A Dimensionally Deconstructed Holographic Superconductor

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September 10, 2013

AdS/CFT

Jumping right in..

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AdS/CFT is a specific duality:

Anti-de Sitter		Conformal Field Theory
Bulk, weakly-coupled gravitational theory in $(d+1)$-dimensional spacetime	$\leftrightarrow$	Boundary, strongly-coupled gauge theory in d-dimensional spacetime

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Provides a framework:

Symmetries and quantum numbers match on both sides.

AdS/CFT

AdS metric (g):

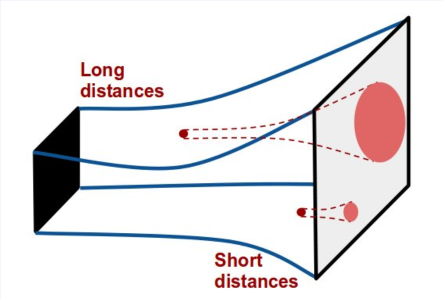
$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

Fields in AdS, $\Phi(x,z)$, break up into two pieces:

Normalizable	$\leftrightarrow$	$\mathcal{O}(x)$
Non-normalizable	$\leftrightarrow$	$\phi_0(x)$

(E.g. $A_{0_\mu}(x)$ sources $J^\mu(x)$)

AdS Space



From Hartnoll arXiv:1106.4324

AdS/CFT

Recipe for model building:

AdS	$\leftrightarrow$	CFT
Fields	$\leftrightarrow$	Operators
Gauge fields in bulk	$\leftrightarrow$	Global Symmetry
Black hole	$\leftrightarrow$	Finite temperature

Holographic Superconductors?

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AdS_4 black hole can develop a condensate, spontaneously breaking a $U(1)$ gauge symmetry.

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$\rightarrow$ Holographic interpretation?

Constructing Holographic Superconductor

Ingredients [Hartnoll, Herzog, and Horowitz]:

- Consider U(1) gauge field F.
- Add charged scalar $\Psi(x,z)$, dual to $\mathcal{O}$, Cooper pair operator.
- Background for gauge field $\rightarrow$ Chemical potential.
- Black hole background $\rightarrow$ System at temperature.

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We have an AdS_4-Schwarzschild background (g):

$$ds^2 = \frac{1}{z^2}\left(f(z)dt^2 - d\vec{x}^2 - \frac{dz^2}{f(z)}\right), \quad \epsilon \leq z \leq z_{\mathrm{H}}$$

where $f(z) = 1 - (z/z_{\mathrm{H}})^3$.

Holographic Superconductor

The action for a holographic superconductor:

$$\mathcal{S} = \int d^4x \sqrt{g} \left\{ |D\Psi|^2 - m^2|\Psi|^2 - \frac{1}{4} F_{MN} F^{MN} \right\}$$

Strategy:

Holographic Superconductor

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Strategy:

Vary the temperature. Find nonvanishing solution for Ψ.

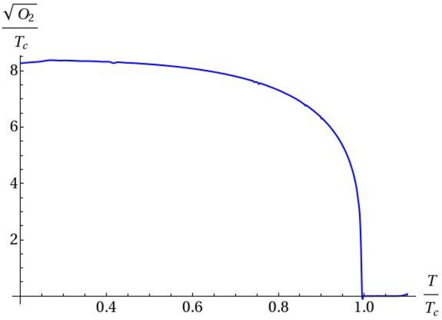
$\rightarrow \langle \mathcal{O} \rangle \neq 0$.

A Holographic Superconductor

Some features:

- $\langle \mathcal{O} \rangle \propto (1 - T/T_c)^{1/2}$
- $2\Delta \equiv \sqrt{\langle \mathcal{O} \rangle} \approx 8.4 T_c$

Condensate $\langle O \rangle$



A Holographic Superconductor

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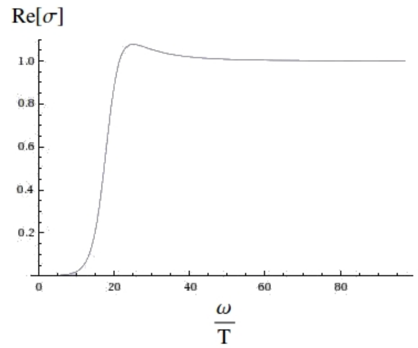
- $\langle \mathcal{O} \rangle \propto (1 - T/T_c)^{1/2}$
- $2\Delta \equiv \sqrt{\langle \mathcal{O} \rangle} \approx 8.4 T_c$
- The normal phase has $\mathrm{Re}[\sigma(\omega)] = 1$.
- Delta function $\delta(\omega)$

Conductivity



Deconstructing Superconductivity

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$\rightarrow$ Many scalar fields, but now 3D model.

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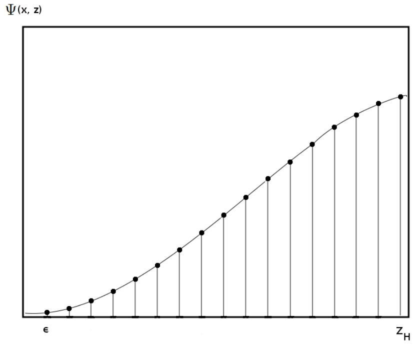
→ Basically, turning the extra dimension into a lattice.

→ Many scalar fields, but now 3D model.

One scalar at each site



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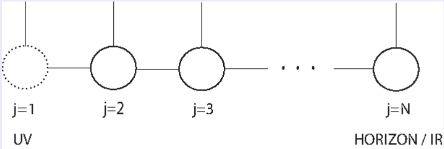
Moose diagram



Deconstructing Superconductivity

The Lagrangian for the moose diagram:

$$\mathcal{L} = \sum_{j=2}^{N-1} \left[-\frac{1}{4}(F_{\mu\nu})_j (F^{\mu\nu})_j + Z_j |D_\mu \Psi_j|^2 \right] + \sum_{j=1}^{N-1} \left[|D_\mu \Sigma_j|^2 - Z_j V_j \right].$$

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Leaving out the details...

- Couplings change from site to site.
- Link fields get a vev and are “integrated out”.

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Leaving out the details...

- Couplings change from site to site.
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$$\left(\mathcal{S} = \int d^4x \sqrt{g} \left\{ -\frac{1}{4} F_{MN} F^{MN} + |D\Psi|^2 - m^2 |\Psi|^2 \right\} \right)$$

Similar strategy to continuum case: Solve set of equations for nonvanishing Ψ_j.

Deconstructing Superconductivity

Boundary conditions:

- Chosen to best match continuum result.
- Nondynamical first site and last site.

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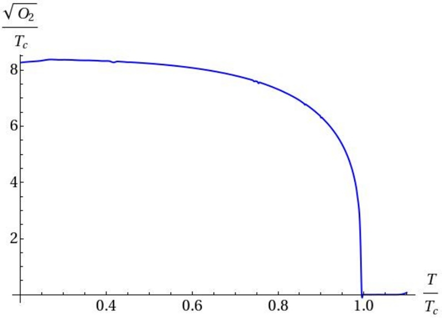
Boundary conditions:

- Chosen to best match continuum result.
- Nondynamical first site and last site.
- Continuum ingoing wave BC presents a challenge.
  $\rightarrow$ Ingoing wave BC closer to UV.

Deconstructing Superconductivity

What do we find?

Continuum $\langle \mathcal{O} \rangle$



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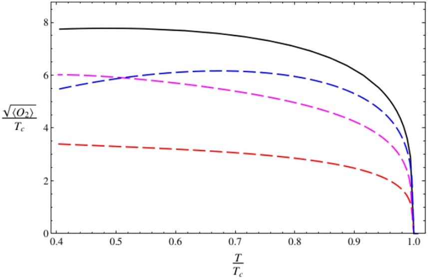
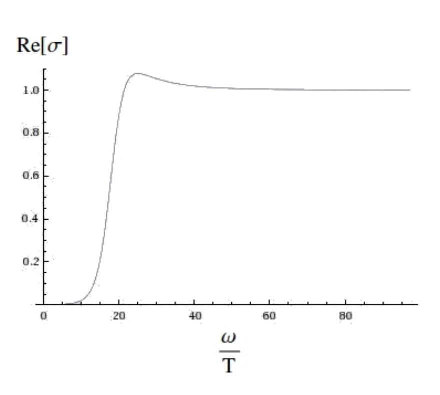


Deconstructing Superconductivity

What do we find? ($N = 1000$ and $N = 100$).

Continuum $\langle \mathcal{O} \rangle$



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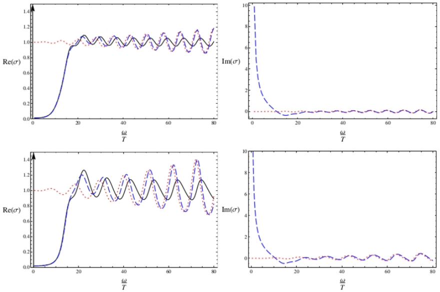

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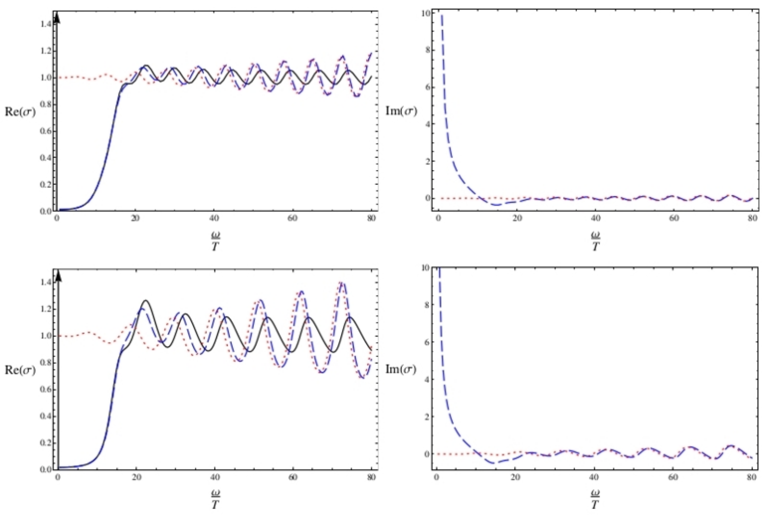

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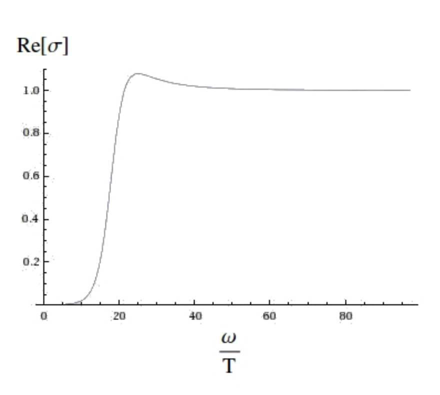

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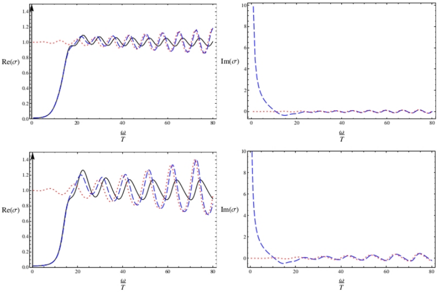
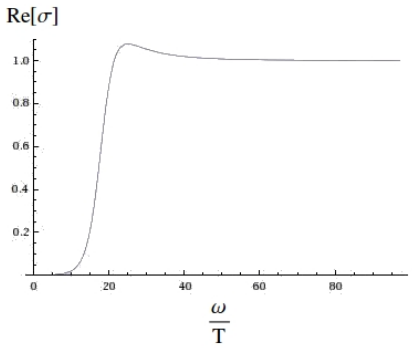


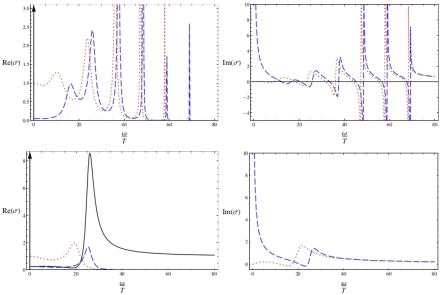
Deconstructing Superconductivity

What do we find? ($N = 10$ and $N = 5$).

Continuum $\langle \mathcal{O} \rangle$



Deconstructed $\langle \mathcal{O} \rangle$



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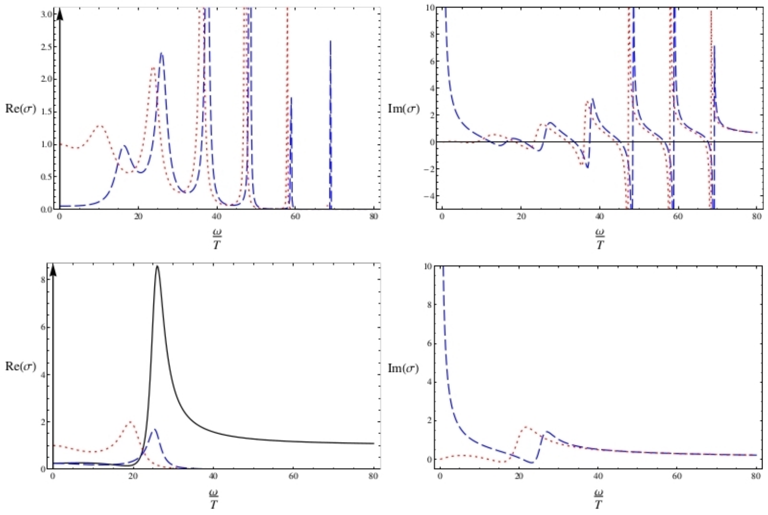

To Conclude

- Deconstruction provides a framework for building lower-dimensional models that can mimic higher-dimensional physcs.
- Some calculations are difficult to match in a natural way.
- Interpreting ingoing wave BCs.
- Excitons governed by hidden local symmetries?

The End.