Toward the real time dynamics of periodically driven holographic superconductor

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1. Introduction and motivation
2. Holographic model of superconductors
3. Numerics for differential equations
4. Numerical results for the real time dynamics
5. Low lying QNMs in the time averaged approximation
6. Conclusion and outlook

1 Introduction and motivation

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- Q: Why AdS/CFT?
- A: It is a machine, mapping a hard problem to a easy one.

Figure: AdS/CFT as a simplifying machine

- Q: Why AdS/CFT in the dynamical setting?
- A: Non-equilibrium phenomenon is ubiquitous around us. In particular, various non-equilibrium behaviors can now be managed in a controllable way.

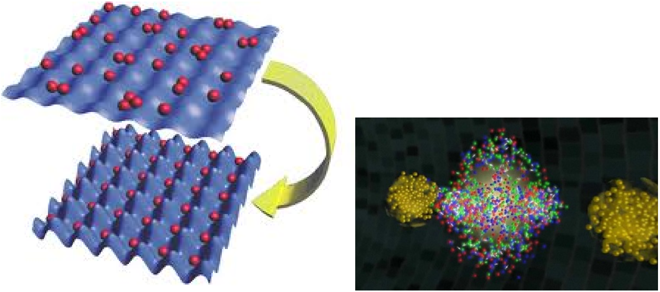
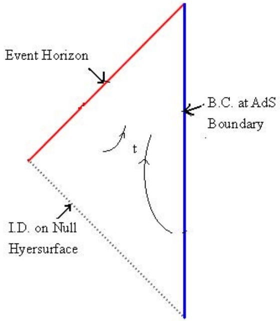


Figure: Two prestigious examples: Cold atoms trapped in optical lattices and quark gluon plasma produced in LHC

- Q: How to implement the boundary non-equilibrium physics by the bulk dynamics?
- A: It is better to work in the infalling Eddington coordinates.



- Computing time is obviously saved by the causality manifest.
- Numerical code is simplified by the 1st differential equations.

- Degree of difficulty for the numerical calculation
  - Within the probe limit.
  - To the regime of numerical relativity.
- Possibilities for the holographic setup
  - Non-equilibrium state as I.D. with source free B.C..
  - Equilibrium state as I.D. with B.C. modeling various protocols such as quantum quench and periodic driving.

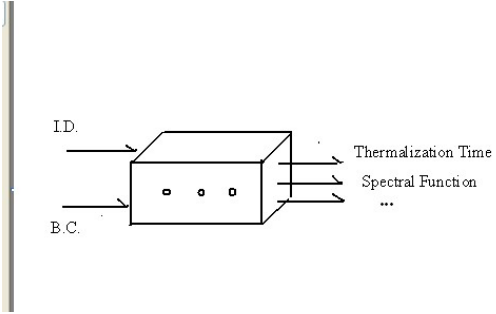


Figure: The holographic machine is something like a black box.

Why our work?

- Compared to [Bao, Dong, Silverstein, and Torroba, arXiv:1104.4098].
  - Driven by the monochromatically varying chemical potential vs. driven by a monochromatically varying electric field.
  - Homogeneous and isotropic vs. homogeneous and anisotropic.
  - Only in the large frequency limit vs. at the various frequencies.
  - Only the final would-be steady state vs. the real time dynamics towards the final state as well as the linear perturbation of the final steady state.
- Compared to [Bhaseen, Gauntlett, Simons, Sonner, and Wiseman, arXiv:1207.4194].
  - Quantum quench vs. periodic driving.
  - Inclusion of back reaction vs. in the probe limit.
  - Perturbed by the source of the scalar field vs. irradiated by the alternating electric field.
  - Homogeneous and isotropic vs. homogeneous and anisotropic.
  - One dimensional dynamical phase diagram vs. two dimensional dynamical phase diagram.
  - No time averaged vs. time averaged.

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- Action of model [Hartnoll, Herzog, and Horowitz,arXiv:0803.3295,0810.6513]

$$S=\int_{\mathcal{M}} d^4x\sqrt{-g}[R+\frac{6}{L^2}+\frac{1}{q^2}(-\frac{1}{4}F_{ab}F^{ab}-|D\Psi|^2-m^2|\Psi|^2)]. \tag{1}$$

- Background metric

$$ds^2=\frac{L^2}{z^2}[-f(z)dt^2-2dtdz+dx^2+dy^2]. \tag{2}$$

- Heat bath temperature

$$T=\frac{3}{4\pi z_h}. \tag{3}$$

- Asymptotical behavior at AdS boundary

$$A_\nu=a_\nu+b_\nu z+o(z), \tag{4}$$

$$\Psi=\frac{1}{L}[\phi z+z^2\varphi+o(z^2)]. \tag{5}$$

AdS/CFT dictionary

$$\langle J^\nu \rangle = \frac{\delta S_{ren}}{\delta a_\nu} = \lim_{z\to 0} \frac{\sqrt{-g}}{q^2} F^{z\nu}, \tag{6}$$

$$\langle O \rangle = \frac{\delta S_{ren}}{\delta \phi} = \lim_{z\to 0} [\frac{z\sqrt{-g}}{Lq^2}(D^z\Psi)^* - \frac{z\sqrt{-\gamma}}{L^2 q^2}\Psi^*]$$

$$= \frac{1}{q^2}(\varphi^* - \dot{\phi}^* - i a_t \phi^*), \tag{7}$$

where

$$S_{ren} = S - \frac{1}{Lq^2}\int_{\mathcal{B}} \sqrt{-\gamma}|\Psi|^2 \tag{8}$$

is the renormalized action by holography.

Phase transition to a superconductor

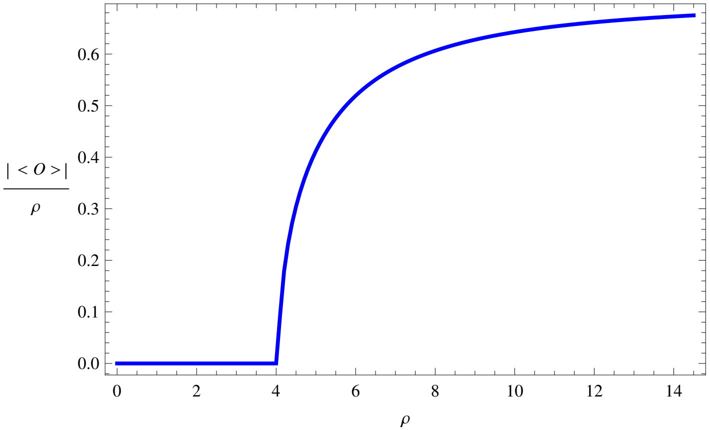

Figure: The condensate as a function of charge density with the critical charge density $\rho_c = 4.0637$.

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Pseudo-spectral method

By expanding the solution in terms of some sort of spectral functions, plugging it into eoms, and validating eoms at some grid points, the differential equations are replaced by a set of algebraic equations.

- The resultant solution thus has an analytical expression.
- The numerical error goes like $\propto e^{-N}$ with N the number of grid points.

complemented by two caveats

- The resultant algebraic equations are generically non-linear, so here comes Newton-Raphson method.
- It turns out to be extremely time consuming to apply it in the time direction, if not impossible. Instead the finite difference methods such as Runge-Kuta or Crank-Nicolson method are often adopted.

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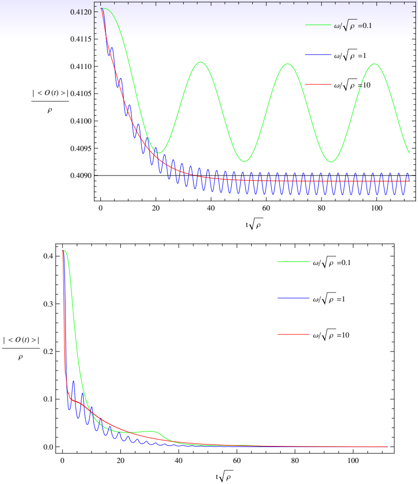


Figure: The real time dynamics of condensate for the charge density $\rho = 5$, where the upper panel is for $\frac{E}{\omega\sqrt{\rho}} = 0.1$ and the lower panel is for $\frac{E}{\omega\sqrt{\rho}} = 5$.

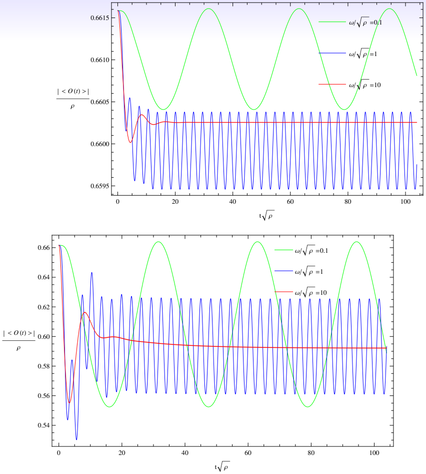
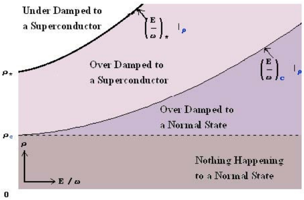


Figure: The real time dynamics of condensate for the charge density $\rho = 12$, where the upper panel is for $\frac{E}{\omega\sqrt{\rho}} = 0.1$ and the lower panel is for $\frac{E}{\omega\sqrt{\rho}} = 1$.

- The condensate is suppressed and decreased with the increase of the driving amplitude.
- The final state is an oscillating state with the oscillation frequency twice of the driving one.
- In the large frequency limit, the real time dynamics exhibits three distinct routes towards the final steady state.



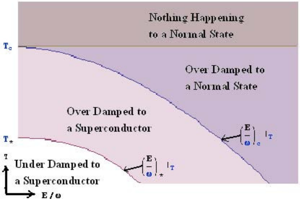


Figure: The dynamical phase cartoon diagram towards the final steady state driven periodically by an electric field in the large frequency limit, where ρ_* is around 8.850.

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Quasi-normal modes

- Definition

$$\delta\psi = e^{-i\Omega t}\delta(z) \tag{9}$$

as a solution to the linear perturbation equations of motion

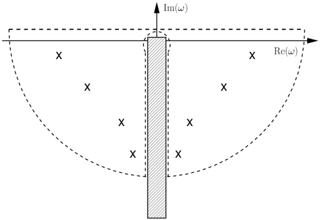
$$D\delta\psi = 0 \tag{10}$$

on top of a static background with $\delta(z)$ regular on the horizon and vanishing at the AdS boundary.

- Roles
  - One loop partition function in the bulk and $\frac{1}{N}$ correction on the boundary[Denef,Hartnoll,and Sachdev,arXiv:0908.1788,0908.2657].

$$\frac{1}{\det D} = e^{\mathrm{Pol}(\Delta)} \prod_{\Omega} \frac{|\Omega|}{4\pi^2 T} |\Gamma(\frac{i\Omega}{2\pi T})|^2 \tag{11}$$

- Poles of retarded Green function for both the bulk and boundary theory[Leaver, PRD34,384(1986)].



- High frequency arcs give rise to a prompt response.
- Branch cuts correspond to a power law tail.
- Quasi-normal modes

$$G(t;z,z') = \sum_n c_n e^{-i\Omega_n t}\delta_n(z)\delta_n(z') \Rightarrow$$
$$\delta\psi = \alpha_n e^{-i\Omega_n t}\delta_n(z). \quad (12)$$

The late time behavior is captured by the lowest lying quasi-normal mode.

Large frequency limit and time averaged approximation

Figure: The large scale structure can be acquired by averaging the small scale ripples.

$$|\langle O(t)\rangle| = |\langle O_f\rangle| + \delta e^{-i\Omega_L t} + \delta^* e^{i\Omega_L^* t} \qquad (13)$$

for the final condensate $\langle O_f\rangle \neq 0$ and

$$|\langle O(t)\rangle| = |\langle O_f\rangle + \delta e^{-i\Omega_L t}| = |\delta| e^{\mathrm{Im}(\Omega_L)t} \qquad (14)$$

for $\langle O_f\rangle = 0$, where Ω_L denotes the lowest lying quasi-normal mode on top of the late time averaged background.

Numerical results

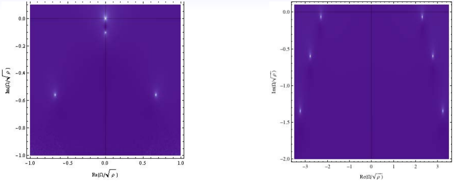


Figure: The low lying quasi-normal modes for the charge density $\rho = 5$, where the left plot is for $\frac{E}{\omega\sqrt{\rho}} = 0.1$ and the right plot is for $\frac{E}{\omega\sqrt{\rho}} = 5$.

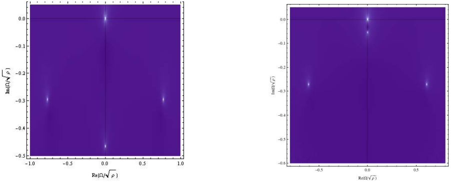


Figure: The low lying quasi-normal modes for the charge density $\rho = 12$, where the left plot is for $\frac{E}{\omega\sqrt{\rho}} = 0.1$ and the right plot is for $\frac{E}{\omega\sqrt{\rho}} = 1$.

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Conclusion

- The first step of holographic investigation of the real time dynamics of periodically driven systems at strong coupling, compared to the previous work.
- The strategy we have developed is applicable to any other periodically driven system which has a gravity dual.

Outlook

- Develop an analytical method for critical lines and the condensate behaviors near the critical lines.
- Scan the parameter space to look for the highly non-linear phenomena like chaos[Basu and Ghosh, arXiv:1304.6349].
- Go to the two point correlation function related quantities such as the conductivity[Balasubramanian, Bernamonti, Craps, Keranen, Keski-Vakkruri, Muller, Thorlacius, Vanhoof, arXiv:1212.6066].
- Go to the regime of numerical relativity by including the back reaction.
- Play with the more relevant scenarios for the non-equilibrium physics of holographic superconductors by the numerics we have developed. One example is to search for the scenario in which the branch cut shows up, leading to the power law tail decay[Barankov and Levitov, PRL96, 230403(2006)].

Thanks for your attention!

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Gauge dependent formalism of bulk dynamics

$$2\partial_t\partial_z\psi + \frac{2}{z^2}f\psi - \frac{f'}{z}\psi - f'\partial_z\psi - f\partial_z^2\psi - i\partial_z A_t\psi$$
$$-2iA_t\partial_z\psi + A_x^2\psi - \frac{2}{z^2}\psi = 0, \quad (15)$$
$$\partial_z^2 A_t = i(\psi^*\partial_z\psi - \psi\partial_z\psi^*), \quad (16)$$
$$\partial_t\partial_z A_t = -i(\psi^*\partial_t\psi - \psi\partial_t\psi^*) - 2A_t\psi^*\psi$$
$$+if(\psi^*\partial_z\psi - \psi\partial_z\psi^*), \quad (17)$$
$$f\partial_z^2 A_x + f'\partial_z A_x - 2\partial_t\partial_z A_x = 2A_x\psi^*\psi. \quad (18)$$

Gauge invariant formalism of bulk dynamics

$$2\partial_t\partial_z\chi + \frac{2}{z^2}f\chi - \frac{f'}{z}\chi - f'\partial_z\chi - f\partial_z^2\chi + M_x^2\chi$$
$$-2M_tM_z\chi + fM_z^2\chi - \frac{2}{z^2}\chi = 0, \tag{19}$$
$$\partial_z^2M_t - \partial_t\partial_zM_z = 2M_z\chi^2, \tag{20}$$
$$\partial_t\partial_zM_t - \partial_t^2M_z = -2M_t\chi^2 + 2fM_z\chi^2, \tag{21}$$
$$f\partial_z^2M_x + f'\partial_zM_x - 2\partial_t\partial_zM_x = 2M_x\chi^2. \tag{22}$$

Local identification by the conserved current[Yu Tian, Xiaoning Wu, and HZ, arXiv:1204.2029]

$$
\begin{aligned}
&T^{ab} = (F^{ac}F^{b}{}_{c} - \frac{1}{4}g^{ab}F_{cd}F^{cd}) + [D^{a}\Psi(D^{b}\Psi)^{*} \\
&+(D^{a}\Psi)^{*}D^{b}\Psi - \frac{1}{2}g^{ab}(|D\Psi|^{2} - 2|\Psi|^{2})] \\
&\Rightarrow \nabla_{a}j^{a} = \nabla_{a}[T^{ab}(\frac{\partial}{\partial t})_{b}] = 0 \Rightarrow \\
&T\delta S_{BH} = \int_{\mathcal{H}} T^{ab}(\frac{\partial}{\partial t})_{a}(\frac{\partial}{\partial t})_{b} = \int_{\mathcal{B}} T^{ab}n_{a}(\frac{\partial}{\partial t})_{b} = \int_{\mathcal{B}} E_{i}J^{i} \\
&\Rightarrow \delta S_{BH} = \frac{\int_{\mathcal{B}} E_{i}J^{i}}{T} = \frac{\delta Q}{T} = \delta S_{HB}.
\end{aligned}
\tag{23}
$$

This suggests a natural local identification for entropy production between the bulk and boundary by the integral curves of the above conserved current j^a, which is generically distinct from the one by the ingoing null geodesics.