Inhomogeneous Holographic Superconductors

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Motivation for a holographic point of view

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  - Physics near quantum critical points as suspected by simple models? (Hertz/Millis). Or should we expect more into the simple picture?

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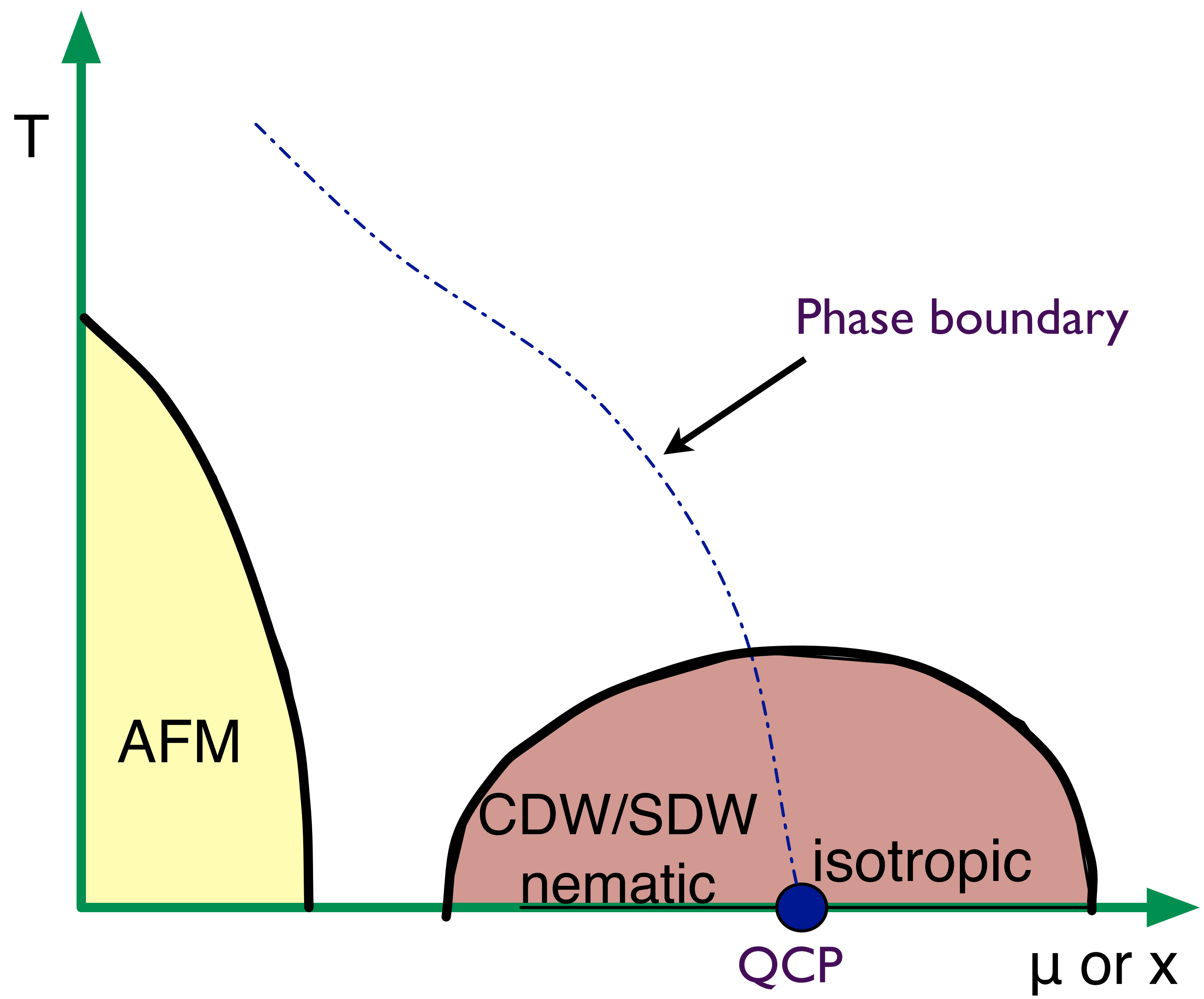
- Is the condensed matter phenomenology complete ?
  - Physics near quantum critical points as suspected by simple models? (Hertz/Millis). Or should we expect more into the simple picture?
  - Is a holographic superconductor “the same” as a BCS superconductor ?

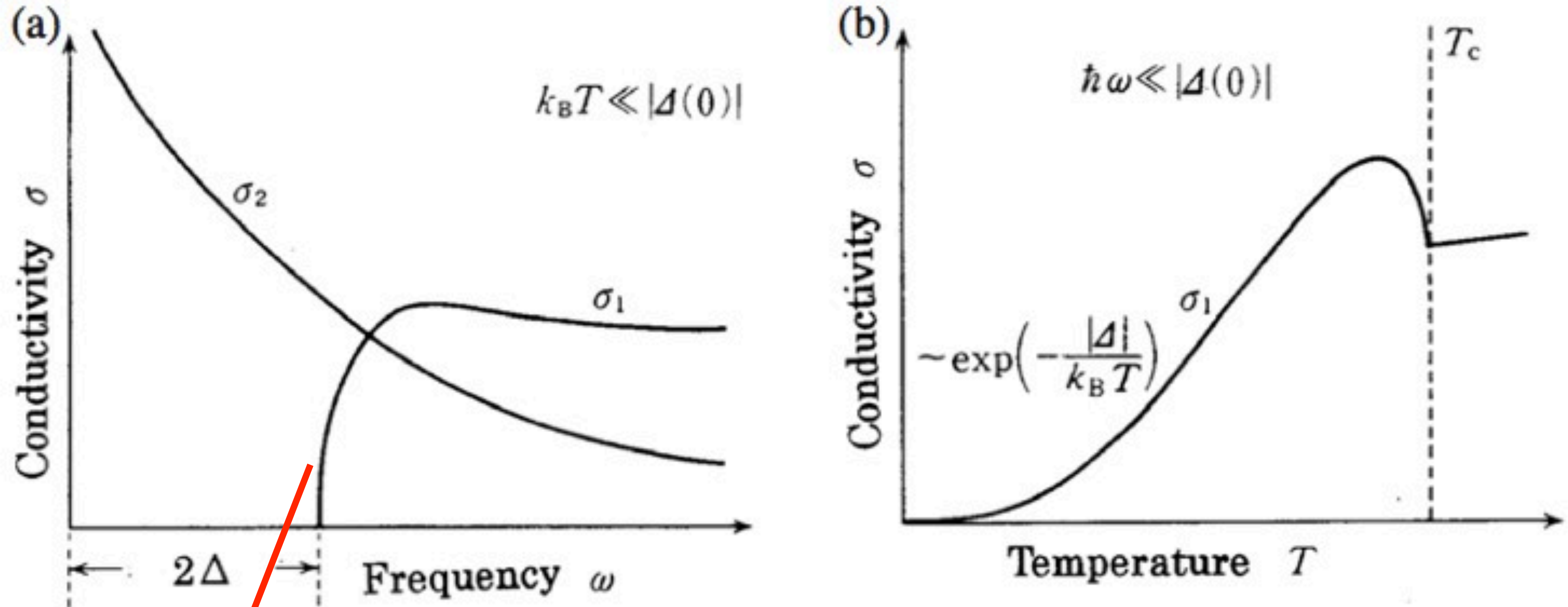
Physics near quantum critical points

- Superconductivity in cuprates hides a putative quantum critical point
- Its location and signatures are masked by superconductivity
- Are there predictions that can guide experiments inside the superconductor?

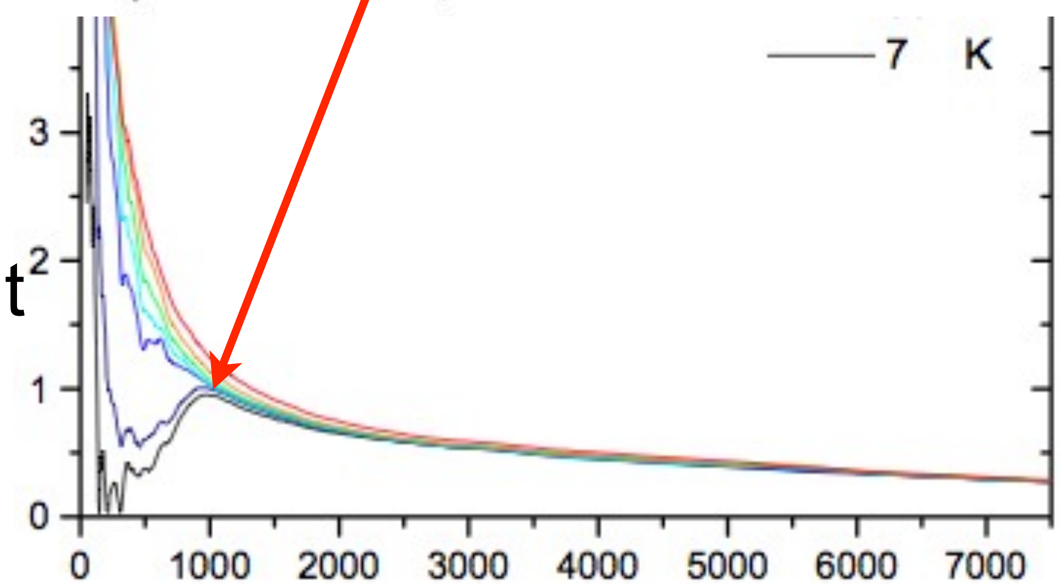


The problem of the superconducting AC conductivity

- Typical theory



- BSCCO experiment



[D. van der Marel *et al.* 2003]

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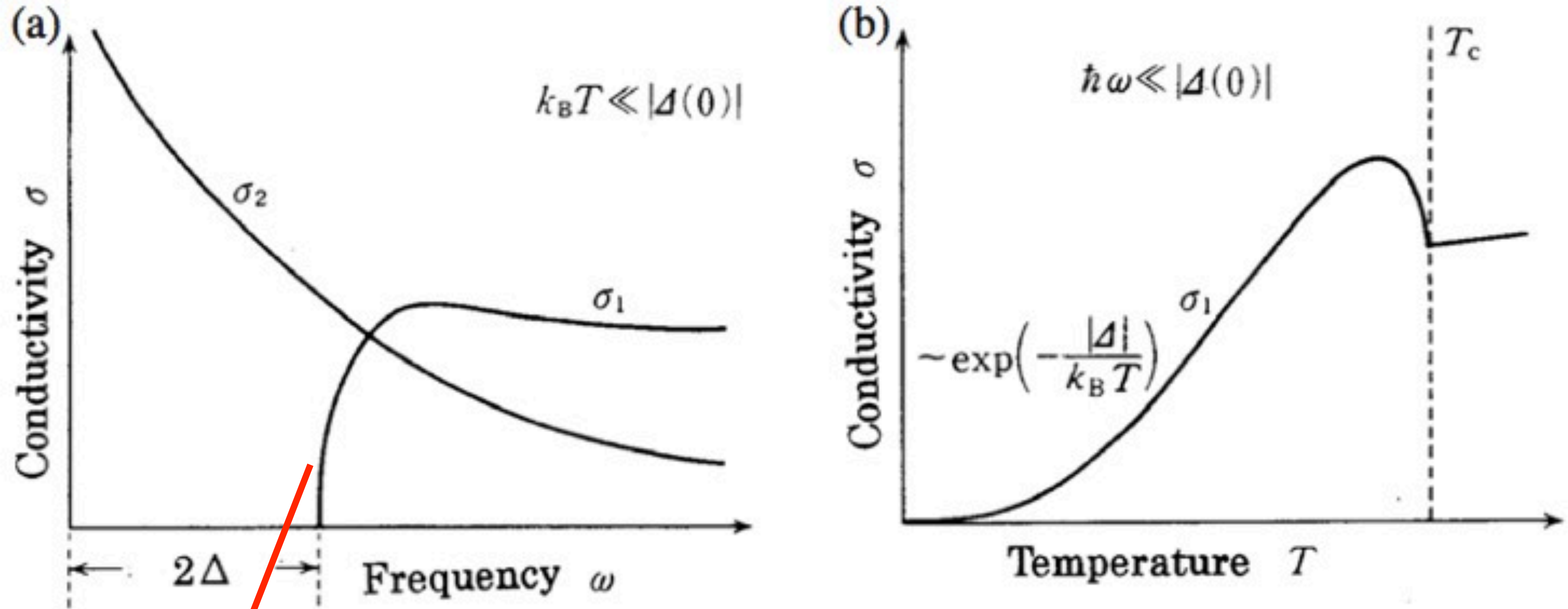

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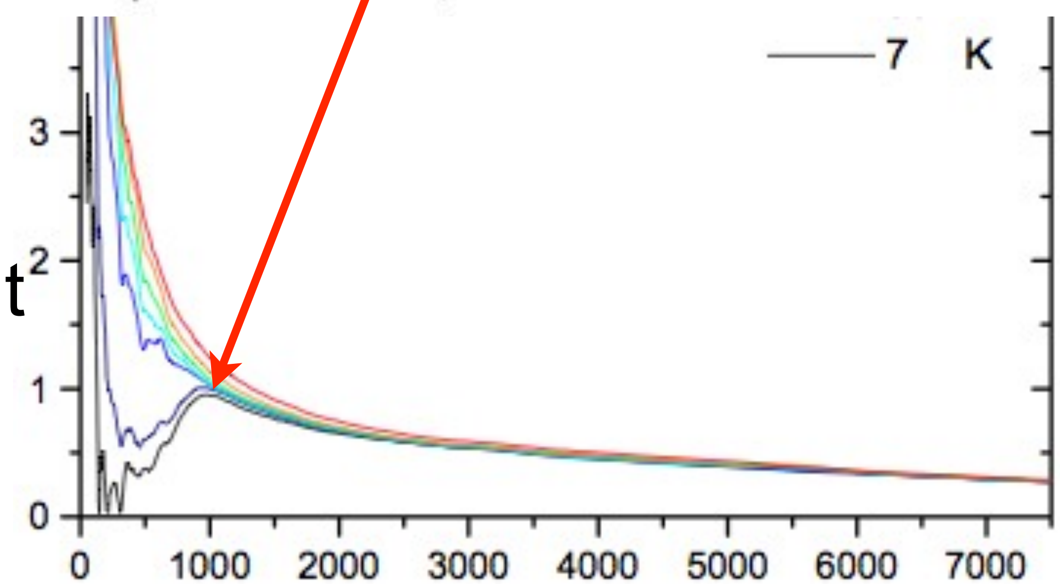

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- Holographic systems provide a quantum critical point, are there fundamentally different expectations in a superconductor?

AdS/CFT Dictionary

Gravity (AdS)	Superconductor (CFT)
Black hole	Temperature
Charged scalar field (hair)	Condensate (Cooper pair)
field mass	scaling dimension

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |\partial\phi - iqA\phi|^2 - m^2|\phi|^2$$

at finite temperature and charge density, for AdS Reissner-Nordstrom black holes:

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2)$$

$$g(r) = r^2 - \frac{1}{r}(r_+^3 + \frac{\rho^2}{4r_+}) + \frac{\rho^2}{4r^2}$$

Intuitively, $m_{\rm eff}^2 \sim m^2 - q^2 A_0^2$ and it can lead to instabilities if too negative.

However: 2 ways of condensing φ: i) through the gravitational environment, ii) through the usual way. We typically neglect the first.

Limit: For large q, saddle-point solutions become exact (no backreaction)

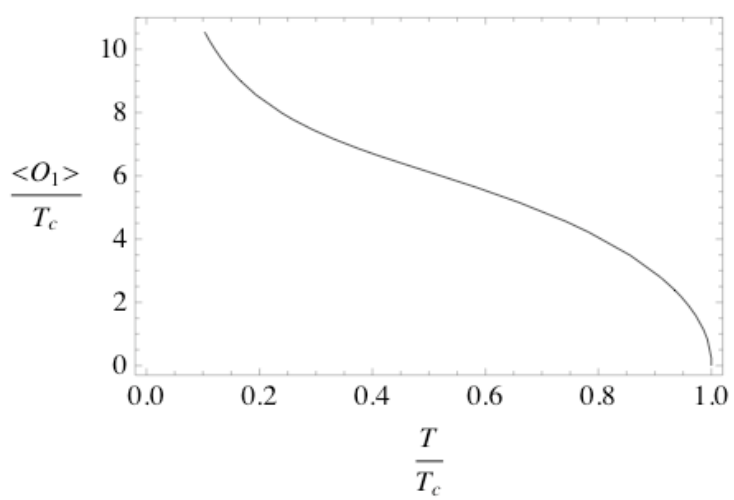
Basics of holographic superconductors

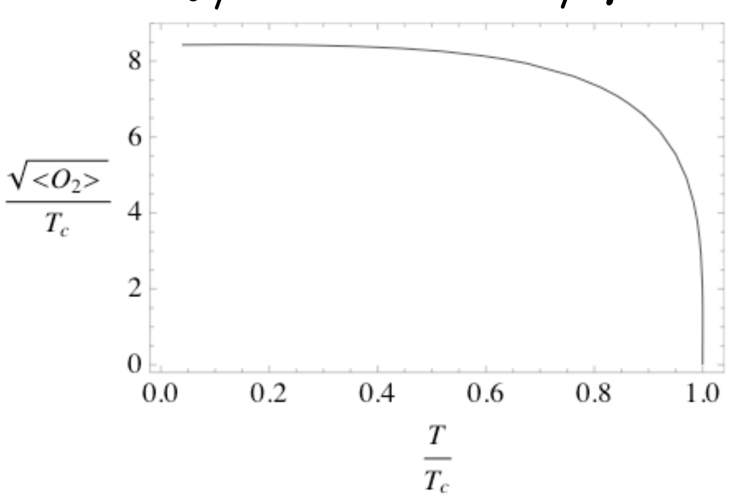
- Field Equations to solve (2+1 CFT, U(1), T) [Hartnoll, Herzog, Horowitz]:

$$\phi'' + \left(\frac{f'}{f} + \frac{2}{r}\right)\phi' + \frac{A^2}{f^2}\phi - \frac{m^2}{f}\phi = 0$$

$$A'' + \frac{2}{r}A' - \frac{2\phi^2}{f}A = 0$$

- At the horizon $f(r_0) = 0$, A must vanish, in order to have finite norm. Then, ϕ and ϕ' are not independent.
- Asymptotically: $A = \mu - \frac{\rho}{r}$, $\phi = \frac{\phi^{(1)}}{r} + \frac{\phi^{(2)}}{r^2}$
- For ϕ, either falloff is normalizable, and if one of them is set to zero, we have a one-parameter family of solutions. $\langle O_i \rangle = \sqrt{2}\,\phi^{(i)}$
- O_i has dimension i, μ has dimension 1, O_i/T and T/μ dimensionless.





Basics of *regular* superconductors (BCS- Landau-Ginsburg)

- BCS superconductors： Instability of the Fermi surface due to attractive interactions; pairs of electrons with opposite spin bind to form charged bosons.

- Traditional approach: Landau-Ginsburg Theory for T~Tc, for $\phi \equiv \langle O \rangle$:

$$F_{L-G} = \frac{1}{2m^*}|(\nabla + iqA)\,\phi|^2 + \alpha(T - T_c)|\,\phi|^2 + \frac{\beta}{2}|\,\phi|^4 + \ldots$$

- Experimental quantities of interest: length scales

superconducting coherence length

$$\xi \sim \frac{1}{(\alpha(T - T_c)m^*)^{1/2}}$$

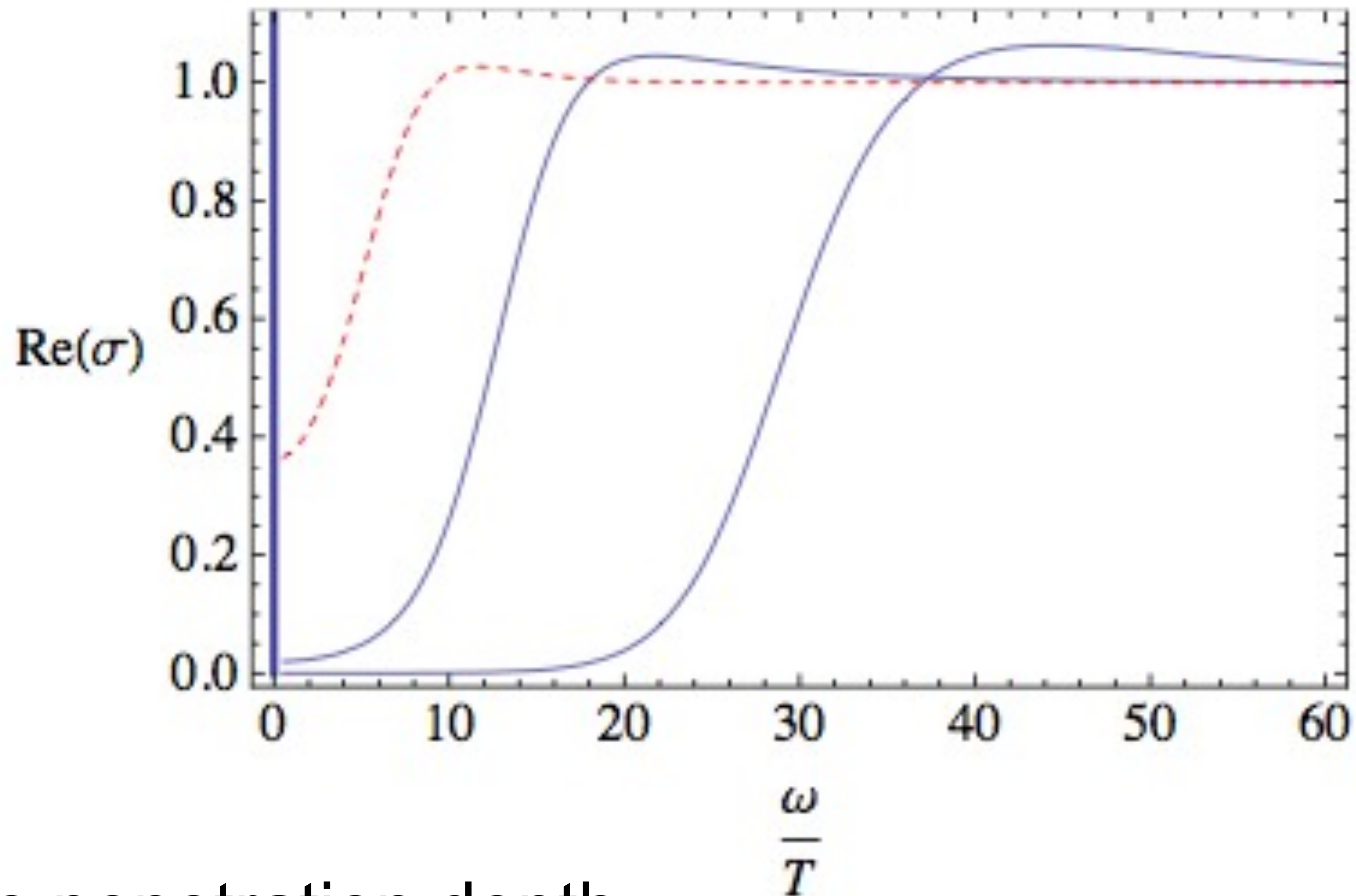
magnetic penetration depth

$$\lambda \sim \frac{(m^*)^{1/2}}{q^2\phi_0}$$

- ratio ξ/λ defines type-I from type-II superconductors.

- Tc is intrinsically connected to the coherence length scale: Pinning potentials with a length scale l, affect Tc according to the ratio l/ξ

Properties of holographic superconductors



- A holographic superconductor:
  - is extreme type-II, with infinite penetration depth,
  - has a conductivity that does not vanish at small ω ($\sigma(\omega) \neq e^{-\Delta/T}$, $\omega \to 0$),
  - has an interesting ratio that appears large ($\frac{\omega_g}{T_c} \approx 8$),
  - has infinite dc conductivity in the normal state (translationally invariant)

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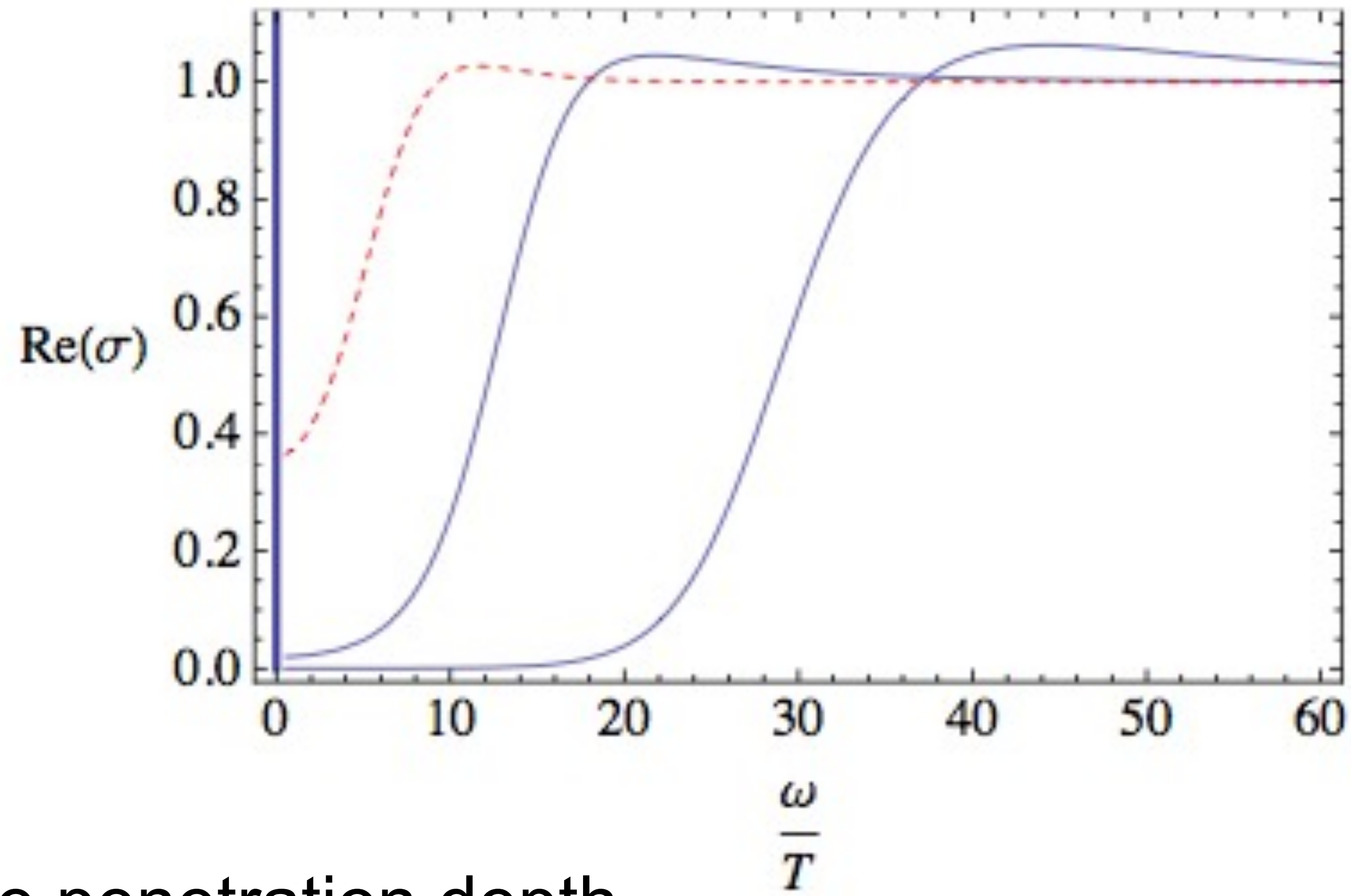

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Target: understand quantum critical aspects of the conductivity profile and make it more realistic

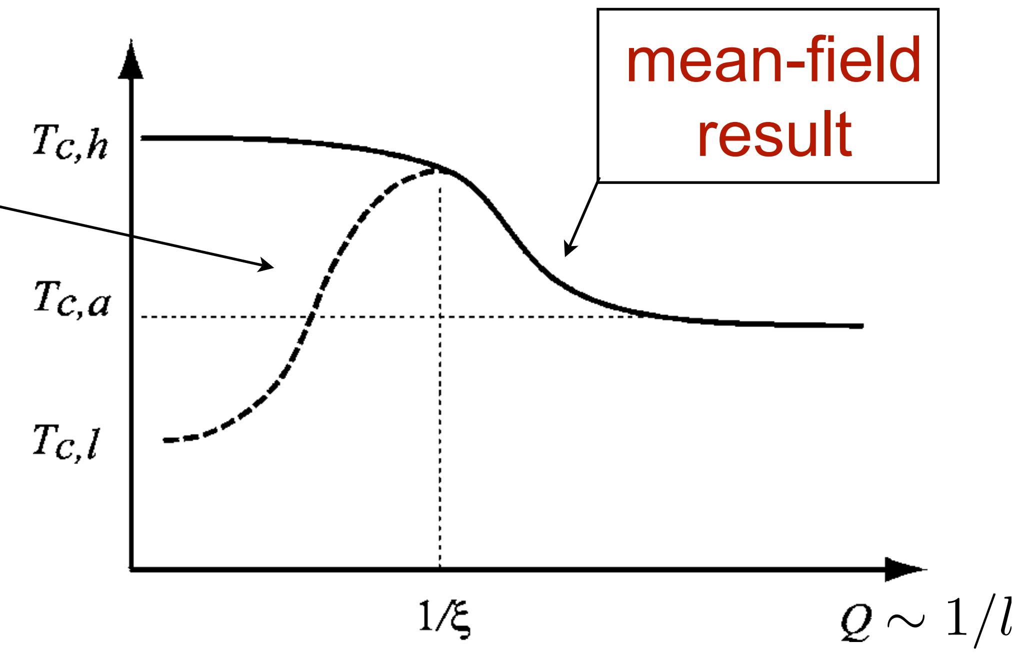
Inhomogeneous holographic pinning potentials

- Let's assume a pinning potential for the charge density:

$$\mu = \mu_0 + \delta \cos(2\pi x/l)$$

- BCS result: Tc increases for any non-zero δ (fixed l) and increases as $l \to \infty$ (fixed δ) [Martin, Podolsky, Kivelson]

heuristic phase-fluctuation result



- Holographic superconductors' result?

- Re-define μ on the AdS boundary, making A_0 inhomogeneous. Additional relevant scales in the problem δ and l .

- Now, x-derivatives become important...

Possible numerical techniques

- Equations to be solved in the *inhomogeneous case:*

$$
\begin{aligned}
\frac{A_{xx}}{r^2 f} + A_{rr} + \frac{2}{r}\partial_r A - 2\frac{\phi^2}{f}A &= 0 \\
\frac{\phi_{xx}}{r^2 f} + \phi_{rr} + \left(\frac{2}{r} + \frac{f'}{f}\right)\partial_r \phi + \left(\frac{2}{fL^2} + \frac{A^2}{f^2}\right)\phi &= 0
\end{aligned}
$$

- Ways to solve them:

    - Numerically, either by solving the equations in real-space (2D grid with non-trivial boundary conditions) or by expanding the solution in Fourier modes and solving explicitly an equation hierarchy (both methods efficient at Q $\rightarrow$ 0)

    - variational approach

Comparison of Numerical Methods

Momentum based

- Fourier decomposition:

$$\psi(x,z) = \sum_{n=0}^{\infty} \psi_n(z)\cos(nQx)$$

- Solve a hierarchy of ODEs:

$$-\Psi'' + \left(zh(z) - A_0^2\right)\Psi - A_1^2 \mathbb{A}_{11}\Psi - 2A_0A_1\mathbb{A}_{01} + h(z)Q^2\mathbb{Q}\Psi = 0$$

- A is analytically solved in an approximately exact manner.
- Efficient at large Q

Real-Space based

- Finite-difference Gauss-Seidel:

$$\psi(x,z) = \Psi(x_n, z_n)$$

- Beginning from a random profile, solve for $\Psi = z\tilde{R}/\sqrt{2}$ and A

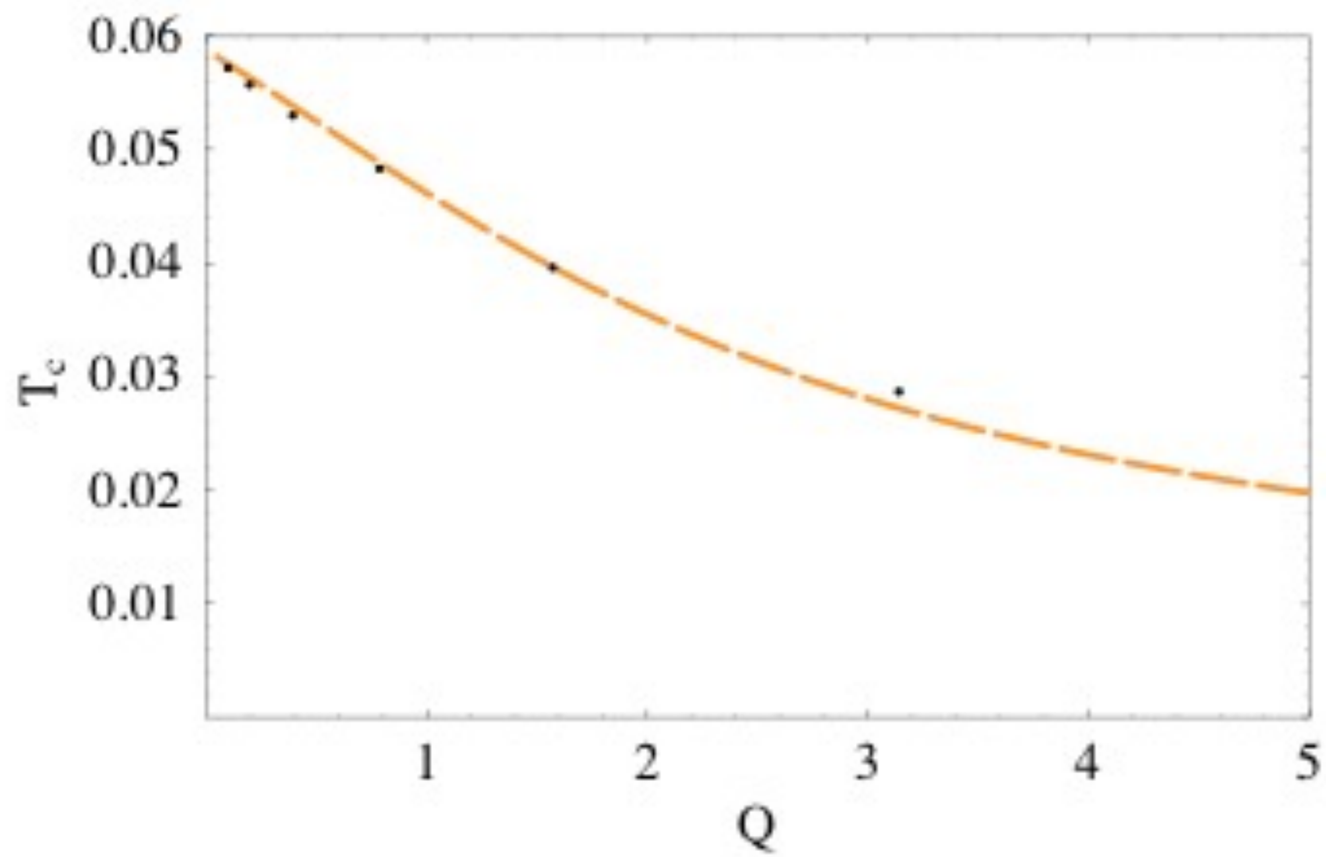
$$\tilde{R}_{00} = \frac{\left(\left(\frac{f}{h_z^2} + \frac{f'}{2h_z}\right)\tilde{R}_{+0} + \left(\frac{f}{h_z^2} - \frac{f'}{2h_z}\right)\tilde{R}_{-0} + \frac{\tilde{R}_{0+}+\tilde{R}_{0-}}{h_x^2}\right)}{\left(z - \frac{A_{00}^2}{f} + \frac{2}{h_x^2} + \frac{2f}{h_z^2}\right)}$$

$$A_{00} = \frac{\frac{f}{h_z^2}(A_{+0} + A_{-0}) + \frac{1}{h_x^2}(A_{0+} + A_{0-})}{\left(\tilde{R}_{00}^2 + \frac{2}{h_x^2} + \frac{2f}{h_z^2}\right)}$$

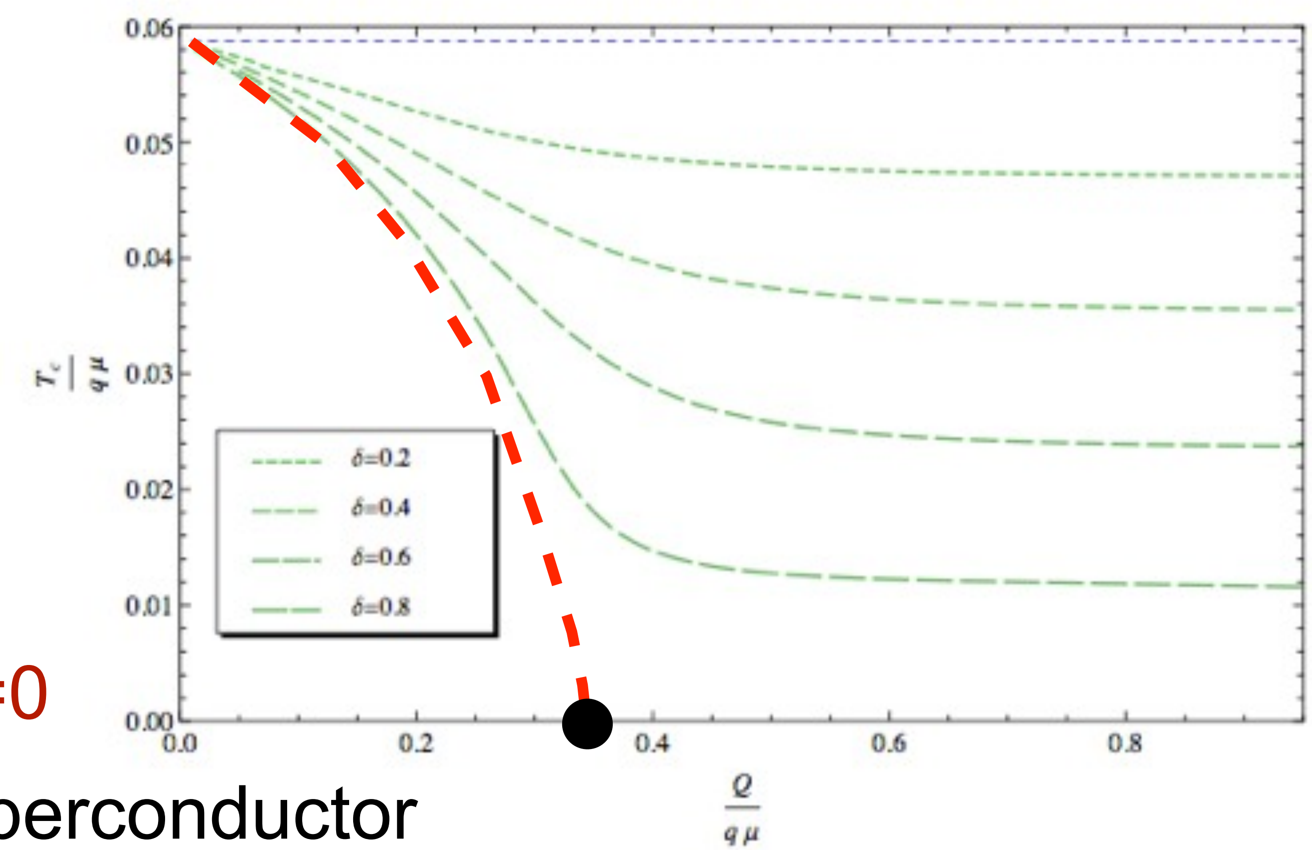
- until self-consistency is achieved
- in principle, exact approach

Comparison of Numerical Methods

- agreement of the possible Tc’s for different modulations

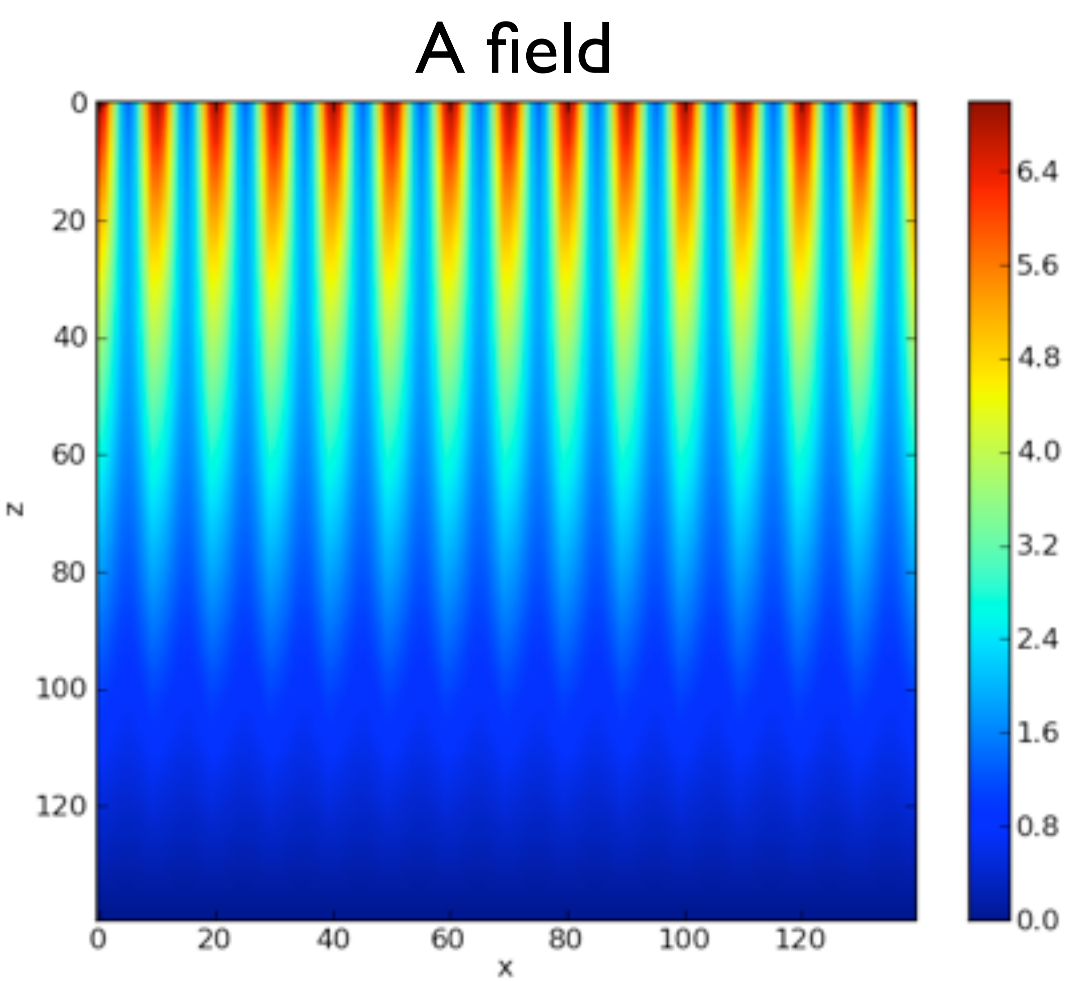


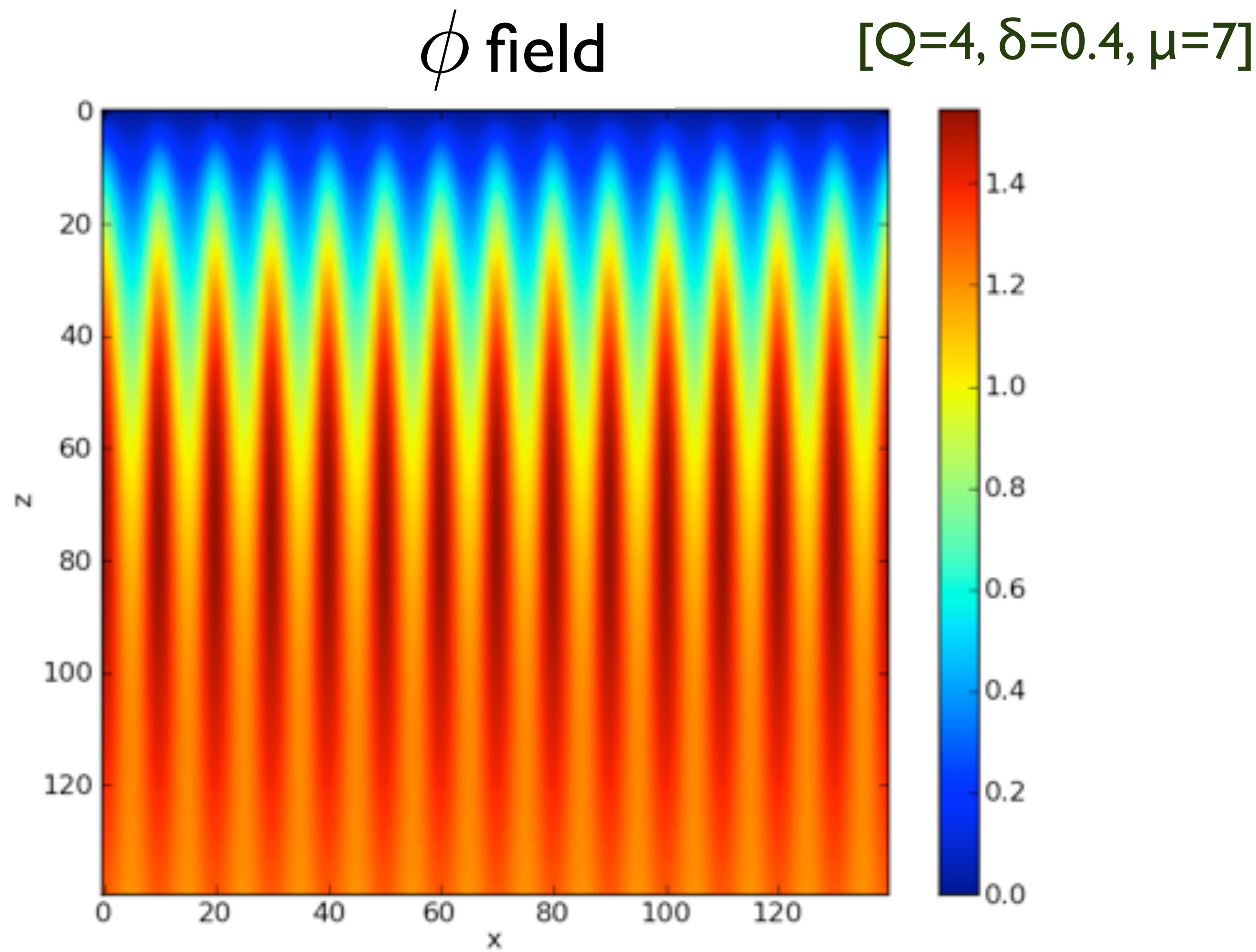
- agreement with homogeneous result at small δ



- Limiting behavior for δ=1 -- Tc = 0 for $Q_{\text{coherence}}$ -- estimate for T=0 coherence length of the parent superconductor

Profiles of inhomogeneous solutions



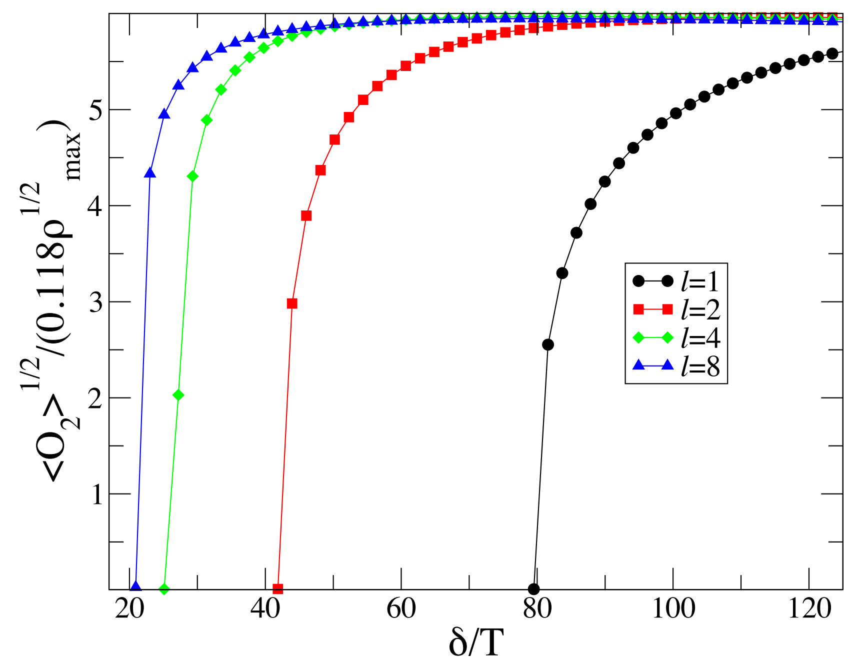


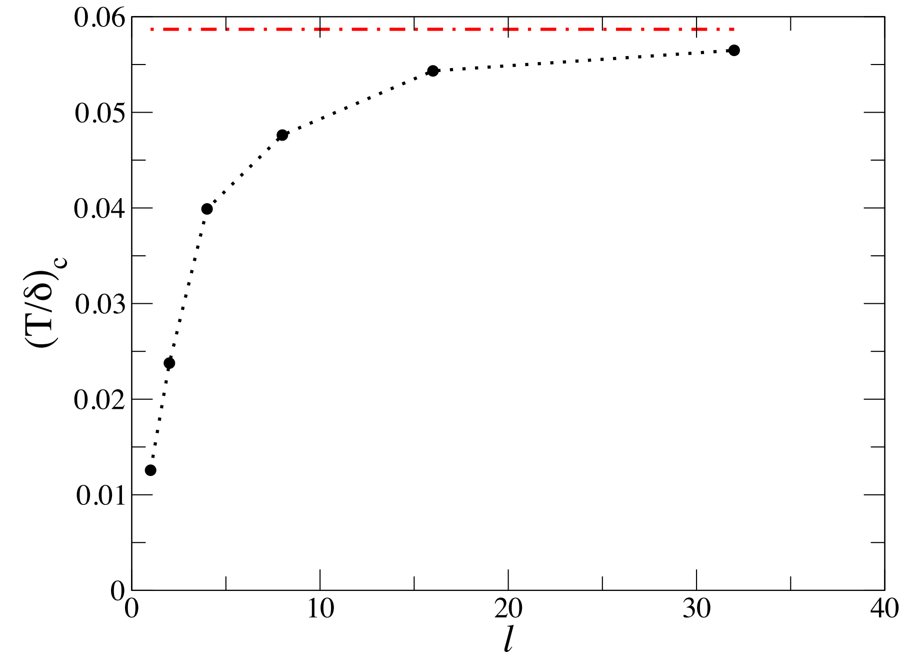
- derivatives of the scalar field at z=0 give the static expectation of the condensate $\langle O \rangle$
- derivatives of the A-field at z=0 give the charge density of the condensate
- Modulations of the condensate follow the imposed chemical potential --- nodeless solution

- μ_c when average in x of condensate is non-zero.

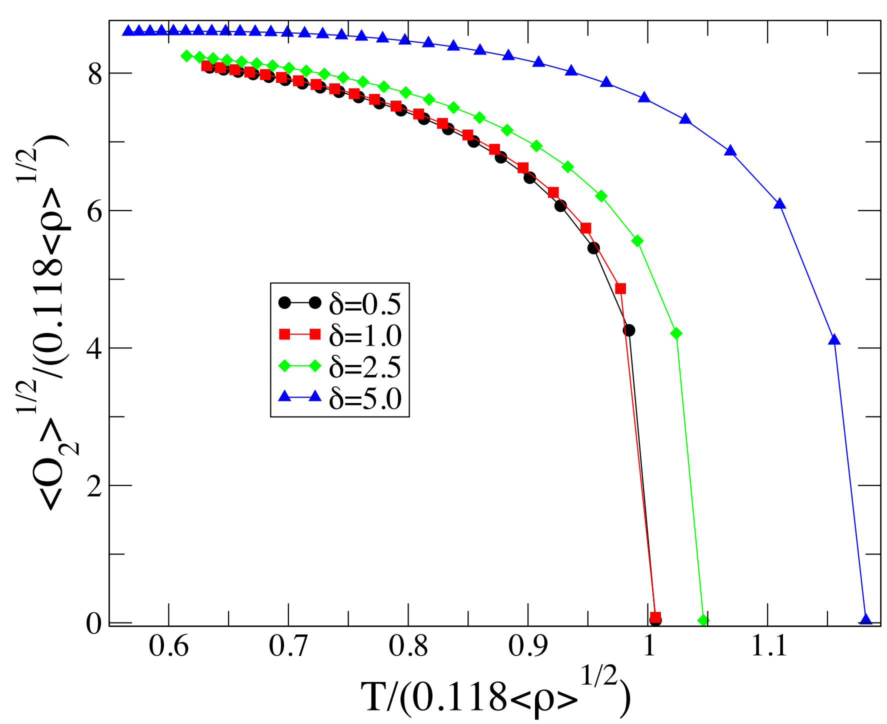
Inhomogeneous holographic pinning potentials

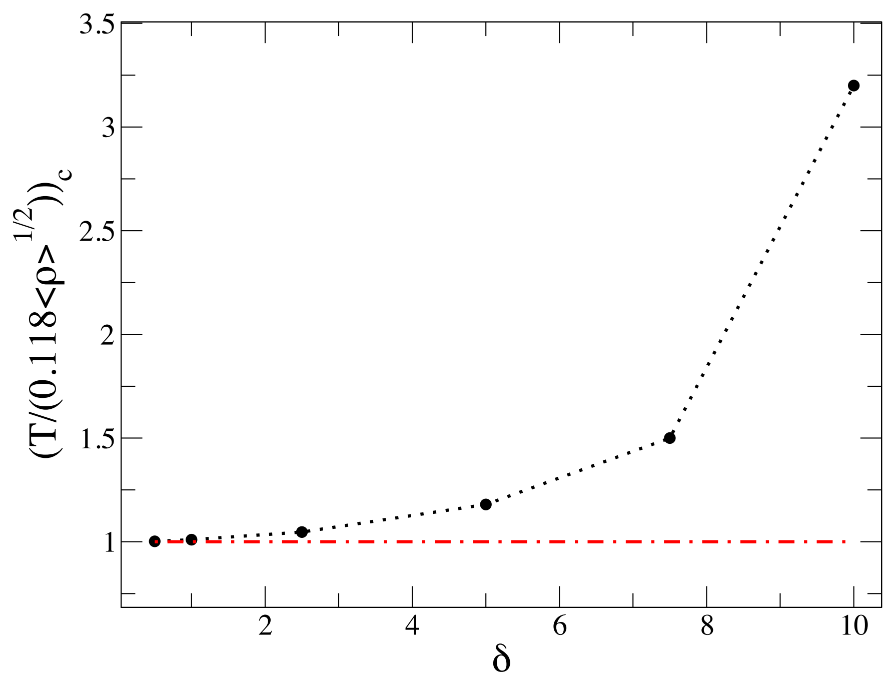
- Transition with zero average charge density, for a modulation:





- Transition for fixed modulation, changing μ_0 :





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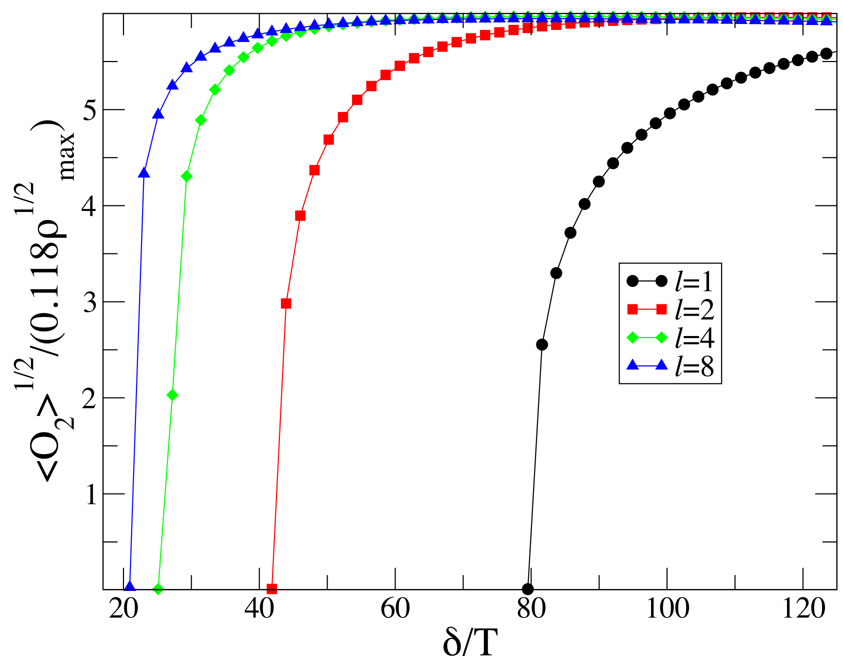

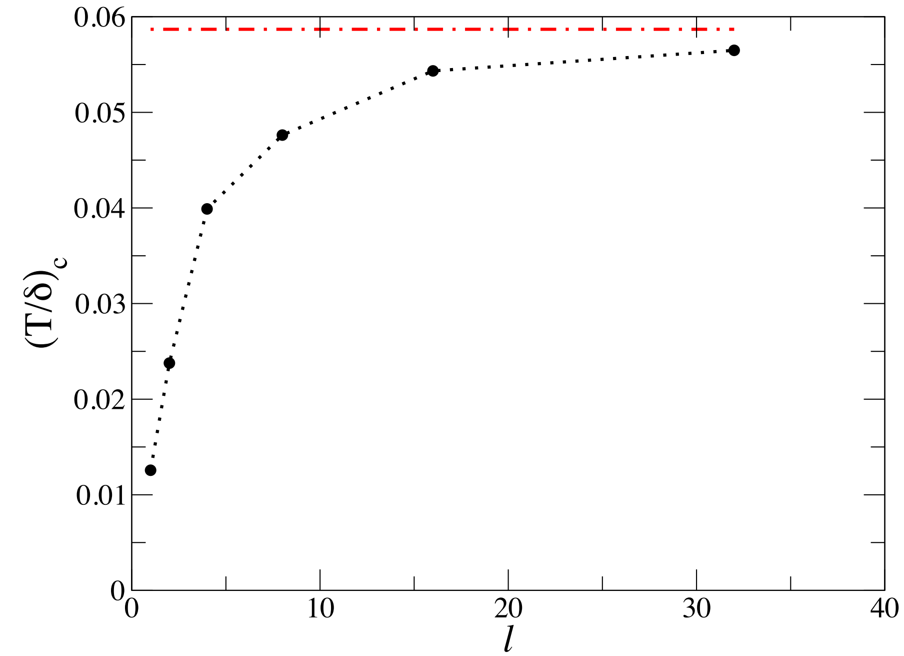

Tc increases as $l \to \infty$

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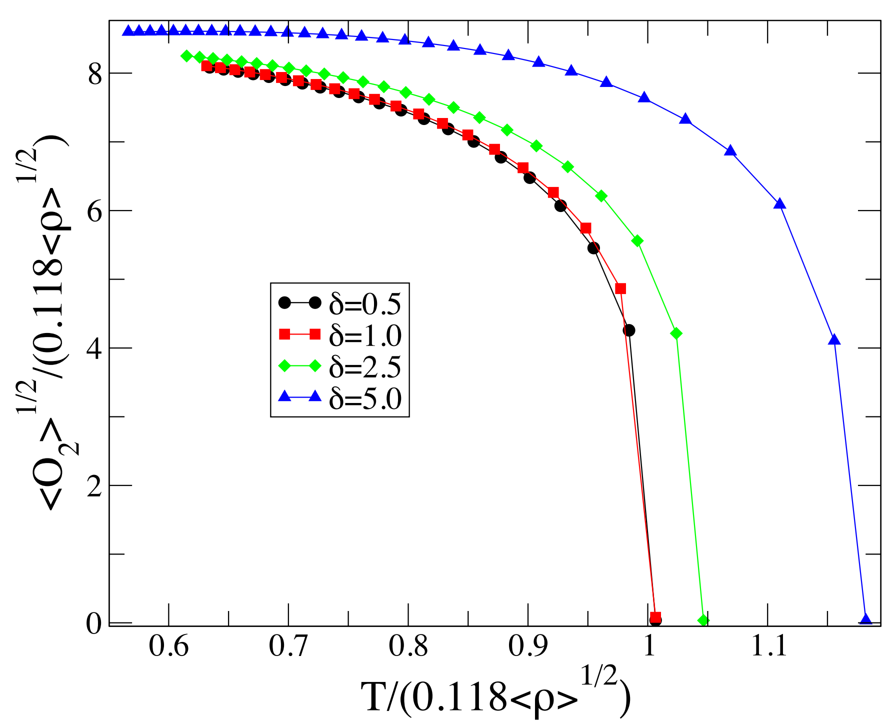

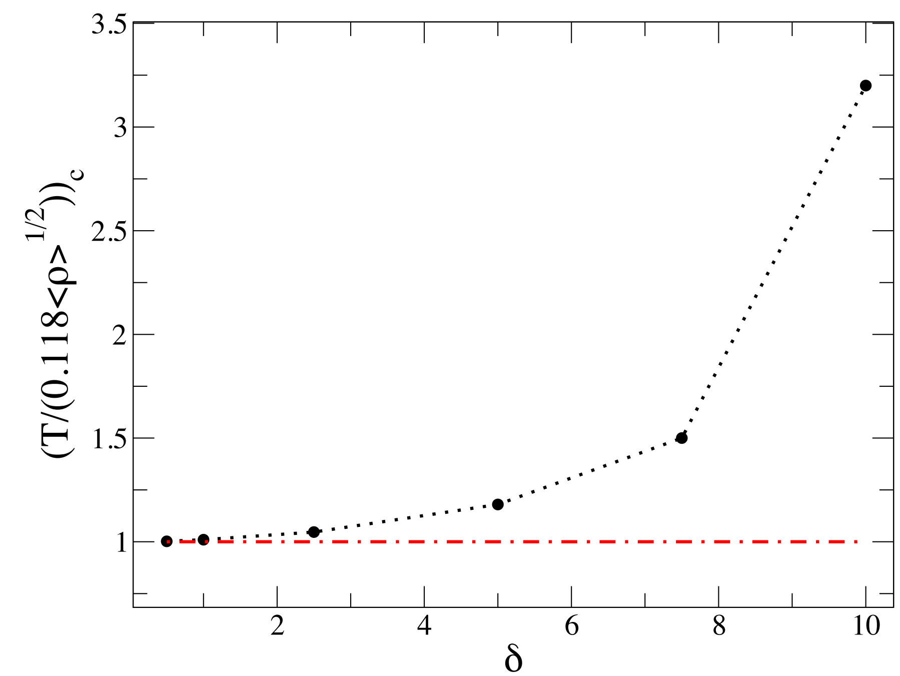

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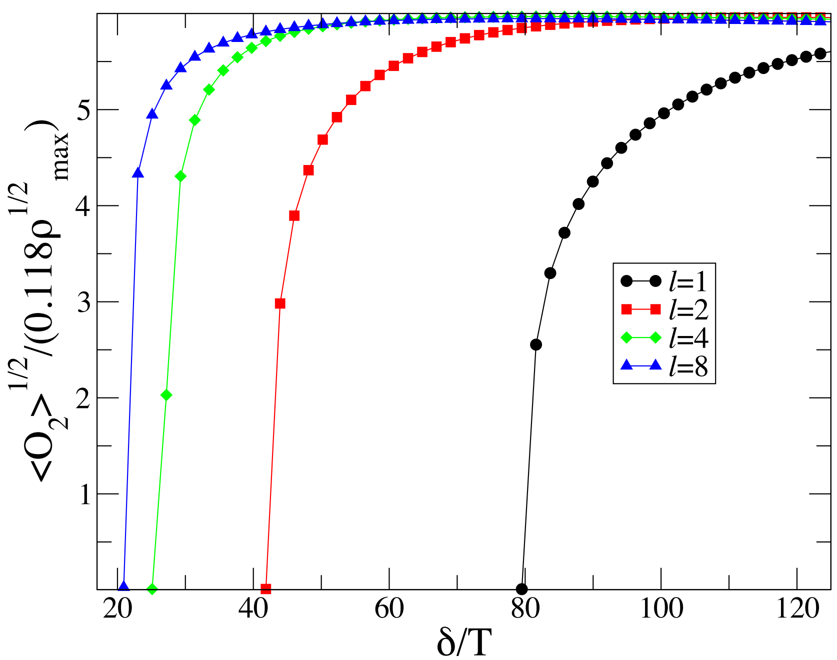

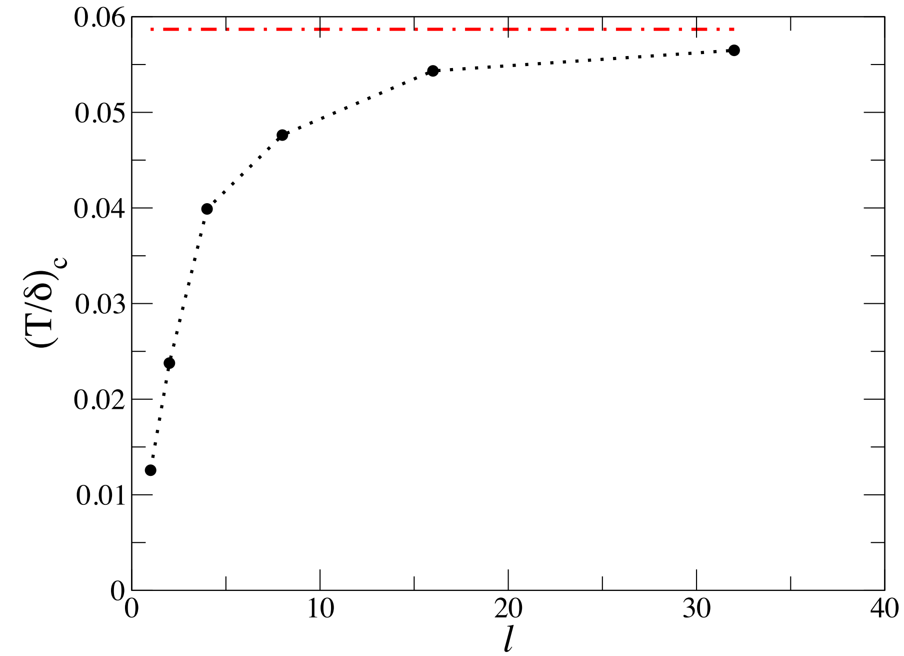

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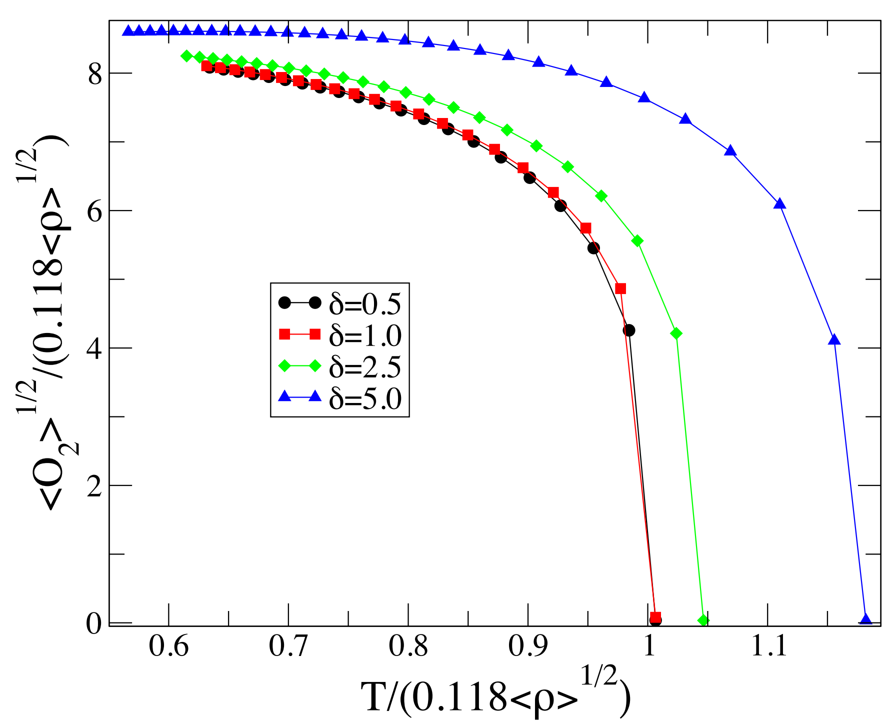
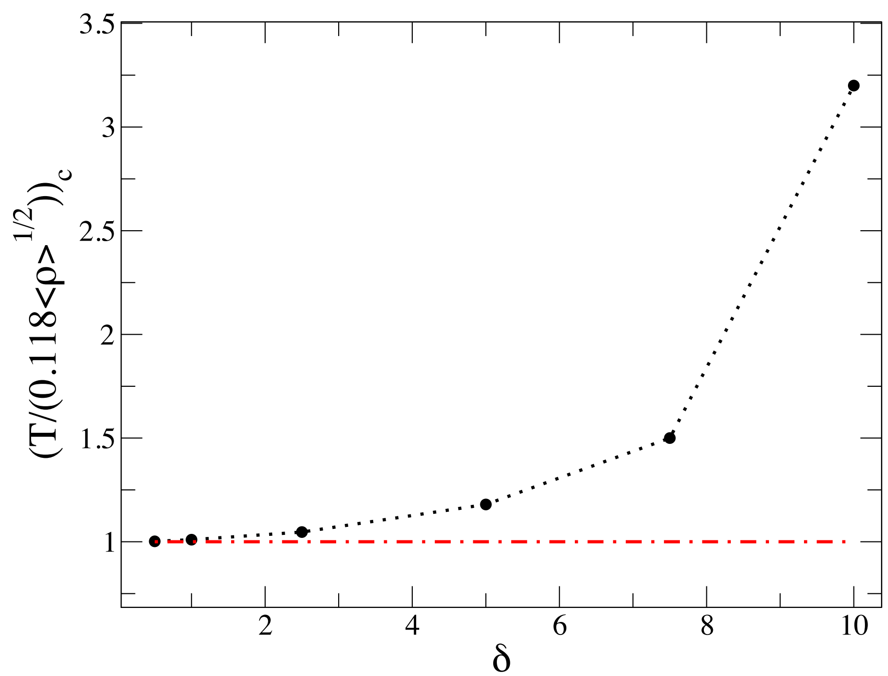




Tc increases for any non-zero δ

Conductivity profile

- easiest component to calculate $\sigma_y(\omega, x)$:

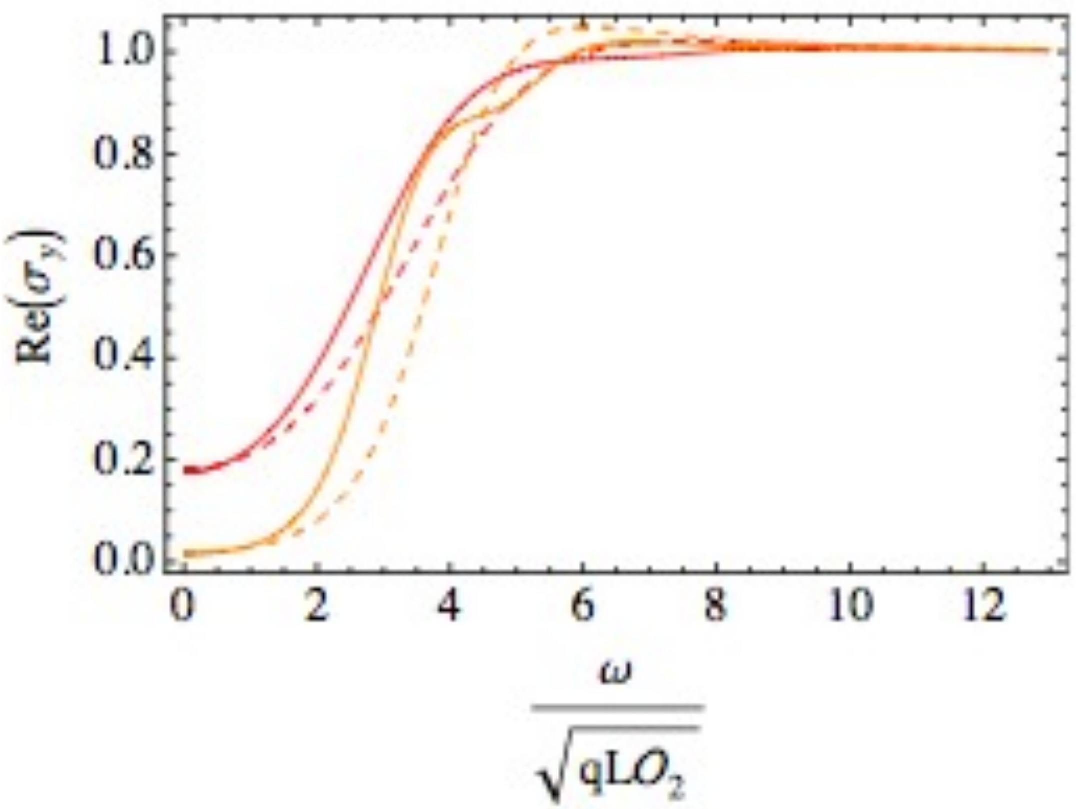
$$A_y(x,z) = A_y^{(0)}(x) + zA_y^{(1)}(x) + \dots .$$

$$\sigma_y(\omega, x) = \frac{J_y}{E_y} = -i\frac{A_y^{(1)}(x)}{\omega A_y^{(0)}(x)}$$

- Solve the equation for A_y on the background of the other solutions:

$$h\partial_z\left(h\partial_z A_y\right) + h\partial_x^2 A_y + \left(\omega^2 - 2|\phi|^2\frac{h}{z^2}\right)A_y = 0$$

Q=4



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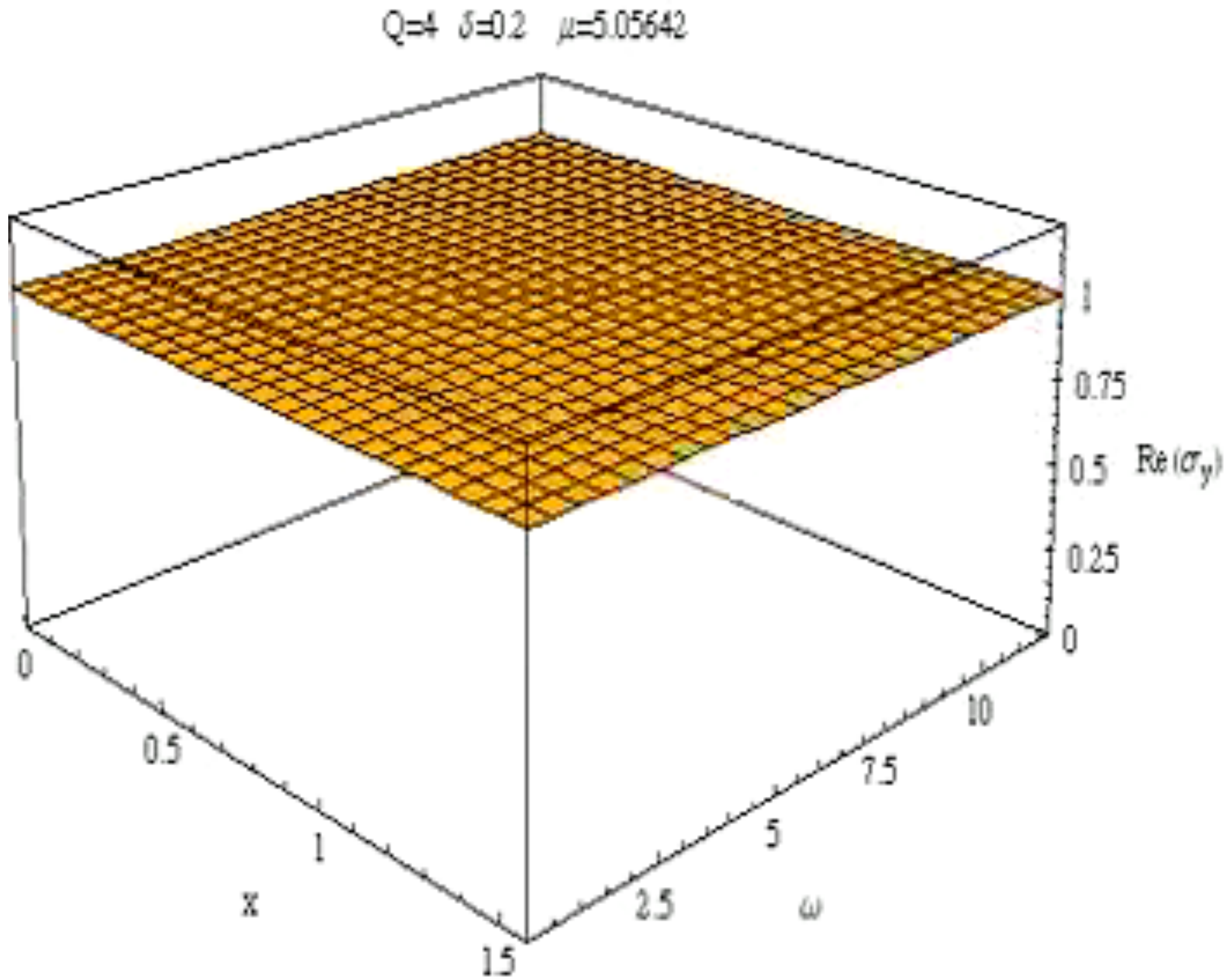

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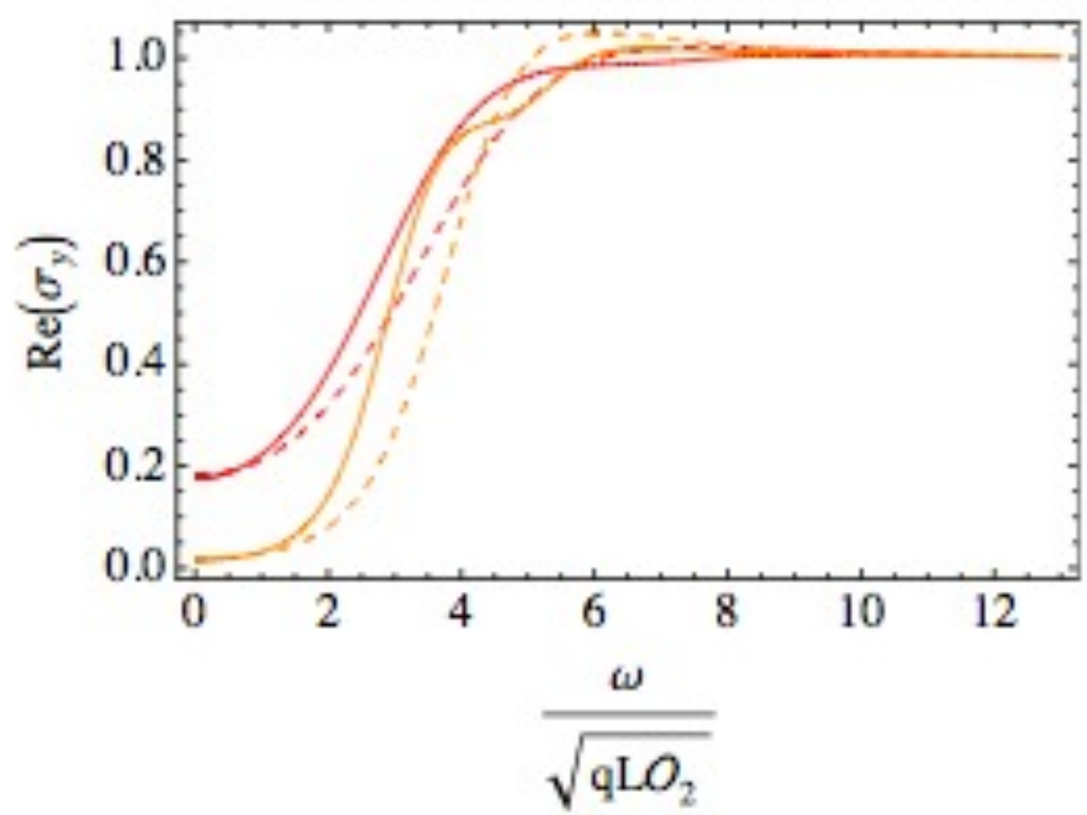

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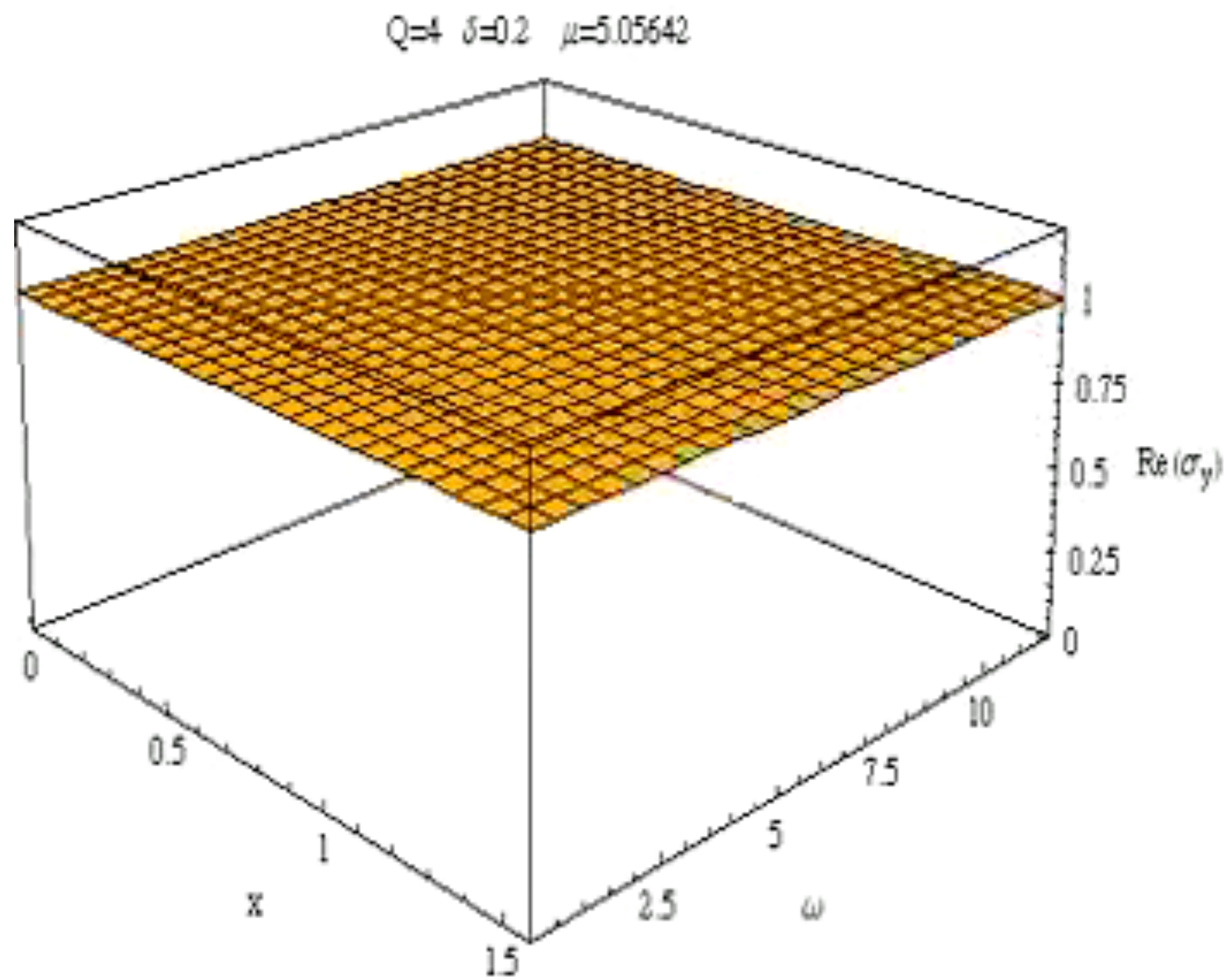

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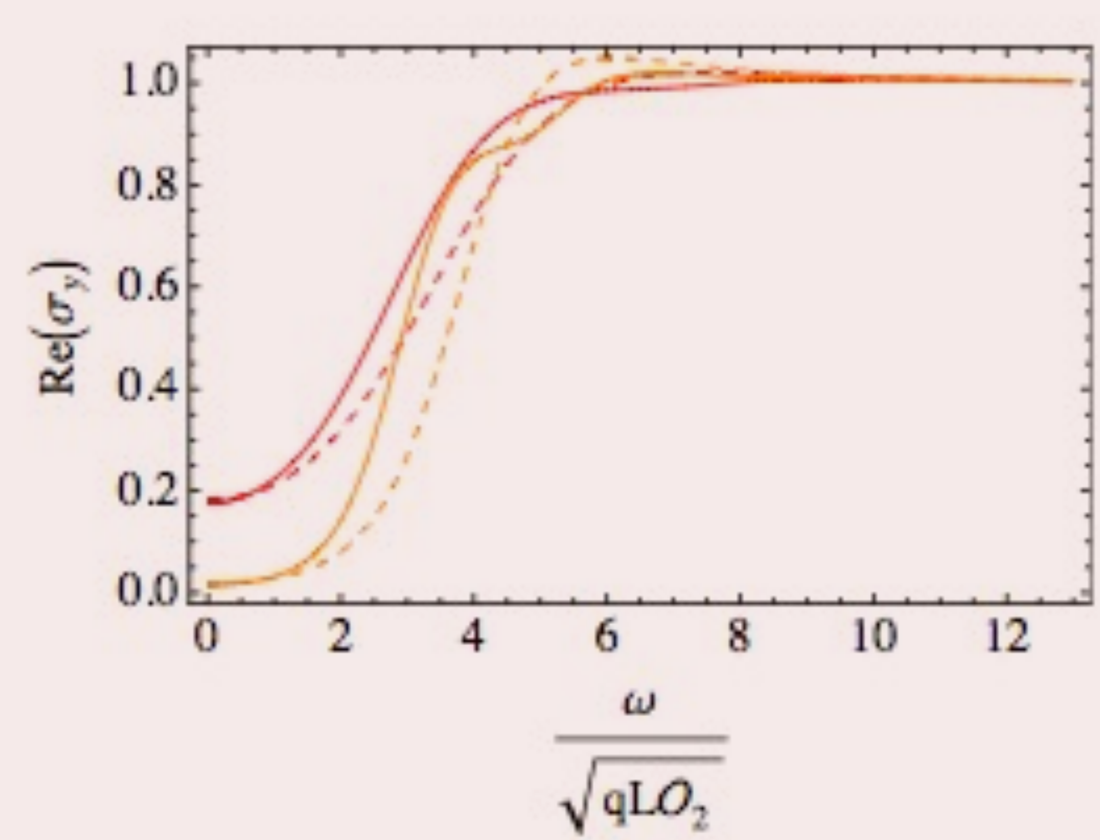
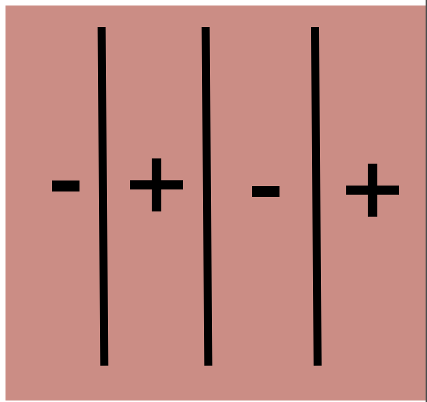
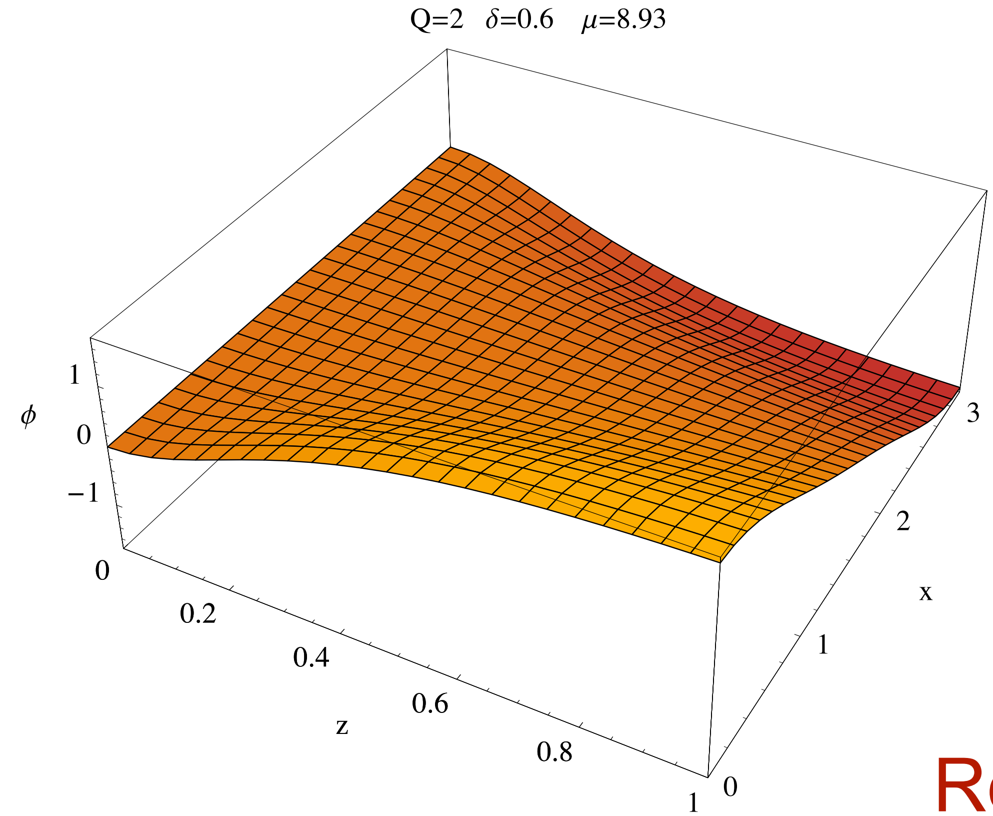


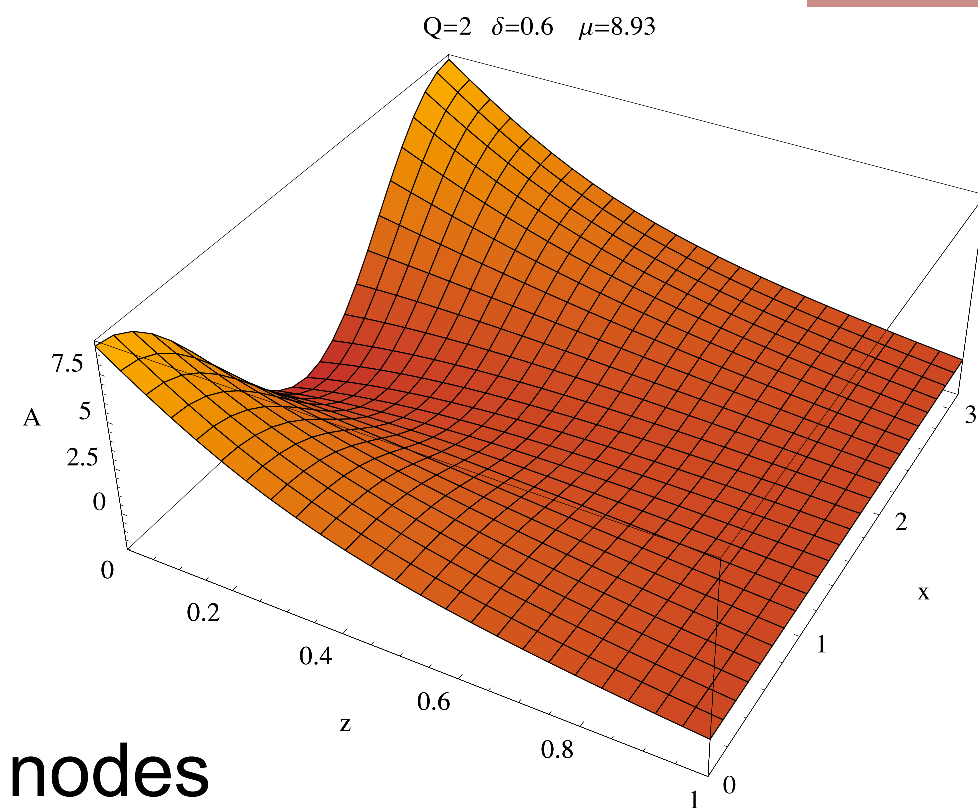
Possibility for a holographic LO-*like* / PDW phase

- Consider the class of solutions for the condensate with antiperiodic boundary conditions:

$$\phi(x+2\pi/Q) = -\phi(x)$$

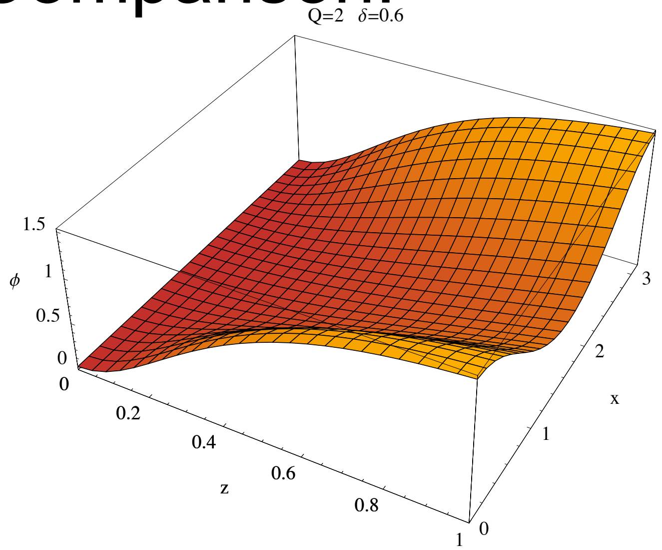


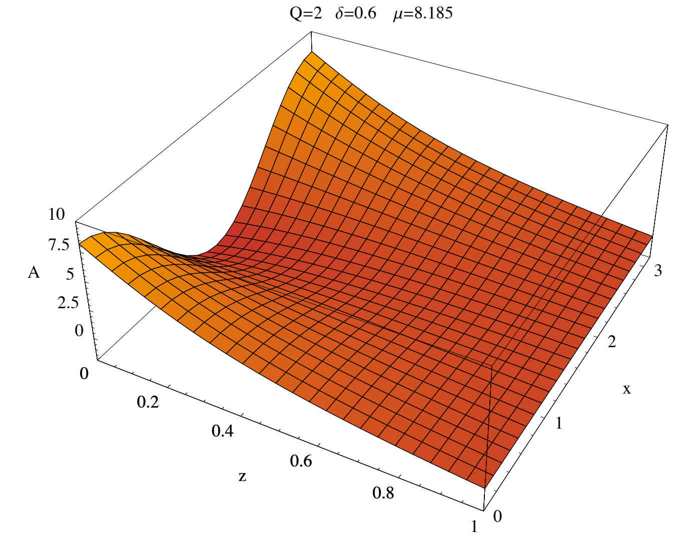




Real-space nodes
mean
Larkin - Ovchinikov
or
Pair Density Wave
states

- Comparison:

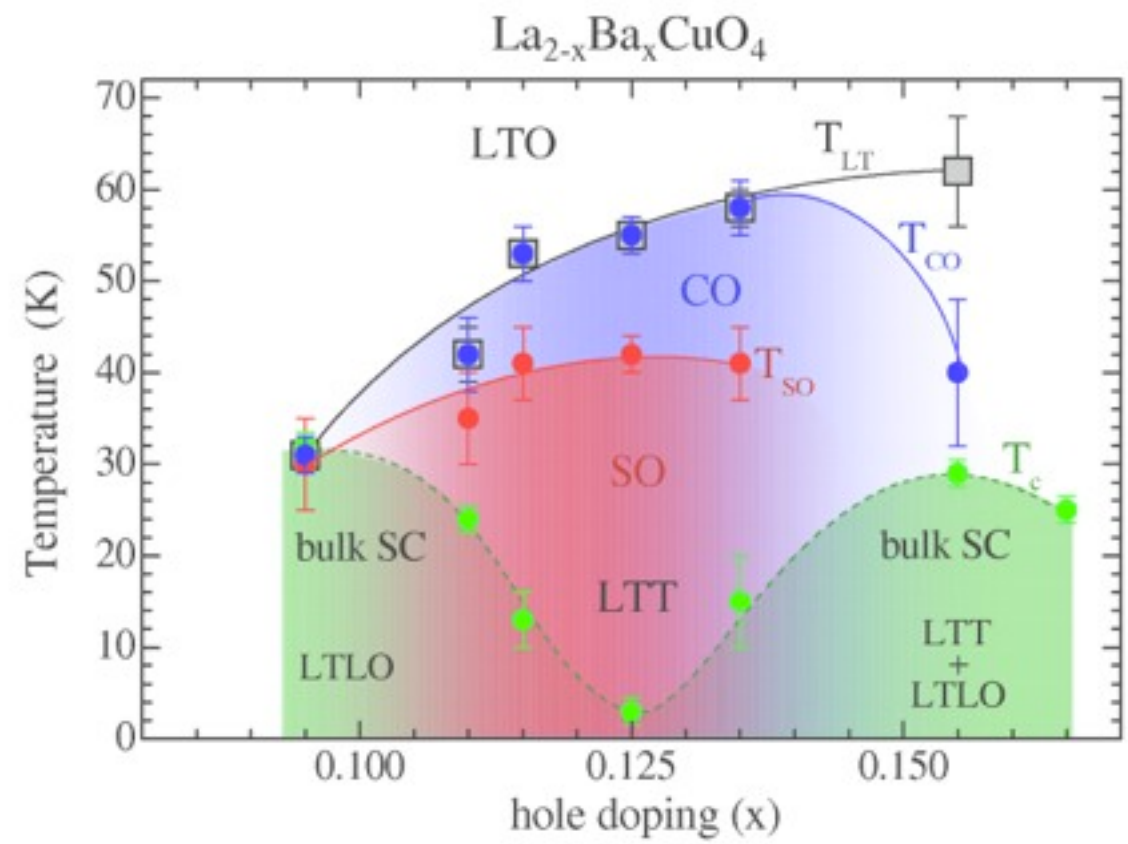




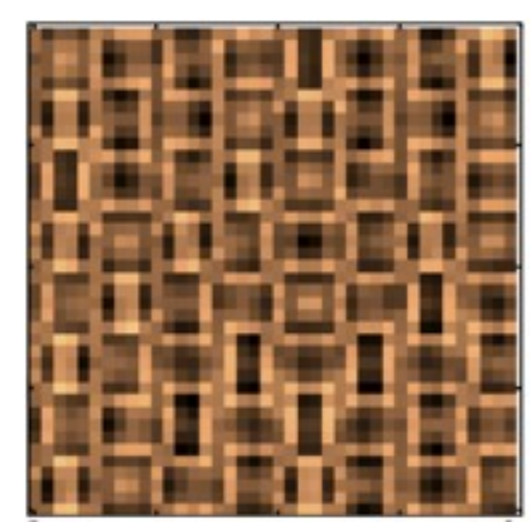
Why holographic PDW states ?

• relevant to the physics of the cuprates

• LBCO around 1/8 doping and suggestions for PDW states [Tranquada, Fradkin, Kivelson]



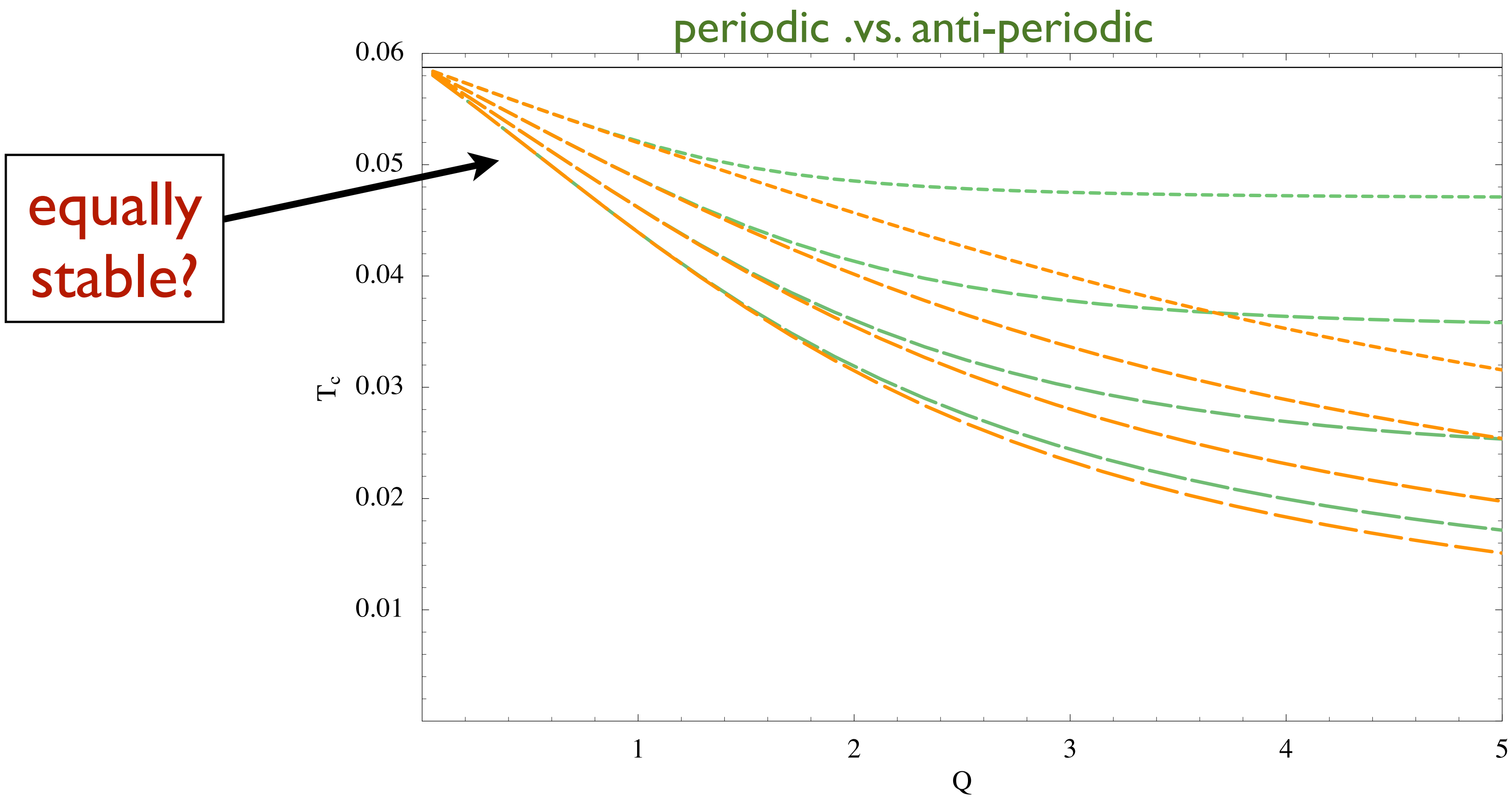
• CCOC, BSCCO are suggested to have strong d$\pm$s order which recently was shown to display PDW states at moderate magnetic fields [Galanakis,SP 2010] and [Liu,Zhou arxiv: 1106.0115, 2011]



• expected quantum critical points to such states as magnetic field changes

Which phase is stable at low temperatures?

- Antiperiodic solutions have similar dependence of T_c on Q and δ.

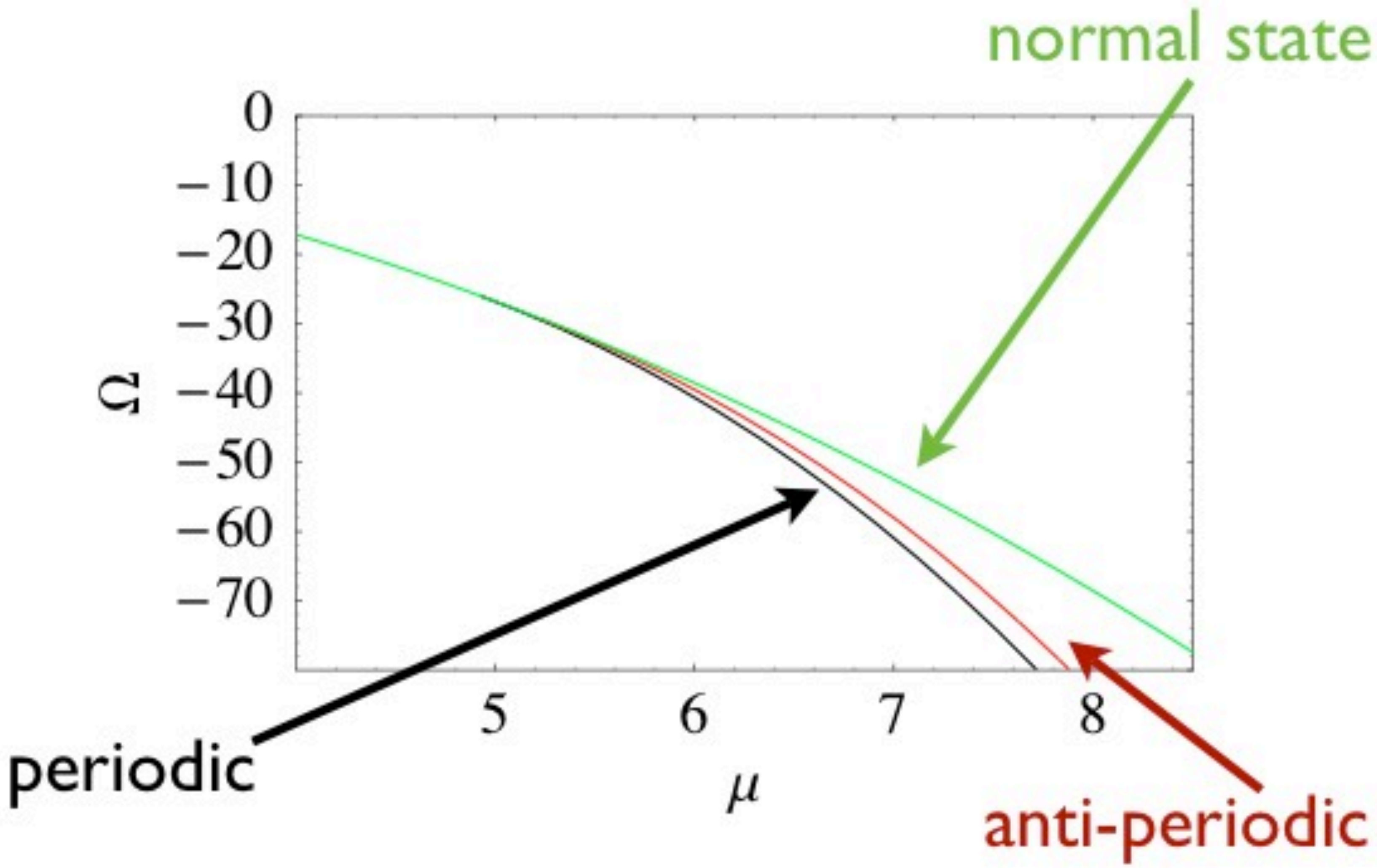


- The fate of the low temperature phase of the inhomogeneous superconductor is decided by the free energy:

$$\Omega_{us} = \int_{z=0} d^3x \frac{1}{2} A A' + \int d^4x \frac{A^2}{hz^2}\phi^2$$

Free energy competition

- Competition between the periodic and antiperiodic solutions as Q→ 0



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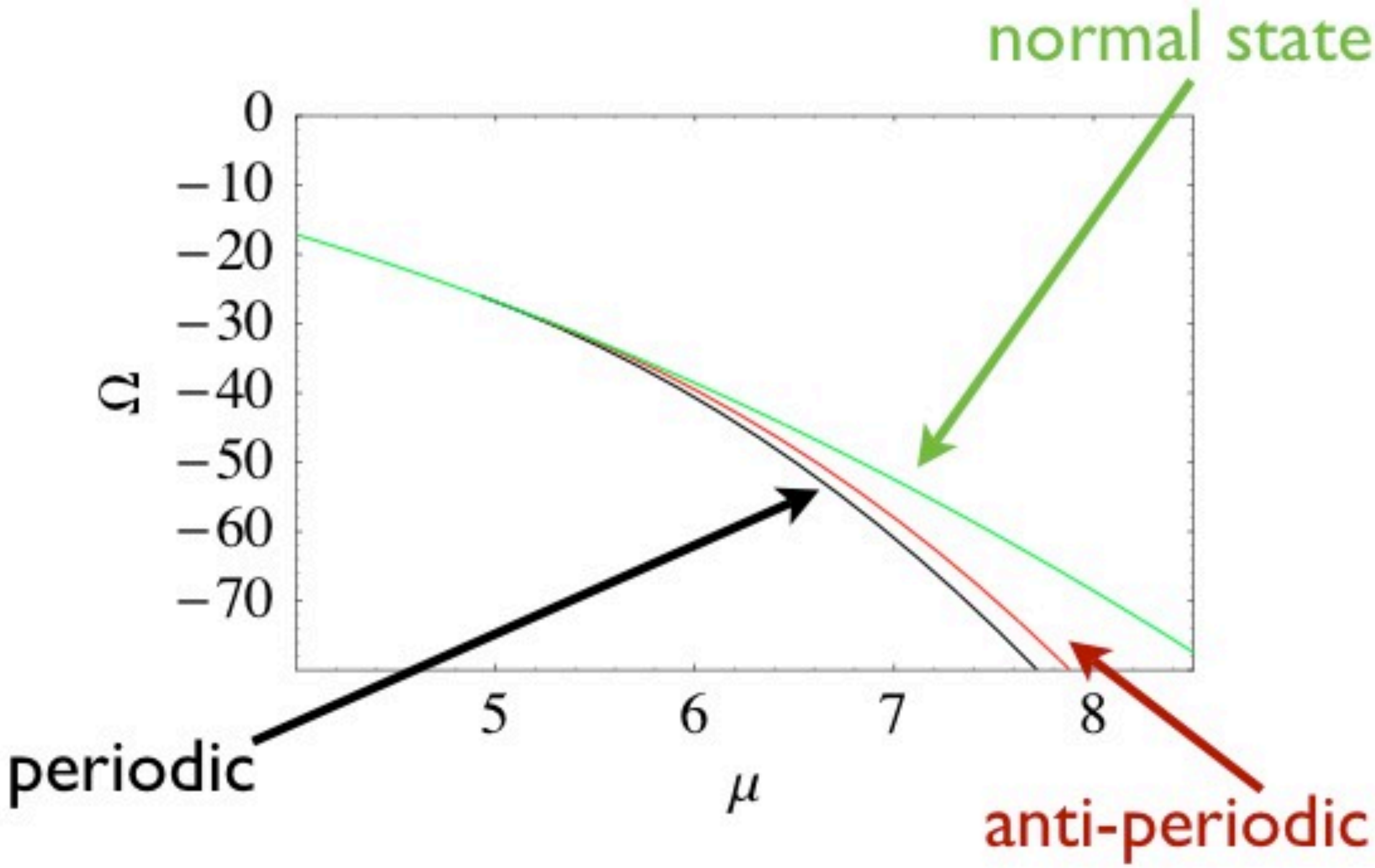

- However, the antiperiodic solution never wins in this model!

Way of stabilizing a PDW ground state

- Generic Ginzburg - Landau free energy for a superconductor in a magnetic field [Buzdin, Kashkachi 1996]:

$$F = \alpha\left|\psi\right|^2 + \beta\left|\partial\psi\right|^2 + \gamma\left|\psi\right|^4 + \delta\left|\partial^2\psi\right|^2 + \mu\left|\psi\right|^2\left|\partial\psi\right|^2$$
$$+\eta\left[(\psi^+)^2(\partial\psi)^2 + \psi^2(\partial\psi^+)^2\right] + \nu\left|\psi\right|^6$$

- Solve for the free energy:

$$\psi(x) = \psi_0\ e^{iq\cdot x}$$

$$F_{\exp} = \left(\alpha + \beta q^2 + \delta q^4\right)\cdot\left|\psi_0\right|^2 + \left(\gamma + (\mu - 2\eta)q^2\right)\cdot\left|\psi_0\right|^4$$

- Finite-q solutions crucially dependent on the δ-term - everything else being details:

$$q^2_{\max} = -\frac{\beta}{2\delta}, \qquad \alpha = \alpha_0 = \frac{\beta^2}{4\delta}$$

Analogs of Holographic PDW states

- Consider a RN black hole with an external magnetic field in AdS:

$$S = \int d^4x\sqrt{-g}\left[\frac{R+6/L^2}{16\pi G} - \frac{1}{4}F_{AB}F^{AB}\right] \qquad A_t = A_z = A_x = 0 \ , \qquad A_y = \mathcal{B}x$$

$$ds^2 = \frac{1}{z^2}\left[-h(z)dt^2 + \frac{dz^2}{h(z)} + dx^2 + dy^2\right] \ , \qquad h(z) = 1 - \left(1 + \frac{\mathcal{B}^2 z_0^4}{4}\right)\frac{z^3}{z_0^3} + \frac{\mathcal{B}^2}{4}z^4$$

- Temperature has a dependence on the magnetic field:

$$T = -\frac{h'(z_0)}{4\pi} = \frac{3}{4\pi z_0}\left[1 - \frac{\mathcal{B}^2 z_0^4}{12}\right]$$

- Charged, massive scalar shows typical BCS-like vortex solutions:

$$S = \int d^4x\sqrt{-g}\left[|D_A\phi|^2 - m^2|\phi|^2\right]$$

- PDW states need additional potential terms which should be Lorentz invariant:

$$S' = -\int d^4x\sqrt{-g}V(\phi) \qquad V = \beta|D_2\phi|^2 + |\gamma D_2^2\phi|^2$$

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non-zero Tc for extended solutions at finite *B*

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Conclusions/Future Directions

- Non-trivial consistency of BCS with holographic superconductivity,
- Construction of more realistic holographic superconducting ground-states
- Future:
    - Understanding of quantum critical properties of holographic PDWs
    - Quantum critical fermions in CDW environments

- R. Flauger, E. Pajer and SP, Phys. Rev. D, 83, 064009 (2011)
- SP and G. Siopsis (to be submitted)