The holographic fluid dual to vacuum Einstein gravity

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Plan

Any gravitational theory is expected to be holographic, *i.e.* it should have a description in terms of a non-gravitational theory in one dimension less.

- If gravity is indeed holographic, one should be able to recover generic features of quantum field theories through gravitational computations.

Holography and asymptotics

- Indeed, in the cases we understand holography, *i.e.* for asymptotically AdS spacetimes and spacetimes conformal to that, one can prove that the divergences are local in boundary data. [Henningson, Skenderis (1998)], [Kanitscheider, Skenderis, M.T. (2008)]
- Conversely, if the IR divergences of a gravitational theory are non-local, the dual quantum theory cannot be a local QFT.
- Asymptotically flat spacetimes fall into this category. The structure of the asymptotic solutions shows that the divergences of the on-shell action are non-local in boundary data. [de Haro, Solodukhin, Skenderis (2001)].
- Holography for such spacetimes is more difficult to understand ... as the dual theory should be non-local.

Holography and long wavelength behavior

- Another generic feature of QFTs is the existence of a *hydrodynamic description* capturing the long-wavelength behavior near to thermal equilibrium.
- One then expects to find the same feature on the gravitational side, *i.e.*, there should exist a bulk solution corresponding to the thermal state, and nearby solutions corresponding to the hydrodynamic regime.
- Global solutions corresponding to non-equilibrium configurations should be well-approximated by the solutions describing the hydrodynamic regime at sufficiently long distances and late times.

Hydrodynamics and AdS/CFT

This picture is indeed beautifully realized in AdS/CFT:

Thermal state	$\Leftrightarrow$	AdS black hole
Relativistic hydrodynamics	$\Leftrightarrow$	Relativistic gradient expansion solution of bulk

- Solutions describing non-equilibrium configurations are well approximated by hydrodynamics at late times.

[Witten (1998)] ... [Policastro, Son, Starinets (2001)] ... [Janik, Peschanski (2005)] ... [Bhattacharyya, Hubeny, Minwalla, Rangamani (2007)] ... [Chestler, Yaffe (2010)] ...

Hydrodynamics and vacuum Einstein gravity

We will see that a similar picture can be developed for vacuum Einstein gravity:

Thermal state	$\Leftrightarrow$	Rindler space
Incompressible Navier-Stokes expansion $+$ corrections	$\Leftrightarrow$	Non-relativistic gradient solution of bulk

One may then use the properties of these solutions in order to obtain clues about the nature of the dual theory.

References

- The talk is based on

  Geoffrey Compère, Paul McFadden, Kostas Skenderis, M.T.,
  The holographic fluid dual to vacuum Einstein gravity,
  [arXiv:1104.3894].
  along with work in progress.
- Key related works:

  I. Bredberg, C. Keeler, V. Lysov, A. Strominger,
  [arXiv:1101.2451]; V. Lysov, A. Strominger [arXiv:1104.5502],
  along with subsequent follow ups.

- Earlier works:

  T. Damour, PhD thesis, 1979; K. Thorne, R. Prince, D. Macdonald, “Black Holes: the membrane paradigm" (1986).

  I. Fouxon, Y. Oz, [arXiv:0809.4512]; C. Eling, I. Fouxon, Y. Oz, [arXiv:0905.3638].

  S. Bhattacharyya, S. Minwalla, S. Wadia, [arXiv: 0810.1545].

  I. Bredberg, C. Keeler, V. Lysov, A. Strominger, “Wilsonian approach to Fluid/Gravity duality", [arXiv:1006.1902].

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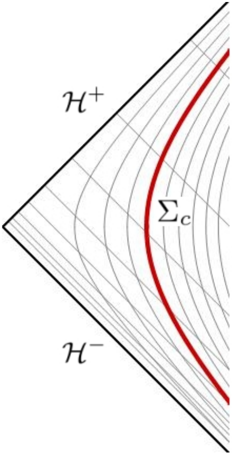
Rindler spacetime

- Flat spacetime in ingoing Rindler coordinates is give by:

$$ds^2 = -r d\tau^2 + 2 d\tau dr + dx_i dx^i$$

  i.e. Minkowski space parametrised by timelike hyperbolae $X^2 - T^2 = 4r$ and ingoing null geodesics $X + T = e^{\tau/2}$.

- We will consider the portion of spacetime between $r = r_c$ and the future horizon, $\mathcal{H}^+$, the null hypersurface $X = T$.



Rindler spacetime: properties

- The induced metric γ_{ab} on Σ_c $(r = r_c)$ is flat.
- The Rindler horizon has constant Unruh temperature,

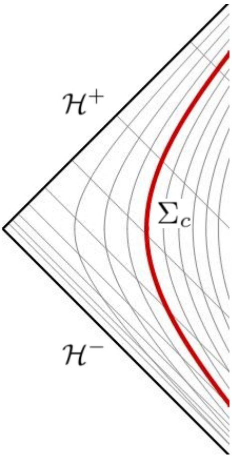
$$T = \frac{1}{4\pi\sqrt{r_c}}$$

- The Brown-York stress energy tensor takes the perfect fluid form:

$$T_{ab} = \rho u_a u_b + p h_{ab}$$

with

$$\rho = 0, \quad p = \frac{1}{\sqrt{r_c}}, \quad u^a = (\frac{1}{\sqrt{r_c}}, \vec{0}).$$



Equilibrium configurations

We now want to obtain a family of equilibrium configurations parametrized by arbitrary constants that would become the hydrodynamic variables in the hydrodynamic regime.

We require three properties:

1. There exists a co-dimension one hypersurface Σ_c on which the fluid lives, with flat induced metric:

$$\gamma_{ab} dx^a dx^b = -r_c d\tau^2 + dx_i dx^i$$

$\sqrt{r_c}$ is speed of light (arbitrary)

2. The Brown-York stress tensor on Σ_c takes the perfect fluid form

$$T_{ab} = \rho u_a u_b + p h_{ab},$$

where $h_{ab} = \gamma_{ab} + u_a u_b$ is spatial metric in local rest frame of fluid.

3. Stationary w.r.t. ∂_τ and homogeneous in x^i directions.

Equilibrium configurations

- One configuration satisfying properties ❶, ❷, ❸ is Rindler spacetime.
- We generate metrics with arbitrary constant p and u^a by acting on Rindler spacetime with diffeomorphisms.
- There are the only *two infinitesimal diffeomorphisms* that preserve the properties ❶, ❷, ❸.

Equilibrium configurations

Exponentiating, these are:

- A constant boost

$$\sqrt{r_c}\tau \to \gamma\sqrt{r_c}\tau - \gamma\beta_i x^i, \qquad x^i \to x^i - \gamma\beta^i\sqrt{r_c}\tau + (\gamma - 1)\frac{\beta^i\beta_j}{\beta^2}x^j,$$

  where $\gamma = (1-\beta^2)^{-1/2}$ and $\beta_i = v_i/\sqrt{r_c}$.

- A constant linear shift of r and re-scaling of τ,

$$r \to r - r_h, \qquad \tau \to (1 - r_h/r_c)^{-1/2}\tau.$$

  This second transformation shifts the position of the horizon to $r = r_h < r_c$.

Equilibrium configurations

Applying these two transformations, the resulting metric is

$$ds^2 = -p^2(r - r_c)u_a u_b dx^a dx^b - 2pu_a dx^a dr + \gamma_{ab} dx^a dx^b.$$

- The induced metric on Σ_c is still γ_{ab}.
- The Brown-York stress tensor is that of a perfect fluid with

$$\rho = 0, \qquad p = \frac{1}{\sqrt{r_c - r_h}}, \qquad u^a = \frac{1}{\sqrt{r_c - v^2}}(1, v_i).$$

- The Unruh temperature is given by

$$T = \frac{1}{4\pi\sqrt{r_c - r_h}}$$

and all thermodynamic identities are satisfied, with the entropy density given by $s = 1/4G$.

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From equilibrium to hydrodynamics

We now wish to consider near-equilibrium configurations.

- We consider the pressure field p and velocities v_i as slowing varying functions of the coordinates.
- We will further consider the limit,

$$v_i^{(\epsilon)}(\tau, \vec{x}) = \epsilon v_i(\epsilon^2 \tau, \epsilon \vec{x}), \qquad P^{(\epsilon)}(\tau, \vec{x}) = \epsilon^2 P(\epsilon^2 \tau, \epsilon \vec{x}), \qquad \epsilon \to 0$$

  where P is the pressure fluctuation around the background value p.
- Keeping terms through order ϵ^2, one finds that the resulting metric satisfies Einstein's equations to $O(\epsilon^3)$, provided one imposes,

$$\partial_i v^i = O(\epsilon^3)$$

Solution to order ϵ^3

- At next order, one can add a new term, $g_{\mu\nu}^{(n)}$, of order ϵ^3 such that the resulting metric solves Einstein equations though order ϵ^3.
- In order for the metric to be Ricci-flat one must impose

$$\partial_\tau v_i + v^j \partial_j v_i - \eta \partial^2 v_i + \partial_i P = O(\epsilon^4),$$

which is precisely the Navier-Stokes equation!
- The metric up to this order was obtained first by Bredberg, Keeler, Lysov, Strominger [arXiv:1101.2451]

Incompressible Navier-Stokes

The incompressible Navier-Stokes equations read

$$\partial_\tau v_i + v^j \partial_j v_i - \eta \partial^2 v_i + \partial_i P = 0, \qquad \partial_i v^i = 0.$$

- The incompressible Navier-Stokes equation captures the universal long-wavelength behavior of essentially any $(d+1)$-dimensional fluid.
- They have an interesting scaling symmetry

$$v_i \to \epsilon v_i(\epsilon^2 \tau, \epsilon \vec{x}), \qquad P \to \epsilon^2 P(\epsilon^2 \tau, \epsilon \vec{x}).$$

- Higher-derivative correction terms are then naturally organized according to their scaling with ϵ.

Solving to all orders

We will now show to construct the solution to arbitrarily high order in ϵ.

- Suppose we have a solution at order ϵ^{n-1}. Let's now add a new term to the metric $g^{(n)}_{\mu\nu}$ at order ϵ^n. The Ricci tensor is

$$R^{(n)}_{\mu\nu} = \delta R^{(n)}_{\mu\nu} + \hat{R}^{(n)}_{\mu\nu}.$$

Here, $\delta R^{(n)}_{\mu\nu}$ is the contribution at order ϵ^n due to the new term $g^{(n)}_{\mu\nu}$, while $\hat{R}^{(n)}_{\mu\nu}$ is the nonlinear contribution at order ϵ^n from the metric at lower orders.

Solving to all orders

- We know $\delta R^{(n)}_{\mu\nu}$ from the usual linearized formula. Moreover, since

$$\partial_r \sim 1, \qquad \partial_i \sim \epsilon, \qquad \partial_\tau \sim \epsilon^2,$$

  we need only keep r-derivatives in this formula, since the rest are higher order.

- The key idea is just that of a gradient expansion:
  The ϵ-expansion filters out the hydrodynamic modes for which $\partial_r \sim 1$, $\partial_i \sim \epsilon$ and $\partial_\tau \sim \epsilon^2$. This assumed hierarchy in derivatives splits the PDE $R_{\mu\nu} = 0$ into a series of coupled ODEs in r.

- We can now set $R^{(n)}_{\mu\nu} = \delta R^{(n)}_{\mu\nu} + \hat{R}^{(n)}_{\mu\nu} = 0$ and try to solve for $g^{(n)}_{\mu\nu}$ in terms of the metric at lower orders.

Integrability conditions

- For this to be possible, however, the following integrability conditions must be satisfied:

$$0 = \partial_r(\hat{R}_{ii}^{(n)} - r\hat{R}_{rr}^{(n)}) - \hat{R}_{rr}^{(n)}, \qquad 0 = \hat{R}_{\tau a}^{(n)} + r\hat{R}_{ra}^{(n)}.$$

- To establish this, we first examine the Bianchi identity at order ϵ^n

$$0 = \partial_r(\hat{R}_{ii}^{(n)} - r\hat{R}_{rr}^{(n)}) - \hat{R}_{rr}^{(n)},$$

$$0 = \partial_r(\hat{R}_{\tau a}^{(n)} + r\hat{R}_{ra}^{(n)}) \quad \Rightarrow \quad \hat{R}_{\tau a}^{(n)} + r\hat{R}_{ra}^{(n)} = f_a^{(n)}(\tau, \vec{x}).$$

- The integrability conditions are therefore satisfied provided the arbitrary function $f_a^{(n)}(\tau, \vec{x})$ vanishes. This in turn follows from conservation of the Brown-York stress tensor on Σ_c. Using the Gauss-Codazzi identity,

$$\nabla^b T_{ab}\Big|_{\Sigma_c}^{(n)} = [2\nabla^b(K\gamma_{ab} - K_{ab})]^{(n)} = -\frac{2}{\sqrt{r_c}} f_a^{(n)}(\tau, \vec{x}).$$

Summary

- Thus, conservation of the Brown-York stress tensor on Σ_c is necessary for the bulk equations to be integrated.
- From the perspective of the dual fluid, conservation of the Brown-York stress tensor is equivalent to incompressibility (at ϵ^2 order) and the Navier-Stokes equation (at ϵ^3 order). At higher orders in ϵ we obtain corrections to these equations.
- To complete our integration scheme, we choose the gauge

$$g^{(n)}_{r\mu} = 0$$

and impose boundary conditions such that:
  - the metric on Σ_c is preserved
  - the solution is regular on the future horizon $\mathcal{H}^+$.

Integration scheme

- Our final integration scheme is thus

$$g^{(n)}_{r\mu} = 0,$$
$$g^{(n)}_{\tau\tau} = (1 - r/r_c)F^{\nu}_{\tau}(\tau, \vec{x}) + \int_r^{r_c} dr' \int_{r'}^{r_c} dr''(\hat{R}^{(n)}_{ii} - r\hat{R}^{\nu}_{rr} - 2\hat{R}^{\nu}_{r\tau}),$$
$$g^{(n)}_{\tau i} = (1 - r/r_c)F^{\nu}_{i}(\tau, \vec{x}) - 2\int_r^{r_c} dr' \int_{r'}^{r_c} dr''\hat{R}^{\nu}_{ri},$$
$$g^{(n)}_{ij} = -2\int_r^{r_c} dr' \frac{1}{r'} \int_0^{r'} dr''\hat{R}^{\nu}_{ij},$$

where the arbitrary functions F^{ν}_{τ} and F^{ν}_{i} encode the freedom to redefine P and v_i at order ϵ^n.

Fluid gauge conditions

The remaining freedom may be fixed by choosing appropriate gauge conditions for the dual fluid.

- F_i^ν may be fixed by imposing Landau gauge on the fluid:

$$0 = u^a T_{ab} h^b_c$$

  i.e. the momentum density $T_{\tau i}$ vanishes in the local rest frame. This is effectively a definition of the fluid velocity u^a.

- F_τ^ν is fixed by imposing that there are no corrections to the isotropic part of the stress tensor:

$$T^{\rm iso}_{ij} = \left(\frac{1}{\sqrt{r_c}} + \frac{P}{r_c^{3/2}} \right) \delta_{ij}.$$

  This effectively defines the pressure fluctuation to be exactly P.

- With all gauge freedom now fixed, we have a unique solution for the bulk metric in terms of v_i and P.

Bulk solution

We computed this bulk solution explicitly through to ϵ^5 order, for arbitrary spacetime dimension.

- For example, at ϵ^3 order, the only nonzero term is:

$$g^{(3)}_{\tau i} = \frac{(r - r_c)}{2r_c} \left[(v^2 + 2P) \frac{2v_i}{r_c} + 4\partial_i P - (r + r_c)\partial^2 v_i \right].$$

- At ϵ^4 order, the nonzero terms are $g^{(4)}_{\tau\tau}$ and $g^{(4)}_{ij}$.
- At ϵ^5 order, only $g^{(5)}_{\tau i}$ is nonzero. [See arXiv:1103.3022]

➢ This behavior makes sense since all scalars and tensors constructed from v_i, P and their derivatives are of even order in ϵ, while all vector quantities are odd.

➢ Interestingly, [arXiv:1101.2451] noted the solution is Petrov type II at leading non-trivial order. This appears *not* to extend to higher order however. ($I^3 - 27J^2$ is nonzero at order ϵ^{14}.)

Recovering Navier-Stokes and incompressibility

From our unique bulk solution, we recover the Navier-Stokes and incompressibility equations, along with a unique set of corrections.

These arise from the momentum constraint on Σ_c:

$$0 = \nabla^b T_{ab}\Big|_{\Sigma_c} = 2\nabla^b (K\gamma_{ab} - K_{ab})$$

- At even orders in ϵ we recover the incompressibility equation plus corrections,

$$\partial_i v_i = \frac{1}{r_c} v_i \partial_i P - v_i \partial^2 v_i + 2\partial_{(i} v_{j)} \partial_i v_j + O(\epsilon^6),$$

- At odd orders we recover Navier-Stokes plus corrections,

$$\partial_\tau v_i + v_j \partial_j v_i - r_c \partial^2 v_i + \partial_i P = (-\frac{3r_c^2}{2}\partial^4 v_i + 2r_c v_k \partial^2 \partial_k v_i + \ldots) + O(\epsilon^7).$$

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The underlying *relativistic* fluid

As the ϵ-expansion is non-relativistic, T_{ab} appears to be non-relativistic. In fact, however, there is an underlying *relativistic* stress tensor which, when expanded out in ϵ, reproduces our above results.

- This is in agreement with the expectation that the dual holographic theory should be relativistic.
- The relativistic stress tensor is much simpler: all information is encoded in only a few transport coefficients. In general,

$$T_{ab} = \rho u_a u_b + p h_{ab} + \Pi^{\perp}_{ab}, \qquad u^a \Pi^{\perp}_{ab} = 0,$$

where $\Pi^{\perp}_{ab}$ represents dissipative corrections and may be expanded in fluid gradients.

Characterizing the dual fluid

- One unusual feature compared to standard relativistic hydrodynamics, however, is that the equilibrium energy density vanishes.
- From our bulk solution, the energy density in the local rest frame is given by

$$\rho = T_{ab}u^a u^b = -\frac{1}{2\sqrt{r_c}}\sigma_{ij}\sigma_{ij} + O(\epsilon^6), \qquad \sigma_{ij} = 2\partial_{(i}v_{j)}.$$

  This vanishes when v_i is constant, and is otherwise negative!
- We note that the Hamiltonian constraint on Σ_c imposes

$$dT_{ab}T^{ab} = T^2.$$

  This determines ρ in terms of p and $\Pi^{\perp}_{ab}$.
- The Hamiltonian constraint therefore plays a role analogous to an equation of state.

First order relativistic hydrodynamics

- At first order in fluid gradients,

$$\Pi^{\perp}_{ab} = -2\eta \mathcal{K}_{ab} + O(\partial^2), \qquad \mathcal{K}_{ab} = h^c_a h^d_b \partial_{(c} u_{d)},$$

  Note there is no bulk viscosity term $\zeta \mathcal{K} h_{ab}$, because $\mathcal{K} = \partial_a u^a$ and the fluid is incompressible: $0 = u^a \partial^b T_{ab} = -p \partial_a u^a + O(\partial^2)$.

- Expanding T_{ab} in ϵ we get

$$\eta = 1, \qquad \eta/s = 1/(4\pi)$$

- The 'equation of state' then fixes

$$\rho = -\frac{2\eta^2}{p} \mathcal{K}_{ab} \mathcal{K}^{ab} + O(\partial^3).$$

  and upon expanding in ϵ we recover

$$\rho = -\frac{1}{2\sqrt{r_c}} \sigma_{ij} \sigma_{ij} + O(\epsilon^6).$$

Second-order relativistic hydrodynamics

The full expansion for $\Pi^{\perp}_{ab}$ to second order in gradients is

$$\Pi^{\perp}_{ab} = -2\eta\mathcal{K}_{ab} + c_1\mathcal{K}_a^c\mathcal{K}_{cb} + c_2\mathcal{K}^c_{(a}\Omega_{|c|b)} + c_3\Omega_a{}^c\Omega_{cb} + c_4 h_a^c h_b^d \partial_c\partial_d \ln p + c_5\mathcal{K}_{ab}\, D\ln p + c_6 D^{\perp}_a \ln p\, D^{\perp}_b \ln p + O(\partial^3),$$

where $D^{\perp}_a = h_a^b\partial_b$ and $D = u^a\partial_a$ and the vorticity $\Omega_{ab} = h_a^c h_b^d \partial_{[c}u_{d]}$.

- There are six second-order transport coefficients: c_1, c_2, etc.
- Expanding this expression in ϵ and comparing with our T_{ab} from our gravity calculation we find:

$$\eta = 1, \qquad 2c_1 = c_2 = c_3 = c_4 = -4\sqrt{r_c}.$$

These five simple terms encode our entire T_{ab} to ϵ^5 order! To fix c_5 and c_6 we need to go beyond ϵ^5 order.

Non-universality of higher order transport coefficients

In a subsequent paper Chirco, Eling and Liberati [arXiv:1105.4482] analyzed the Gauss-Bonnet case:

$$S = \int d^{d+1}x\sqrt{-g}\left[R + \alpha(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})\right], \qquad d \geq 3.$$

While η, c_2 and c_4 stay the same, c_1 and c_3 change:

$$c_1 = -2\sqrt{r_c}\left(1 + \frac{2\alpha}{r_c}\right), \qquad c_3 = -4\sqrt{r_c}\left(1 + \frac{3\alpha}{r_c}\right).$$

Non-universality of higher order transport coefficients

- Since $R_{\mu\nu\rho\sigma} \sim \epsilon^2$, curvature-squared corrections to the field equations don't change the metric until ϵ^4 order, and in fact this holds for all higher-derivative corrections. Hence up to ϵ^3 order the metric is universal.
- This universal part generates the incompressible Navier-Stokes equations, which are themselves universal.
- The non-universal part of the metric generates the higher-order correction terms to the incompressible Navier-Stokes equations; as expected, these are not universal.

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A model for the dual fluid

- We now propose a simple Lagrangian model for the dual fluid. We focus on the non-dissipative part of the stress tensor,

$$T_{ab} = ph_{ab} = p(\gamma_{ab} + u_a u_b),$$

  describing a fluid with nonzero pressure but vanishing energy density in the local rest frame.
- To get the dissipative part would need to couple to a heat bath.

A model for the dual fluid

$$S = \int d^{d+1}x\sqrt{-\gamma}\sqrt{-(\partial\phi)^2}.$$

- The field equations describe *potential flow*

$$\nabla^a u_a = 0, \qquad u_a = \frac{\partial_a \phi}{\sqrt{X}}, \qquad X = -(\partial\phi)^2.$$

- The stress tensor is

$$T_{ab} = \sqrt{X}\gamma_{ab} + \frac{1}{\sqrt{X}}\partial_a\phi\partial_b\phi = \sqrt{X}h_{ab}, \qquad \text{i.e.} \quad p = \sqrt{X}.$$

- One way to obtain this sqrt action is to start with $\mathcal{L}(X,\phi)$ then impose

$$0 = \rho = 2X\frac{\delta\mathcal{L}}{\delta X} - \mathcal{L}$$

A model for the dual fluid

- The equilibrium configuration with $p = 1/\sqrt{r_c}$ in the rest frame corresponds to taking

$$\phi = \tau,$$

so $v_i \sim \partial_i \phi = 0$. This breaks Lorentz invariance, as does any choice of u_a.

- To model small fluctuations about this background we set

$$\phi = \tau + \delta\phi(\tau, \vec{x}).$$

One can then solve for the 3-velocity v_i and pressure fluctuation P:

$$v_i = -\frac{r_c \delta\phi_{,i}}{(1 + \delta\dot{\phi})}, \qquad P = r_c \Big[(1 + 2\delta\dot{\phi} + \delta\dot{\phi}^2 - r_c \delta\phi_{,i} \delta\phi_{,i})^{1/2} - 1 \Big].$$

Remarks

- The action is non-local: the expansion around the background solution involves an infinite number of derivatives.
- One can easily couple to other types of matter (Ψ, Φ, A_a), provided they don't have a background value.
- Connection with brane action? e.g. $(d+1)$-dim brane embedded in $(d+2)$-dim Minkowski target space. In static gauge this is

$$S = -T \int d^{d+1}x \sqrt{1 + (\partial Y)^2},$$

where Y is the transverse coordinate to the brane. Taking the tensionless limit $T \to 0$ while keeping $\phi = TY$ fixed,

$$S = -\int d^{d+1}x \sqrt{(\partial \phi)^2}.$$

Still missing minus sign inside sqrt ... use target space signature $(d, 2)$?

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Summary

➢ We established a direct relation between $(d+2)$-dimensional Ricci-flat metrics and $(d+1)$-dimensional fluids satisfying the incompressible Navier-Stokes equations, corrected by specific higher-derivative terms.

➢ The dual fluid has vanishing equilibrium energy density but nonzero pressure. There is an underlying relativistic hydrodynamic description. We computed the viscosity and four of the six second-order transport coefficients ‘holographically’.

➢ A simple sqrt Lagrangian captures the non-dissipative properties of the fluid.

Open questions

- Is there a manifestly relativistic construction of the bulk metric? Does the solution exist globally? What if we add matter to the bulk?
- Does the correspondence extend beyond the hydrodynamic regime on the field theory side, and/or the classical gravitational description on the bulk side? Is there a string embedding? Can we get the sqrt action from branes?
- How far can flat space holography be developed? Is there a holographic dictionary relating bulk computations to quantities in the dual field theory on Σ_c?
- By the equivalence principle, our construction should hold locally in any small neighbourhood. Can one patch together such a 'local' holographic description of neighbourhoods to obtain a global holographic description of general spacetimes?

Open questions: recent progress

- Is there a manifestly relativistic construction of the bulk metric?

- **YES.** Work to appear soon with Compère, McFadden and Skenderis.

Relativistic construction

- We start with a seed metric

$$ds^2 = -2pu_a dx^a dr + [\eta_{ab} + (1-\theta)u_a u_b]dx^a dx^b, \qquad \theta = 1 + p^2(r-1)$$

  where $\eta^{ab}u_a u_b = -1$ and r_c is scaled to unity.
- The velocity and pressure are functions of $x^a = (\tau, x^i)$, and we work out an expansion in derivatives with $\partial_r \sim 1$ and $\partial_a \sim \epsilon$.
- Elegant and compact rederivation of previous results.

Open questions: recent progress

➢ Do solutions exist globally? What if we add matter to the bulk?

- Partial answers, in Bredberg/Strominger, Lysov/Strominger and in progress.
- Regularity at future horizon apparently related to algebraically special (Petrov) condition on surface Σ_c.
- We can start from more general seed static metrics such as

$$ds^2 = 2d\tau dr - r_c d\tau^2 + \gamma_{ij} dy^i dy^j,$$

in which bulk stress energy tensor is non zero, and make boosts using isometries of γ.

Open questions

➢ Flat space holography à la GKPW? Local holography?