The characteristics of thermalization of boost-invariant plasma from holography

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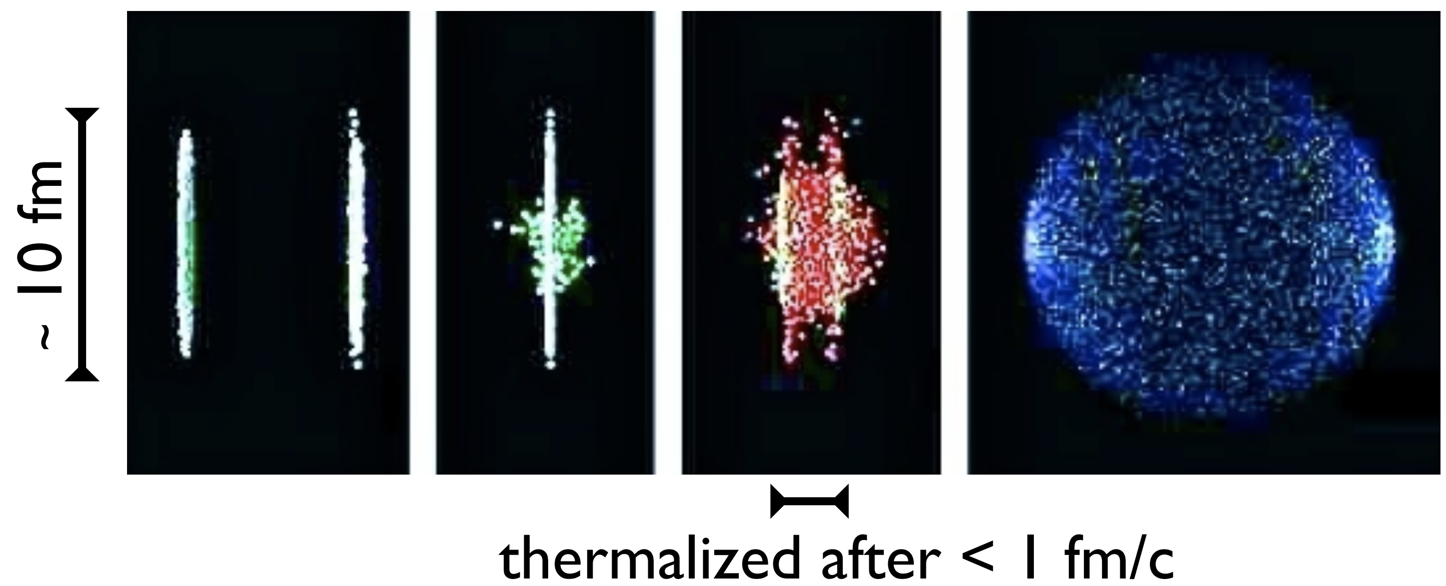
based on
1103.3452 [hep-th] MPH, R. A. Janik & P. Witaszczyk
111x.xxxx [hep-th] MPH, R. A. Janik & P. Witaszczyk

Motivation: fast thermalization at RHIC

Heinz [nucl-th/0407067]

There are overwhelming evidences that relativistic heavy ion collision program at RHIC (now also at the LHC) created strongly coupled quark-gluon plasma (sQGP)

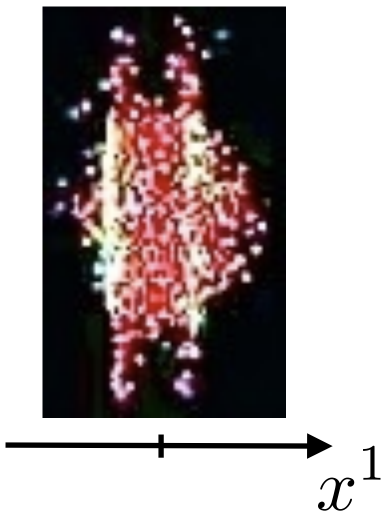
Successful description of experimental data is based on hydrodynamic simulations of an almost perfect fluid of $\eta/s = \mathrm{O}\,(1/4\pi)$ starting on very early (< 1 fm/c)



This very fast thermalization (understood as time after the collision when the stress tensor is to a very good accuracy described by hydrodynamics) is a puzzle

AdS/CFT naturally leads to such short thermalization times, which motivated us to scan through a large set of far-from-equilibrium initial conditions searching for *generic* features of thermalization of (holographic) strongly coupled media

Model: boost-invariant flow [Bjorken 1982]



The simplest, yet phenomenologically interesting field theory dynamics is the **boost-invariant flow** with **no transverse expansion.**

|| relevant for central rapidity region

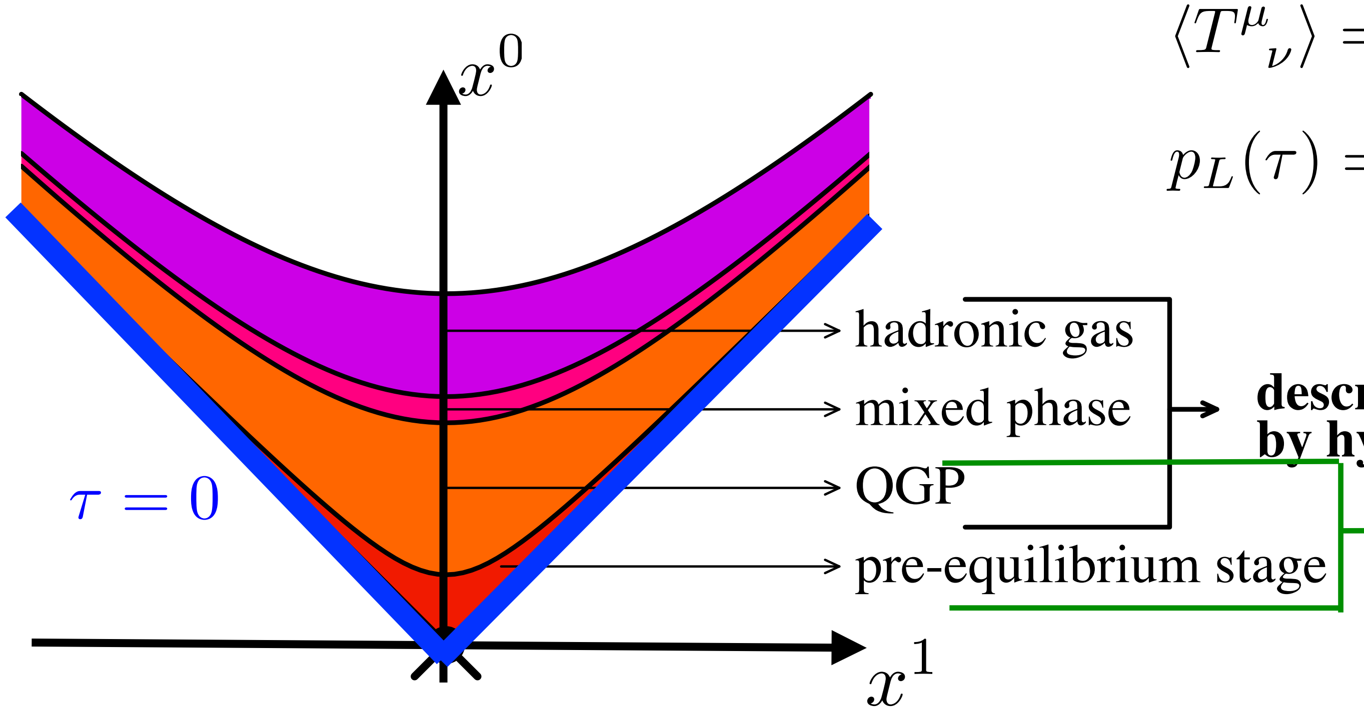
|| no elliptic flow (~ central collision)

In Bjorken scenario dynamics depends only on proper time $\tau = \sqrt{(x^0)^2 - (x^1)^2}$

and stress tensor (in conformal case) is entirely expressed in terms of energy density

$$\langle T^{\mu}{}_{\nu} \rangle = \text{diag}\,\{\epsilon(\tau),\, p_L(\tau),\, p_T(\tau),\, p_T(\tau)\} \text{ with}$$

$$p_L(\tau) = -\epsilon(\tau) - \tau\epsilon'(\tau) \text{ and } p_T(\tau) = \epsilon(\tau) + \frac{1}{2}\tau\epsilon'(\tau)$$



We are interested in setting strongly coupled non-equilibrium initial states at $\tau = 0$ and letting them evolve unforced to achieve local equilibrium (all using AdS/CFT).

Tool: AdS/CFT correspondence

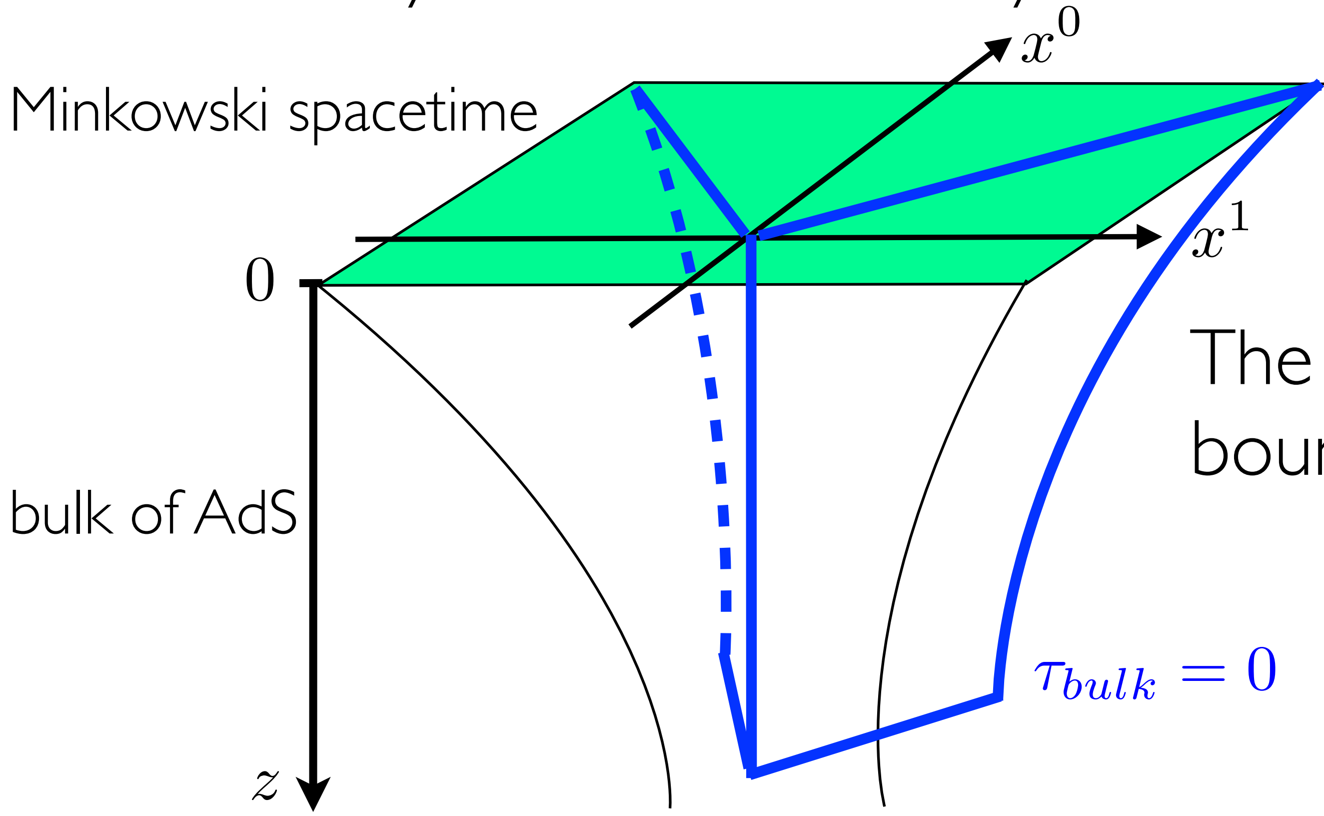
Maldacena [hep-th/9711200]
review: Mc Greevy 0909.0518 [hep-th]

From applicational perspective AdS/CFT is a tool for computing correlation functions in certain strongly coupled gauge theories, such as $\mathcal{N}=4$ SYM at large N_c and λ

In its simplest instance AdS/CFT maps the dynamics of the stress tensor of a holographic CFT_{1+3} into (1+4)-dimensional AdS geometry being a solution of

$$R_{ab} - \frac{1}{2} R\, g_{ab} - 6\, g_{ab} = 0$$

Of interest are geometries which interpolate between far-from-equilibrium states at the boundary at $\tau = 0$ and locally thermalized ones at (some) larger τ



The stress tensor is read off from near-boundary expansion of dual solution

Skenderis et al. [hep-th/0002230]

Geometries dual to equilibration describe gravitational collapse in AdS spacetime

Chesler & Yaffe 0812. 2053 [hep-th]

Initial state and the choice of bulk coordinantes

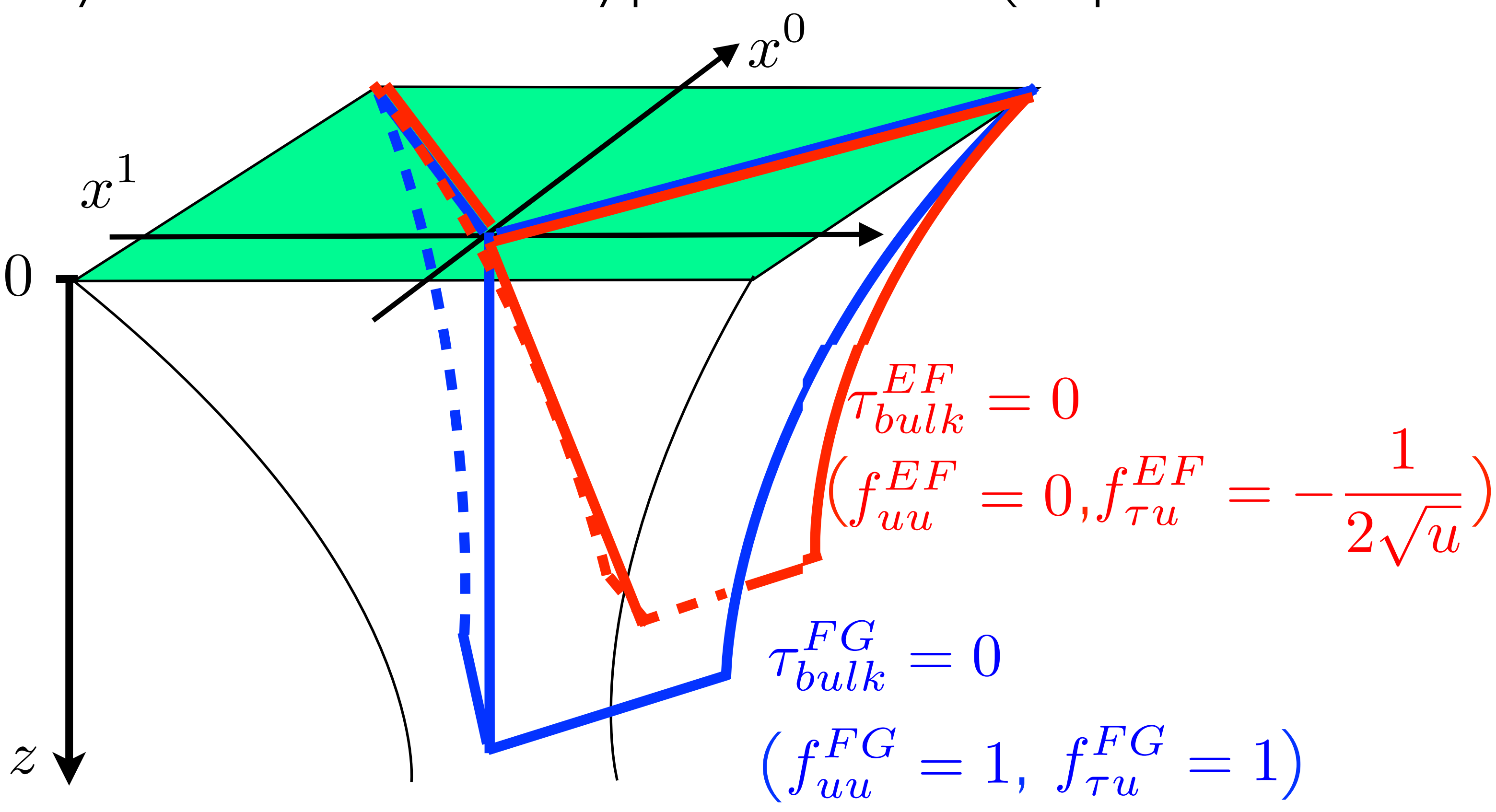
Initial states are solutions of gravitational constraints on chosen $\tau_{bulk} = 0$ hypersurface

Symmetries of a stress tensor dictate metric ansatz

$$ds^2 = \frac{1}{u}\Big\{\frac{1}{4u} f_{uu} du^2 + 2 f_{\tau u} d\tau du + f_{\tau\tau} d\tau^2 + \tau^2 f_{yy} dy^2 + f_{\perp\perp} dx_\perp^2\Big\}$$

Diffeomorphism freedom = one is free to choose 2 functions out of f_{zz}, $f_{\tau z}$ and $f_{\tau\tau}$ leaving 3 dynamical warp factors

Different choices cover different patches of spacetime and lead to different foliations by constant time hypersurfaces (in particular, different bulk initial time hypersurface)



In 0906.4423 [hep-th] we chose $f_{uu}^{FG} = 1$, $f_{\tau u}^{FG} = 0$, and looked at constraint equations at $\tau_{bulk}^{FG} = 0$

Obtained warp factors served as initial data for numerical simulations in 1103.3452 [hep-th]

Energy density at early time

Beuf, MPH, Janik, Peschanski 0906.4423 [hep-th]

Einstein's equations when viewed as evolution equations in radial direction require specifying two „initial" data: boundary metric and dual stress tensor (here $\epsilon(\tau)$)

In general this problem is very hard to solve, but one can analytically obtain the bulk metric close to the boundary in the small u expansion, e.g.

$$\ldots + \frac{1}{u} f_{\perp\perp} du^2 = \ldots + \frac{1}{u}\left\{1 + (\epsilon + \frac{1}{2}\tau\epsilon')u^2 + \boxed{(\frac{1}{8\tau}\epsilon'} + \frac{5}{24}\epsilon'' + \frac{1}{24}\tau\epsilon''')u^3 + \ldots\right\}$$

Plugging the general expression for regular early time power series for energy density

$$\epsilon(\tau) = \epsilon_0 + \epsilon_1\tau + \epsilon_2\tau^2 + \epsilon_3\tau^3 + \ldots$$

one finds out that all odd terms need to vanish for metric to be regular at $\tau^{FG}_{bulk} = 0$

$$\ldots + \frac{1}{u} f_{\perp\perp} du^2 = \ldots + \frac{1}{u}\left\{1 + (\epsilon_0 + \frac{3}{2}\epsilon_1\tau + 2\epsilon_2\tau^2 + \frac{5}{2}\epsilon_3\tau^3 + \ldots)u^2 + \boxed{(\frac{1}{8\tau}\epsilon_1} + \frac{2}{3}\epsilon_2 + \frac{15}{8}\epsilon_3\tau + \ldots)u^3 + \ldots\right\}$$

This implies that at early time

A) $\epsilon(\tau)$ has an expansion in even powers of proper time: $\epsilon(\tau) = \epsilon_0 + \epsilon_2\tau^2 + \epsilon_4\tau^4 + \ldots$

B) FG bulk time derivatives of warp factors vanish at $\tau^{FG}_{bulk} = 0$

cf. Kovchegov & Taliotis 0705.1234 [hep-ph]

„FG" bulk initial state

Beuf, MPH, Janik, Peschanski 0906.4423 [hep-th]

We choose the following metric ansatz

$$ds^2 = \frac{1}{u}\Big\{\frac{1}{4u}du^2 - a^2 d\tau^2 + \tau^2 b^2 dy^2 + c^2 dx_\perp^2\Big\}$$

Denoting $a(\tau = 0, u) = a_0(u)$, $b(\tau = 0, u) = b_0(u)$ and $c(\tau = 0, u) = c_0(u)$ and keeping in mind that time derivatives of warps vanish at $\tau_{bulk}^{FG} = 0$ leads to constraints

$$b_0(u) = a_0(u) \quad \text{and} \quad \frac{a_0''(u)}{a_0(u)} = -\frac{c_0''(u)}{c_0(u)}$$

The latter eqn can be solved for $a_0(u)$ once a regular $c_0(u)$ is provided

Typical example of a solution is $c_0(u) = \cosh(u)$ and $a_0(u) = \cos(u)$, we took 20 different initial conditions

As shown in the original paper, timelike warp always has a zero for some u, which is a coordinate singularity of the FG chart

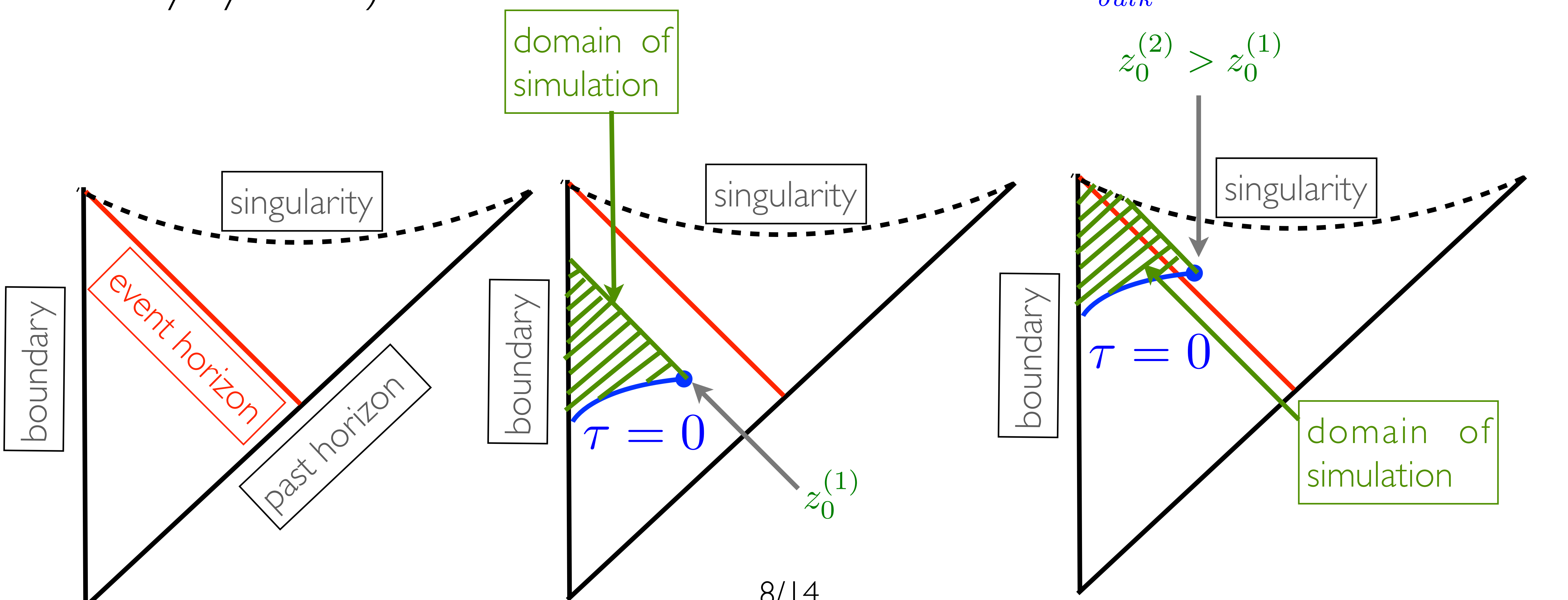
The idea behind solving initial value problem

FG coordinates are not well suited for numerics, because it is hard to find natural boundary conditions to cut off radial integration

For this reason we considered another coordinate patch, which coincides with FG one at $\tau_{bulk}^{FG} = 0$, but otherwise it is different: $f_{\tau\tau}^{N} \sim (z_0 - z)^2$ instead of $f_{zz}^{FG} = 1$

$f_{\tau\tau}$ measures the flow of coordinate time so if it vanishes at fixed position, this point does not evolve providing natural boundary conditions and nice bulk cut-off

By increasing z_0 and running numerics one recovers more and more bulk (and so boundary dynamics) until on crosses the event horizon at $\tau_{bulk}^{FG} = 0$!



Metric parametrization and numerics

Instead of $ds^2 = \frac{1}{u}\left\{\frac{1}{4u}du^2 - a^2 d\tau^2 + \tau^2 b^2 dy^2 + c^2 dx_\perp^2\right\}$ we are using the following metric ansatz

$$ds^2 = \frac{1}{u}\left\{\frac{1}{4u}d^2 du^2 - \alpha^2 a^2 d\tau^2 + \tau^2 a^2 b^2 dy^2 + c^2 dx_\perp^2\right\}$$

where $a = \cos(u/u_0)$, u_0 is a cut-off for radial integration and $\alpha^2 a^2$ is the lapse

Now dynamical warp-factors are b,c and d and α expressed in terms of this trio measures distances between constant time slices. We choose α`s giving stable code

(caveat: for generic α time on the boundary does not coincide with simulation (so bulk) time)

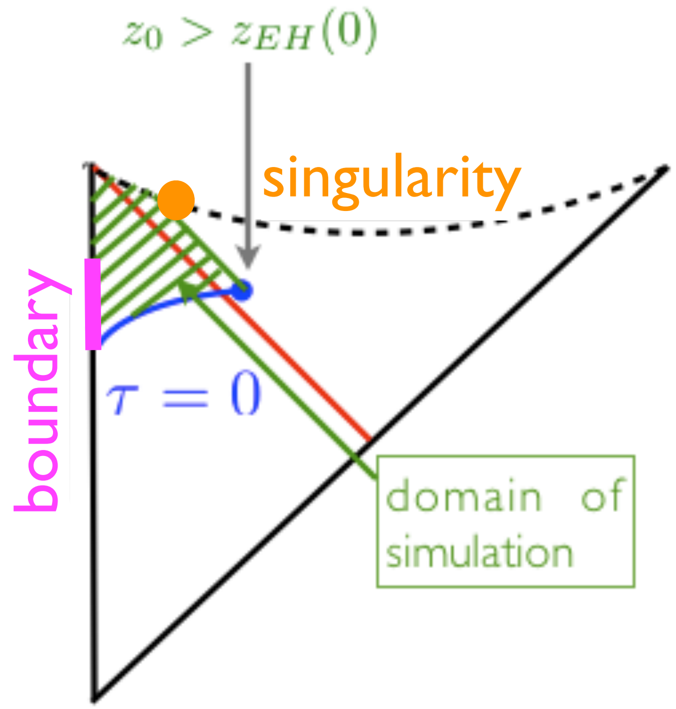
We are using unconstrained ADM evolution for b, c and d and their time derivatives with initial data taken as $c_0(u)$ from FG coordinates and u_0 chosen for each of them by examining the behavior of reproduced spacetime

Numerical implementation relies on spectral discretisation in the radial direction and high order adaptive Runge-Kutta time stepping.

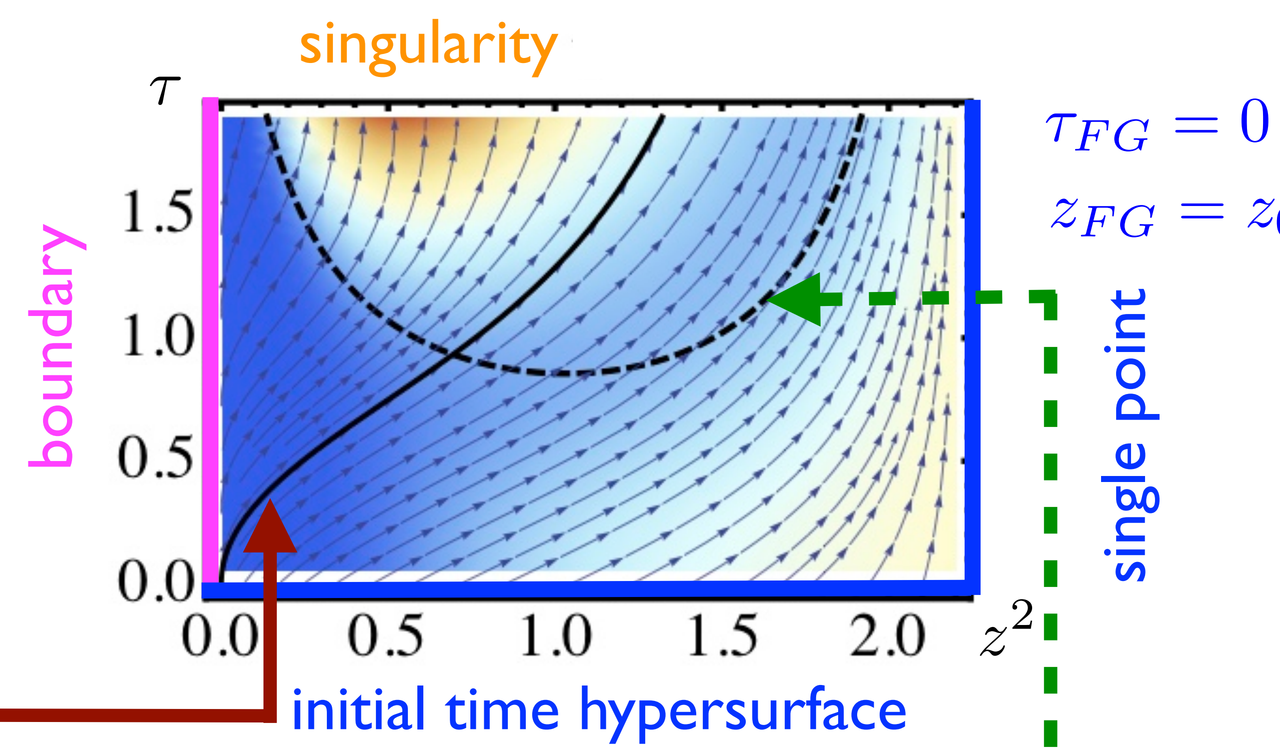
Non-equilibrium entropy

Rangamani et al. 0902.4696 [hep-th]
Booth, MPH, Spalinski 0910.0748 [hep-th]

Theory



Numerics



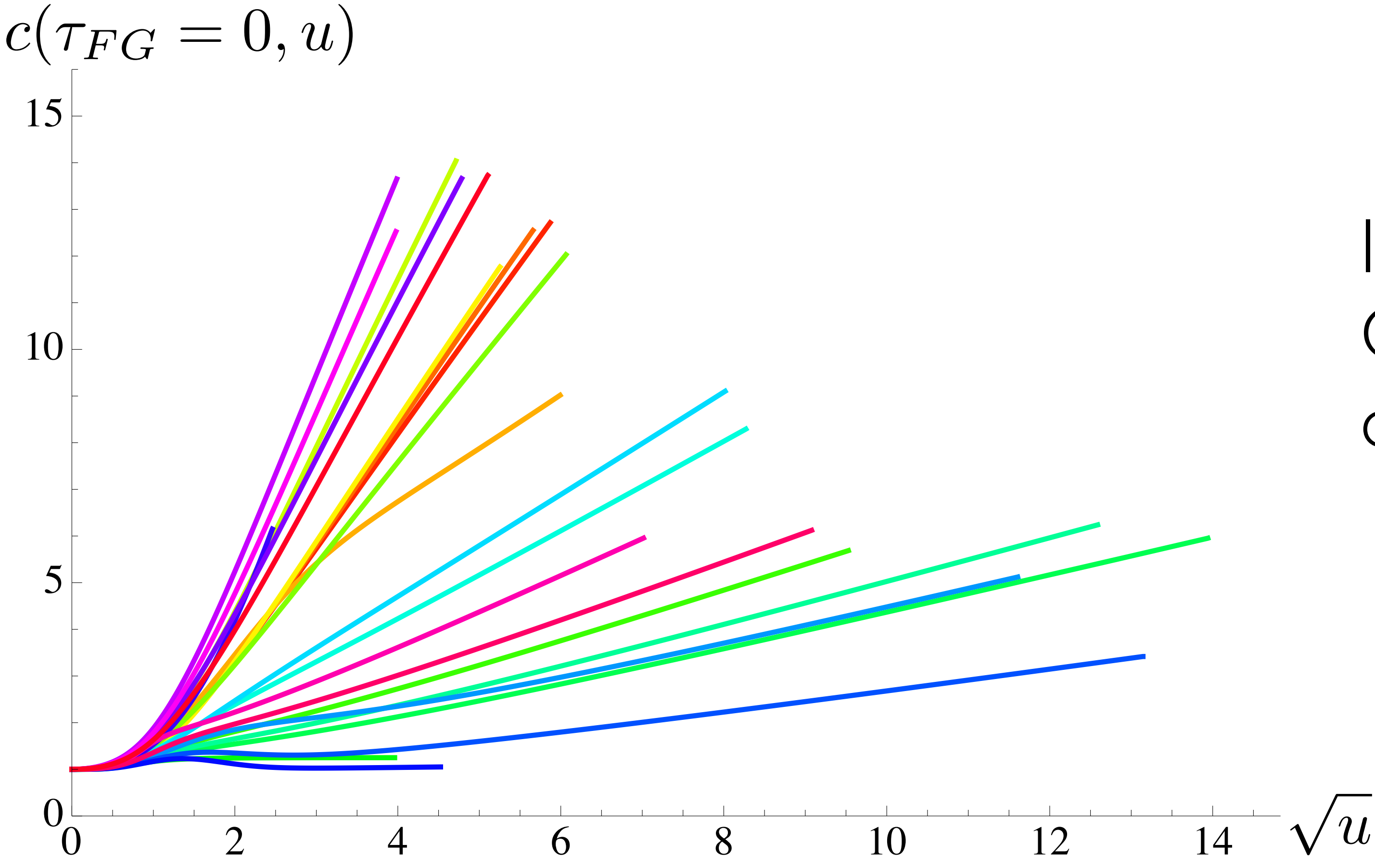
Beyond equilibrium event horizon is not the right notion of entropy.

In the gravity dual to boost-invariant flow it seems sensible to associate non-equilibrium entropy with unique translationally-invariant **apparent horizon**

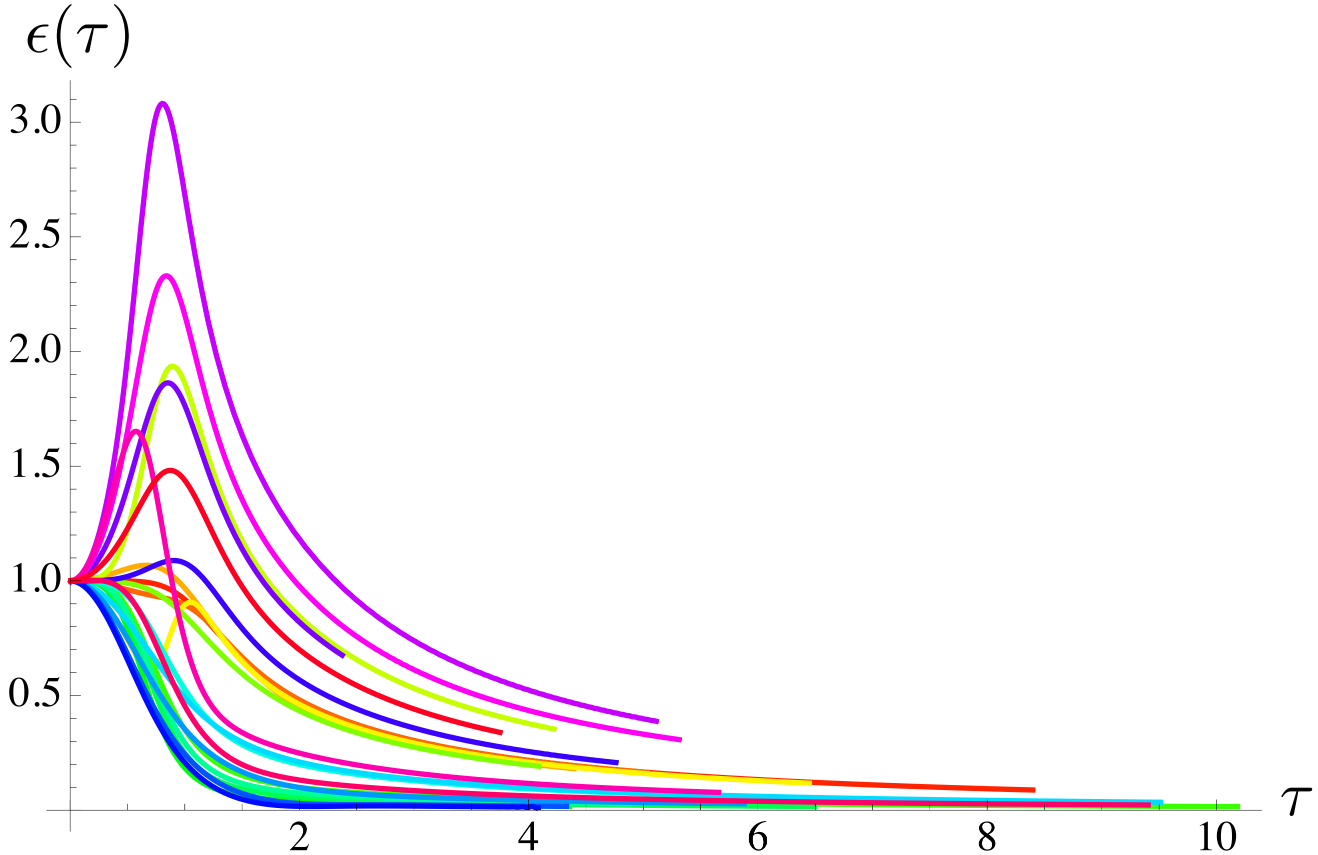
Its area element is associated with points on the boundary lying on the same **ingoing radial null geodesic** (bulk-boundary map)

All considered initial data had a non-zero non-equilibrium entropy at $\tau_{boundary} = 0$, thus **thermalization is not horizon formation, but rather horizon equilibration!**

Initial data and corresponding energy densities



Initial warp factors in Fefferman-Graham coordinates with radial cut off enabling seeing thermalization



Energies densities as functions of proper time (each corresponds to a warp factor of a given color)

Thermalization time

Local thermalization: $T_{\mu\nu}$ obeys eqns of hydrodynamics

In conformal hydrodynamics $T_{\mu\nu}$ can be expressed in terms of gradients of u^μ only

But here due to symmetries $u^\mu \partial_\mu = \partial_\tau$, so its gradients are trivial

Because of this $\nabla_\mu T^{\mu\nu} = 0$ in the boost-invariant hydro is a 1st order ODE for $\epsilon(\tau)$

We define T_{eff} by $\epsilon(\tau) = \frac{3}{8} N_c^2 \pi^2 T_{eff}(\tau)^4$ and use dimensionless qty $w = \tau T_{eff}$

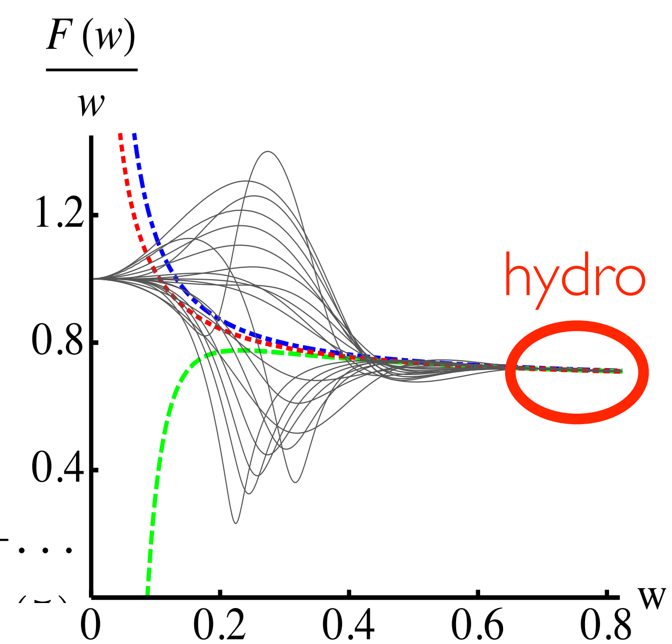
Equations of hydro: $\frac{\tau}{w}\frac{d}{d\tau} w = \frac{F_{hydro}(w)}{w}$

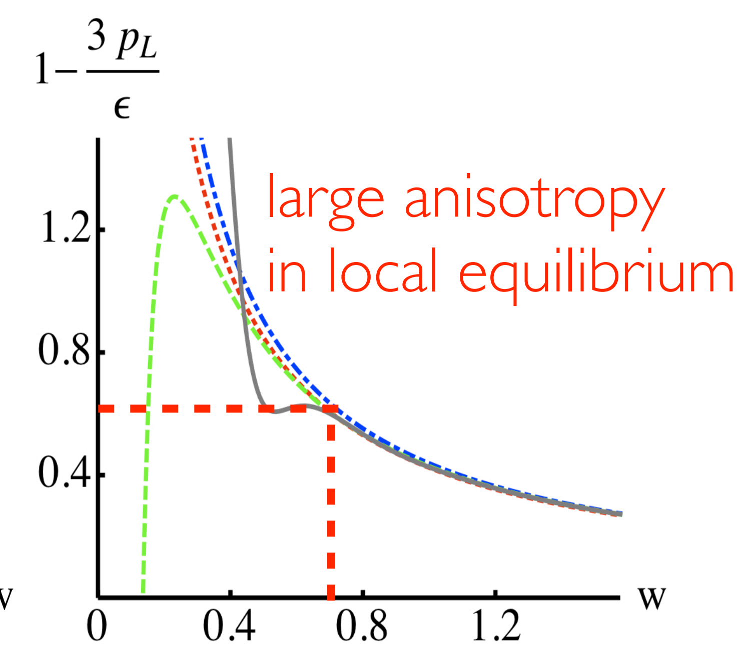
||

perfect fluid

$$\frac{2}{3} + \frac{1}{9\pi w} + \frac{1-\log 2}{27\pi^2 w^2} + \frac{15 - 2\pi^2 - 45\log 2 + 24\log^2 2}{972\pi^3 w^3} + \ldots$$

1st 2nd 3rd order hydro

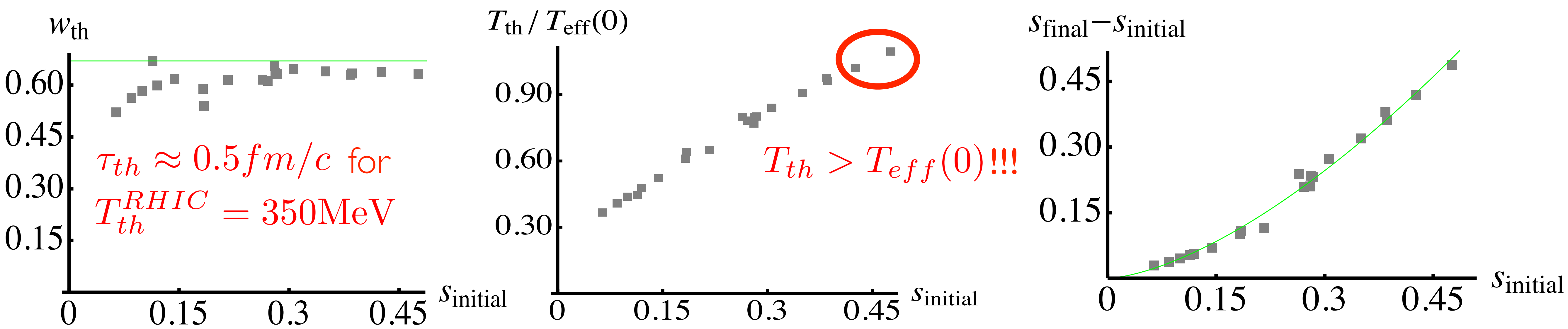




Characteristics of local thermalization

We choose $\left\| \frac{\tau \frac{d}{d\tau} w}{F_{hydro}^{3^{rd}\,order}(w)} - 1 \right\| < 0.5\%$ as a criterium for thermalization

Below are the plots of various non-equilibrium characteristics of plasma as a function of dimensionless entropy density defined by $S \cdot T_{eff}(0)^{-2} = N_c^2 \cdot \frac{1}{2}\pi^2 \cdot s$



Although initial far-from-equilibrium state is specified by infinitely many numbers (infinite number of derivatives of energy density at $\tau = 0$), **its energy density and non-equilibrium entropy seem to be the main characteristics** determining crude features of thermalization!

Summary

AdS/CFT naturally leads to short thermalization times

Holographic thermalization = bulk black hole equilibration

Key novelty - scanning through a large (20) set of initial data revealing rich dynamics

Holographic system can be very anisotropic $(\epsilon - 3p_L)/\epsilon \approx 0.6$, but locally thermalized

The most surprising observation is that initial non-equilibrium entropy predetermines crude features of boost-invariant thermalization at strong coupling

Open directions

Is there a simple model behind discovered phenomenological relations?

Do similar relations hold for less symmetric (more realistic) dynamics?

What are the properties of thermalization in the presence of transverse dynamics?