Transport in Anisotropic Superfluids: A Holographic Description

Hansjörg Zeller

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based on 1011.5912 and 1110.0007 in collaboration with J. Erdmenger and P. Kerner

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- hydrodynamics: system characterised by a few transport coefficients
- strongly interacting systems: transport properties inaccessible from underlying microscopic theories by standard methods
 examples: Quark-Gluon-Plasma, high T_c superconductors
- Gauge/Gravity duality can help!
- ⇒ Holographic Hydrodynamics

Motivation - Gauge/Gravity Duality and Anisotropic Systems

- anisotropic systems: more transport coefficients in systems with lower symmetry
- Gauge/Gravity Duality + Anisotropy
- \Rightarrow holographic p-wave superfluid

Gauge/Gravity Duality

- 2 Hydrodynamics & Shear Viscosity
- 3 P-Wave Superfluid with Backreaction
- 4 Perturbations about the Background

Gauge/Gravity Duality – brief reminder

• Maldacena in 1997:

 $\mathcal{N} = 4 \ SU(N_c)$ Super-Yang-Mills theory in D=4 \Leftrightarrow AdS₅×S⁵ type IIB Superstring theory in

• Generalisation:

Large N gauge theory in D spacetime dimensions

$$\langle e^{\int d^{D} x \mathcal{O} \phi_{bdy}} \rangle_{FT} = \mathcal{Z}_{GRAV}[\Phi \to \phi_{bdy}]$$

 \Leftrightarrow

• finite temperature \Leftrightarrow Black Hole solution

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• chemical potential (\Rightarrow finite density)

\Leftrightarrow

Einstein-Maxwell theory with A_t \neq 0 at the AdS boundary
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holographic superfluid (arXiv:0810.1563)
 SSB global U(1) + condensate
 ⇔
 SSB gauged U(1) + condensate hovering over black hole horizon

long wavelength, small frequency fluctuations

- effective theory: macroscopic behaviour of the system
- response of system: transport coefficients e.g. shear viscosity, bulk viscosity, diffusion coefficient, conductivity
- constitutive equations

$$T^{\mu\nu} = T^{\mu\nu}_{eq.} + \Pi^{\mu\nu}$$
 and $J^{\mu} = J^{\mu}_{eq.} + \Upsilon^{\mu}$
with $\Pi_{ij} \sim \eta \left(\partial_i u_j + \partial_j u_i\right)$

What is the Shear Viscosity?

- momentum diffusion transverse to the momentum
- viscosity tensor $\eta_{\alpha\beta\gamma\delta}$
- anisotropic systems: 21 components
- isotropic systems:
 1 shear viscosity
- transversely isotropic systems:
 2 shear viscosities



What is the Shear Viscosity?



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Shear Viscosity of Weakly Coupled Theories

- shear vicosity $\eta \sim \epsilon I_{\rm mfp}$
- mean free path $\mathit{I}_{\rm mfp} \sim \frac{1}{\mathit{n}\sigma}$
- in $\lambda \phi^4$ theory:

$$\eta \sim rac{\mathcal{T}^3}{\lambda^2}\,,\,\, ext{with}\,\,\,\lambda \ll 1$$

• using $s \sim T^3$

$$\Rightarrow \frac{\eta}{s} \sim \frac{1}{\lambda^2}$$

• universal property of strongly coupled isotropic field theories with classical, two-derivative gravity dual:

$$rac{\eta}{s}=rac{1}{4\pi}\simeq 0.079$$



 λ

arXiv:0709.1523

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• universal property of strongly coupled isotropic field theories with classical, two-derivative gravity dual:

$$\frac{\eta}{s} = \frac{1}{4\pi} \simeq 0.079$$

• conjecture by Kovtun, Son and Starinets (arXiv:hep-th/0405231):

 $rac{\eta}{s} \geq rac{1}{4\pi}$ for substances found in Nature



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• RHIC experiment:

Quark-Gluon-Plasma
$$rac{\eta}{s} \sim [0-0.2]$$

Gravitational Setup for holographic p-wave Superfluid

• *SU*(2) Einstein-Yang-Mills theory in (4+1)-dimensional asymptotically AdS Space

$$S = \frac{1}{2\kappa_5^2} \int \mathrm{d}^5 x \sqrt{-g} \left[R - \Lambda - \frac{\alpha^2}{2} F^a_{MN} F^{aMN} \right] + S_{\mathrm{bdy}}$$

with

$$\alpha \equiv \frac{\kappa_5}{g_{\rm YM}}$$

• α measures the backreaction

metric ansatz

$$ds^{2} = -N(r)\sigma(r)^{2}dt^{2} + \frac{1}{N(r)}dr^{2} + \frac{r^{2}}{f(r)^{4}}dx^{2} + r^{2}f(r)^{2}(dy^{2} + dz^{2})$$

with

$$N(r) = -\frac{2m(r)}{r^2} + r^2$$

AdS boundary $r = r_{bdy} \rightarrow \infty$ & black hole horizon $r = r_h$

• gauge field ansatz

$$A = \phi(r)\tau^3 \mathrm{d}t + w(r)\tau^1 \mathrm{d}x$$

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Field Theory	\Leftrightarrow	Gravity
chemical potential μ		$A_t^3 = \phi(r) \neq 0$
$SU(2) ightarrow U(1)_3$		$SU(2) ightarrow U(1)_3$
$egin{aligned} &\langle \mathcal{J}_1^x angle eq 0 \ &U(1)_3 ightarrow \mathbb{Z}_2, \ \mathcal{SO}(3) ightarrow \mathcal{SO}(2) \end{aligned}$		$egin{aligned} &\mathcal{A}_{\chi}^{1}=w(r) eq 0\ &U(1)_{3} ightarrow\mathbb{Z}_{2},\ &SO(3) ightarrow SO(2) \end{aligned}$

• $w(r_{bdy}) = 0 \Rightarrow SSB \ U(1)_3 \rightarrow \mathbb{Z}_2 \& SO(3) \rightarrow SO(2)$

⇒ holographic p-wave superfluid with backreaction (Ammon, Erdmenger, Grass, Kerner, O'Bannon: arXiv:0912.3515)

- w(r) = 0: AdS Reissner-Nordström black hole solution
- $w(r) \neq 0$: numerical solution (shooting method)

The $w(r) \neq 0$ Solution

- 5 fields: $\{\phi(r), w(r), \sigma(r), f(r), m(r)\}$
- 4 independent parameter at the horizon
- expansion of fields at the boundary:

$$\begin{split} \phi(r \to r_{\rm bdy}) &\simeq \mu + \frac{r_h^2}{r^2} \phi_1^b + \dots \\ w(r \to r_{\rm bdy}) &\simeq 0 + \frac{r_h^2}{r^2} w_1^b + \dots \\ f(r \to r_{\rm bdy}) &\simeq 1 + \frac{r_h^4}{r^4} f_2^b + \dots \\ \sigma(r \to r_{\rm bdy}) &\simeq 1 + \mathcal{O}\left(r^{-6}\right) \\ m(r \to r_{\rm bdy}) &\simeq m_0^b + \mathcal{O}\left(r^{-2}\right) \end{split}$$

- fix chemical potential $\mu \Rightarrow {\rm grand}$ canonical ensemble
- Gauge/Gravity dictionary:
- \Rightarrow Hawking temperature \Leftrightarrow temperature in field theory

 $T\propto r_h$

 $\Rightarrow\,$ black hole entropy density $\Leftrightarrow\,$ entropy density of field theory

$$s \propto r_h{}^3 \propto T^3$$

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variation of on-shell action at AdS boundary + GGD dictionary:

• energy-momentum tensor

$$\langle \mathcal{T}_{\mu\mu} \rangle \propto \mathcal{T}^4 \cdot \operatorname{Func}(m_0^b, f_2^b), \text{ with: } \langle \mathcal{T}_{yy} \rangle = \langle \mathcal{T}_{zz} \rangle \neq \langle \mathcal{T}_{xx} \rangle$$

 $\langle \mathcal{T}_{\mu\nu} \rangle = 0 \text{ for } \mu \neq \nu$

density current

$$\langle \mathcal{J}_3^t \rangle \propto T^3 \phi_1^b$$

condensate

$$\langle \mathcal{J}_1^x
angle \propto T^3 w_1^b$$

properties of the energy-momentum tensor:

- traceless (\leftarrow CFT)
- zero net momentum

$$\langle \mathcal{T}_{tx} \rangle = \langle \mathcal{T}_{ty} \rangle = \langle \mathcal{T}_{tz} \rangle = 0$$

grand canonical potential

$$\Omega = \lim_{r_{
m bdy} o \infty} TS_{
m on-shell} = -V \langle \mathcal{T}_{yy}
angle$$

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colour coding: $\alpha = 0.316$ and $\alpha = 0.447$ $\langle \mathcal{J}_1^{\chi} \rangle \propto (1 - T/T_c)^{1/2}$ for $\alpha = 0.316$

arXiv:0912.3515

Ω and s for $\alpha=$ 0.316



colour coding: Reissner-Nordström solution and $w(r) \neq 0$ solution arXiv:0912.3515

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Ω and s for $\alpha=$ 0.447



colour coding: Reissner-Nordström solution and $w(r) \neq 0$ solution arXiv:0912.3515

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Order of the Phase transition



preferred ground state: blue: broken phase, $w(r) \neq 0$; white: Reissner-Nordström black hole

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- $\alpha < \alpha_c = 0.365 \pm 0.001$: second order
- $\alpha > \alpha_c$: first order
- $\alpha < \alpha_{\max} = 0.628$

arXiv:0912.3515

$$\langle {T_\mu}^\mu
angle \propto rac{1}{\kappa_5^2} \propto \#$$
 of total degrees of freedom

$$\langle J
angle \propto {1 \over g_{
m YM}^2} \propto \#$$
 of charged degrees of freedom

$$\Rightarrow \alpha^2 = \frac{\kappa_5^2}{g_{\rm YM}^2} \propto \frac{\# \text{ of charged degrees of freedom}}{\# \text{ of total degrees of freedom}}$$

Perturbations about the Thermodynamical Equillibrium

small perturbations:

• metric $\hat{g}_{MN} = g_{MN}(r) + h_{MN}(x^{\mu}, r)$

• gauge field
$$\hat{A}^a_M = A^a_M(r) + a^a_M(x^\mu, r)$$

- x^{μ} -spacetime translational invariance still unbroken
- \Rightarrow Fourier decomposition of fluctuations possible:

$$egin{aligned} h_{MN}(x^{\mu},r) &= \int rac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{\mathrm{i} k_{\mu} x^{\mu}} h_{MN}(k^{\mu},r) \ a^a_M(x^{\mu},r) &= \int rac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{\mathrm{i} k_{\mu} x^{\mu}} a^a_M(k^{\mu},r) \end{aligned}$$

• SO(2) symmetry \Rightarrow two distinct momenta needed: k_{\parallel} and k_{\perp}

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• set $k_{\perp} = 0$

 \Rightarrow classification under *SO*(2) rotational symmetry around *x*-axis possible:

	dynamical fields	constraints	# physical modes
helicity 2	$h_{yz}, h_{yy} - h_{zz}$	none	2
helicity 1	$h_{ty}, h_{xy}; a_y^a$	h _{yr}	4
	$h_{tz}, h_{xz}; a_z^a$	h _{zr}	4
helicity 0	$h_{tt}, h_{xx}, h_{yy} + h_{zz}, h_{xt};$	$h_{tr}, h_{xr}, h_{rr}; a_r^a$	4
	a_t^a, a_x^a		

gauge choice $h_{Mr} = 0$ and $a_r^a = 0 \Rightarrow 14$ physical modes

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- find numerical solution to equations of motions and constraints using incoming boundary conditions at horizon
- determine on-shell action at AdS boundary
- determine Green's functions using Son's and Starinets' recipe (arXiv:hep-th/0205051)
- determine transport coefficients using Kubo formula

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$$\begin{split} G^{\mu\nu,\rho\sigma}(\omega,\vec{k}) &= \int \mathrm{d}^{4}x \, \mathrm{e}^{-ik_{\mu}x^{\mu}}\theta(t) \, \left\langle \left[T^{\mu\nu}(t,\vec{x}), T^{\rho\sigma}(0)\right] \right\rangle \\ G^{\mu\nu,\rho}_{a}(\omega,\vec{k}) &= \int \mathrm{d}^{4}x \, \mathrm{e}^{-ik_{\mu}x^{\mu}}\theta(t) \, \left\langle \left[T^{\mu\nu}(t,\vec{x}), J^{\rho}_{a}(0)\right] \right\rangle \\ G^{\rho,\mu\nu}_{a}(\omega,\vec{k}) &= \int \mathrm{d}^{4}x \, \mathrm{e}^{-ik_{\mu}x^{\mu}}\theta(t) \, \left\langle \left[J^{\rho}_{a}(t,\vec{x}), T^{\mu\nu}(0)\right] \right\rangle \\ G^{\mu,\nu}_{a,b}(\omega,\vec{k}) &= \int \mathrm{d}^{4}x \, \mathrm{e}^{-ik_{\mu}x^{\mu}}\theta(t) \, \left\langle \left[J^{\mu}_{a}(t,\vec{x}), J^{\nu}_{b}(0)\right] \right\rangle \end{split}$$

in the following set $\vec{k} = 0$

- only 1 non-trivial helicity 2 mode
- minimal coupled scalar $\Xi = g^{yy} h_{yz}$

•
$$G^{yz,yz}(\omega,\vec{k}=0) = rac{2r_h^A}{\kappa_5^2} rac{\Xi_2^b(\omega)}{\Xi_0^b(\omega)} - \langle \mathcal{T}_{yy}
angle + \mathcal{O}(\omega^4)$$

Hydrodynamics from the Helicity 2 Mode

- from isotropic case: $\langle T^{yz}
 angle = -(P + i\omega \eta_{yz})h_{yz}$, with $P = \langle \mathcal{T}_{yy}
 angle$
- Kubo formula:

$$\eta_{yz} = -\lim_{\omega \to 0} \frac{1}{\omega} \operatorname{Im}(G^{yz,yz}) = -\lim_{\omega \to 0} \frac{1}{\omega} \frac{2r_h^4}{\kappa_5^2} \frac{\Xi_2^b(\omega)}{\Xi_0^b(\omega)}$$

$$rac{\eta_{yz}}{s}=rac{1}{4\pi}\pm0.5\%$$

Hydrodynamics from the Helicity 2 Mode

• Ξ minimally coupled scalar

• with
$$\partial_r \Pi = 0 + \mathcal{O}(\omega)$$
 and $\partial_r(\omega \Xi) = 0 + \mathcal{O}(\omega)$

 \Rightarrow lqbal *et. al.* (arXiv:0809.3808):

$$\frac{\eta_{yz}}{s} = \frac{1}{4\pi}$$

using:

$$\eta_{yz} = \lim_{\omega \to 0} \frac{\Pi}{\mathrm{i}\omega\Xi} \bigg|_{r=r_h}$$

• in
$$\vec{k} \to 0$$
 limit additional symmetry:
 $\mathbb{Z}_2: x \to -x, w \to -w$

⇒ helicity 1 modes decouple in 2 blocks:
even parity: {
$$\Psi_t = g^{yy} h_{t\perp}, a_{\perp}^3, h_{r\perp}$$
}
odd parity: { $\Psi_x = g^{yy} h_{x\perp}, a_{\perp}^1, a_{\perp}^2$ }

3 equations of motion + 0 constraint

- \Rightarrow 3 independent fields: $\Psi_{x}, a_{\perp}^{1}, a_{\perp}^{2}$
- \Rightarrow Green's function: 3 \times 3 matrix

Hydrodynamics from the Helicity 1 Modes with odd Parity

• choose basis:
$$a_{\perp}^{\pm}=a_{\perp}^{1}\pm\mathrm{i}a_{\perp}^{2}$$

 \Rightarrow transform in fundamental repr. of unbroken $U(1)_3$

• field theory:

$$\begin{pmatrix} \langle J_{+}^{\perp} \rangle \\ \langle J_{-}^{\perp} \rangle \\ \langle T^{\times \perp} \rangle \end{pmatrix} = \begin{pmatrix} G_{+,+}^{\perp,\perp} & G_{+,-}^{\perp,\perp} & G_{+}^{\perp^{\times \perp}} \\ G_{-,+}^{\perp,\perp} & G_{-,-}^{\perp,\perp} & G_{-}^{\perp^{\times \perp}} \\ G^{\times \perp \perp}_{+} & G^{\times \perp \perp}_{-} & -\langle T_{xx} \rangle - \mathrm{i}\omega\eta_{x\perp} \end{pmatrix} \begin{pmatrix} a_{\perp}^{+} \\ a_{\perp}^{-} \\ h_{x\perp} \end{pmatrix}$$

with

$$\eta_{x\perp} = -\lim_{\omega \to 0} \frac{1}{\omega} \operatorname{Im} \left(\boldsymbol{G}^{x\perp,x\perp} \right)$$

Non-universal $\frac{\eta_{x\perp}}{s}$ for $\alpha < \alpha_c$



colour coding: $\frac{\eta_{yz}}{s} = \frac{1}{4\pi}$ and $\frac{\eta_{x\perp}}{s}$: $\alpha_c > \alpha_1 > \alpha_2 > \alpha_3$

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Non-universal $\frac{\eta_{\star\perp}}{s}$ for $\alpha < \alpha_c$



confirmed analytically by Basu, Oh (arXiv:1109.4592)

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Non-universal $\frac{\eta_{s\perp}}{s}$ for $\alpha > \alpha_c$



expectation: maximal deviation from $1/(4\pi)$ for α_{max}

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$\operatorname{Im}({\sf G}_{\mp,\mp}^{\perp,\perp})$ for lpha= 0.316





colour coding: $T = \infty > T = 3.02T_c > T = 1.00T_c > T = 0.88T_c > T = 0.50T_c$

 $G_{+,-}^{\perp,\perp}$ for $\alpha = 0.316$





colour coding: $T = \infty > T = 3.02T_c > T = 1.00T_c > T = 0.88T_c > T = 0.50T_c$

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Flexoelectric effect

• strain leads to effective polarization

• electric field leads to stress



de Gennes

• our system: coupling between flavour fields and strain

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$$G_{-}^{\perp^{\times\perp}}$$
 for $\alpha = 0.316$



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$$G_{-}^{\perp^{\chi\perp}}$$
 for $\alpha = 0.316$



colour coding: $T = \infty > T = 3.02T_c > T = 1.00T_c > T = 0.88T_c > T = 0.50T_c$

$2 \mbox{ equations of motion } + 1 \mbox{ constraint }$

- \Rightarrow 1 independent field: a_{\perp}^3
- \Rightarrow Green's function: 2 \times 2 matrix with 1 independent component

Hydrodynamics from the Helicity 1 Modes with even Parity

• Hartnoll (arXiv:0903.3246):

$$E_{\perp} = \mathrm{i}\omega \left((a_{\perp}^3)_0^b + \mu(\Psi_t)_0^b
ight)$$
 and $- \frac{
abla_{\perp} T}{T} = \mathrm{i}\omega(\Psi_t)_0^b$

• field theory (Thermoelectric effect):

$$\begin{pmatrix} \langle J^{\perp} \rangle \\ \langle Q^{\perp} \rangle \end{pmatrix} = \begin{pmatrix} \sigma^{\perp \perp} & T \alpha^{\perp \perp} \\ T \alpha^{\perp \perp} & T \bar{\kappa}^{\perp \perp} \end{pmatrix} \begin{pmatrix} E_{\perp} \\ -(\nabla_{\perp} T)/T \end{pmatrix}$$

with $\langle Q^{\perp} \rangle = \langle T^{t\perp} \rangle - \mu \langle J^{\perp} \rangle$

• identification with gravity side in our system:

$$\sigma^{\perp\perp} = -\frac{\mathrm{i}\,G_{3,3}^{\perp,\perp}}{\omega} = -\frac{\alpha^2 r_h}{\kappa_5^2} \frac{\mathrm{i}}{\tilde{\omega}} \left(\frac{2\left(\tilde{a}_{\perp}^3\right)_1^b}{\left(\tilde{a}_{\perp}^3\right)_0^b} - \frac{\tilde{\omega}^2}{2}\right)$$
$$T\alpha^{\perp\perp} = \frac{\mathrm{i}}{\omega} \langle \mathcal{J}_3^t \rangle - \mu \sigma^{\perp\perp} \quad , \quad T\bar{\kappa}^{\perp\perp} = \frac{\mathrm{i}}{\omega} \langle \mathcal{T}_{tt} \rangle + \mu^2 \sigma^{\perp\perp}$$

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Electrical conductivity at $\alpha < \alpha_c$



colour coding: $T = \infty > T = 1.00T_c > T = 0.88T_c > T = 0.50T_c > T = 0.19T_c$

$\omega \operatorname{Im}(\sigma^{\perp \perp})$ at $\alpha < \alpha_c$



colour coding: $T = \infty > T = 1.00T_c > T = 0.88T_c > T = 0.50T_c > T = 0.19T_c$

Conclusion

- holographic hydrodynamics: useful tool to compute transport coefficients
- p-wave superfluid with backreaction \Rightarrow two shear modes:



• flexoelectric effect, thermoelectric effect

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- What are the bumps in the odd parity helicity 1 block?
- What effects can we see in the helicity 0 modes?
- What happens if we turn on momentum?

Thank you!

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