Supersymmetry on three-manifolds

Cyril Closset

Weizmann Institute of Sciences

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Outline:

- Introduction and motivation
- Basics of rigid supersymmetry on curved space
- Classification of supersymmetric three-manifolds (locally)
- SUSY multiplets and Lagrangians
- Comments on metric dependence
- Application: τ_{rr} from squashed sphere
- Conclusions and outlook

Based on:

- C.C., T. Dumitrescu, G.Festuccia, Z. Komargodski, [To appear]
- C.C., T. Dumitrescu, G.Festuccia, Z. Komargodski, N. Seiberg, 1206.5218 and 1205.4142

Rigid supersymmetry on curved manifolds

Given a d-dimensional supersymmetric field theory $\mathcal{T}$ in flat space and a Riemannian manifold $(\mathcal{M}, g_{\mu\nu})$, can we define a corresponding supersymmetric theory on $(\mathcal{M}, g_{\mu\nu})$?

$$\begin{array}{ccc} (\mathcal{T}, \mathbb{R}^d, \delta_{\mu\nu}) & \rightarrow & (\mathcal{T}', \mathcal{M}, g_{\mu\nu}) \\ \delta\mathcal{T} & \rightarrow & \delta'\mathcal{T}' \end{array}$$

- Until recently, there was no systematic method. Case-by-case studies for $\mathcal{M}$ simple enough. E.g. supersymmetric theories are known on S^4, $S^3 \times S^1$, S^3, $S^2 \times S^1$,... [Sen, 1987; Romelsberger 2007; Pestun 2007; Kapustin, Willett, Yaakov, 2010; Jafferis, 2010; Hama, Hosomichi, Lee, 2010, 2011; Imamura, Yokoyama, 2011; Benini, Cremonesi, 2012; Doroud, Gomis, Le Floch, Lee, 2012; ...]
- General method has been proposed, based on background supergravity fields [Festuccia, Seiberg, 2011]

Motivation: Exact results for supersymmetric theories

In recent literature, there has been some intense study of supersymmetric theories on *spheres*.

- Exact calculation of the partition functions $Z(S^3)$, $Z(S^2 \times S^1)$, $Z(S^2)$, $\cdots$ are known, using localisation. [See refs above]
- One can generalise such results to more general manifolds. Exact formulas for $Z(\mathcal{M})$? (I will not discuss this.) [Work in progress.]
- The more general approach allows to understand better previous results on spheres, and extract more information from them.

Curved space rigid supersymmetry

Consider a supersymmetric quantum field theory described by some UV Lagrangian $\mathcal{L}_0$.

$$\delta_0 \mathcal{L}_0 = \partial_\mu (\cdots)\,, \qquad \tilde{\delta}_0 \mathcal{L}_0 = \partial_\mu (\cdots)\,.$$

$$\{\delta_0, \tilde{\delta}_0\} \sim P_\mu\,.$$

We can put this theory on a Riemannian manifold by the usual covariantisation: $\delta_{\mu\nu} \to g_{\mu\nu}$, etc.
Recall that such a procedure is not unique: We can always add terms involving the curvature, that vanish in the flat space limit.

Example: Massless chiral supermultiplet (3d, $\mathcal{N} = 2$ SUSY):

$$\mathcal{L}_0 = g^{\mu\nu}\partial_\mu\phi\partial_\nu\tilde{\phi} - i\tilde{\psi}\gamma^\mu\nabla_\mu\psi - F\tilde{F} + \alpha R\tilde{\phi}\phi + \cdots .$$

This is not supersymmetric. We need additional corrections. The supersymmetry algebra itself is going to be modified.

Remark: We do *not* require nor use conformal invariance.

The procedure obviously relies on diffeomorphism invariance.
Even though we fix the metric once and for all. We should think of the metric as a background field.

Possible because we consider theories with a conserved energy-momentum operator $T_{\mu\nu}$.
(It is like using $U(1)$ gauge invariance to determine the correct coupling to a background magnetic field in a theory with a conserved current.)

Background supergravity fields

In any supersymmetric theory, we have a conserved supercurrent $S_{\mu\alpha}$, which sits in the same supersymmetry multiplet as $T_{\mu\nu}$.

$$\mathcal{S}_\mu \sim \cdots + \theta S_\mu + \theta\gamma^\nu\tilde{\theta}\, T_{\mu\nu} + \cdots$$

The detailed structure of the supercurrent multiplet can vary. The general supermultiplet $\mathcal{S}$ can often be improved to a simpler supercurrent. [Komargodski, Seiberg, 2010; Dumitrescu, Seiberg, 2011]

Festuccia-Seiberg proposal: To describe rigid supersymmetry in curved space, we should "weakly gauge" the supercurrent multiplet.
$\Rightarrow$ Consider background supergravity [Festuccia, Seiberg, 2011]

Metric and its superpartners form a "background superfield".

- We can think of rigid supersymmetry as some $M_p \to \infty$ of a full-fledged supergravity theory.
- For any given supercurrent there exists a corresponding supergravity multiplet $(g_{\mu\nu}, \Psi_\mu, X)$. E.g. "old-minimal" or "new-minimal" in 4d. [Stelle, West, 1978; Ferrara, van Nieuwenhuizen, 1978; Sohnius, West, 1981]
- We should not impose any gravitational equation of motion. Need to consider off-shell formalism for the supergravity of interest.
- In the rigid limit, $\Psi_\mu = 0$, $\delta\Psi_\mu = 0$. (*Much* simpler than SUGRA.)
- Given a set of background fields $(g_{\mu\nu}, X)$, we have one rigid supersymmetry for each spinor ζ solving the generalised Killing spinor equation

$$\delta_\zeta \Psi_\mu = D(g, X)\zeta = 0\,.$$

- For $\mathcal{N}=1$ supersymmetric theories in four dimensions, this program has been recently completed. [Dumitrescu, Festuccia, Seiberg, 2012; Klare, Tomassielo, Zaffaroni, 2012; Dumitrescu, Festuccia, 2012]
  Complete classification of supersymmetric backgrounds.
  Rigid supersymmetry $\leftrightarrow$ hermitian structure on $\mathcal{M}$.

We will apply the backround supergravity formalism to *R-symmetric* $\mathcal{N}=2$ supersymmetric theories in three dimensions.

A technical difficulty to tackle is that the corresponding $\mathcal{N}=2$ off-shell supergravity has not been worked out, to date. (At least in components.)

Linearised supergravity is good enough. [C.C., Dumitrescu, Festuccia, Komargodski, Seiberg, 2012]

3d R-multiplet and supergravity background fields

In a 3d $\mathcal{N} = 2$ theory with an R-symmetry, we have an R-multiplet

$$\begin{aligned}\mathcal{R}_\mu &= j_\mu^{(R)} - i\theta S_\mu - i\tilde{\theta}\tilde{S}_\mu - (\theta\gamma^\nu\tilde{\theta})\left(2T_{\mu\nu} + i\epsilon_{\mu\nu\rho}\partial^\rho J^{(Z)}\right)\\ &\quad -i\theta\tilde{\theta}\left(2j_\mu^{(Z)} + i\epsilon_{\mu\nu\rho}\partial^\nu j^{(R)\rho}\right) + \cdots\ .\end{aligned}$$

There exists a metric multiplet

$$\left(g_{\mu\nu}, \Psi_\mu, A_\mu, C_\mu, H\right), \qquad\qquad V_\mu \equiv -\epsilon_{\mu\nu\rho}\partial^\nu C^\rho\ .$$

The linearised coupling to the R-mutiplet operators is

$$-T_{\mu\nu}h^{\mu\nu} + j_\mu^{(R)}\left(A^\mu - \frac{3}{2}V^\mu\right) - ij_\mu^{(Z)}C^\mu + J^{(Z)}H + \Psi^\mu S_\mu + c.c.$$

(here $g_{\mu\nu} = \delta_{\mu\nu} + 2h_{\mu\nu}$)

Further remarks:

- The linear submultiplet

$$\mathcal{J}^{(Z)} = J^{(Z)} - \frac{1}{2}\theta\gamma^\mu S_\mu + \frac{1}{2}\tilde{\theta}\gamma^\mu \tilde{S}_\mu + i\theta\tilde{\theta}T^\mu_\mu - (\theta\gamma^\mu\tilde{\theta})j^{(Z)}_\mu + \cdots ,$$

  can be improved to zero in a superconformal theory.
  The background fields H and C_μ couple to redundant operators in a CFT.
- All the supergravity background fields would be real in a unitary theory.
- We allow for complex H, A_μ, V_μ. Metric is real.

3d Killing spinor equations

For R-symmetric theories, 3d rigid supersymmetry is governed by:

$$
\begin{aligned}
(\nabla_\mu - iA_\mu)\zeta_\alpha &= -\frac{1}{2}H(\gamma_\mu\zeta)_\alpha - \frac{1}{2}\epsilon_{\mu\nu\rho}V^\nu(\gamma^\rho\zeta)_\alpha - iV_\mu\zeta_\alpha\,, \\
(\nabla_\mu + iA_\mu)\tilde{\zeta}_\alpha &= -\frac{1}{2}H(\gamma_\mu\tilde{\zeta})_\alpha + \frac{1}{2}\epsilon_{\mu\nu\rho}V^\nu(\gamma^\rho\tilde{\zeta})_\alpha + iV_\mu\tilde{\zeta}_\alpha\,.
\end{aligned}
$$

The spinors ζ_α, $\tilde{\zeta}_\alpha$ are sections of $S \otimes L$, $S \otimes L^{-1}$.
Real part of A_μ is a $U(1)_R$ connection.

Note that these equations subsume all the Killing spinor equations used in recent literature on 3d. In particular the round 3-sphere corresponds to $H = -i$, $A_\mu = V_\mu = 0$.

One supercharge: almost contact structure

An almost contact structure on $\mathcal{M}$ is the triplet (ξ, η, Φ) such that

$$\eta(\xi) = 1\,, \qquad \Phi \circ \Phi = -\mathbf{1} + \xi \otimes \eta$$

It is metric-compatible if $g(X, Y) = g(\Phi(X), \Phi(Y)) + \eta(X)\eta(Y)\,.$

On a three-dimensional Riemannian manifold, any real (co)-vector field η_μ of unit norm defines such a structure:

$$\xi^\mu = \eta^\mu\,, \qquad \Phi_\mu{}^\nu = \epsilon_\mu{}^{\nu\rho}\eta_\rho\,.$$

There always exists such a structure on $(\mathcal{M}, g_{\mu\nu})$.
The frame bundle structure group is restricted to $U(1)$.

Consider a solution ζ of the first Killing spinor equation on $(\mathcal{M}, g_{\mu\nu})$. It is nowhere vanishing, and completely determined by its value at a point.

Useful bilinear

$$\eta^\mu = \frac{\zeta^\dagger \gamma^\mu \zeta}{\zeta^\dagger \zeta}$$

Satisfies $\eta_\mu \eta^\mu = 1$.

Supersymmetry (one supercharge) on $\mathcal{M}_3$
$\Leftrightarrow$ metric-compatible almost contact structure $(\mathcal{M}_3, g_{\mu\nu}, \eta_\mu)$

Using the Killing spinor equation, one can solve explicitly for the supergravity background fields in term of the almost contact structure:

$$H = \frac{1}{2}\nabla_\mu \eta^\mu + \frac{i}{2}\Phi^{\mu\nu}\nabla_\mu \eta_\nu + i\lambda_{(\eta)} \,,$$

and similar expressions for V_μ, A_μ.

Two supercharges: Seifert manifold

If we have one ζ and one $\tilde{\zeta}$, we can define the two almost contact structures η_μ, $\tilde{\eta}_\mu$, and also the Killing vector

$$K^\mu = \tilde{\zeta}\gamma^\mu\zeta\,.$$

We restrict our attention to the case where K^μ is real. That implies $\eta_\mu = -\tilde{\eta}_\mu = \Omega^{-1}K_\mu$.

We can introduce local coordinates $(\tau, z, \bar{z})$ and

$$ds^2 = c(z,\bar{z})^2 dz d\bar{z} + \eta^2\,, \qquad \eta = \Omega(z,\bar{z})(d\tau + b(z,\bar{z})dz + \bar{b}(z,\bar{z})d\bar{z})$$

$U(1)$ bundle over Riemann surface: Seifert manifold.

Four supercharges

Maximally supersymmetric background requires $A_\mu = V_\mu$, $\partial_\mu H = 0$, $\partial_\mu V^2 = 0$. Several cases:

- $V_\mu = 0$, $\mathcal{M}_3 = S^3$, T^3 or H^3.
- $H = 0$, $\mathcal{M} = \mathbb{R} \times \Sigma$.
- $H = ih$, $h \in \mathbb{R}$. $\mathcal{M}_3$ is a particular $U(1)$-fibration over a surface of constant curvature.

The last case includes the "Imamura-Yokoyama three-sphere" [Imamura, Yokoyama, 2011], which is a $SU(2) \times U(1)$-isometric squashed sphere.

$$ds^2 = (\mu^1)^2 + (\mu^2)^2 + h^2(\mu^3)^3$$

with $H = ih$ and $V_\mu dx^\mu = 2\sqrt{h^2 - \frac{1}{r^2}} e^3$.

Supersymmetry algebra and supermultiplets

Consider a supersymmetric manifold $(\mathcal{M}_3, g_{\mu\nu}, A_\mu, V_\mu, H)$, with some supersymmetries ζ and/or $\tilde{\zeta}$.

One can work out the generalisation of the off-shell supersymmetry multiplets from $\mathcal{N} = 2$ flat-space supersymmetry to our case.

The supersymmetry algebra is

$$\delta_\zeta^2 \varphi = 0\,, \qquad \delta_{\tilde{\zeta}}^2 \varphi = 0\,,$$

$$\{\delta_\zeta, \delta_{\tilde{\zeta}}\}\varphi = -2i\mathcal{L}_K^{(A-\frac{1}{2}V)}\varphi + 2i\tilde{\zeta}\zeta\,(Z - \Delta_\varphi H)\,\varphi$$

on a field φ of R-charge Δ_φ. Here $K = \tilde{\zeta}\gamma\zeta$.

Consider a ζ-sypersymmetry ($\tilde{\zeta}$ is similar). The real multiplet transformation rules become

$$
\begin{aligned}
\delta_\zeta\, C &= i\zeta\chi \\
\delta_\zeta\, \chi_\alpha &= \zeta_\alpha M \\
\delta_\zeta\, \tilde{\chi}_\alpha &= -(\gamma^\mu\zeta)_\alpha(\partial_\mu C - i a_\mu) - \zeta_\alpha \sigma \\
\delta_\zeta\, M &= 0 \\
\delta_\zeta\, \tilde{M} &= 2\zeta\lambda - 2iD_\mu(\zeta\gamma^\mu\tilde{\chi}) + 4iH\zeta\tilde{\chi} \\
\delta_\zeta\, a_\mu &= -i\zeta\gamma_\mu\tilde{\lambda} + \partial_\mu(\zeta\chi) \\
\delta_\zeta\, \sigma &= -\zeta\tilde{\lambda} \\
\delta_\zeta\, \lambda_\alpha &= i\zeta_\alpha(D + \sigma H) - i(\gamma^\mu\zeta)_\alpha(\epsilon_{\mu\nu\rho}\nabla^\nu a^\rho + iV_\mu\sigma + \partial_\mu\sigma) \\
\delta_\zeta\, \tilde{\lambda}_\alpha &= 0 \\
\delta_\zeta\, D &= \nabla_\mu(\zeta\gamma^\mu\tilde{\lambda}) - iV_\mu\zeta\gamma^\mu\tilde{\lambda} - H\zeta\tilde{\lambda}
\end{aligned}
$$

where $D_\mu = \nabla_\mu - i\Delta_\varphi(A_\mu - \frac{1}{2}V_\mu)$. Similarly we work out the rules for chiral multiplets.

Supersymmetric Lagrangians

From a real multiplet, we have the D-term action

$$S = \int d^3x\sqrt{g}(D - \sigma H - a_\mu V^\mu)$$

In particular, for a vector multiplet this is the FI term.

From this we can derive the vector multiplet kinetic term

$$\mathcal{L}_{YM} = \frac{1}{4} f^{(V)}_{\mu\nu} f^{(V)\mu\nu} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - i\tilde{\lambda}\gamma^\mu(D_\mu + \frac{i}{2}V_\mu)\lambda - \frac{1}{2}(D+\sigma H)^2 + \frac{i}{2}H\lambda\tilde{\lambda}\,,$$

with $f^{(V)}_{\mu\nu} = f_{\mu\nu} + i\epsilon_{\mu\nu\rho}V^\rho\sigma$.

The Chern-Simons term is simply

$$\mathcal{L}_{CS} = i\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho + 2i\lambda\bar{\lambda} - 2\sigma D\,.$$

We can similarly work out the matter Lagrangian (chiral multiplet of R-charge Δ coupled to vector multiplet):

$$\begin{aligned}\mathcal{L} &= \mathbf{D}_\mu \phi \mathbf{D}^\mu \tilde{\phi} - i\tilde{\psi}\gamma^\mu \mathbf{D}_\mu \psi - F\tilde{F} + \phi D\tilde{\phi} + \phi\sigma^2\tilde{\phi} - i\psi\sigma\tilde{\psi} \\ &\quad + i\sqrt{2}(\phi\tilde{\lambda}\tilde{\psi} + \tilde{\phi}\lambda\psi) + H(\Delta - \frac{1}{2})(2\phi\sigma\tilde{\phi} - i\psi\tilde{\psi}) \\ &\quad + \left(\Delta(\Delta - \frac{1}{2})H^2 - \frac{\Delta}{4}R + \frac{\Delta - \frac{1}{2}}{2}V_\mu V^\mu\right)\phi\tilde{\phi}\end{aligned}$$

with

$$\mathbf{D}_\mu = \nabla_\mu - i\Delta\left(A_\mu - \frac{3}{2}V_\mu\right) - i(\Delta - \Delta_0)V_\mu - ia_\mu \,.$$

- Note that the couplings depend heavily on the R-charge.
- Superconformal value at $\Delta = \frac{1}{2}$ (free field).
- $\mathcal{L}$ reproduces expected R-multiplet operators around flat space.

Metric dependence at first order

One can show that at first order around flat space, $g_{\mu\nu} = \delta_{\mu\nu} + 2h_{\mu\nu}$,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{O}(h_{\mu\nu}^2)\,, \qquad \delta = \delta_0 + \delta_1 + \cdots$$

we must have

$$\mathcal{L}_1 = -h^{\mu\nu}\mathcal{O}_{\mu\nu} = -h^{\mu\nu}(T_{\mu\nu} + \cdots)\,, \qquad \delta_0\mathcal{O}_{\mu\nu} = 0$$

to preserve one supercharge. Such δ_0-closed operators in the R-multiplet are easily classified. In fact they are all δ_0-exact.

- Matching $\mathcal{L}_1$ with the linearized SUGRA Lagrangian, we have a nice check of our solution for the supergravity background fields.
- This δ_0-exactness suggests that the partition function $Z(\mathcal{M}_3, g_{\mu\nu})$ is "quasi-topological' '. This is indeed borne out by the known examples.

Application: τ_{rr} from the squashed three-sphere

In any $\mathcal{N}=2$ superconformal theory, we have

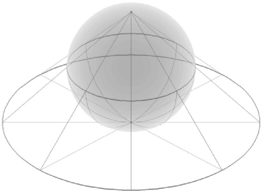
$$\begin{aligned}\langle T_{\mu\nu}(x)T_{\rho\sigma}(0)\rangle &= -\frac{\tau_{rr}}{64\pi^2}(\delta_{\mu\nu}\partial^2-\partial_\mu\partial_\nu)(\delta_{\rho\sigma}\partial^2-\partial_\rho\partial_\sigma)\frac{1}{x^2}\\ &\quad+\frac{\tau_{rr}}{64\pi^2}\left((\delta_{\mu\rho}\partial^2-\partial_\mu\partial_\rho)((\delta_{\nu\sigma}\partial^2-\partial_\nu\partial_\sigma)+(\mu\leftrightarrow\nu)\right)\frac{1}{x^2}\,,\\ \langle j_\mu^{(R)}(x)j_\nu^{(R)}(0)\rangle &= \frac{\tau_{rr}}{16\pi^2}\left(\delta_{\mu\nu}\partial^2-\partial_\mu\partial_\nu\right)\frac{1}{x^2}\end{aligned}$$

determined by a unique parameter τ_{rr}, *at separated points*.
($\tau_{rr}=\frac{1}{4}$ for a free chiral multiplet.)
We would like to compute τ_{rr} as

$$\tau_{rr}\sim\frac{\delta^2}{\delta g_{\mu\nu}\delta g_{\rho\sigma}}Z\sim\frac{\delta^2}{\delta A_\mu\delta A_\nu}Z\,,$$

using the exact results for $Z(S^3)$.

S^3 is conformal to flat space.



Correlations functions in $\mathbb{R}^3$ and S^3 are related by Weyl rescaling. In particular, for a conserved current

$$\langle j_a(x) j_b(y) \rangle_{S^3} = \Omega(x)^{-2} \Omega(y)^{-2} \langle j_a(x) j_b(y) \rangle_{\mathbb{R}^3}$$

with $\Omega(x) = \frac{2}{(1+x^2)}$.

To bring down $j_\mu^{(R)}$ on S^3, we consider a one-parameter family of supersymmetric squashing. The most convenient one is the maximally supersymmetric squashing of [Imamura, Yokoyama, 2011] we discussed before.

The supergravity background fields are

$$H = ih\,, \quad A_\mu = V_\mu = v\,K_\mu\ (K = e^3), \qquad h = \frac{b + b^{-1}}{2}\,, \qquad v = b - b^{-1}$$

with $b > 0$ ($b = 1$ for the round sphere).

We have the coupling $(A_\mu - \frac{3}{2} V_\mu)\, j^\mu_{(R)} = -\frac{1}{2} v\, K_\mu j^\mu_{(R)}$.

All other couplings of the squashing to the CFT are through the parameter h. We can use that $\partial_b h|_{b=1} = 0$ and $\partial_b v|_{b=1} = 2$ to isolate the R-symmetry current. One can see that ($F_b = -\ln Z(S^3_b)$)

$$\partial_b^2 F_b\Big|_{b=1} = -\int_{S^3} d^3x\sqrt{g} \int_{S^3} d^3y\sqrt{g}\, K^\mu(x) K^\nu(y) \langle j_\mu^{(R)}(x) j_\nu^{(R)}(y)\rangle_{S^3}$$

One can evaluate that last integral, to arrive at

$$\mathrm{Re}\,\frac{\partial^2}{\partial b^2} F_b\Big|_{b=1} = \frac{\pi^2}{2}\tau_{rr}$$

in term of the free energy F_b of the $\mathcal{N}=2$ SCFT on the squashed sphere.

- Since an exact formula is known for F_b, (at least) for any SCFT described in the UV by a YM-CS-matter theory, the above is an exact and explicit formula for τ_{rr}.
- There can be *contact term contributions* to the integrated two-point functions. But one can show [C.C., Dumitrescu, Festuccia, Komargodski, Seiberg] that contact terms contribute only to the *imaginary part* of F_b. That is why we must consider the real part in the above.

Some simple examples:

- Free chiral multiplet. A very explicit form of F_b is

$$F_b = i\int_0^\infty \frac{dx}{x}\left(\frac{\sin(2x(z-\omega)}{\sin(\omega_1 x)\sin(\omega_2 x)} - \frac{z-\omega}{\omega_1\omega_2 x}\right)$$

  with $z = \frac{i}{2}h$. One can compute $\partial_b^2 F_b|_{b=1} = \frac{\pi^2}{8}$.

- Large N theories with a $AdS_4 \times X_7$ dual. In this case, F_b simplifies to [Imamura, Yokoyama, 2011; Martelli, Passias, Sparks, 2005]

$$F_b = \frac{(b+b^{-1})^2}{4}F\,, \qquad F \equiv F_{b=1}\,.$$

  Moreover we have that $F = \frac{\pi^2}{4}\tau_{rr}$ at large N [Barnes, Gorbatov, Intriligator, Wright, 2005]. Our relation follows.

Conclusions

Summary

- Any orientable three-manifold preserve (at least) one supercharge (we can put any $\mathcal{N}=2$ supersymmetric R-symmetric theory on $\mathcal{M}_3$ supersymmetrically).
- Supersymmetry on $\mathcal{M}_3$ is associated to an almost contact structure.
- We developed a general formalism for curved space rigid supersymmetry: supermultiplets, Lagrangians,... These results can be seen as a rigid limit of some as-yet unwritten off-shell "new-minimal" supergravity in 3d.
- As a first physical application of these technical results, we presented an exact formula for the two-point function of $T_{\mu\nu}$ in any $\mathcal{N}=2$ SCFT in 3d.

Outlook

Further applications of our formalism :

- Our results set the ground for a general discussion of localisation. The next question is: Can we write down an exact formula for $Z(M_3, g_{\mu\nu}, \eta_\mu)$? [Work in progress.]

  The localisation locus on a general almost contact manifold is still relatively simple. The vector multiplet localises to solutions of the Bogomolny equation (BPS monopole configurations).
- Possible to understand systematically the metric-dependence (or independence) of $Z(\mathcal{M}_3)$.
- It is easy to dimensionally reduce to 2d $\mathcal{N} = (2,2)$ theories with an $U(1)_V$ R-symmetry. This makes the link with the recent work on the round S^2 [Benini, Cremonesi, 2012; Doroud, Gomis, Le Floch, Lee, 2012] and allows to generalise it in various directions. [In progress]