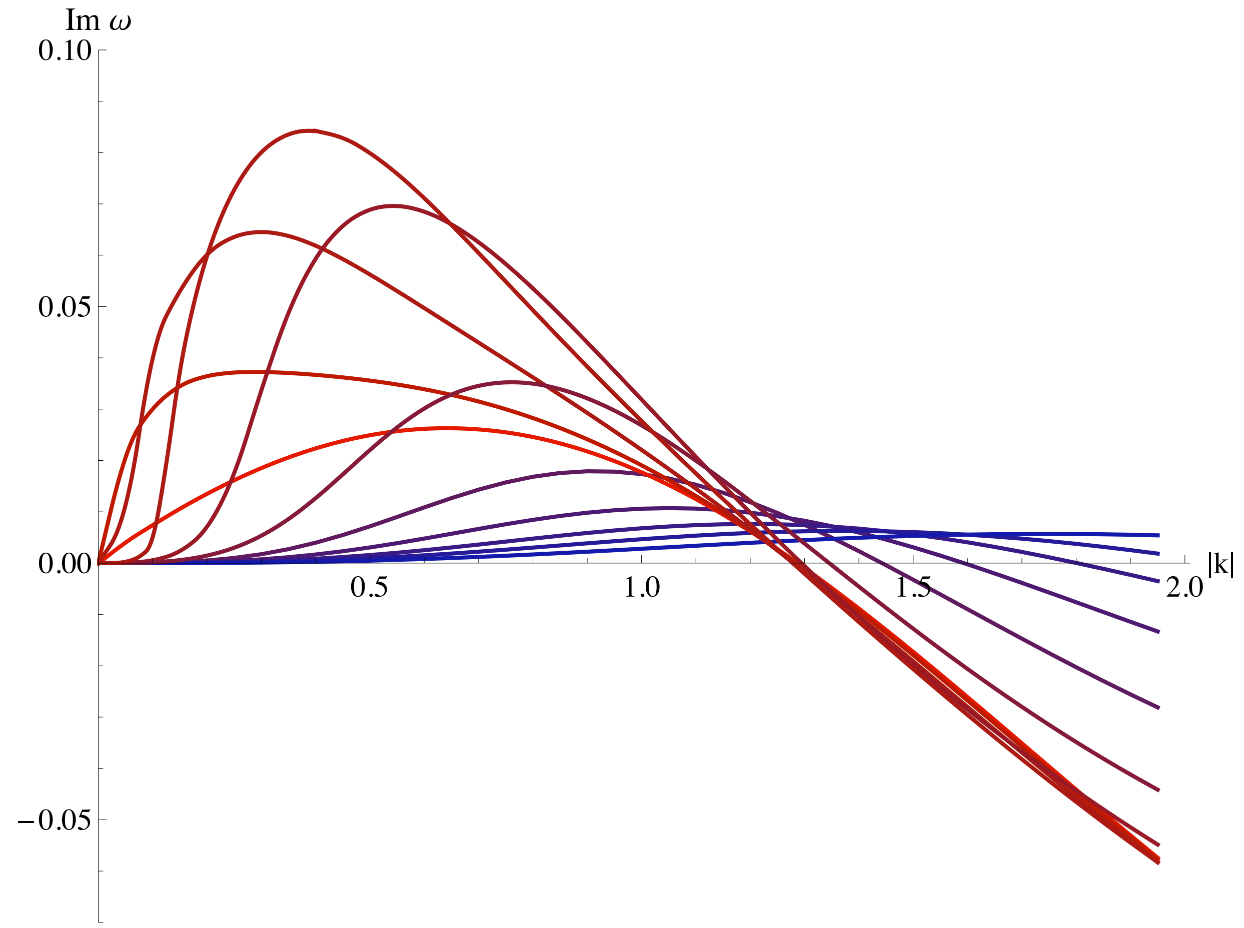
Type I Goldstone



Landau criterion

Type II Goldstone

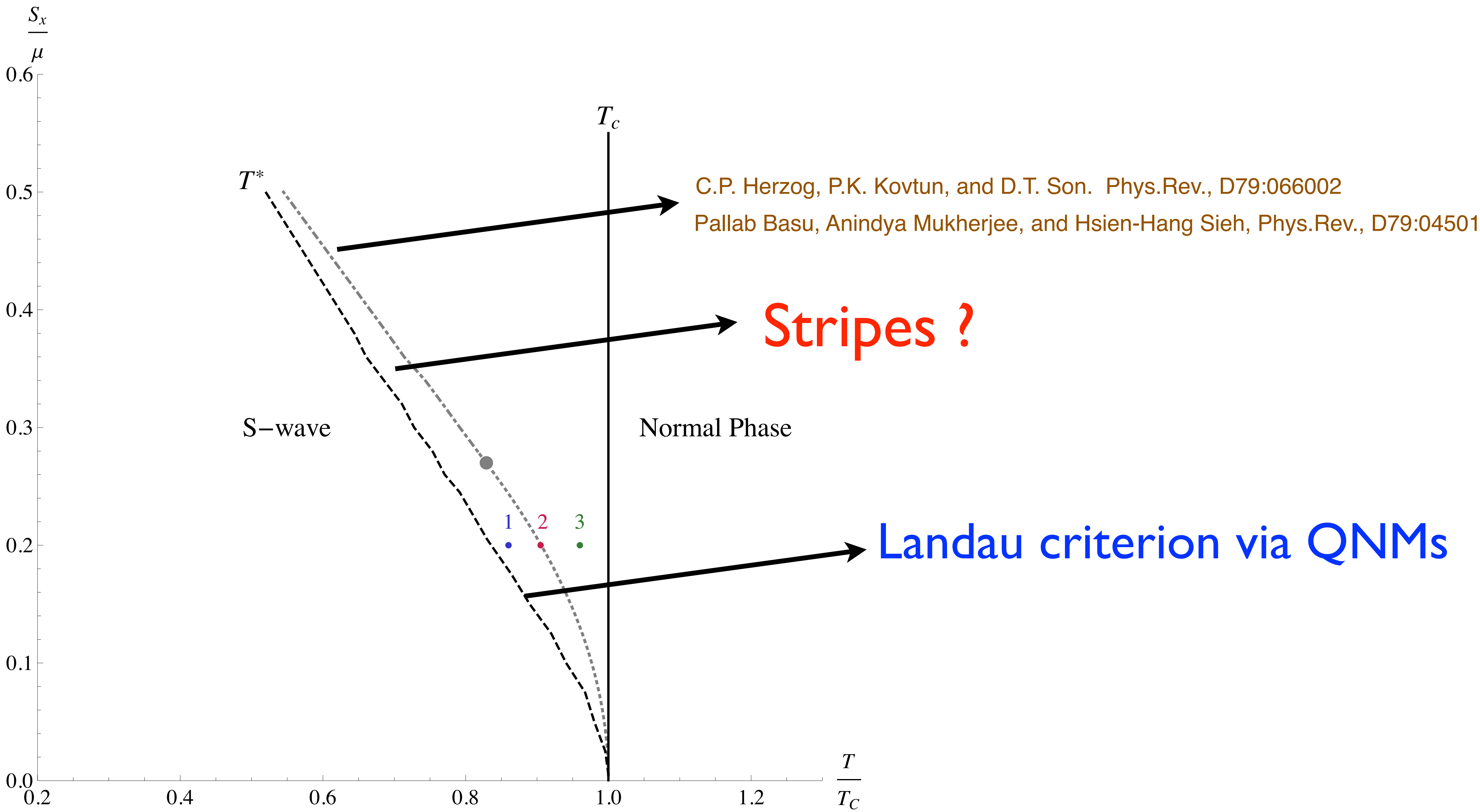




Does not support finite superflow!
Superconductor but not Superfluid

Landau criterion

Phase diagram based on Landau criterion



Ungauged Model

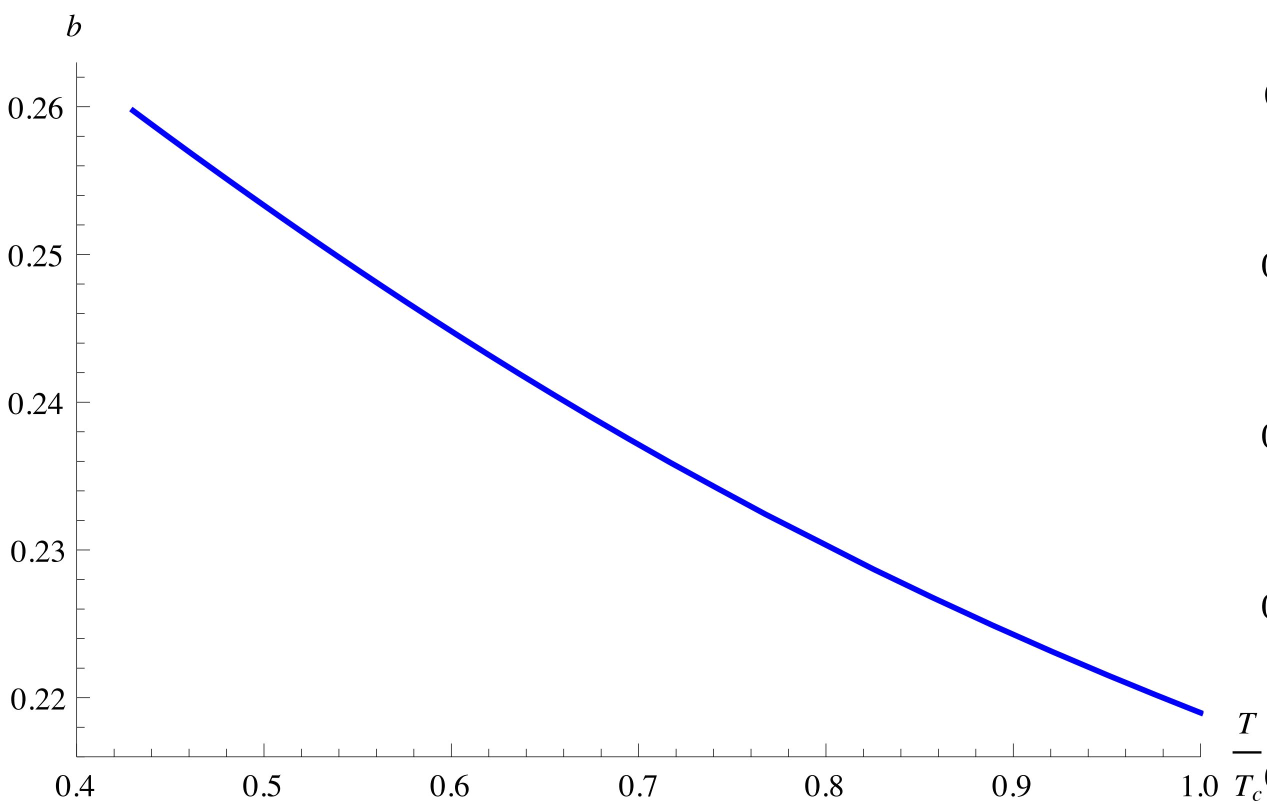
What if SU(2) only global in AdS bulk?

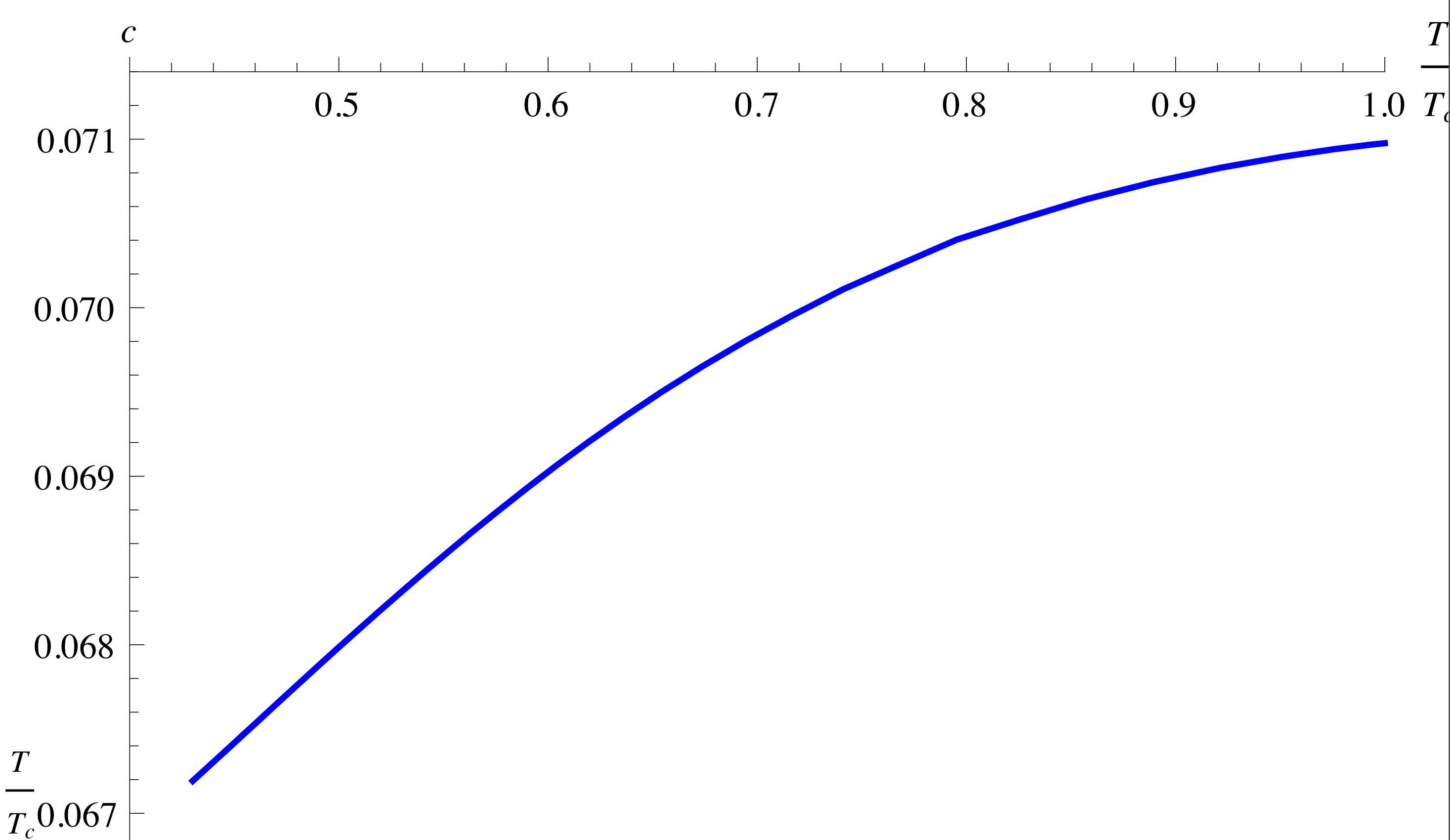
- U(1) gauge field + scalar doublet
- Global symmetry sufficient to chose vacuum
- SU(2) = “Outer Automorphism”
- Decomposition: HHH-superfluid + scalar
- Still type II Goldstone?
- Massive ?

Ungauged Model

Massless mode in 2nd scalar!

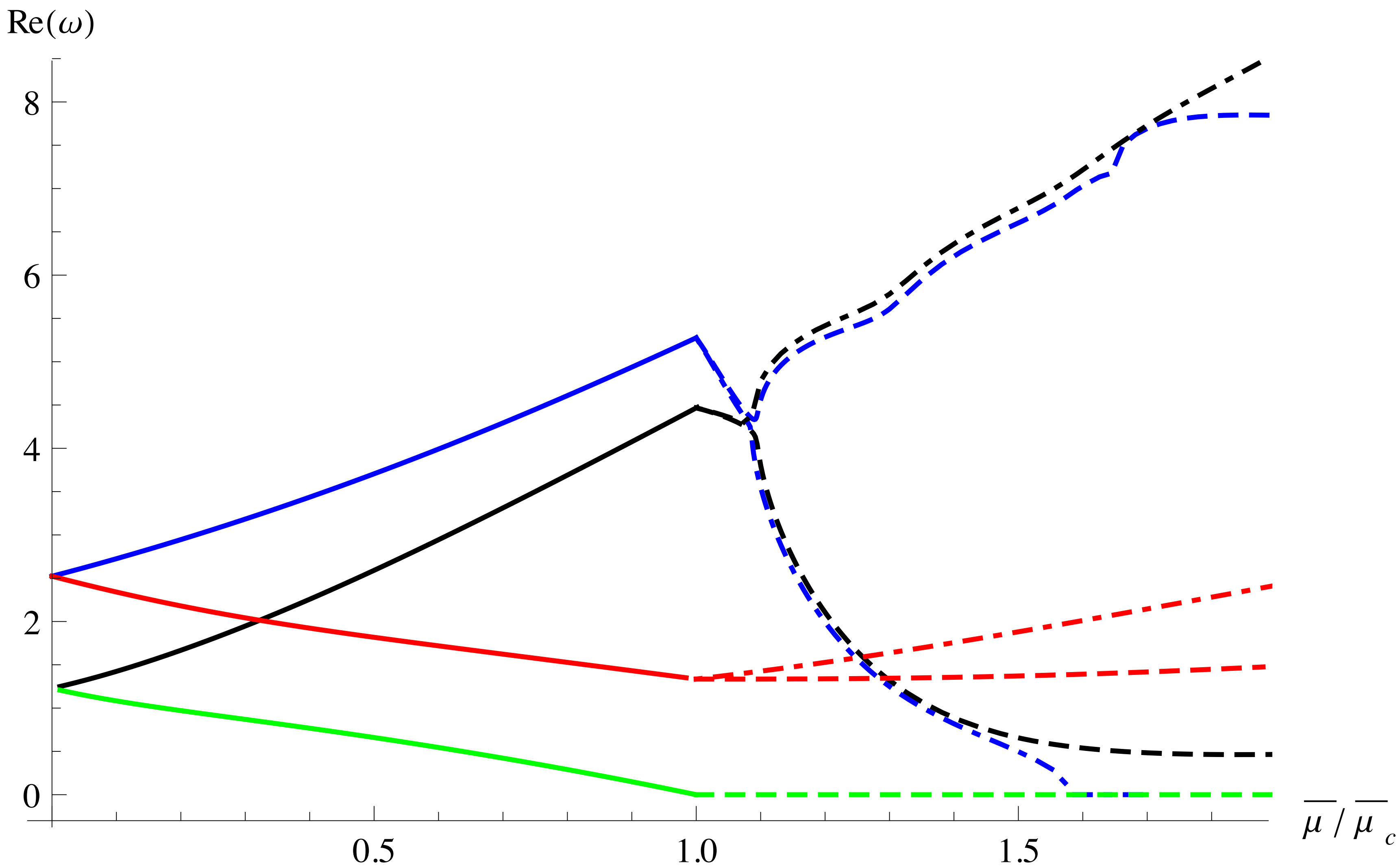
Dispersion relation: $\omega = (b + ic)k^2$





Ungauged Model

Massive mode:



Does not obey theory!! $\omega \neq q\mu$

Effective U(2) symmetric action only for lower Energies!

Summary

- Type II Goldstone QNMs !
- Compare to weak coupling
- Universality of Pseudo-diffusion ?
- "Un-gauged" model:
  no SU(2) gauge fields, violates some Theorems
- Backreacted models
  1st, 2nd, 4th sound modes etc.
- Superflow: striped phases ?

Summary

- Type II Goldstone QNMs !
- Compare to weak coupling
- Universality of Pseudo-diffusion ?
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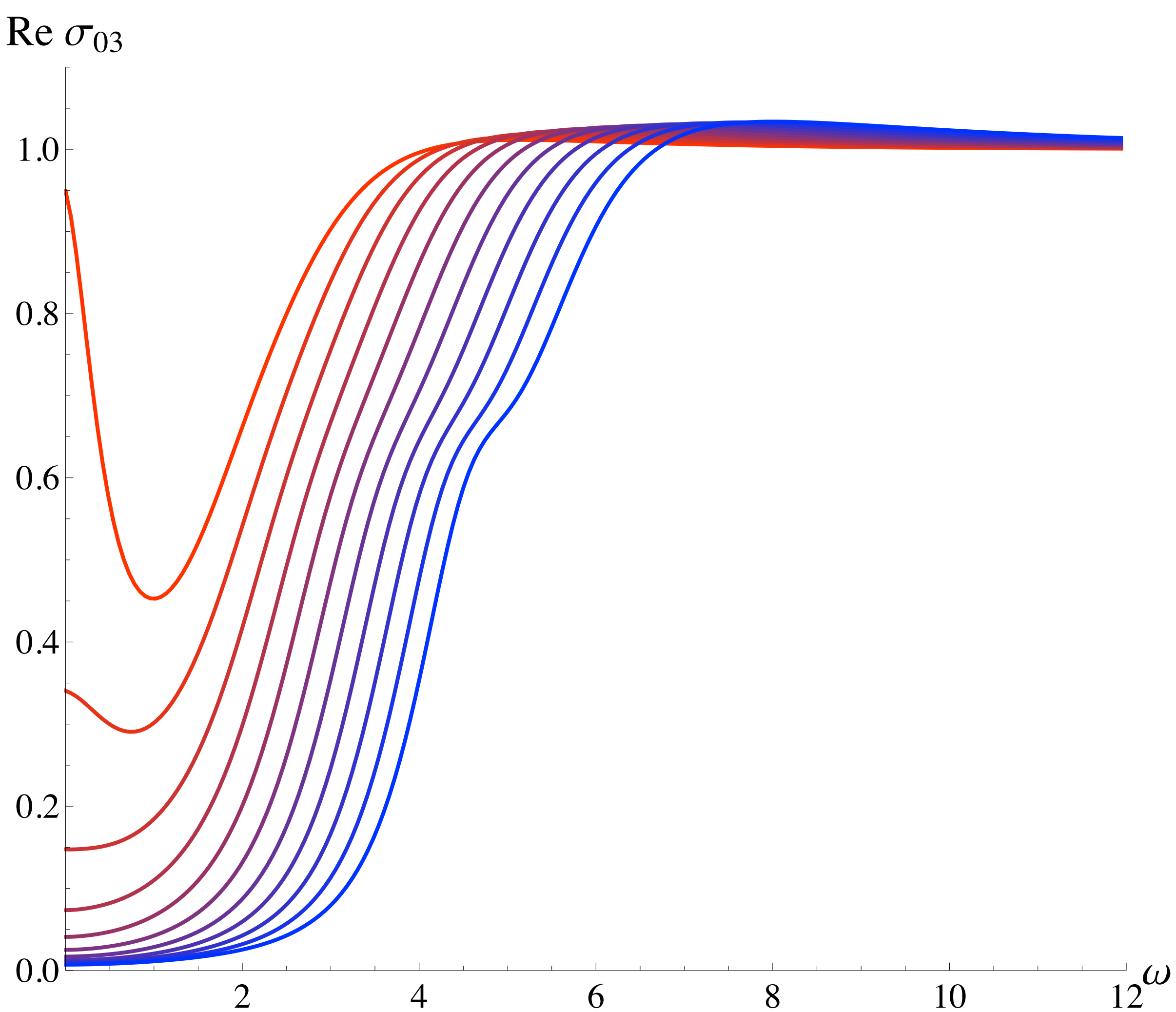
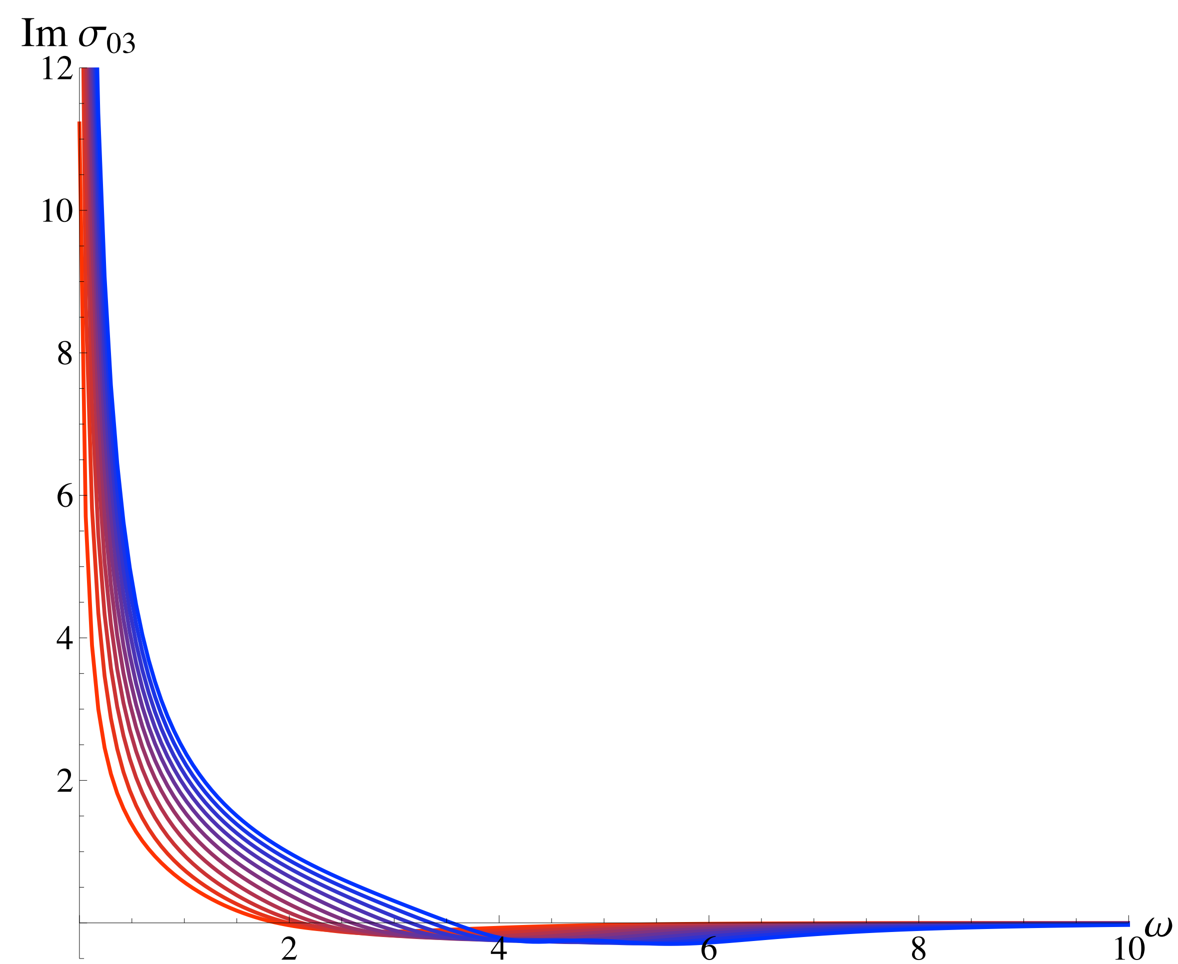
THANK YOU!

Drude Peak

Due to gapped "Pseudo" diffusion mode !





Holography

Conductivities related to type II, off-diagonal

no Superconductor !