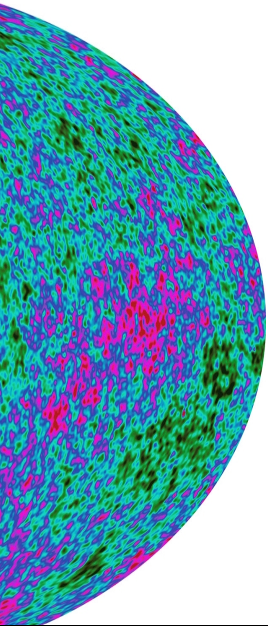


Holographic Inflation: an update

Kostas Skenderis

Institute for Theoretical Physics,
Gravitation and AstroParticle Physics
Amsterdam (GRAPPA),
KdV Institute for Mathematics
University of Amsterdam

References

- The talk is based on
  - P. McFadden, KS, Cosmological 3-point correlators from holography, arXiv:1104.3894
  - R. Easther, R. Flauger, P. McFadden, KS, Constraining holographic inflation with WMAP, arXiv:1104.2040.
  - A. Bzowski, P. McFadden, KS, to appear
- Earlier work with Paul McFadden
  - Holography for Cosmology, arXiv:0907.5542
  - The Holographic Universe, arXiv:1007.2007
  - Observational signatures of holographic models of inflation, arXiv:1010.0244
  - Holographic Non-Gaussianity, arXiv:1011.0452
- Related work
  - J. Maldacena, G. Pimentel, On graviton non-Gaussianities during inflation, arXiv:1104.2846
  - M. Dias, Cosmology at the boundary of de Sitter using the dS/QFT correspondence, arXiv:1104.0625

Introduction

- Any gravitational theory should be holographic, i.e. it should have a description in terms of a non-gravitational theory in one dimension less.
- There is no holographic construction that works in general to date; explicit examples depend on the form of asymptotics.
- The properties of the dual theory depend also on the asymptotics.
  - The best-understood examples originate from string theory via decoupling limits of branes:
  - D3, M5 etc. branes $\longrightarrow$ asymptotically AdS spacetimes $\longrightarrow$ local QFT that in the UV becomes conformal.
  - D2, D4 etc. $\longrightarrow$ asymptotically power-law spacetimes $\longrightarrow$ local QFT with a generalized conformal structure.

Holographic inflation

Here I will describe a holographic framework for inflationary spacetimes that:

1. approach de Sitter spacetime at late times,

$$ds^2 \to ds^2 = -dt^2 + e^{2t} dx^i dx^i, \qquad \text{as} \quad t \to \infty$$

2. approach power-law scaling solutions at late times ,

$$ds^2 \to ds^2 = -dt^2 + t^{2n} dx^i dx^i, (n > 1) \qquad \text{as} \quad t \to \infty$$

These backgrounds have the property that they are in 1-1 correspondence with backgrounds that describe holographic RG flows, either asymptotically AdS or asymptotically power-law.

Holographic universe

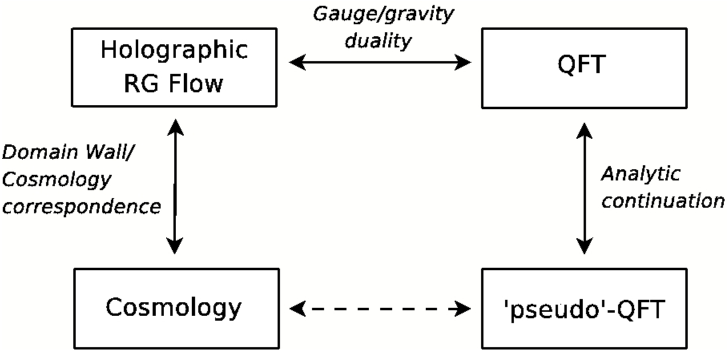
- At the end of this period we arrive at a FRW spacetime and small inhomogeneities with super-horizon correlations. The latter originate from correlations in the dual QFT.
- The end of this period in the beginning of hot Big Bang cosmology.
- The inhomogeneities are the initial conditions for the subsequent cosmological evolutions that leads to structure formation (stars, galaxies).

Domain-wall cosmology correspondence

- The correspondence between inflationary spacetimes and holographic RG flows is a special case of a more general Domain-wall/Cosmology correspondence (DW/C) [KS, Townsend (2006)] and can be understood as analytic continuation.
- For holographic RG flows, there is a well-established holographic dictionary.
- One may express the analytic continuation associated with DW/C in QFT terms: one analytically continues the momenta q and the rank of the gauge group N:

$$q \to \bar{q} = -iq, \qquad N \to \bar{N} = -iN$$

Holographic framework [McFadden, KS (2009)]



Weakly coupled gravity

To test this framework we analyzed the case where gravity is weakly coupled, so standard treatments are available.

- We worked out the cosmological 2- and 3-point functions of scalar and tensor perturbations around a general single scalar inflationary background.
- We worked out the 2- and 3-point functions of the stress energy tensor of the corresponding domain-wall background, using standard gauge/gravity duality.
- **We found that the cosmological correlators can be expressed in terms of the QFT correlation functions at strong coupling upon analytic continuation.**

$\Rightarrow$ Standard inflation is holographic.

Holographic formulae: preliminaries

- The 2-point function of T_{ij} in a flat spacetime has the form

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl},$$

  where $\Pi_{ijkl} = \frac{1}{2}(\pi_{ik}\pi_{lj} + \pi_{il}\pi_{kj} - \pi_{ij}\pi_{kl}), \quad \pi_{ij} = \delta_{ij} - \bar{q}_i \bar{q}_j / \bar{q}^2.$

- We found useful to use a helicity basis.

$$T_{ij} \to T, T^{(s)}, \quad s = \pm 1$$

  where T is the trace of T_{ij} and the transverse traceless part of T_{ij} is traded for $T^{\pm}$.

- In cosmology, the physical degrees of freedom are a scalar mode, ζ, and transverse traceless tensor modes $\hat{\gamma}_{ij}$. In a helicity basis:

$$(\delta\phi, h_{ij}) \to \zeta, \hat{\gamma}^{(s)}, \qquad s = \pm 1$$

Holographic formulae: 2-point functions

- The cosmological 2-point functions are given by

$$\langle\zeta(q)\zeta(-q)\rangle = \frac{-1}{8\mathrm{Im}[B(\bar{q})]}, \qquad \langle\hat{\gamma}^{(s)}(q)\hat{\gamma}^{(s')}(-q)\rangle = \frac{-\delta^{ss'}}{\mathrm{Im}[A(\bar{q})]},$$

- The 2-point functions determine the power spectra (measured by WMAP and other experiments)

$$\Delta^2_{\mathcal{R}}(q) = \frac{q^3}{2\pi^2}\left(\frac{-1}{8\mathrm{Im}B(-iq)}\right), \quad \Delta^2_{T}(q) = \frac{2q^3}{\pi^2}\left(\frac{-1}{\mathrm{Im}A(-iq)}\right),$$

[McFadden, KS (2009)]

Holographic formulae: 3-point functions

- $\langle\zeta(q_1)\zeta(q_2)\zeta(q_3)\rangle$

$$= -\frac{1}{256}\Big(\prod_i \mathrm{Im}[B(\bar{q}_i)]\Big)^{-1} \times \mathrm{Im}\Big[\langle T(\bar{q}_1)T(\bar{q}_2)T(\bar{q}_3)\rangle + (\text{semi}-\text{local terms})\Big],$$

- $\langle\zeta(q_1)\zeta(q_2)\hat{\gamma}^{(s_3)}(q_3)\rangle$

$$= -\frac{1}{32}\Big(\mathrm{Im}[B(\bar{q}_1)]\mathrm{Im}[B(\bar{q}_2)]\mathrm{Im}[A(\bar{q}_3)]\Big)^{-1}$$

$$\times\ \mathrm{Im}\Big[\langle T(\bar{q}_1)T(\bar{q}_2)T^{(s_3)}(\bar{q}_3)\rangle + (\text{semi}-\text{local terms})\Big],$$

[McFadden, KS (2010), (2011)]

Holographic formulae: 3-point functions

- $\langle \zeta(q_1)\hat{\gamma}^{(s_2)}(q_2)\hat{\gamma}^{(s_3)}(q_3)\rangle$

$$= -\frac{1}{4}\Big(\mathrm{Im}[B(\bar{q}_1)]\mathrm{Im}[A(\bar{q}_2)]\mathrm{Im}[A(\bar{q}_3)]\Big)^{-1}$$

$$\times\ \mathrm{Im}\Big[\langle T(\bar{q}_1)T^{(s_2)}(\bar{q}_2)T^{(s_3)}(\bar{q}_3)\rangle + (\text{semi}-\text{local terms})\Big],$$

- $\langle \hat{\gamma}^{(s_1)}(q_1)\hat{\gamma}^{(s_2)}(q_2)\hat{\gamma}^{(s_3)}(q_3)\rangle$

$$= -\Big(\prod_i \mathrm{Im}[A(\bar{q}_i)]\Big)^{-1} \times\ \mathrm{Im}\Big[2\langle T^{(s_1)}(\bar{q}_1)T^{(s_2)}(\bar{q}_2)T^{(s_3)}(\bar{q}_3)\rangle + (\text{semi}-\text{local terms})\Big]$$

[McFadden, KS (2011)]

Remarks

- The cosmological 3-point functions are related with non-gaussianities and the Planck satellite (with data expected to be released in 2012) and other missions are expected to constrain them significantly.
- The semi-local terms correspond to contributions where two of the three insertion points in the 3-point function are coincident. Such terms contribute to the so-called local type non-gaussianity.
- The holographic formulae are consistent with the interpretation of such duality as computing the wavefunction of the universe, as discussed for de Sitter by [Maldacena (2002)]

New holographic models

- While conventional inflationary models are related with strongly coupled QFT, new models arise when we consider the QFT at weak coupling (but still at large N).
- In these models, the very early universe is in a non-geometric phase. This phase should have a description in string theory in terms of a strongly coupled sigma model. Here we use holography to describe it.
- The end of this period is the beginning of hot big bang cosmology.

Dual QFT

To make predictions we need to specify the dual QFT. The two classes of asymptotic behaviors correspond to two classes of dual QFT's.

- asymptotically de Sitter $\rightarrow$ QFT is deformation of a CFT
- asymptotically power-law $\rightarrow$ QFT has generalized conformal structure

Here we discuss theories of the second type.

Dual QFT

We require that the theory has the following properties:

1. admits a large N limit
2. all fields are massless
3. it has a dimensionful coupling constant
4. all terms in the Lagrangian have the same scaling dimension, which should be different from three.

Properties (2)-(4) imply that the theory admits a generalized conformal structure: the theory would be conformal if the coupling constant is promoted to a background field that transforms under conformal transformations [Jevicki et. al. (1998)] [Kanitscheider, Taylor, KS (2008)].

Dual QFT

- A class of models exhibiting these properties is given by the following super-renormalizable theory:

$$S = \frac{1}{g_{\mathrm{YM}}^2} \int d^3x \mathrm{tr} \left[\frac{1}{2} F_{ij}^I F^{Iij} + \frac{1}{2} (D\phi^J)^2 + \frac{1}{2} (D\chi^K)^2 + \bar{\psi}^L \not{D} \psi^L \right.$$
$$\left. + \lambda_{M_1 M_2 M_3 M_4} \Phi^{M_1} \Phi^{M_2} \Phi^{M_3} \Phi^{M_4} + \mu_{M L_1 L_2}^{\alpha\beta} \Phi^M \psi_\alpha^{L_1} \psi_\beta^{L_2} \right].$$

  $\Phi^M = \{\phi^I, \chi^K\}$, χ^K: conformal scalars, ϕ^I: minimally coupled scalars, ψ^L: fermions

- To extract predictions we need to compute n-point functions of the stress energy tensor analytically continue the result and insert them in the holographic formulae.

Holographic power spectrum

- The relevant correlation function is the 2-point function of the trace of the stress energy tensor T. The form of this 2-point function is fixed by generalized conformal structure [Kanitscheider, KS, Taylor (2008)]. At large N,

$$\langle T(q)T(-q)\rangle = q^3 N^2 f(g_{\text{eff}}^2),$$

  where $g_{\text{eff}}^2 = g_{\text{YM}}^2 N/q$ is the effective dimensionless 't Hooft coupling and $f(g_{\text{eff}}^2)$ is a general function of g_{eff}^2.
- The analytic continuation acts as

$$g_{\text{eff}}^2 \to g_{\text{eff}}^2, \qquad N^2 q^3 \to -iN^2 q^3$$

  and therefore

$$\Delta_{\mathcal{R}}^2(q) = -\frac{q^3}{4\pi^2}\frac{1}{\text{Im}\langle T(q)T(-q)\rangle} = \frac{1}{4\pi^2 N^2}\frac{1}{f(g_{\text{eff}}^2)}$$

- It remains to compute $f(g_{\text{eff}}^2)$ ...

Holographic power spectrum

- When g_{eff}^2 is small, one finds that the function $f(g_{\text{eff}}^2)$ has the form

$$f(g_{\text{eff}}^2) = f_0(1 - f_1 g_{\text{eff}}^2 \ln g_{\text{eff}}^2 + f_2 g_{\text{eff}}^2 + O[g_{\text{eff}}^4]).$$

$\longrightarrow$ f_0 is determined at 1-loop in perturbation theory. It has been computed in [McFadden, KS (2009)].

$\longrightarrow$ f_1 is determined at 2-loop in perturbation theory. It has *not* been computed to date.

$\longrightarrow$ f_2 is related with an infrared generated scale $q_{IR} \sim g_{\text{YM}}^2$ [Jackiw,Templeton (1981)][Appelquist, Pisarski(1981)]. As long as one probes the theory at scales large compared with the IR scale, this term is negligible.

Holographic power spectrum

- Redefining variables, $f_1 g_{\mathrm{YM}}^2 N = g q_*$, where q_* is a reference scale that is taken to be $q_* = 0.05\ \mathrm{Mpc}^{-1}$ (the WMAP momentum range is $10^{-4} \lesssim q \lesssim 10^{-1}\ \mathrm{Mpc}^{-1}$), we obtain the final formula:

$$\Delta_{\mathcal{R}}^2(q) = \Delta_{\mathcal{R}}^2 \frac{1}{1 + (g q_*/q) \ln |q/g q_*|},$$

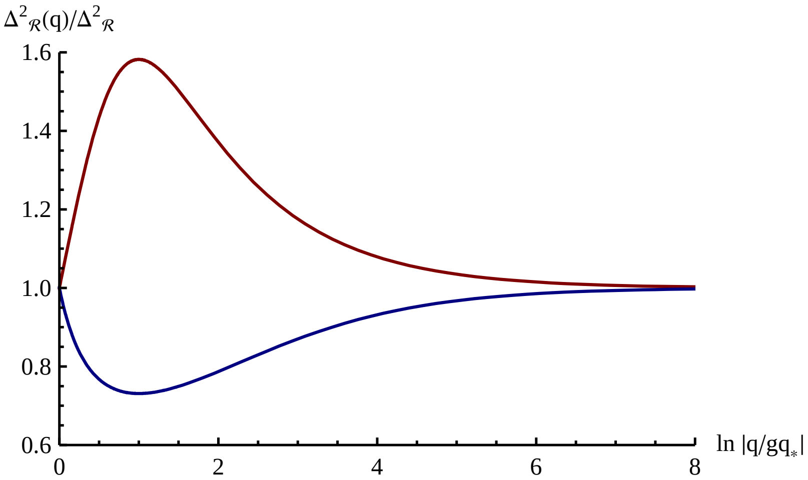
$\rightarrow$ $\Delta_{\mathcal{R}}^2 = 1/(4\pi^2 N^2 f_0)$. Smallness of the amplitude is related with the large N limit: matching with observations implies $N \sim 10^4$.

$\rightarrow$ When $(g q_*/q) \ll 1$ one may rewrite the spectrum in the power-law form

$$\Delta_{\mathcal{R}}^2(q) = \Delta_{\mathcal{R}}^2 q^{n_s - 1}, \qquad n_s(q) - 1 = g q_*/q$$

Thus the small deviation from scale invariance appears to be related with the smallness of the coupling constant of the dual QFT.

Holographic power spectrum



Blue curve: $g > 0$, Red curve: $g < 0$

Holographic model vs slow-roll inflation

- This is a rather different spectrum than that of generic slow-roll models. In such models the dependence of n_s on q is rather weak. The "running" $\alpha_s = dn_s/d\ln q$ is higher order in slow-roll than $(n_s - 1)$,

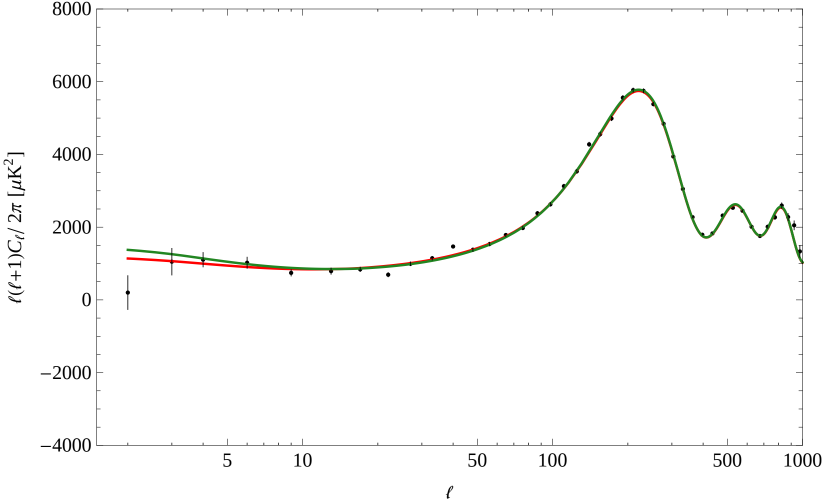
$$\alpha_s/(n_s - 1) \sim \epsilon$$

  In contrast, in the holographic model, $\alpha_s \sim (n_s - 1)$. In fact, all $d^k n_s(q)/d\ln q^k$ are of the same order in the holographic model.
- Given that the power-law ΛCDM model, in which n_s is a constant, fits remarkably well the WMAP and other astrophysical data, one may wonder whether already existing data are sufficient to rule out this class of holographic models.
- We thus undertook the task to make a dedicated data analysis [Easther, Flauger, McFadden, KS (2011)]. Related work appeared in [Dias (2011)].

Holographic model vs ΛCDM

- The power-law ΛCDM model depends on six parameters. Four describe the composition and expansion of the universe and the other two are the tilt n_s and the amplitude $\Delta^2_{\mathcal{R}}$ of primordial curvature perturbations.
- The holographic ΛCDM model depends on the same set of parameters, except that the tilt n_s is replaced by the parameter g.
- We determined the best-fit values for all parameters for both models and used Bayesian evidence in order to make a model comparison.

Angular power spectrum: ΛCDM vs holographic model



Red: ΛCDM, Green: holographic model

Parameter estimation

- The estimated values for the five common parameters of the two models are roughly within one standard deviation of each other.
- The data favor negative values of g (red spectrum) with central value $g = -1.27 \times 10^{-3}$.

$\rightarrow$ This indeed leads to a small effective coupling, except potentially for the very low wavelength modes. Since $g_{\text{eff}}^2 = (1/f_1)(gq_*/q)$ one needs to know the value of the 2-loop factor f_1 when (gq_*/q) itself is not very small.

$\rightarrow$ A related issue is that the infrared scale q_{IR} may be inside the WMAP range. In such case the precise value of the parameter f_2 is important and the power spectrum is modified:

$$\Delta_{\mathcal{R}}^2(q) = \Delta_{\mathcal{R}}^2 \frac{1}{1 + (gq_*/q)\ln|q/\beta g q_*|}$$

$\rightarrow$ If g_{eff}^2 is not small for all relevant momenta, one must include higher order terms in the computation of the 2-point function.

Information criteria

- The holographic model is compatible with current data, but the overall fit is somewhat better for ΛCDM:

	Holographic Model	ΛCDM	$\Delta \ln \mathcal{L}_{\text{best}}$
WMAP7	3735.5	3734.3	1.2
WMAP+BAO+H_0	3737.3	3735.7	1.6
WMAP+CMB	3815.0	3812.5	2.5

Table: Best-fit values for $-\ln \mathcal{L}$ for both the holographic model and ΛCDM, as well as the difference between them. Positive numbers in the last column favor ΛCDM.

- One often uses the value of the likelihood at the best-fit point as the criterion for a model selection. However, this is the probability for obtaining the data given a model with specific parameter values.
- For model comparison we would like to know the probability for a model given the data.

Model Comparison

- The probability for a model given the data is computed by the Bayesian evidence E,

$$E = \int d\alpha_M P(\alpha_M)\mathcal{L}(\alpha_M)$$

  $P(\alpha_M)$ is the probability that a choice of parametes α_M is realized and $\mathcal{L}(\alpha_M)$ is the probability for the data given these parameters.

- A rough guide for model selection is Jeffreys scale:

$\Delta \ln E$	Strength of evidence
< 1	Inconclusive
> 1	weak evidence
> 2.5	Moderate evidence
> 5	Strong evidence

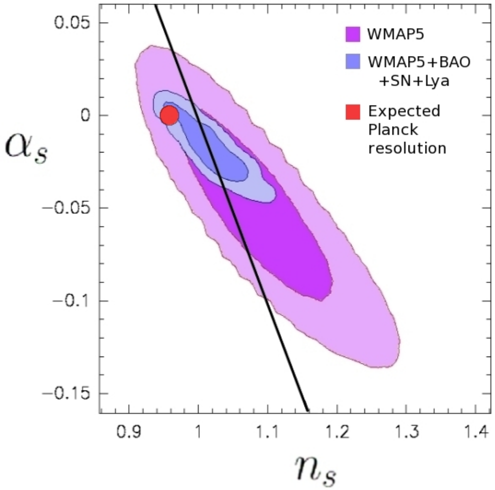
ΛCDM vs Holographic model

- The computation of the evidence depends on the prior probability $P(\alpha_M)$. The priors ought to reflect the underlying assumptions and knowledge of the problem before the data came along.
- For the holographic model, the prior for g is clear: the power spectrum was obtained from a perturbative computation $\rightarrow g$ must not be very large.
- ΛCDM is an empirical model and the choice of priors for n_s is more subjective. We considered two choices: (i) a nearly optimal choice for ΛCDM, $0.92 < n_s < 1$, and (ii) a symmetric choice around the scale invariant spectrum $0.9 < n_s < 1.1$.
- **With choice (i) there is a weak evidence for ΛCDM. With choice (ii) the evidence in inconclusive.**

More data is required to decide between the two models.

WMAP and Planck data

WMAP Cosmological Parameter Plotter



Solid line:

$$\alpha = -(n_s - 1)$$

Non-Gaussianity

Non-Gaussianity implies non-zero higher-point correlation functions. The lowest order is the 3-point function, or bispectrum, of curvature perturbations ζ:

$$\langle\zeta(q_1)\zeta(q_2)\zeta(q_3)\rangle = (2\pi)^3\delta(\sum q_i)B(q_i)$$

- Non-Gaussianity is important as it potentially provides a *very strong* test of inflationary models. The amplitude of the bispectrum is parametrised by f_{NL}:

$$B(q_i) = f_{NL} \times (\text{shape function})$$

- Different inflationary models give different predictions for f_{NL} and shape function.

The shape of non-Gaussianity

- Local form [Verde etal(2000)] ; Komatsu & Spergel (2001)]

$$B_{\rm local}(q_1,q_2,q_3) = f_{NL}^{\rm local} \frac{6A^2}{5q_1^3q_2^3q_3^3} \sum_{i=1}^{3} q_i^3, \qquad A = 2\pi^2\Delta_S^2(q)$$

$\rightarrow$ WMAP7: $f_{NL}^{\rm local} = 32 \pm 21 (68\% CL)$
$\rightarrow$ Single scalar slow-roll inflation: $f_{NL}^{\rm local} \sim O(\epsilon,\eta) \sim 0.01$

- Equilateral form [Creminelli etal, astro-ph/0509029]]

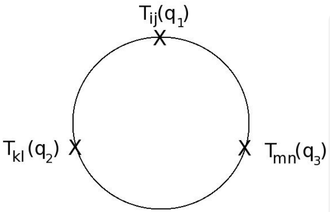
$$B_{\rm equil}(q_1,q_2,q_3) = f_{NL}^{\rm equil} \frac{18A^2}{5q_1^3q_2^3q_3^3} \left(-2q_1q_2q_3 - \sum_{i=1}^{3} q_i^3 + (q_1q_2^2 + 5\ perm)\right)$$

$\rightarrow$ WMAP7: $f_{NL}^{\rm equil} = 26 \pm 140 (68\% CL)$

- The Planck data (expected next year) should be sensitive to just

$$f_{NL} \sim 5.$$

Holographic Non-Gaussianity [McFadden, KS, (2010)]



Direct computation gives

$$\langle T(\bar{q}_1)T(\bar{q}_2)T(\bar{q}_3)\rangle + \sum_i \langle T(\bar{q}_i)T(-\bar{q}_i)\rangle$$
$$-2\big(\langle T(\bar{q}_1)\Upsilon(\bar{q}_2,\bar{q}_3)\rangle + \text{cyclic perms}\big)$$
$$= 2C_B\bar{N}^2\big(2\bar{q}_1\bar{q}_2\bar{q}_3 + \sum_i \bar{q}_i^3 - (\bar{q}_1\bar{q}_2^2 + 5\,\text{perms})\big)$$

- Using the holographic formula one finds

$$B(q_1,q_2,q_3) = B_{NL}^{\text{equil}}(q_1,q_2,q_3)$$

with

$$f_{NL}^{\text{equil}} = \mathbf{5/36}$$

- This is independent of all details of theory.
- This value is larger than the f_{NL} for slow-roll inflation, but probably still too small to be detected by Planck.

Tensor non-Gaussianities [Bzowski, McFadden, KS] (to appear)

What about non-Gaussianities involving tensors?

- Perhaps not observable ... but ...
- there are interesting theoretically ...
- How does the holographic prediction compare with that of slow-roll inflation?

Tensor non-Gaussianities from 3-point functions

To compute them we need to compute the general 3-point function

$$\langle T_{ij}(q_1) T_{kl}(q_2) T_{mn}(q_3) \rangle$$

- The leading order computation amounts to a free field computation.
- Even this, however, is challenging ...
- ... one needs to evaluate integrals of the form ...

$$\int [dq] \, \frac{q_a q_c (q-q_1)_b (q-q_1)_e (q+q_2)_d (q+q_2)_f}{q^2 (q-q_1)^2 (q+q_2)^2}$$

- ... which (somewhat to our surprise) were not available in the literature.

Computation of $\langle T_{ij}(q_1) T_{kl}(q_2) T_{mn}(q_3) \rangle$

We developed 3 different ways to evaluate all relevant integrals:

- Helicity projection into scalar integrals.
- Tensor integrals via Davidychev recursion.
- Tensor integrals via Feynman parametrization.

Using these results we computed $\langle T_{ij}(q_1) T_{kl}(q_2) T_{mn}(q_3) \rangle$.

Ward identities

- Fermions and conformal scalars are CFTs so their correlators should satisfy conformal Ward identities.
- Minimal scalars and Maxwell fields are dual to each other in $d = 3$.
- Correlation functions of T_{ij} for the minimal scalars can be obtained from CFT correlators of the conformal scalar:

$$T_{ij}^{min} = T_{ij}^{conf} - \frac{1}{8}\left(g^{ij}\partial^2 - \partial^i\partial^j\right)\phi^2$$

- It follows that all correlation functions can be built from conformal correlators, even though not all constituents are CFTs!
- Verification of the Ward identities provides a very non-trivial check of our direct computation.

Holographic predictions

$$\langle\langle \hat\gamma^{(+)}(q_1)\hat\gamma^{(+)}(q_2)\hat\gamma^{(+)}(q_3)\rangle\rangle = \frac{1024}{\sqrt{2}N^4\mathcal{N}_{(A)}^2}\frac{\lambda^2 a_{123}^2}{c_{123}^5}\times$$

$$\times\Big[(a_{123}^3 - a_{123}b_{123} - c_{123}) - \Big(1-4\frac{\mathcal{N}_\psi}{\mathcal{N}_{(A)}}\Big)\frac{64c_{123}^3}{a_{123}^6}\Big],$$

$$\langle\langle \hat\gamma^{(+)}(q_1)\hat\gamma^{(+)}(q_2)\hat\gamma^{(-)}(q_3)\rangle\rangle = \frac{1024}{\sqrt{2}N^4\mathcal{N}_{(A)}^2}\frac{\lambda^2}{a_{123}^2 c_{123}^5}\times$$

$$\times\big[(q_3 - a_{12})^4(a_{123}^3 - a_{123}b_{123} - c_{123})\big],$$

$$\langle\langle \zeta(q_1)\zeta(q_2)\hat\gamma^{(s)}(q_3)\rangle\rangle = \cdots$$

$$\langle\langle \zeta(q_1)\hat\gamma^{(s_1)}(q_2)\hat\gamma^{(s_2)}(q_3)\rangle\rangle = \cdots$$

where

$$\mathcal{N}_{(A)} = \mathcal{N}_A + \mathcal{N}_\phi + \mathcal{N}_\chi + 2\mathcal{N}_\psi, \qquad \mathcal{N}_{(B)} = \mathcal{N}_A + \mathcal{N}_\phi.$$

$$a_{123} = q_1 + q_2 + q_3, \quad b_{123} = q_1q_2 + q_2q_3 + q_3q_1, \quad c_{123} = q_1q_2q_3, \quad etc.$$

$$\lambda^2 = (q_1+q_2+q_3)(-q_1+q_2+q_3)(q_1-q_2+q_3)(q_1+q_2-q_3)$$

Slow-roll vs Holography

$$\langle\!\langle \hat{\gamma}^{(+)}(q_1)\hat{\gamma}^{(+)}(q_2)\hat{\gamma}^{(+)}(q_3)\rangle\!\rangle = \frac{\kappa^4 H_*^4}{64\sqrt{2}}\frac{\lambda^2 a_{123}^2}{c_{123}^5}(a_{123}^3 - a_{123}b_{123} - c_{123})$$

$$\langle\!\langle \hat{\gamma}^{(+)}(q_1)\hat{\gamma}^{(+)}(q_2)\hat{\gamma}^{(-)}(q_3)\rangle\!\rangle = \frac{\kappa^4 H_*^4}{64\sqrt{2}}\frac{\lambda^2}{a_{123}^2 c_{123}^5}\times$$
$$\times\Big[(q_3 - a_{12})^4(a_{123}^3 - a_{123}b_{123} - c_{123})\Big]$$

- These correlators exactly match the holographic ones if

$$2\mathcal{N}_\psi = \mathcal{N}_\phi + \mathcal{N}_A + \mathcal{N}_\chi, \qquad \frac{1}{256}N^2\mathcal{N}_{(A)} = \frac{1}{\kappa^2 H_*^2}.$$

- The blue condition is different than in [Maldacena, Pimentel].
- The remaining slow-roll correlators are different from the holographic ones, but surprisingly not too different.

Shape functions for tensors

- To quantify the similarity/difference we generalize the notion of a shape to 3-point functions involving tensors.
- Define the dimensionless combinations:

$$\mathcal{A}(\zeta\zeta) = q^3 \langle\langle \zeta(q)\zeta(-q) \rangle\rangle, \qquad \mathcal{A}(\hat{\gamma}\hat{\gamma}) = q^3 \langle\langle \hat{\gamma}^{(+)}(q)\hat{\gamma}^{(+)}(-q) \rangle\rangle$$

and

$$\mathcal{A}(\zeta\zeta\hat{\gamma}^{(+)}) = q_1^2 q_2^2 q_3^2 \, \langle\langle \zeta(q_1)\zeta(q_2)\hat{\gamma}^{(+)}(q_3) \rangle\rangle, \quad etc.$$

Then the shape functions are defined by:

$$\begin{aligned}
\mathcal{A}(\zeta\zeta\hat{\gamma}^{(s_3)}) &= \mathcal{A}(\zeta\zeta)\mathcal{A}(\hat{\gamma}\hat{\gamma})\mathcal{S}(\zeta\zeta\hat{\gamma}^{(s_3)}), \\
\mathcal{A}(\zeta\hat{\gamma}^{(s_2)}\hat{\gamma}^{(s_3)}) &= \mathcal{A}^2(\hat{\gamma}\hat{\gamma})\mathcal{S}(\zeta\hat{\gamma}^{(s_2)}\hat{\gamma}^{(s_3)}), \\
\mathcal{A}(\hat{\gamma}^{(s_1)}\hat{\gamma}^{(s_2)}\hat{\gamma}^{(s)}) &= \mathcal{A}^2(\hat{\gamma}\hat{\gamma})\mathcal{S}(\hat{\gamma}^{(s_1)}\hat{\gamma}^{(s_2)}\hat{\gamma}^{(s)}),
\end{aligned}$$

Slow-roll vs Holography

$$\mathcal{S}(\zeta\zeta\hat{\gamma}^{(+)}) = \mathcal{S}_{SR}(\zeta\zeta\hat{\gamma}^{(+)}) - \frac{1}{4\sqrt{2}}\frac{\lambda^2}{a_{123}c_{123}},$$

$$\mathcal{S}(\zeta\hat{\gamma}^{(+)}\hat{\gamma}^{(+)}) = \mathcal{S}_{SR}(\zeta\hat{\gamma}^{(+)}\hat{\gamma}^{(+)}) - \frac{1}{16a_{123}c_{123}}(q_1^2 - a_{23}^2)^2\left(1 + \frac{4c_{123}}{a_{123}^3}\right),$$

$$\mathcal{S}(\zeta\hat{\gamma}^{(+)}\hat{\gamma}^{(-)}) = \mathcal{S}_{SR}(\zeta\hat{\gamma}^{(+)}\hat{\gamma}^{(-)}) - \frac{1}{16a_{123}c_{123}}(q_1^2 - a_{23}^2 + 4b_{23})^2,$$

$$\mathcal{S}(\hat{\gamma}^{(+)}\hat{\gamma}^{(+)}\hat{\gamma}^{(+)}) = \mathcal{S}_{SR}(\hat{\gamma}^{(+)}\hat{\gamma}^{(+)}\hat{\gamma}^{(+)}) - \frac{\lambda^2}{\sqrt{2}\,a_{123}^4}\left(1 - \frac{4\mathcal{N}_\psi}{\mathcal{N}_{(A)}}\right),$$

$$\mathcal{S}(\hat{\gamma}^{(+)}\hat{\gamma}^{(+)}\hat{\gamma}^{(-)}) = \mathcal{S}_{SR}(\hat{\gamma}^{(+)}\hat{\gamma}^{(+)}\hat{\gamma}^{(-)}).$$

Why the results are so similar?

This is clear for the 3-point function of only tensors:

- Slow-roll: These correlators do not depend on the slow-roll parameter: isometries of de Sitter imply they are essentially determined by conformal invariance.
- Holographic model: The correlators are determined by the 3-point function of the transverse traceless part of T_{ij}.

It is less clear why the other correlators are so similar.

- A holographic model based on free QFTs leads to the exact Harrison-Zel'dovich scale invariant spectrum. Our results for the bispectrum are the exact results for such model. In a sense slow-roll is close to Harrison-Zel'dovich ...

Conclusions

- Standard inflation is holographic: standard observables such as power spectra and non-Gaussianities can be expressed in terms of (analytic continuation of) correlation functions of a dual QFT.
- There are new holographic models based on perturbative QFT that describe a universe that started in a non-geometric strongly coupled phase.
- A class of such models based on a super-renormalizable QFT was custom-fit to data and shown to provide a competitive model to ΛCDM.
- Data from the Planck satellite should permit a definitive test of this holographic scenario.