Universal scaling properties of holographic cohesive phases

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Based on [1308.2084]
and on [1107.2116], [1212.2625] with E. Kiritsis
and [1005.4690] with C. Charmousis, B.S. Kim, E. Kiritsis and R. Meyer

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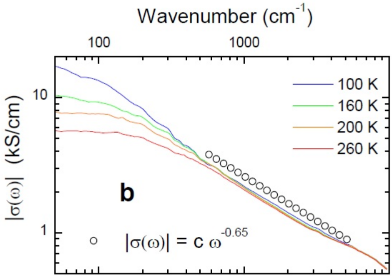
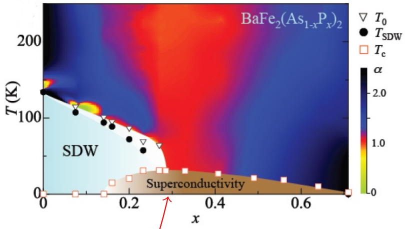
Scaling in high critical temperature superconductors



[van der Marel&al'03]



[Kasahara&al'10]

- **Non-Fermi Liquid excitations** (no weakly-coupled quasiparticles).
- Interesting scaling behaviour of transport coefficients:

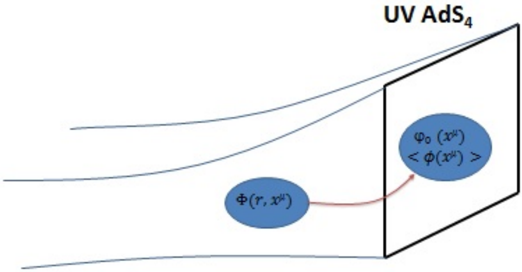
$$\sigma_{DC}^{-1} \sim T \quad (T \ll \mu)\,, \qquad Re\,(\sigma_{AC}) \sim \omega^{-2/3} \quad (T \lesssim \omega \ll \mu)$$

- A Quantum Critical Point at strong coupling is conjectured to be responsible for these scaling properties.
  Can holography help to understand the nature of QCPs at strong coupling and their transport properties?

Mapping the holographic quantum critical landscape

Holography enables us to describe strongly-coupled phases of matter with a UV conformal fixed point (though strong effort to generalise to other UV asymptotics).

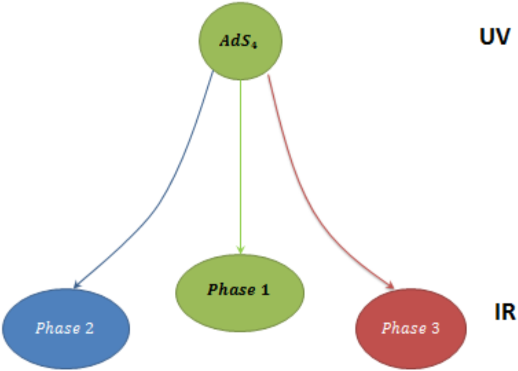
It gives a clear prescription to do so: asymptotics of bulk fields are mapped to sources and vevs of the dual field theory.



This picture however does not constrain the IR of the theory much, which might display **universal scaling behaviour** and give us information about the ground state.

Mapping the holographic quantum critical landscape

In particular, **many phases might be competing** in the IR.



How do we characterise these phases?
How do we determine the dominant one, **the ground state**?

To determine the ground state, it is particularly important to have a reliable map of the possible IR phases, the flows to the UV as well as possible quantum phase transitions.

Outline

1. Philosophy of the classification
2. Classes of solutions
3. Stability of RG flow
4. Optical conductivity and spectrum
5. Charge transport and holographic lattices

Mapping the holographic quantum critical landscape

One tool we can use is the concept of **effective holographic theories**, [Charmousis, Goutéraux, Kim, Kiritsis & Meyer '10]:

- introduce a minimal set of operators irrelevant in the UV but which will drive the IR dynamics (vector, scalar, etc.);
- write down an effective action describing these IR dynamics;
- figure out possible (extremal) IR phases, according to their symmetries: zero temperature, scaling backgrounds;

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Mapping the holographic quantum critical landscape (2)

- work out the nature of the deformations around them: a relevant nonzero temperature deformation must exist; enough irrelevant deformations are needed to connect to the UV;
- construct flows to the UV (sometimes analytically, more often numerically);
- determine the most stable in various regions of the phase diagram (thermal/quantum phase transitions with/without explicit/spontaneous symmetry breaking).

Symmetries: hyperscaling

- We keep **translation and rotation invariance** (for now).

- Break Poincaré symmetry,
  but retain scaling symmetries $t \to \lambda^z t$, $x^i \to \lambda x^i$

  $$\mathrm{d}s^2 = -r^{-2z}\mathrm{d}t^2 + L^2 r^{-2}\mathrm{d}r^2 + r^{-2}\mathrm{d}\vec{x}^2_{(d)}$$

  which are supported by p-forms, massive vector fields or runaway scalars [Kachru&al'08, Taylor'08, Goldstein&al'09].

  $z = 1$: AdS_4;
  $z \to +\infty$: $AdS_2 \times R^2$;
  $z < +\infty$ Lifshitz.

  $\Rightarrow$ Hyperscaling solutions: $S \sim T^{\frac{d}{z}}$

Symmetries (2): hyperscaling violation

- Break scale invariance in the metric Ansatz

$$\mathrm{d}s^2 = r^{\frac{2}{d}\theta}\left(-r^{-2z}\mathrm{d}t^2 + L^2 r^{-2}\mathrm{d}r^2 + r^{-2}\mathrm{d}\vec{x}^2_{(d)}\right)$$

This metric is only covariant under $t \to \lambda^z t$, $x^i \to \lambda x^i$

There is an **effective spatial dimensionality** $d_\theta = d - \theta$ such that:

$$S \sim T^{\frac{d-\theta}{z}} \sim T^{\frac{d_\theta}{z}}$$

hyperscaling violation [Goutéraux&Kiritsis'11, Huijse&al'11, Dong&al'12]

Using KK lifts, d_θ can be traced back to the higher-dimensional spacetime [Goutéraux&Kiritsis'11].

The effective holographic action

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[R - \partial\phi^2 - Z(\phi)F^2 + V(\phi) + A_\mu J^\mu_{eff}(A_\nu,\phi)\right]$$

- Contains gravity, a gauge field (finite density) and a neutral scalar [Charmousis, Goutéraux, Kim, Kiritsis & Meyer'10].
- The effective scalar potential has several competing terms

$$V_{eff}(\phi) = V(\phi) - Z(\phi)F^2 + A_\mu J^\mu_{eff}(A_\nu,\phi)$$

- The scalar field can either **settle to a constant** extremizing V_{eff}: hyperscaling solutions
- Or display **logarithmic running**, and then the scalar couplings can be approximated by exponentials (for instance)

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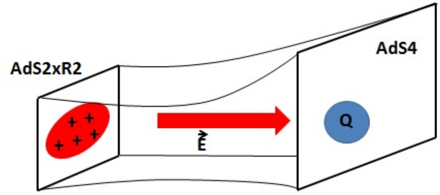
Cohesion/Fractionalisation in Holography

Zero density, [Witten'98]: **Event horizon ⇔ Deconfinement**

Finite density, [Liu&al'11,Hartnoll'11]:

Charged horizon ⇔ **Fractionalisation**

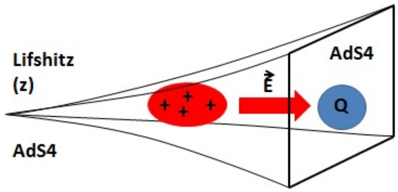
Separate contributions to the boundary charge density



Reissner-Norström black hole

[MIT, Leiden'09]

Fractionalised phase



Electron star [Hartnoll&al'10],

Superfluid [Gubser&Nellore, Horowitz&Roberts'09]

Cohesive phase

How to source charge density in the bulk

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[R - \partial\phi^2 - Z(\phi)F^2 + V(\phi) + A_\mu J^\mu_{eff}(A_\nu,\phi)\right]$$

- Effective source to the right hand side of Gauss's law:

$$\nabla_\mu\left(Z(\phi)F^{\mu\nu}\right) = J^\nu_{eff}(A_\mu,\phi)$$

which might break or not the U(1) symmetry.

- $A_\mu J^\mu_{eff} \sim W(\phi)A^2$, massive vector fields, effective description of **holographic superfluids** [Goutéraux&Kiritsis'12];
- $A_\mu J^\mu_{eff} \sim -\tau(\phi)p(\mu_{loc})$ describing a **charged ideal fluid of fermions** in the Thomas-Fermi limit [Hartnoll&al'10];
- $A_\mu J^\mu_{eff} \sim \vartheta(\phi)F\wedge F$, **Chern-Simons coupling** [Donos&Gauntlett'11]

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IR dynamics: relevant vs irrelevant operators

$$S = \int \mathrm{d}^{d+2}x\sqrt{-g}\left[R - \partial\phi^2 - Z(\phi)F^2 + V(\phi) + A_\mu J^\mu_{eff}(A_\nu, \phi)\right]$$

The behaviour of the IR phase under scaling actions is determined by the dimension of the IR operators

- A **relevant current** breaks Poincaré symmetry $\Rightarrow$ time and space are anisotropic $z \neq 1$
  In [Gubser&Nellore'09], interplay between AdS_4 and Lifshitz IR for the superfluid phase.

- A **relevant scalar** operator breaks scale invariance $\Rightarrow$ hyperscaling violation with $\theta \neq 0$, along with a runaway scalar in the IR.

- A **relevant source** for the current breaks the conservation of the electric flux $\Rightarrow$ cohesive phases. Let us introduce a 'cohesion' exponent:

$$\int_{\mathbf{R}^d} Z(\phi) \star F \underset{IR}{\sim} r^\xi$$

The final ingredient: the conduction exponent

Up until now, we have discussed the scaling behaviour of the metric, the scalar and the electric flux. There is a final ingredient, related to the scaling of the electric component of the vector

$$A_t \sim r^{\zeta-\xi-z}\,\mathrm{d}t$$

ζ is the conduction exponent (for reasons shortly apparent).

For fractionalised phases $\xi = 0$, it parameterizes the violation of the Lifshitz scaling $t \to \lambda t^z$, $x^i \to \lambda x^i$ by A_t.

For cohesive phases $\xi \neq 0$, ξ also participates in the violation of the Lifshitz scaling.

In that sense, it has a role **similar to** θ for the metric.

Overall classifications of solutions

Whether (partially) fractionalised ($\xi = 0$) or not ($\xi \neq 0$), hyperscaling (θ=0) or not ($\theta \neq 0$), with or without a runaway scalar, solutions organise themselves into two classes:

- The current is relevant in the IR: dynamical exponent z is arbitrary but the conduction exponent $\zeta = -d_\theta$.
- The current is irrelevant in the IR: dynamical exponent $z = 1$, but the conduction exponent ζ is arbitrary.

Details can be found in in

[Goutéraux&al'10,Goutéraux&Kiritsis'11,'12,Goutéraux'13]

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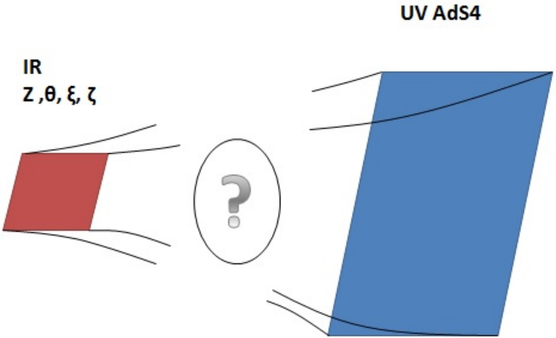
Question

Given that

- the form of the metric is universal in terms of θ and z
- the temperature scaling of entropy/free energy also depends only on θ and z
- the entanglement entropy only knows about θ

Can we find observables which are sensitive to the origin of the boundary charge density, e.g. to the behaviour of the electric flux (ξ) and gauge field (ζ)?

From the IR to the UV



Recipe: perturb around the zero temperature solution with purely radial, static deformations, and work out the modes.

$$\mathrm{d}s^2 = (\mathrm{d}s^2)_{(0)} \left(1 + \eta \left(\frac{r}{r_\beta}\right)^\beta\right) , \qquad \eta \ll 1$$

Irrelevant vs relevant modes

They can be of two kinds

- relevant: they grow towards the IR, and if turned on prevent from reaching the IR geometry.
- irrelevant: they decay towards the IR, and allow to shoot out to the UV.

This is equivalent to working out whether the IR geometry is a **stable 'fixed point' of the RG flow** or not.

They come by **pairs** $\beta_\pm$, and correspond (loosely) to the insertion in the effective IR field theory of

$$\int \mathrm{d}^{d_\theta} x \,\mathrm{d}t\, g_{\mathcal{O}}\, \mathcal{O}$$

Remember the scaling $t \to \lambda^z t$, $\vec{x} \to \lambda \vec{x}$.
By dimensional analysis, we expect

$$\beta_+ + \beta_- = d + z - \theta = d_\theta + z$$

IR scaling dimensions

There is always **a universal pair**: $\beta_0 = 0$ (rescalings of time) and $\beta_u = d_\theta + z$ (nonzero temperature).

- $z \neq 1$, $\zeta = -d_\theta$: all other pairs read

$$\beta^i_\pm = \frac{1}{2}(d_\theta + z) \pm \frac{1}{2}\nu(z, \theta, \xi)$$

with $\nu(z, \theta, \xi)$ a nonuniversal piece.

- $z = 1$, $\zeta \neq -d_\theta$:
  1. Massive vectors: same as $z \neq 1$
  2. Electron stars: $\beta^{el}_+ + \beta^{el}_- = \zeta - 1$
  3. Chern-Simons: $\beta^{el}_+ + \beta^{el}_- = 2 + \zeta - \theta$

Pair of modes with anomalous dimension

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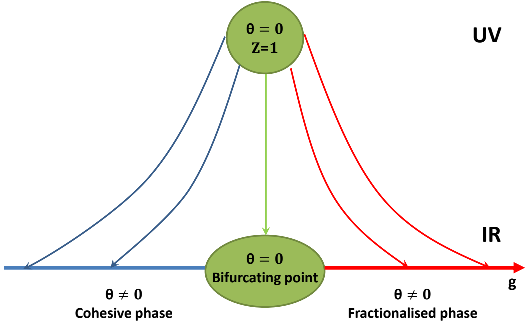
Quantum fractionalisation transitions

[Hartnoll&Huijse'11], [Sonner,Withers&al'12], [Goutéraux&Kiritsis'12]



If there is a **scale invariant fixed point** $(\theta = 0)$ with a **relevant deformation**, there is a bifurcation in the RG flow: to reach this point from the UV, the flow must be fine tuned.

Away from the critical value, the flow picks up the **relevant deformation** and lands into a collection of stable hyperscaling violation fixed points: **a quantum critical line.**

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The electric perturbation problem $\equiv$ Schrödinger problem

Turn on a small electric field along x on the boundary:

$$A_x \sim a_x(r)e^{-i\omega t}\,, \qquad \vec{k} = 0$$

This usually couples to other perturbations of the metric and other fields which couple to the vector field.

With a little work, the linearised, perturbed equations can be decoupled [Horowitz&Roberts'09, Goldstein &al'09, Charmousis&al'10, Hartnoll&Tavanfar'11...] and take the form of a **one-dimensional Schrödinger equation**

$$-\frac{d^2\tilde{a}_x}{\mathrm{d}\rho^2} + \tilde{V}\tilde{a}_x = \omega^2\tilde{a}_x\,, \qquad \tilde{V}_{IR} = \frac{\tilde{V}_0}{\rho^2}$$

Only when no magnetic fields.

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Spectrum of electric fluctuations

By examining the behaviour of $\tilde{V}$, one can determine if the spectrum is gapless or gapped.

In the UV, $\tilde{V} \to +\infty$

- if $d > 2$;
- if $d = 2$ and $1/2 < \Delta_\phi < 1$.

If $\tilde{V} \to +\infty$ also in the IR, the spectrum is gapped.

- $\tilde{V} \to \pm\infty$ in the IR iff $\rho \to 0 \Rightarrow z\, d_\theta < 0$.

- In the (locally) **thermodynamically stable** region $d_\theta / z > 0$ this never happens, the spectrum is **always gapless** and the system a conductor.

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AC conductivity scaling

The conductivity can now be written as a reflexion coefficient in the Schrödinger potential.

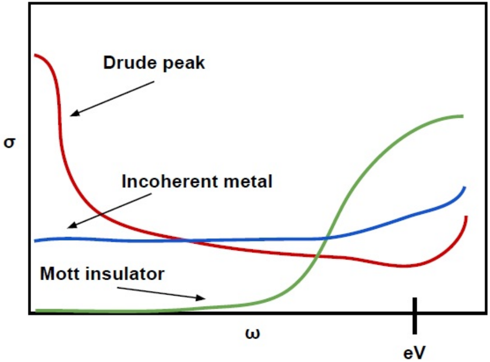
Net amount of charge + conservation of momentum = $\delta(\omega)$

- $z \neq 1$, $\zeta = -d_\theta$: $\quad Re(\sigma) \sim \omega^{|3-\frac{2-d_\theta}{z}|-1}$
- $z = 1$, $\zeta \neq -d_\theta$: $\quad Re(\sigma) \sim \omega^{|1-\zeta|-1}$

These exponents are always positive when the system is gapless, negative when it is gapped.

It is very tempting to conjecture that

$$z \neq 1\,,\ \zeta \neq -d_\theta : \qquad Re(\sigma) \sim \omega^{|3-\frac{2+\zeta}{z}|-1} \quad ?$$



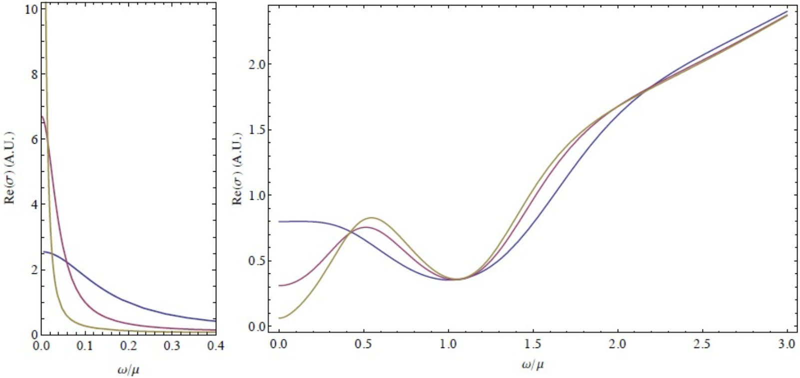
[DONOS&HARTNOLL'12]

- **Metals**: translation invariant IR ground states with irrelevant lattice deformations (UV lattice) The resistivity decreases with the temperature, the optical conductivity displays a Drude peak.
- **Insulators**: localized phases, IR ground state has a relevant lattice and the resisitivity increases when T decreases, while the optical conductivity vanishes.
- **Incoherent metals**: conduct at low ω, no well-defined Drude peak

Holographic realization [Donos&Hartnoll'12]

Metallic phase: $AdS_2 \times \mathbf{R}^3$ with irrelevant lattice deformations.

Insulating phase: Bianchi VII spatial symmetry (ODEs)



Continuous phase transition between the metallic (high pitch) and insulating phase (low pitch), simulatneously quantum fractionalisation phase transition.

Ongoing work with A. Donos and E. Kiritsis

- What controls the insulating behaviour from a geometrical perspective?
- Can we have more freedom in the scaling of the transport observables?
- Can this be detuned from a fractionalisation transition?

Ongoing work with A. Donos and E. Kiritsis:
Write down an effective holographic action for a running scalar, no Chern-Simons term, 2 gauge fields

4 scaling exponents:
θ, z (anisotropy between time and axis of the helix), ζ and z_2 (spatial anisotropy between the plane and the axis of the helix)

$$\mathrm{d}s^2 = r^{\frac{2\theta}{3}}\left[-\frac{\mathrm{d}t^2}{r^{2z}} + \frac{L^2\mathrm{d}r^2 + \omega_1^2}{r^2} + \frac{\omega_2^2 + \lambda r^{-2}\omega_3^2}{r^{2z_2}}\right]$$

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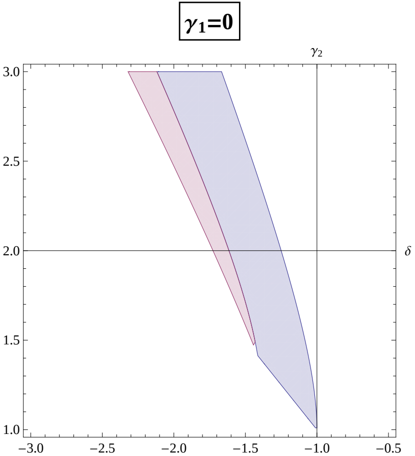
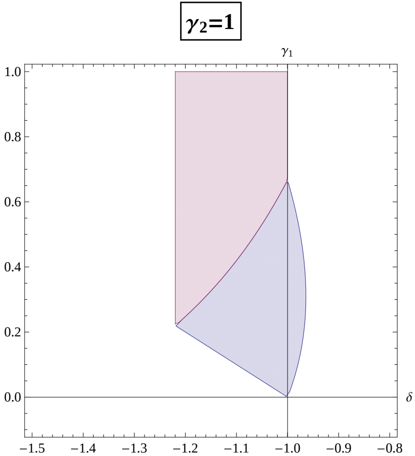
Phases with (ir)relevant current at zero temperature



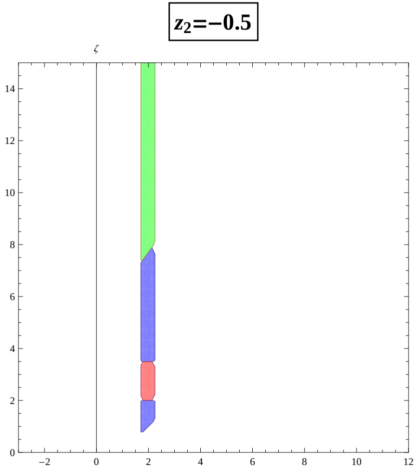
Blue region: relevant current (fixed ζ)



Red region: irrelevant current (fixed z)

Incoherent metals and insulators at zero temperature

Phases with relevant currents are always insulators.



Phases with irrelevant currents can be insulators (blue, green) or incoherent metals (red)

Summary and outlook

- Unified framework for translation-invariant extremal backgrounds, whether scale invariant or hyperscaling violating: **'cohesion' and 'conduction' exponents**.

- Scaling dimensions of IR operators typically depend on z, θ, ζ and ξ, can sum anomalously, can trigger quantum fractionalisation transitions.

- $$z \neq 1\,, \zeta \neq -d_\theta \qquad Re(\sigma) \sim \omega^{|3-\frac{2+\zeta}{z}|-1>0} \quad ?$$

- Local thermodynamic stability $\Rightarrow$ gapless spectrum

- How do such scaling exponents show up in other observables?

- Generalisation to phases breaking translation invariance? Ongoing for Bianchi VII, which has interesting phenomenology.