Heavy quarks in a magnetic field

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The boundary-bulk correspondence

AdS/CFT correspondence

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- It is tested to a large extent in the case of the $4d$ $\mathcal{N} = 4$ sYM theory, which is dual to 5d supergravity in an AdS_5 background.

- The AdS/CFT correspondence provides us with a calculational tool for large-N_c gauge theories at strong coupling (Maldacena).
- It is tested to a large extent in the case of the $4d$ $\mathcal{N} = 4$ sYM theory, which is dual to 5d supergravity in an AdS_5 background.
- It is used in order to obtain an indicative picture in the strong coupling regime in non-supersymmetric, non-conformal theories with some string-inspired potentials and interactions, but a rigorous correspondence can't be established. The fields and the symmetries of the gauge theory give some elements for a quite succesful correspondence.

Drag-force calculations

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- In particular a heavy quark moving in the vacuum of N=4 sYM, would correspond to a string end-point, attached to a flavor brane at some radial position $r = \Lambda \rightarrow 0$ (near the boundary) and moving in AdS space.
- Alternatively, if the quark moves in a plasma of temperature T, then the geometric background is replaced by the AdS-Schwartzschild black hole, with Hawking temperature T.
- A drawback is that these strings describing external quarks and their gluonic fields cannot break. This is because $\mathcal{N} = 4$ sYM has no fundamental charges, so there isn't the limit of $2m_q$ in the energy of the gluonic flux tube where it can break into a pair $q\bar{q}$. Perhaps the effective mass of the quarks in the QGP at temperature T is big enough so that there might be some region of validity for such calculations.

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- The string profile codifies the gluonic degrees of freedom $tr\mathcal{F}^2, t^{\mu\nu}$ through the AdS/CFT correspondence.
- At finite temperature T, there are two main contributions in the energy absorbed by the quark. One part is the drag force which is the force provided externally to the quark and gluonic degrees of freedom in order for the system to keep its energy and the other part is the radiation emitted by the quark. These two can be distincted in the limit of $v \to 0$.

Equations of motion

Drag-Force calculations

- In the case of drag force calculations, in principle we must solve the highly nonlinear Euler-Lagrange equations coming from the variation of the Nambu-Goto action and impose the boundary conditions that are relevant to the physical problem we want to solve. For example, in the simplest case of an AdS-Schwarzschild background metric, the equations for a motion for a linear motion read:

$$
\begin{aligned}
- & \quad 2\left(r^4T^4-1\right)^2 x'^3 + r\left(r^4T^4-1\right)\ddot{x}x'^2 + 2\Big(r^8T^8 + \left(2r^4T^4+1\right)\dot{x}^2 + \\
+ & \quad r\left(1-r^4T^4\right)\dot{x}\dot{x}' - 1\Big)x' + r\left(\left(r^4T^4-1\right)x''\left(r^4T^4+\dot{x}^2-1\right)-\ddot{x}\right) = 0,
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and are third order equations in $x(t,r)$. In the case we have motion on a plane $x-y$, then square-roots of powers of $\dot{x}, x'$ appear making the problem too complicated.

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  and are third order equations in $x(t,r)$. In the case we have motion on a plane $x-y$, then square-roots of powers of $\dot{x}, x'$ appear making the problem too complicated.
- Therefore, we make an ansatz, e.g. $x(t,r) = x[f(r), g(r), h[t], \ldots]$, we substitute it into the Nambu-Goto action and minimize it w.r.t. the functions $f(r), g(r), \ldots$. If this is possible, we are left with a problem involving o.d.es.

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- The most usual ansatz used describe linear uniform motion and circular motion with constant speed and are

$$x(t,r) = ut + \xi(r),$$

$$x(t,r) = R(r)cos(\omega t - \phi(r)), y(t,r) = R(r)sin(\omega t - \phi(r))$$

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correspondingly.
- Then the Nambu-Goto action is minimized w.r.t. the one variable functions $\xi(r)$ or $R(r), \phi(r)$ and almost always there is some point $r_c < r_h$ in the radial coordinate where a worldsheet horizon appears before the Einstein metric horizon r_h.

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- The two pieces of the string before and after the worldsheet horizon are causally disconnected, i.e. the part of the string behind the worldsheet horizon doesn't affect the part in front of it.

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- The ansatz made was $x(t,r) = vt + \xi(r)$ considering that the string vibrations have practically disappeared for large times $t \to \infty$.
- The background metric is

$$ds^2 = -\frac{r^2}{L^2}\left(1 - \left(\frac{r_h}{r}\right)^4\right) dt^2 + \frac{r^2}{L^2} d\vec{x}^2 + \frac{L^2}{r^2} \frac{dr^2}{\left(1 - \left(\frac{r_h}{r}\right)^4\right)}$$

and the Nambu-Goto langrangian becomes

$$S_{NG} = -\frac{1}{2\pi \ell_s^2} \int drdt \sqrt{1 - \frac{v^2}{\left(1 - \left(\frac{r_h}{r}\right)^4\right)} + \frac{\left(1 - \left(\frac{r_h}{r}\right)^4\right) r^4}{L^4} \xi'(r)}.$$

- The quantity $\pi_\xi = \frac{\partial L}{\partial \xi'}$ is a constant and when this equation is solved for ξ' we have

$$\xi'(r) = \pm \frac{L^4}{r^4\left(1-\left(\frac{r_h}{r}\right)^4\right)} \sqrt{\frac{\left(1-\left(\frac{r_h}{r}\right)^4\right) - v^2}{\left(1-\left(\frac{r_h}{r}\right)^4\right) - \pi_\xi^2 \frac{L^4}{r^4}}}.$$

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- In order for both the nominator and the denominator to change sign at the same point we have

$$\pi_\xi = \frac{v}{\sqrt{1-v^2}}\frac{r_h^2}{L^2}, \xi'(r) = v\frac{r_h^2 L^2}{r^4\left(1-\left(\frac{r_h}{r}\right)^4\right)} = v\frac{r_h^2 L^2}{r^4 - r_h^4}$$

$$\xi(r) = -\frac{L^2}{2r_h}v\left(\tan^{-1}\frac{r}{r_h} + log\sqrt{\left(\frac{r+r_h}{r-r_h}\right)}\right).$$

- Then the rate at which momentum is subtracted is

$$\frac{dp}{dt} = -\frac{r_h^2}{2\pi\alpha' L^2}\frac{v}{\sqrt{1-v^2}}.$$

Motion with constant velocity at nonzero temperature

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$$\frac{dp}{dt} = -\frac{r_h^2}{2\pi\alpha' L^2}\frac{v}{\sqrt{1-v^2}}.$$

- Then from the relations $L^4 = g_{YM}^2 N_c \alpha'^2$, $T = \frac{r_h}{\pi L^2}$ we have

$$\frac{dp}{dt} = -\frac{\pi\sqrt{g_{YM}^2 N_c}}{2}T^2\frac{v}{\sqrt{1-v^2}} = -\frac{\pi\sqrt{g_{YM}^2 N_c}}{2}T^2\frac{p}{m}.$$

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- This rate of momentum loss is constant during the motion which keeps its constant velocity for $t \to \infty$.
- The realistic situation is a motion without an external force keeping the velocity constant. This is actually a problem in such a kind of calculations because we can't observe the slowdown of the quark.

- Nevertheless, if we consider that the rate of momentum loss is quite small, we can consider the above result as a quasi-static situation and therefore we obtain $p(t) = p(0)e^{-\frac{t}{t_0}}$, with $t_0 = \frac{2}{\pi\sqrt{\lambda}}\frac{m}{T^2}$.

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- These predictions can be checked for the heavy quarks bottom and charm and the estimations compared with the experiments at RHIc show that they are in the range achieved in reality.
- However, the more established models use destructive interference effects between the quarks and a radiated gluon and the energy loss depends on the square of the distance traveled.

Circular motion with constant speed at zero temperature

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- The solution obtained is

$$R(z) = \sqrt{z^2\gamma^2 v^2 + R_0^2}, \phi(z) = -z\gamma\omega_0 + \arctan(z\gamma\omega_0).$$

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- There is also a worldsheet horizon at $z_c = \frac{1}{v\gamma^2\omega}$ with Hawking temperature $T = \frac{1}{2\pi\gamma z_c}$ at the point where the velocity of the string is c.

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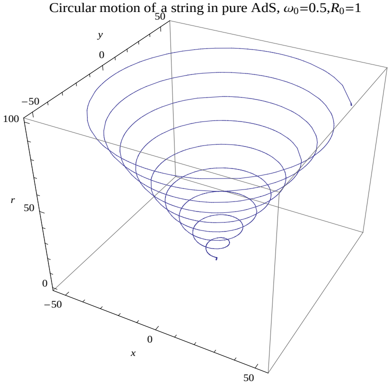
- The total power radiated is $P = \frac{\sqrt{\lambda}}{2\pi} v^2 \gamma^4 \omega_0^2$.
- We observe that the power radiated isn't a function of v for small velocities but also depends on the angular velocity ω_0. This is important to remember because it seperates the drag-force power absorbed with the power radiated from the quark to the gluonic degrees of freedom. When $\omega >> 1$ and $v << 1$ we can have radiation emitted at very low velocity v. This has the interpretation that at low temperatures the radiation emition is through the emission of glueballs with gapped mass.

- The profile of the string is



- For small temperatures T satisfying $\omega_0^2\gamma^3 >> \pi^2 T^2$ the vacuum result for the total energy radiated still holds.We also expect the radiation pattern to be similar to that of the zero temperature for distances $r < 1/T$ around the quark, then it will begin to thermalize converting into hydrodynamic excitations moving at the speed of sound, broaden and dissipate due to the presence of the plasma.

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- At low temperatures generically the radiation absorbed by the quark is much more than the energy absorbed by the plasma due to the drag force.
- Contrary, at high temperatures, generically the drag force absorption of energy is much larger than the radiation emitted by the quark.
- ''Generically'' means keeping the same ω, v while changing the temperature.

Motivation of the calculation

Motion in a magnetic field at zero temperature

- We will calculate the motion of a heavy charged quark moving in a constant magnetic field. The charge can be a flavor charge (electric charge is a special case of this), and the magnetic field should be thought as being imposed on the flavor brane. Here we will treat this by imposing this at the end point of the string, at $r = \Lambda$.

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- The interest in this configuration stems from various contexts. Magnetic fields induce the chiral magnetic effect in strongly coupled matter, and this may have implications both for heavy ion experiments as well as neutron stars.

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- The interest in this configuration stems from various contexts. Magnetic fields induce the chiral magnetic effect in strongly coupled matter, and this may have implications both for heavy ion experiments as well as neutron stars.
- Magnetic fields are also one of the most important environments in condensed matter experiments. In view of the potential applications of holography to strongly coupled condensed matter systems, it is interesting to understand the physics of heavy colored objects in magnetic fields. The Hall conductivity one of the main observables in this contex can be calculated in this way.

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- In that case $X^{\mu}(\tau, r) = x^{\mu}(\tau) \pm r\frac{dx^{\mu}}{d\tau}$, with τ the proper time on the boundary, r the radial coordinate in the Poincare patch and x^{μ} the coordinates of the endpoint at the boundary at $r = 0$.

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- The solutions of Mikhailov have the intepretation in global AdS_5 space that the fluctuation of the string in one brane are absorbed completely by the anti-brane.

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- The solutions with the $+$ sign are the retarded ones, which we will consider.
- The solutions of Mikhailov have the intepretation in global AdS_5 space that the fluctuation of the string in one brane are absorbed completely by the anti-brane.
- We will consider a flavour brane at $r = \Lambda$ and the motion of the endpoint corresponds to the motion of a quark with mass $m_q = \frac{\sqrt{\lambda}}{2\pi\Lambda}$ with $\lambda = g_{YM}^2 N_c$ the t′ Hooft coupling.

- Then the solutions of Mikhailov read:

$$X^{\mu}(\tau, r) = \left(\frac{r - \Lambda}{\sqrt{1 - \Lambda^4 \frac{4\pi^2}{\lambda} \mathcal{F}^2}} \right) \left(\frac{dx^{\mu}}{d\tau} - \frac{2\pi}{\sqrt{\lambda}} \Lambda^2 \mathcal{F}^{\mu} \right) + x^{\mu}(\tau),$$

with τ, x^{μ} the coordinates of the endpoint at $r = \Lambda$, the position of the flavour brane and $\mathcal{F}^{\mu}$ the 4-force on the quark.

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  with τ, x^μ the coordinates of the endpoint at $r = \Lambda$, the position of the flavour brane and $\mathcal{F}^\mu$ the 4-force on the quark.

- We consider a constant magnetic field on the boundary $\vec{B} = B\hat{z}$ and we request the boundary variation of the action

$$S = -\frac{1}{2\pi\ell_s^2}\int dr\, d\tau \sqrt{-detg} + e\int d\tau A^\mu \frac{dx^\mu}{d\tau}$$

  to be zero. The bulk variation is automatically zero for the solutions of Mikhailov in the case of pure AdS_5.

Equations of motion for the endpoint

- When an external force $\mathcal{F}^\mu$ is exerted on the quark, its equation of motion reads:

$$\frac{d}{d\tau}\left(\frac{m\frac{dx^\mu}{d\tau}-\frac{\sqrt{\lambda}}{2\pi m}\mathcal{F}^\mu}{\sqrt{1-\frac{\lambda}{4\pi^2 m^4}\mathcal{F}^2}}\right)=\frac{\mathcal{F}^\mu-\frac{\sqrt{\lambda}}{2\pi m^2}\mathcal{F}^2\frac{dx^\mu}{d\tau}}{1-\frac{\lambda}{4\pi^2 m^4}\mathcal{F}^2}$$

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- When the external force is due to the magnetic field then the force components become

$$\mathcal{F}^0=0 \quad , \quad \mathcal{F}^x(\tau)=-B\dot{y}(\tau) \quad , \quad \mathcal{F}^y(\tau)=B\dot{x}(\tau)$$

and its equation of motion reads:

$$\frac{d}{dt}\left(\gamma\frac{\frac{d\vec{x}}{dt}-s\frac{d\vec{x}}{dt}\times\hat{z}}{\sqrt{1-s^2\gamma^2\left(\frac{d\vec{x}}{dt}\right)^2}}\right)=\frac{s\frac{d\vec{x}}{dt}\times\hat{z}-s^2\gamma^2\left(\frac{d\vec{x}}{dt}\right)^2\frac{d\vec{x}}{dt}}{1-s^2\gamma^2\left(\frac{d\vec{x}}{dt}\right)^2},$$

where we have considered $\Lambda=1, B=s\frac{m}{\Lambda}$.

Equations of motion for the endpoint

- When an external force $\mathcal{F}^\mu$ is exerted on the quark, its equation of motion reads:

$$\frac{d}{d\tau}\left(\frac{m\frac{dx^\mu}{d\tau}-\frac{\sqrt{\lambda}}{2\pi m}\mathcal{F}^\mu}{\sqrt{1-\frac{\lambda}{4\pi^2 m^4}\mathcal{F}^2}}\right)=\frac{\mathcal{F}^\mu-\frac{\sqrt{\lambda}}{2\pi m^2}\mathcal{F}^2\frac{dx^\mu}{d\tau}}{1-\frac{\lambda}{4\pi^2 m^4}\mathcal{F}^2}$$

- When the external force is due to the magnetic field then the force components become

$$\mathcal{F}^0=0 \quad , \quad \mathcal{F}^x(\tau)=-B\dot{y}(\tau) \quad , \quad \mathcal{F}^y(\tau)=B\dot{x}(\tau)$$

and its equation of motion reads:

$$\frac{d}{dt}\left(\gamma\frac{\frac{d\vec{x}}{dt}-s\frac{d\vec{x}}{dt}\times\hat{z}}{\sqrt{1-s^2\gamma^2\left(\frac{d\vec{x}}{dt}\right)^2}}\right)=\frac{s\frac{d\vec{x}}{dt}\times\hat{z}-s^2\gamma^2\left(\frac{d\vec{x}}{dt}\right)^2\frac{d\vec{x}}{dt}}{1-s^2\gamma^2\left(\frac{d\vec{x}}{dt}\right)^2},$$

where we have considered $\Lambda=1, B=s\frac{m}{\Lambda}$.

- The maximum initial velocity is given by $v_{max}=\frac{1}{\sqrt{1+s^2}}$.

Equations of motion for the endpoint

- This happens because as we increase v_0 the worldsheet horizon comes closer to the flavour brane. When $v_0 \approx v_{max}$ then $r_h \approx \Lambda$ and $v_0^{r=0} \approx c$.

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- Then the acceleration for the endpoint is given by

$$\ddot{\vec{x}} = -\frac{s\sqrt{1-(\gamma^2-1)s^2}}{\gamma(1+s^2)}\left(s\dot{\vec{x}} + \hat{z} \times \dot{\vec{x}}\right).$$

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- The above equation multiplied with $\dot{\vec{x}}$ indicates the motion is damped.
- The motion can't be found analytically, apart from the case of small initial velocity compared to v_{max}, i.e. $v_0 << v_{max}$. In this case the motion is

$$\begin{pmatrix} x^{(0)}(t) \\ y^{(0)}(t) \end{pmatrix} = R_0 e^{-\frac{s^2}{1+s^2}t}\begin{pmatrix} \cos(\frac{s}{1+s^2}t+\phi_0) \\ \sin(\frac{s}{1+s^2}t+\phi_0) \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}.$$

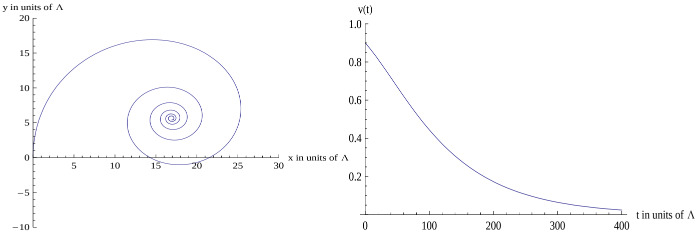
Trajectory of the quark

The motion of the quark

The velocity of the quark is

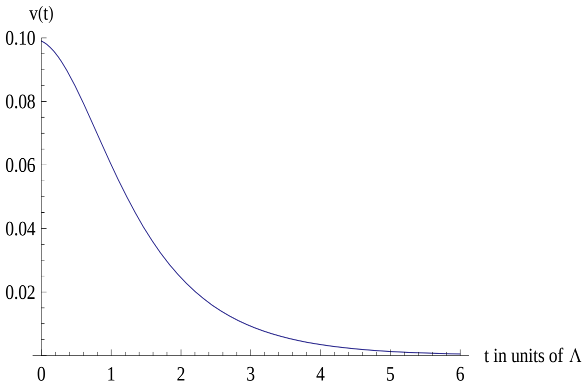
$$v^2(t) = \frac{sech^2\left(\frac{s^2 t}{s^2+1} + \tanh^{-1}\left(\sqrt{1-(s^2+1)\,v_0^2}\right)\right)}{s^2+1}$$

and for small $s = 0.1$ and $v_0 = 0.9c$ its trajectory and velocity are:



and for large times there is a characteristic time $t^* = \frac{s^2+1}{s^2}$ which gives $t^* = 100$ for $s = 0.1$.

For large $s = 10$ the velocity of the quark is



and for large times there is a characteristic time $t^* = \frac{s^2+1}{s^2}$ which gives $t^* \approx 1$ for $s = 10$.

- The four-momentum of the quark and the rate at which four-momentum is carried away are

$$
\begin{aligned}
p_q^\mu &= \frac{m\frac{dx^\mu}{d\tau} - \frac{\sqrt{\lambda}}{2\pi m}\mathcal{F}^\mu}{\sqrt{1 - \frac{\lambda}{4\pi^2 m^4}\mathcal{F}^2}}, \\
\frac{dP^\mu_{rad}}{d\tau} &= \frac{\sqrt{\lambda}\mathcal{F}^2}{2\pi m^2}\left(\frac{\frac{dx^\mu}{d\tau} - \frac{\sqrt{\lambda}}{2\pi m^2}\mathcal{F}^\mu}{1 - \frac{\lambda}{4\pi^2 m^4}\mathcal{F}^2}\right).
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- In our case this can be written as

$$\frac{dE_{rad}}{dt} = \frac{m s^2 \dot{\vec{x}}(t)^2}{\sqrt{1-\dot{\vec{x}}(t)^2}\left(1-\left(s^2+1\right)\dot{\vec{x}}(t)^2\right)}$$

$$\frac{d\vec{P}_{rad}}{dt} = \left(\dot{\vec{x}}(t) + s\dot{\vec{x}}(t)\times\hat{z}\right)\frac{dE_{rad}}{dt}.$$

Energy loss by the quark

The motion of the quark

- The bulk and boundary actions are time independent and therefore the total energy is conserved. This means that the energy lost by the quark is stored in the gluonic degrees of fredom and propagates to infinity.

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- Then we obtain

$$\frac{dE_{rad}}{dt} = \frac{ms^2 csch^2\left(\frac{s^2 t}{s^2+1} + \tanh^{-1}\left(\sqrt{1-(s^2+1)\,v_0^2}\right)\right)}{\sqrt{(s^2+1)\left(sech^2\left(\frac{s^2 t}{s^2+1} + \tanh^{-1}\left(\sqrt{1-(s^2+1)\,v_0^2}\right)\right) + s^2 + 1\right)}}$$

and we observe that for late times $t >> \lambda^{-1} = \frac{s^1+1}{s^2}$

$$\frac{dE_{rad}}{dt} \propto e^{-\frac{2s^2 t}{1+s^2}},$$

has an exponential damping as the squared velocity $v(t)^2$.

- In the following figures we show the exponential damping of the energy of the quark in units of its mass for larga $s = 10$ and small $s = 0.1$.

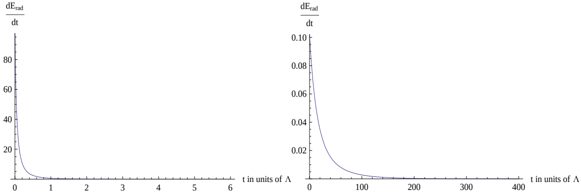

Figure: **Left**:Rate of energy transfer per unit mass from the particle for initial velocity $v_0 = 0.099c \approx v_{max}$ and large $s = 10$. **Right**:Rate of energy transfer per unit mass from the particle for initial velocity $v_0 = 0.099c \approx v_{max}$ and small $s = 0.1$.

Horizon of the induced metric

The motion of the quark

- The induced metric elements are

$$g_{\tilde{\tau}\tilde{\tau}} = -\frac{L^2}{r^2}\left(1 - r^2 \ddot{\tilde{x}}^{\mu}(\tilde{\tau})\ddot{\tilde{x}}_{\mu}(\tilde{\tau})\right), g_{\tilde{\tau}r} = -\frac{L^2}{r^2} \quad , \quad g_{rr} = 0$$

where $\tilde{x}^{\mu} = \{\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z}\}$ and $\tilde{\tau}$ is the proper time on the boundary at $r = 0$.
There is a horizon when

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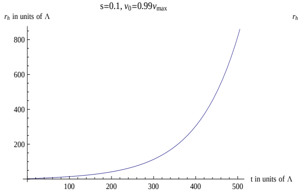
- For our case for a constant magnetic field we have:

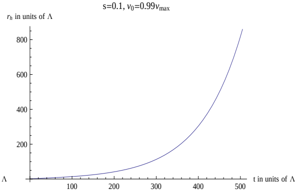
$$r_h = \frac{1}{s\gamma||\vec{v}||}.$$

Horizon of the induced metric

The motion of the quark

- In the left and right following figures, we show the horizon as a function of time on the boundary t for small $s = 0.1$ and for large $s = 10$. The horizons therefore start from $r_h(t = 0) \approx 1$ as the initial velocity of the quark v_0 in near v_{max} and have an exponential increase with time as expected because for large times in the regime $||\vec{v}(t)|| << v_{max}$ the velocity has the form $||\vec{v}(t)|| \approx v_{t_0} e^{-\frac{s^2}{1+s^2}(t-t_0)}$, and therefore $\gamma \approx 1$ and then $r_h \approx \frac{1}{||\vec{v}(t)||} \propto e^{\frac{s^2}{1+s^2}t}$.





- By calculating the curvature invariant we have a constant curvature $R = -\frac{2}{L^2}$ everywhere and for all times, the curvature of a hyperboloid with radius L in two dimensions (AdS_2). This constant negative curvature is true for every solution of Mikhailov.

$$X^\mu(\tilde{\tau}, r) = \tilde{x}^\mu(\tilde{\tau}) + r\frac{d\tilde{x}^\mu(\tilde{\tau})}{d\tilde{\tau}}.$$

Profile of the string

- The motion of the string is given by:

$$X^\mu(\tau, r) = \left(\frac{r - \Lambda}{\sqrt{1 - \Lambda^4 \frac{4\pi^2}{\lambda} \mathcal{F}^2}} \right) \left(\frac{dx^\mu}{d\tau} - \frac{2\pi}{\sqrt{\lambda}} \Lambda^2 \mathcal{F}^\mu \right) + x^\mu(\tau),$$

w.r.t. the coordinates at $r = \Lambda$. However, in oder to draw the string profile we need the function $\vec{X}(X^0, r)$. This can be done numerically, and we assume linear motion with constant speed for the quark before it enters the area with the magnetic field. This induces a discontinuity in the first derivative of $\vec{X}(X^0, r)$ at the point where the propagating waves of the two motions meet.

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- By the above solution $X^{\mu}(\tau, r)$ for the string, we expect roughly a linear increase of the propagating wave w.r.t. r.

Profile of the string

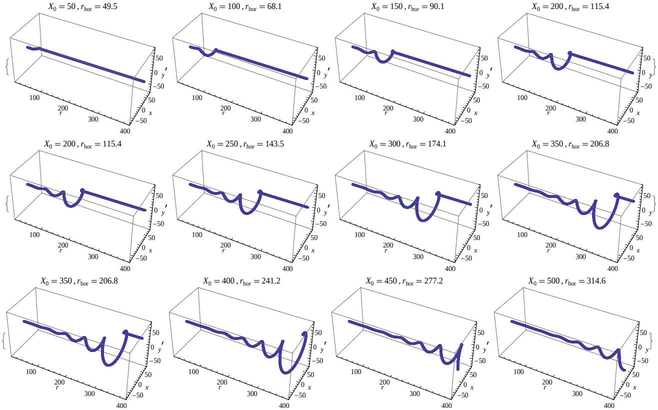

Figure: Profile of the string in the (more interesting) case of small $s = 0.1$ and initial velocity $v_0 = 0.2 v_{max} \approx 0.2c$ for different times X^0 in the range $r \in (1, 400)$ for the radial coordinate.

- We name $x^\mu = \{t, x, y, z\}$ the coordinates and τ the proper time on the boundary at $r = 0$. The metric elements are

$$g_{\tau\tau} = -\frac{L^2}{r^2}\left(1 - r^2 \ddot{x}^\mu(\tau)\ddot{x}_\mu(\tau)\right) \quad , \quad g_{\tau r} = -\frac{L^2}{r^2} \quad , \quad g_{rr} = 0.$$

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- There is a horizon when

$$g_{\tau\tau} = 0 \Rightarrow r_h = \frac{1}{\sqrt{\ddot{x}^\mu(\tau)\ddot{x}_\mu(\tau)}} = \frac{\left(1 - \vec{v}^2\right)^{\frac{3}{2}}}{\sqrt{\vec{a}^2\left(1 - \vec{v}^2\left(1 - \frac{(\vec{v}\vec{a})^2}{\vec{v}^2\vec{a}^2}\right)\right)}},$$

Then the metric elements become

$$g_{\tau\tau} = -\frac{L^2}{r^2}\left(1 - \frac{r^2}{r_h^2}\right) \quad , \quad g_{\tau r} = -\frac{L^2}{r^2} \quad , \quad g_{rr} = 0.$$

The induced world metric

Properties of the solution of Mikhailov for pure AdS spacetime

- There is also a point r^* till where we can draw the profile of the string $\vec{X}(X^0, r)$ in the static gauge given by

$$\left.\frac{\partial X^0}{\partial \tau}\right|_r = 0 \Rightarrow \frac{dt}{d\tau} + r\frac{d^2 t}{d\tau^2} = 0 \Rightarrow \gamma + r\frac{d\gamma}{dt}\gamma = 0 \Rightarrow$$

$$r^* = -\gamma^{-3}\frac{1}{\vec{v}\vec{a}} = -\frac{\left(1-\vec{v}^2\right)^{\frac{3}{2}}}{\vec{v}\vec{a}}$$

which means that r^* is positive only when $\vec{v}\vec{a} < 0$. We also observe that $r^* > r_h$, this point r^* is hidden by the horizon:

$$r_h = \frac{\left(1-\vec{v}^2\right)^{\frac{3}{2}}}{\sqrt{\vec{a}^2\left(1-\vec{v}^2\right) + \left(\vec{v}\vec{a}\right)^2}} < \frac{\left(1-\vec{v}^2\right)^{\frac{3}{2}}}{\vec{v}\vec{a}} = r^*.$$

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- The curvature is constant

$$R = -\frac{2}{L^2}.$$

Cases of a static horizon

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- We can also have a particle with acceleration $\vec{a}$ codirectional with the velocity. If they are both in the x-direction

$$\begin{aligned} \vec{a} \quad & \nearrow\nearrow \quad \vec{v} \Rightarrow \\ r_h^2 \quad &= \quad \frac{(1-v^2)^3}{a^2} = const. \Rightarrow \frac{dv}{dt} = \pm\frac{(1-v^2)^{3/2}}{r_h} \Rightarrow \\ v(t) \quad &= \quad \frac{2(v_0-1)}{-v_0+e^{\frac{2t}{r_h}}(v_0+1)+1}+1, v(t) = -\frac{2(v_0+1)}{e^{\frac{2t}{r_h}}(v_0-1)-v_0-1}-1 \end{aligned}$$

The horizon lies at any r_h we like.

- The quadratic term of the Nambu-Goto action is

$$\begin{aligned} L_2^{NG} &= -\sqrt{-detg_2} = \\ &- \frac{L^2}{2r^2}\Bigg[Y'^{\mu}Y'^{\nu}\Big[-r^2 a_\mu a_\nu - r^4 a^4 U_\mu U_\nu - 2r^3 a^2 a_\mu U_\nu + \\ &+ (U + r\, a)_\mu (U + r\, a)_\nu + \left(1 - r^2 a^2\right)\eta_{\mu\nu}\Big] + \\ &+ \frac{L^2}{r^2} Y'^{\nu}\dot{Y}^{\mu}\Big[-\left(1 + r^2 a^2\right) U_\mu U_\nu - \eta_{\mu\nu} - 2r\big(a_\mu U_\nu + a_\nu U_\mu\big)\Big], \end{aligned}$$

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&+ \left.\left. (U + r\, a)_\mu (U + r\, a)_\nu + \left(1 - r^2 a^2\right)\eta_{\mu\nu}\right] + \right. \\
&+ \left. \frac{L^2}{r^2} Y'^{\nu}\dot{Y}^{\mu}\left[-\left(1 + r^2 a^2\right) U_\mu U_\nu - \eta_{\mu\nu} - 2r\left(a_\mu U_\nu + a_\nu U_\mu\right)\right]\right],
\end{aligned}
$$

where U, a the 4-velocity and 4-acceleration correspondingly.

- In the case of linear motion with constant velocity the diagonalized metric in $Y_1, Y2$ becomes

$$
S_2^{imm} = \frac{L^2\left(Y_1'^2 - 2\dot{Y}_1 Y_1' + Y_2'\left(Y_2' - 2\dot{Y}_2\right)\right)}{2r^2}.
$$

Fluctuations around the solution

Properties of the solution of Mikhailov for pure AdS spacetime

- Then the fully diagonal metric becomes

$$S_2 = -\frac{L^2}{4\pi\ell_s^2\left(\frac{\sqrt{5-\sqrt{5}\tau'}-\sqrt{5+\sqrt{5}r'}}{\sqrt{10}}\right)^2}\times$$
$$\times \left(-\left(\left(\sqrt{5}-1\right)\dot{Y}_1^2+\left(1+\sqrt{5}\right)Y_1'^2\right)+\left(-\left(\sqrt{5}-1\right)\dot{Y}_2^2+\left(1+\sqrt{5}\right)Y_2'^2\right)\right)$$

where we have used new worldsheet coordinates

$$r = \frac{\sqrt{5-\sqrt{5}\tau'}-\sqrt{5+\sqrt{5}r'}}{\sqrt{10}} \quad , \quad \tau = \frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(\sqrt{5}-1\right)r'+2\tau'\right).$$

Hall conductivity

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- We describe the motion of the charge carriers in a strongly coupled vacuum with the motion of the endpoint of the strings at $r = \Lambda$, where Λ^{-1} is proportional to the mass of the carriers.
- Pure AdS spacetime has Lorentz invariance under boosts and rotations in the x, y, z, t directions. The electric and magnetic field transform under boosts:

$$\begin{aligned} \vec{E}_{||} &= \vec{E}_{||} \quad , \quad \vec{E}_{\perp} = \gamma\left(\vec{E}_{\perp} + \vec{v} \times \vec{B}\right) \\ \vec{B}_{||} &= \vec{B}_{||} \quad , \quad \vec{B}_{\perp} = \gamma\left(\vec{B}_{\perp} - \vec{v} \times \vec{E}\right) . \end{aligned}$$

By doing a boost with velocity $v_y = -E_x/B_z$ we have in the boosted frame $\vec{E}' = \vec{0} \quad , \quad \vec{B}' = \hat{z}\sqrt{B_z^2 - E_x^2}$.

Hall conductivity

- Therefore in this frame we have only a constant magnetic field B_z' in the z-direction and the motion of the string will be a spiral towards a fixed point as we have seen already. Therefore for large times, the velocity of the particle will be that of the boosted frame $v_y = -E_x / B_z$. From this, we deduce that we have the Hall conductivity

$$\sigma_{xy} = \frac{j_y}{E_x} = -\frac{q}{B_z}.$$

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- Since the motion is a special case of the retarded solutions of Mikhailov, the absorption of energy by the gluonic degrees of freedom simulated by the string obeys the Lienard formula.

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- The motion for the quark is a spiral, as the radiation emmited absorbs continuousky its energy. The motion of the particle for late times is exponentially damped.
- There is a maximum initial velocity of the particle, beyond which the classical string description breaks down.
- The induced string metric has a horizon that is time dependent and its position moves exponentially fast at late times towards the center of AdS. The induced string metric is locally that of AdS_2 like in any other string motion in bulk AdS.
- Since the motion is a special case of the retarded solutions of Mikhailov, the absorption of energy by the gluonic degrees of freedom simulated by the string obeys the Lienard formula.
- The embedding of the string in the static gauge $\vec{X}(X^0, r)$ stops at a point which is hidden by the worldsheet horizon.

Conclusions

- In order for the particle to move on a cycle with constant velocity, there must also be a time dependent electric field codirectional with the velocity of the particle, so that it provides energy to the quark, equal to the energy absorbed by the drag force per unit time.

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- The Hall conductivity is the same with the classical case.