

Tests and theories of Lorentz symmetry violation  
APC, 2 March 2012

# *Lorentz violation, dissipation and holography*

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# Introduction

- Lorentz invariance is one of the most important pillars of modern day physics, experimentally unchallenged for more than a hundred years.
- It has been tested to unprecedented accuracy reaching  $10^{-21}$  in several contexts, and down to  $10^{-29}$  recently in the neutron sector.

*Romalis et al. 2011*

- Its reign coincided with the demise of “aether”. **Michelson and Morley and Einstein (1905) eventually banished aether as a physical theory.**
- Few physicists realized that a new kind of “aether” made it back into physics after 1915, with the general theory of relativity: the gravitational field.
- **Only rarely, a few theorists have questioned the universal applicability of Lorentz invariance.** The most prominent idea was due to H. Nielsen and collaborators, that **sought to explain LI as an emergent IR symmetry** with inconclusive results.

*Nielsen et al. 1984*

- Related ideas were applied to other low-energy symmetries, prompting the term **anti-grand-unification**.

*Iliopoulos+Nanopoulos+Tomaras, 1985*

- **Kostelecky and Samuel** argued that in open string theory, unusual tachyon vevs can trigger **vevs of tensors and therefore LV**. We know now that this cannot happen as such in tachyon condensation. This triggered a long term effort to parametrize LV QFTs.

*Kostelechy+Samuel, 1989*

- A new series of investigations saw the light in the 90's prompted by vague claims of "quantum gravity and spacetime foams". **Their solid outcome were parametrizations of LV dispersion relations and comparison with data.**

- It was noted in late 90's that **the velocity of light on probe D-branes in string theory can be variable, and smaller from the speed of light in the bulk**, being affected by the bulk gravitational fields.

*Kiritsis, 1999*

- It was suggested that it may be used to solve the horizon problem in cosmology.

*Kiritsis, 1999, Chung+Freese, 1999*

- It was also suggested that **LV is generic in string theory orientifolds** and the idea was brought to its natural conclusion giving rise to "**Mirage Cosmology**" in string theory.

*Kehagias+Kiritsis, 1999*

- **Varying speed of light theories using branes were constructed by stabilizing branes like planets in the solar system, around black branes.**

*Kiritsis, 1999, Alexander 1999, Burgess+Martineau+Quevedo+Rabadan, 2003*

- It was shown in general that **the speed of light on branes is always smaller or equal to that of the bulk,**

*Gibbons+Herdeiro, 2000*

- Many years later, Gubser constructed a braneworld solution with varying speed of light, but carrying no entropy.

*Gubser, 2008*

- Such solutions appear today in **holographic backgrounds describing strongly coupled holographic systems at finite density.**

*Charmousis+Gouteraux+Kim+Kiritsis+Meyer, 2010*

- It was pointed out that strong violations of LI in the UV can allow for a resolution of the hierarchy problem, **modifying significantly the power counting of interactions.**

*Anselmi+Halat, 2007*

- A similar idea applied to gravity gave rise to a power-counting renormalizable class of LV theories of gravity, with interesting properties.

*Hořava, 2009*

- The original theory suffers from several issues, including strong coupling and phenomenological problems. But this class of theories **solves several of the problems we need inflation for, automatically.**

*Kiritsis+Kofinas, 2009, Mukohyama, 2009*

- Phenomenologically acceptable alternatives were proposed.

*Blas+Pujolas+Sibiryakov, 2010*

# Questions to be considered

- It is important to investigate LV, even in the absence of experimental hints.

## Questions

- Is Lorentz invariance an accident of low energies? If it is, what is the dependence of the breaking scale with energy?
- Under what conditions the breaking of LI is "natural" in the standard sense of Quantum field theory?
- Can LI be spontaneously (dynamically) broken?
- What is the general connection between LV and dissipation? In the simplest cases this is Cerenkov radiation.
- How is LV compatible with diffeomorphism invariance and gravity as we know it? Is gravity coupled to a LV QFT a consistent theory?
- ♠ One might think that LV in QFT and gravity are completely decoupled subjects. We will argue they are intimately related.

# Lorentz Violation in QFT

- Parametrizing LV in QFT is a simple exercise
- It amounts to provide sources for **all** tensor operators:

$$L = V(\phi) + b^\mu \partial_\mu \phi + c^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + d^{\mu\nu\rho} \partial_\mu \phi \partial_\nu \phi \partial_\rho \phi + \dots$$

- For constant generic  $b^\mu, c^{\mu\nu}, d^{\mu\nu\rho}$  the theory breaks LI.  
When  $b^\mu = d^{\mu\nu\rho} = 0, c^{\mu\nu} \sim \eta^{\mu\nu}$  the theory is Lorentz Invariant.
- When  $b^\mu, c^{\mu\nu}, d^{\mu\nu\rho}$  are functions of the spacetime point, translational invariance also breaks.
- If the theory is in a non-trivial metric then

$$c^{\mu\nu} = \sqrt{-g} g^{\mu\nu}$$

- LI is spontaneously broken for non-trivial  $g_{\mu\nu}$

- The LV couplings can be systematically parametrized using the previous idea.
  - String theory suggests that all couplings of QFT are dynamical fields in a gravitational sector.
  - Gauge and Yukawa couplings are scalars (sometime moduli)
  - Kinetic tensors translate into the spacetime metric.
  - $\theta$ -angles translate to pseudoscalars (axions), like  $\theta F \wedge F$ .
  - Chemical potentials translate into graviphotons.
- 

We can make a rather general claim motivated by string theory and the AdS/CFT correspondence

- Gravity/string theory is the dynamics of sources of QFT.



- This is explicitly realized in holography.

However in general, the collection of an infinite number of sources for a QFT matches (multi)string states in a string theory.

If the QFT is at large  $N$ , the string theory is semiclassical.

If it is also strongly coupled then the string is stiff and the theory is approximated by a gravitational theory.

- Holography states that the effective action for sources of a QFT, is a gravity theory in higher dimensions.

- In this language, LV is the existence of non-trivial tensor backgrounds in the gravitational sector (vectors, 2-tensors, etc).

There are two ways LV can appear in this context:

♠ “Environmental LV”: Energy and charges generate gravitational and other classical fields, and they break LI.

♠ “Dynamical LV”: The effective potential for tensor operators in QFT has non-trivial minima, that break LI. It is unlikely this can happen at weak coupling, but it might at strong coupling. Holography provides a formalism to compute such effective potentials

*Kiritsis+Niarchos ongoing work*

# Coupling LV to gravity

- There is an obvious puzzle that generalizes similar statements for other symmetries.
- If a global symmetry is broken in QFT, how can it couple to a “gauge theory” where this symmetry is “gauged”?
- It is well known from the examples of standard gauge symmetry, and supersymmetry, that such consistent couplings exist, **only if the breaking of the symmetry in the QFT is spontaneous/environmental.**
- Equivalently, the QFT is invariant under the symmetry if the symmetry violating couplings, transform under the symmetry, so that the symmetry is formally restored.
- A simple example of a U(1) symmetry:

$$L = |\partial\psi|^2 + \text{Re}[g\psi^2] \quad , \quad \psi \rightarrow \psi e^{i\epsilon} \quad , \quad g \rightarrow g e^{-2i\epsilon}$$

is U(1) invariant if  $g$  transforms appropriately.

- It can be promoted to a new scalar field  $g \rightarrow ge^{-2ia}$

$$L' = |\partial\psi|^2 + \text{Re}[g\psi^2 e^{-2ia}] - \frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}(\partial_\mu a + A_\mu)^2$$

$$A_\mu \rightarrow A_\mu - \partial_\mu \epsilon \quad , \quad a \rightarrow a + \epsilon \quad , \quad \psi \rightarrow \psi e^{i\epsilon}$$

- Gravity can be thought of as a gauge theory of translations, or Lorentz transformations (or both) depending on the formalism.
- Therefore a LV theory can consistently couple to gravity iff the breaking is spontaneous/environmental.
- Translated in plain words: a LV QFT is a QFT coupled to a generalized gravitational sector (with possibly many tensor fields), some of which have non-trivial vevs/classical values.
- Note, that in this language, the theory of fluctuating fields, both QFT and gravitational, is fully diff+Lorentz invariant. Only the presence of background fields break the symmetries.
- In particular, the gravitational interaction is diff-invariant and not of the Hořava type.

# Hořava(-Lifshitz) gravity

- According to the previous conclusion, there is no place for Horava gravity coupled to LV QFT. This does not look reasonable.
- There is a **hidden assumption** in our previous arguments: the QFT is defined by perturbations of standard CFTs with full conformal symmetry.
- If we change "frame" and start from scaling QFTs with Lifshitz-symmetries

$$t \rightarrow \lambda^z t \quad , \quad x^i \rightarrow \lambda x^i$$

example :  $L = \dot{\phi}^2 - \phi \square^2 \phi + \dots \quad , \quad z = 2$

then the gravity they will couple to will be **(generalized) Hořava-Lifshitz gravity**.

- A quick way to see this is to check that the **renormalization counterterms of holographic Lifshitz theories generate Hořava-Lifshitz gravity**.
- This in retrospect says that HL gravity can be written as standard diff-invariant gravity coupled to more fields that have vevs. Indeed, this is known to be true.
- The two cases on the QFT side seem to be related with a basis rearrangement of operators, and the UV scaling symmetry.

# LV on the brane

- We will therefore couple QFT to non-trivial background fields to generate LV.

- A natural setup to do this is to embed brane-worlds in bulk LV metrics.

*Kiritsis 1999*

- The concrete example is a black-3 brane metric

$$ds^2 = e^{2A(r)} \left[ -f(r)dt^2 + \frac{dr^2}{f(r)} + dx^i dx^i \right]$$

with  $f(r)$  the “blackness” function:  $0 \leq f \leq c_{UV}^2$ .

- $c_{UV}$  is the UV speed of light and also the speed of gravitons in the bulk.

- Embedding a **D3 brane** at  $r = r_*$ , with a gauge interaction on it, we obtain the effective action

$$S_{DBI} \sim \int d^4\xi \sqrt{-\det(\hat{g}_{\mu\nu} + F_{\mu\nu})} \simeq -\frac{\sqrt{-\det(g)}}{4} F_{\mu\nu}^2 + \dots$$

with

$$\hat{g}_{ab} = G_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b}$$

- The effective speed of light is

$$c_{eff}^2 = f(r_*) \leq c_{UV}^2$$

- Near the bulk horizon,  $f(r_h) = 0$ , the world-volume theory approaches the “Carolean limit”:  $c_{eff} \rightarrow 0$ .

- In more complicated embeddings (like the brane moving with constant velocity in an internal dimension), the effective speed of light on the brane depends on more data, both the bulk fields and the motion parameters.

$$c_{eff}^2 = f(r_*) - V^2$$

- The world-volume theory has a horizon at  $r_w$  such that

$$c_{eff}^2 = f(r_w) - V^2 = 0$$

- The speed of light here is “position dependent” and this breaks LI.

# The holographic dual view

- The previous setup corresponds to a **holographic large-N theory**  $\Theta$  (that generates the bulk black-brane metric) coupled to a (weakly coupled) theory  $\Theta'$  (the theory on the brane).
- In the bulk gravity solution,  $r \rightarrow \infty$  corresponds to the boundary (= the UV of the dual large-N QFT). The presence of the blackness factor, implies a non-trivial uniform energy density in the dual large-N QFT  $\Theta$ .
- The presence of a bulk horizon implies that such a state is thermal.
- It is the non-trivial energy density that triggers LV.
- The bare action of the  $\Theta'$  theory, corresponds to the brane being placed at the UV boundary  $r \rightarrow \infty$  of the geometry.
- As usual in holography, the radial scale stands for the RG scale  $r \sim E$  of the QFT.

- The induced action of the brane placed at an intermediate radial position,  $r_*$  corresponds to the **effective action of theory  $\Theta'$** , at energy  $E = r_*$ , after **integrating out the quantum corrections of the theory  $\Theta$** .

- Therefore

$$c_{eff}^2 = f(r_*) = f(E)$$

gives the RG dependence of the speed of light of  $\Theta'$  due to the quantum effects of  $\Theta$ .

- There are other "sources" of LV in such metrics. An important one, in theories  $\Theta$  with a conserved charge, is **the presence of finite charge density**. It was shown in general that at zero temperature, the generic solution is a solution with a generalized Lifshitz symmetry

*Charmousis+Gouteraux+Kim+Kiritsis+Meyer, 2010, Gouteraux+Kiritsis, 2011*

$$ds^2 = u^c \left[ -\frac{dt^2}{u^{2z}} + \frac{du^2 + dx^i dx^i}{u^2} \right], \quad t \rightarrow \lambda^z t, \quad u \rightarrow \lambda u, \quad x^i \rightarrow \lambda x^i$$

with dynamical exponent  $z$ . Putting the metric in the standard form

$$ds^2 \sim r^{2a} \left[ -f(r) dt^2 + \frac{dr^2}{f(r)} + dx^i dx^i \right], \quad f(r) \sim r^{2\frac{1-z}{2-z}}$$



We obtain

$$c_{eff} \sim r^{\frac{1-z}{2-z}} \sim E^{z-1}$$

- Since  $z \geq 1$ ,  $c_{eff} \rightarrow \infty$  when  $E \rightarrow \infty$  and vanishes at  $E \rightarrow 0$ .
- The black brane solution has finite entropy, as the horizon area is finite.
- The charge-density Lifshitz solutions have no entropy.

# Dispersion and energy loss

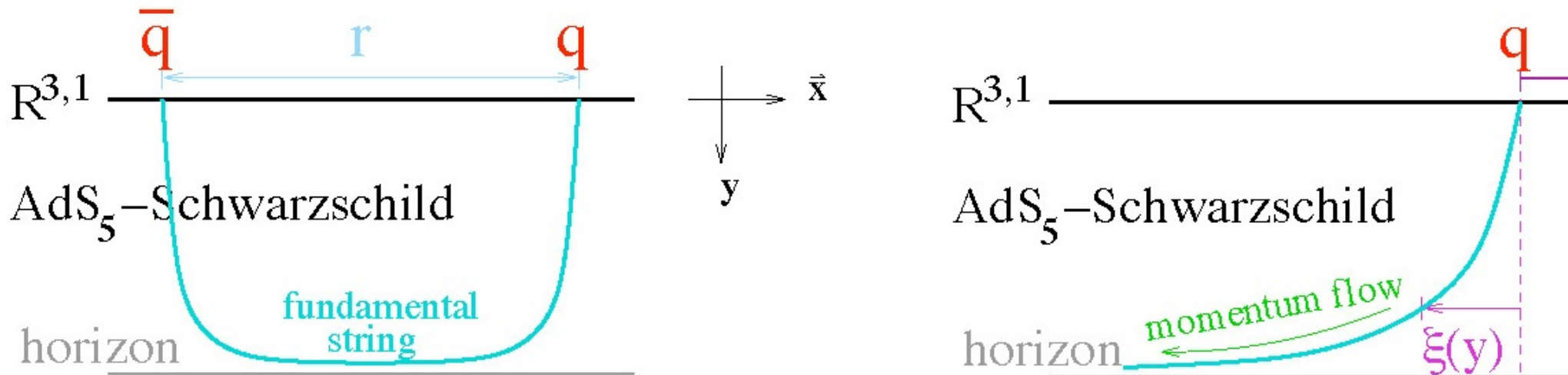
- Cerenkov radiation is a generic phenomenon in the presence of superluminal particles. It is generated by three point vertices of light/massless particles that allow decays, as the relevant speeds of light are not equal.
- This is a mechanism studied at weak coupling.
- It is an important constraint on experimental tests of superluminality.  
*Cohen+GLashow, 2011*
- The loss rate depends on the relevant interactions.  $\frac{dE}{dx} \sim E^3$  for neutrinos.
- Such a rate can be also computed in the braneworld set up. A relevant class particles in that case are "quarks": **interacting particles living on the brane world.**
- It exists only if the LI is violated by the background metric.

# Energy loss

- We must find a solution to the string equations with

$$x^1 = vt + \xi(r) \quad , \quad x^{2,3} = 0 \quad , \quad \sigma^1 = t \quad , \quad \sigma^2 = r$$

Herzog+Karch+kovtun+Kozcac+Yaffe, 2006 Gubser, 2006  
Casaldelrrey-Solana+Teaney, 2006, Gursoy+Kiritsis+Mazzanti+Nitti, 2010



For a LV metric (in string frame)  $ds^2 = b(r)^2 \left[ \frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x} \cdot d\vec{x} \right]$  the solution profile is

$$\xi'(r) = \frac{C}{f(r)} \sqrt{\frac{f(r) - v^2}{b^4(r)f(r) - C^2}} \quad , \quad C = vb(r_s)^2 \quad , \quad f(r_s) = v^2$$

- The induced metric on the world-sheet is a 2d black-hole with horizon at the turning point  $r = r_s$  ( $t = \tau + \zeta(r)$ ).

$$ds^2 = b^2(r) \left[ -(f(r) - v^2)d\tau^2 + \frac{1}{(f(r) - \frac{b^4(r_s)}{b^4(r)}v^2)}dr^2 \right]$$

- We can calculate the drag force:

$$F_{\text{drag}} = P_\xi = -\frac{b^2(r_s)\sqrt{f(r_s)}}{2\pi\ell_s^2}$$

- In AdS-bh it is given by a universal result.

$$F_{\text{drag}} = -\frac{\pi\ell_s^2}{2\ell_s^2} T^2 \frac{v}{\sqrt{1-v^2}} = -\frac{1}{\tau} \frac{p}{M}, \quad \tau = \frac{2M}{\pi\sqrt{\lambda} T^2}$$

- For a generalized Lifshitz metric  $ds^2 = u^c \left[ -\frac{dt^2}{u^{2z}} + \frac{du^2 + dx^i dx^i}{u^2} \right]$ .

$$F_{\text{drag}} \sim v \frac{z+1-c}{z-1}$$

- There is also Cerenkov emission of on-the-brane particles. This is sub-leading at large  $N_c$ .

# Fluctuations

- Superluminal particles are subject to dispersion and energy loss, that we calculated.
- In some cases such processes can be thermal, while in other cases they are not.

*Kiritsis+Pavlopoulos, 2011*

- Thermal cases are associated to world-volume horizons.
- In such cases the fluctuations are important observables.
- The fluctuation-dissipation relation can be used to define temperature.
- Fluctuations generate a Langevin diffusion of superluminal particles.
- They should be related to Hawking radiation associated with the world-volume horizons

*deBoer+Hubeny+Rangamani+Shigemori, 2008, Son+Teaney, 2009*

# Outlook

- LV in QFT is intimately connected to gravity.
- The scaling symmetries of the QFT decide the type of gravity theory that is relevant
- All LV protocols can be therefore generated with non-trivial background fields (metric etc).
- Probe branes on such background field are model LV QFTs.
- They also have a dual holographic interpretation: weakly coupled QFTs in interaction with a large N sector that breaks LI.
- Cerenkov energy loss is generic and has analogs at strong coupling, that were calculated.
- Is spontaneous LV possible in QFT?

# Detailed plan of the presentation

- Title page 1 minutes
- Introduction 5 minutes
- Questions 7 minutes
- LV in QFT 10 minutes
- Coupling LV to gravity. 13 minutes
- Hořava(-Lifshitz) gravity 15 minutes
- LV on the brane 17 minutes
- The holographic dual view 22 minutes
- Dispersion and energy loss 23 minutes
- Energy loss 27 minutes
- Fluctuations 28 minutes
- Outlook 29 minutes