Holographics models for QCD in the Veneziano limit

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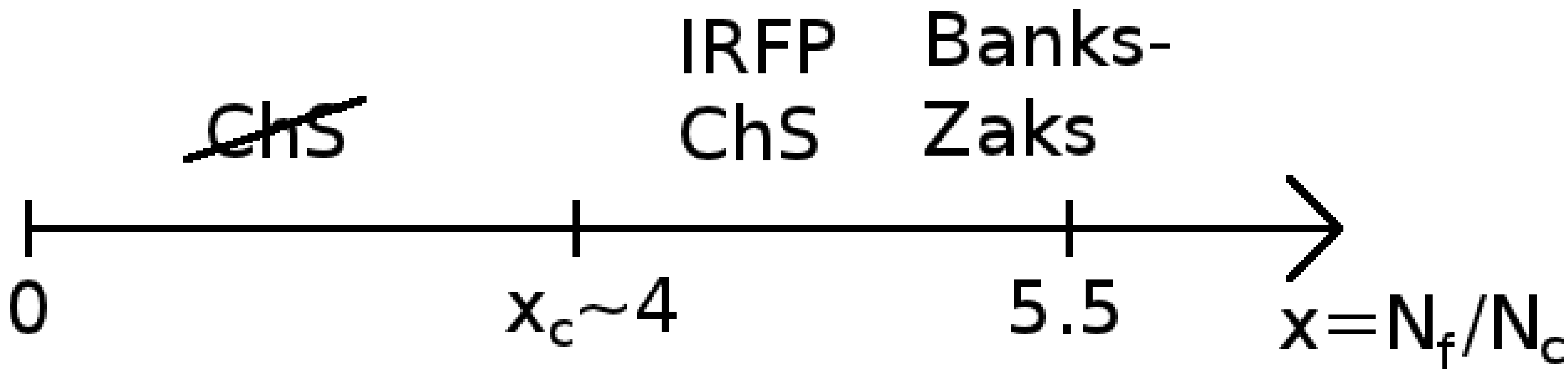
Outline

- Motivation
- Setup of the framework
  - Identifying models with desired phase structure
- Numerical results
  - Background
  - Free energy
  - Chiral condensate and BKT scaling

Motivation: QCD in the Veneziano limit

Veneziano limit: large N_f, N_c with $x = N_f/N_c$ fixed

"Phase diagram" for massless quarks



- $x \ll x_c$: running theory $\leftrightarrow$ standard QCD
- As $x \to x_c-$: quasiconformal or "walking" region
- $x_c < x < 5.5$: Conformal window

Motivation: Quarks in 5D dilaton gravity

$$\mathcal{S}_{\mathrm{g}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\phi)^2 + V_g(\phi) \right]$$

$$ds^2 = e^{2A}(dr^2 + \eta_{\mu\nu}x^\mu x^\nu)$$

- Potential $V_g \leftrightarrow$ QCD β-function
  - $A \to \log\mu$ energy scale
  - $e^\phi \to \lambda$ 't Hooft coupling

Fundamental quarks: probe $D4$-$\bar{D}4$ branes with $N_f \ll N_c \Rightarrow$

$$\mathcal{S}_{\mathrm{TDBI}} = -N_f N_c M^3 \int d^5x V_f(T) e^{-\phi} \sqrt{-\det(g_{ab} + \partial_a T \partial_b T)}$$

- $T \leftrightarrow \bar{q}q$

The Veneziano limit: backreaction

This work: Ansatz

$$\mathcal{S} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$-N_f N_c M^3 \int d^5x V_f(\lambda, T) \sqrt{-\det(g_{ab} + h(\lambda)\partial_a T \partial_b T)}$$

with $V_f(\lambda, T) = V_0(\lambda) \exp(-a(\lambda)T^2)$

- $N_c \to \infty$ with $x = N_f/N_c$ fixed: backreacted system
- Probe limit $x \to 0 \Rightarrow V_g$ fixed as before
- How to choose V_0, a, and h, in order to have the desired phase structure?

Effective potential

Assuming a constant tachyon

$$\mathcal{S} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) - x V_f(\lambda, T) \right]$$

$$V_{\mathrm{eff}}(\lambda) = V_g(\lambda) - x V_f(\lambda, T)$$

- In the UV or if ChS conserved, $T \to 0$ (since $T \sim mr + \sigma r^3$):

  $V_{\mathrm{eff}}(\lambda) = V_g(\lambda) - x V_f(\lambda, 0) = V_g(\lambda) - x V_0(\lambda)$

  - Glue dynamics modified by the presence of the tachyon

- In the IR require $T \to \infty$: $V_{\mathrm{eff}}(\lambda) = V_g(\lambda)$ (if ChSB)

Effective potential: IR fixed point

$$V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_0(\lambda)$$

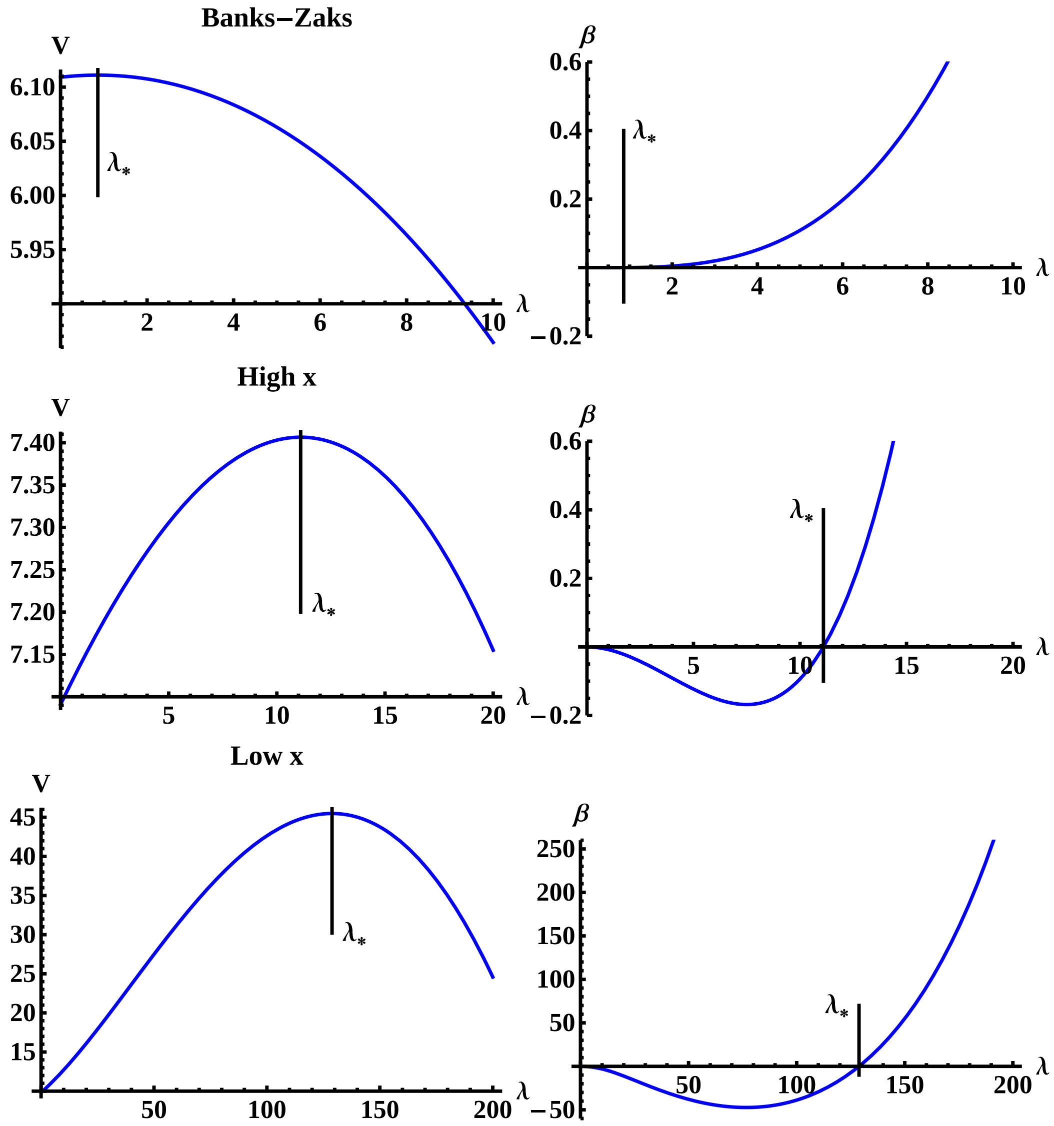
Fixed point must appear in the Banks-Zaks region, and move to higher λ with decreasing x

$\longrightarrow$ Match UV expansion

$V_{\text{eff}}(\lambda) = v_0 + v_1\lambda + v_2\lambda^2 \cdots$

to QCD beta function

$\beta(\lambda) = -\frac{2}{3}\frac{(11-2x)}{(4\pi)^2}\lambda^2 - \frac{(34-13x)}{3(4\pi)^4}\lambda^3 + \cdots$



Condensate dimension at IR fixed point

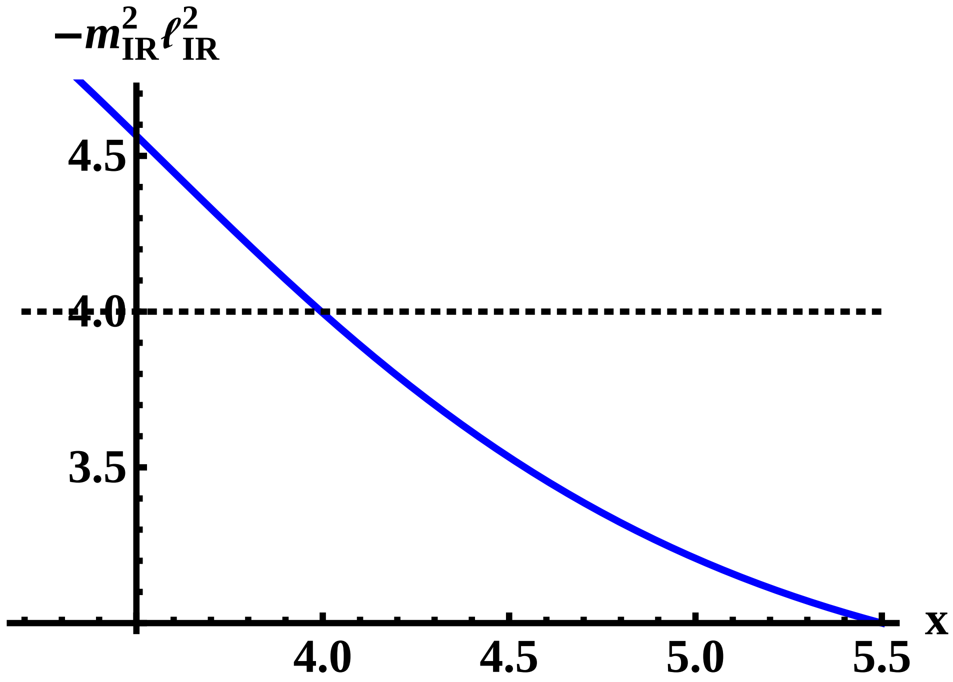
Tachyon IR mass at $\lambda = \lambda_*$ $\leftrightarrow$ chiral condensate dimension

$$-m_{\rm IR}^2 \ell_{\rm IR}^2 = \Delta_{\rm IR}(4-\Delta_{\rm IR}) = \frac{24 a(l_*)}{h(l_*)(V_g(l_*) - x V_0(l_*))}$$

Must reach the Breitenlohner-Freedman (BF) bound (horizontal line) at some x_c

$\longrightarrow$ constrains $a(\lambda)/h(\lambda)$

x_c marks the conformal phase transition



Saturating the BF bound (sketch)

Why is the BF bound saturated at the phase transition (massless quarks)??

$$\Delta_{\mathrm{IR}}(4-\Delta_{\mathrm{IR}}) = \frac{24a(l_*)}{h(l_*)(V_g(l_*) - xV_0(l_*))}$$

- For $\Delta_{\mathrm{IR}}(4-\Delta_{\mathrm{IR}}) < 4$:
  $T(r) \sim m_q r^{4-\Delta_{\mathrm{IR}}} + \sigma r^{\Delta_{\mathrm{IR}}}$
- For $\Delta_{\mathrm{IR}}(4-\Delta_{\mathrm{IR}}) > 4$:
  $T(r) \sim Cr^2 \sin\left[(\mathrm{Im}\Delta_{\mathrm{IR}})\log r + \phi\right]$
- Saturating the BF bound, the tachyon solutions will engtangle
  $\longrightarrow$ required to satisfy boundary conditions
- Nodes in the solution appear trough UV $\longrightarrow$ massless solution

Saturating the BF bound (sketch)

Does the nontrivial (ChSB) massless tachyon solution exist?

Two possibilities:

- $x > x_c$: BF bound satisfied at the fixed point $\Rightarrow$ only trivial massless solution ($T \equiv 0$, ChS intact, fixed point hit)
- $x < x_c$: BF bound violated at the fixed point $\Rightarrow$ a nontrivial massless solution exist, which drives the system away from the fixed point

Conclusion: phase transition at $x = x_c$

As $x \to x_c$ from below, need to approach the fixed point to satisfy the boundary conditions $\Rightarrow$ nearly conformal, “walking” dynamics

Matching with QCD

As $\lambda \to 0$ we can match

- $V_g(\lambda)$ with (two-loop) Yang-Mills beta function
- $V_g(\lambda) - xV_0(\lambda)$ with QCD beta function
- $a(\lambda)/h(\lambda)$ the with anomalous dimension of the quark mass/chiral condensate

Matching quite in general guarantees also the correct phase structure

Choices in the IR

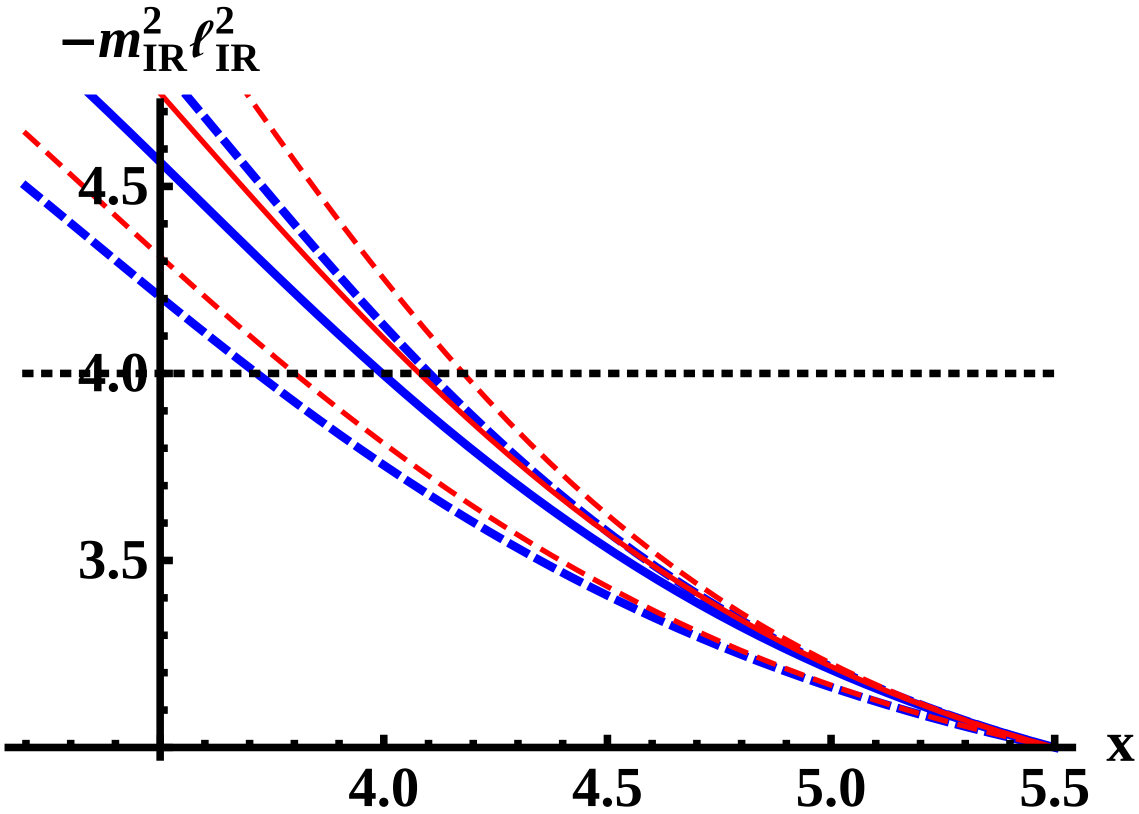
In the IR, the tachyon has to diverge $\Rightarrow$ the tachyon action $\propto e^{-T^2}$ becomes small

- $V_g(\lambda)$ chosen as for Yang-Mills, so that a “good” IR singularity exists
- $V_0(\lambda)$, $a(\lambda)$, and $h(\lambda)$ chosen to produce tachyon divergence: several possibilities
- Phase structure essentially independent of IR choices

Matching with QCD: "prediction" for x_c

After fixing UV coefficients from QCD, there is still freedom in choosing the leading coefficient of V_0 at $\lambda \to 0$ and the IR asymptotics of the potentials

Resulting variation of the edge of conformal window $x_c = 3.7 \ldots 4.2$



Summary of setup

- In order to have the correct phase structure:
  - $V_g(\lambda) - xV_0(\lambda)$ has a fixed point, at least for large x
  - Condensate dimension at the fixed point must decrease with decreasing $x \lesssim 5.5$ and reach the BF bound
- Potentials matched with QCD in the UV are ok
- (Reasonable) potential choices in IR do not affect the phase structure

Numerics: a choice of potentials

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda, T) &= V_0(\lambda)e^{-a(\lambda)T^2} \\
V_0(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473 - 2726x + 92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x) \\
h(\lambda) &= \frac{1}{\left(1 + \frac{115-16x}{288\pi^2}\lambda\right)^{4/3}}
\end{aligned}
$$

Numerics: degrees of freedom

For fixed x:

- EoMs for λ, A and T: two second order and one first order $\Rightarrow$ five integration constants
- "Conformal transformation" $r \to \Lambda(r - r_0)$: eliminate two
- Remaining three constants can be chosen to be m_q, σ, and F (the free energy)
- Good IR singularity fixes all parameters expect one: tachyon normalization in the IR T_0
- $\Rightarrow$ Can solve $\sigma = \sigma(m)$, $F = F(m)$ as expected

Numerics: some details

Two distinct backgrounds:

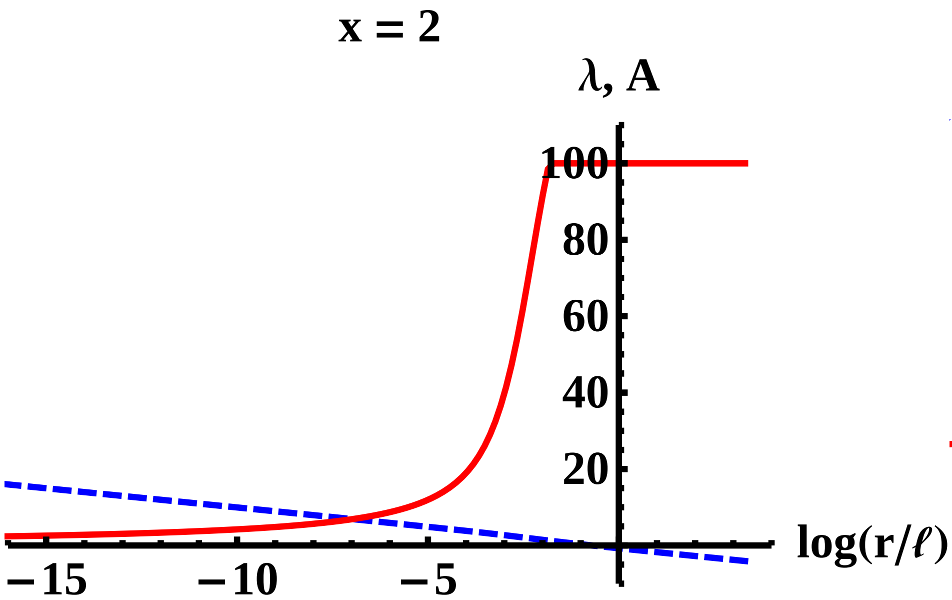
- $T \equiv 0$: zero quark mass, chiral symmetry conserved
- Tachyon diverging at $r \to \infty$: generic mass, chiral symmetry broken

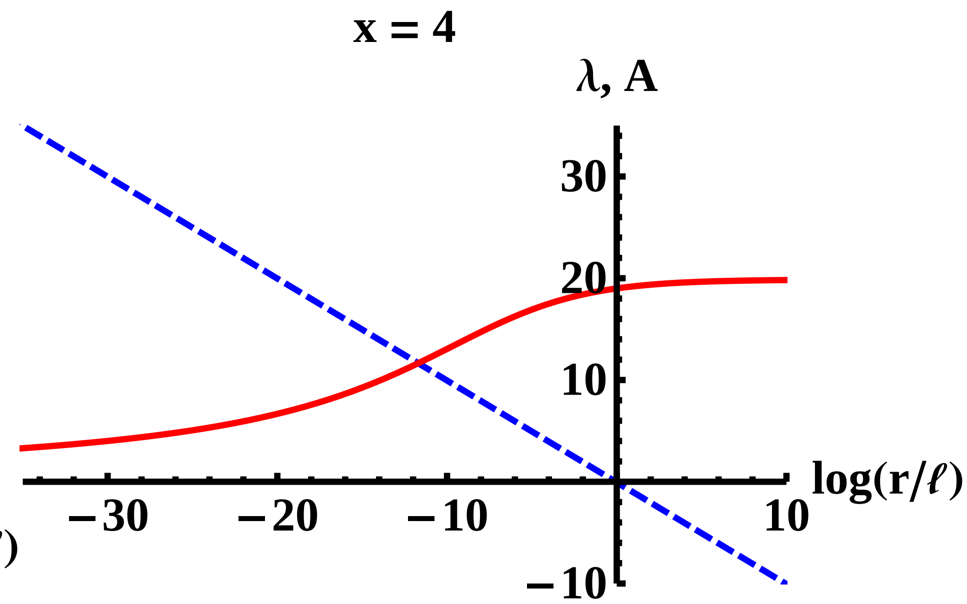
In order to construct the latter:

- Shooting from IR is numerically stable
- Match with IR expansions at a cutoff $r < \infty$
- Desired UV singularity ($\lambda \to 0$, $T \sim mr + \sigma r^3$) reached quite in general

Numerics: results

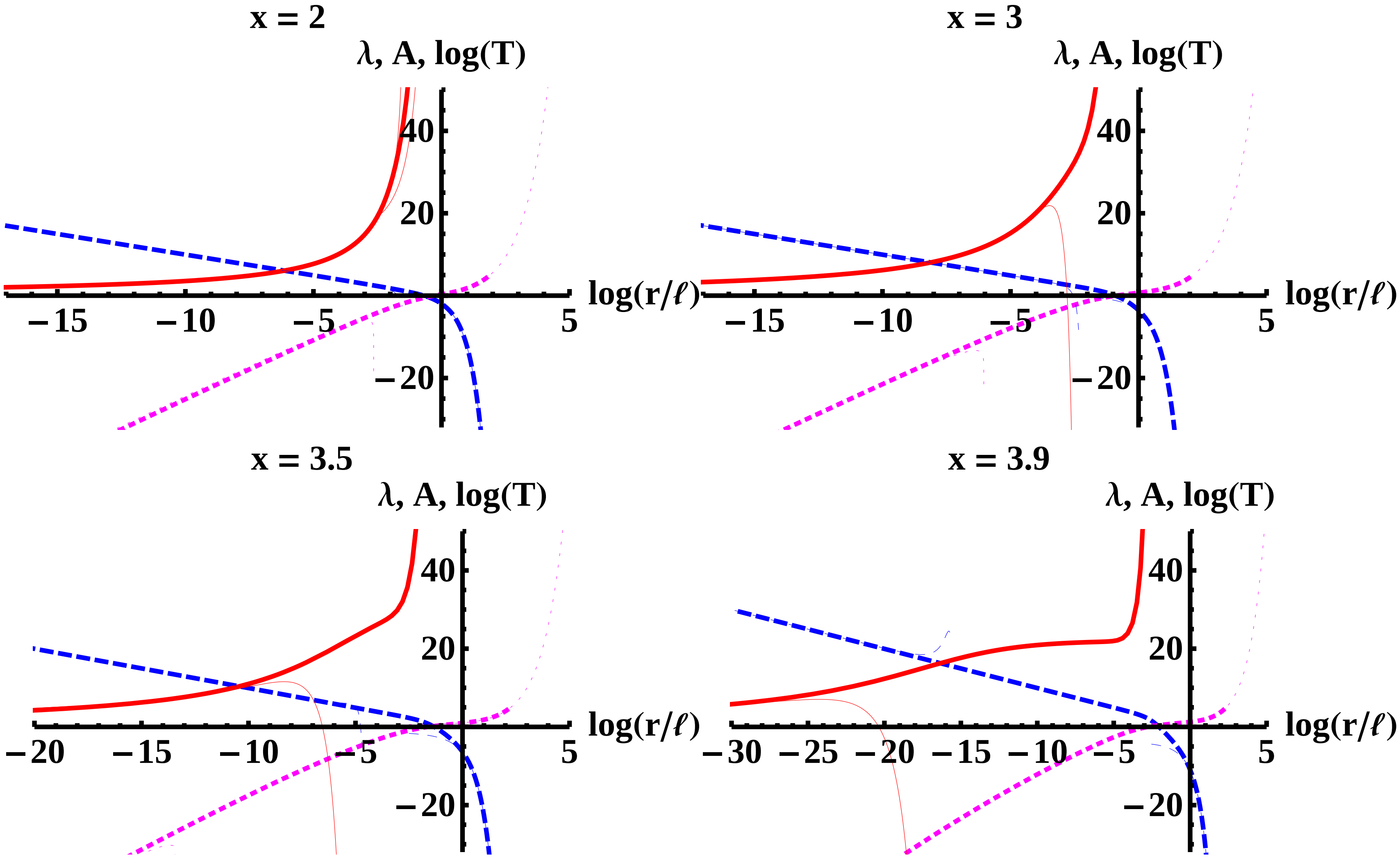
$T \equiv 0$ backgrounds (color codes λ, A)





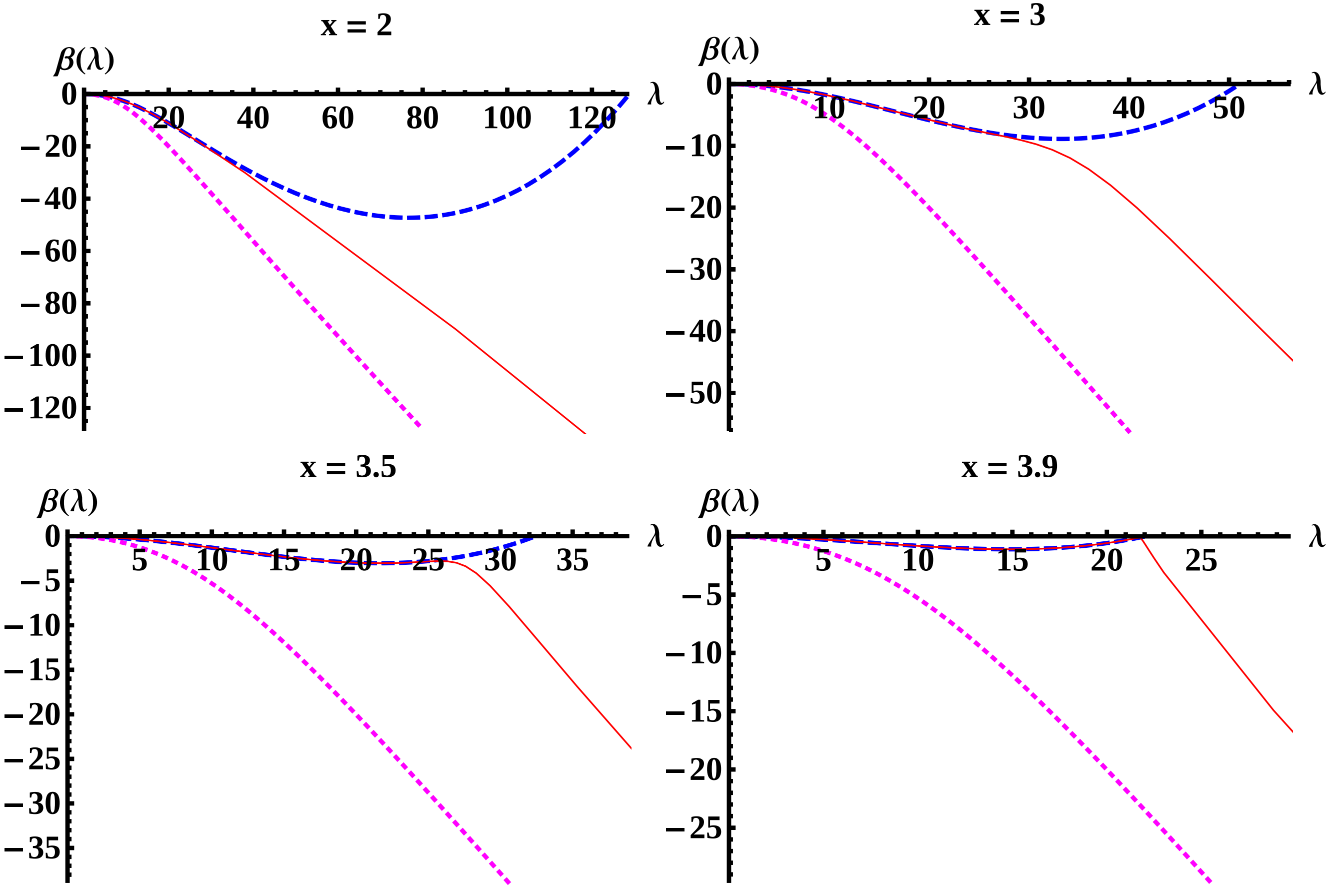
Numerics: results

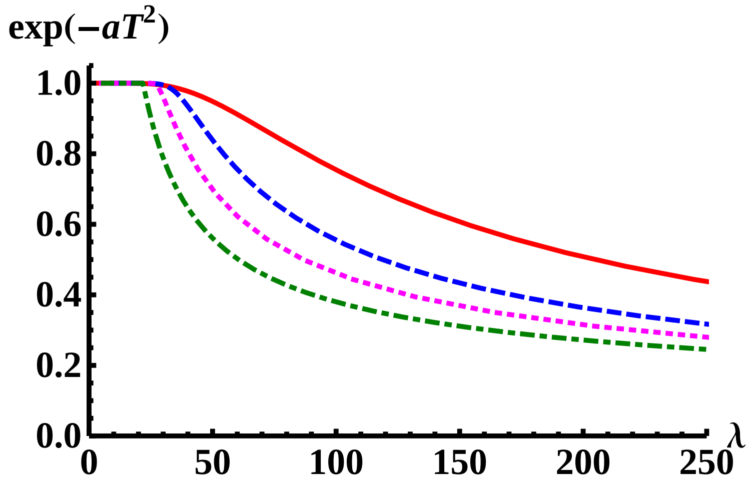
Massless backgrounds with $x < x_c \simeq 3.9959$ (λ, A, T)



Numerics: results

Massless backgrounds: beta functions $\beta = \frac{d\lambda}{dA}$ ($x_c \simeq 3.9959$)

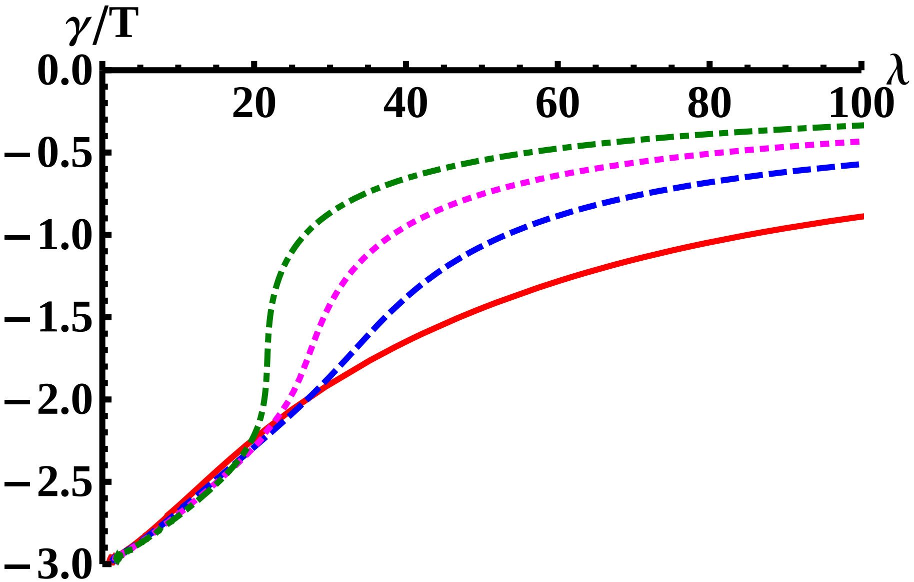


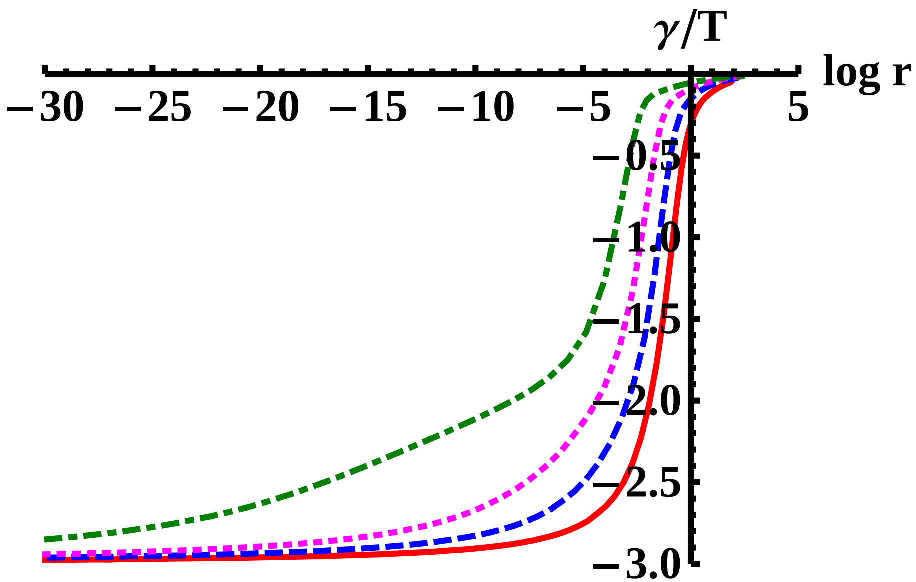


$x = 2, 3, 3.5, 3.9$

Numerics: results

Massless backgrounds: gamma functions $\frac{\gamma}{T} = \frac{d \log T}{dA}$



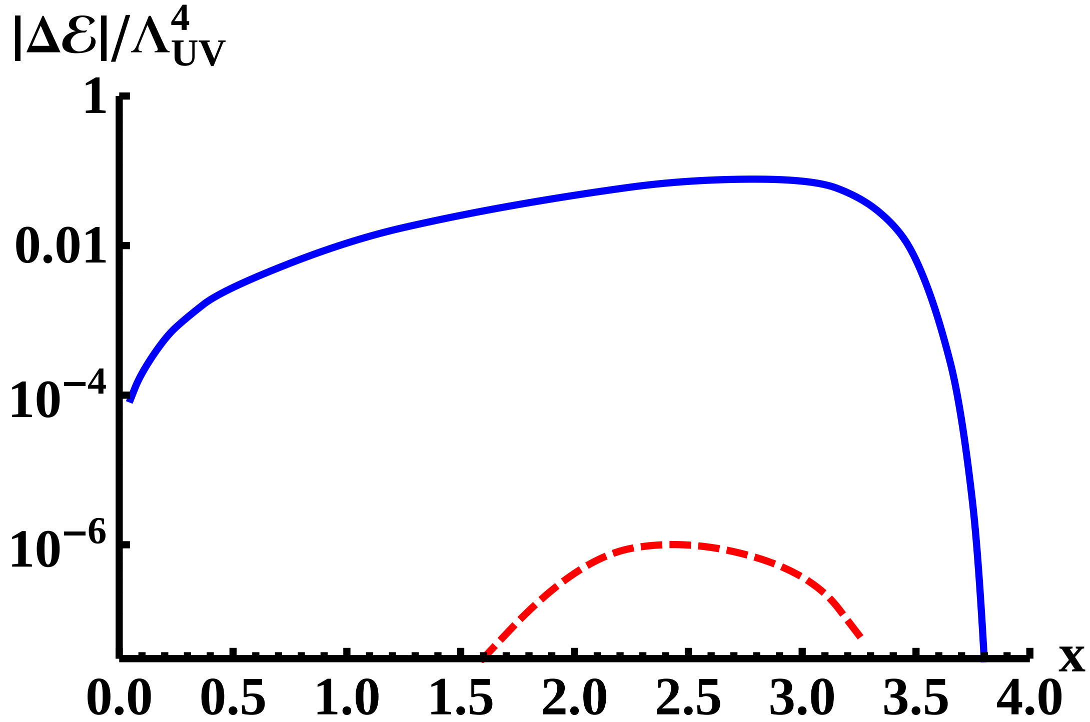


$x = 2, 3, 3.5, 3.9$

Free energy

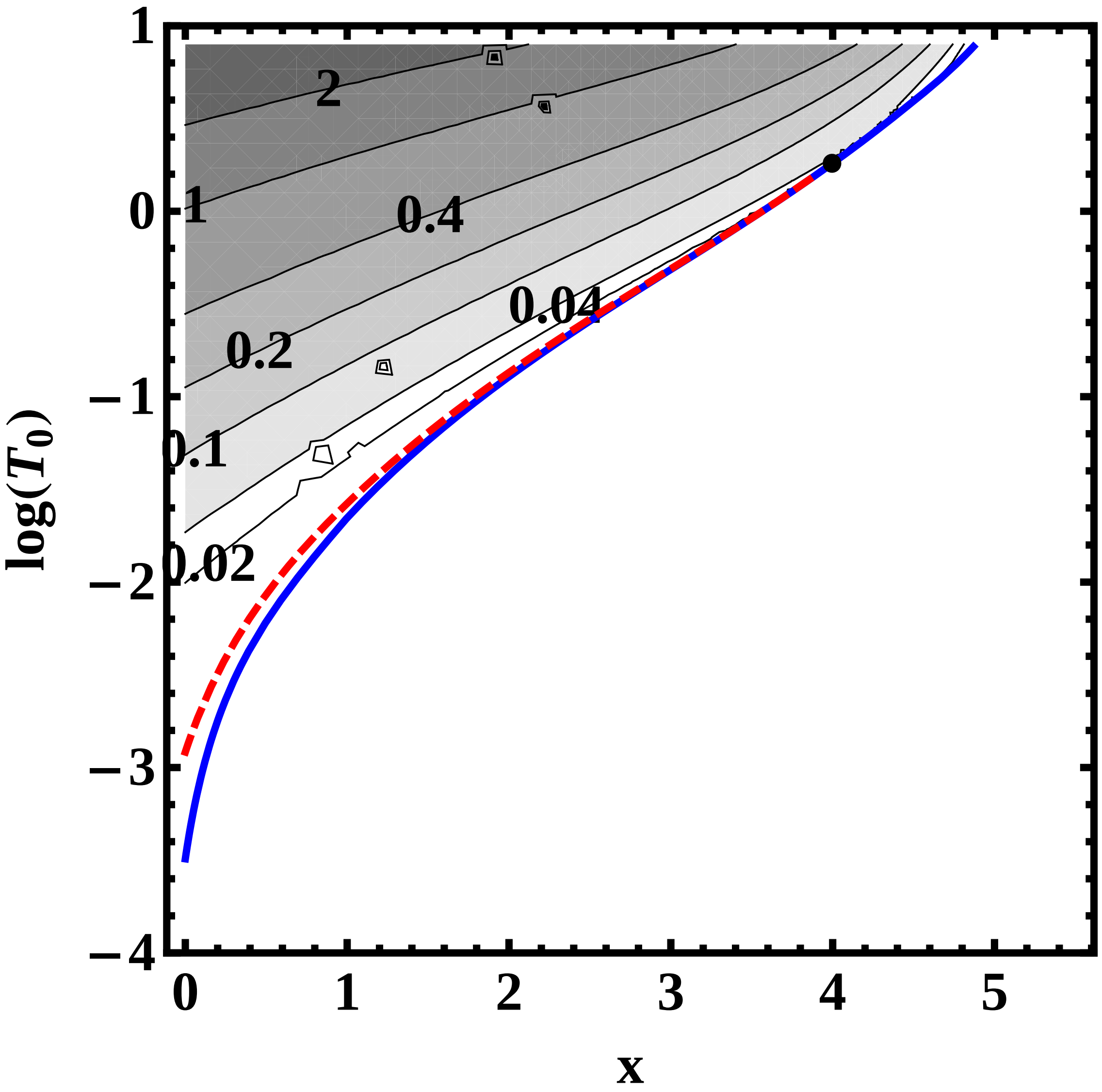
The free energy difference between the ChS and ChSB $m_q = 0$ solutions

Chiral symmetry breaking solution favored whenever it exists ($x < x_c$)



Numerics: adding quark mass

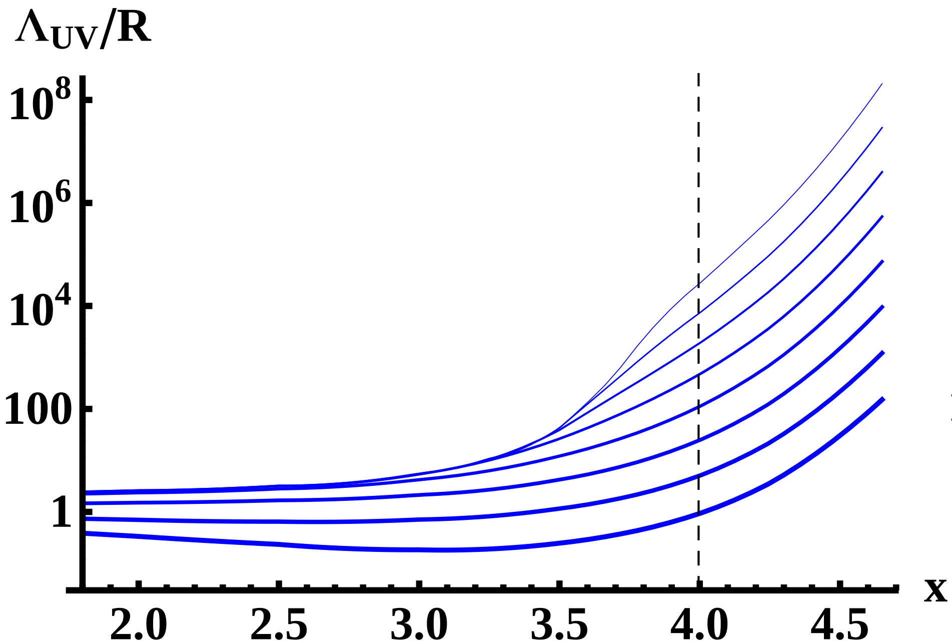
Mass dependence on x and T_0 (tachyon normalization)

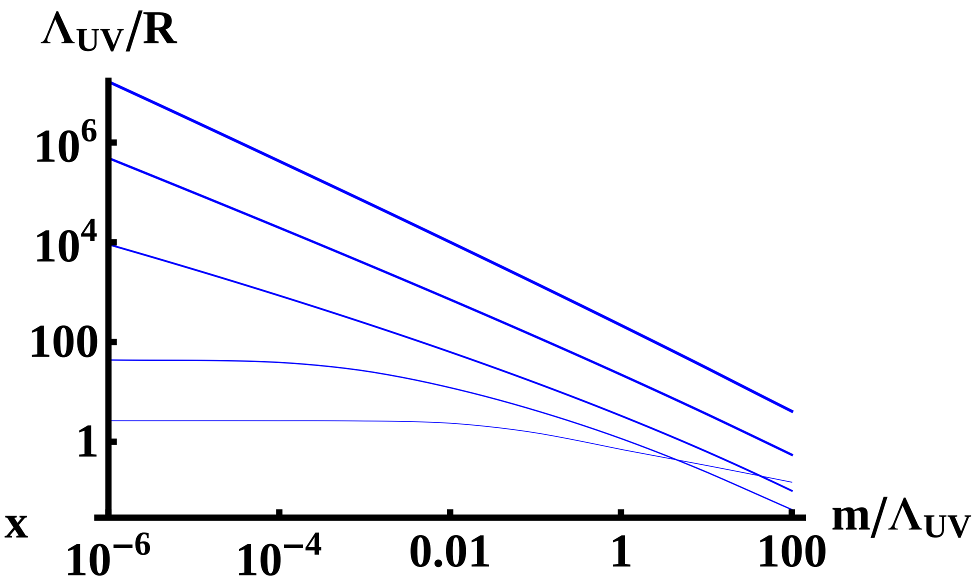


Numerics: adding quark mass

Qualitatively similar background as before, but the scale separation $\Lambda_{\rm UV}/\Lambda_{\rm IR}$ varies in a natural way

$m/\Lambda_{\rm UV} = 10^{-6}, 10^{-5}, \ldots, 10 \qquad x = 2, 3.5, 3.9, 4.25, 4.5$

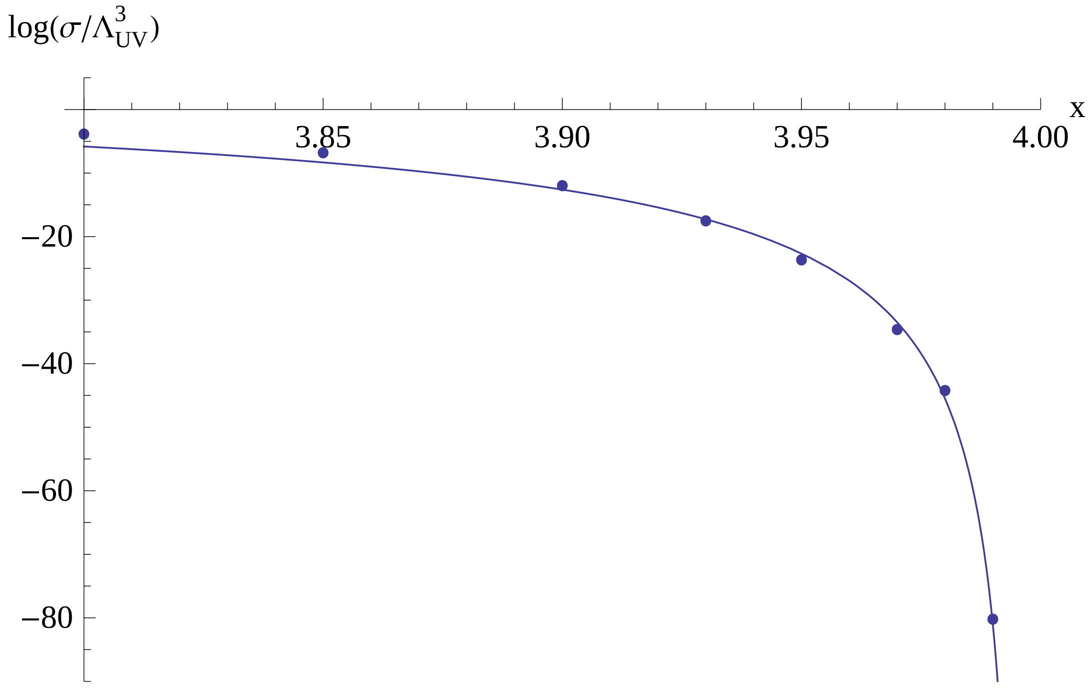




BKT scaling

As $x \longrightarrow x_c$, the chiral condensate $\sim \sigma$ (for $m_q = 0$) and the scale separation $\Lambda_{\mathrm{UV}}/\Lambda_{\mathrm{IR}}$ have the peculiar scaling $\sim \exp(-\#/\sqrt{x_c - x})$

- Dots: numerical extraction of σ
- Line: BKT scaling fit
- Can be derived analytically from the tachyon behavior



Conclusion

- We identified a class of holographic bottom-up models that shows the qualitative features of the QCD phase structure (varying $x = N_f/N_c$ and m_q)
- Next steps: fluctuation and finite temperature analysis