ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Higgs-Dilaton Cosmology: From the Early to the Late Universe

Mikhail Shaposhnikov

Heraklion, 8 October 2012

Outline

- ETOE
- Dilaton-Higgs Cosmology
- Higgs mass, stability, inflation and asymptotic safety
- Conclusions

An alternative to SUSY, large extra dimensions, technicolor, etc

Effective

Theory

Of

Everything

Definitions

"Effective": valid up to the Planck scale, quantum gravity problem is not addressed. No new particles heavier than the Higgs boson.

"Everything":

- neutrino masses and oscillations
- dark matter
- baryon asymmetry of the Universe
- inflation
- dark energy

Particle content of ETOE

Particles of the SM

+

graviton

+

dilaton

+

3 Majorana leptons

Symmetries of ETOE

- gauge: SU(3)$\times$SU(2)$\times$U(1) – the same as in the Standard Model

Symmetries of ETOE

- Restricted coordinate transformations: TDIFF, $det[-g] = 1$ (Unimodular Gravity).

Equations of motion for Unimodular Gravity:

$$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = 8\pi G_N (T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T)$$

Perfect example of “degravitation" - the “$g_{\mu\nu}$" part of energy-momentum tensor does not gravitate. Solution of the “technical part" of cosmological constant problem - quartically divergent matter loops do not change the geometry. But - no solution of the “main" cosmological constant problem - why $\Lambda \ll M_P^4$? Scale invariance can help!

Symmetries of ETOE

- Exact quantum scale invariance
  - No dimensionful parameters
  - Cosmological constant is zero
  - Higgs mass is zero
  - these parameters cannot be generated radiatively, if regularisation respects this symmetry
- Scale invariance must be spontaneously broken
  - Newton constant is nonzero
  - W-mass is nonzero
  - Λ_{QCD} is nonzero

Lagrangian of ETOE

Scale-invariant Lagrangian

$$\mathcal{L}_{\nu\mathrm{MSM}} = \mathcal{L}_{\mathrm{SM[M\to 0]}} + \mathcal{L}_G + \frac{1}{2}(\partial_\mu \chi)^2 - V(\varphi,\chi)$$
$$+ \left(\bar{N}_I i\gamma^\mu \partial_\mu N_I - h_{\alpha I}\, \bar{L}_\alpha N_I \tilde{\varphi} - f_I \bar{N_I}^c N_I \chi + \mathrm{h.c.}\right)\ ,$$

Potential (χ - dilaton, φ - Higgs, $\varphi^\dagger\varphi = 2h^2$):

$$V(\varphi,\chi) = \lambda\left(\varphi^\dagger\varphi - \frac{\alpha}{2\lambda}\chi^2\right)^2 + \beta\chi^4,$$

Gravity part

$$\mathcal{L}_G = -\left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger\varphi\right)\frac{R}{2}\ ,$$

For $\lambda > 0, \ \ \beta = 0$ the scale invariance can be spontaneously broken. The vacuum manifold:

$$h_0^2 = \frac{\alpha}{\lambda}\chi_0^2$$

Particles are massive, Planck constant is non-zero:

$$M_H^2 \sim M_W \sim M_t \sim M_N \propto \chi_0, \ \ M_{Pl} \sim \chi_0$$

Phenomenological requirement:

$$\alpha \sim \frac{v^2}{M_{Pl}^2} \sim 10^{-38} \lll 1$$

Absence of gravity: the only choice leading to interacting particles is $\beta = 0$. With gravity this argument is lost. Still, the choice of $\beta = 0$ will be made.

Roles of different particles

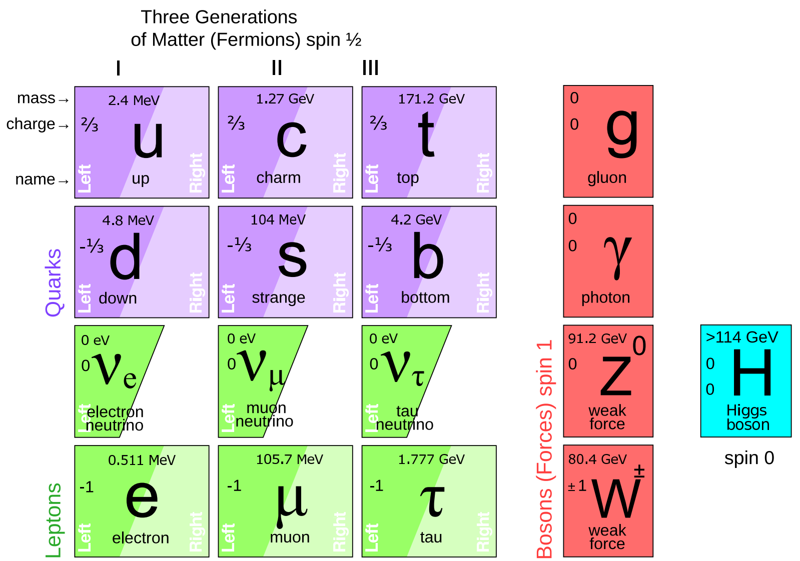
The roles of dilaton:

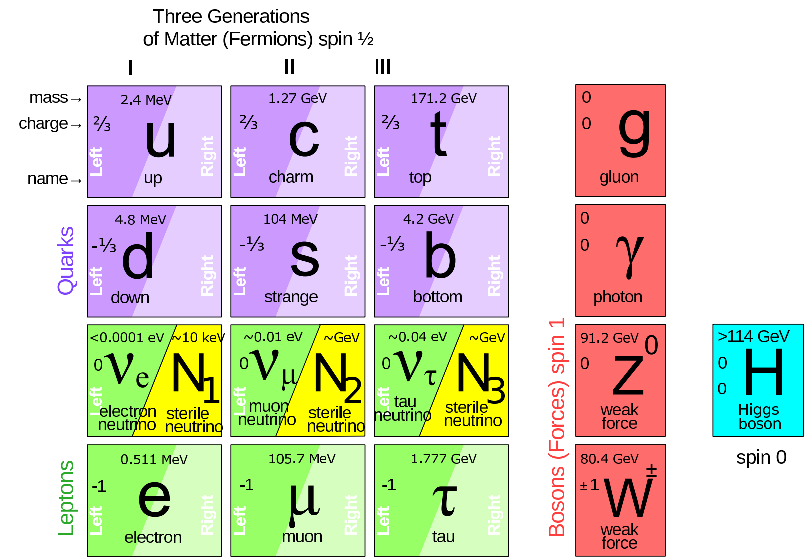
- determine the Planck mass
- give mass to the Higgs
- give masses to 3 Majorana leptons
- lead to dynamical dark energy
- Note: dilaton is a Goldstone boson of broken dilatation symmetry
  $\Longrightarrow$ only derivative couplings to matter, no fifth force!

Roles of the Higgs boson:

- give masses to fermions and vector bosons of the SM
- provide inflation

New fermions: the νMSM





Role of N_1 with mass in keV region: dark matter

Role of $N_2,\ N_3$ with mass in 100 MeV – GeV region: “give” masses to neutrinos and produce baryon asymmetry of the Universe

The couplings of the νMSM

Particle physics part, accessible to low energy experiments: the νMSM. Mass scales of the νMSM:

$M_I < M_W$ (No see-saw)

Consequence: small Yukawa couplings,

$$F_{\alpha I} \sim \frac{\sqrt{m_{atm} M_I}}{v} \sim (10^{-6} - 10^{-13}),$$

here $v \simeq 174$ GeV is the VEV of the Higgs field,

$m_{atm} \simeq 0.05$ eV is the atmospheric neutrino mass difference.

Small Yukawas are also necessary for stability of dark matter and baryogenesis (out of equilibrium at the EW temperature).

Scale invariance + unimodular gravity

Solutions of scale-invariant UG are the same as the solutions of scale-invariant GR with the action

$$S = -\int d^4x\sqrt{-g}\left[\left(\xi_\chi \chi^2 + 2\xi_h \varphi^\dagger \varphi\right)\frac{R}{2} + \Lambda + ...\right] ,$$

Physical interpretation: Einstein frame

$$g_{\mu\nu} = \Omega(x)^2 \tilde{g}_{\mu\nu} \ , \ \ (\xi_\chi \chi^2 + \xi_h h^2)\Omega^2 = M_P^2$$

Λ is not a cosmological constant, it is the strength of a peculiar potential!

Relevant part of the Lagrangian (scalars + gravity) in Einstein frame:

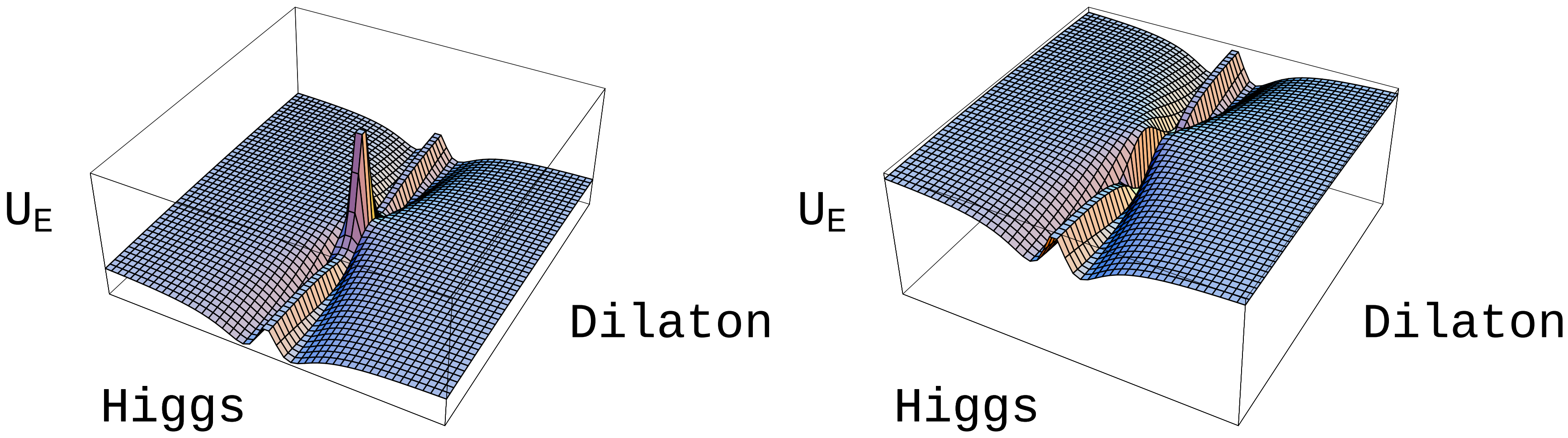
$$\mathcal{L}_E = \sqrt{-\tilde{g}} \left(-M_P^2 \frac{\tilde{R}}{2} + K - U_E(h,\chi) \right) ,$$

K - complicated non-linear kinetic term for the scalar fields,

$$K = \Omega^2 \left(\frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2}(\partial_\mu h)^2 \right) - 3M_P^2 (\partial_\mu \Omega)^2 .$$

The Einstein-frame potential $U_E(h,\chi)$:

$$U_E(h,\chi) = M_P^4 \left[\frac{\lambda \left(h^2 - \frac{\alpha}{\lambda}\chi^2 \right)^2}{4(\xi_\chi \chi^2 + \xi_h h^2)^2} + \frac{\Lambda}{(\xi_\chi \chi^2 + \xi_h h^2)^2} \right] ,$$



Potential for the Higgs field and dilaton in the Einstein frame.

Left: $\Lambda > 0$, right $\Lambda < 0$.

50% chance ($\Lambda < 0$): inflation + late collapse

50% chance ($\Lambda > 0$): inflation + late acceleration

Inflation

Chaotic initial condition: fields χ and h are away from their equilibrium values.

Choice of parameters: $\xi_h \gg 1, \;\; \xi_\chi \ll 1$ (will be justified later)

Then - dynamics of the Higgs field is more essential, $\chi \simeq const$ and is frozen. Denote $\xi_\chi \chi^2 = M_P^2$.

Redefinition of the Higgs field to make canonical kinetic term

$$\frac{d\tilde{h}}{dh} = \sqrt{\frac{\Omega^2 + 6\xi_h^2 h^2/M_P^2}{\Omega^4}} \quad \Longrightarrow \begin{cases} h \simeq \tilde{h} & \text{for } h < M_P/\xi \\ h \simeq \frac{M_P}{\sqrt{\xi}} \exp\left(\frac{\tilde{h}}{\sqrt{6}M_P}\right) & \text{for } h > M_P/\sqrt{\xi} \end{cases}$$

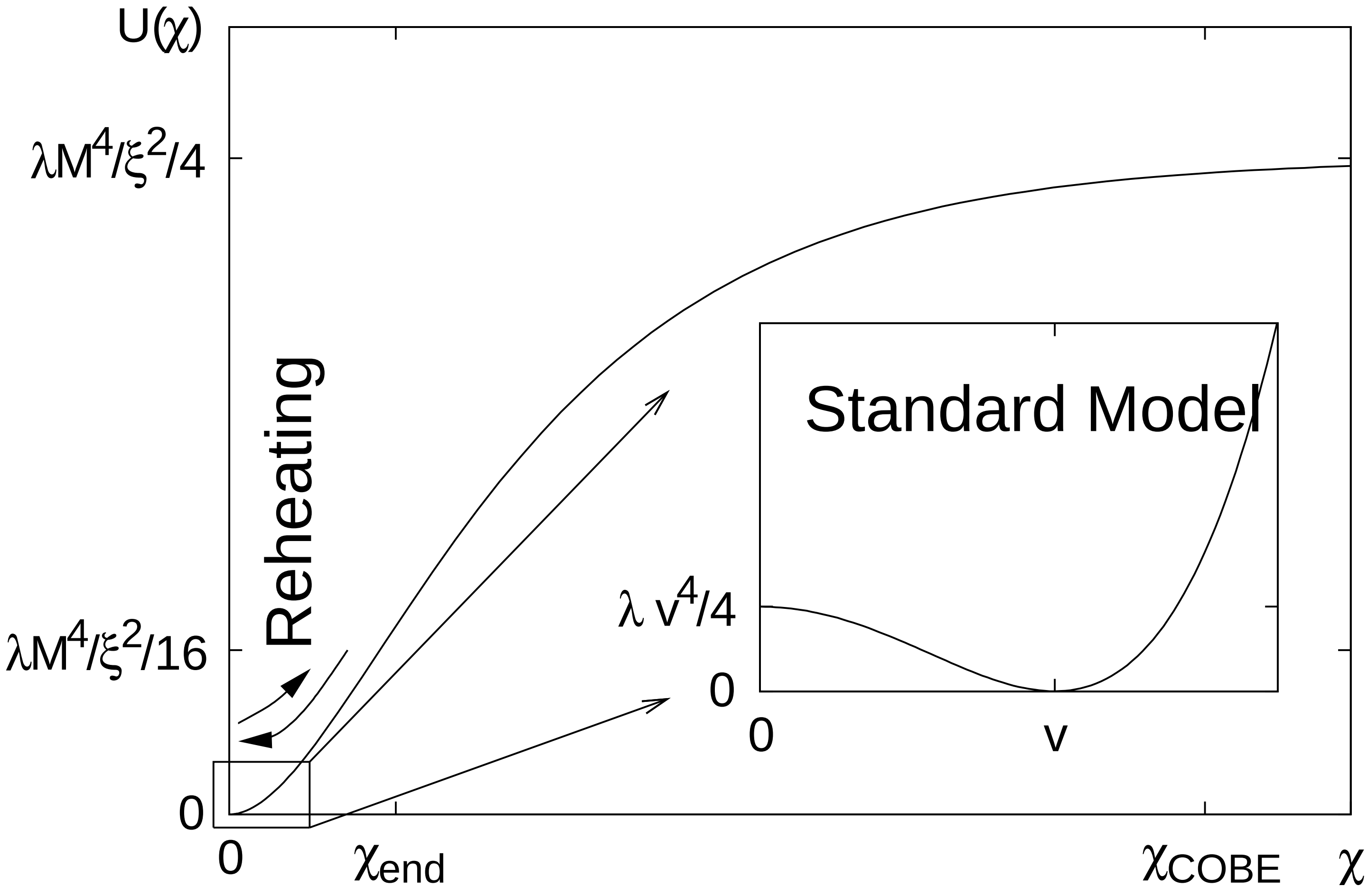
Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \tilde{h} \partial^\mu \tilde{h}}{2} - \frac{1}{\Omega(\tilde{h})^4} \frac{\lambda}{4} h(\tilde{h})^4 \right\}$$

Potential:

$$U(\tilde{h}) = \begin{cases} \frac{\lambda}{4}\tilde{h}^4 & \text{for } h < M_P/\xi \\ \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\tilde{h}}{\sqrt{6}M_P}}\right)^2 & \text{for } h > M_P/\xi \end{cases} .$$

Potential in Einstein frame



Slow roll stage

$$\epsilon = \frac{M_P^2}{2}\left(\frac{dU/d\chi}{U}\right)^2 \simeq \frac{4}{3}\exp\left(-\frac{4\chi}{\sqrt{6}M_P}\right)$$

$$\eta = M_P^2 \frac{d^2U/d\chi^2}{U} \simeq -\frac{4}{3}\exp\left(-\frac{2\chi}{\sqrt{6}M_P}\right)$$

Slow roll ends at $\chi_{\text{end}} \simeq M_P$

Number of e-folds of inflation at the moment h_N is $N \simeq \frac{6}{8}\frac{h_N^2 - h_{\text{end}}^2}{M_P^2/\xi}$

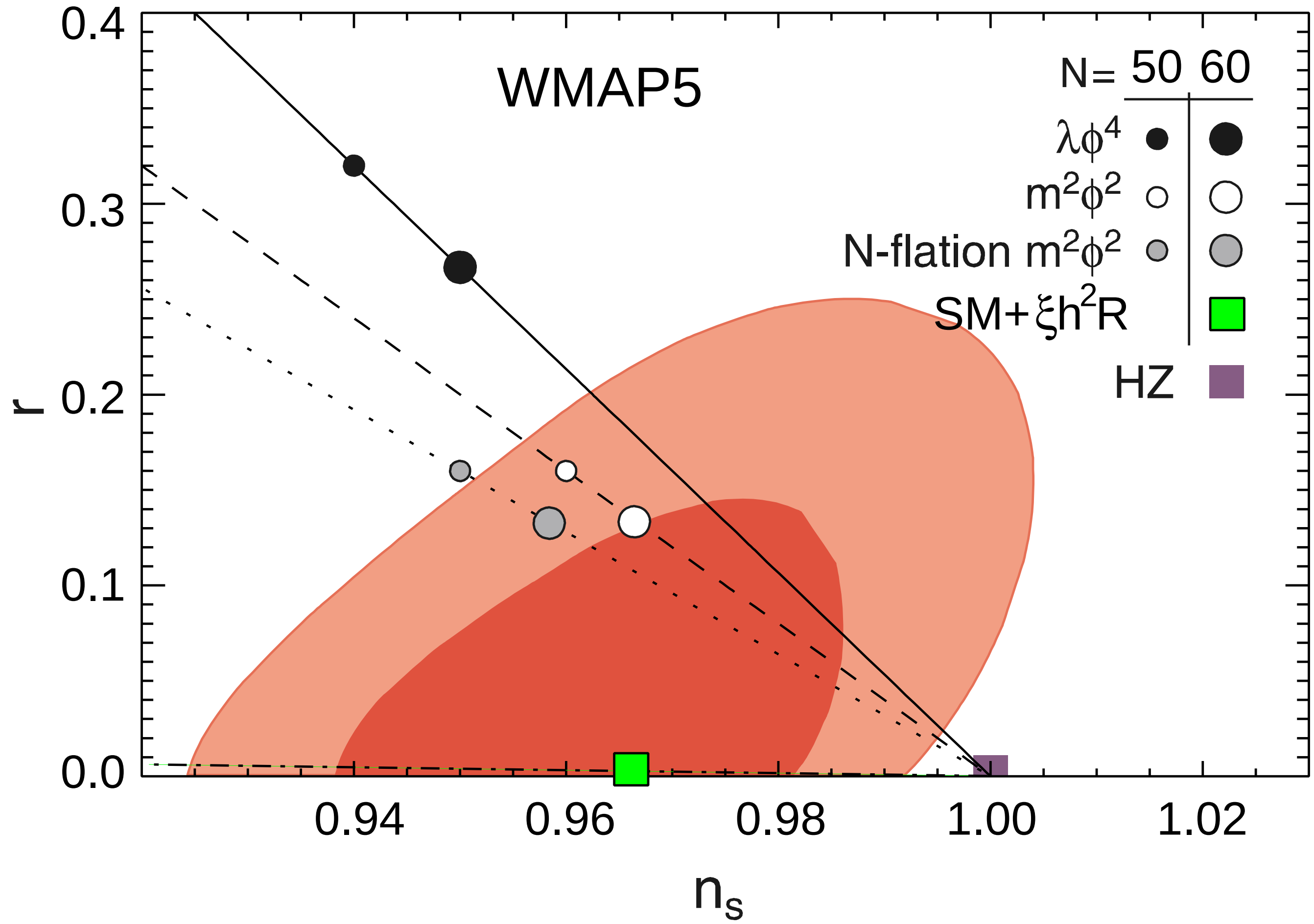
$$\chi_{60} \simeq 5M_P$$

COBE normalization $U/\epsilon = (0.027M_P)^4$ gives

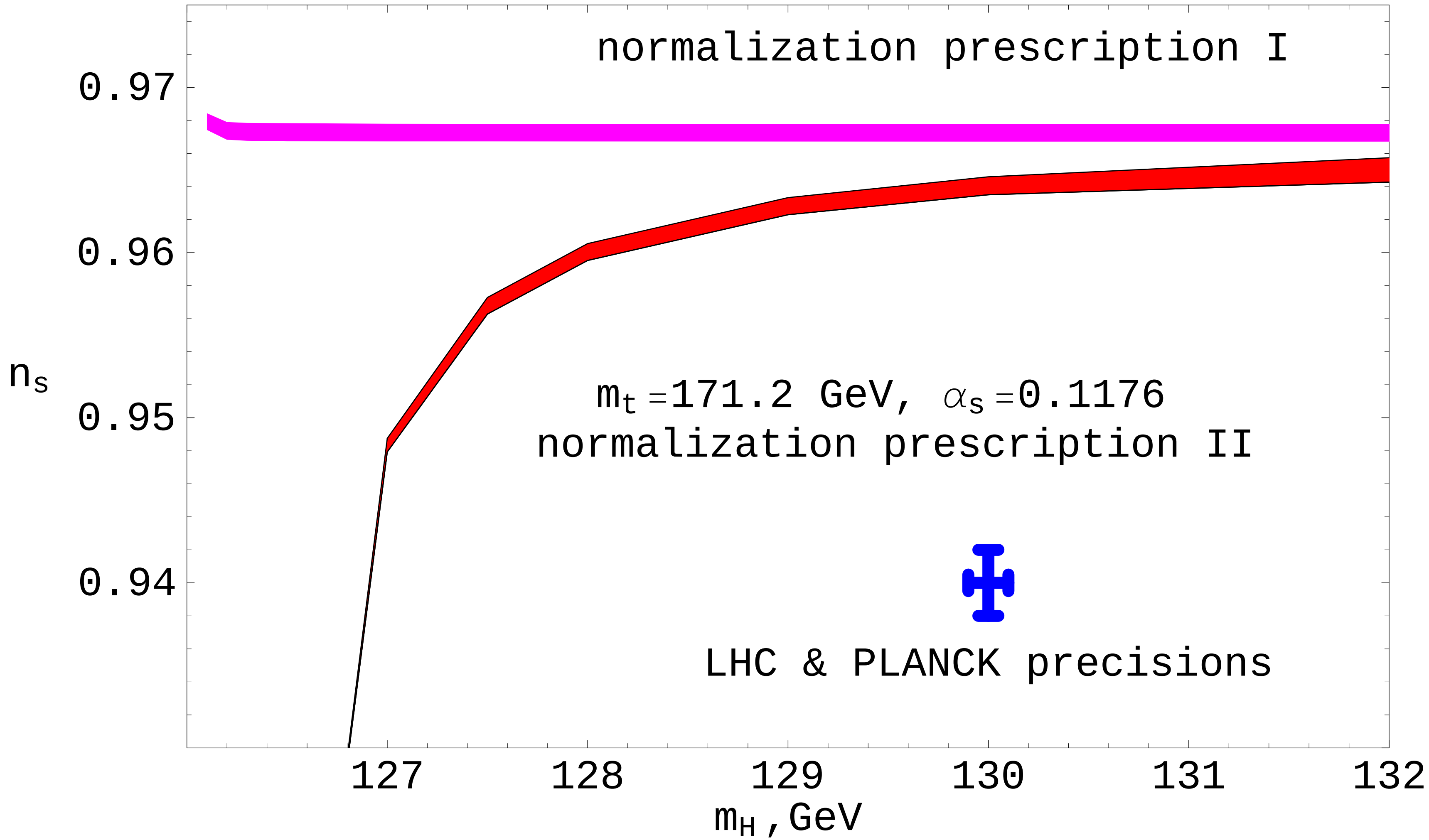
$$\xi \simeq \sqrt{\frac{\lambda}{3}\frac{N_{\text{COBE}}}{0.027^2}} \simeq 49000\sqrt{\lambda} = 49000\frac{m_H}{\sqrt{2}v}$$

Connection of ξ and the Higgs mass!

CMB parameters—spectrum and tensor modes



Experimental precision



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Real physics question is not whether this or that theory is “natural” but whether it is realised in Nature...

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Real physics question is not whether this or that theory is "natural" but whether it is realised in Nature...

If ξ is large then chaotic inflation is inevitable in the Standard model, $V_{\text{inf}} \propto \lambda M_P^4/\xi^2$.

What happens at large ξ?

Sibiryakov, ’08; Burgess, Lee, Trott, ’09; Barbon and Espinosa, ’09

Tree amplitudes of scattering of scalars above electroweak vacuum hit the unitarity bound at energies

$$E > \Lambda \sim \frac{M_P}{\xi}$$

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What does it mean?

Option 1: The theory fails and must be replaced by a more fundamental one

Option 2: A theorist fails and must work harder to figure out what happens

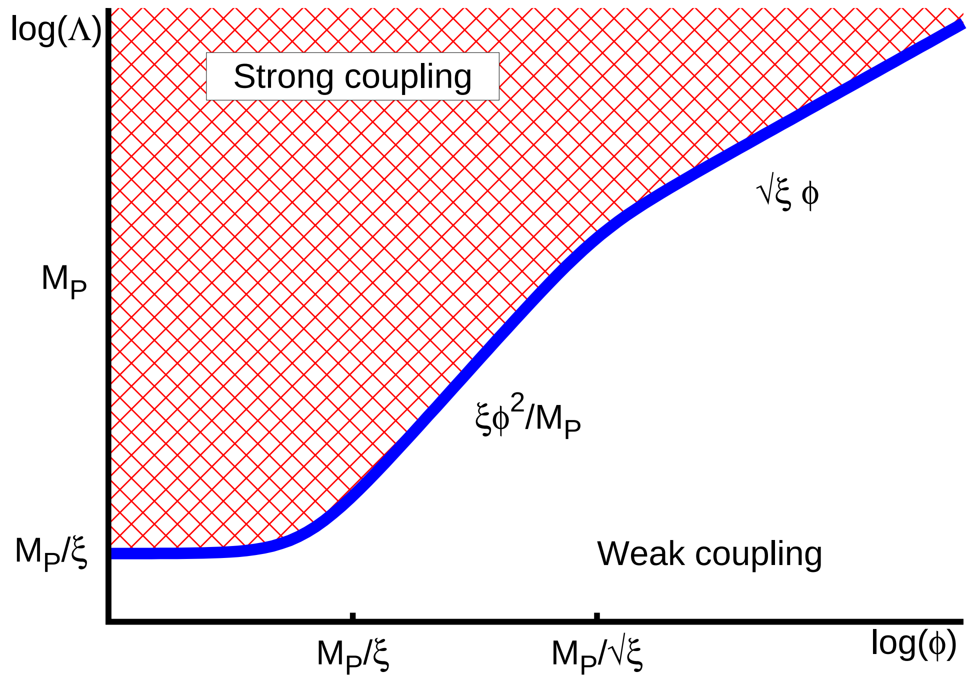
Effective theory

We do not know the more fundamental theory. So, lets add to the SM all sorts of higher dimensional operators suppressed by powers of cutoff Λ. Cutoff is background dependent: Bezrukov, Magnin, M.S., Sibiryakov; Ferrara, Kallosh, Linde, A. Marrani, Van Proeyen

$$\Lambda(h) \simeq \begin{cases} \frac{M_P}{\xi} \,, & \text{for } h \lesssim \frac{M_P}{\xi} \,, \\ \frac{h^2\xi}{M_P} \,, & \text{for } \frac{M_P}{\xi} \lesssim h \lesssim \frac{M_P}{\sqrt{\xi}} \,, \\ \sqrt{\xi}h \,, & \text{for } h \gtrsim \frac{M_P}{\sqrt{\xi}} \,. \end{cases}$$

Important: scale invariance in Jordan frame = shift symmetry in Einstein frame

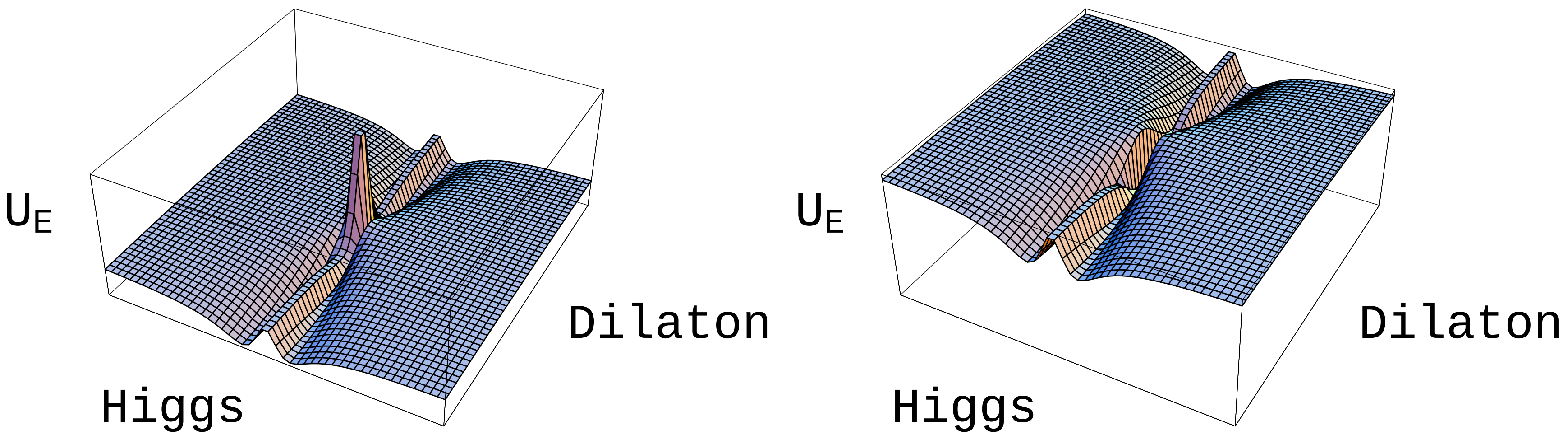
Higgs-dependent cutoff



Cutoff is higher than the relevant dynamical scales throughout the whole history of the Universe, including the inflationary epoch and reheating!!

The Higgs-inflation is “natural” in the Standard Model.

Dark energy



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Dark energy

Late time evolution of dilaton ρ along the valley, related to χ as

$$\chi = M_P \exp\left(\frac{\gamma\rho}{4M_P}\right), \quad \gamma = \frac{4}{\sqrt{6 + \frac{1}{\xi_\chi}}} .$$

Potential: Wetterich; Ratra, Peebles

$$U_\rho = \frac{\Lambda}{\xi_\chi^2} \exp\left(-\frac{\gamma\rho}{M_P}\right) .$$

From observed equation of state: $0 < \xi_\chi < 0.09$

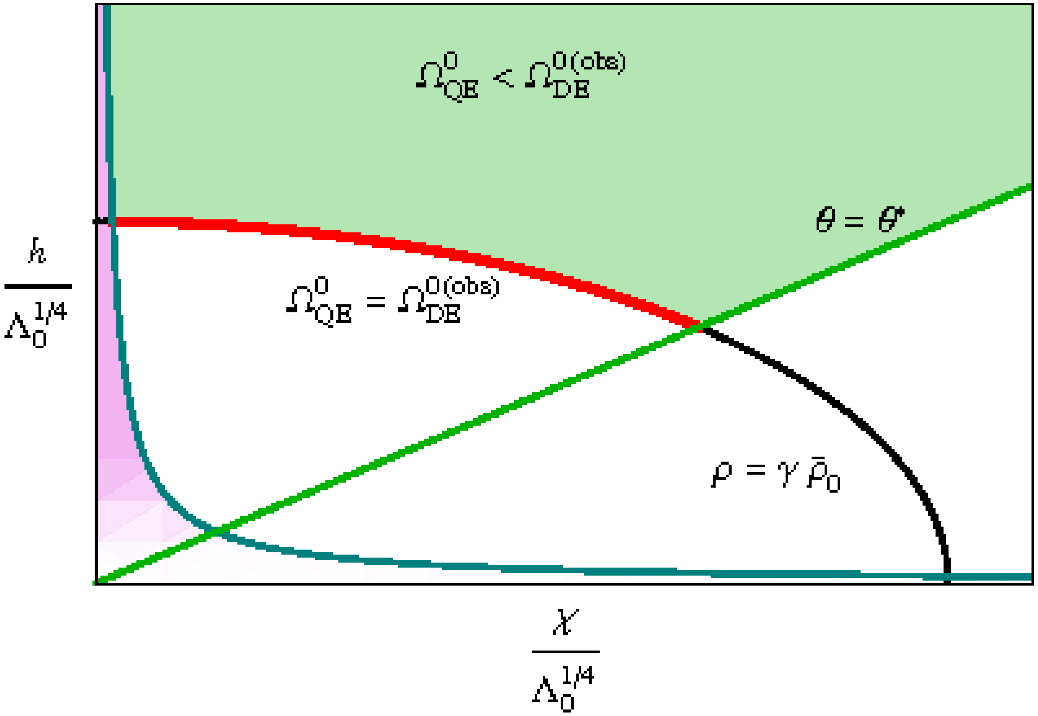
Result: equation of state parameter $\omega = P/E$ for dark energy must be different from that of the cosmological constant, but $\omega < -1$ is not allowed.

Higgs-dilaton inflation

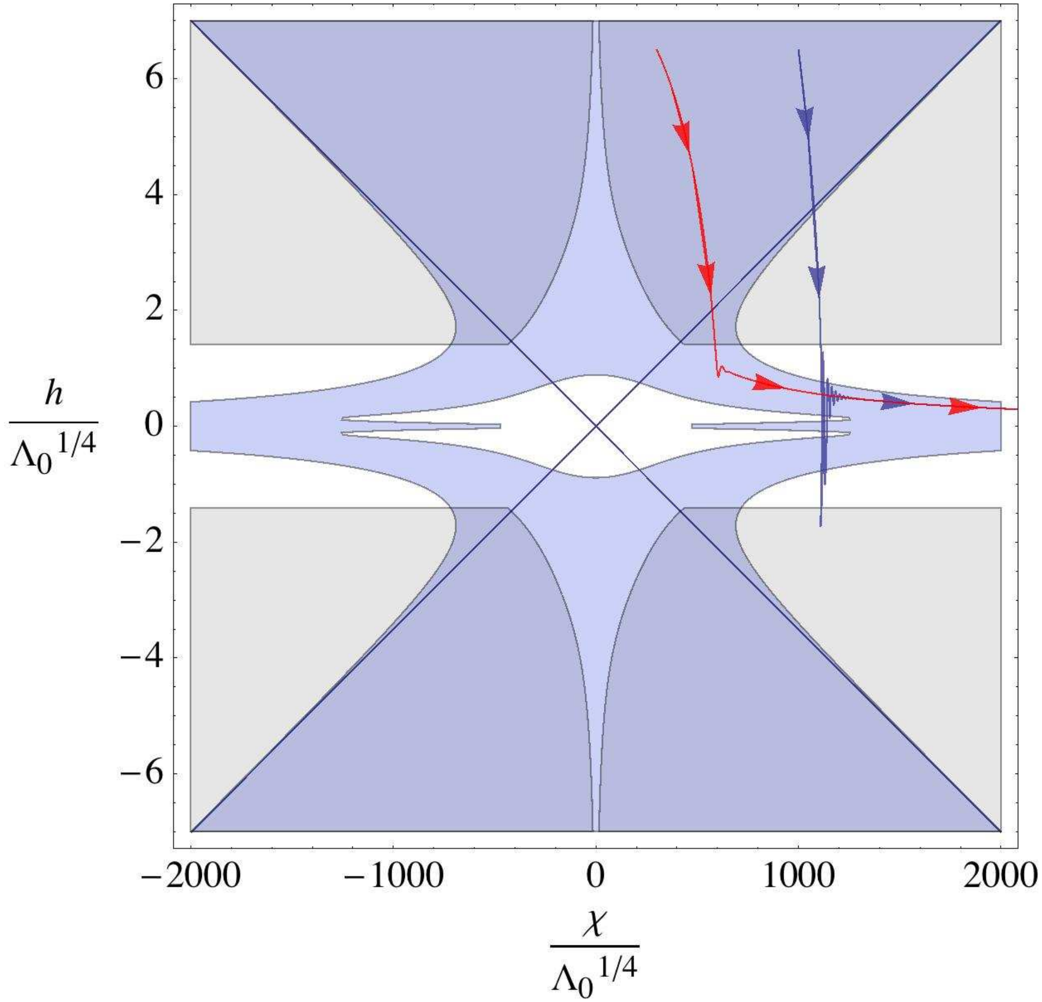
Juan García-Bellido, Javier Rubio, M.S., Daniel Zenhäusern

- Take arbitrary initial conditions for the Higgs and the dilaton
- Find the region on the $\{\chi, h\}$ plane that lead to inflation
- Find the region on the $\{\chi, h\}$ plane that lead to exit from inflation
- Find the region on the $\{\chi, h\}$ plane that lead to observed abundance of Dark Energy

Initial conditions



Trajectories



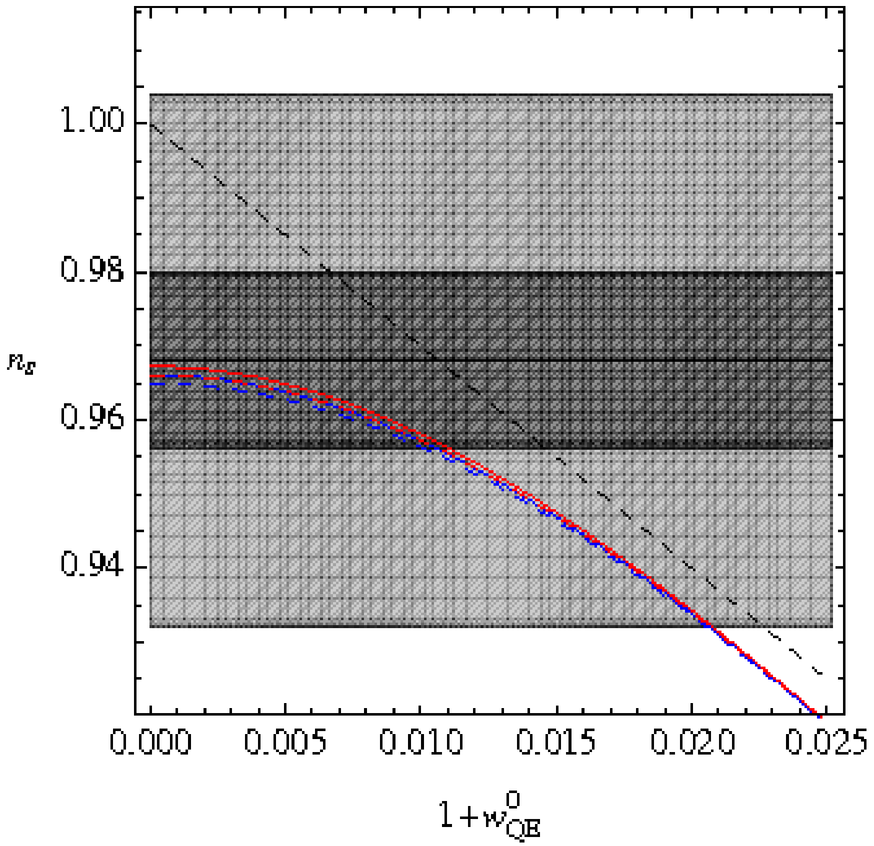
Generic semiclassical initial conditions lead to:

- the Universe, which was inflating in the past
- the Universe with the Dark Energy abundance smaller, than observed

Quantum initial state to explain the DM-DE coincidence problem?

Inflation-dark energy relation

Value of n_s is determined by ξ_h and ξ_χ, and equation of state of DE ω by $\xi_\chi \Longrightarrow n_s - \omega$ relation:



Higgs mass, stability, inflation and asymptotic safety

Radiative corrections are essential for validity of ETOE (and thus for the Higgs-dilaton cosmology). ETOE must be self-consistent up to inflationary scale. This gives a direct relation to the Higgs mass.

Definition: "$\overline{MS}$ benchmark Higgs mass M_{crit}" is defined from equations

$$\lambda(\mu_0) = 0, \quad \beta_\lambda^{\rm SM}(\mu_0) = 0$$

together with parameter μ_0, assuming that all parameters of the SM, except the Higgs mass, are fixed.

Then:

- Electroweak vacuum is stable for $M_H > M_{crit} + \Delta M_{stab}$
- Higgs or Higgs-dilaton inflation can take place at $M_H > M_{crit} + \Delta M_{infl}$
- Prediction of the Higgs mass from asymptotic safety of the SM is $M_H = M_{crit} + \Delta M_{safety}$

All ΔM_I are small (few hundred MeV).

Value of M_{crit} as of 2009 (one-loop matching at the EW scale and 2-loop running up to high energy scale):

$$m_{crit} = [126.3 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5]\ \mathrm{GeV}\ ,$$

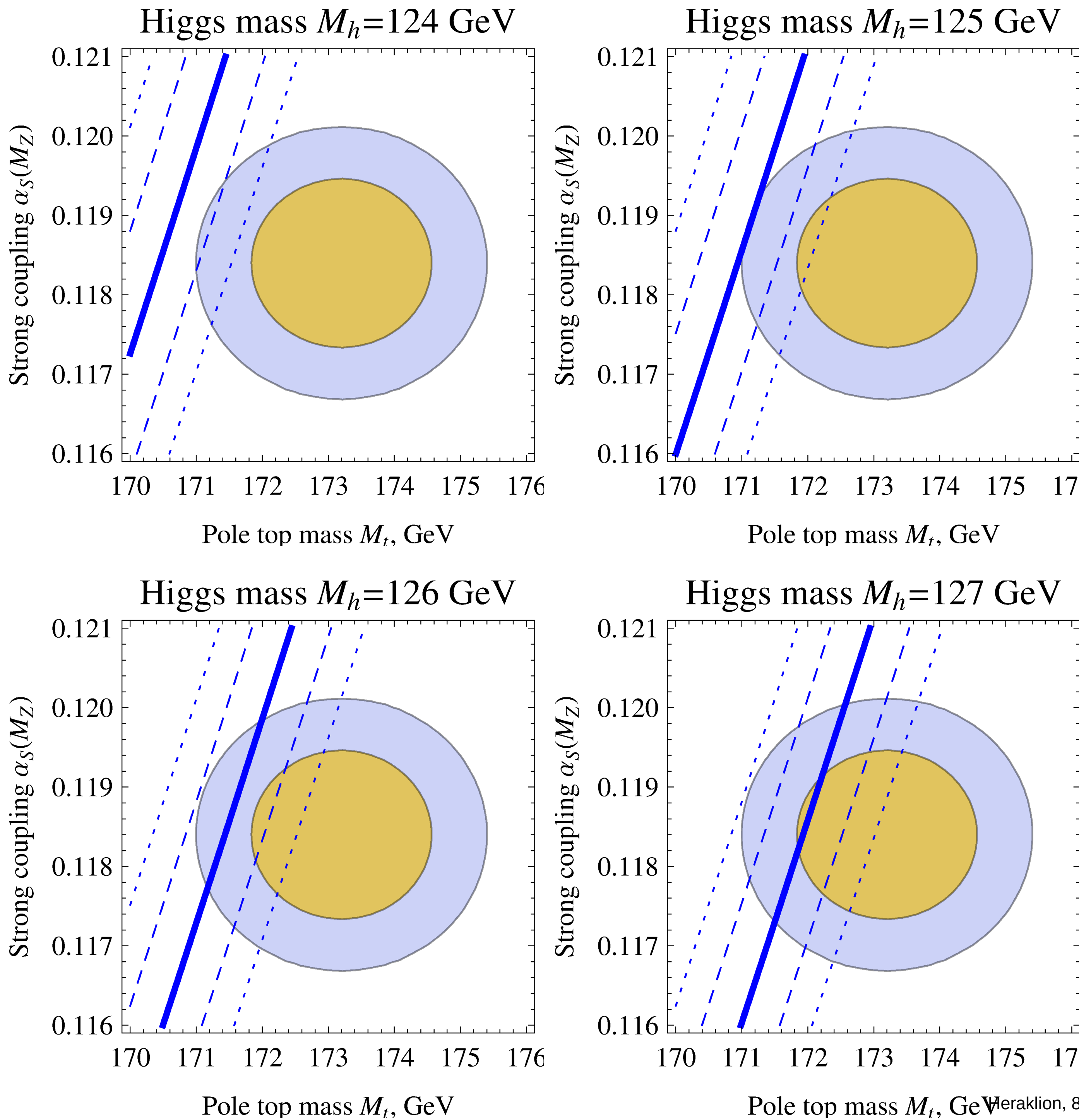
Theoretical uncertainties: ± 2.5 GeV (different sources are summed quadratically) or ± 5 GeV (different sources are summed linearly).

Updated computation of M_H (Bezrukov, Kalmykov, Kniel, M.S., May 13, 2012), incorporating $\mathcal{O}(\alpha\alpha_s)$ two-loop matching and 3-loop running of coupling constants (Chetyrkin, Zoller)

$$m_{crit} = [129.0 + \frac{m_t - 172.9}{1.1} \times 2.2 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.56]\ \mathrm{GeV}\ ,$$

Theoretical uncertainties: ± 1.2 GeV (different sources are summed quadratically) or ± 2.3 GeV (different sources are summed linearly).

Effect of contributions $\propto y_t^4, y_t^2\lambda^2, \lambda^4$ (Degrassi et al., May 29, 2012): shift of the Higgs mass by $100 - 200$ MeV. Quadratic theoretical uncertainty is reduced to ~ 0.8 GeV.


Higgs mass M_h=124 GeV
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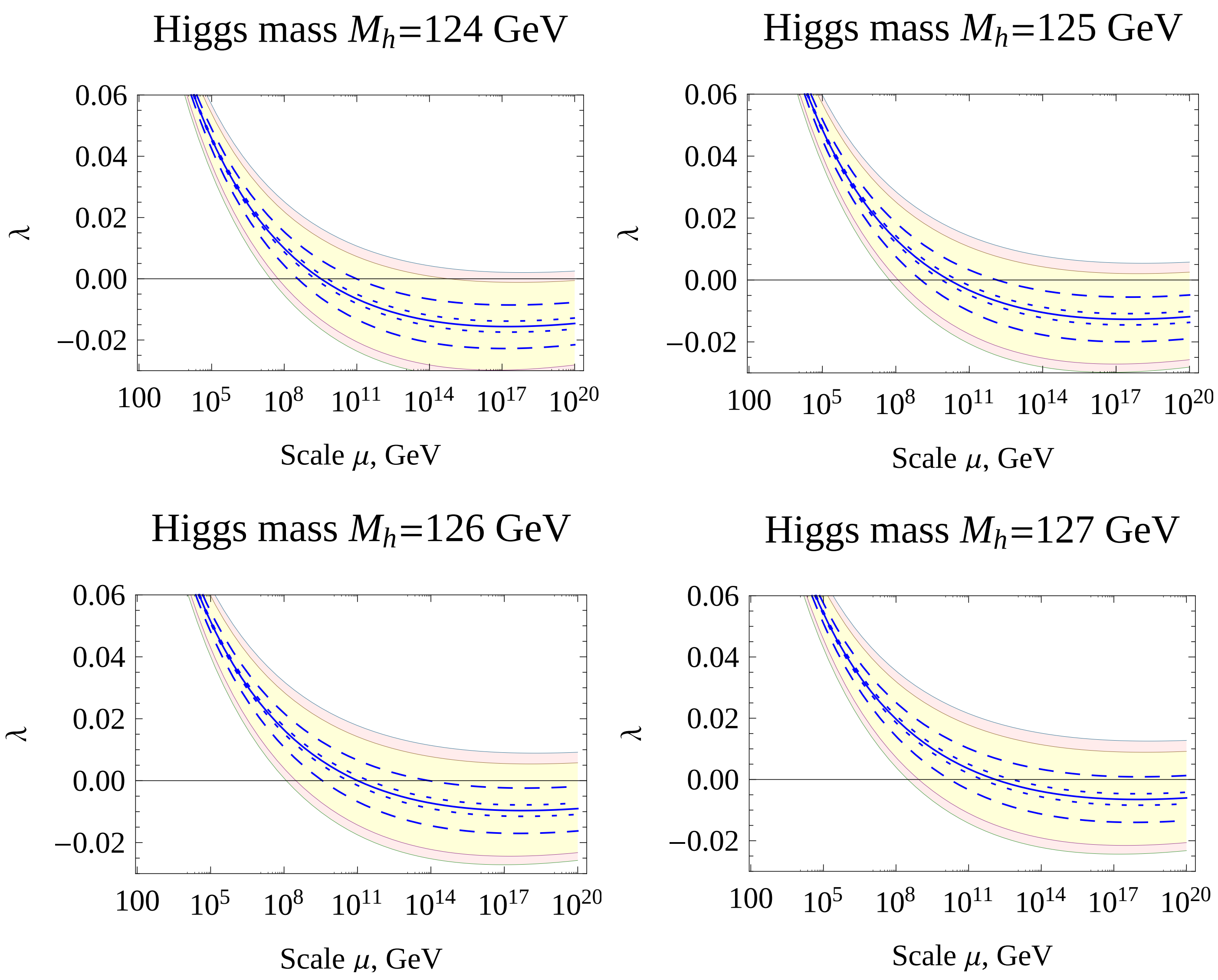
Strong coupling $\alpha_S(M_Z)$
0.121
0.120
0.119
0.118
0.117
0.116
170
171
172
173
174
175
176
Pole top mass M_t, GeV

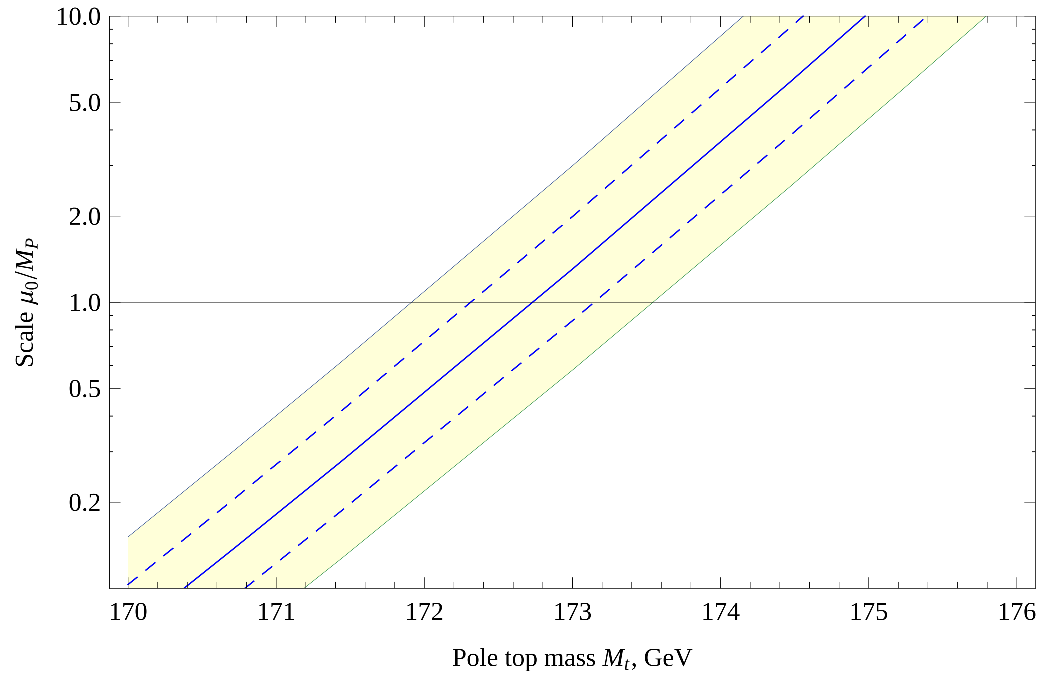
To decrease uncertainty: (the LHC accuracy can be as small as 200 MeV!)

- Compute remaining two-loop $\mathcal{O}(\alpha^2)$ corrections to pole - $\overline{\mathrm{M}S}$ matching for the Higgs mass and top masses. Theoretical uncertainty can reduced to ~ 0.5 GeV, due to irremovable non-perturbative contribution $\sim \Lambda_{QCD}$ to top quark mass.
- Measure better t-quark mass (present error in m_H due to this uncertainty is $\simeq 4$ GeV at 2σ level): construct t-quark factory – e^+e^- or $\mu^+\mu^-$ linear collider with energy $\simeq 200 \times 200$ GeV - proposal for the European high energy strategy committee
- Measure better α_s (present error in m_H due to this uncertainty is $\simeq 1$ GeV at 2σ level)

Behaviour of the Higgs self-coupling



Scale from equations: $\lambda(\mu_0) = 0$ and $\beta_\lambda^{SM}(\mu_0) = 0$



μ_0 determined by the EW physics gives the Planck scale!

Numerical coincidence?

Fermi scale is determined by the Planck scale (or vice versa)?

Possible explanation - asymptotic safety of the SM+gravity

Conclusions. ETOE gives:

- Dynamical origin of all mass scales
- Hierarchy problem gets a different meaning - an alternative (to SUSY, techicolor, little Higgs or large extra dimensions) solution of it may be possible.
- Cosmological constant problem acquires another formulation.
- Natural chaotic cosmological inflation
- Low energy sector contains a massless dilaton
- There is Dark Energy even without cosmological constant
- There is direct relation between inflation and DE equation of state
- Agreement with LHC indications of the Higgs existence and of absence of evidence of new physics right above the EW scale

Problems to solve

- Though the stability of the electroweak scale against quantum corrections may be achieved, it is unclear *why* the electroweak scale is so much smaller than the Planck scale (or why $\zeta \lll 1$).

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Based on works with

- Takehiko Asaka, Niigata U.
- Fedor Bezrukov, Connecticut U.
- Steve Blanchet, EPFL
- Diego Blas, CERN
- Alexey Boyarsky, Leiden
- Laurent Canetti, EPFL
- Marco Drewes, Aachen U.
- Juan Garcia-Bellido, Madrid U.
- Dmitry Gorbunov, INR Moscow
- Mikhail Kalmykov, Hamburg U.
- Bernd Kniel, Hamburg U.
- Mikko Laine, Bern U.
- Amaury Magnin, EPFL
- Andrii Neronov, Versoix
- Javie Rubio, EPFL
- Oleg Ruchayskiy, CERN
- Sergei Sibiryakov, INR Moscow
- Igor Tkachev, INR Moscow
- Christof Wetterich, Heidelberg U.
- Daniel Zenhausern, EPFL

Back up slides

Hot, Warm and Cold

Abazajian, Fuller, Patel

The mass inside sterile neutrino free streaming length λ_{FS}:

$$M_{FS} \simeq 2.6 \times 10^{11} M_{\odot} (\Omega_N h^2) \left(\frac{1\text{keV}}{M_N}\right)^3 \left(\frac{\langle p/T \rangle}{3.15}\right)^3$$

$p/T \simeq 3.15$ for thermal spectrum of sterile neutrino. In reality $0.3 < \frac{\langle p/T \rangle}{3.15} < 0.9$ (Asaka, Laine, MS)

Joel Primack: “WDM producing less structures than CDM at the scales $10^6 - 10^8 M_{\odot}$ is excluded”.

If $10^8 M_{\odot}$: $M_N > 2 - 5$ KeV, depending on the spectrum

If $10^6 M_{\odot}$: $M_N > 8 - 25$ KeV, depending on the spectrum

Quantum scale invariance

Quantum scale invariance

Common lore: quantum scale invariance does not exist, divergence of dilatation current is not-zero due to quantum corrections:

$$\partial_\mu J^\mu \propto \beta(g) G^a_{\alpha\beta} G^{\alpha\beta\; a} \,,$$

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Everything above does not make any sense???!!!

Standard reasoning

Dimensional regularisation $d = 4 - 2\epsilon$, $\overline{MS}$ subtraction scheme:

mass dimension of the scalar fields: $1 - \epsilon$,

mass dimension of the coupling constant: 2ϵ

Counter-terms:

$$\lambda = \mu^{2\epsilon} \left[\lambda_R + \sum_{k=1}^{\infty} \frac{a_n}{\epsilon^n} \right] ,$$

μ is a dimensionfull parameter!!

One-loop effective potential along the flat direction:

$$V_1(\chi) = \frac{m_H^4(\chi)}{64\pi^2} \left[\log \frac{m_H^2(\chi)}{\mu^2} - \frac{3}{2} \right] ,$$

Result: explicit breaking of the dilatation symmetry. Dilaton acquires a nonzero mass due to radiative corrections.

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Idea: Replace $\mu^{2\epsilon}$ by combinations of fields χ and h, which have the correct mass dimension:

$$\mu^{2\epsilon} \to \chi^{\frac{2\epsilon}{1-\epsilon}} F_\epsilon(x) \ ,$$

where $x = h/\chi$. $F_\epsilon(x)$ is a function depending on the parameter ϵ with the property $F_0(x) = 1$.

Zenhäusern, M.S

Englert, Truffin, Gastmans, 1976

Example of computation

Natural choice:

$$\mu^{2\epsilon} \to \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \ , (\xi_\chi \chi^2 + \xi_h h^2) \equiv \omega^2$$

Potential:

$$U = \frac{\lambda_R}{4} \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \left[h^2 - \zeta_R^2 \chi^2\right]^2 \ ,$$

Counter-terms

$$U_{cc} = \left[\omega^2\right]^{\frac{\epsilon}{1-\epsilon}} \left[A h^2 \chi^2 \left(\frac{1}{\bar{\epsilon}} + a\right) + B \chi^4 \left(\frac{1}{\bar{\epsilon}} + b\right) + C h^4 \left(\frac{1}{\bar{\epsilon}} + c\right)\right],$$

To be fixed from conditions of absence of divergences and presence of spontaneous breaking of scale-invariance

$$U_1 = \frac{m^4(h)}{64\pi^2}\left[\log\frac{m^2(h)}{v^2} + \mathcal{O}\left(\zeta_R^2\right)\right]$$

$$+ \frac{\lambda_R^2}{64\pi^2}\left[C_0 v^4 + C_2 v^2 h^2 + C_4 h^4\right] + \mathcal{O}\left(\frac{h^6}{\chi^2}\right),$$

where $m^2(h) = \lambda_R(3h^2 - v^2)$ and

$$C_0 = \frac{3}{2}\left[2a - 1 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) + \frac{4}{3}\log 2\lambda_R + O(\zeta_R^2)\right],$$

$$C_2 = -3\left[2a - 3 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) + O(\zeta_R^2)\right],$$

$$C_4 = \frac{3}{2}\left[2a - 5 + 2\log\left(\frac{\zeta_R^2}{\xi_\chi}\right) - 4\log 2\lambda_R + O(\zeta_R^2)\right].$$

Consider the high energy ($\sqrt{s} \gg v$ but $\sqrt{s} \ll \chi_0$) behaviour of scattering amplitudes on the example of Higgs-Higgs scattering (assuming, that $\zeta_R \ll 1$). In one-loop approximation

$$\Gamma_4 = \lambda_R + \frac{9\lambda_R^2}{64\pi^2}\left[\log\left(\frac{s}{\xi_\chi \chi_0^2}\right) + \text{const}\right] + \mathcal{O}\left(\zeta_R^2\right) \,.$$

This implies that at $v \ll \sqrt{s} \ll \chi_0$ the effective Higgs self-coupling runs in a way prescribed by the ordinary renormalization group!

For QCD:

$$\Lambda_{QCD} = \chi_0 e^{-\frac{1}{2b_0\alpha_s}}, \quad \beta(\alpha_s) = b_0\alpha_s^2$$

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- Renormalizability: Can we remove all divergences with the similar structure counter-terms? The answer is “no" (Tkachov, MS). However, this is not essential for the issue of scale invariance. We get scale-invariant effective theory

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Problems

- Renormalizability: Can we remove all divergences with the similar structure counter-terms? The answer is “no" (Tkachov, MS). However, this is not essential for the issue of scale invariance. We get scale-invariant effective theory
- Unitarity and high-energy behaviour: What is the high-energy behaviour $(E > M_{Pl})$ of the scattering amplitudes? Is the theory Unitary? Can it have a scale-invariant UV completion?

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- Higgs mass is stable against radiative corrections (in dimensional regularisation)
- Requirement of spontaneous breakdown of scale invariance - cosmological constant is tuned to zero in all orders of perturbation theory

Dilaton as a part of the metric

Previous discussion - ad hoc introduction of scalar field χ. It is massless, as is the graviton. Can it come from gravity?

Yes - it automatically appears in scale-invariant TDiff gravity as a part of the metric!

Consider arbitrary metric $g_{\mu\nu}$ (no constraints). Determinant g of $g_{\mu\nu}$ is TDiff invariant. Generic scale-invariant action for scalar field and gravity:

$$\mathcal{S} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2}\phi^2 f(-g) R - \frac{1}{2}\phi^2 \mathcal{G}_{gg}(-g)(\partial g)^2$$

$$-\frac{1}{2}\mathcal{G}_{\phi\phi}(-g)(\partial\phi)^2 + \mathcal{G}_{g\phi}(-g)\phi\,\partial g\cdot\partial\phi - \phi^4 v(-g)\Big]\,.$$

Equivalence theorem

This TDiff theory is equivalent (at the classical level) to the following Diff scalar tensor theory:

$$\frac{\mathcal{L}_e}{\sqrt{-g}} = -\frac{1}{2}\phi^2 f(\sigma)R - \frac{1}{2}\phi^2 \mathcal{G}_{gg}(\sigma)(\partial\sigma)^2 - \frac{1}{2}\mathcal{G}_{\phi\phi}(\sigma)(\partial\phi)^2$$

$$-\mathcal{G}_{g\phi}(\sigma)\phi\,\partial\sigma\cdot\partial\phi - \phi^4 v(\sigma) - \frac{\Lambda_0}{\sqrt{\sigma}}\,.$$

Transformation to Einstein frame:

$$\frac{\mathcal{L}_e}{\sqrt{-\tilde{g}}} = -\frac{1}{2}M^2\tilde{R} - \frac{1}{2}M^2\mathcal{K}_{\sigma\sigma}(\sigma)(\partial\sigma)^2 - \frac{1}{2}M^2\mathcal{K}_{\phi\phi}(\sigma)(\partial\ln(\phi/M))^2$$

$$-M^2\mathcal{K}_{\sigma\phi}(\sigma)\,\partial\sigma\cdot\partial\ln(\phi/M) - M^4V(\sigma) - \frac{M^4\Lambda_0}{\phi^4 f(\sigma)^2\sqrt{\sigma}}\,,$$

As expected, ϕ is a Goldstone boson with derivative couplings only (except the term containing Λ_0).

So, TDiff scale invariant theory automatically contains a massless dilaton. All previous results can be reproduced in it.

Towards to Physics at All Scales

If gravity (Weinberg, M. Reuter)

and the Standard Model (M.S., Wetterich)

are asymptotically safe then

ETOE may appear to be a fundamental theory

To be true: all the couplings of the SM must be asymptotically safe or asymptotically free

Problem for:

- U(1) gauge coupling $g_1, \quad \mu \frac{dg_1}{d\mu} = \beta_1^{\mathrm{SM}} = \frac{41}{96\pi^2} g_1^3$
- Scalar self-coupling $\lambda, \quad \mu \frac{d\lambda}{d\mu} = \beta_\lambda^{\mathrm{SM}} =$

$$= \frac{1}{16\pi^2} \left[(24\lambda + 12h^2 - 9(g_2^2 + \frac{1}{3} g_1^2))\lambda - 6h^4 + \frac{9}{8} g_2^4 + \frac{3}{8} g_1^4 + \frac{3}{4} g_2^2 g_1^2 \right]$$

- Fermion Yukawa couplings, t-quark in particular $h, \quad \mu \frac{dh}{d\mu} = \beta_h^{\mathrm{SM}} =$

$$= \frac{h}{16\pi^2} \left[\frac{9}{2} h^2 - 8 g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 \right]$$

Landau pole behaviour

Gravity contribution to RG running

Let x_j is a SM coupling. Gravity contribution to RG:

$$\mu \frac{dx_j}{d\mu} = \beta_j^{\mathrm{SM}} + \beta_j^{grav} \ .$$

On dimensional grounds

$$\beta_j^{grav} = \frac{a_j}{8\pi} \frac{\mu^2}{M_P^2(\mu)} x_j \ .$$

where

$$M_P^2(\mu) = M_P^2 + 2\xi_0 \mu^2 \ ,$$

with $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}$ GeV, $\xi_0 \approx 0.024$
from a numerical solution of FRGE

Remarks

- The couplings are not in $\overline{\mathrm{M}S}$ scheme
- The couplings are not in MOM scheme
- Pretty vague definition based on physical scattering amplitudes at large momentum transfer - never actually worked out in details

Thus, computations of a_j are ambiguous and controversial.

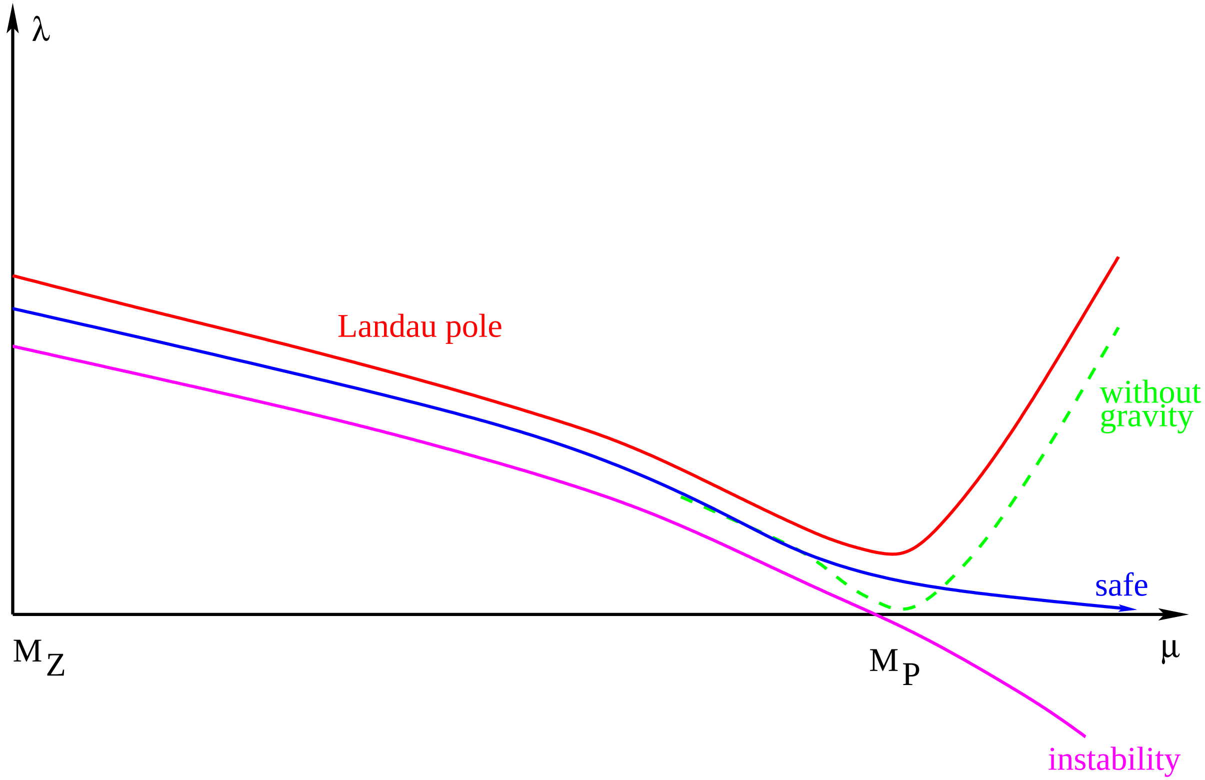
Still, even without exact knowledge of a_j a lot can be said about the Higgs mass

Robinson and Wilczek '05, Pietrykowski '06, Toms '07&'08, Ebert, Plefka and Rodigast '07, Narain and Percacci '09, Daum, Harst and Reuter '09, Zanusso et al '09, ...

- Most works get for gauge couplings a universal value $a_1 = a_2 = a_3 < 0$: U(1) gauge coupling get asymptotically free in asymptotically safe gravity
- $a_\lambda \simeq 2.6 > 0$ according to Percacci and Narain '03 for scalar theory coupled to gravity
- $a_h >< 0$?? The case $a_h > 0$ is not phenomenologically acceptable - only massless fermions are admitted

Suppose that indeed $a_1 < 0,\ a_h < 0,\ a_\lambda > 0$. Then the Higgs mass can be predicted :

$$m_{\rm H} = [126.3 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5]\ \mathrm{GeV}\ ,$$



Possible understanding of the amazing fact that $\lambda(M_P) = 0$ and $\beta_\lambda^{\rm SM}(M_P) = 0$ simultaneously at the Planck scale.