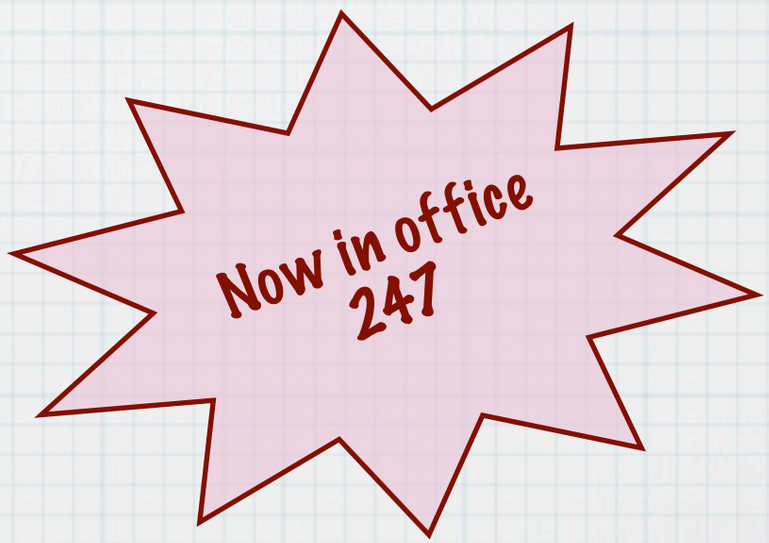
Fermi Surfaces in N=4 SYM

Some recent calculations by **Christopher Rosen**, Oliver DeWolfe, and Steve Gubser



Fun

Time

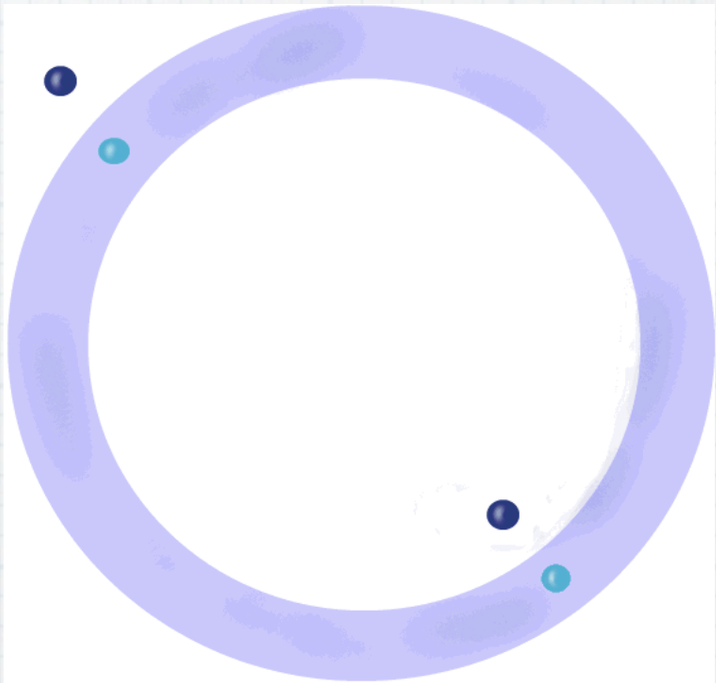
Lately...

...but Fermi Surfaces:

In Field Theory

A Fermi surface is the place, k_F, where

$$G_R{}^{-1}(\omega = 0, k = k_F) = 0$$



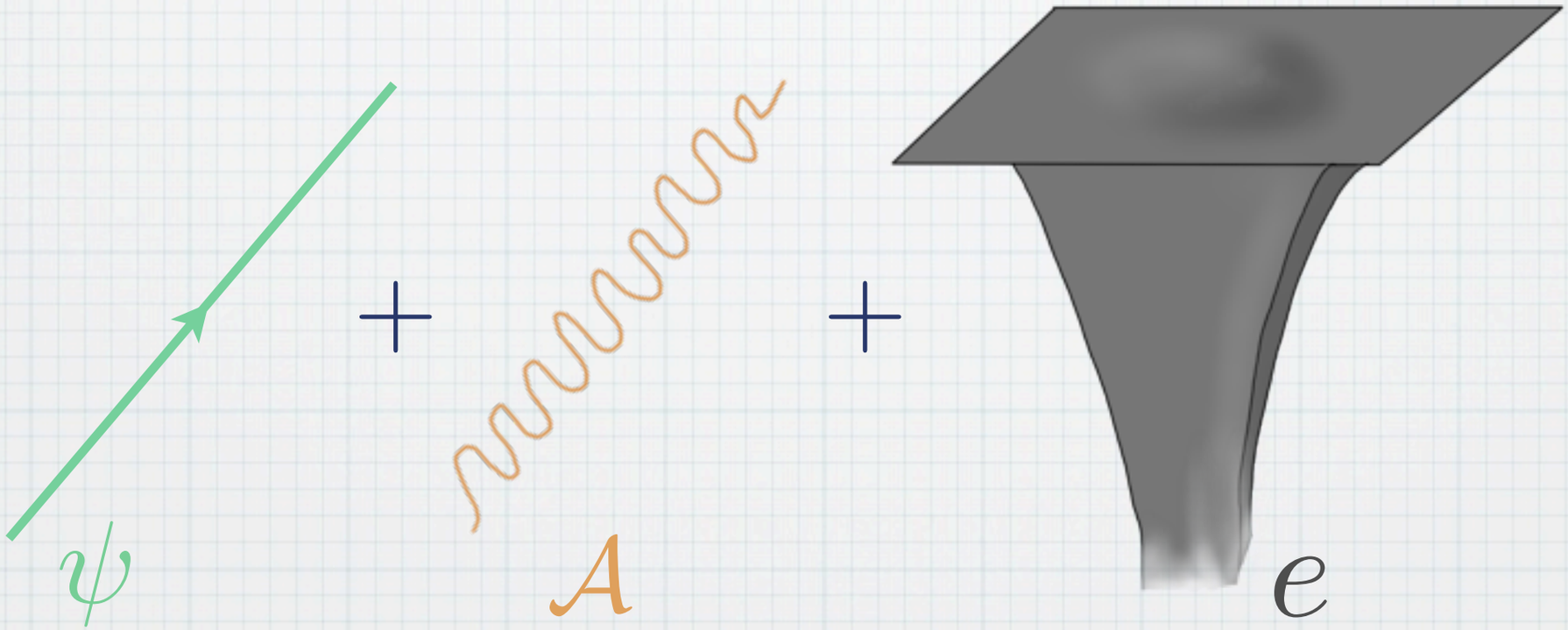
Low energy excitations about this surface can be characterized by

$$G_R = \frac{Z}{\omega - v_F(k - k_F) + \Sigma(\omega, k)}$$

...but Fermi Surfaces:

In Gravity Theories

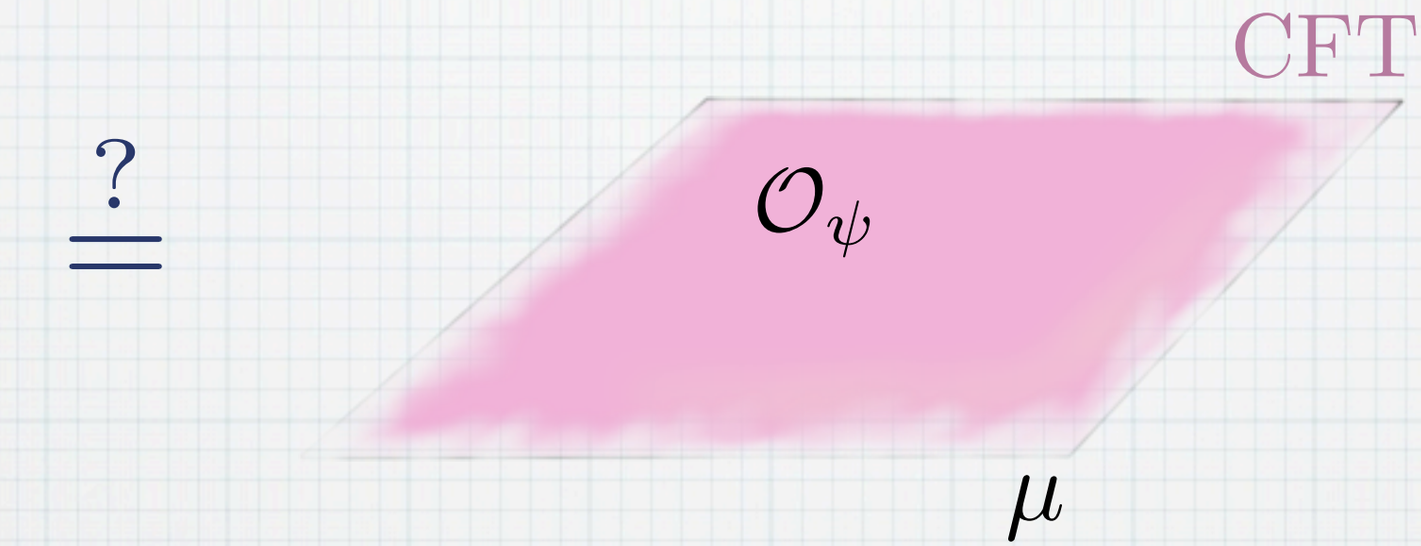
Bottom up finite density physics



...but Fermi Surfaces:

In Gravity Theories

Bottom up finite density physics

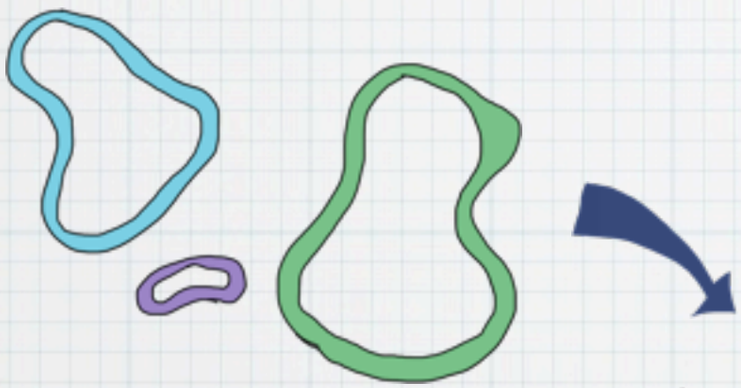


$$(i\Gamma^\mu \nabla_\mu + q\,\Gamma^\mu \mathcal{A}_\mu - \ m \ + \ldots)\,\psi = 0$$

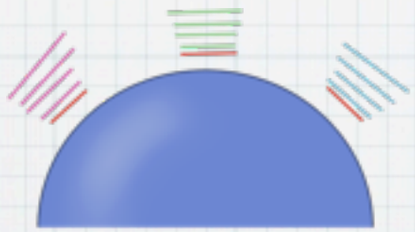
...but Fermi Surfaces:

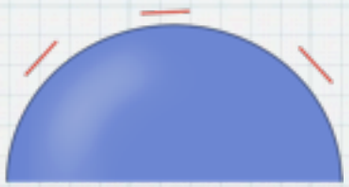
In Gravity Theories

The top down alternative



IIB SUGRA in D=10





N=8 gauged SUGRA in D=5

$$\mathcal{L}_B\left[e, \mathcal{A}_i, \varphi_j\right]$$

...but Fermi Surfaces:

In Gravity Theories

What about the fermions?

Find a background, study spin-1/2 fluctuations...

$$\left(i\Gamma^{\mu}\nabla_{\mu} + q_j\Gamma^{\mu}\mathcal{A}^j_{\mu} - m(\varphi) + ip_j(\varphi)\mathcal{F}^j_{\mu\nu}\Gamma^{\mu\nu}\right)\psi = 0$$

Important: fermion properties are no longer arbitrary

Fermi Surface Embeddings

In Gravity Theories

Which Background?

Try the "2+1 Q" BHs ⟷ N=4 SYM at (2x) finite density, T

$$\mathrm{d}s^2 = e^{2A(r)}\left(-h(r)\,\mathrm{d}t^2 + \mathrm{d}\vec{x}^2\right) - \frac{e^{2B(r)}}{h(r)}\mathrm{d}r^2$$

$$a = \Phi_1(r)\mathrm{d}t \qquad \mathcal{A} = \Phi_2(r)\mathrm{d}t$$

$$\varphi = \phi(r)$$

The functions A, h, B, Φ1, Φ2, and ϕ are cumbersome but explicitly known

Fermi Surface Embeddings

In Gravity Theories

Workflow

Fix Background

Fermi Surface Embeddings

In Gravity Theories

Workflow

Fix Background → **Solve Dirac EQ at ω=0**

$$\psi_{r\to\infty} \sim A(k)\sqrt{r} + B(k)r^{-3/2}$$

where

$$\delta S_{\mathrm{CFT}} = \int \mathrm{d}^4x\, A(x)\mathcal{O}_\psi(x)$$

Fermi Surface Embeddings

In Gravity Theories

Workflow

Fix Background

Solve Dirac EQ at ω=0

Tune k to look for a kF

$$\psi_{r\to\infty} \sim A(k)\sqrt{r} + B(k) r^{-3/2}$$

where

$$\delta S_{\mathrm{CFT}} = \int \mathrm{d}^4x\, A(x)\mathcal{O}_\psi(x)$$

$$G_R(\omega = 0, k) \sim \frac{B(k)}{A(k)}$$

so

$$A(k_F) = 0$$

Fermi Surface Embeddings

In Gravity Theories

Finite frequency fluctuations

$$G_R = \frac{Z}{\omega - v_F(k - k_F) + \Sigma(\omega, k)}$$

In the extremal 2+1 system, controlled by IR AdS2:

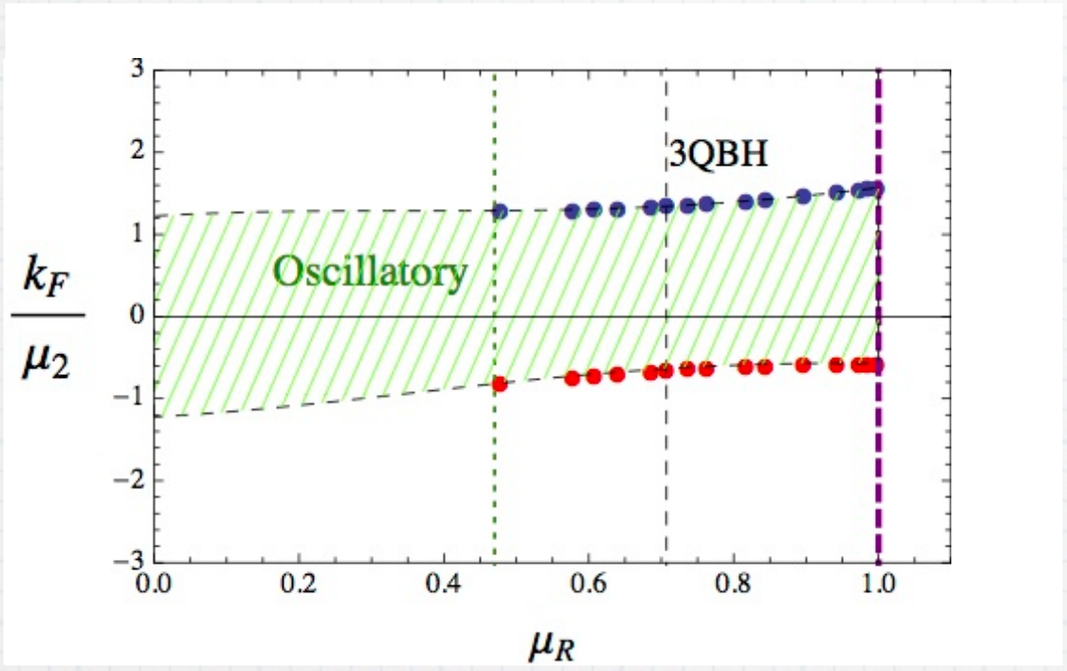
$$\Sigma(\omega, k) \sim e^{i\gamma_{k_F}} \omega^{2\nu_{k_F}}$$

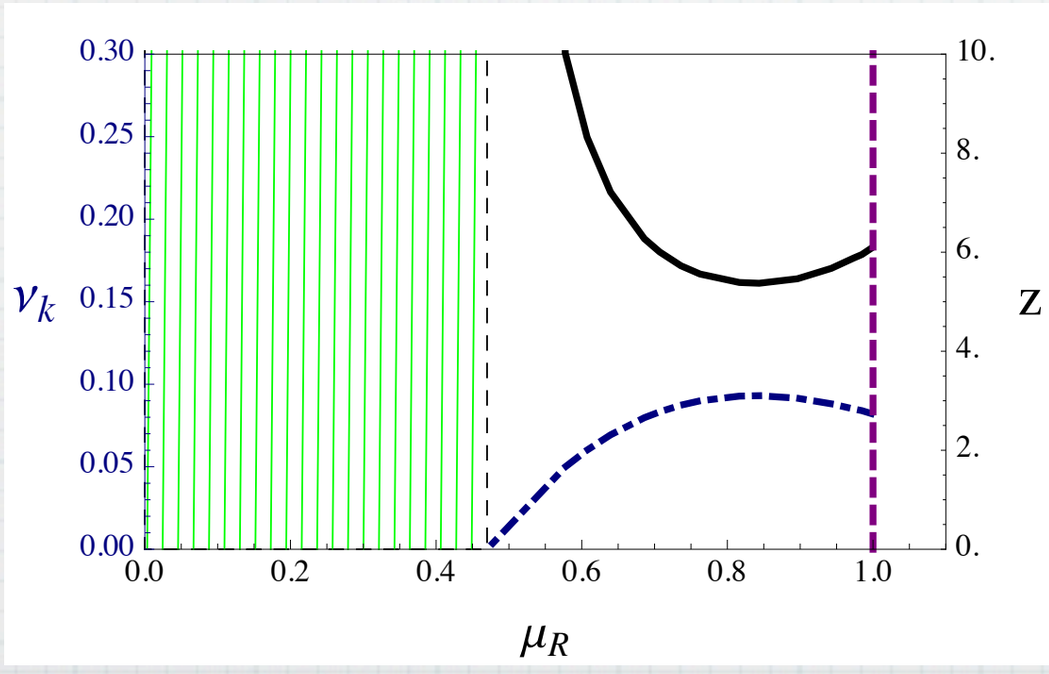
In these embeddings, self energy dominates...

$$\omega_* \sim (k - k_F)^z \qquad \text{where} \qquad z \equiv \frac{1}{2\nu_{k_F}}$$

Fermi Surface Embeddings

Lessons from the 2+1 Charge Zoo





- Fermion modes dual to Tr Fλ have no Fermi surface

- Fermion modes dual to Tr λZ may have 0 or 1 or 2 Fermi surfaces, depending on their charge

- These systems are almost all non-Fermi liquids, but there exists one case resembling a MFL

Fermi Surface Embeddings

The extremal 2-Charge Solution

Background

$$A(r) = \log\frac{r}{L} + \frac{1}{3}\log\left(1+\frac{Q^2}{r^2}\right)$$

$$B(r) = -\log\frac{r}{L} - \frac{2}{3}\log\left(1+\frac{Q^2}{r^2}\right)$$

$$h(r) = 1 - \frac{Q^4}{(r^2+Q^2)^2}$$

$$\boxed{\phi(r) = \sqrt{\frac{2}{3}}\log\left(1+\frac{Q^2}{r^2}\right)}$$

This is important!

$$\Phi(r) = \frac{Q}{2L}\left(1 - \frac{Q^2}{r^2+Q^2}\right)$$

Fermi Surface Embeddings

The extremal 2-Charge Solution

Fermi surfaces exist

So do novel features at finite ω...

Near the horizon, bulk fermions are "gapped":

$$\psi_{r\to 0} \sim e^{-\frac{1}{2r}\sqrt{\frac{Q^2}{2}-\omega^2}}$$

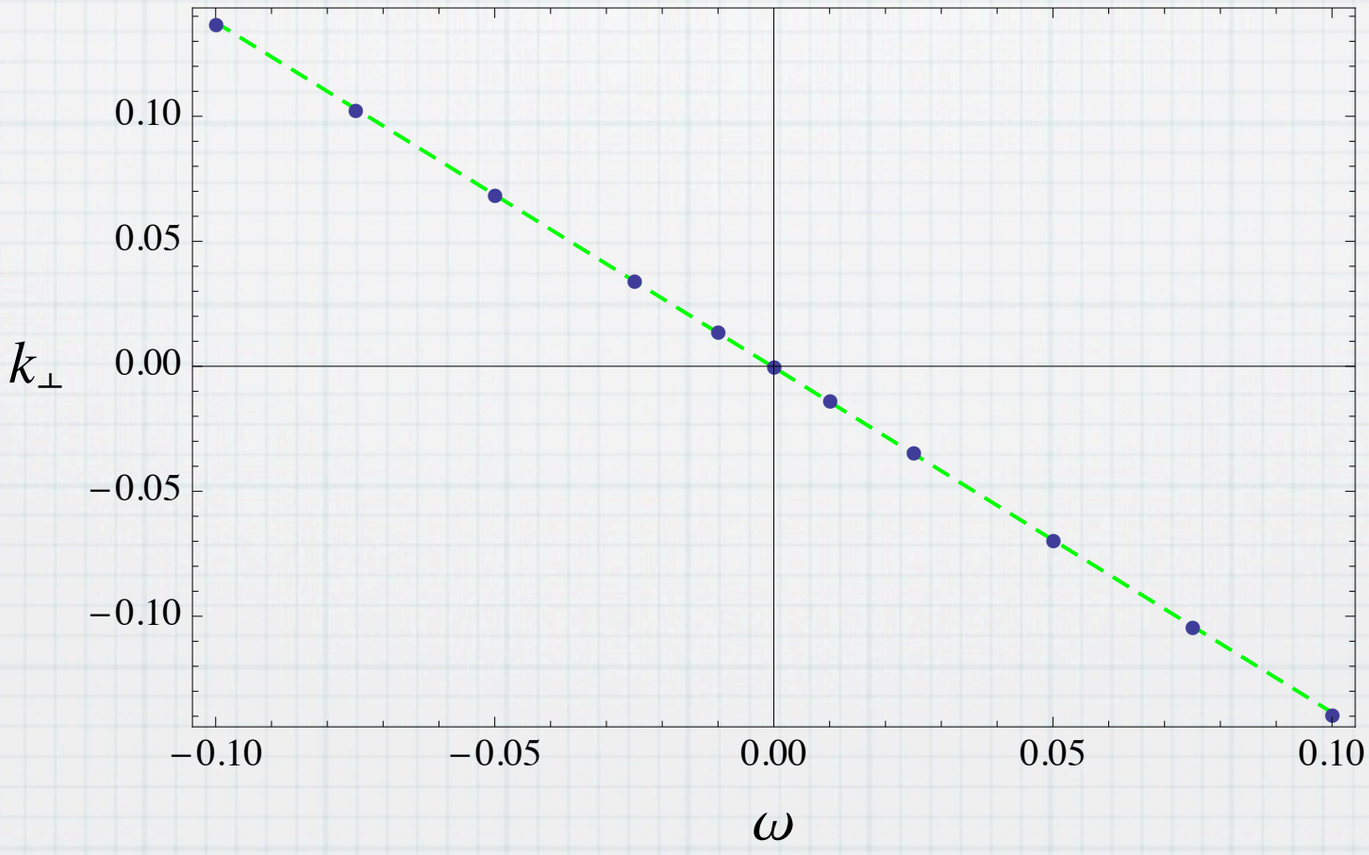
for ω < ω* bulk modes damped, expect qp's

for ω > ω* bulk modes oscillatory, expect field theory dissipation

Fermi Surface Embeddings

The extremal 2-Charge Solution

Low Energy Excitations

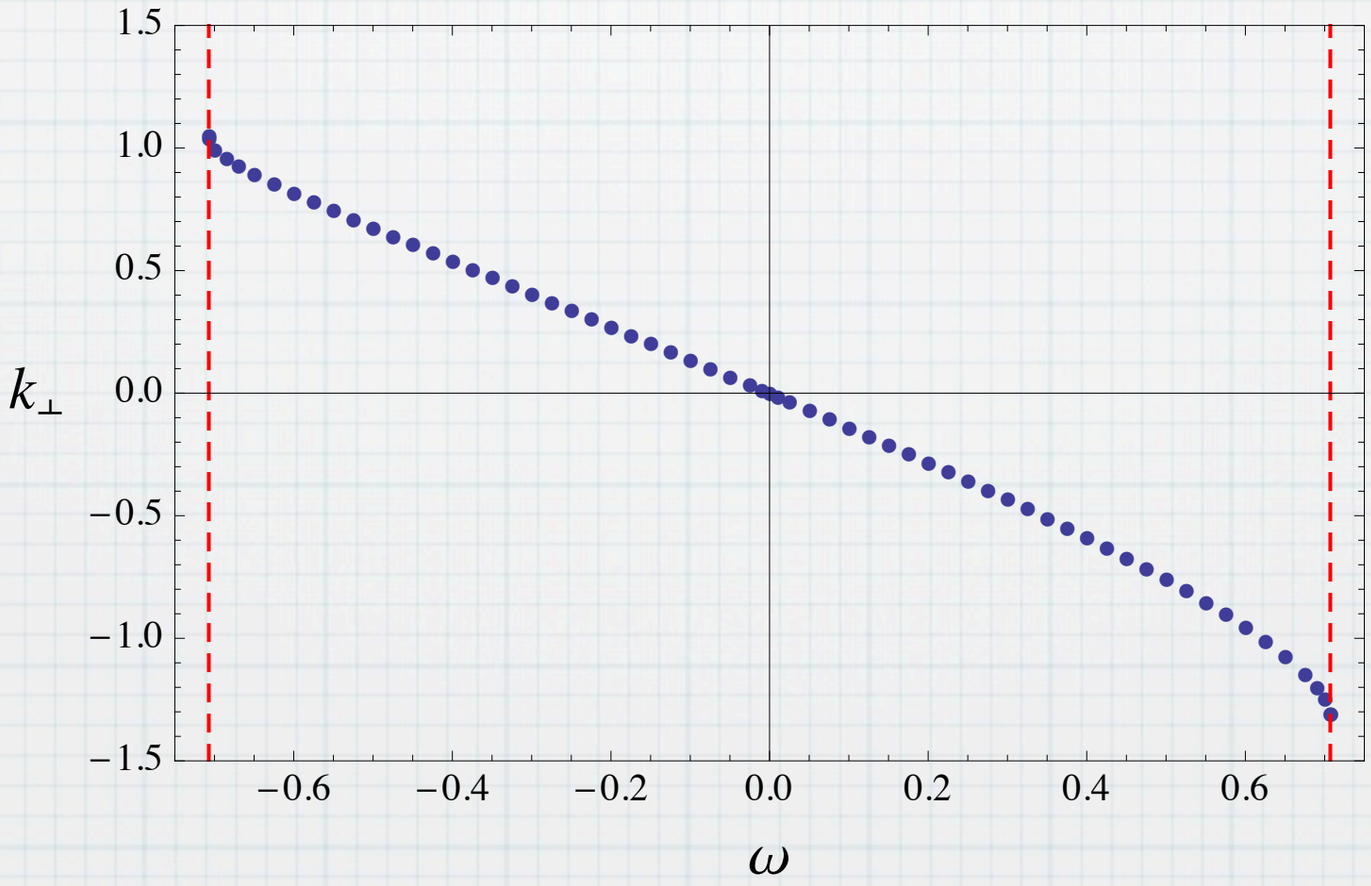


More like a Fermi liquid?

Fermi Surface Embeddings

The extremal 2-Charge Solution

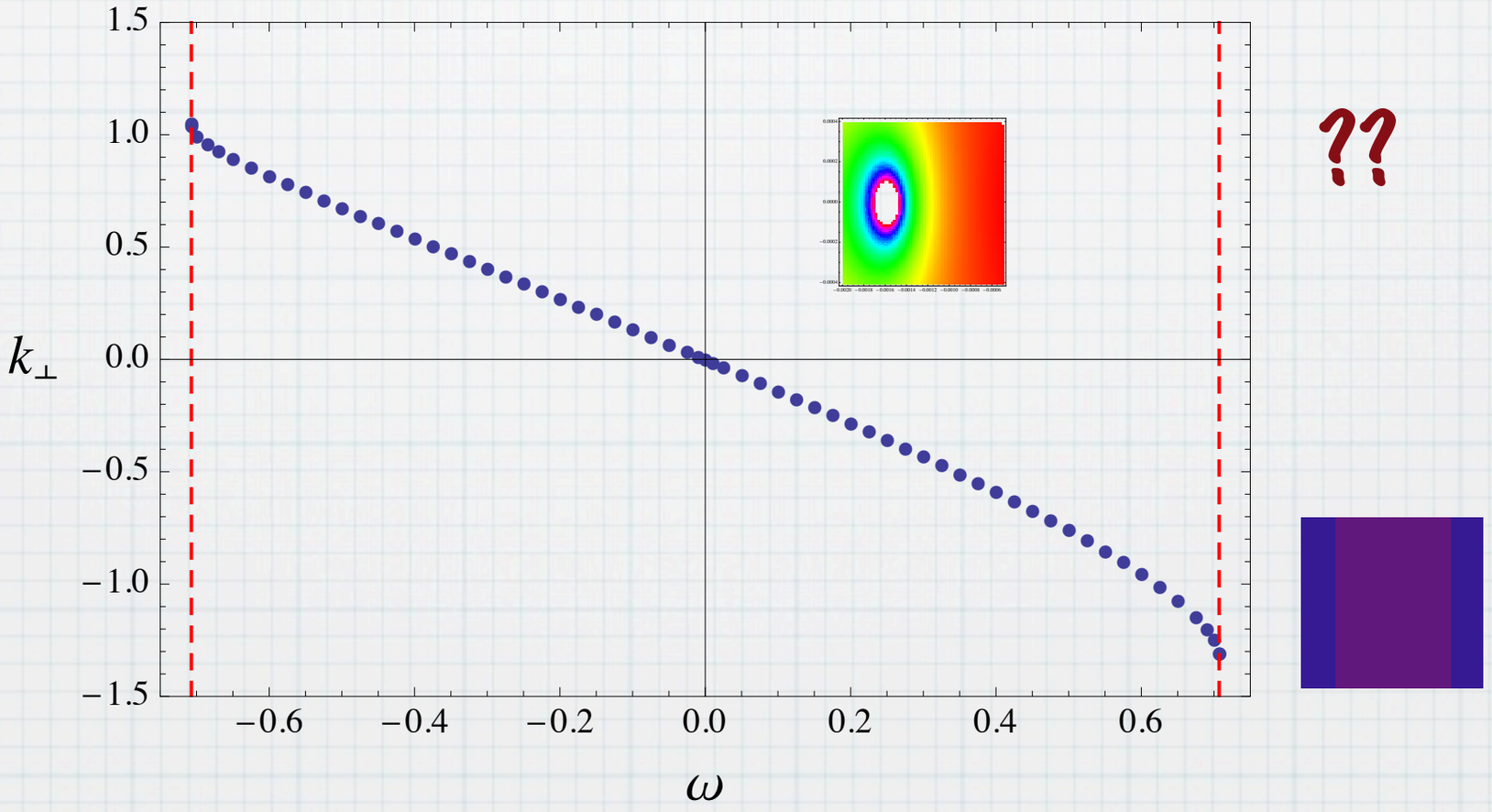
Low Energy Excitations



Fermi Surface Embeddings

The extremal 2-Charge Solution

Low Energy Excitations



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Up and Coming

In the 2+1-Q BHs

Need to understand instabilities better

In the 2-Q BH

Where are the quasi-normal modes??

Can a resolution of the singularity in terms of a higher dimension theory teach us anything important?

Lots more to do...