The Trailing String in Confining Holographic Theories

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Work with E. Kiritsis, L. Mazzanti, to appear soon

Introduction: AdS/CFT

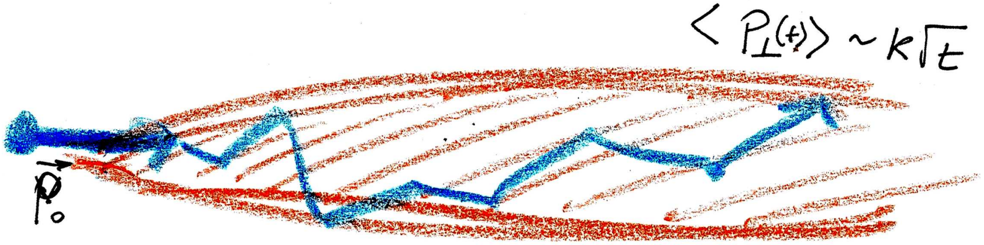
Since 15 years, the AdS/CFT correspondence or gauge/gravity duality, offered us another way to think about QFTs

- The QFT degrees of freedom and dynamics can be recast in the language of a theory of gravity in a higher dimensional, curved space-time.
- The higher dimensional (bulk) theory becomes simple e.g. classical GR in the regime where the number of degrees of the QFT becomes large, and the couplings become strong.
- The deconfined, high-temperature phase of gauge theories is mapped to a black hole solution.

Strongly coupled quantum field theories may be described using the tools of classical General Relativity

Introduction: Heavy quarks in QGP

- in a heavy-ion collision experiment, a collecive state (quark-gluon plasma) is form that undergoes fast thermalization and can be described by hydrodimamics.
- A heavy quark can be created out of equilibrium in the QGP produced by a heavy ion collision. It then undergoes a diffusion process governed by the interactions with the medium.



The brownaian-like dynamics causes viscous energy loss and a spread in transverse momentum

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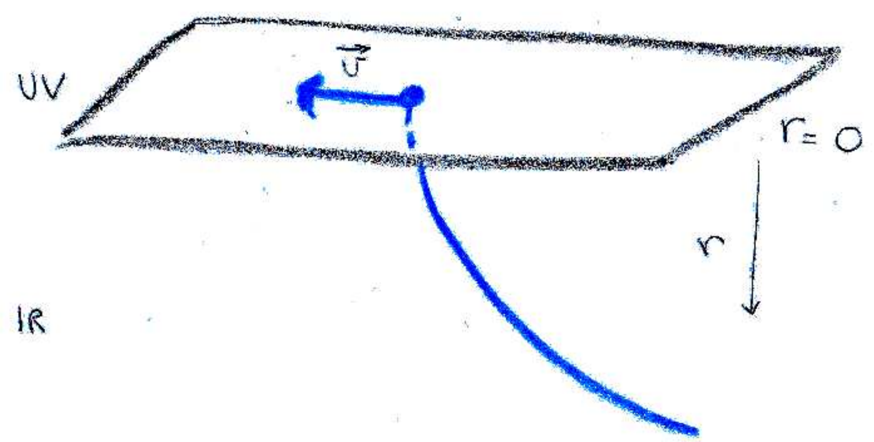

Dual dual to a trailing string with the quark as its endopoint.

Motivation

I will discuss the trailing string solution in $T = 0$ vacuum geometries, in particular those dual to a confining theory.

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- From a practical point of view, in order to correctly obtain the dynamics of the probe in the deconfined medium, one needs a subtraction procedure to make basic quantities (Boundary retarded correlators) well defined. The natural way to operate this subtraction is through the vacuum correlator. Whence the need of the vacuum trailing string solution.

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- The vacuum trailing string in confining theories, and its fluctuations, have an interesting and non-trivial structure
  - presence of a confining horizon
  - long time dissipation effects

Outline

- Confinement in AdS/CFT
- The trailing string picture of a probe quark, and connection to Brownian motion
- Review of trailing string in a black hole solution
- Static trailing string in a confining background
- Dragged confining trailing string
- Conclusion

AdS/CFT and Confinement

Confinement in AdS/CFT is decided via the dual of the Wilson loop test: In confining gauge theories, the Wilson Loop operator

$$W(\gamma) = \mathcal{P} \exp i \oint_\gamma A$$

exhibits an Area Law: $W \sim \exp \sigma_c Area$

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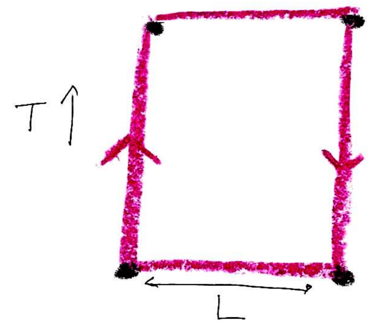


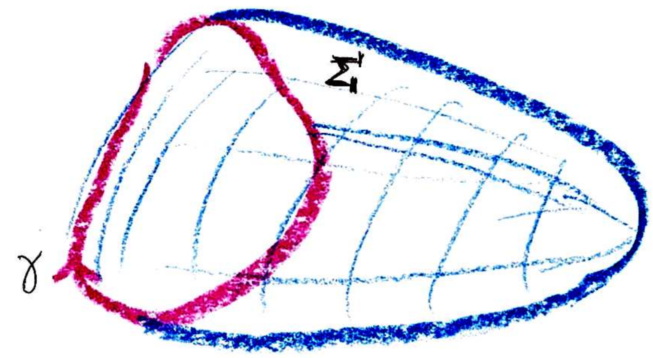
Area law implies a linear potential between two static quarks,

$$S_\gamma = TV(L) \sim \sigma_c TL \Rightarrow V(L) = \sigma_c L$$

AdS/CFT and Confinement

The holographic dual of the Wilson Loop is the action of a string attaching to the contour on the boundary, and closing into the interior.

$\Leftrightarrow$



$$S_\gamma = \frac{1}{2\pi \ell_s^2} Area_\Sigma$$

AdS/CFT and Confinement

Confinement if:

- The higher-dimensional space ends regularly (or at a hard wall) at some coordinate r_m
- The space is non-compact in the IR but the metric functions have a non-zero minimum at some point r_m

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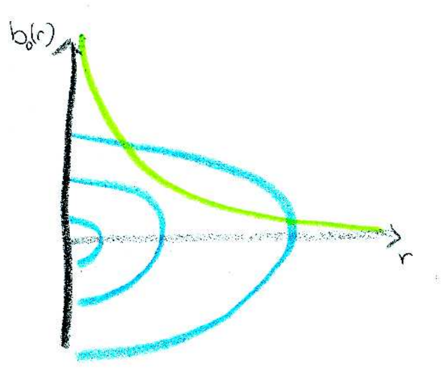
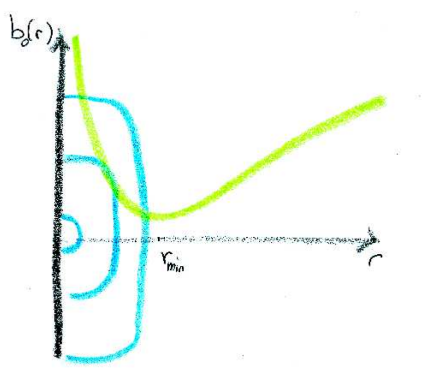
$$ds^2 = b(r)\left[dr^2 + dx_\mu^2\right]$$

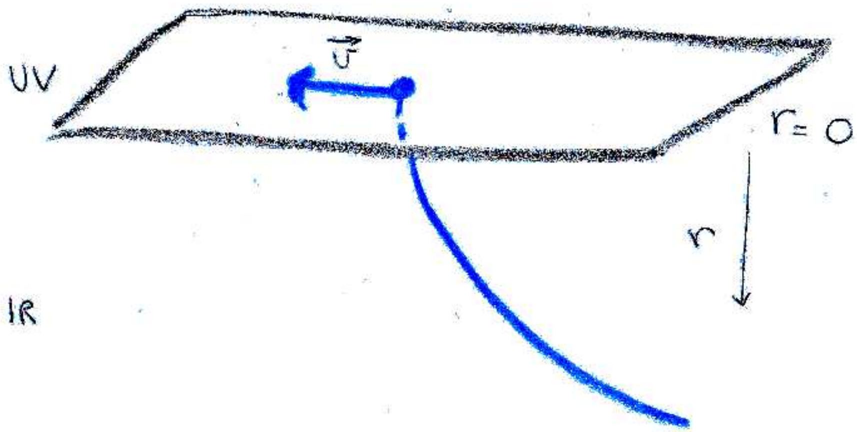


non-confining, $\sigma_c = 0$



confining, $\sigma_c = b^2(r_m)$

The Traling String



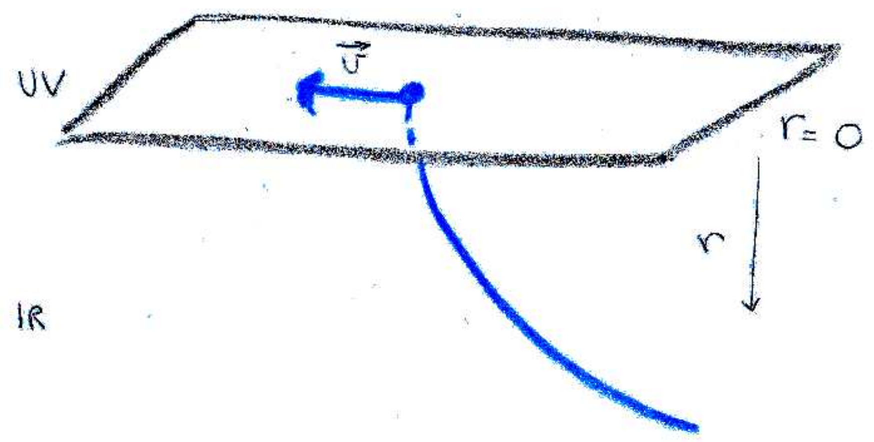
Probe quark on the boundary a 5D asymptotically AdS spacetime

$\Updownarrow$

Classical string attached at the boundary and extending in the interior.

(Gubser '06)

The Trailing String



The string profile is found by extremizing the surface spanned by the string

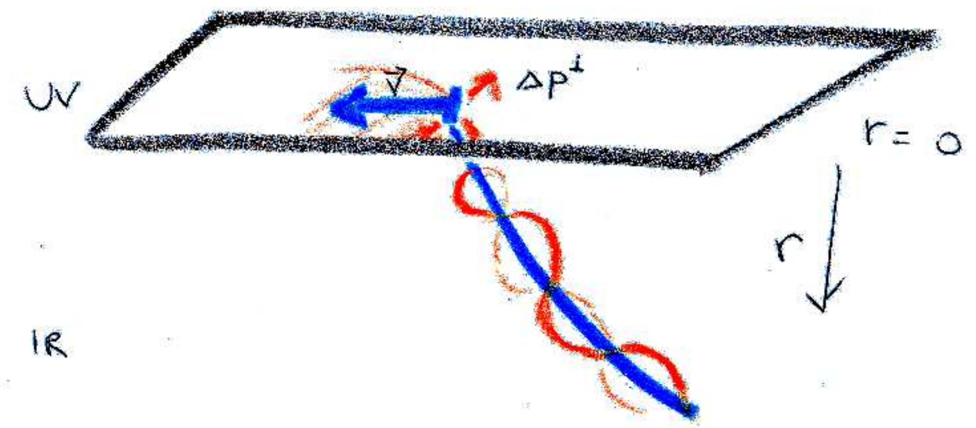
$$S = \frac{1}{2\pi\ell_s^2}\int dt dr \sqrt{-\det g_{ind}}\,,$$

with respect to the embedding coordinates: $\vec{X}(t,r) = \vec{v}t + \vec{\xi}(r)$.

The string exerts a drag force which causes the quark to lose energy:

dual description of in-medium energy loss

The Trailing String Fluctuations



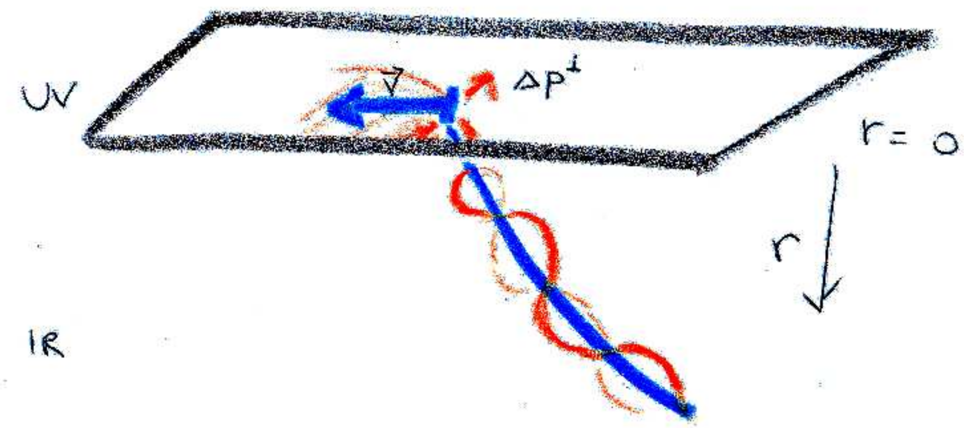
Add small fluctuations along the string:

$$X(t,r) = \xi(r) + \delta X(t,r)$$

they induce a Brownian-like dynamics for the boundary quark, governed by a Langevin equation, and leading to a spread in momentum. (Gubser '05, De Boer *et al* 06, Herzog *et al* 06, Son and Teaney 09) .

Dual description of transverse momentum broadening.

The Trailing String Fluctuations



$$M\delta\ddot{X}(t)+\int dt'\, G_R(t-t')\delta X(t') = \zeta(t), \qquad \langle\zeta(t)\zeta(t')\rangle = G_s(t-t')$$

Generalized Langevin equation.

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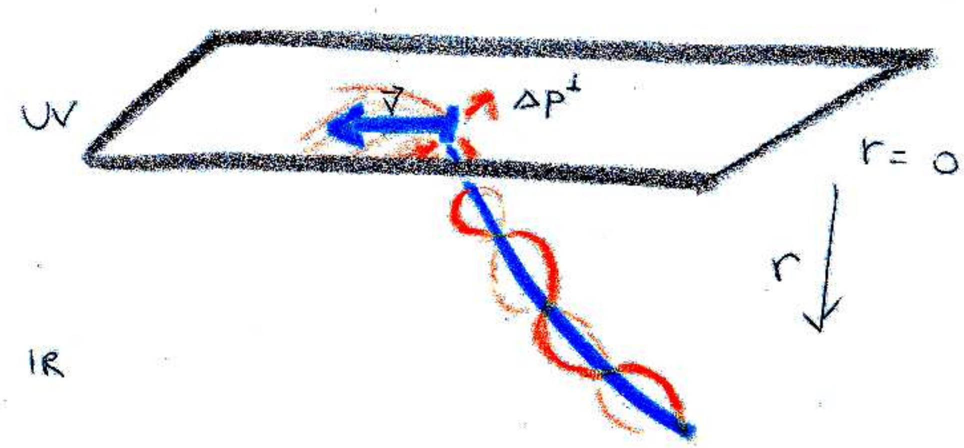

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Generalized Langevin equation. Two force terms:

- a classical force with retardation effects;
- a stochastic force with a Gaussian distribution.

Both terms arise from the same underlying physics.

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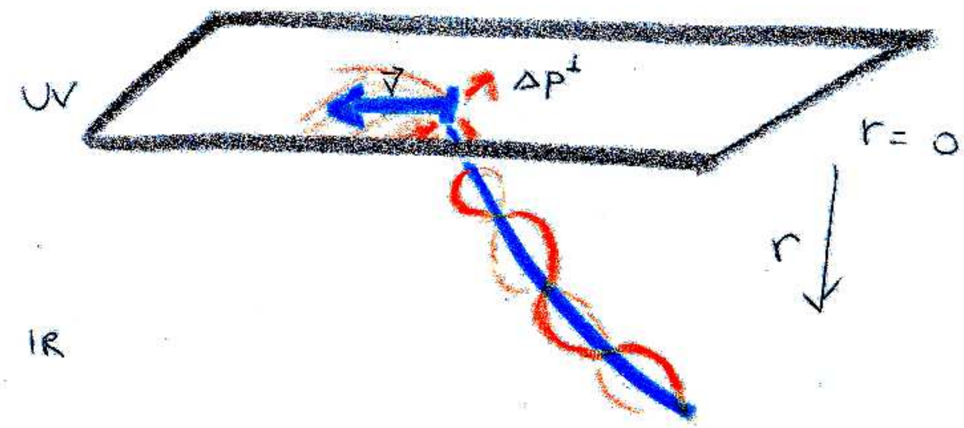

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- $G_R(t)$ is the retarded boundary correlator associated to the fluctuations $\delta X(t,r)$ around the classical trailing string.
- $G_s(t)$ is the associated the symmetric correlator, obtained from $G_R(t)$ via a Fluctuation-Dissipation relation, characteristic of the ensemble.

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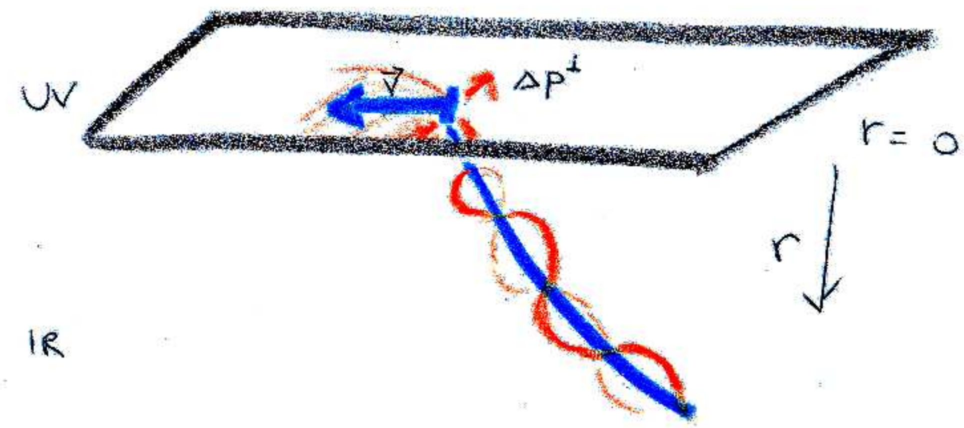

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long-time limit:

$$M\delta\ddot{X}(t) + \eta\dot{X}(t) = \zeta(t), \qquad \langle\zeta(t)\zeta(t')\rangle = k\delta(t-t')$$

$$\eta = \lim_{\omega\to 0}\frac{\mathrm{Im}\ \mathrm{G_R}(\omega)}{\omega}, \qquad \kappa = \lim_{\omega\to 0} G_s(\omega)$$

Trailing string in 5D black hole

Consider a generic asymptotically AdS 5D black hole:

$$ds^2 = b^2(r)\left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx_i\right]$$

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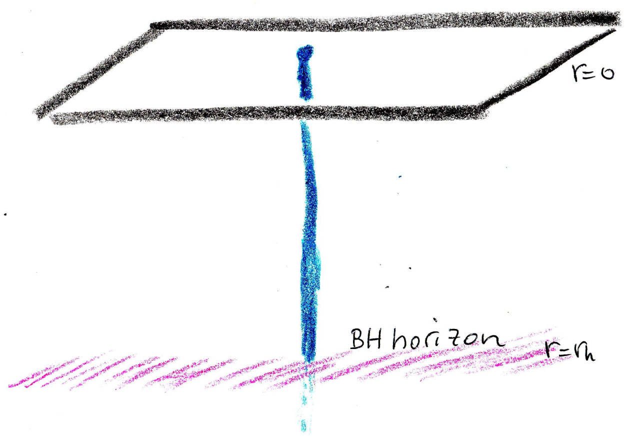
- Dual to a non-conformal gauge theory in thermal equilbrium at a temperature T_h, in a deconfined phase.

Trailing String in a 5D black hole

Let us focus on the static case.



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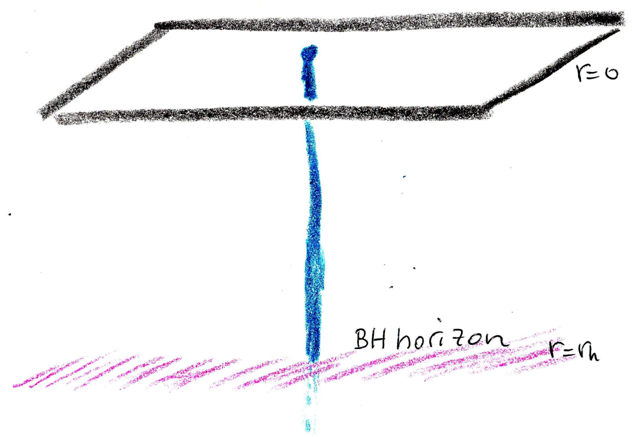

The string falls straight down into the horizon.
The induced 2D worldsheet metric is a 2D black hole with horizon r_h and temperature T_h.

$$ds^2_{ind} = b^2(r)\left[-f(r)dt^2 + f^{-1}(r)dr^2\right],$$

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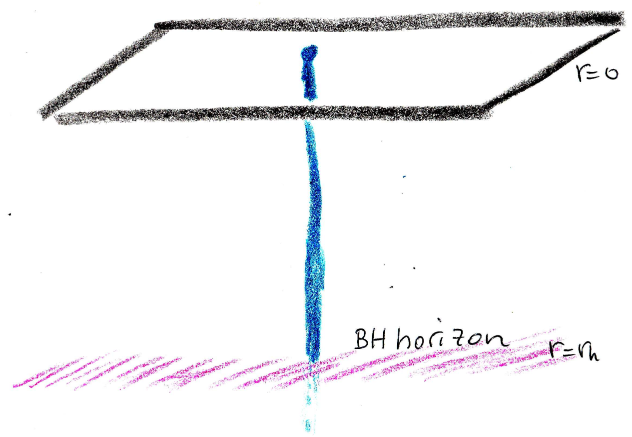

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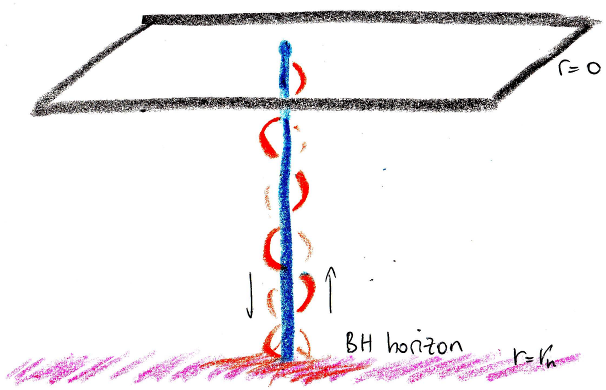
$$ds^2_{ind} = b^2(r)\left[-f(r)dt^2 + f^{-1}(r)dr^2\right], \quad f(r) \simeq 4\pi T_h(r_h - r),\ r \simeq r_h$$

Fluctuations

Add fluctuations: modes going in/out of the horizon.



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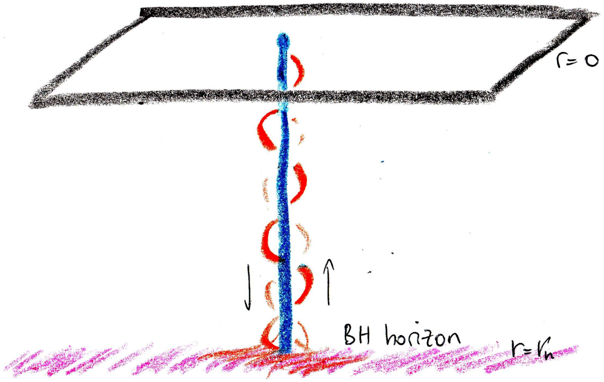

The fluctuation equation close to the horizon is

$$\delta X'' - \frac{1}{(r_h - r)}\delta X' + \frac{\hat{\omega}^2}{(r_h - r)^2}\delta X = 0, \qquad \hat{\omega} \equiv \frac{\omega}{4\pi T_h}$$

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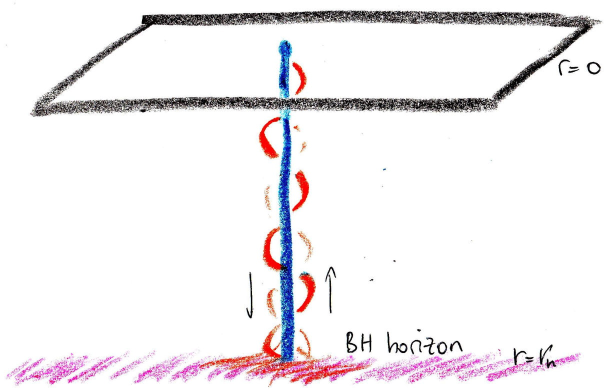

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The solutions have infalling/outgoing behavior near r_h,

$$\delta X(\omega, r) \simeq (r_h - r)^{\pm i\hat{\omega}}$$

Correlators

The retarded correlator is found by the Policastro-Son-Starinets prescription

$$G_R(\omega) = \left[\mathcal{G}(r)\,\delta X_R'(\omega,r)\right]_{r\to 0}, \qquad \delta X_R(\omega,r) \to \begin{cases} 1 & r \to 0 \\ (r-r_h)^{-i\hat{\omega}} & r \to r_h \end{cases}$$

The coefficient $\mathcal{G}(r)$ depends only on the background bulk geometry and is explicitely known.

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The long-time Langevin dynamics is governed by the viscosity coefficient η that appears as the leading $\omega \to 0$ term in $G_R(\omega)$

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Gursoy, Mazzanti, Kiritsis, FN 1006.3261

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At long times, the dynamics is encoded by the Langevin coefficient

$$\kappa = \lim_{\omega\to 0} G_s(\omega) = 2T_h\eta$$

Green's functions: High frequency limit

The large ω limit obtained via WKB approximation

Gursoy, Mazzanti, Kiritsis, FN 1006.3261:

$$Im\, G_R(\omega) \simeq \omega^3\, h\left(\frac{\sqrt{2}}{\gamma\omega}\right) \qquad b(r) \sim \frac{\ell}{r} h(r)$$

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This diverges too fast for G_R to be physical:

- Dispersion relations that allow to write

$$G_R = \int \frac{Im\, G_R(\omega')}{\omega' - \omega' - i\epsilon}$$

  require $G(\omega) \sim 1/\omega$

- Real time dynamics completely dominated by the shortest time interval with big random kicks as $\Delta t \to 0$.

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- Remark: the leading behavior is temperature-independent.

Dressed spectral density

UV-safe spectral densities can be defined: Subtract the correlator obtained from the vacuum background. Mazzanti, Kiritsis, FN, 1111.1008:

$$G_R^{(ph)}(\omega) = G_R(\omega) - G_R^{(vac)}(\omega)$$

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- This prescription can be obtained with a change of variable in the quark path integral.

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Two qualitatively very different cases:

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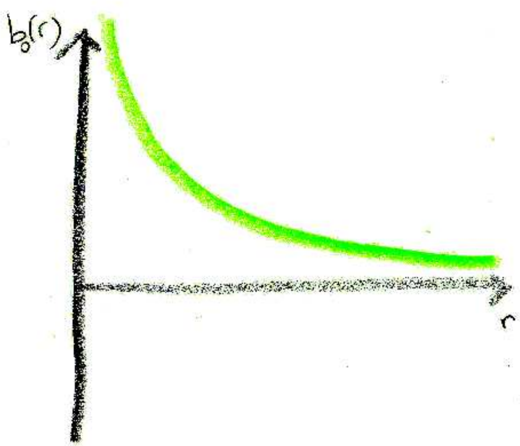
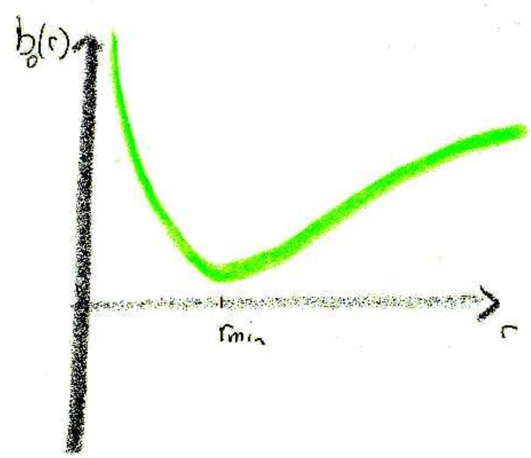
Confinement is essentially equivalent to the presence of a minimum of the bulk scale factor $b(r)$ (cfr. J .Sonnenschein's talk)





non-confining

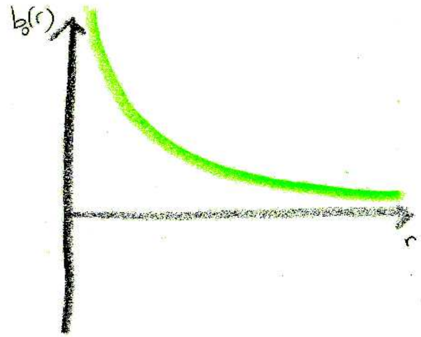
confining $\sigma_c = b^2(r_m)$

Non-confining case

$$ds^2 = b^2(r)\left[dr^2 + dx^\mu dx_\mu\right],$$

the string profile satisfies:

$$\xi'(r) = \frac{C}{\sqrt{b^4(r) - C^2}}$$



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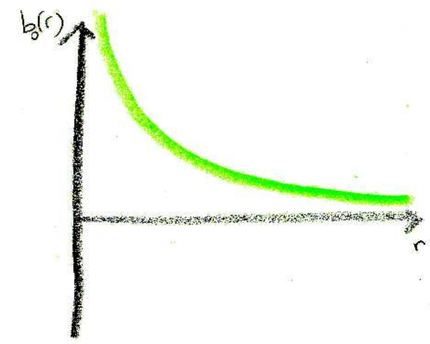
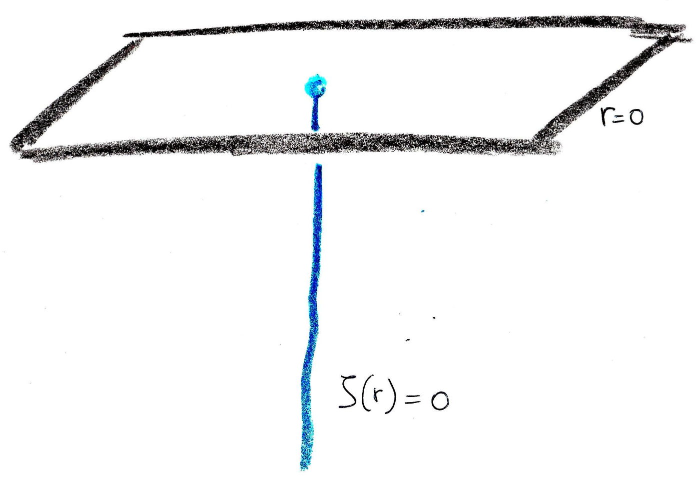


As $b \to 0$, regularity requires $C = 0$: the embedding is trivial, $\xi = 0$



Confining Trailing String

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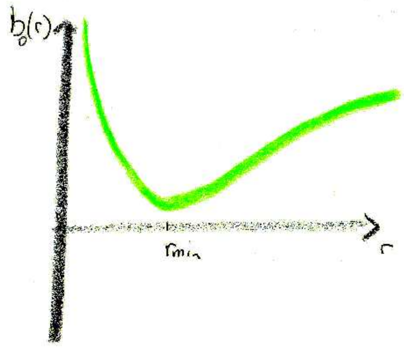

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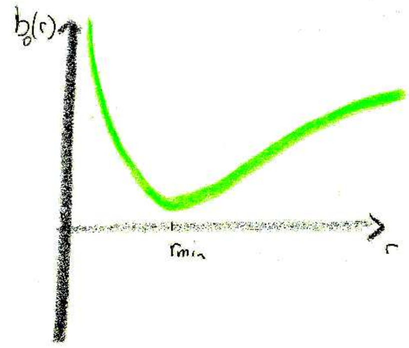

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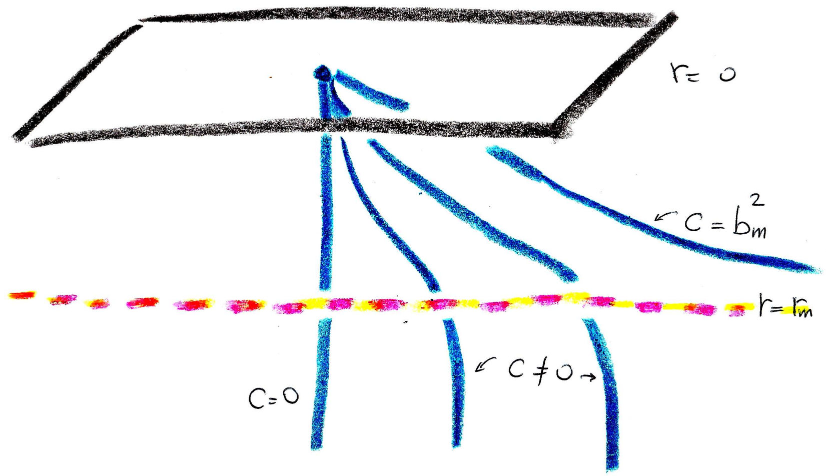
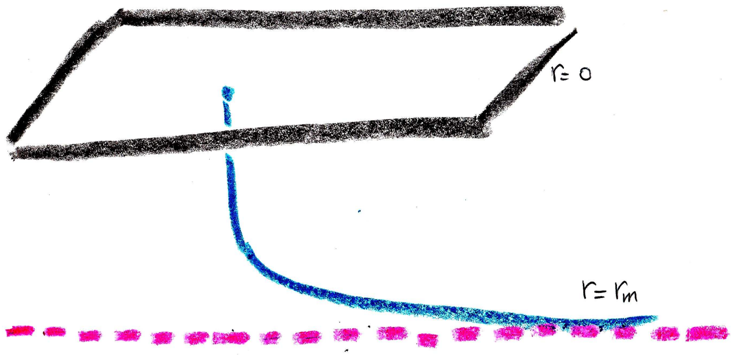
One-parameter family of solutions with $0 \leq C \leq b^2(r_m)$



Confining Trailing String

The extremal $C = b_m^2$ string is the one with lowest action. It does not extend beyond the *confining horizon* $r = r_m$, and it extend to infinity along one of the spatial directions.



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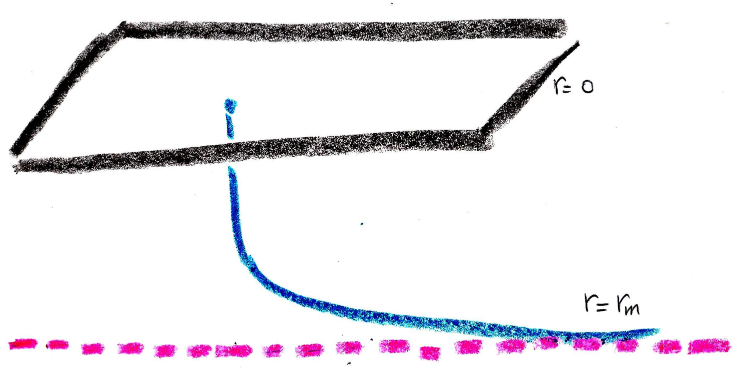

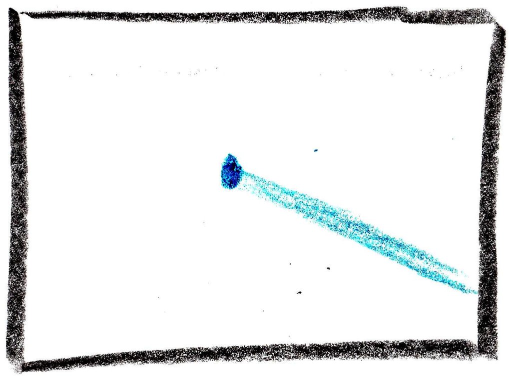

Asymptotically it looks like a straight string with fixed tension b_m^2 i.e. the confining string tension of the dual theory: it is the QCD flux-tube.

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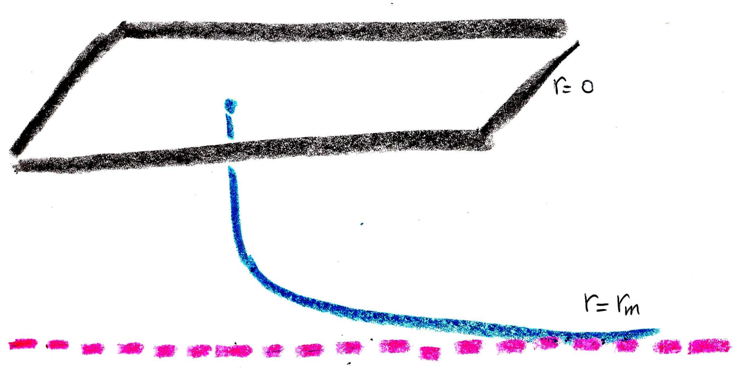
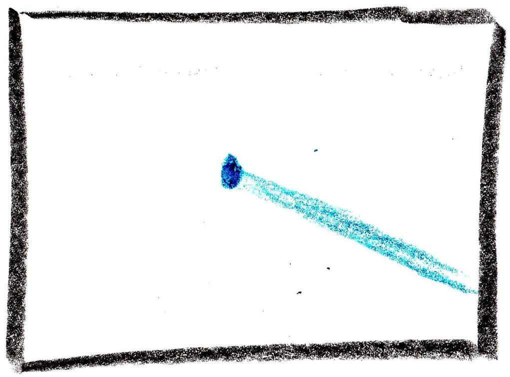
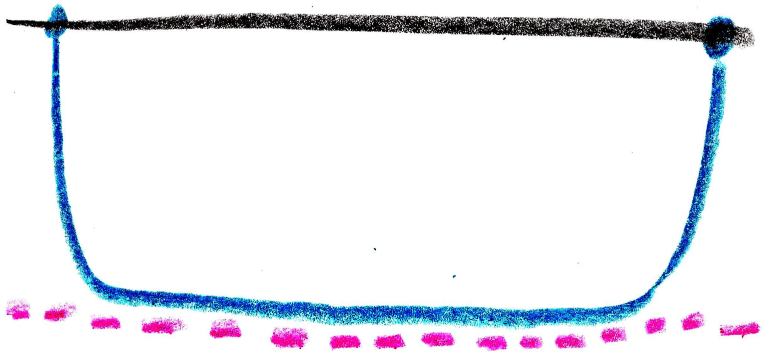




The string breaks rotational invariance. It can be recovered summing over different directions.

Physical picture: the Shadow Quark

This has a simple physical interpretation:



look at the trailing string as half of the confining string connecting two quarks, one of which is observed, the other (shadow quark) infinitely far.

Confining string geometry

Worldsheet induced metric:

$$ds^2 = b^2(r)\left[-dt^2 + \frac{b^4}{R^2}\,dr^2\right], \qquad R = \sqrt{b^4 - b_m^4}$$

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$$ds^2 \sim (\ell/r)^2\left[-dt^2 + dr^2\right] \quad r \to 0$$

- close to the confining horizon r_m it reduces to

$$ds^2 \sim b_m^2\left[-dt^2 + (4\pi T_m)^2 \frac{dr^2}{(r_m - r)^2}\right] \quad T_m = (4\pi)^{-1}\sqrt{b_m''/b_m}$$

Confining horizon geometry

Metric close to the confining horizon r_m:

$$ds^2 \sim b_m^2 \left[-dt^2 + (4\pi T_m)^2 \frac{dr^2}{(r_m - r)^2} \right]$$

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$$ds^2 \sim b_m^2 \left[-dt^2 + (4\pi T_m)^2 \frac{dr^2}{(r_m - r)^2} \right] = b_m^2 [-dt^2 + dz^2]$$

$$z \sim \; -4\pi T_m \log(r_m - r) \to +\infty.$$

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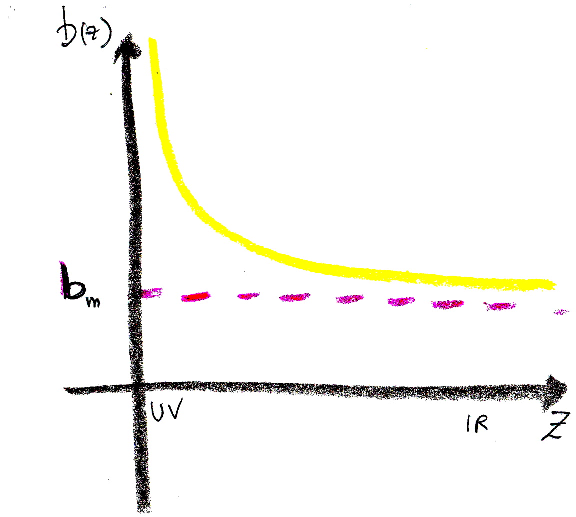
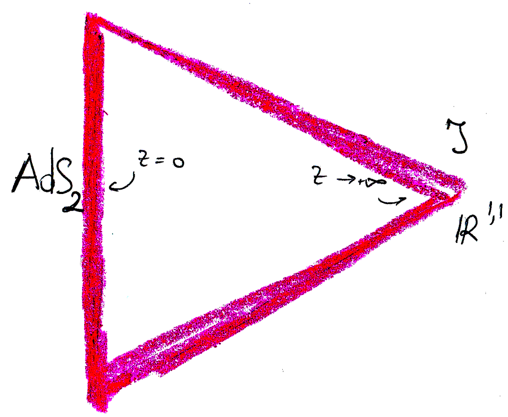
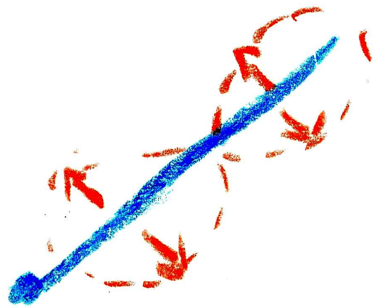


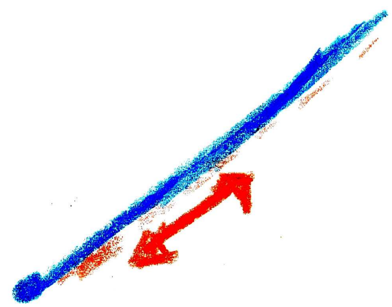


Transverse vs. longitudinal fluctuations

We can distinguish between fluctuations longitudinal and transverse to the boundary direction of the string:



Transverse



Longitudinal

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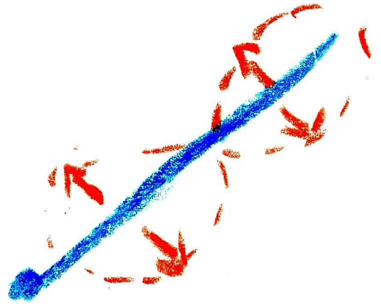

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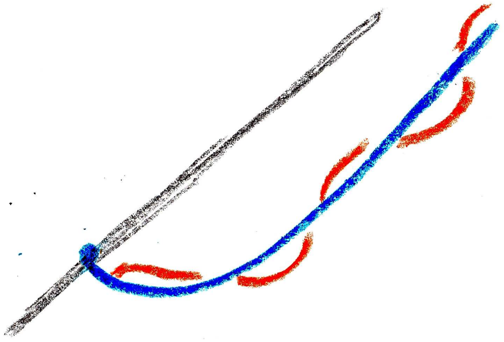

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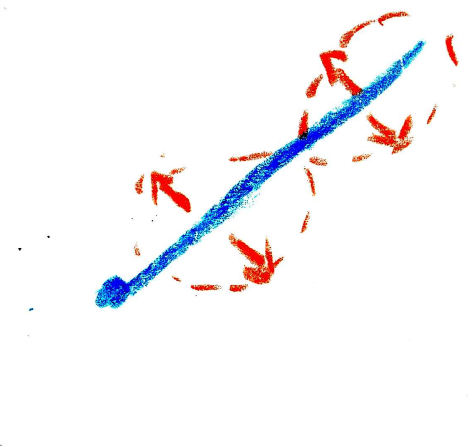

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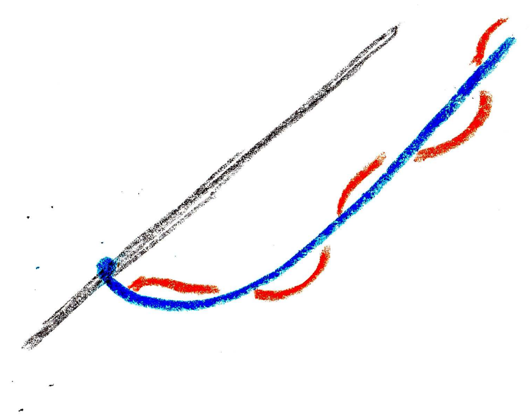

Longitudinal

We require no information come from the quark at infinity.
$\Rightarrow$ we impose infalling boundary conditions as $r \to r_m$.

Transverse vs. longitudinal fluctuations

The $\perp$ and $\|$ fluctuations behave differently:

$$\partial_r \left[R\, \partial_r \left(\delta X^{\perp} \right) \right] + \frac{\omega^2 b^4}{R} \delta X^{\perp} = 0,$$

$$\partial_r \left[\frac{R^3}{b^4} \partial_r \left(\delta X^{\|} \right) \right] + \omega^2 R \delta X^{\|} = 0$$

$$R(r) = \sqrt{b^4(r) - b_m^2}$$

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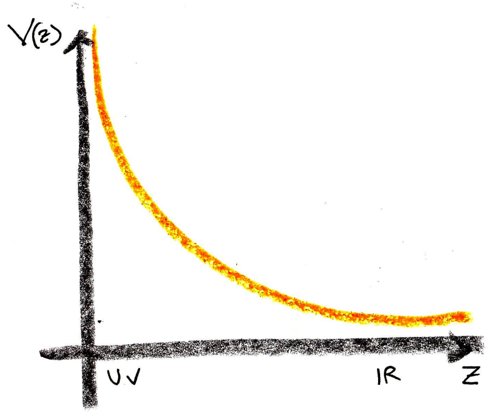
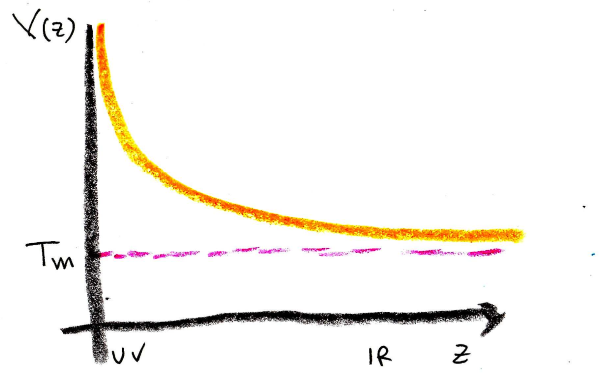
$$R(r) = \sqrt{b^4(r) - b_m^2} \sim b_m^2\, 4\pi T_m \left(r_m - r \right), \qquad r \simeq r_m .$$

Angular dependence

The fluctuations $\perp$ and $\|$ to the direction of the string behave differently.
One can transform the equation into the form of a Schrödinger problem:



Transverse



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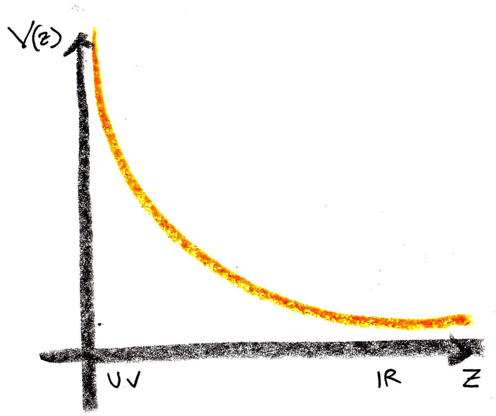

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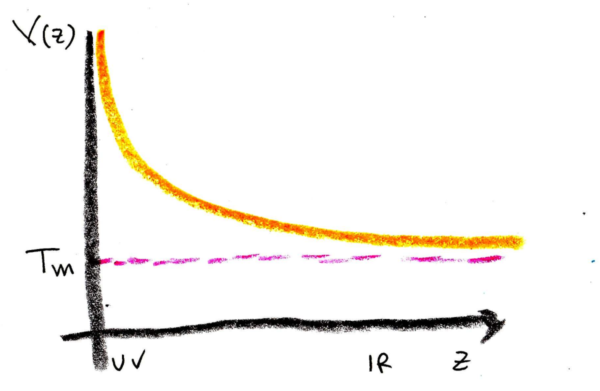

Longitudinal

The transverse modes have a continuous spectrum starting at $\omega = 0$,

the longitudinal modes are gaped and start at $\omega = 4\pi T_m$

An Effective Temperature?

The equation for transverse fluctuations close to r_m is:

$$\delta X'' - \frac{1}{(r_m - r)}\delta X' + \frac{\hat{\omega}^2}{(r_m - r)^2}\delta X = 0, \qquad \hat{\omega} \equiv \frac{\omega}{4\pi T_m}$$

same as close to a black hole horizon with temperature T_m.

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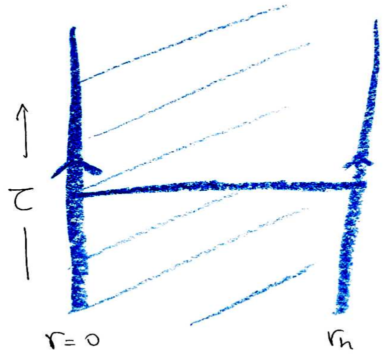
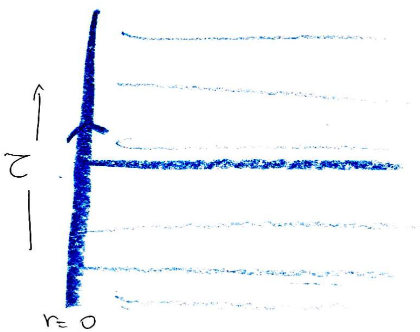
Are we seeing the emergence of an *effective temperature set by the confinement scale* ?

An Effective Temperature?

BH: finite length



Confining : ∞ length



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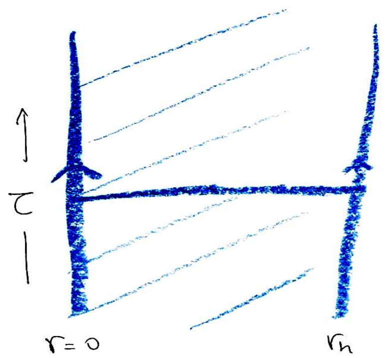
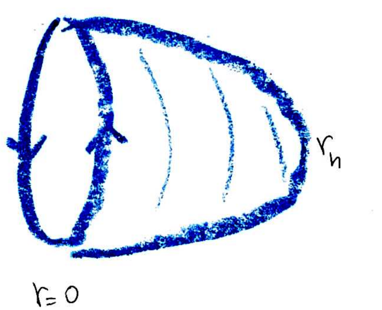


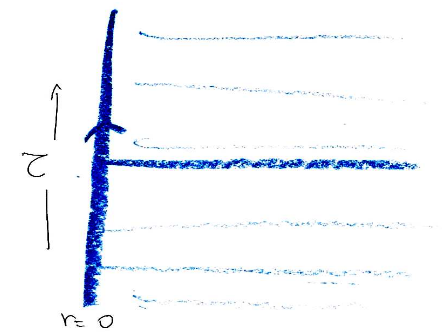
$\Downarrow$



Periodic euclidean time

$T = T_h$

Confining : ∞ length



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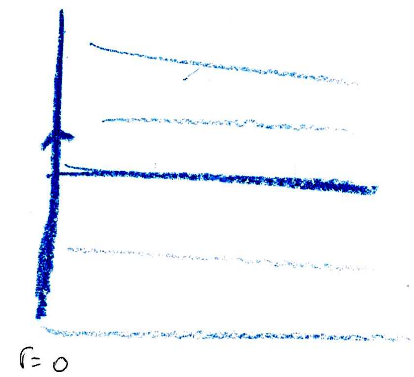

Non-compact Euclidean time

T=0

Dissipation at zero temperature

The transverse boundary correlator at small frequency behaves as:

$$G_R^T(\omega) \simeq i b_m^2 \omega + O(\omega^2), \quad b_m^2 = \sigma_c \quad the\, confining\, string\, tension$$

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The confining vacuum is dissipative for the fluctuations of a single quark and the dissipation time scale is again set by the confinement scale.

Dissipation at zero temperature

Because of the gap in the longitudinal mode, the imaginary part of the longitufinal boundary correlator vanishes identically at small frequency:

$$G_R^L(\omega) = 0 \qquad \omega < 4\pi T_m$$

$$G_R^L(\omega) \sim \sqrt{\left(\frac{\omega}{4\pi T_m}\right)^2 - 1} \qquad \omega \gtrsim 4\pi T_m$$

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$[ImG_R^L/\omega]_{\omega\to 0} = 0$ and there is no viscous friction for longitudinal modes.

Averaging over angles

The string solution breaks spontaneously spatial rotations $SO(3) \to SO(2)$. For a given solution, the correlator will be anisotropic:

$$G^{ij} = G^L(\omega) n^i n^j + G^T(\omega)(\delta^{ij} - n^i n^j) \qquad \vec{n} = \vec{n}(\theta, \varphi)$$

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To quadratic order in the fluctuations:

$$Z = \int \frac{d\Omega}{4\pi} \exp i \int d\omega\, \delta X^i(\omega) G_{ij}(\omega, \Omega) \delta X^j(-\omega)$$

$$\simeq \exp i \int \frac{d\Omega}{4\pi} \int d\omega\, \delta X^i(\omega) G_{ij}(\omega, \Omega) \delta X^j(-\omega)$$

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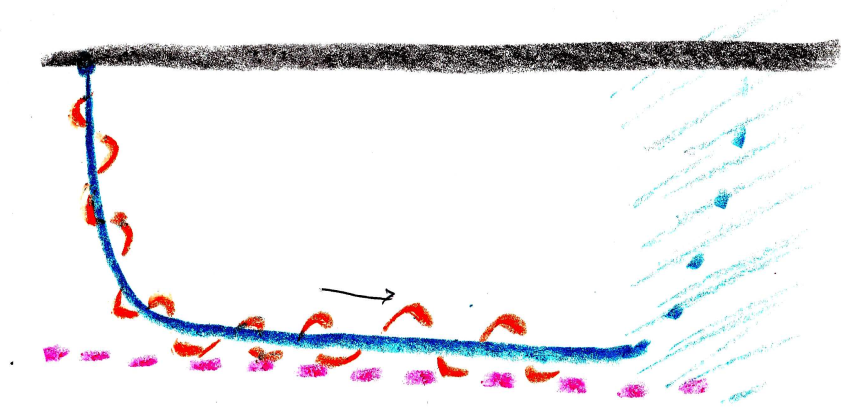
Physical picture: the Shadow Quark

The friction coefficient arises because of infalling boundary condition.



All calculation done on a single (observed) quark should be done by assuming that no information is available or comes from the shadow quark. E.g. the infalling wave condition at $z \longrightarrow \infty$

Physical picture: the Shadow Quark

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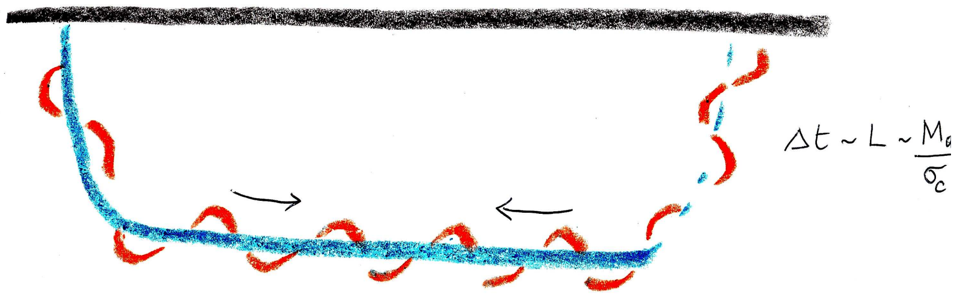


In practice, for any finite string, sooner or later information will come back and the system will become non-dissipative: a finite length of the string destroys the small-ω linear term in ImG_R. A finite quark mass M will introduce an IR cutoff to the string length.

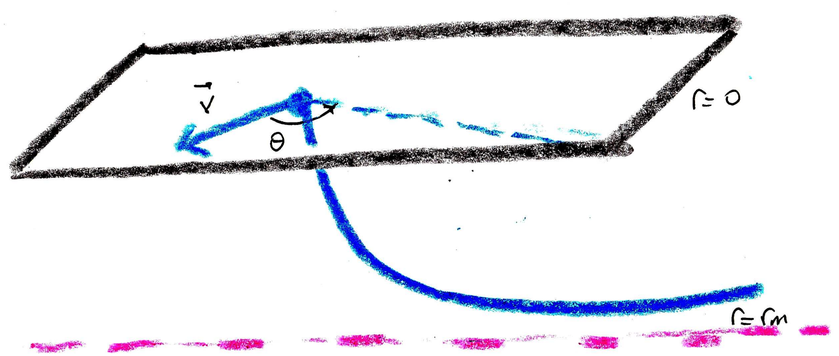
Physical picture: the Shadow Quark

The friction coefficient arises because of infalling boundary condition.

Trailing String at finite velocity

Suppose the boundary quark has a constant velocity $\vec{v}$. The more general solution is now:

$$X(r,t) = \vec{v}t + \vec{\xi}(r), \quad \xi'(r) = \frac{\vec{c}}{\sqrt{b^4 - C^2}}, \quad |\vec{c}|^2 + \frac{(\vec{v}\cdot\vec{c})^2}{1-v^2} = C^2$$



Again, the action does not depend on the string direction, and the preferred solution is the one with $C = b_m^2$.

Drag Force

From the classical solution we can compute the net force exherted by the string, as the worldsheet momentum. This correspodns to the vev of the force operator dual to δX:

$$\langle \vec{\mathcal{F}} \rangle = \frac{\sigma_c}{(1-v^2\sin^2\theta)^{3/2}} \begin{pmatrix} \cos\theta \\ (1-v^2)\sin\theta\cos\varphi \\ (1-v^2)\sin\theta\sin\varphi \end{pmatrix}$$

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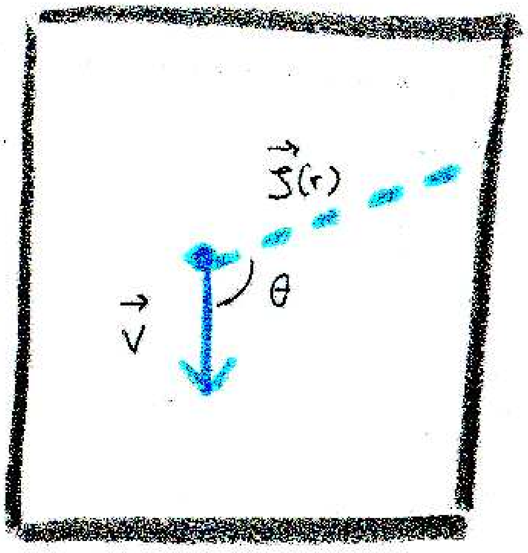
For $v \ll 1$ it is directed along the string, and it represents the constant force applied by a string with tension σ_c on its endpoint.

Fluctuations

Now we have two ways of defining transverse and longitudinal modes:

- With respect to $\vec{v}$
- With respect to the string direction.



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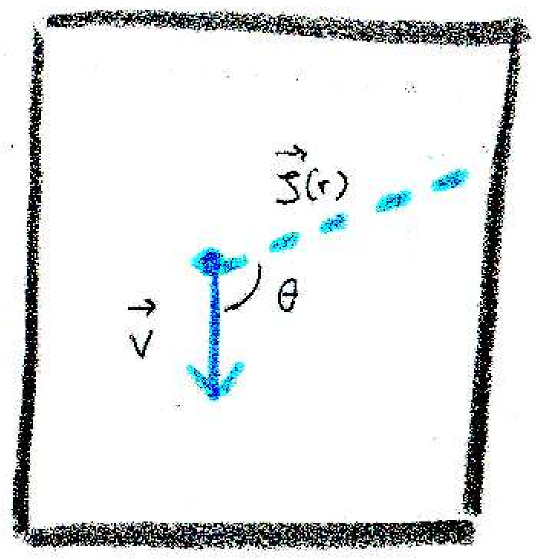
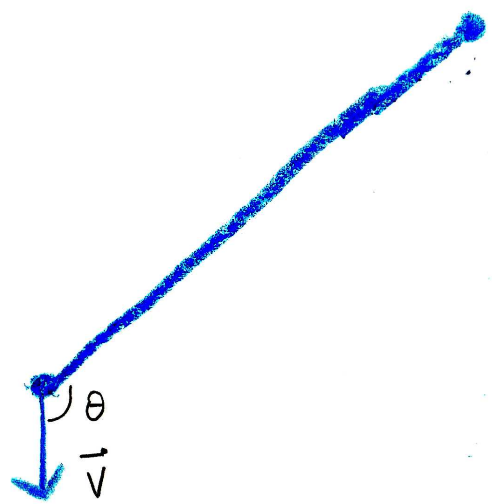


Only the first distinction has a meaning on the boundary.

Integrating over angles

The way to perfom the angular average is not unique.

- We can average with the usual meausre on the unit sphere. This means assuming that for each choice of the angle, the shadow-quark has the same velocity as the observed quark.



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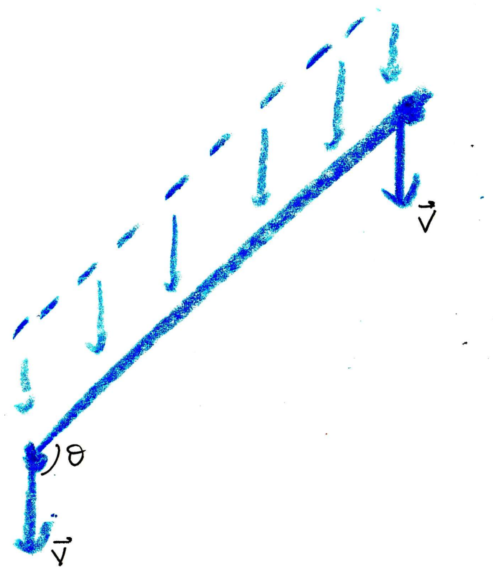

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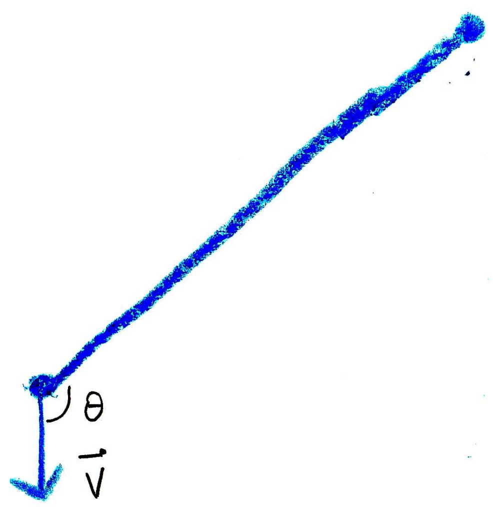

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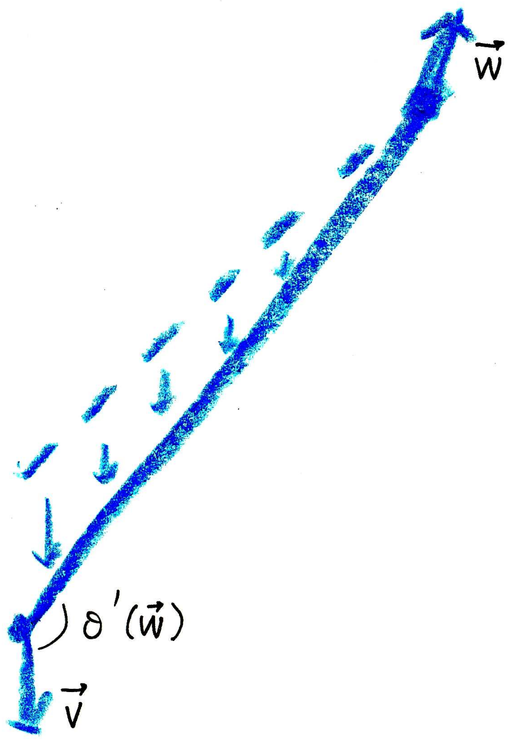

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$$cos\theta = \frac{\vec{v}}{|v|} \frac{\vec{w} - \vec{v}}{|\vec{w} - \vec{v}|}$$

Averaged correlators

Both averaging procedures can be carried out analytically. The result is always in the form:

$$\langle G^{\|} \rangle = A(v) G^L(\omega) + B(v) G^T(\omega), \quad \langle G^{\perp} \rangle = C(v) G^T(\omega) + D(v) G^L(\omega)$$

Averaged correlators

Both averaging procedures can be carried out analytically. The result is always in the form:

$$\langle G^{\|}\rangle = A(v)G^L(\omega)+B(v)G^T(\omega), \quad \langle G^{\perp}\rangle = C(v)G^T(\omega)+D(v)G^L(\omega)$$

- $G^{T,L}(\omega)$: (essentially) the same correlators we found in the static case;
- A, B, C, D : simple functions of v which depend on the kind of average.
- In the static limit we obtain the expected isotropic result:

$$\langle G^{\|,\perp}\rangle \to \frac{1}{3}G^L + \frac{2}{3}G^T \qquad v \to 0$$

- The large-ω behavior is universal.

Consistency condition

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$$\frac{d\langle \mathcal{F} \rangle}{dv} = -i \left[\frac{dG_R(\omega)}{d\omega} \right]_{\omega=0}$$

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- the simple angular average over the string direction DOES NOT satisfy this relation;
- the average over the velocity of the shadow quark $\vec{w}$ DOES.
- This is a consistency check that the shadow-quark picture (complete ignorance about the string boundary at infinity) is the correct one if we want to describe a consistent ensemble

Conclusion

- We computed the trailing string solution and corresponding fluctuation in confining holographic backgrounds.
- The solution exhibits new features (e.g. spontanous breaking of isotropy), and interesting dynamics (viscous drag and low frequency modes at zero temperaure)
- Integration over moduli gives correlators which can be used to define physical Langevin correlators for the diffusion problem at finite temperature.
- The consistent interpratation of the vacuum correlator is in terms of a quark pair, one of which is very far and unobserved.
- Next: finite mass quarks, comparison with experimental results.