Chiral symmetry breaking and meson physics from Sen's tachyon action

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Papers:

Phys.Rev.D81:115004,2010 [ArXiv:1003.2377][hep-ph]

and

JHEP 11 (2010) 123 [ArXiv:1010.1364][hep-ph]

Introduction

• There are several holographic approaches to flavor.

• Most work in the quenched approximation.

• This involves flavor branes that are "light": they do not backreact on the geometry of glue in the bulk.

• There has been discussion of flavor dynamics beyond the quenched approximation, but this effort is yet incomplete.

Kuperstein, Sonnenschein,Klebanov, Maldacena, Cotrone, Bigazzi, Casero, Paredes, Kiritsis,Nunez,Ramallo,Mas,Arean,Chen,Hashimoto,Matsuura,..... 2006-2010

• At the bottom up level, M. Jarvinnen will present a model for QCD, where back-reaction is included, and describe the full phase diagram.

Top-down flavor

• The state of the art in top-down models involves the Sakai-Sugimoto model: $D8 + \bar{D}8$ branes in the black-D4 glue geometry.

It exhibits $U(N_f)_L \times U(N_f)_R \to U(N_f)_V$ chiral symmetry breaking, and very good and extensive phenomenology both for mesons and baryons.

• Its weak points are that:

(a) Quarks are always massless

(b) The dual of the quark mass operator is not explicit/easily usable in the model

(c) The tower of 0^{++} scalar mesons is missing

(d) The asymptotic meson trajectories have quadratic masses $m_n^2 \to n^2$.

Bottom-up flavor

- Bottom up approaches involve the hard-wall model,

Erlich+Katz+Son+Stephanov, Da Rold+Pomarol

and its sequel, the soft-wall model

Karch+Katz+Son+Stephanov

which is rather successful but for the following points that can be improved:

(a) The background glue solution cannot be obtained from any consistent dynamics/action

(b) No consistent thermodynamics can be defined.

(c) The magnetic quarks are confined instead of screened.

(d) Chiral symmetry breaking is input by hand as an IR boundary condition.

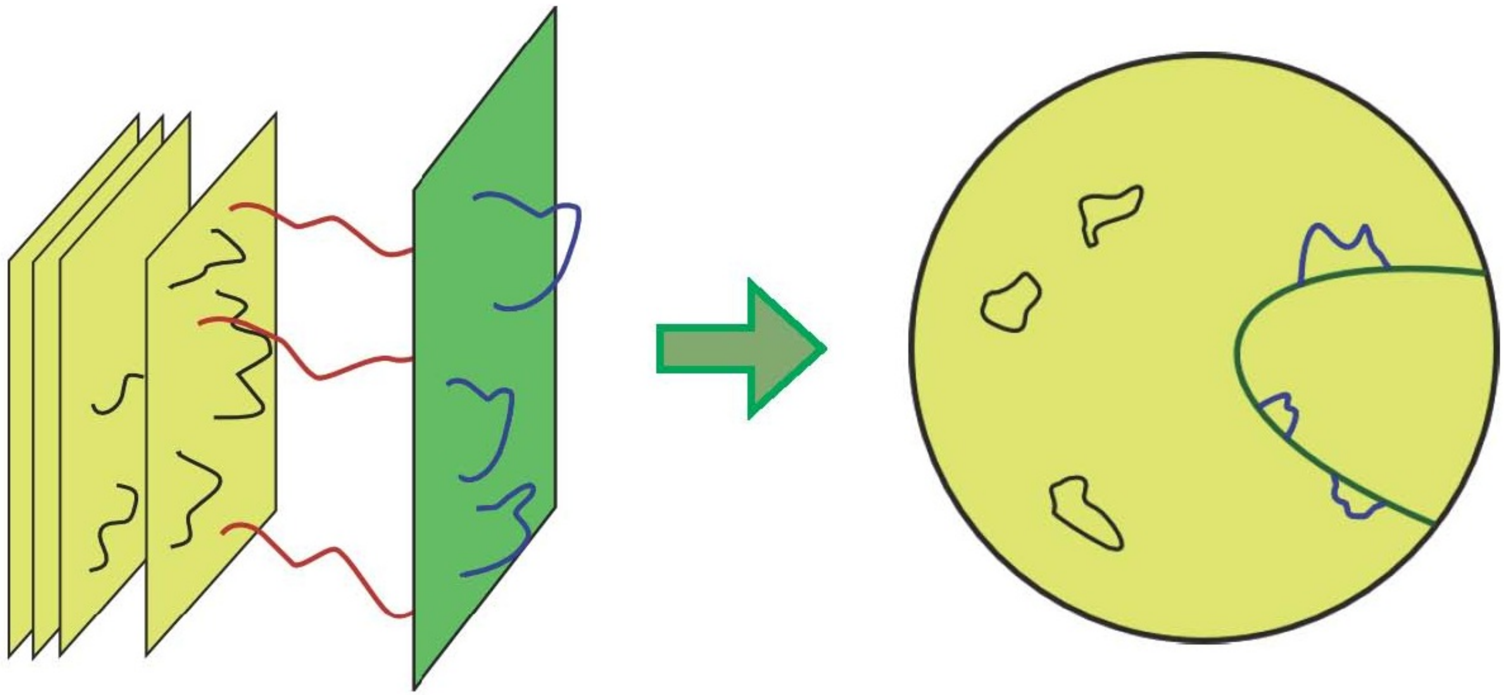
(e) The mass of the ρ meson is insensitive to the quark (or pion) mass.

(f) The finite density physics is insensitive to quark masses.

The price for its simplicity is that many features do not correspond to QCD.

The general setup for flavor

• In the minimal setup the bulk has 5 dimensions.



• To add N_f quarks q_L^I and antiquarks $q_R^{\bar{I}}$ we must add space-filling N_f D_4 and N_f $\bar{D}_4$ branes.

• The q_L^I are the "zero modes" of the $D_3 - D_4$ strings while $q_R^{\bar{I}}$ are the "zero modes" of the $D_3 - \bar{D}_4$

• The low-lying fields on the D_4 branes ($D_4 - D_4$ strings) are U$(N_f)_L$ gauge fields A^L_μ. They are dual to the left flavor currents

$$J^{\mu,IJ}_L \quad = \quad \bar{q}^I_L \gamma^\mu q^J_L \qquad \leftrightarrow \qquad A^{\mu,IJ}_L$$

• The low-lying fields on the $\overline{D_4}$ branes ($\overline{D_4} - \overline{D_4}$ strings) are U$(N_f)_R$ gauge fields A^R_μ. They are dual to the right flavor currents

$$J^{\mu,\bar{I}\bar{J}}_R \quad = \quad \bar{q}^{\bar{I}}_R \gamma^\mu q^{\bar{J}}_R \qquad \leftrightarrow \qquad A^{\mu,\bar{I}\bar{J}}_R$$

• There are also the low-lying fields of the ($D_4 - \overline{D_4}$ strings), essentially the complex string-theory "tachyon" $T_{I\bar{J}}$ transforming as $(N_f, \bar{N}_f)$ under the chiral symmetry $U(N_f)_L \times U(N_f)_R$. It is dual to the quark mass terms

$$T_{I\bar{J}} \sim \bar{q}^I_L \; q^{\bar{J}}_R$$

• Integrating out the quarks, generates an effective action $S_{flavor}(A^{L,R}_\mu, T)$, so that $A^{L,R}_\mu, T$ can be thought as effective $q\bar{q}$ composites, that is : mesons

• On the string theory side: integrating out $D_3 - D_4$ and $D_3 - \bar{D}_4$ strings gives rise to the DBI action for the $D_4 - \overline{D_4}$ branes in the D_3 background:

$$S_{flavor}(A^{L,R}_\mu, T) \qquad \longleftrightarrow \qquad S_{DBI}(A^{L,R}_\mu, T) \qquad \text{holographically}$$

• This is the tachyon effective action discussed in flat space by A. Sen and others. Its implementation in asymptotically AdS space-times is the appropriate setup for fundamental flavor dynamics

Casero+Kiritsis+Paredes (07)

• In the "vacuum" only T may have a non-trivial profile: $T^{I\bar{J}}(r)$. Near an asymptotically AdS_5 boundary ($r \to 0$)

$$T^{I\bar{J}}(r) = M_{I\bar{J}}\, r + \cdots + \langle \bar{q}^I_L\ q^{\bar{J}}_R \rangle r^3 + \cdots$$

• If $M_{I\bar{J}} = 0$ then the theory is invariant under chiral symmetry.

• If the preferred solution has $\langle \bar{q}^I_L\ q^{\bar{J}}_R \rangle \neq 0$, then chiral symmetry is spontaneously (and dynamically) broken. As $T = 0$ is always a solution, chiral symmetry breaking is a consequence of dynamics.

• If there are no trivial flavor anomalies, with the associated 5d CS terms, then the flavor branes can have no IR boundary. the only option is tachyon condensation $T \neq 0$ $\leftrightarrow$ "chiral-symmetry breaking" (holographic Coleman-Witten theorem)

Casero+Kiritsis+Paredes (07)

Sen's tachyon DBI action

• The flavor action is the (coinciding) $D_4 - \bar{D}_4$ action:

$$S[T,A^L,A^R] = S_{DBI} + S_{WZ}$$

$$S_{DBI} = \int drd^4x\, \frac{N_c}{\lambda}\, \mathbf{Str}\left[V(T)\left(\sqrt{-\det\left(g_{\mu\nu} + D_{\{\mu}T^\dagger D_{\nu\}}T + F^L_{\mu\nu}\right)} + \right.\right.$$

$$\left.\left.+\sqrt{-\det\left(g_{\mu\nu} + D_{\{\mu}T^\dagger D_{\nu\}}T + F^R_{\mu\nu}\right)}\right)\right]$$

$$D_\mu T \equiv \partial_\mu T - iTA^L_\mu + iA^R_\mu T \quad , \quad D_\mu T^\dagger \equiv \partial_\mu T^\dagger - iA^L_\mu T^\dagger + iT^\dagger A^R_\mu$$

transforming covariantly under flavor gauge transformations

$$T \to V_R\, T\, V_L^\dagger \quad , \quad A^L \to V_L(A^L - iV_L^\dagger dV_L)V_L^\dagger \quad , \quad A^R \to V_R(A^R - iV_R^\dagger dV_R)V_R^\dagger$$

• For the vacuum structure and spectrum $Str = Tr$.

• The tachyon potential in flat space can be computed from boundary CFT.

A simple glue background

Take a simple non-critical confining background:

$$S = \int d^6x \sqrt{g_{(6)}} \left[e^{-2\phi} \left(\mathcal{R} + 4(\partial\phi)^2 + \frac{c}{\alpha'} \right) - \frac{1}{2}\frac{1}{6!} F_{(6)}^2 \right] ,$$

Consider the AdS_6 soliton, a solution of non-critical string theory

$$ds_6^2 = \frac{R^2}{z^2} \left[dx_{1,3}^2 + f_\Lambda^{-1} dz^2 + f_\Lambda \, d\eta^2 \right] \quad , \quad f_\Lambda = 1 - \frac{z^5}{z_\Lambda^5} \quad , \quad z \in [0, z_\Lambda]$$

$$F_{(6)} = \frac{Q_c}{\ell_s} \sqrt{-g_{(6)}} \, d^6x \quad , \quad e^\phi = \frac{1}{Q_c} \sqrt{\frac{2c}{3}}$$

Sonnenschein+Kuperstein (04)

- η is periodic

$$\eta \sim \eta + \delta\eta \, , \qquad \delta\eta = \frac{4\pi}{5} z_\Lambda = \frac{2\pi}{M_{KK}} \, . \quad , \quad R^2 = \frac{30}{c} \ell_s^2$$

The deconfined phase

• We consider the theory at non-zero temperature by compactifying to Euclidean time t_E. When both circles t_E and η are compactified, there is a second solution competing with the thermal gas solution :

$$ds_6^2 = \frac{R^2}{z^2}\left[-f_T dt^2 + dx_3^2 + \frac{dz^2}{f_T} + d\eta^2\right] \quad , \quad f_T = 1 - \frac{z^5}{z_T^5}$$

• z_T is related to the temperature as:

$$t_E \sim t_E + \delta t_E \ , \qquad \delta t_E = \frac{4\pi}{5} z_T = \frac{1}{T} \ .$$

• There is a deconfining first order phase transition at

$$T_c = \frac{M_{KK}}{2\pi} = \frac{5}{4\pi\, z_\Lambda}$$

• For $T < T_c$, the confining solution is preferred and, conversely the black-hole solution dominates for $T > T_c$.

A "warmup" bottom-up model of flavor

• We consider N_f $D_4 + \bar{D}_4$ branes at a fixed η, and we will neglect the coordinate of the branes transverse to the η circle.

• We will take $T = \tau \cdot 1$

$$V = \mathcal{K}\, e^{-\frac{\mu^2}{2}\tau^2}$$

$$\mathbf{A}_{MN} = g_{MN} + \frac{2\pi\ell_s^2}{g_V^2} F^{(i)}_{MN} + \pi\ell_s^2 \lambda \left((D_M T)^*(D_N T) + (D_N T)^*(D_M T)\right)$$

• Parameters: R, z_Λ, ℓ_s, g_V, λ, $\mathcal{K}$, μ and β.

$$m_q = \beta\; c_1 \quad , \quad \tau(z) \sim c_1\; z + \mathcal{O}(z^3)$$

μ can be eliminated by redefining τ , and we also have

$$\frac{R^2\mu^2}{2\pi\ell_s^2\lambda} = 3 \quad , \quad \frac{(2\pi\ell_s^2)^2\mathcal{K}R}{g_V^4} = \frac{N_c}{12\pi^2} \quad , \quad \frac{(2\pi\ell_s^2)^2\mathcal{K}R^2\lambda}{\beta^2} = \frac{N_c}{8\pi^2}$$

• We are left with 2+1 parameters that affect the spectra, decay constants and vacuum structure: z_Λ, m_q and $k = \frac{4R^4 g_V^4}{3(2\pi\ell_s^2)^2}$

The chiral vacuum structure

- Set $A_L = A_R = 0$ and derive the scalar $\tau(r)$ equation:

$$\tau'' - \frac{4\pi\, z\, f_\Lambda}{3}\tau'^3 + (-\frac{3}{z} + \frac{f'_\Lambda}{2f_\Lambda})\tau' + \left(\frac{3}{z^2 f_\Lambda} + \pi\,\tau'^2\right)\tau = 0$$

- Near the boundary $z = 0$, the solution can be expanded in terms of two integration constants as:

$$\tau = c_1 z + \frac{\pi}{6} c_1^3 z^3 \log z + c_3 z^3 + \mathcal{O}(z^5)$$

- c_1, c_3 are related to the quark mass and condensate.

- At the tip of the cigar, the generic behavior of solutions is

$$\tau \sim constant_1 + constant_2 \sqrt{z - z_\Lambda}$$

- With special tuned condition there is a one-parameter family of diverging solutions in the IR depending on a single parameter:

$$\tau = \frac{C}{(z_\Lambda - z)^{\frac{3}{20}}} - \frac{13}{6\pi C}(z_\Lambda - z)^{\frac{3}{20}} + \ldots$$

• This is the correct "regularity condition" in the IR as τ is allowed to diverge only at the tip. This is implied by the holographic Coleman-Witten theorem and indicates that the brane-antibrane pair "fuses" at the IR tip.

• To obtain it we must correlate the condensate c_3 to the mass c_1.

• There are always two values of c_3 for a given c_1 that reach the proper solution in the IR, and have opposite signs.

• One of them is always unstable (negative fluctuation masses^2) and is therefore discarded.

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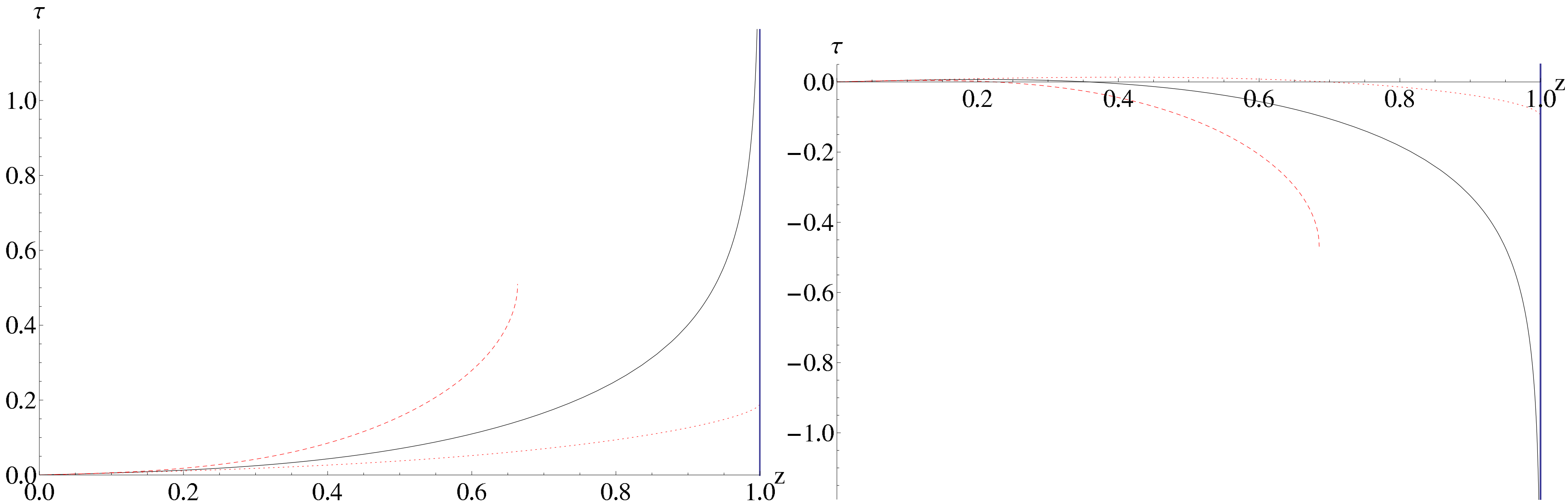
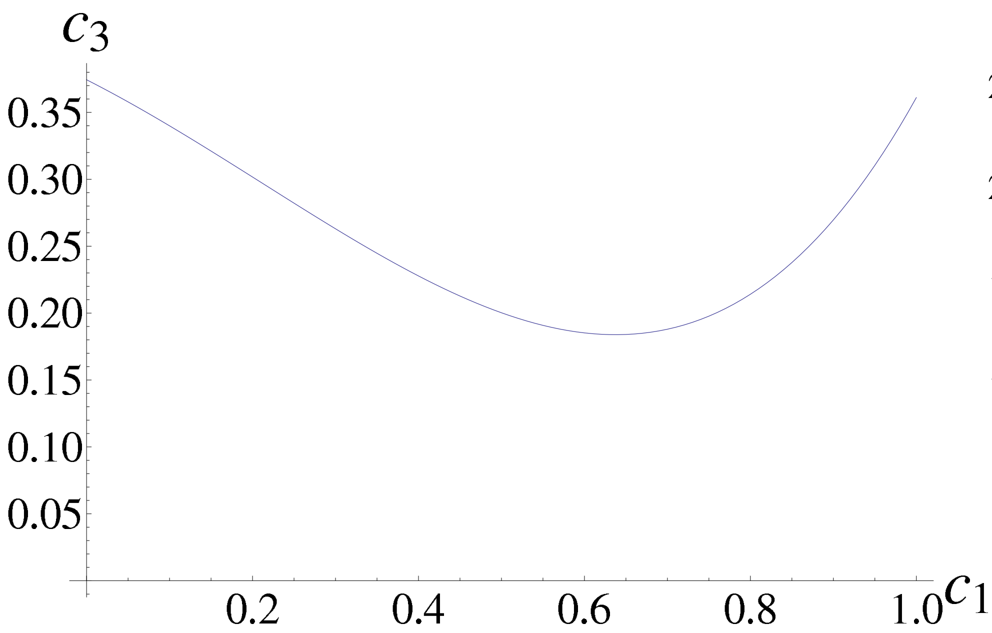
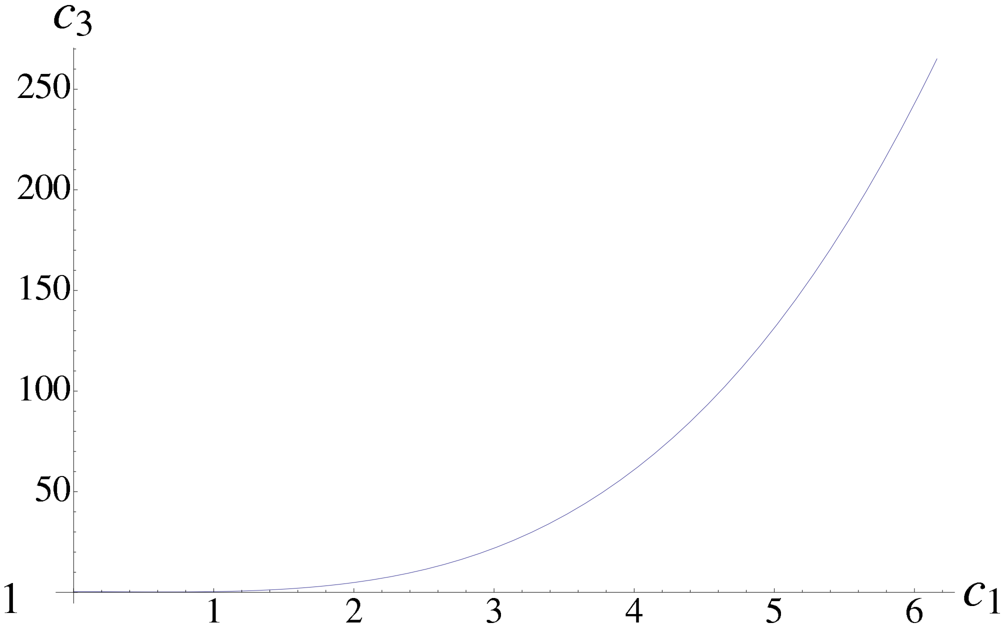
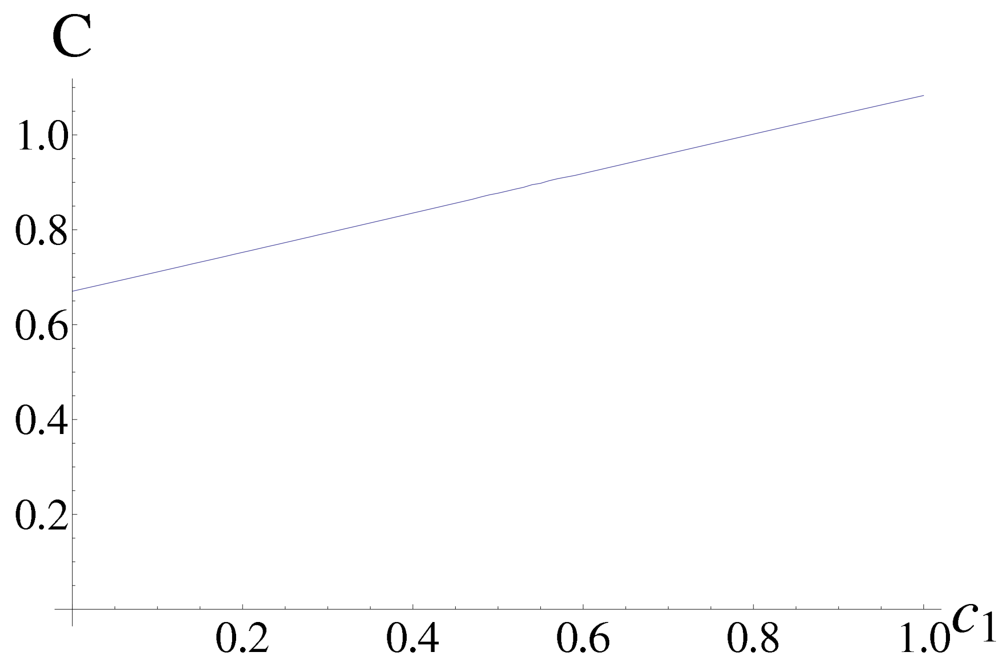


All the graphs are plotted using $z_\Lambda = 1$, $\mu^2 = \pi$ and $c_1 = 0.05$. The tip of the cigar is at $z = z_\Lambda = 1$. On the left, the solid black line represents a solution with $c_3 \approx 0.3579$ for which τ diverges at z_Λ. The red dashed line has a too large c_3 ($c_3 = 1$) - such that there is a singularity at $z = z_s$ where $\partial_z \tau$ diverges while τ stays finite. This is unacceptable since the solution stops at $z = z_s$ where the energy density of the flavor branes diverges. The red dotted line corresponds to $c_3 = 0.1$; this kind of solution is discarded because the tachyon stays finite everywhere. The plot in the right is done with the same conventions but with negative values of $c_3 = -0.1, -0.3893, -1$. For $c_3 \approx -0.3893$ there is a solution of the differential equation such that τ diverges to $-\infty$. This solution is unstable.







- Chiral symmetry breaking is manifest.

Chiral restauration at deconfinement

• In the deconfined phase, the bulk metric is that of a bh, and the tachyon equation becomes

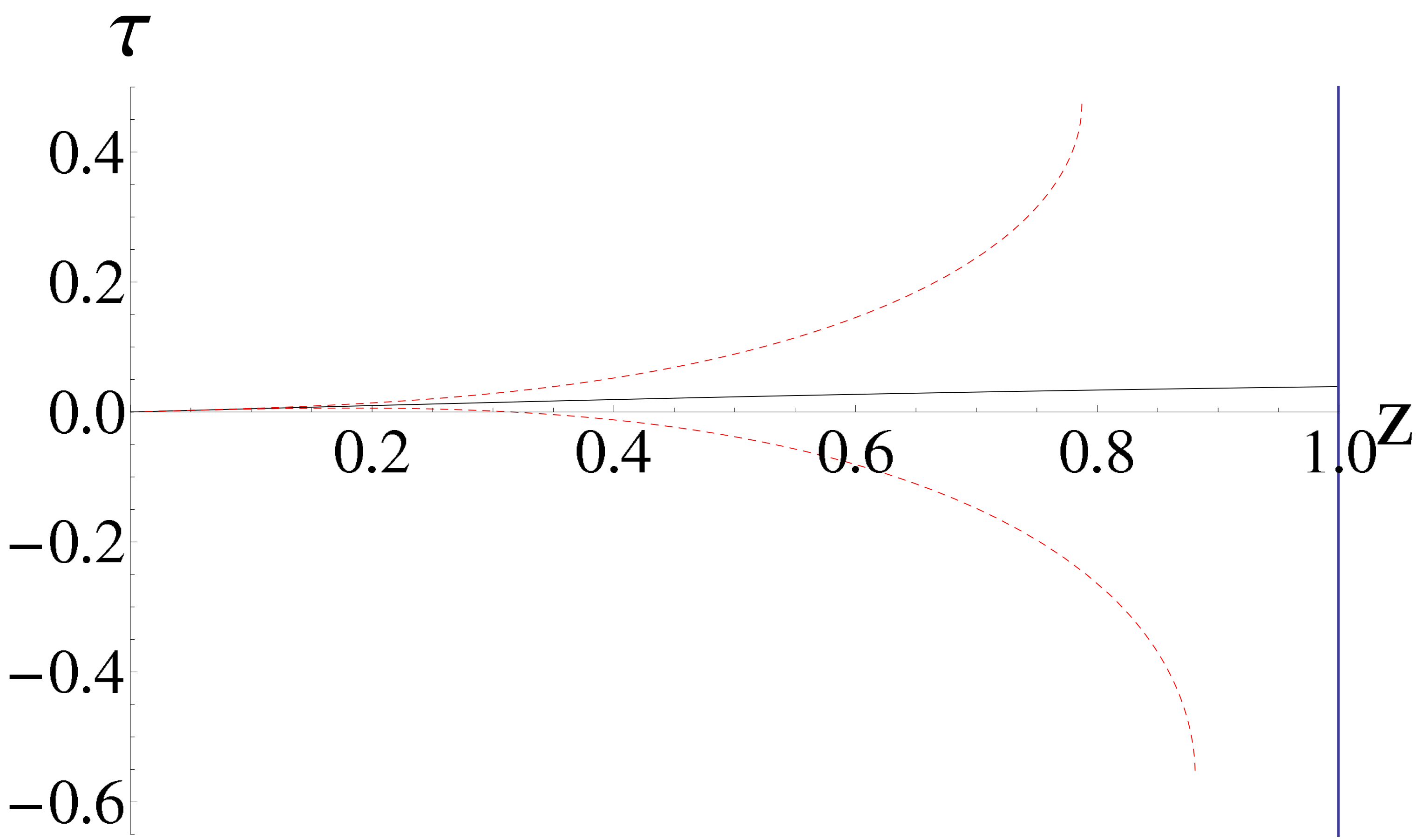
$$\tau'' + \frac{\mu^2 z^2 f_T}{3}\tau'^3\left(-\frac{4}{z}+\frac{f'_T}{2f_T}\right) + (-\frac{3}{z}+\frac{f'_T}{f_T})\tau' + \left(\frac{3}{z^2 f_T}+\mu^2\tau'^2\right)\tau = 0$$

• The branes now are allowed to enter the horizon without recombining.

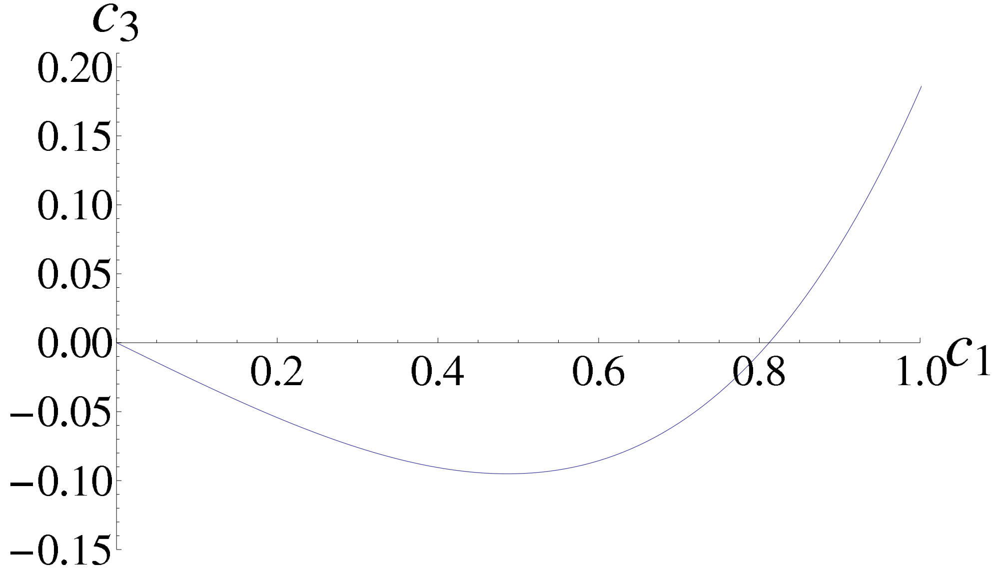
• To avoid intermediate singularities of the solution the boundary conditions must be tuned so that tachyon is finite at the horizon.

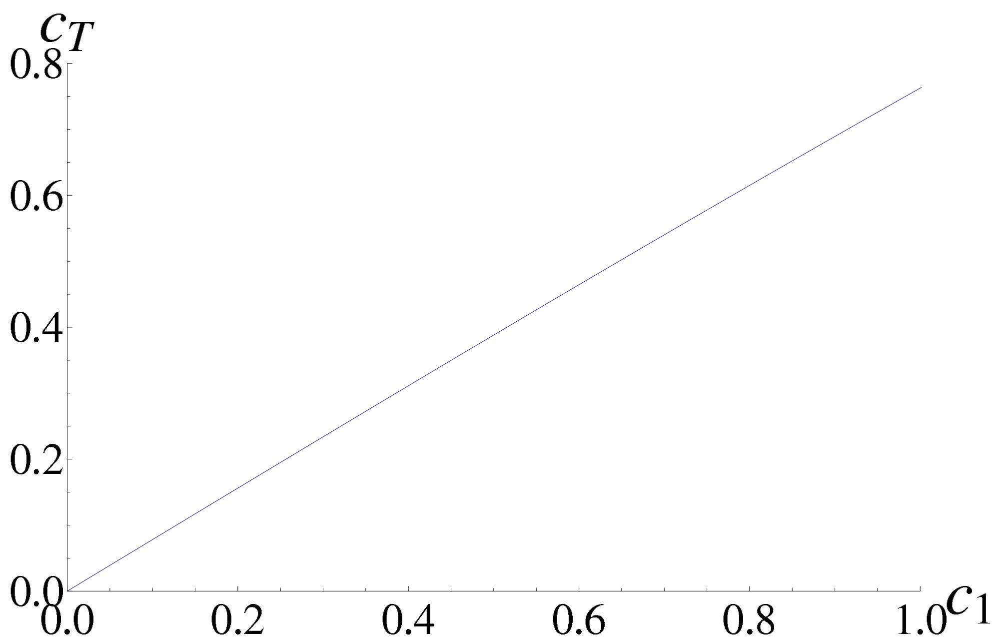
• Near the horizon the correct solution behaves as a one-parameter family

$$\tau = c_T - \frac{3c_T}{5z_T}(z_T - z) - \frac{9c_T}{200z_T}(8+\mu^2 {c_T}^2)(z_T - z)^2 + \ldots$$



Plots corresponding to the deconfined phase. We have taken $c_1 = 0.05$. The solid line displays the physical solution $c_3 = -0.0143$ whereas the dashed lines ($c_3 = -0.5$ and $c_3 = 0.5$) are unphysical and end with a behavior of the type $\tau = k_1 - k_2\sqrt{z_s - z}$.





These plots give the values of c_3 and c_T determined numerically by demanding the correct IR behavior of the solution, as a function of c_1.

Jump of the condensate at the phase transition

- From holographic renormalization we obtain

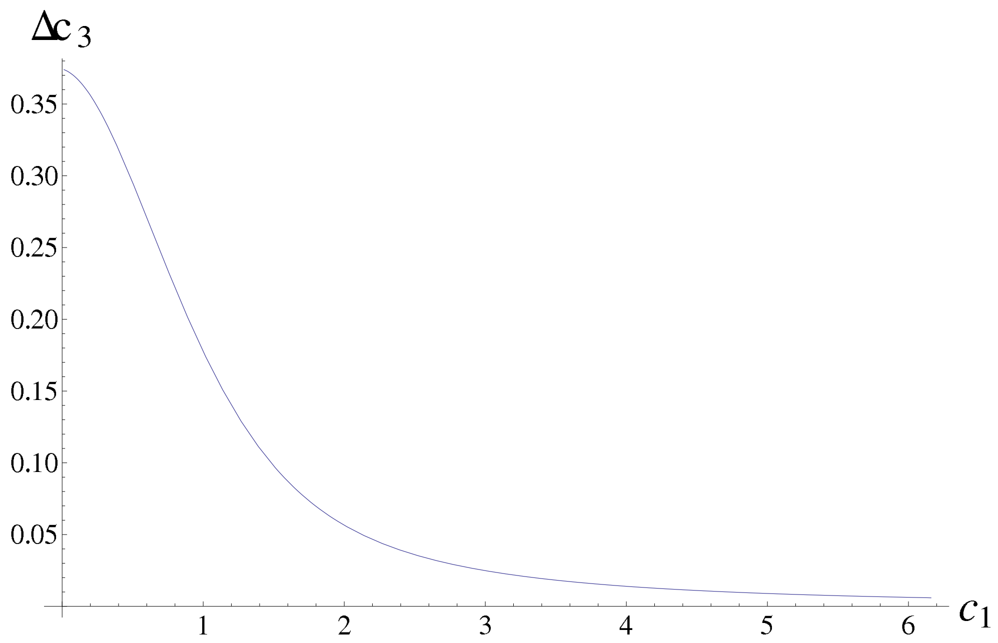
$$\langle \bar{q}q \rangle = \frac{1}{\beta}(2\pi\alpha' \mathcal{K} R^3 \lambda)\left(-4 c_3 + \left(\frac{m_q}{\beta}\right)^3 \mu^2 (1+\alpha) \right) \quad , \quad m_q = \beta \; c_1$$

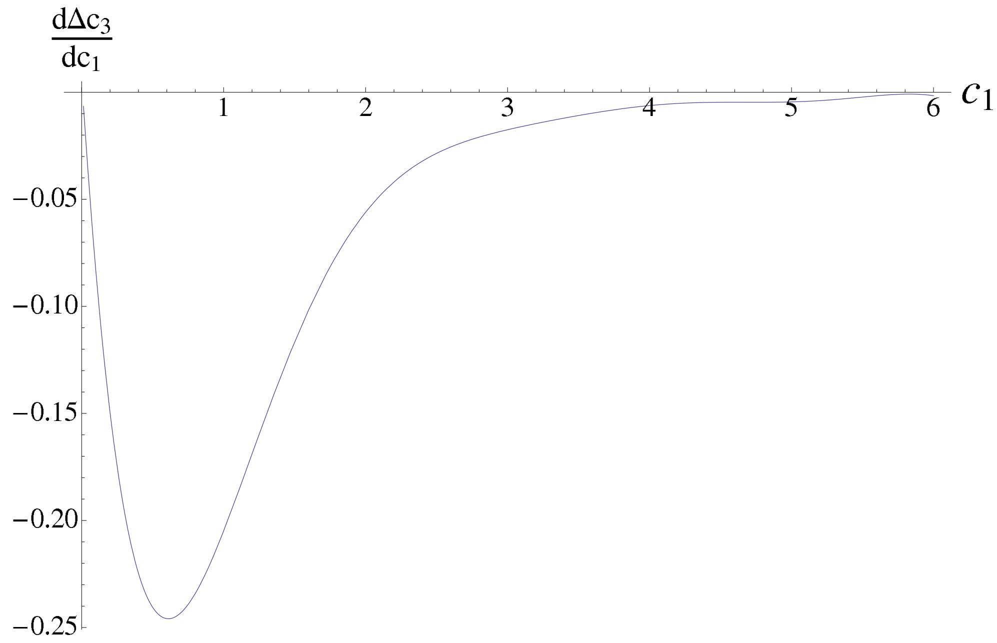
- We calculate the jump at the phase transition that is scheme independent for a fixed quark mass.

$$\Delta \langle \bar{q}q \rangle \equiv \langle \bar{q}q \rangle_{conf} - \langle \bar{q}q \rangle_{deconf} = -4 \frac{1}{\beta}(2\pi\alpha' \mathcal{K} R^3 \lambda) \Delta c_3$$

- This is equivalent to Δc_3

• We plot it as a function of the quark mass.



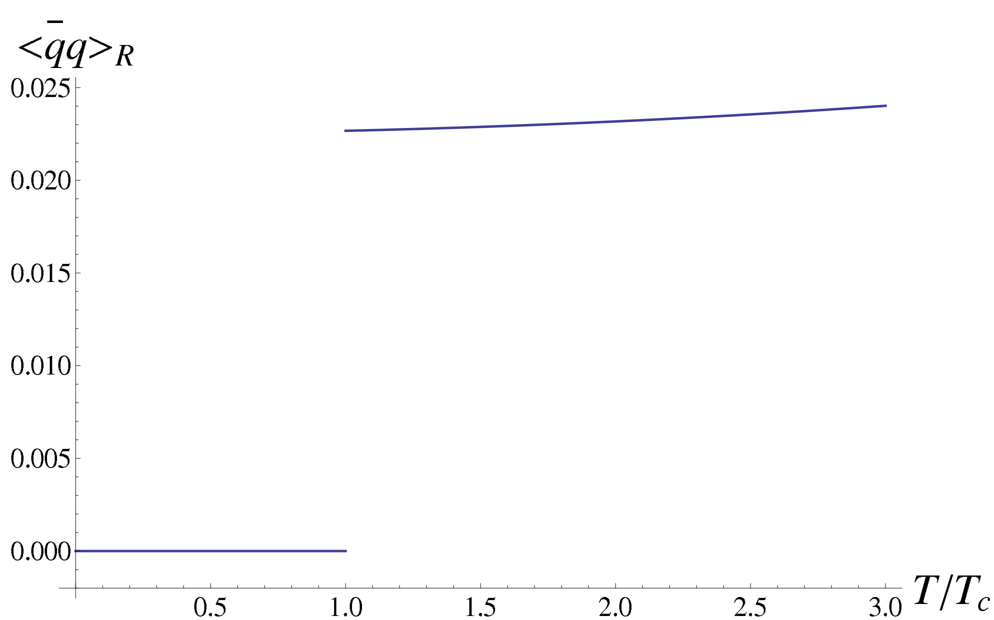


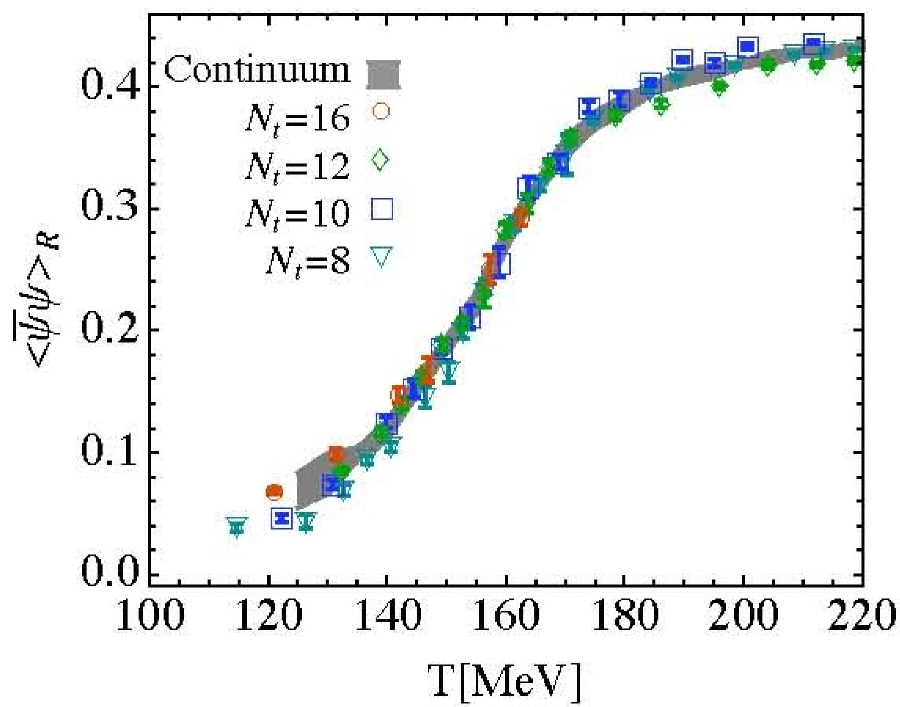
The finite jump of the quark condensate and its derivative with respect to c_1 when the confinement-deconfinement transition takes place. The important features appear when $m_q \sim \Lambda_{QCD}$

• Another interesting quantity is

$$\langle\bar{q}q\rangle_R = \frac{m_q}{T_c^4}(\langle\bar{q}q\rangle_T - \langle\bar{q}q\rangle_0) \approx N_c\frac{m_q}{T_c^4}(0.3\beta\, T_c^3 + 0.09 m_q T^2)\ , \qquad (T > T_c)$$

that tracks the T-dependence of the condensate.





We have taken $\beta = 1$, $m_q/T_c = 1/40$ for the plot.

S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, C. Ratti, K. K. Szabo [Wuppertal-Budapest Collaboration], [ArXiv:1005.3508][hep-lat].

Meson spectra

For the vectors

$$z_\Lambda m_V^{(1)} = 1.45 + 0.718c_1 , \quad z_\Lambda m_V^{(2)} = 2.64 + 0.594c_1 , \quad z_\Lambda m_V^{(3)} = 3.45 + 0.581c_1 ,$$
$$z_\Lambda m_V^{(4)} = 4.13 + 0.578c_1 , \quad z_\Lambda m_V^{(5)} = 4.72 + 0.577c_1 , \quad z_\Lambda m_V^{(6)} = 5.25 + 0.576c_1 .$$

For the axial vectors:

$$z_\Lambda m_A^{(1)} \approx 2.05 + 1.46c_1 , \quad z_\Lambda m_A^{(2)} \approx 3.47 + 1.24c_1 , \quad z_\Lambda m_A^{(3)} \approx 4.54 + 1.17c_1 ,$$
$$z_\Lambda m_A^{(4)} \approx 5.44 + 1.13c_1 , \quad z_\Lambda m_A^{(5)} \approx 6.23 + 1.11c_1 , \quad z_\Lambda m_A^{(6)} \approx 6.95 + 1.10c_1 .$$

For the pseudoscalars:

$$z_\Lambda m_P^{(1)} \approx \sqrt{3.53c_1^2 + 6.33c_1} , \quad z_\Lambda m_P^{(2)} \approx 2.91 + 1.40c_1 , \quad z_\Lambda m_P^{(3)} \approx 4.07 + 1.27c_1 ,$$
$$z_\Lambda m_P^{(4)} \approx 5.04 + 1.21c_1 , \quad z_\Lambda m_P^{(5)} \approx 5.87 + 1.17c_1 , \quad z_\Lambda m_P^{(6)} \approx 6.62 + 1.15c_1 .$$

For the scalars:

$$z_\Lambda m_S^{(1)} = 2.47 + 0.683c_1 , \quad z_\Lambda m_S^{(2)} = 3.73 + 0.488c_1 , \quad z_\Lambda m_S^{(3)} = 4.41 + 0.507c_1 ,$$
$$z_\Lambda m_S^{(4)} = 4.99 + 0.519c_1 , \quad z_\Lambda m_S^{(5)} = 5.50 + 0.536c_1 , \quad z_\Lambda m_S^{(6)} = 5.98 + 0.543c_1 .$$

- Valid up to $c_1 \sim 1$. For the axials and pseudo-scalars, we used $k = \frac{18}{\pi^2}$.
- In qualitative agreement with lattice results

Laerman+Schmidt., Del Debbio+Lucini+Patela+Pica, Bali+Bursa

- The GOR relation is satisfied

$$-4m_q\langle q\bar{q}\rangle = m_\pi^2 f_\pi^2$$

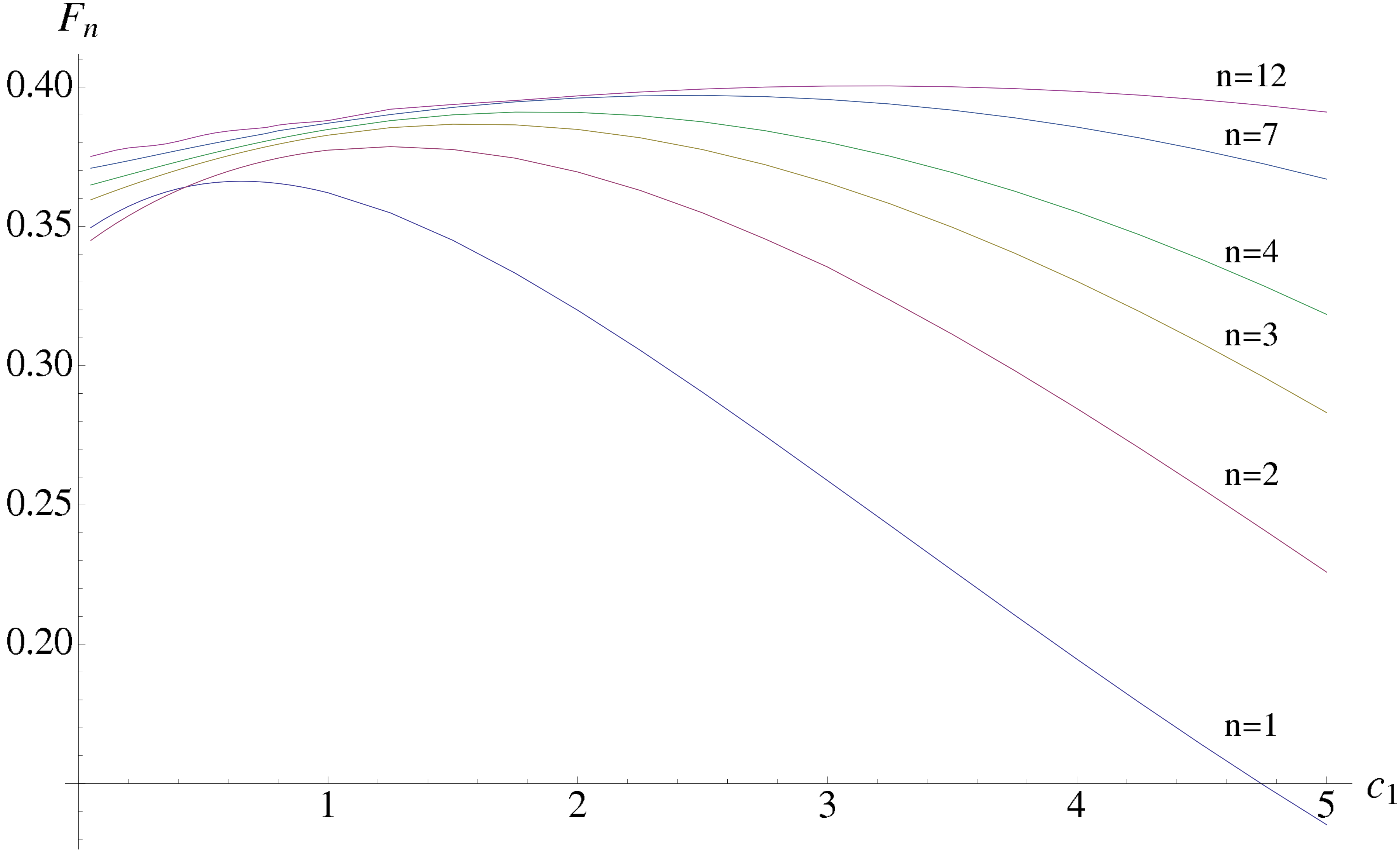
- The vector two-point function has the appropriate form

$$\int d^4x\, e^{iqx}\langle J_\mu(x)J_\nu(0)\rangle = (\eta_{\mu\nu}q^2 - q^\mu q^\nu)\Pi_V(q^2)$$

$$\Pi_V = -\frac{N_c}{12\pi^2}\left[\log\frac{q^2}{z_\Lambda^2} - 1 - \log 4 + 2\gamma - 9\frac{z_\Lambda^4}{q^4} + \cdots\right]$$

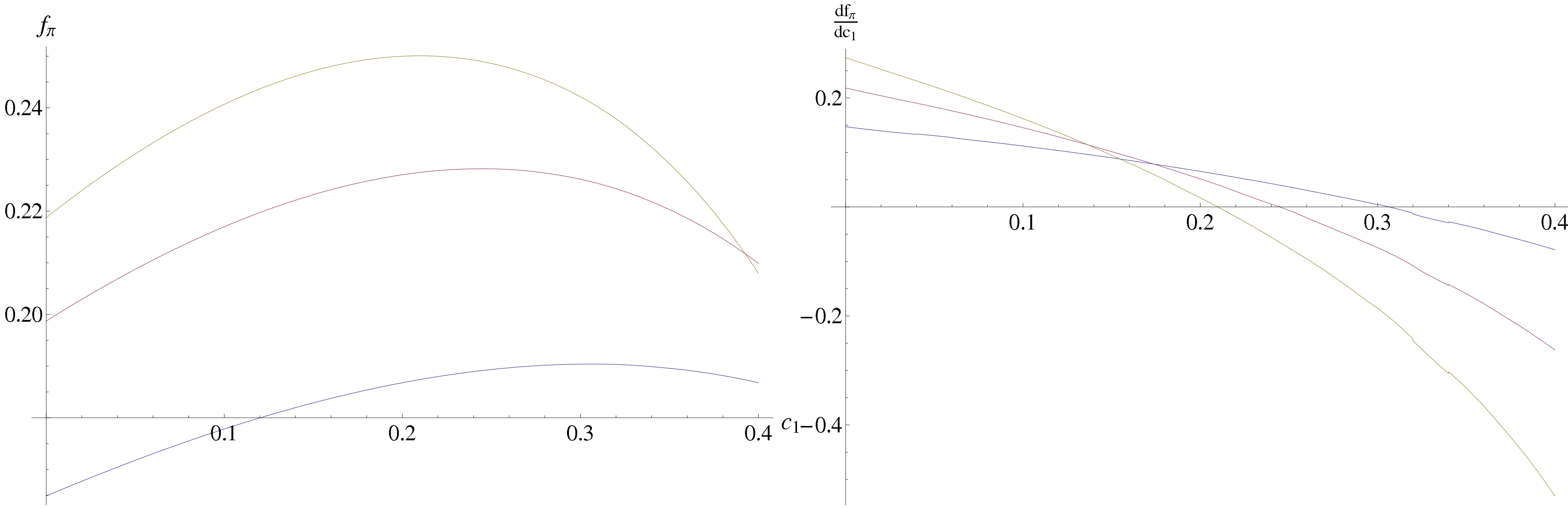
- Decay widths can be calculated from the wave-functions

$$F_n^2 = \frac{N_c}{6\pi^2}\frac{R}{m_n^2}\left(\frac{d^2\psi_V^{(n)}}{dz^2}\Big|_{z=0}\right)^2$$



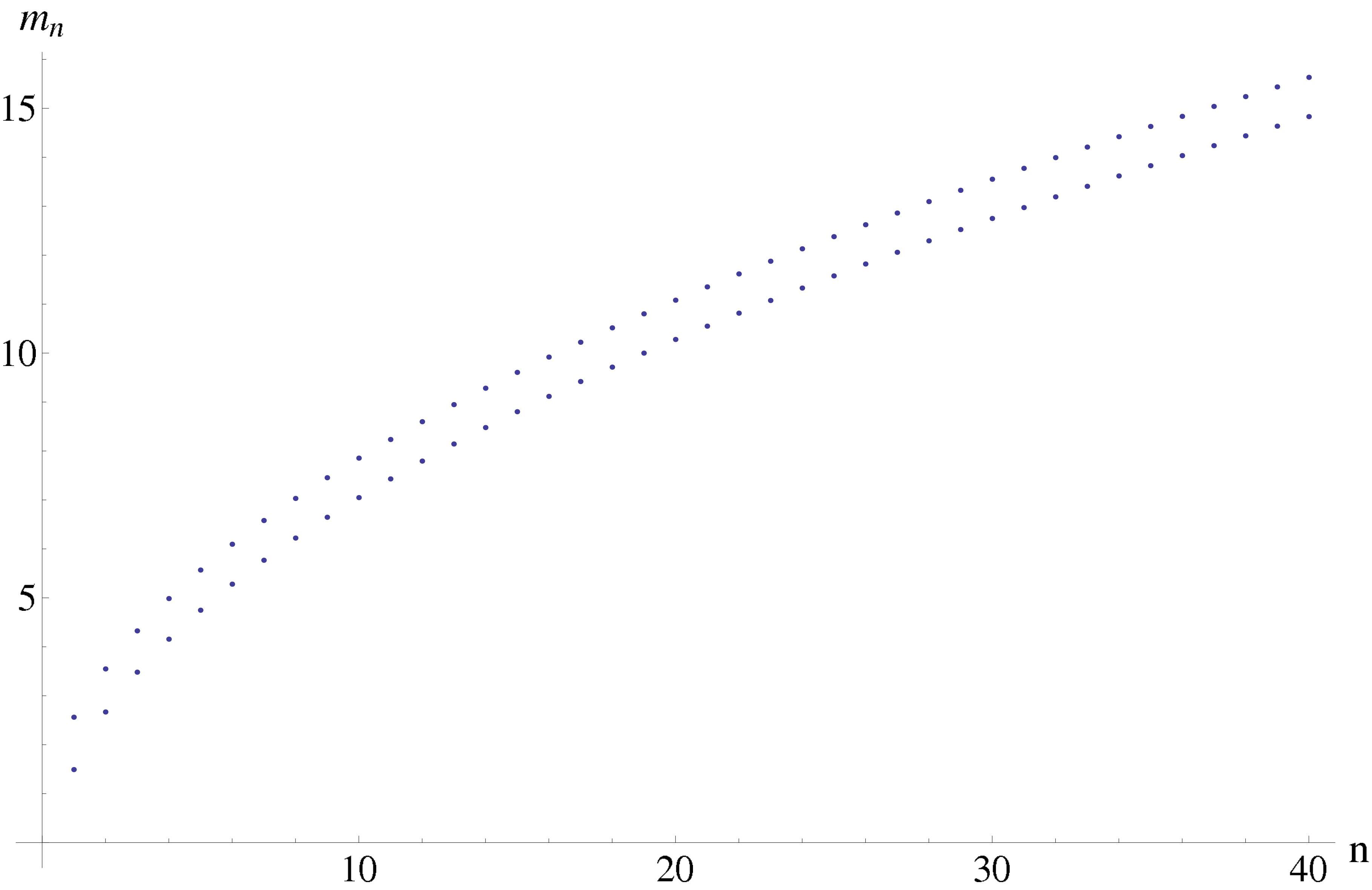
The decay constant, in units of z_Λ^{-1} for the four lowest-lying, the seventh and the twelve-th vector mode (from bottom to top), as a function of c_1. The numerical plot was made by taking $\mu^2 = \pi$ and $N_c = 3$.

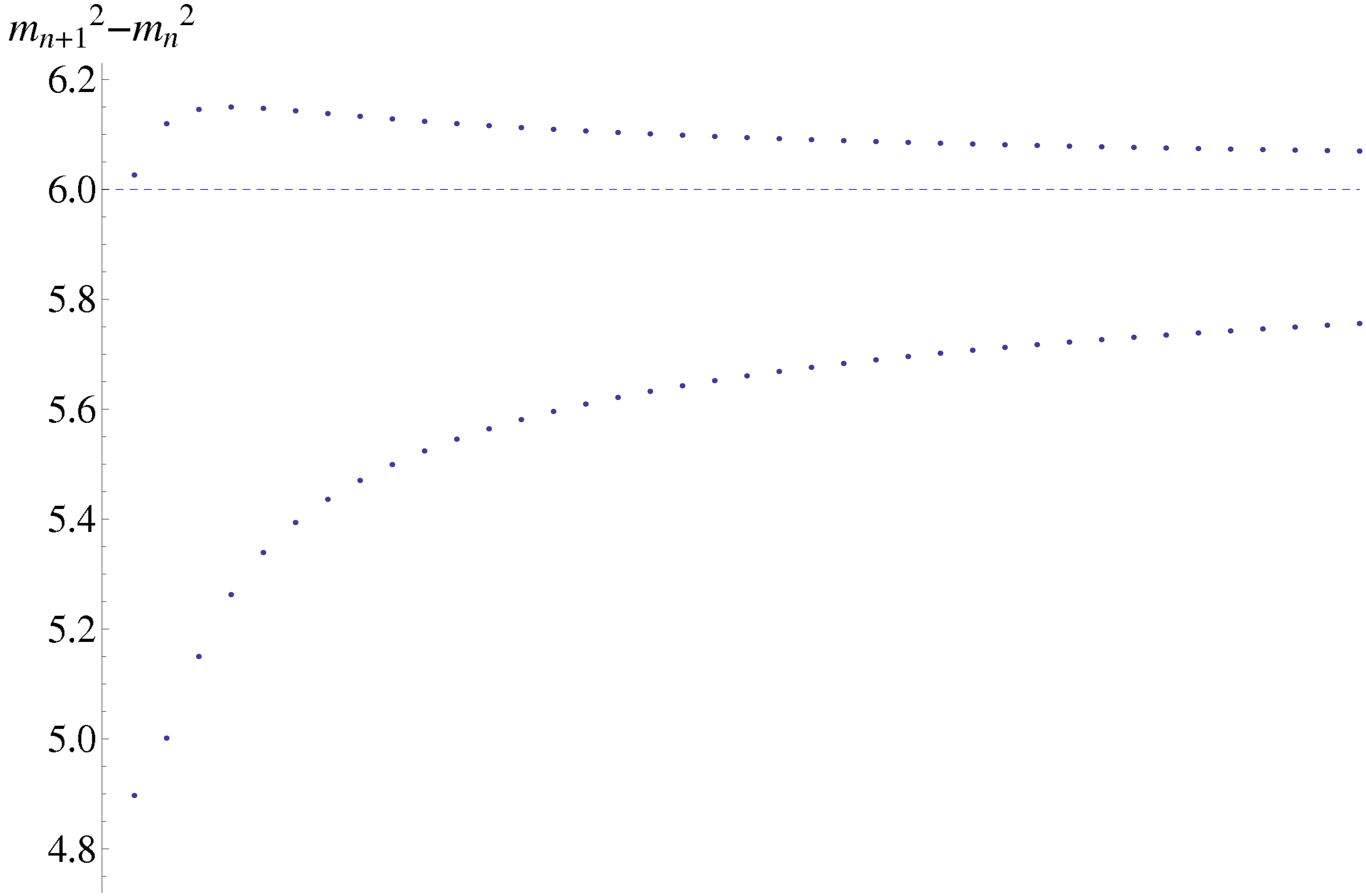
Mass dependence of f_π



The pion decay constant and its derivative as a function of c_1 - the quark mass. The different lines correspond to different values of k. From bottom to top (on the right plot, from bottom to top in the vertical axis) $k = \frac{12}{\pi^2}, \frac{24}{\pi^2}, \frac{36}{\pi^2}$. The pion decay constant comes in units of z_Λ^{-1}.

Linear Regge Trajectories





Results corresponding to the forty lightest vector states with $c_1 = 0.05$ and $c_1 = 1.5$. On the right, the horizontal line signals the asymptotic value 6 of the Regge trajectory, the lower line corresponds to $c_1 = 0.05$ and the upper line to $c_1 = 1.5$. Masses are given in units of z_Λ^{-1}. $m_{n+1}^2 - m_n^2 = \frac{6}{z_\Lambda^2} + \mathcal{O}(1/n)$.

Fit to data

We fit the three parameters to the "confirmed" isospin 1 mesons

$$z_\Lambda^{-1} = 549\ MeV \quad , \quad c_{1l} z_\Lambda = 0.0094 \quad , \quad k = \frac{18}{\pi^2}$$

minimizing

$$\epsilon_{rms} = \left(\frac{1}{n} \sum_i \left(\frac{\delta O_i}{O_i} \right)^2 \right)^{\frac{1}{2}}$$

where n is the number of the observables minus the number of the fitted parameters, $n = 9 - 3$. The rms error then is $\epsilon_{rms} = 14.5\%$

- For masses

J^{CP}	Meson	Measured (MeV)	Model (MeV)	$100\|\delta O\|/O$
1^{--}	$\rho(770)$	775	800	3.2%
	$\rho(1450)$	1465	1449	1.1%
1^{++}	$a_1(1260)$	1230	1135	7.8%
0^{-+}	π_0	135.0	134.2	0.5%
	$\pi(1300)$	1300	1603	23.2%
0^{++}	$a_0(1450)$	1474	1360	7.7%

- For decay constants

J^{CP}	Meson	Measured (MeV)	Model (MeV)	$100\|\delta O\|/O$
1^{--}	$\rho(770)$	216	190	12%
1^{++}	$a_1(1260)$	216	228.5	5.8%
0^{-+}	π_0	127	101.3	20.2%

- Masses of "less confirmed mesons"

J^{PC}	Meson	Measured (MeV)	Model (MeV)
1^{--}	$\rho(2270)$	2270	2649
1^{++}	$a_1(1930)$	1930	2166
	$a_1(2096)$	2096	2591
	$a_1(2270)$	2270	2965
	$a_1(2340)$	2340	3303
0^{-+}	$\pi(2070)$	2070	2406
	$\pi(2360)$	2360	2798
0^{++}	$a_0(2020)$	2025	1883

- The RMS error here is 23%. Axial vector mesons are consistently over-estimated.

"$s\bar{s}$" states

They can be "estimated" using

$$m(\text{"}\eta\text{"}) = \sqrt{2m_K^2 - m_\pi^2} \quad , \quad m(\text{"}\phi(1020)\text{"}) = 2m(K^*(892)) - m(\rho(770)) \quad , \quad .$$

Allton+Gimenez+Giusti+Rapuano

J^{PC}	Meson	"Measured" (MeV)	Model (MeV)
1^{--}	"$\phi(1020)$"	1009	857
	"$\phi(1680)$"	1363	1432
1^{++}	"$f_1(1420)$"	1440	1188
0^{-+}	"η"	691	740
	"$\eta(1475)$"	1620	1608
0^{++}	"$f_0(1710)$"	1386	1365

The "mass" of the s-quark is $c_{1,s} = 0.350$. The rms error for this set of observables ($n = 6 - 1$) is $\varepsilon_{rms} = 11\%$.

- $\frac{2m_s}{m_u+m_d} \simeq \frac{c_{1,s}}{c_{1,l}} \simeq 26$

- $T_{deconf} = \frac{5}{4\pi z_\Lambda} \simeq 215 MeV$.

Steps forward

Advantages of this simple AdS/QCD-like model

• Compared to the SS model it contains all trajectories corresponding to $1^{--}, 1^{++}, 0^{-+}, 0^{++}$ and can accommodate a mass of the quarks. The asymptotic masses of mesons are $m_n^2 \sim n$, as they should.

• Compared to the soft-wall AdS/QCD model:

(a) The background glue solution is a consistent solution with proper thermodynamics.

(b) The magnetic quarks are confined instead of screened.

(c) Chiral symmetry breaking is dynamical.

(d) The mass of the ρ meson depends on the quark (or pion) mass.

(e) The finite density physics is sensitive to quark masses.

Open problems

- Derive the finite density physics.

- Investigate the baryon states and spectra.

- Explore different actions, and in particular investigate the difference of axial and vector asymptotic slopes.

- Investigate the non-abelian case involving both light and heavy quarks with mixing.

- Consider the IHQCD glue backgrounds and study the associated meson physics.

- Proceed beyond the quenched approximation for flavor.

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Thank you for your Patience

Introduction

- Chiral symmetry breaking is a central effect for the nuclear interaction

- The Large-N limit of QCD promised a new approximation scheme at strong coupling.

- In 1997 Maldacena conjectured a precise correspondence for a more symmetric cousin of YM.

There were surprises in this duality and new intuition that developed.

- The conjecture was tested in many contexts but still remains a conjecture. Few doubt it validity.

- This AdS/CFT correspondence has led to important new insights on the problems of the strong force.

- New experimental arenas are available to test strong couplings physics in the deconfined phase (at RHIC and LHC).

The glue

• There are several models to describe the dynamics of glue and important properties, like confinement, and the associated phase transition at finite temperature.

• There are top-down models, like the Witten D4 model, Klebanov-Strassler and Chamsedinne-Volkov-Maldacena-Nunez solutions that emerge from well controlled situations in string theory.

• There are also bottom up models like AdS/QCD, that are phenomenological but sometimes they can address more realistic cases.

• The state of the art for Glue (bottom-up) is Improved Holographic QCD

$$S = M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\phi)^2 + V(\phi) \right] \quad , \quad \lambda \equiv e^{\phi}$$

$$V(\lambda) = \frac{12}{\ell^2} \left[1 + c_1\lambda + c_2\lambda^2 + \cdots \right] \quad , \quad \lambda \to 0 \quad , \quad V \to \lambda^{\frac{4}{3}} \sqrt{\log\lambda} \quad , \quad \lambda \to \infty$$

• It agrees well with pure YM, both a zero and finite temperature.

Gursoy+Kiritsis+Mazzanti+Nitti, 2007-2009

YM Entropy

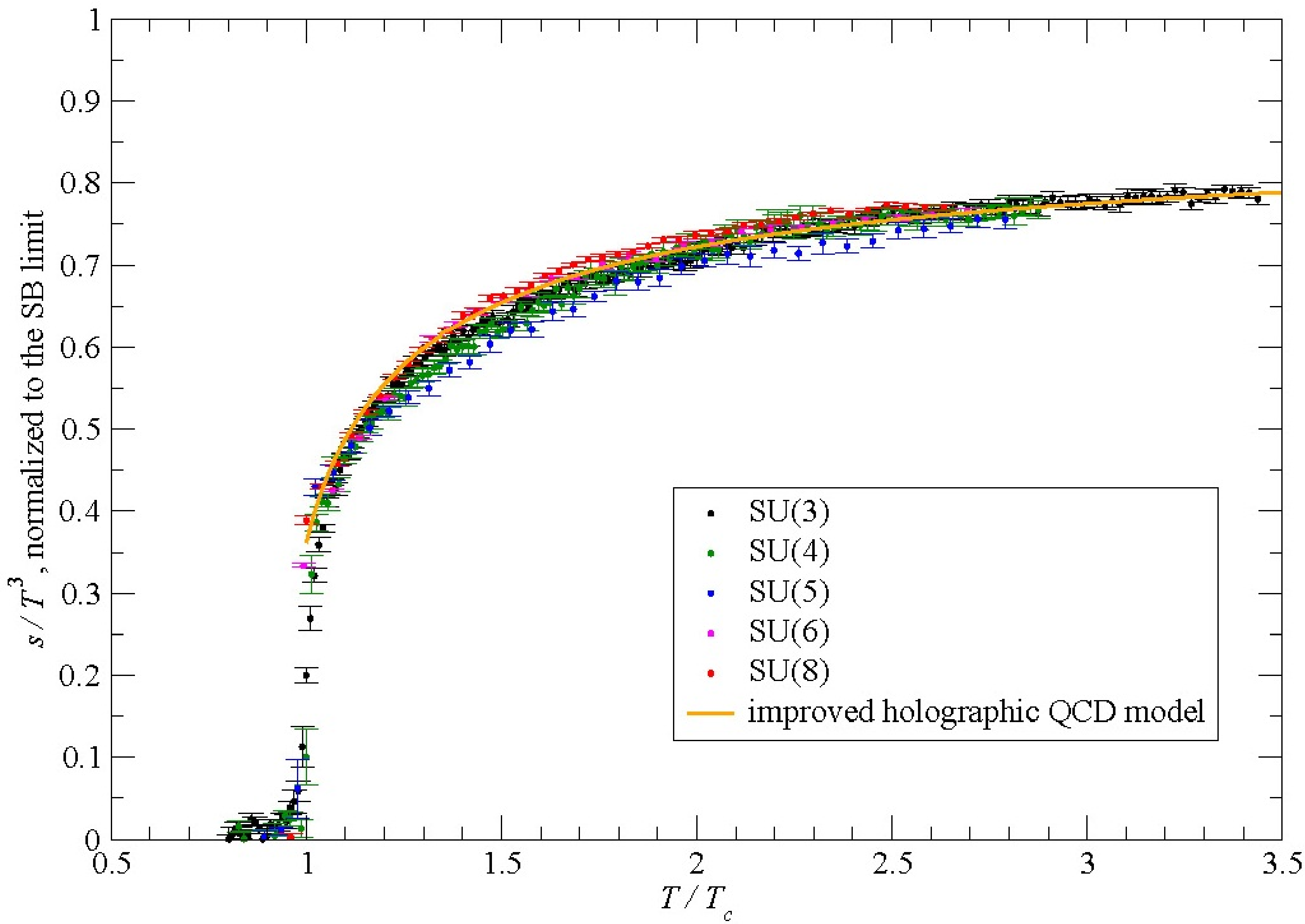


Figure 4: (Color online) Same as in fig. 1, but for the s/T^3 ratio, normalized to the SB limit.

From M. Panero, arXiv:0907.3719

YM Equation of state (interaction measure)

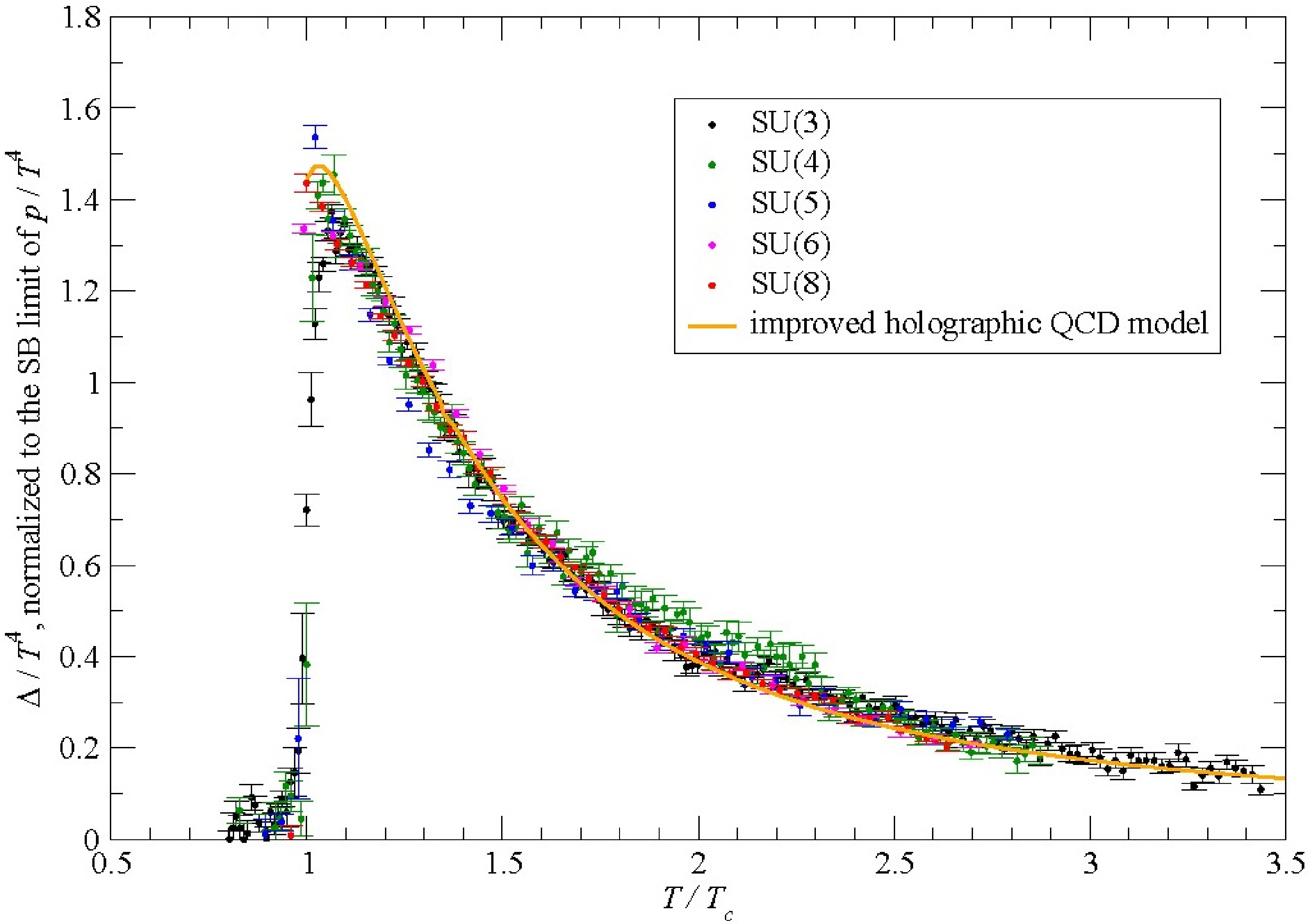


Figure 2: (Color online) Same as in fig. 1, but for the Δ/T^4 ratio, normalized to the SB limit of p/T^4.

From M. Panero, arXiv:0907.3719

The tachyon WZ action

- The WZ action is given by

Kennedy+Wilkins, Kraus+Larsen, Takayanagi+Terashima+Uesugi
Alishahiha+Ita+Oz

$$S_{WZ} = T_4 \int_{M_5} C \wedge \mathrm{Str}\ \exp\left[i2\pi\alpha' \mathcal{F}\right]$$

- M_5 is the world-volume of the D4-$\overline{\text{D4}}$ branes that coincides with the full space-time.

- C is a formal sum of the RR potentials $C = \sum_n (-i)^{\frac{5-n}{2}} C_n$,

- $\mathcal{F}$ is the curvature of a superconnection $\mathcal{A}$:

$$i\mathcal{A} = \begin{pmatrix} iA_L & T^\dagger \\ T & iA_R \end{pmatrix}, \qquad i\mathcal{F} = \begin{pmatrix} iF_L - T^\dagger T & DT^\dagger \\ DT & iF_R - TT^\dagger \end{pmatrix}$$

$$\mathcal{F} = d\mathcal{A} - i\mathcal{A} \wedge \mathcal{A} \quad , \quad d\mathcal{F} - i\mathcal{A} \wedge \mathcal{F} + i\mathcal{F} \wedge \mathcal{A} = 0$$

• Under (flavor) gauge transformation it transforms homogeneously

$$\mathcal{F} \to \begin{pmatrix} V_L & 0 \\ 0 & V_R \end{pmatrix} \mathcal{F} \begin{pmatrix} V_L^\dagger & 0 \\ 0 & V_R^\dagger \end{pmatrix}$$

• Expanding:

$$S_{WZ} = T_4 \int C_5 \wedge Z_0 + C_3 \wedge Z_2 + C_1 \wedge Z_4 + C_{-1} \wedge Z_6$$

where Z_{2n} are appropriate forms coming from the expansion of the exponential of the superconnection.

• $Z_0 = 0$, signaling the global cancelation of 4-brane charge, which is equivalent to the cancelation of the gauge anomaly in QCD.

$$Z_2 = d\Omega_1 \quad , \quad \Omega_1 = iSTr(V(T^\dagger T))Tr(A_L - A_R) - \log\det(T) d(StrV(T^\dagger T))$$

Casero+Kiritsis+Paredes (07)

• This term provides the Stuckelberg mixing between $Tr[A^L_\mu - A^R_\mu]$ and the QCD axion that is dual to C_3. Dualizing the full action we obtain:

$$S_{CP-odd} = \frac{M^3}{2N_c^2}\int d^5x\sqrt{g}Z(\lambda)\left(\partial a + i\Omega_1\right)^2$$

$$= \frac{M^3}{2N_c^2}\int d^5x\sqrt{g}Z(\lambda)\left(\partial_\mu a + \zeta\partial_\mu V(\tau) - \sqrt{\frac{N_f}{2}}V(\tau)A_\mu^A\right)^2$$

$$\zeta = \Im \log\det T \quad , \quad A_L - A_R \equiv \frac{1}{2N_f}A^A\mathbf{I} + (A_L^a - A_R^a)\lambda^a$$

• This term is invariant under the $U(1)_A$ transformations, reflecting the QCD $U(1)_A$ anomaly.

$$\zeta \to \zeta + \epsilon \quad , \quad A_\mu^A \to A_\mu^A - \sqrt{\frac{2}{N_f}}\partial_\mu\epsilon \quad , \quad a \to a - N_f\epsilon V(\tau)$$

• This is responsible for the mixing between the QCD axion and the η' $\to$ we have two scalars a, ζ and an (axial) vector, A_μ^A. Then an appropriate linear combination of the two scalars will become the 0^{-+} glueball field while the other will be the η'. The transverse (5d) vector will provide the tower of $U(1)_A$ vector mesons.

• The term $C_1 \times Z_4 \sim V(\tau)\ C_1\left[F_L\wedge F_L + F_R\wedge F_R\right] + \cdots$ couples the flavor instanton density to the baryon vertex.

• Using $Z_6 = d\Omega_5$ we may rewrite the last term as

$$\int F_0 \wedge \Omega_5 \quad , \quad F_0 = dC_{-1}$$

$F_0 \sim N_c$ is nothing else but the dual of the five-form field strength. This term then provides the correct Chern-Simons form that reproduces the flavor anomalies of QCD. It contains the tachyon non-trivially.

Casero+Kiritsis+Paredes (07)

• The five form Ω_5 is rather complicated and depends non-trivial on the tachyon

$$\Omega_5 = \frac{\text{tr}}{6}\exp\left[-\tau^2\right]\left\{-iA_L\wedge F_L\wedge F_L+\frac{1}{2}A_L\wedge A_L\wedge A_L\wedge F_L+i\frac{A_L\wedge A_L\wedge A_L\wedge A_L\wedge A_L}{10}+\right.$$

$$+iA_R\wedge F_R\wedge F_R-\frac{1}{2}A_R\wedge A_R\wedge A_R\wedge F_R-i\frac{A_R\wedge A_R\wedge A_R\wedge A_R\wedge A_R}{10}+\tau^2\Big[iA_L\wedge F_R\wedge F_R-$$

$$-iA_R\wedge F_L\wedge F_L+\frac{i}{2}(A_L-A_R)\wedge(F_L\wedge F_R+F_R\wedge F_L)+\frac{1}{2}A_L\wedge A_L\wedge A_L\wedge F_L-\frac{1}{2}A_R\wedge A_R\wedge A_R\wedge F_R+$$

$$+\frac{i}{10}A_L\wedge A_L\wedge A_L\wedge A_L\wedge A_L-\frac{i}{10}A_R\wedge A_R\wedge A_R\wedge A_R\wedge A_R\Big]+i\tau^3\,d\tau\wedge$$

$$\wedge\Big[(A_L\wedge A_R-A_R\wedge A_L)\wedge(F_L+F_R)+iA_L\wedge A_L\wedge A_L\wedge A_R-\frac{i}{2}A_L\wedge A_R\wedge A_L\wedge A_R+iA_L\wedge A_R\wedge A_R\wedge A_R\Big]+$$

$$\left.+\frac{i}{20}\tau^4(A_L-A_R)\wedge(A_L-A_R)\wedge(A_L-A_R)\wedge(A_L-A_R)\wedge(A_L-A_R)\right\}$$

Discrete symmetries

- Parity (P):

$$P = P_1 \cdot P_2 \quad , \quad P_1 : A_L \leftrightarrow A_R \quad , \quad P_2 : x^i \to -x^i$$

- The DBI+WZ action is invariant under parity. In particular

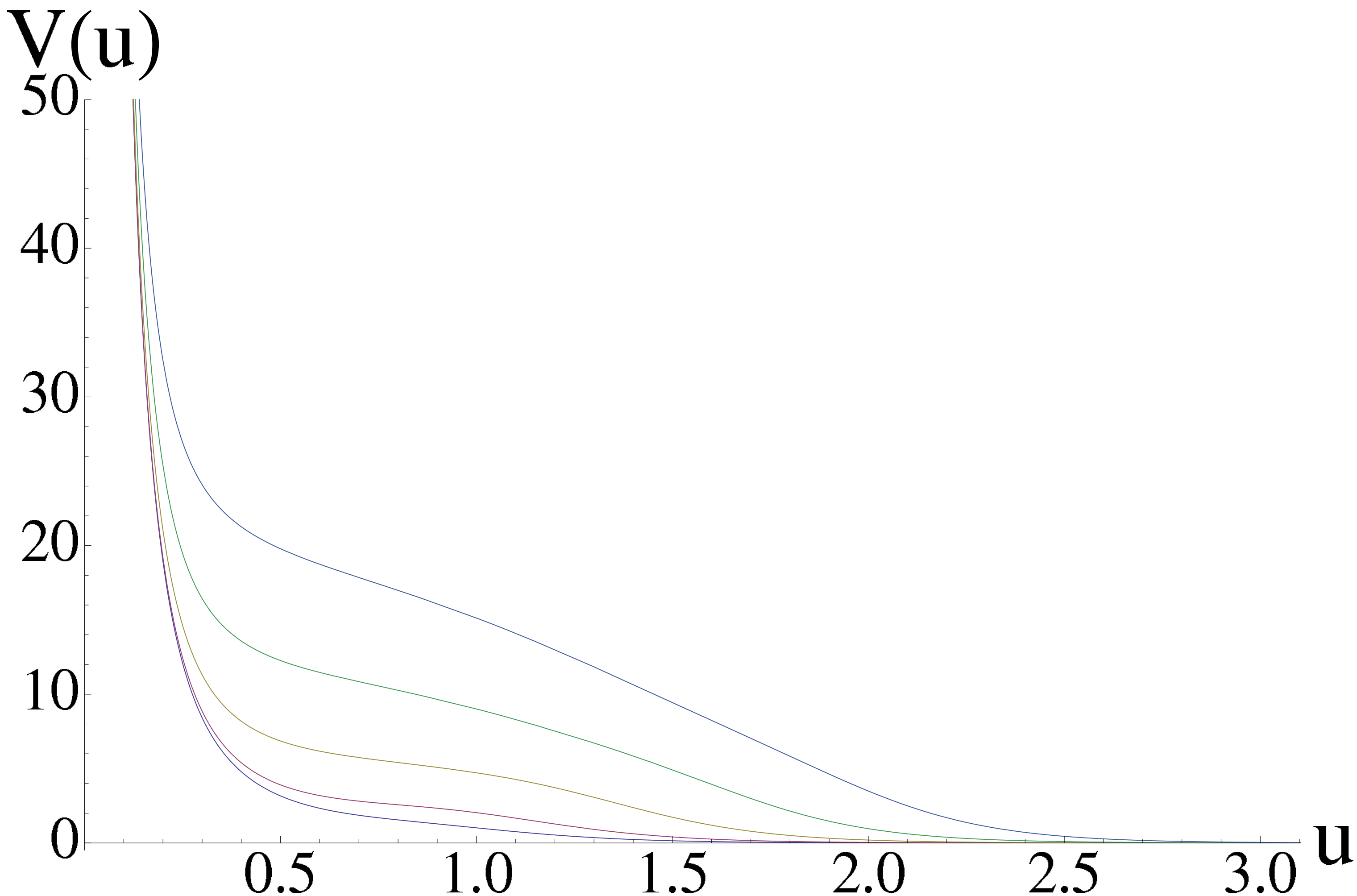
$$P \quad : \quad D_4 \leftrightarrow \bar{D}_4$$

- Charge conjugation is also a symmetry

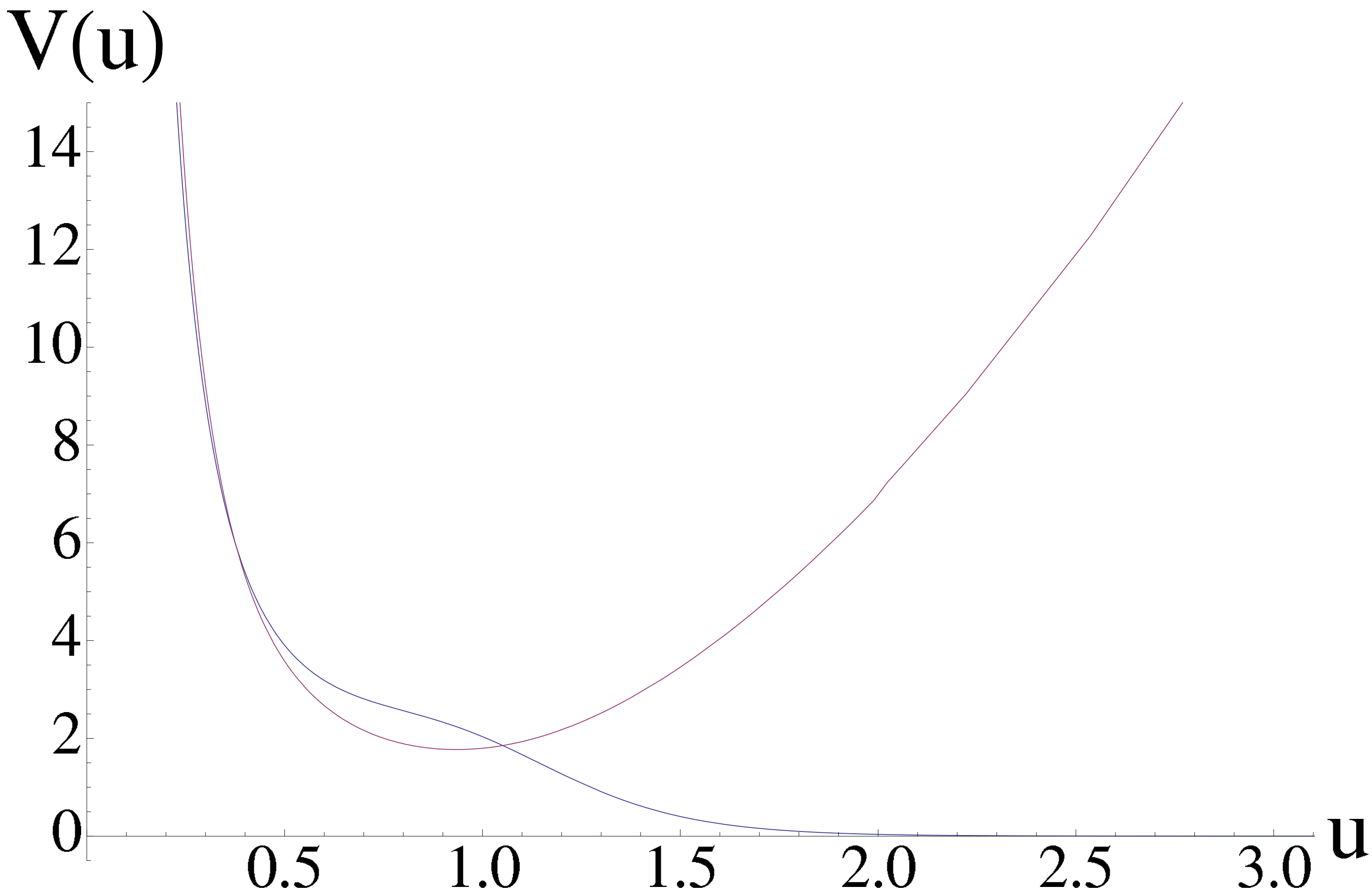
$$C : \qquad A_L \to -A_R^t \,, \quad A_R \to -A_L^t \,, \quad T \to T^t \,, \quad T^\dagger \to \left(T^\dagger\right)^t$$

World-volume field	$T + T^\dagger$	$i(T - T^\dagger)$	$\frac{(A_L + A_R)_\mu}{2}$	$\frac{(A_L - A_R)_\mu}{2}$
J^{PC}	0^{++}	0^{-+}	1^{--}	1^{++}

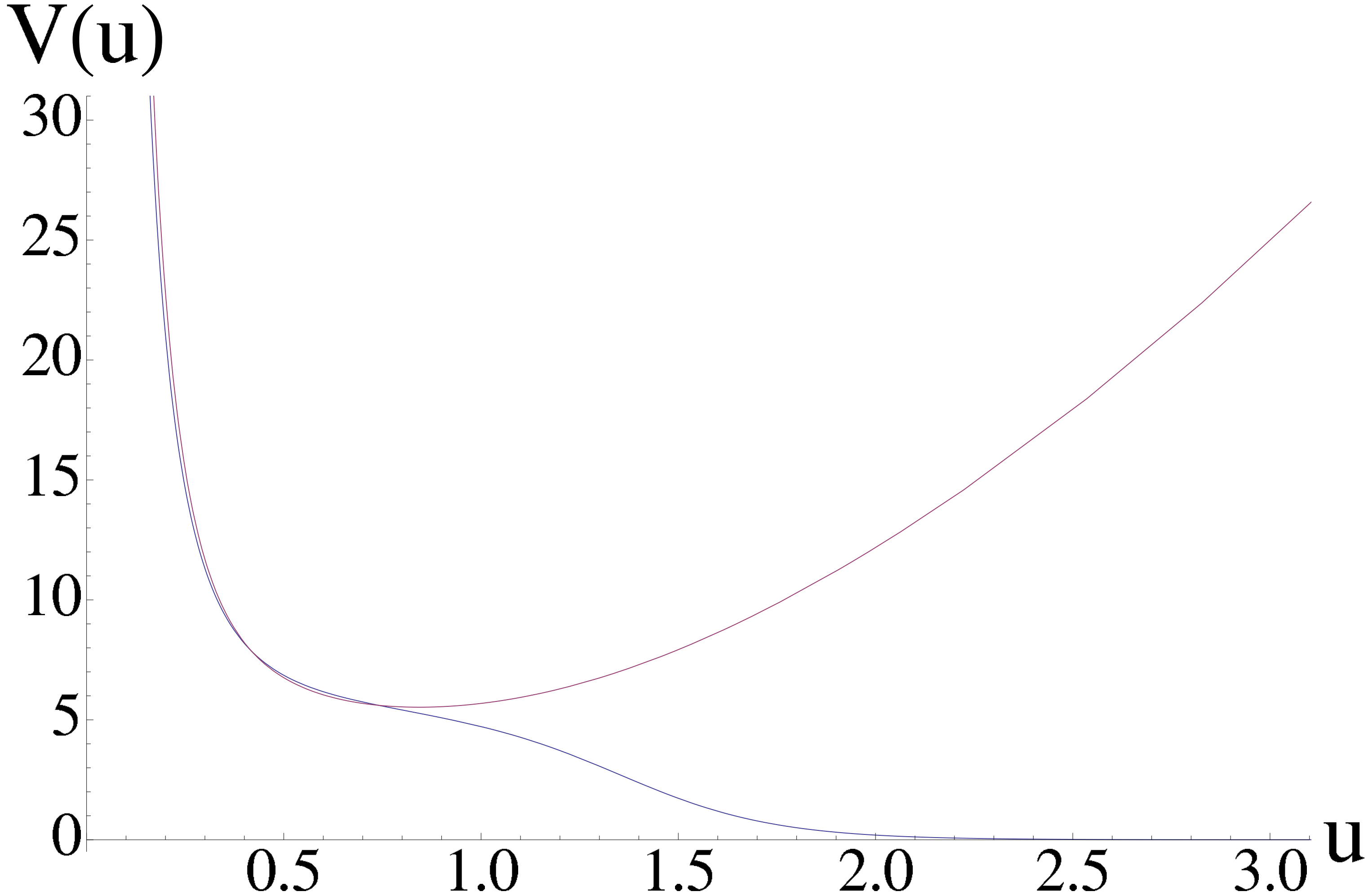
Meson melting



The Schrödinger potentials associated to the vector excitation in the deconfined phase, at zero momentum, for different values of $c_1 \sim m_q/T$. Here $c_1 = 0.01, 1, 2, 3, 4$.



Here $c_1 = 1$, we compare the potentials in the confined phase for the same values of c_1.



Here $c_1 = 2$ and we make a comparison with the potentials in the confined phase for the same values of c_1.

The gauge-theory/string-theory(gravity) duality

• The gauge-theory/gravity duality is a duality that relates a string theory with a (conformal) gauge theory.

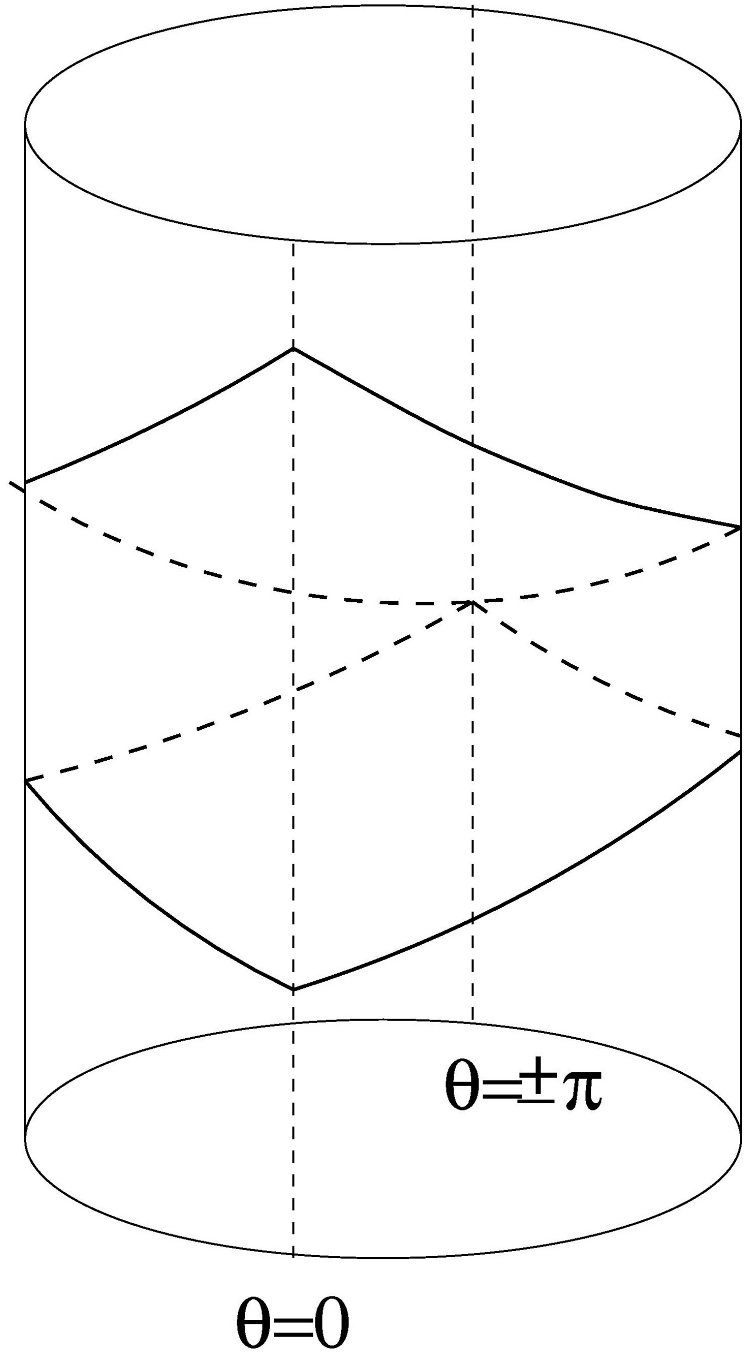
• The prime example is the AdS/CFT correspondence

Maldacena 1997

• It states that N=4 four-dimensional SU(N) gauge theory (gauge fields, 4 fermions, 6 scalars) is equivalent to ten-dimensional IIB string theory on $\mathrm{AdS}_5 \times S^5$

$$ds^2 = \frac{\ell_{AdS}^2}{r^2}\left[dr^2 + dx^\mu dx_\mu\right] + \ell_{AdS}^2\,(d\Omega_5)^2$$

• This space (AdS_5) is non-compact and has a single boundary, at $r = 0$.


θ=±π
θ=0

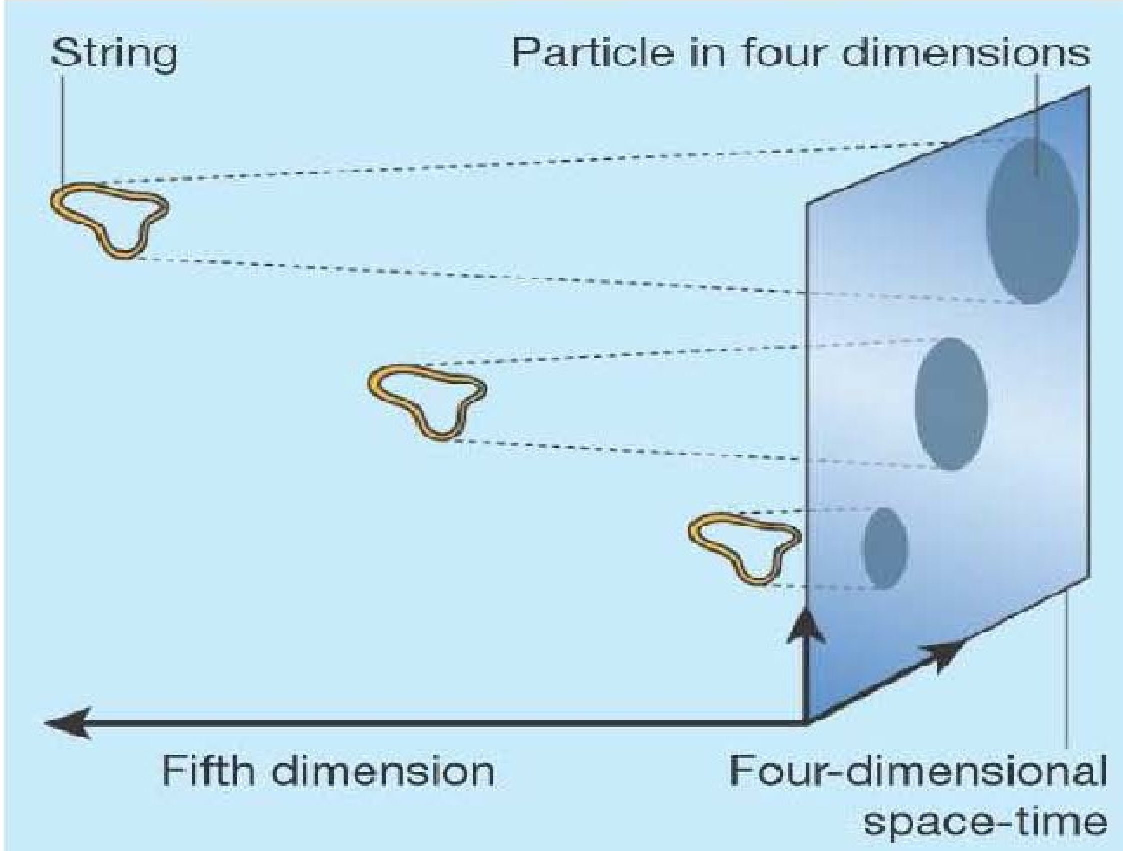
• The string theory has as parameters, g_{string}, ℓ_{string}, ℓ_{AdS}. They are related to the gauge theory parameters as

$$g_{YM}^2 = 4\pi \ g_{\text{string}} \quad , \quad \lambda = g_{YM}^2 \ N = \frac{\ell_{AdS}^4}{\ell_{\text{string}}^4}$$

• As $N \to \infty$, $g_{\text{string}} \sim \frac{\lambda}{N} \to 0$.

• As $N \to \infty$, $\lambda \gg 1$ implies that $\ell_{\text{string}} \ll \ell_{AdS}$ and the geometry is very weakly curved. String theory can be approximated by gravity in that regime and is weakly coupled.

• As $N \to \infty$, $\lambda \ll 1$ the gauge theory is weakly coupled, but the string theory is strongly curved.



• There is one-to-one correspondence between on-shell string states $\Phi(r,x^\mu)$ and gauge-invariant (single-trace) operators $O(x^\mu)$ in the sYM theory

• In the string theory we can compute the "S-matrix" , $S(\phi(x^\mu))$ by studying the response of the system to boundary conditions $\Phi(r=0,x^\mu)=\phi(x^\mu)$

• The correspondence states that this is equivalent to the generating function of c-correlators of O

$$\langle e^{\int d^4x\ \phi(x)\ O(x)}\rangle \ = \ e^{-S(\phi(x))}$$

Gubser+Klebanov+Polyakov, Witten, 1998

Holographic Renormalization

- The tachyon action to be renormalized is

$$\mathcal{L} = -2\mathcal{K}\, e^{-\frac{1}{2}\mu^2\tau^2} g_{tt}^{\frac{1}{2}} g_{xx}^{\frac{3}{2}} \sqrt{g_{zz} + 2\pi\alpha'\lambda(\partial_z\tau)^2}$$

- The quark condensate is defined as:

$$\langle \bar{q}\, q \rangle = -\frac{\delta S_{ren}}{\delta m_q} = -\frac{\delta\tau}{\delta m}\frac{\delta S_{reg}}{\delta\tau}$$

$$\delta S_{reg} = \int_{\epsilon}^{z_\Lambda} \left(\delta\tau \frac{\partial\mathcal{L}}{\partial\tau} + \delta\tau' \frac{\partial\mathcal{L}}{\partial\tau'}\right) dz = \int_{\epsilon}^{z_\Lambda} \frac{d}{dz}\left(\delta\tau \frac{\partial\mathcal{L}}{\partial\tau'}\right)$$

and therefore

$$\frac{\delta S_{reg}}{\delta\tau} = -\frac{\partial\mathcal{L}}{\partial\tau'}\Big|_{z=\epsilon}$$

• We obtain

$$\frac{\delta S_{reg}}{\delta c_1} = \mathcal{K}R^5\mu^2\left(\frac{2c_1}{3\epsilon^2}+\frac{2}{3}c_1^3\mu^2\log\epsilon+2c_3-\frac{1}{3}c_1^3\mu^2+\frac{2}{3}c_1\partial_{c_1}c_3+\mathcal{O}(\epsilon)\right)$$

• The subtracted action is

$$S_{sub}=S_{reg}+S_{ct}\quad,\quad S_{ct}=-\mathcal{K}R\int d^4x\sqrt{-\gamma}\left(-\frac{1}{2}+\frac{\mu^2}{3}\tau^2+\frac{\mu^4}{18}\tau^4\log\epsilon+\frac{\mu^4}{12}\alpha\,\tau^4\right)$$

• The constant α captures the scheme dependence of the condensate and reflects an analogous scheme dependence in field theory.

• The renormalized action is

$$S_{ren}=\lim_{\epsilon\to 0}S_{sub}$$

• With $m_q=\beta\ c_1$, we finally obtain

$$\langle\bar{q}q\rangle=\frac{1}{\beta}(2\pi\alpha'\mathcal{K}R^3\lambda)\left(-4c_3+\left(\frac{m_q}{\beta}\right)^3\mu^2(1+\alpha)\right)$$

RETURN

The Garousi action

• Garousi proposed an effective action for the brane-antibrane system which has a subtle difference with respect to Sen's one.

$$S = -\mathrm{STr}\int d^4x dz\; e^{-\hat{T}^2}\sqrt{-\det(g_{MN} + \hat{F}_{MN} + D_M\hat{T}D_N\hat{T})}$$

where hatted symbols are 2x2 matrices:

$$\hat{T} = \begin{pmatrix} 0 & T \\ T^* & 0 \end{pmatrix}, \quad \hat{F}_{MN} = \begin{pmatrix} F^{(L)}_{MN} & 0 \\ 0 & F^{(R)}_{MN} \end{pmatrix}, \quad D_M\hat{T} = \begin{pmatrix} 0 & D_M T \\ (D_M T)^* & 0 \end{pmatrix}.$$

• The equations for the vectors are not modified with respect to the main text.

• The equations for the axials are different. They still obey a Regge law $m_n^2 \propto n$ for large excitation number n but with different slope compared to the main text.

• This slope is still larger than the one for vectors.

Detailed plan of the presentation

- Title page 1 minutes
- Collaborators 2 minutes
- Introduction 3 minutes
- Top-down flavor 5 minutes
- Bottom-up flavor 7 minutes
- The general setup for flavor 14 minutes
- The tachyon DBI action 17 minutes

- A simple glue background 19 minutes
- The deconfined phase 21 minutes
- A “warmup” bottom-up model of flavor 23 minutes
- The chiral vacuum structure 29 minutes
- Chiral restauration at deconfinement 33 minutes
- Jump of the condensate at the phase transition 36 minutes
- Meson Spectra 40 minutes
- Mass dependence of f_π 41 minutes
- Linear Regge trajectories 42 minutes
- Fit to data 48 minutes
- Steps Forward 49 minutes
- Open problems 51 minutes

- Bibliography 51 minutes
- Introduction 53 minutes
- The glue 55 minutes
- YM entropy 57 minutes
- YM equation of state (interaction measure) 58 minutes
- The tachyon WZ action 66 minutes
- Discrete symmetries 67 minutes
- Meson Melting 68 minutes
- The gauge theory/string theory duality 74 minutes
- Holographic Renormalization 77 minutes
- The Garousi tachyon action 80 minutes