transport phenomena and anomalies

by Francisco Peña-Benitez

based on:
arXiv: 1207.5808 and 1210.????
in collaboration with:
E. Megias, K. Landsteiner, L. Melgar, I. Amado, A. Rebhan, A. Gynther

the motivation

- Sketching the chiral magnetic effect
- anomalies, Kubo formulas and linear response
- weak coupling
- strong coupling
- summary and comparison between regimes

chiral magnetic effect (CME)

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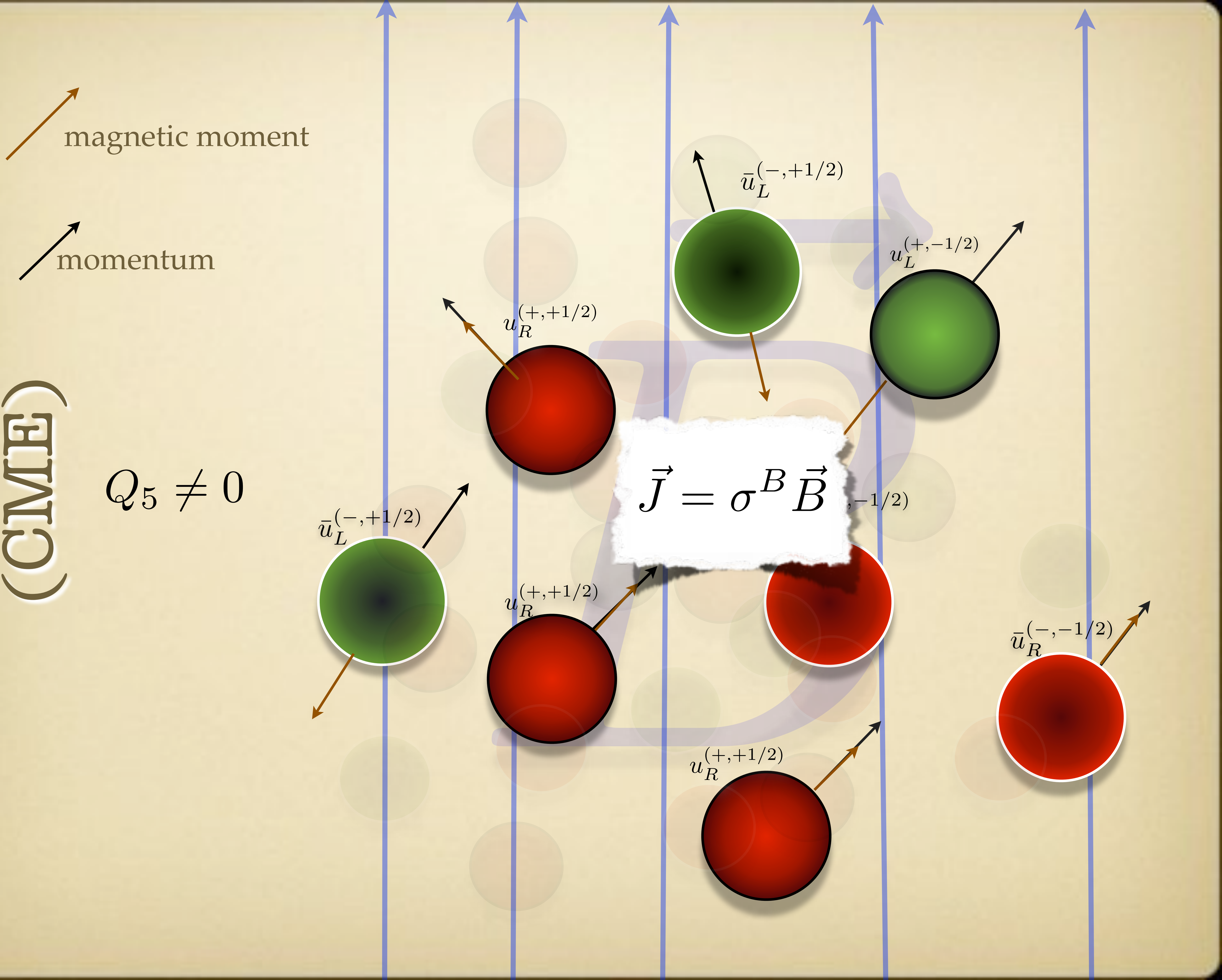
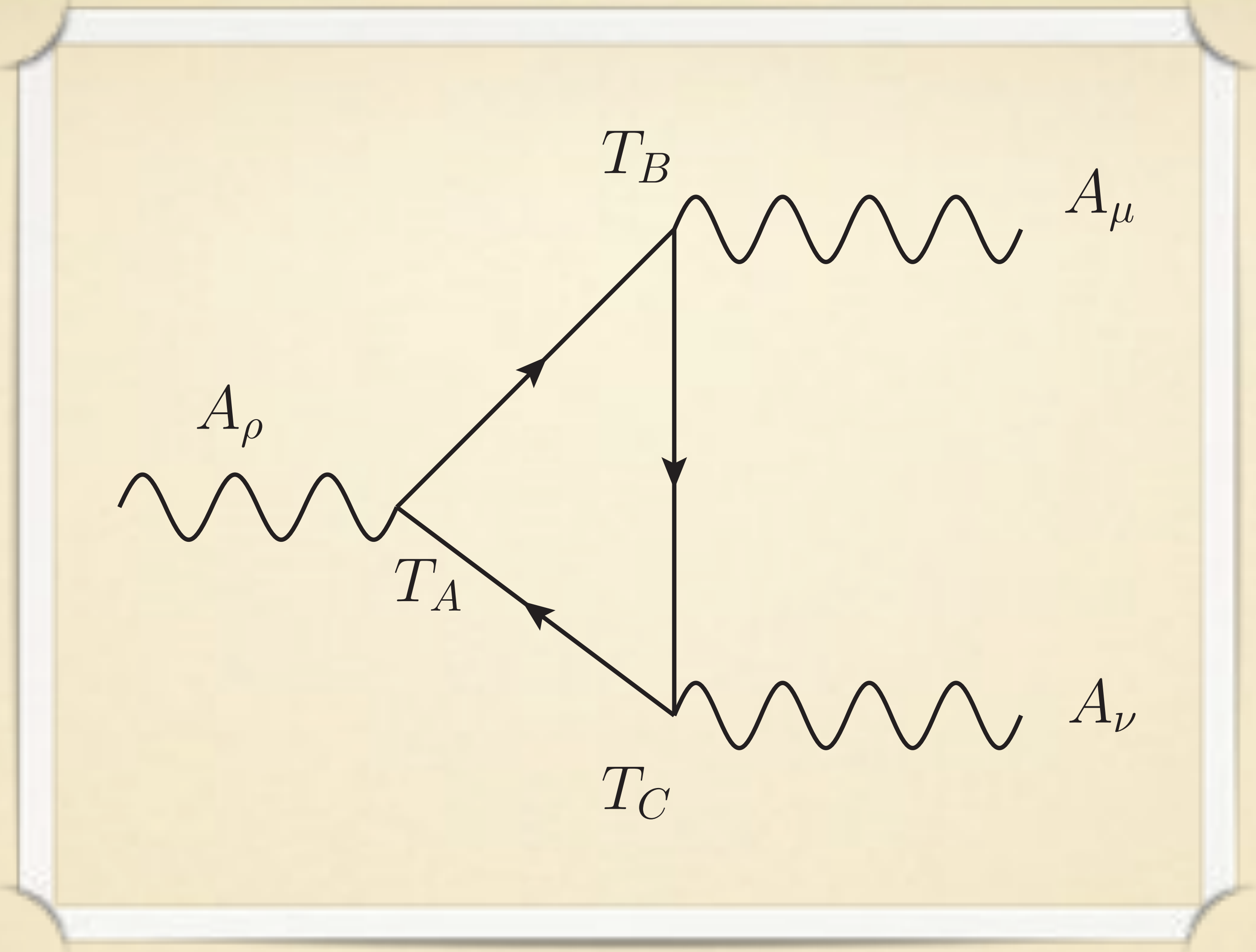


chiral anomalies

chiral anomalies



$$c_{abc} \sim \mathrm{Tr}[T_a\{T_b, T_c\}]$$

$$\nabla_\mu J^\mu_a = \frac{3c_{abc}}{4} F^b_{\mu\nu} \tilde{F}^{c\mu\nu}$$

Kubo formulas and linear response

the basic idea is as follows

imagine we have the Ohm's Law

$$\vec{J}(\omega, q) = \sigma(\omega, q)\vec{E}(\omega, q)$$

but we can understand
the current as

$$\vec{J} \equiv \langle \vec{j} \rangle$$

linear response tells us

$$\langle \vec{j}(\omega, q) \rangle = G^R_{\vec{j}\,\vec{j}}(\omega, q)\vec{A}(\omega, q)$$

Kubo formulas and linear response

$$\vec{E}(\omega, q) = -i\omega \vec{A}(\omega, q)$$

$$\sigma(\omega, q) = -\frac{i}{\omega} G^R_{\vec{j}\,\vec{j}}(\omega, q)$$

he basic idea is as follows

ave the Ohm's Law

$$\vec{J}(\omega, q) = \sigma(\omega, q)\vec{E}(\omega, q)$$

but we can understand the current as

$$\vec{J} \equiv$$

linear response tells

$$\langle \vec{j}(\omega, q)\rangle = G^R_{\vec{j}\,\vec{j}}(\omega, q)\vec{A}(\omega, q)$$

DC conductivity

$$\sigma = -\lim_{\omega \to 0} \frac{i}{\omega} G^R_{\vec{j}\,\vec{j}}(\omega, 0)$$

summarizing the Kubbo formulas

$$j^i_\varepsilon = T^{ti} \qquad \vec{j}_a = \begin{pmatrix} \vec{j}_e \\ \vec{j}_5 \\ \vec{j}_\varepsilon \end{pmatrix}$$

charge transport

chirality transport

energy transport

$$\sigma^B_a = \lim_{q_z \to 0} \frac{i}{q_z} G^R_{j^x_a j^y}(0, q_z)$$

$$\sigma^V_a = \lim_{q_z \to 0} \frac{i}{q_z} G^R_{j^x_a T^{ty}}(0, q_z)$$

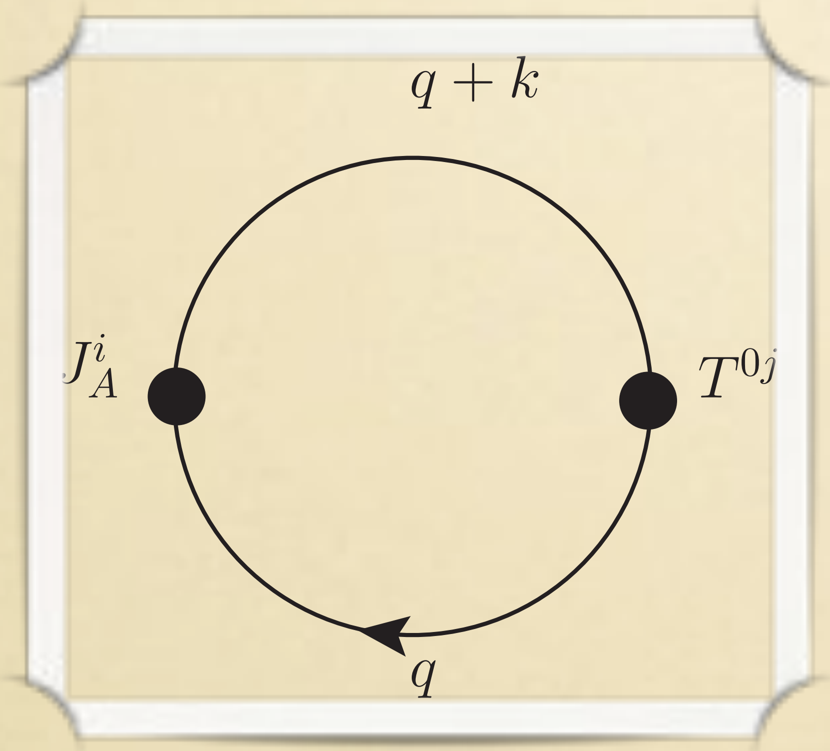
$g = 0$

interaction

free fermions

$$\vec{j}_c = \bar{\psi} T_c \vec{\gamma} \mathcal{P}_+ \psi$$

$$T^{0i} = \frac{i}{4} \bar{\psi} (\gamma^0 \partial^i + \gamma^i \partial^0) \mathcal{P}_+ \psi$$

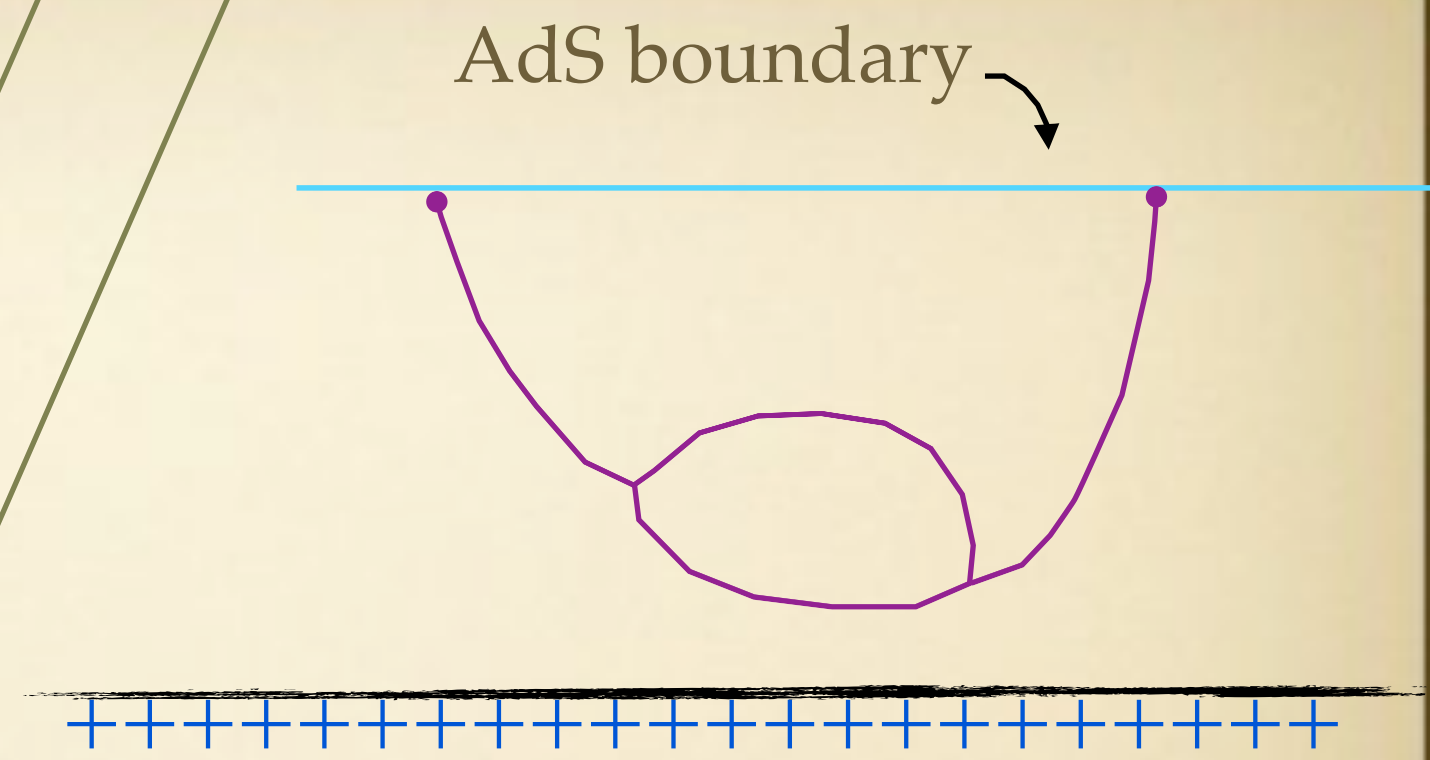


$T \neq 0$

$\mu \neq 0$

$g = \infty$

interaction



$g_{MN} \Leftrightarrow T_{\mu\nu}$

$A_M \Leftrightarrow j_\mu$

thermal state = blackhole geometry

5D

holography

$g = \infty$

free fermions case

free fermions case

[Kharzeev & Warringa, PRD.80 (2009)]

[Landsteiner, Megías & F. P-B, PRL. 107 (2011)]

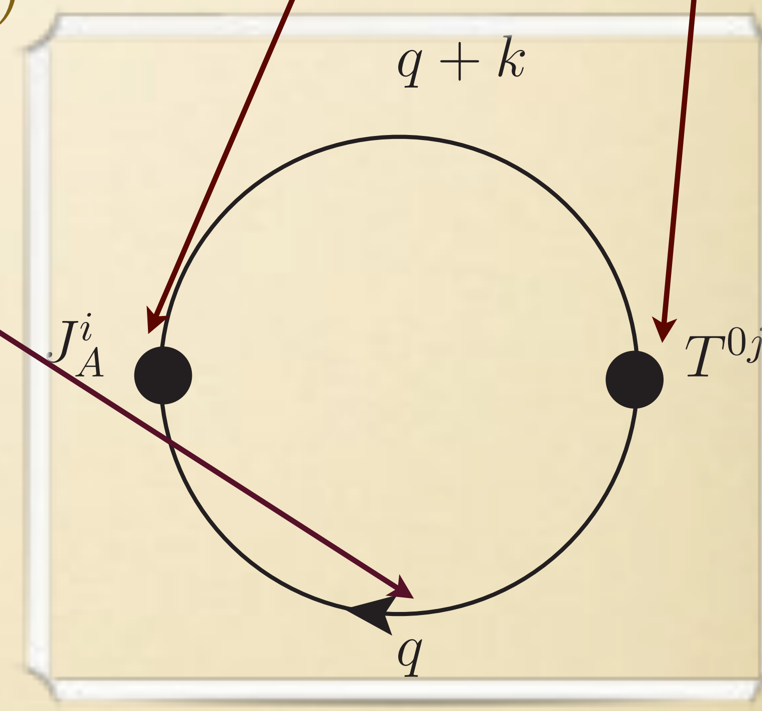
$$T^{0i} = \frac{i}{4}\bar{\psi}(\gamma^0\partial^i + \gamma^i\partial^0)\mathcal{P}_+\psi$$

$$\vec{j}_c = \bar{\psi}\, T_c \vec{\gamma}\mathcal{P}_+\psi$$

$$S(q)_g^f = \frac{\delta_g^f}{2}\sum_{t=\pm}\frac{1}{i\tilde{\omega}_n + \mu^f - t|\vec{q}|}\mathcal{P}_+(\gamma_0 + t\gamma_i\hat{q}^i)$$

$$\tilde{\omega}_n = \pi T(2n+1)$$

$$\sigma = \lim_{k\to 0}\frac{1}{k}$$



$$\sigma_{ab}^B = \frac{1}{4\pi^2}\sum_c \mathrm{Tr}(T_a\{T_b, H_c\})\mu_c$$

$$\sigma_a^V = \frac{1}{16\pi^2}\left[\sum_{b,c}\mathrm{Tr}(T_a\{H_b, H_c\})\mu_b\mu_c + \frac{2\pi^2}{3}T^2\mathrm{Tr}(T_a)\right]$$

free fermions case

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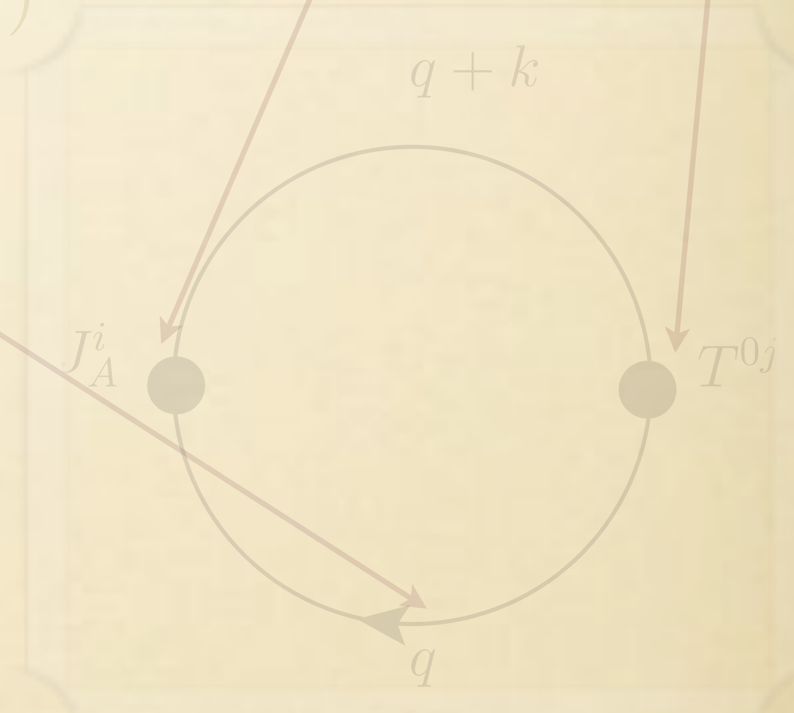

$$\sigma^B_{ab} = \frac{1}{4\pi^2}\sum_c \mathrm{Tr}(T_a\{T_b, H_c\})\mu_c$$

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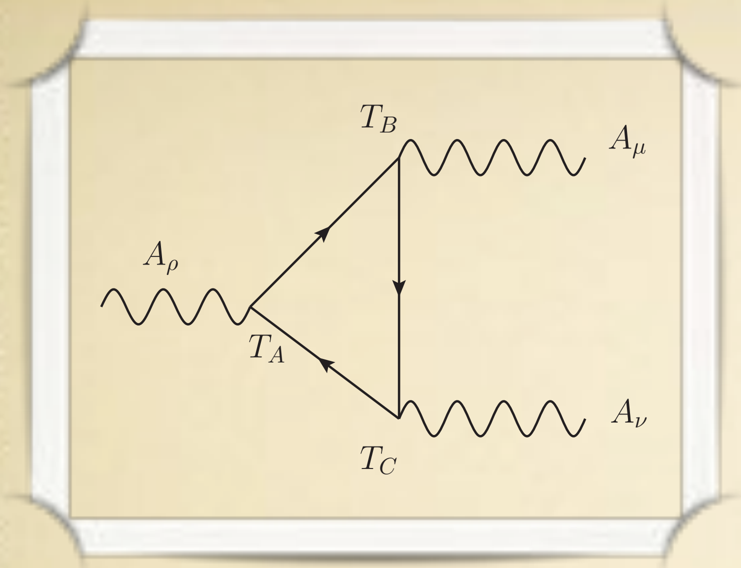
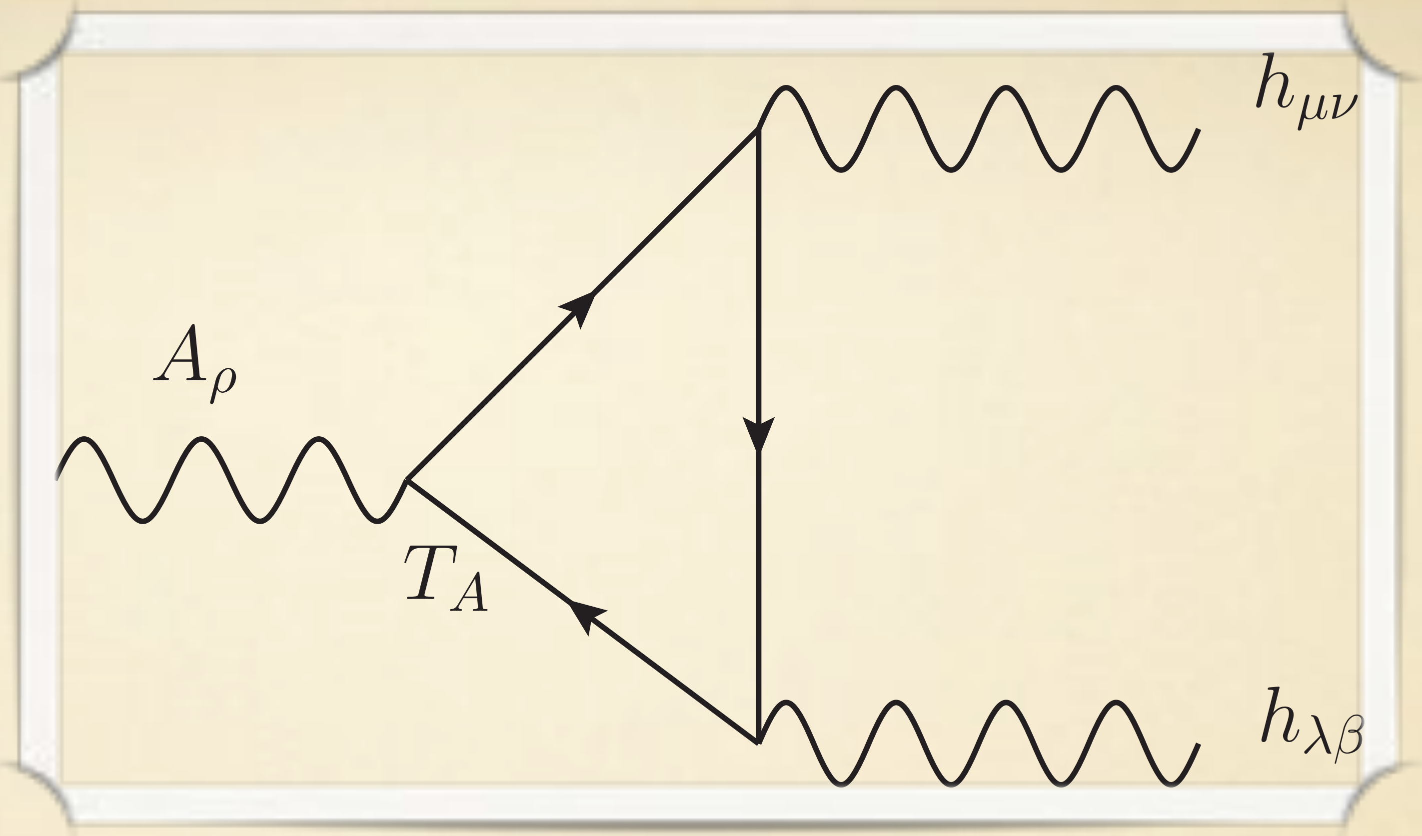
chiral anomalies



$c_{abc} \sim \mathrm{Tr}[T_a\{T_b, T_c\}]$



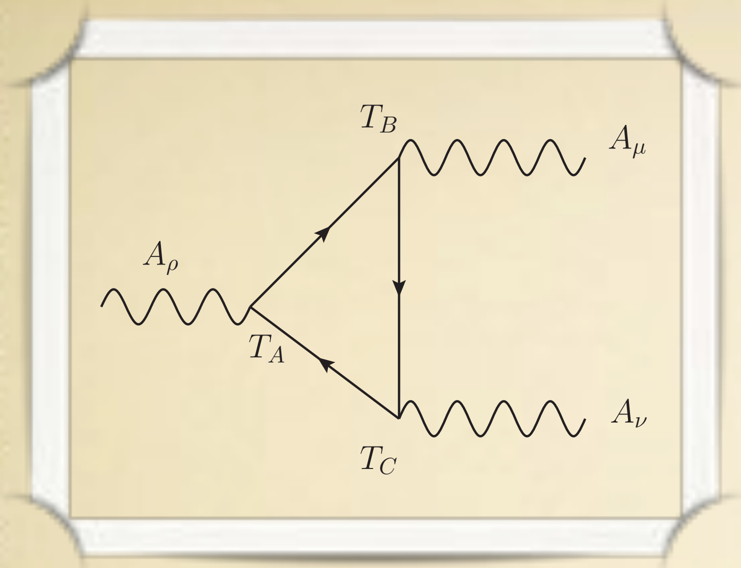
$b_a \sim \mathrm{Tr}[T_a]$

$$\nabla_\mu J^\mu_a = \frac{3c_{abc}}{4} F^b_{\mu\nu}\tilde{F}^{c\mu\nu} + \frac{b_a}{4}\epsilon^{\mu\nu\rho\lambda} R^\alpha_{\ \beta\mu\nu} R^\beta_{\ \alpha\rho\lambda}$$

$$\sigma^B_{ab} = \frac{1}{4\pi^2}\sum_c \mathrm{Tr}(T_a\{T_b, H_c\})\mu_c$$

$$\sigma^V_a = \frac{1}{16\pi^2}\left[\sum_{b,c}\mathrm{Tr}(T_a\{H_b, H_c\})\mu_b\mu_c + \frac{2\pi^2}{3}T^2\mathrm{Tr}(T_a)\right]$$

chiral anomalies



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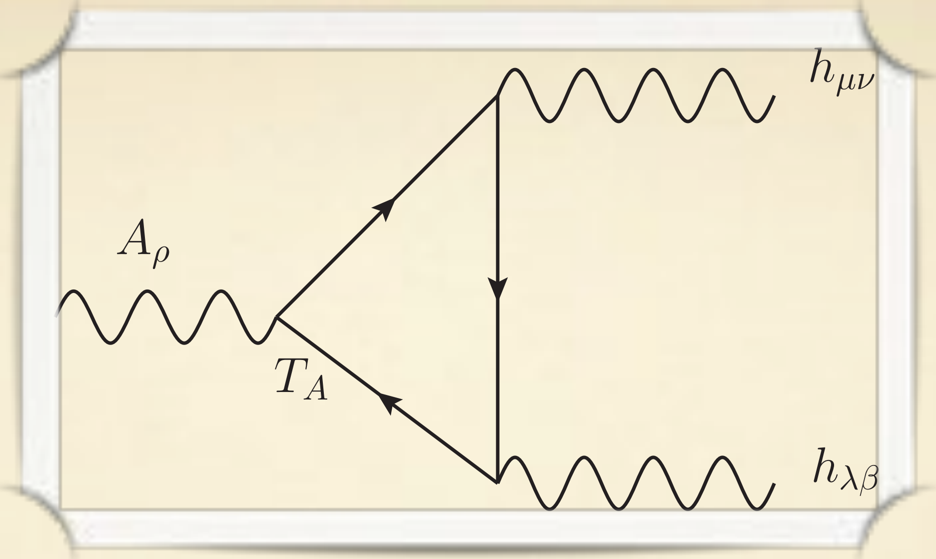


$b_a \sim \mathrm{Tr}[T_a]$

in the case of interest for QCD

$U(1)_V \mathrm{x} U(1)_A$

$$\nabla_\mu j^\mu = 0$$

$$\nabla_\mu j_5^\mu = \frac{1}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{384\pi^2}\epsilon^{\mu\nu\rho\lambda} R^{\alpha}{}_{\beta\mu\nu} R^{\beta}{}_{\alpha\rho\lambda}$$

free fermions case

Solutions for U(1)VxU(1)A

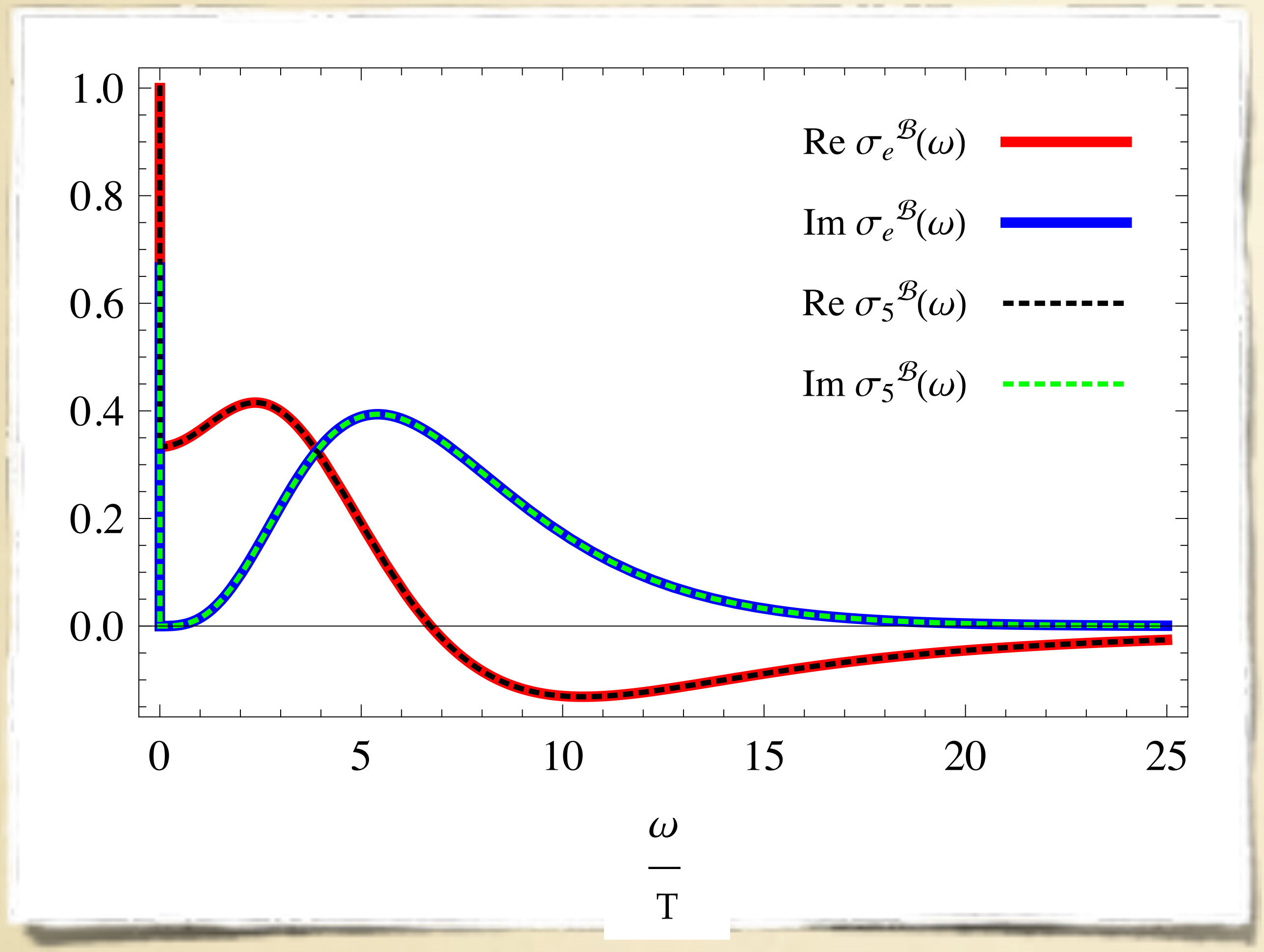
$$\sigma_B = \frac{1}{2\pi^2}\mu_5 \qquad \sigma_5 = \frac{1}{2\pi^2}\mu$$

$$\sigma^V = \frac{\mu\mu_5}{2\pi^2} \qquad \sigma_5^V = \frac{\mu^2+\mu_5^2}{4\pi^2} + \frac{T^2}{12}$$

[Vilenkin, Phys. Rev. D20, 1807 (1979)]

free fermions case

frequency dependence



magnetic conductivies

vortical conductivies

$$\Re[\sigma^{\mathcal{V}}_A(\omega,0)] = \begin{cases} \sigma^{\mathcal{V}}_{A,(0)} & \omega = 0 \\ 0 & \omega \neq 0 \end{cases}$$

$$\Im[\sigma^{\mathcal{V}}_A(\omega,0)] = \pi\sigma^{\mathcal{V}}_{A,(0)}\omega\delta(\omega)$$

strongly coupled theory

holographic model

- we need to build a model with a U(1)vxU(1)A global symmetry
- the vector current must be conserved
- the axial current must be anomalous with the mixed gauge-gravitational anomaly included

$$S = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + \frac{12}{L^2} - \frac{1}{4}\left(F_{MN}F^{MN} + F^{(5)}_{MN}F^{(5)MN}\right)\right]$$

$$S_{ano} = \int d^5x\sqrt{-g}\left[\epsilon^{MNPQR}A^{(5)}_M\left(\frac{\kappa}{3}F^{(5)}_{NP}F^{(5)}_{QR} + \kappa F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)\right]$$

$$\delta_{\xi_5}(S + S_{ano} + S_{bound}) \propto \int_\partial d^4x\sqrt{-h}\xi_5\epsilon^{\mu\nu\rho\beta}\left(\frac{\kappa}{3}F^{(5)}_{\mu\nu}F^{(5)}_{\rho\beta} + \kappa F_{\mu\nu}F_{\rho\beta} + \lambda R^\alpha{}_{\delta\mu\nu}R^\delta{}_{\alpha\rho\beta}\right)$$

$$\kappa = -\frac{1}{16\pi^2} \qquad \lambda = -\frac{1}{384\pi^2}$$

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f(r)}dr^2$$

$$A = \left(\beta - \frac{\mu\, r_{\mathrm{H}}^2}{r^2}\right)dt$$

$$T = \frac{2r_H^2 M - 3(Q^2 + Q_5^2)}{2\pi r_H^5}$$

$$A^{(5)} = \left(\gamma - \frac{\mu_5\, r_{\mathrm{H}}^2}{r^2}\right)dt$$

$$f(r) = 1 - \frac{ML^2}{r^4} + \frac{(Q^2 + Q_5^2)L^2}{r^6} \qquad Q = \frac{r_H^2}{\sqrt{3}}\mu \qquad Q_5 = \frac{r_H^2}{\sqrt{3}}\mu_5$$

AdS Reissner-Nordström blackhole

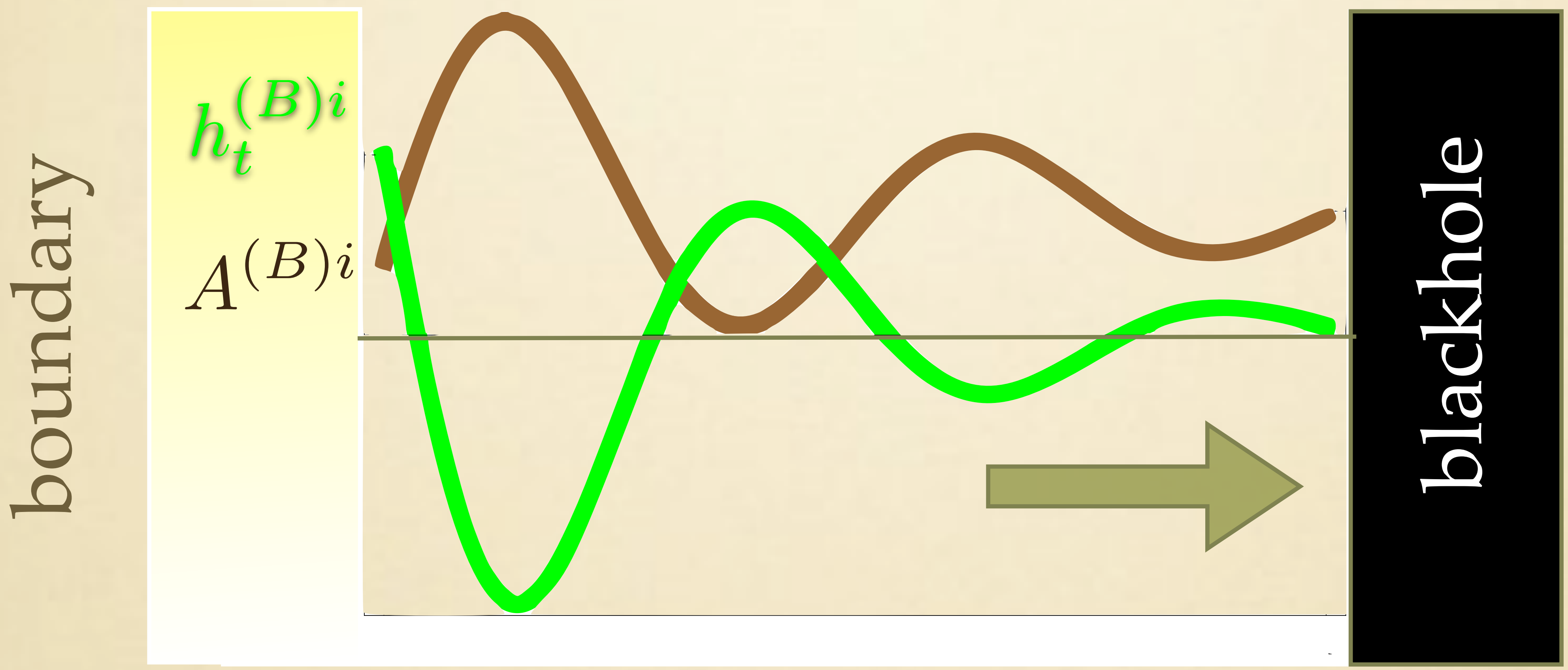
$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \epsilon h_{\mu\nu}$$

$$A^{(5)} = A^{(5)}_0 + \epsilon\, A^{(5)}_1$$

$$A = A_0 + \epsilon\, A_1$$

linear fluctuations

strongly couple theory

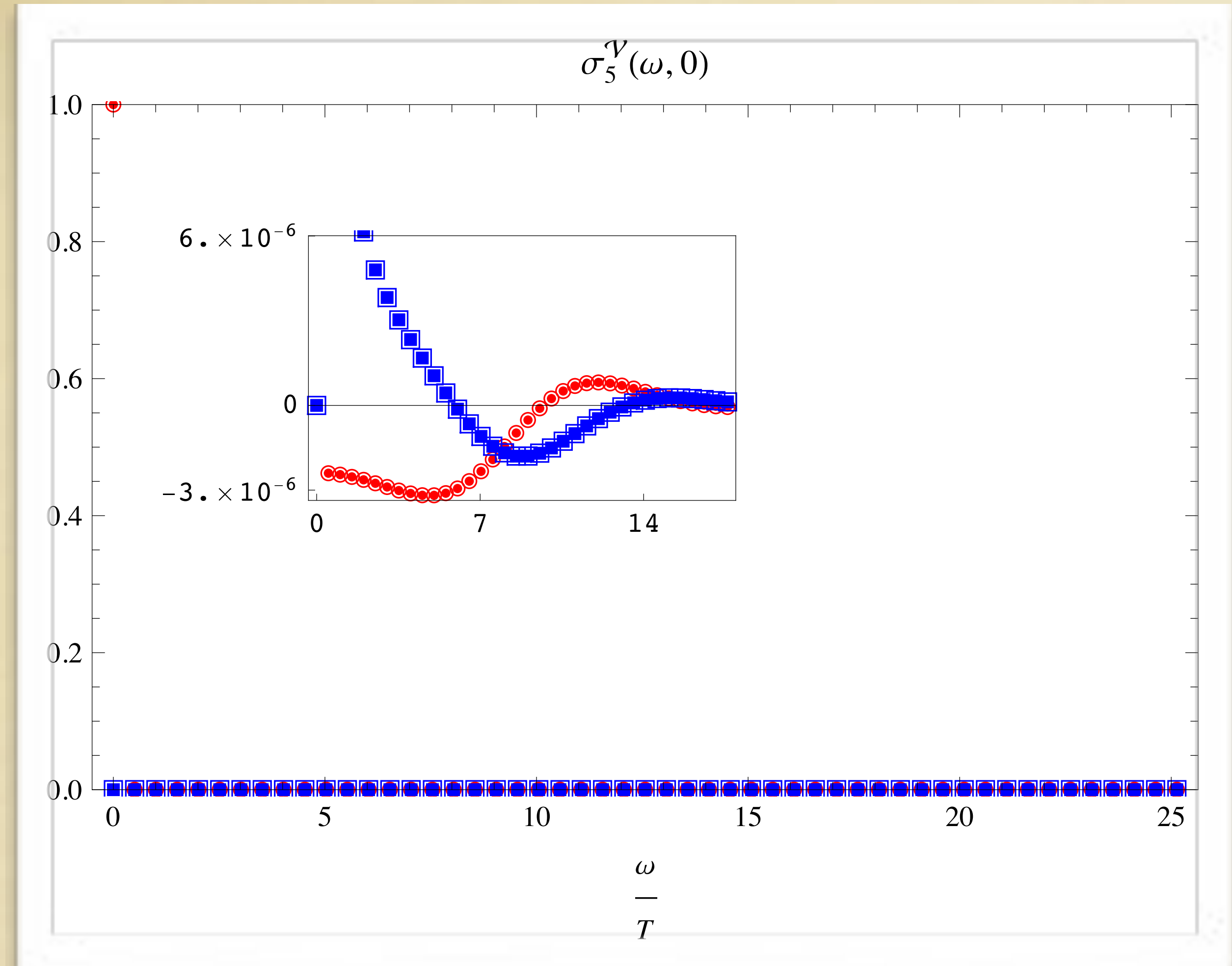


strongly couple theory

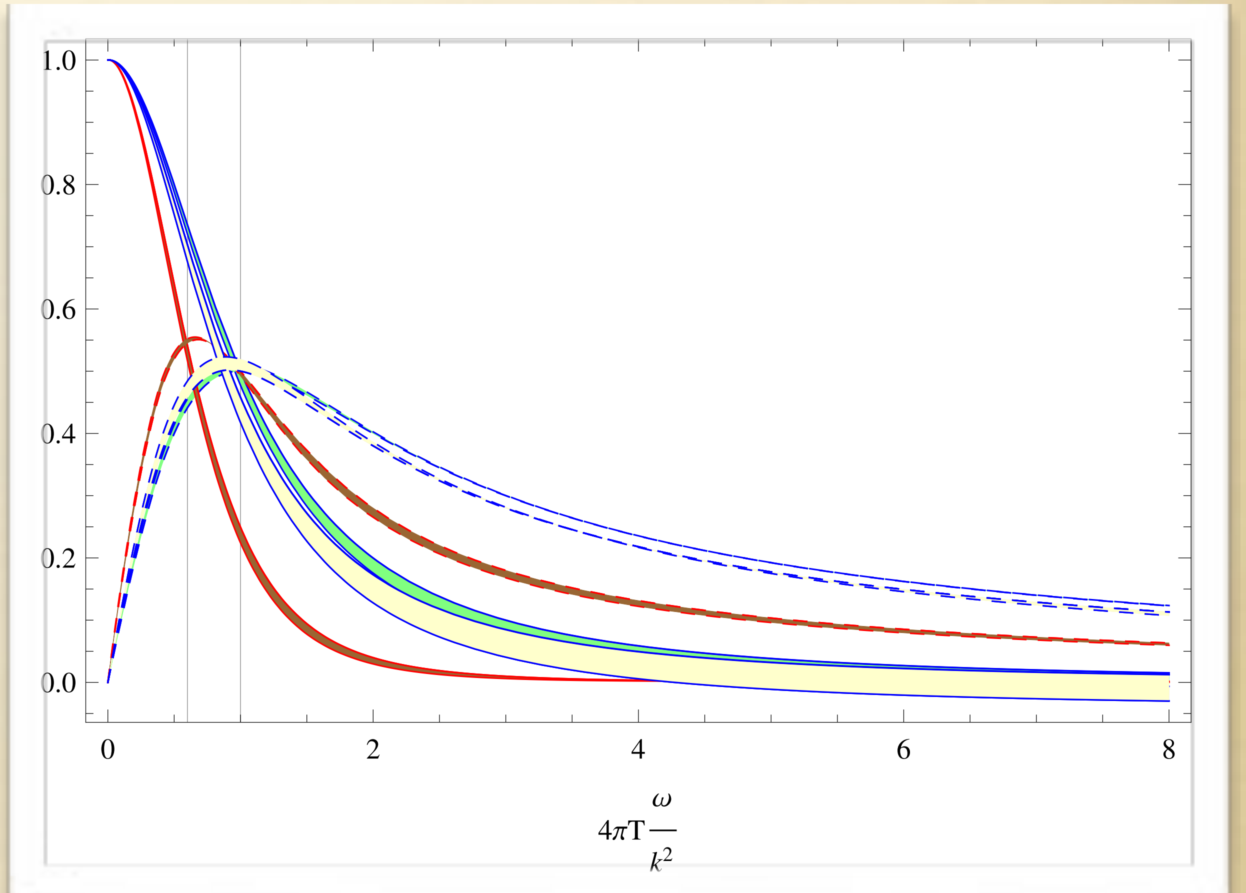
after solving the linearized system of e.o.m. and using the AdS dictionary

$$
G^R(0,k) = ik \begin{array}{cc}
\begin{array}{ccc} \quad A_z \quad & \quad A_z^{(5)} \quad & \quad h_t^z \quad \end{array} & \\
\begin{pmatrix}
\frac{\mu_5}{2\pi^2} & \frac{\mu}{2\pi^2} & \frac{\mu\mu_5}{2\pi^2} \\
\frac{\mu}{2\pi^2} & \frac{\mu_5}{2\pi^2} & \frac{\mu^2+\mu_5^2}{4\pi^2} + \frac{T^2}{12} \\
\frac{\mu\mu_5}{2\pi^2} & \frac{\mu^2+\mu_5^2}{4\pi^2} + \frac{T^2}{12} & \mu_5\left(\frac{\mu_5^2+3\mu^2}{6\pi^2} + \frac{T^2}{6}\right)
\end{pmatrix} & \begin{array}{c} A_x \\ A_x^{(5)} \\ h_t^x \end{array}
\end{array}
$$

strongly couple theory



frequency dependence of the vortical conductivity



cut-off momentum

$$\frac{k}{4\pi T} = 1/10$$

$$\frac{k}{4\pi T} = 1/100$$

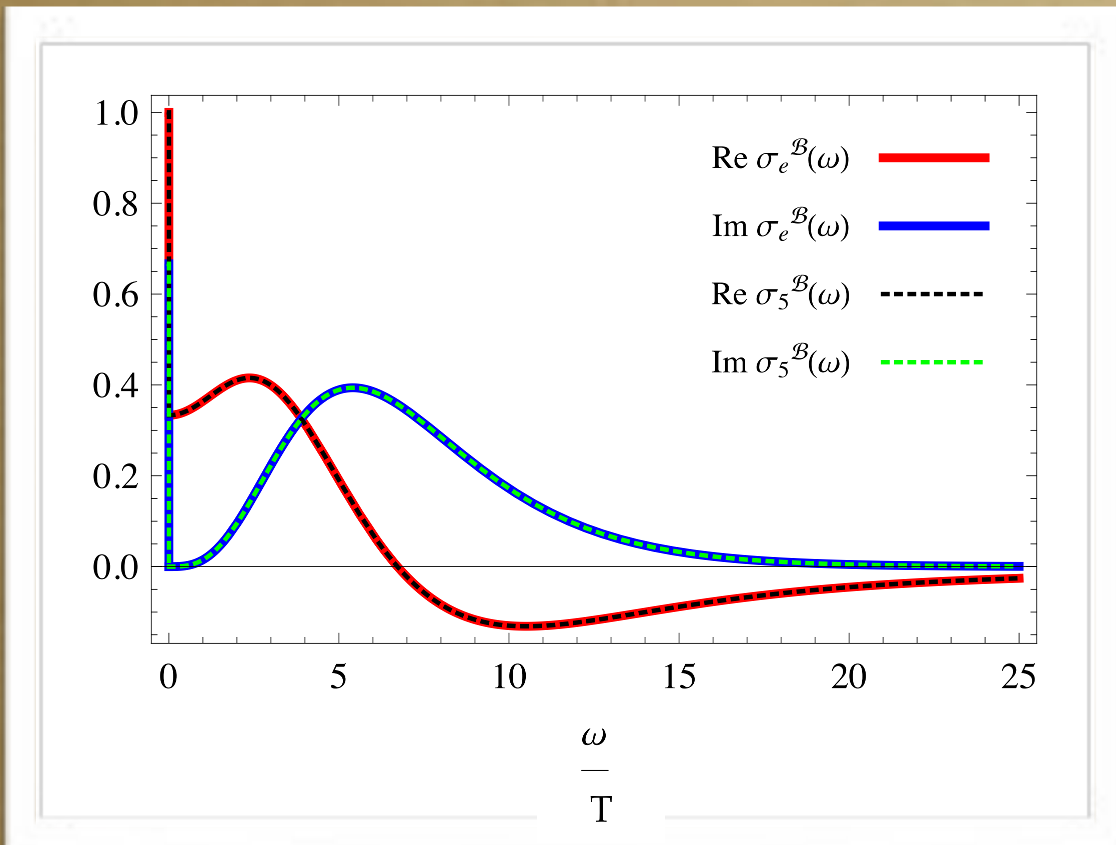
$$\frac{k}{4\pi T} = 1/1000$$

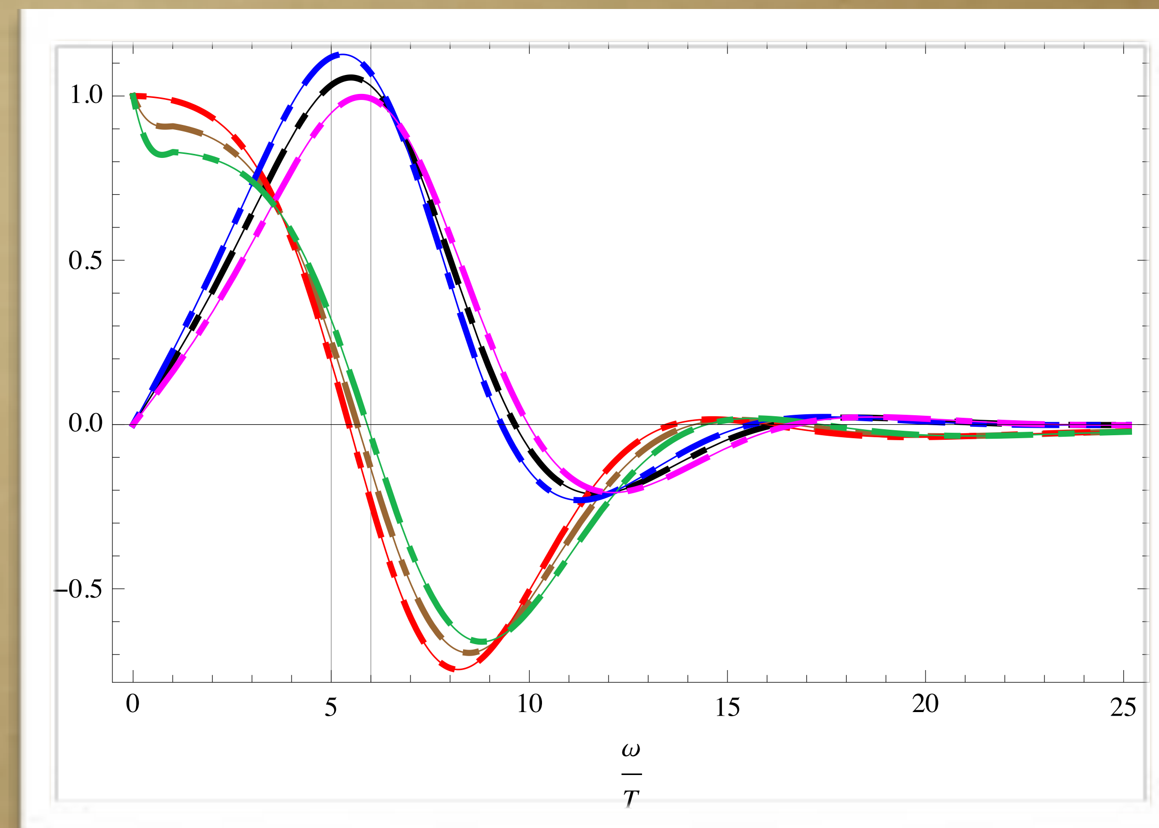
The width goes like

$$\Delta\omega \sim \frac{k^2}{4\pi T}$$

summary

some "universal" behaviour in the magnetic conductivities





weakly coupled

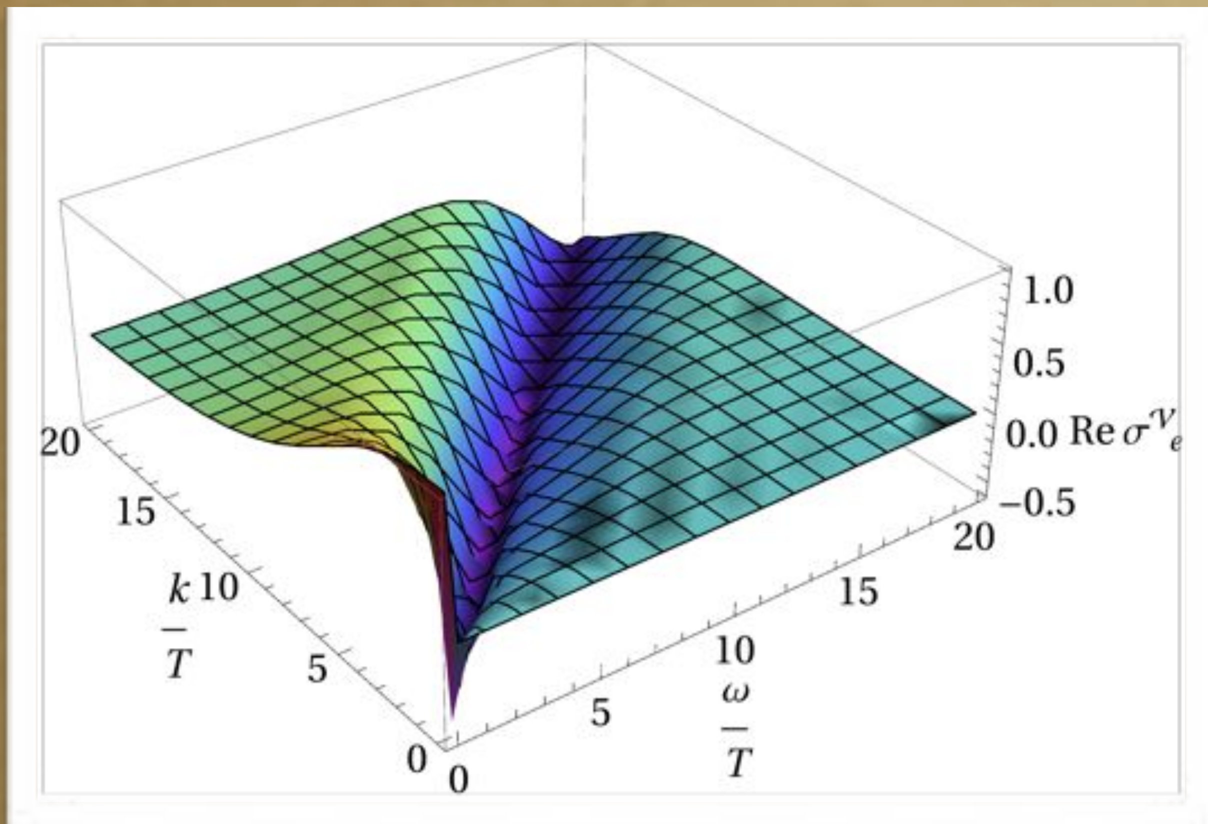
strongly coupled

imaginary part has a maximum at $\omega \sim 5T$

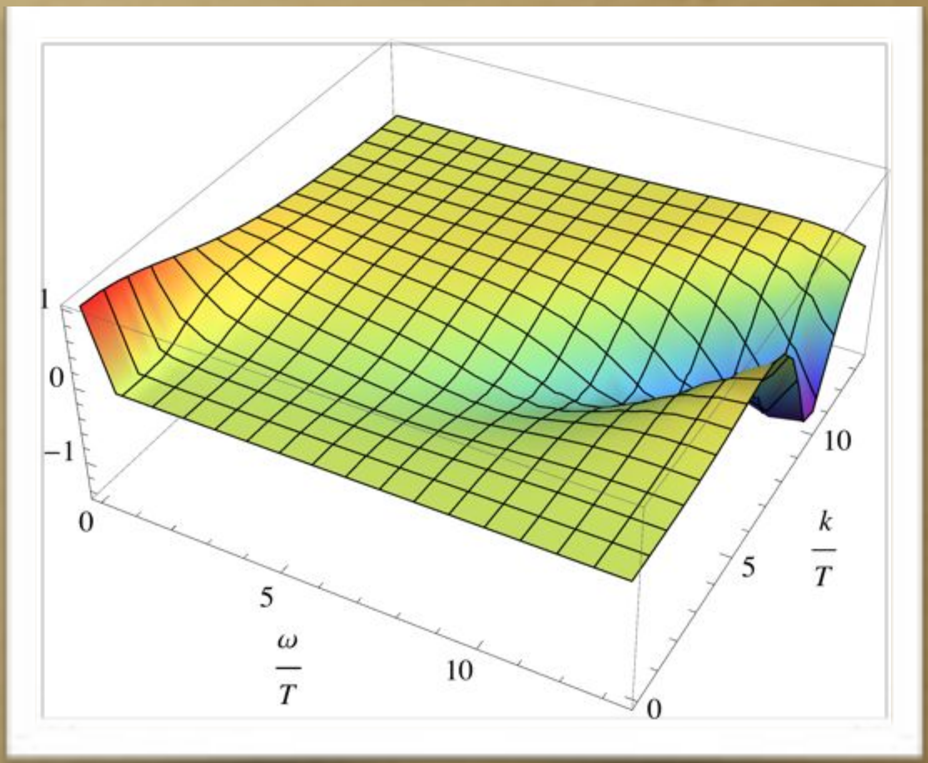
characteristic frequency $\omega \sim 15T$

summary

some "universal" behaviour in the magnetic conductivities



weakly coupled



strongly coupled

vorticiy must be strictly static in the homogenuos case

characteristic momentum $k \sim 5T$

thanks