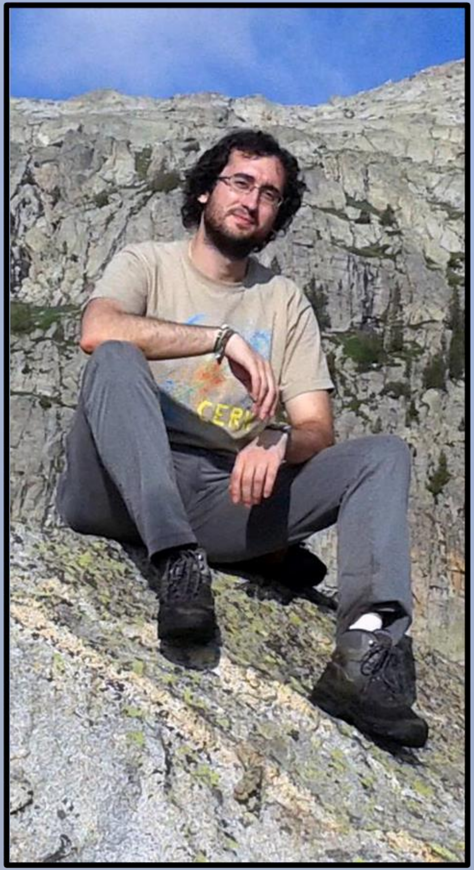
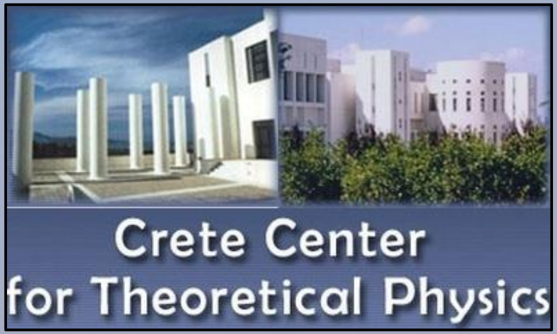


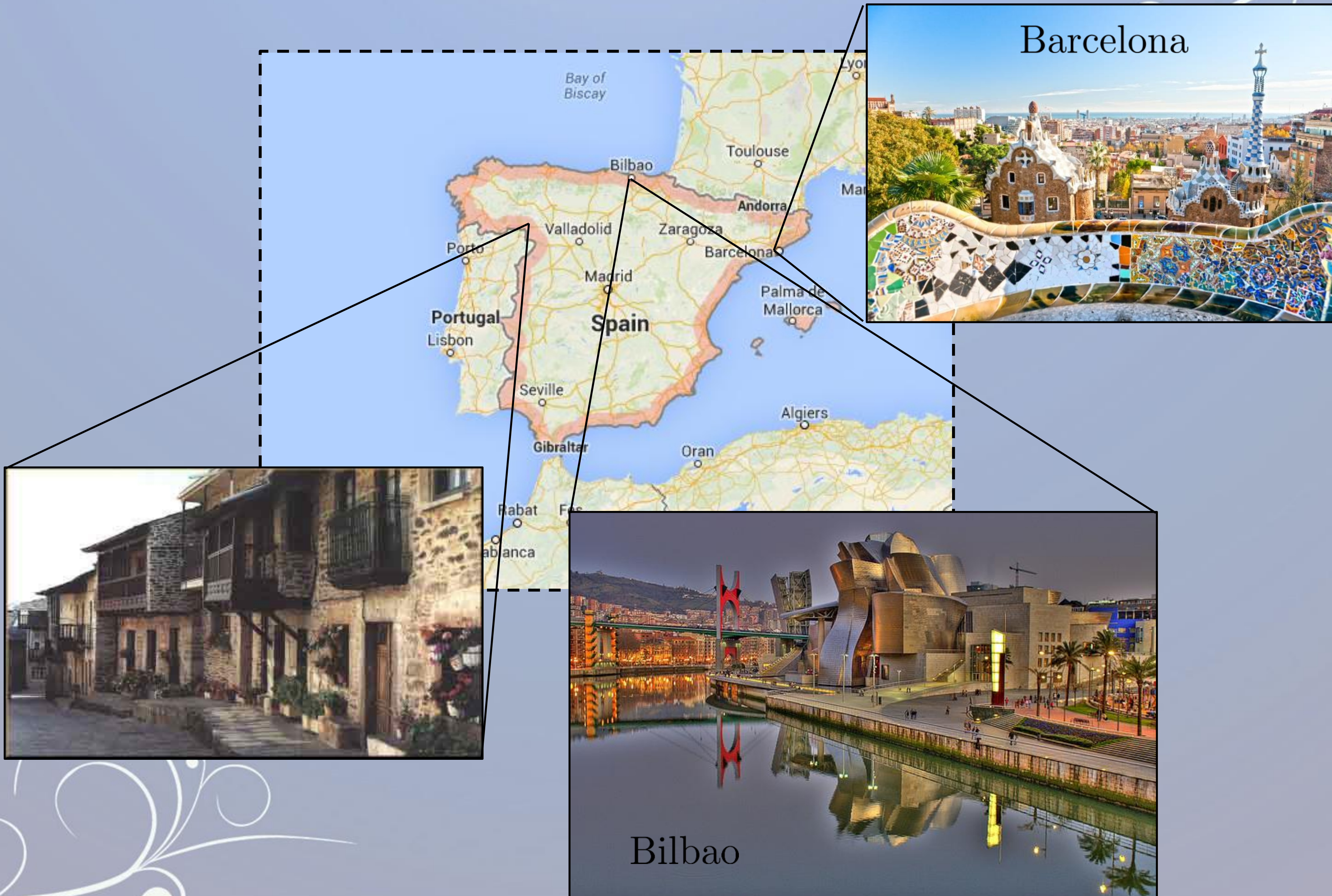
Daniel Fernández

Self-presentation:

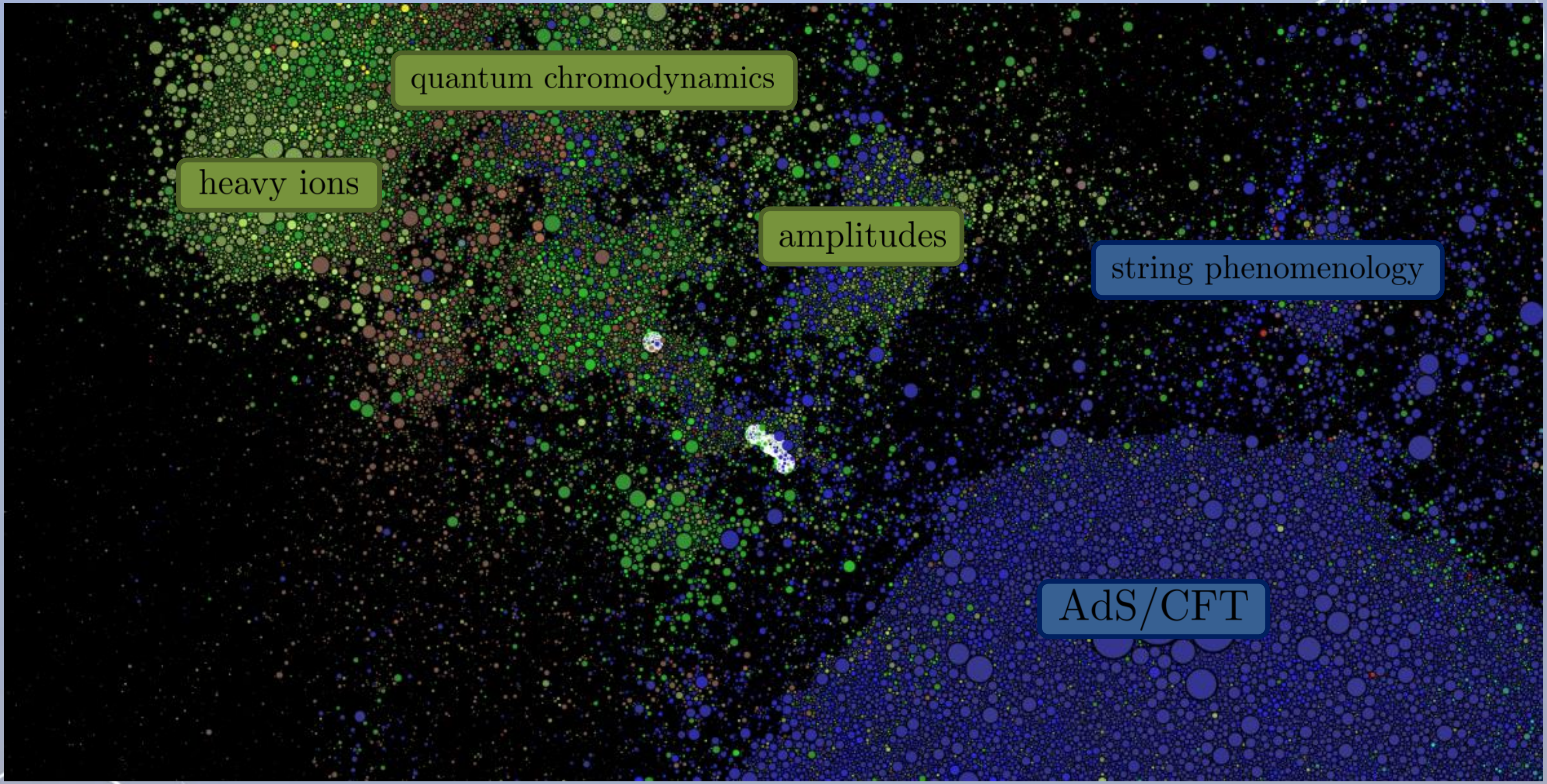
The holographic pathway



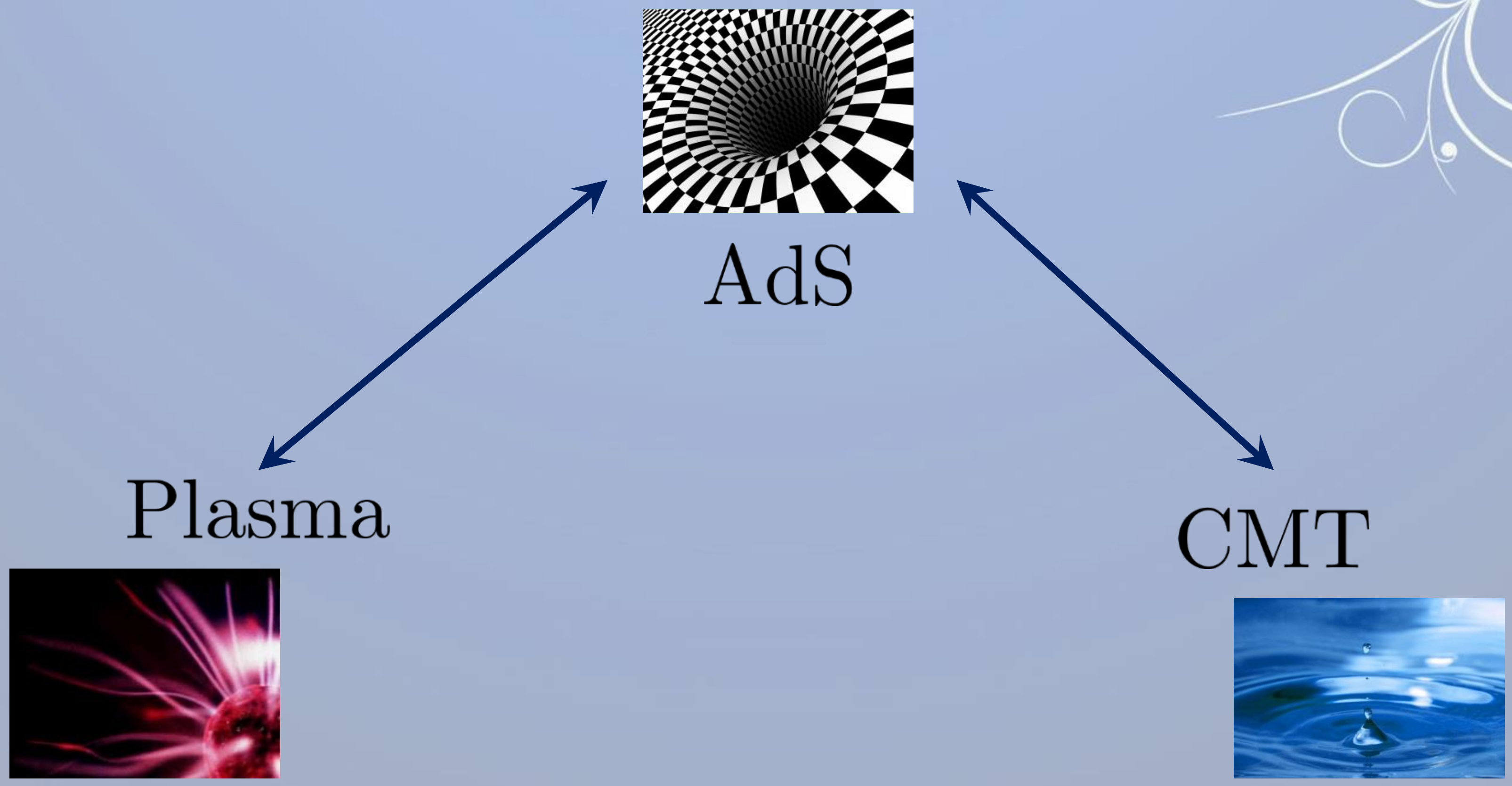
Where do I come from



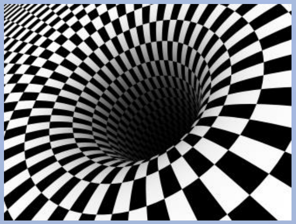
Where do I come from... in the paperscape



A diagram of my research



A diagram of my research



- Not fixed AdS

AdS

Numerical techniques

Plasma

- Probes
- Anisotropy

CMT

- Hydro description
- Something else

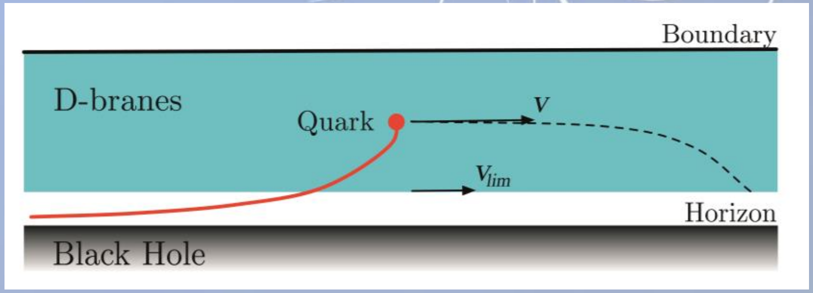
(J. Casalderrey-Solana, M. Chernicoff, DF, D. Mateos, D. Trancanelli, '09-12)

Holographic Plasmas

- **Emission of Cherenkov radiation**

String endpoint moving sufficiently fast radiates mesons via Cherenkov.

- Energy loss $\sim \mathcal{O}(N_c)$ effect.
- Universal mechanism!
- Exactly calculable for $\mathcal{N} = 4$ SYM plasma.



The study of probes is especially useful: they're produced within the same collision that produces the strongly coupled plasma itself.

(Backreaction ignored in the $N_f \ll N_c$ limit)

- **A study of anisotropy**

QGP created in HIC is time dependent and anisotropic:

Anisotropy might play an important role.
This was investigated for several observables.

Holographic Plasmas

- **Thermodynamics of anisotropic brane**

Probe limit: $\epsilon = \frac{N_f}{N_c}$ small.

But asymptotics of metric and embedding are different for $\epsilon = 0$ and for $\epsilon \neq 0$.

$\Longrightarrow$ UV limit does not commute with probe limit.
$\Longrightarrow$ Trouble with counterterms!

- Historical motivation for **holographic renormalization**: Finiteness of the on-shell action.

(I. Papadimitriou, '10)

- Actual motivation: Requirement that the variational problem at infinity be well defined.

Condition that determines S_b : $\frac{\mathrm{d}}{\mathrm{d}r}(S + S_b) \xrightarrow[r\to\infty]{} 0$

which amounts to a phase space transf.:

$$\begin{pmatrix} \pi \\ \phi \end{pmatrix} \longrightarrow \begin{pmatrix} \Pi \\ \Phi \end{pmatrix} = \begin{pmatrix} \pi + \frac{\delta S_b}{\delta \phi} \\ \phi \end{pmatrix}$$

additional requirement that the symplectic form must be preserved:

$$\Omega = \int \mathrm{d}^d x \; \delta\pi \wedge \delta\phi$$

Induced gravity at the boundary

FG-type expansion: $g_{\mu\nu} = g^{(0)}_{\mu\nu} r^2 + g^{(1)}_{\mu\nu} r + \dots$ *(D. Marolf, '08)*

Formal construction: Promoting $g^{(0)}_{\mu\nu}$ to a dynamical field, define "induced gravity" partition function:

$$Z_{\text{bulk}} = \int \mathcal{D}g^{(0)}\, Z^{\text{Dir}}_{\text{bulk}}\left[g^{(0)}\right] = \iint \mathcal{D}\Phi\, \mathcal{D}g\; e^{iS_{\text{bulk}}}$$

$$\left(Z^{\text{Dir}}_{\text{bulk}}\left[g^{(0)}\right] = \iint_{g\to g^{(0)}} \mathcal{D}\Phi\, \mathcal{D}g\; e^{iS_{\text{bulk}}} \right)$$

In the semi-classical limit,

$$\delta S_{\text{bulk}} = \text{EOM} + 2\int T^{\mu\nu}\, \delta g^{(0)}_{\mu\nu} \sqrt{g^{(0)}}$$

bdry. stress tensor

Natural bdry. cond. are such that $g^{(0)}$ free $\Rightarrow T_{\mu\nu} = 0$.

Claim: This theory corresponds to the induced gravity on the CFT.

$\dfrac{\delta S_{\text{CFT}}}{\delta g_{\text{CFT}}} = T_{\mu\nu} = 0$ is the EOM expected in a theory with no EH term.

This corresponds to the limit $G_{\text{N}} \to \infty$.

(J. Erdmenger, DF, H. Zeller, '12)

Holographic Superfluids

- **p-wave transport coefficients**

Holography is useful to study transport properties of strongly correlated systems at finite temperature.

Take SU(2) Einstein-Yang-Mills theory in the bulk:

$$S = \frac{1}{2\kappa_5^2}\int d^5x\,\sqrt{-g}\left[R - \Lambda - \frac{\alpha^2}{2}F^a_{MN}F^{aMN}\right] + S_{\text{bdy}}$$

with ansatz

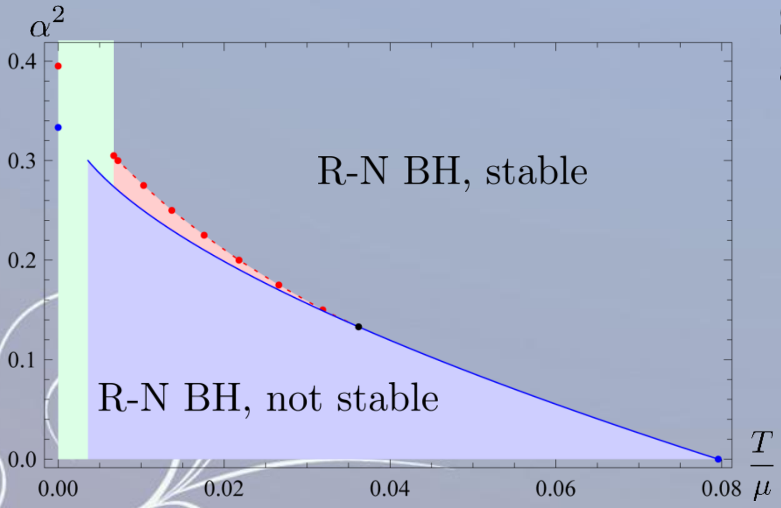
$$A = \phi(r)\,\tau^3\,dt + w(r)\,\tau^1\,dx$$

chemical potential

$$\phi(r) \to \mu$$

spontaneous value acquired in broken phase:

$$w(r) \to w_1^b/r^2\,,\;\; w_1^b \propto \langle J_1^x\rangle \neq 0$$



Study perturbations around this background and relations between sources and vevs to find

$$\begin{pmatrix}\langle J^y\rangle\\ \langle Q^y\rangle\end{pmatrix} = \begin{pmatrix}\sigma^{yy} & T\alpha^{yy}\\ T\alpha^{yy} & T\bar{\kappa}^{yy}\end{pmatrix}\begin{pmatrix}E_y\\ -(\nabla_y T)/T\end{pmatrix}$$

transport coefficients and viscosities

$$\eta_{yz} = -\lim_{\omega\to 0}\frac{1}{\omega}\text{Im}\,\langle T_{yz}T_{yz}\rangle$$

$$\eta_{xy} = -\lim_{\omega\to 0}\frac{1}{\omega}\text{Im}\,\langle T_{xy}T_{xy}\rangle$$

Holographic Superfluids

- **p-wave hydrodynamics**

- Spontaneous symmetry breaking of cotinuous symmetry
- Nambu-Goldstone boson in the spectrum
- New hydrodynamic mode:

$$\{\mu,\, T,\, u_\mu\},\ v_\mu = \partial_\mu \varphi$$

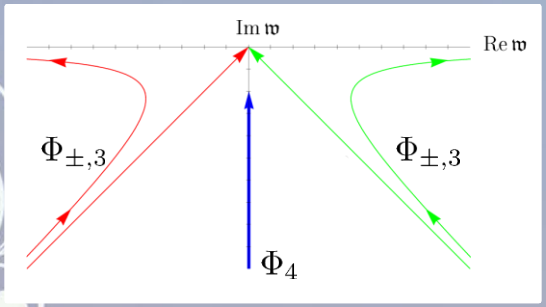
$$\Psi = |\Psi|\, e^{\mathrm{i}\varphi}$$

Eqs. of relativistic hydrodynamics,
derivable from general considerations, consistent with:

(J. Bhattacharya, S. Bhattacharyya, S. Minwalla, A. Yarom, '11)

- Lorentz invariance → $J^\mu_{\mathrm{S}} = J^\mu_{\mathrm{S\ canon}} + \sum_i v_i \mathcal{V}^\mu_i$
- 2^{nd} law of thermodynamics → $\partial_\mu J^\mu_{\mathrm{S}} \geq 0$
- Covariance under time reversal.

Phase φ is coherent superposition in the s-wave condensate. In our case,



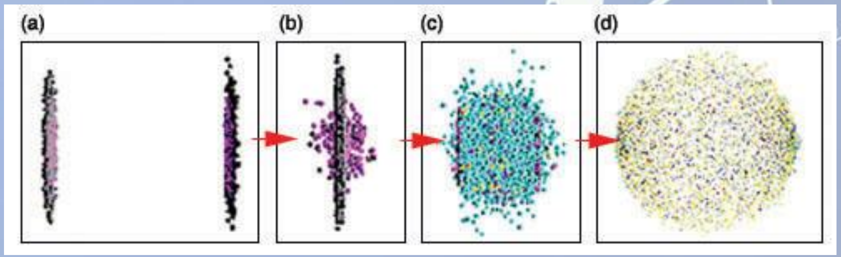
$$U(1)_3 \xrightarrow[\mathrm{SSB}]{} \mathbb{Z}_2$$

$$SO(3) \xrightarrow[\mathrm{SSB}]{} SO(2)$$

3 Goldtone modes $\Longrightarrow$ different hydrodynamics.

Holographic Thermalization

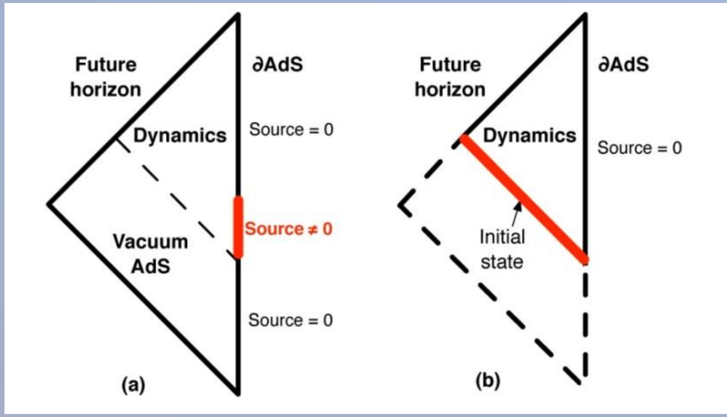
Far from equilibrium evolution of plasma $\Leftrightarrow$ Relaxation of black hole



There are 2 alternatives:

a) Create ffe state on the CFT, with external anisotropic source. *(P. Chesler, L. Yaffe '07)*

b) Study isotropization of initial state in absence of external sources. *(M. Heller, D. Mateos '12)*



Simple ansatz:

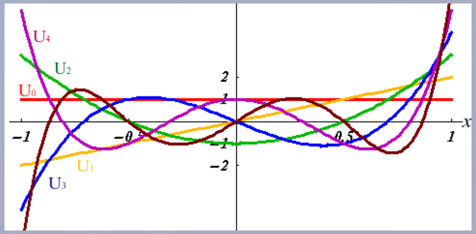
$$ds^2 = 2dt\,dr - A\,dt^2 + \Sigma^2 e^{-2B}\,dx_{\rm L}^2 + \Sigma^2 e^{B}\,d\vec{x}_{\rm T}^2$$

$$\langle T_{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P_{\rm L}(t) & 0 & 0 \\ 0 & 0 & P_{\rm T}(t) & 0 \\ 0 & 0 & 0 & P_{\rm T}(t) \end{pmatrix}$$

Numerical implementation:

1. Spatial grid on holographic direction: pseudo-spectral collocation method.

2. Time evolution via finite differences. (taking care of AH position)



Holographic Thermalization

- **Validity of hydro approximation out of eq.**

Linearizing Einstein's equations around the final equilibrium state reproduces the expectation value of the boundary stress tensor with a 20% accuracy.

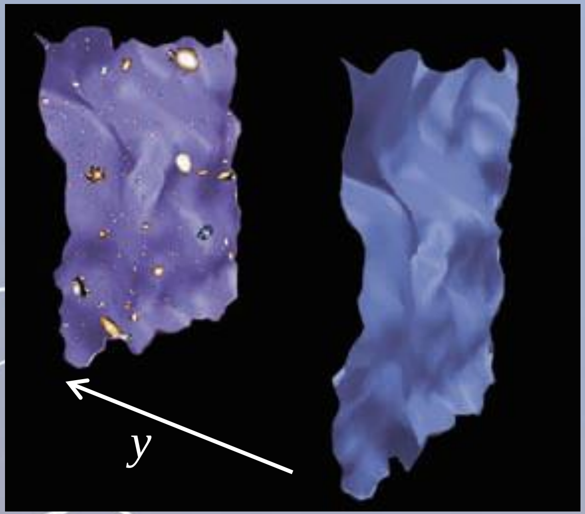
It would be interesting to understand the reason behind this relative precision.

- **Giving initial data**

(P. Chesler, L. Yaffe '10)

Initial data must describe two well-separated planar shocks, moving toward each other.

One shock would be $ds^2 = r^2 \left[-dx_+ \, dx_- + dx_\perp^2\right] + \frac{1}{r^2}\left[dr^2 + h(x_\pm) dx_\pm^2\right]$,
with h a gaussian.



- **Generalization to inhomogeneous case**

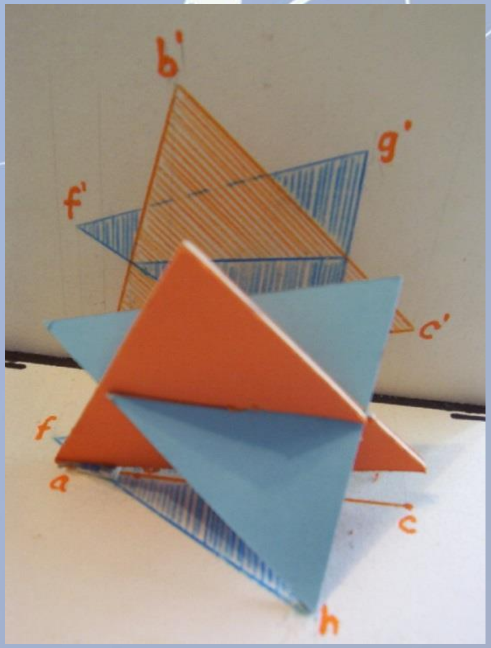
Periodic function in transverse direction:

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{BACKG}}(r,t,y) + e^{\mathrm{i}\,k\,x}\delta g_{\mu\nu}(r,t,y)$$

1. Spatial grid on r: pseudo-spectral method.
2. Spatial grid on y: Fourier spectral method.
3. Time evolution on t: via finite differences.

Numerical Techniques in AdS/CFT

- **Brane intersection solution in Supergravity**



(S. Cherkis, A. Hashimoto '02)

Problem: Find gravity dual to
3 dim. supersym. $SU(N_2)$ gauge th. with N_6 fundamentals.

M-theory uplift of N_6 coincident D6-branes at $r = 0$:

$$\mathbb{R}^7 \times \text{Taub-NUT} \Longrightarrow \text{M2 in this background?}$$

Ansatz for M2-branes spanning t, x_1, x_2:

$$ds^2 = H^{-2/3}\left(-dt^2 + dx_1^2 + dx_2^2\right) + H^{1/3}\left(dy^2 + y^2 d\Omega_3^2 + ds^2_{\text{TN}}\right)$$

$$F_{[4]} = dt \wedge dx_1 \wedge dx_2 \wedge dH^{-1}$$

Taub-NUT metric

Fully backreacted solution found via $H(z, y)$.

Generalization to non-extremal case: include additional functions $\mathbf{f_1}, \mathbf{f_2}, \tilde{\mathbf{H}}$.

(This has not been found, but numerically it could be doable)

- **...and many analogous problems in AdS/CMT.**

Thank you!