Recent Developments in Gravity
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New physics from gravity on asymptotically AdS spacetimes

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Bibliography

Based ongoing work with

T. Alho (Jyvaskyla U.), D. Arean (SISSA), K. Kajantie (Helsinki U.), K. Tuominen (Jyvaskyla U.), I. Iatrakis (Crete), M. Järvinen (Crete)

and published recent work with

C. Charmousis (Orsay) and B. Gouteraux, (APC) arXiv:1206.1499 [hep-th]

V. Niarchos (Crete) arXiv:1205.6205 [hep-th]

M. Jarvinnen (Crete) arXiv:1112.1261 [hep-ph]

B. Gouteraux (APC) arXiv:1012.3464 [hep-th]

B. S. Kim (Crete) and C. Panagopoulos (Crete) arXiv:1012.3464 [cond-mat.str-el]

C. Charmousis, B. Gouteraux (Orsay), B. S. Kim and R. Meyer (Crete)
. arXiv:1005.4690 [hep-th]

ΠΡΟΛΕΓΟΜΕΝΑ

• The Purpose of this talk is to present several interesting problems for gravity in asymptotically AdS spacetimes.

• The motivation is the conjectured AdS/CFT correspondence aka: gauge theory/string theory (or gravity) duality.

• This duality has had important impact in our understanding of strongly coupled gauge theories, but also in string theory/quantum gravity.

• There are several problems on the gravity side that are well motivated by the correspondence , but we do not know the answers of.

• In this talk I will focus more on problems rather than answers.

The plan of the talk

- Introduction: the AdS/CFT correspondence.
- Asymptocally AdS spacetimes and boundary conditions.
- Singularities and their resolution.
- Classification of the gravitational landscape
- Beyond gravity: the string theory landscape.
- Outlook

Gauge theories with many colors

• Gauge theories with N-colors (SU(N) gauge group) have a single continuous parameter: the gauge coupling constant g_{YM}.

• When N is large ($N \to \infty$) there is another way of reorganizing the theory:

't Hooft, 1974

$$N \to \infty \quad , \qquad \text{keep} \quad \lambda \equiv g_{YM}^2 N \quad \text{fixed}$$

• The expansion in powers of $1/N$ is similar to the topological expansion of a string theory with

$$g_{\text{string}} \sim \frac{1}{N}$$

• When $N \to \infty$ and $\lambda \to 0$ we can use perturbation theory to calculate.

• When $N \to \infty$ and λ is large, we are at strong coupling.

The gauge-theory/gravity duality

- The gauge-theory/gravity duality is a duality that relates a string theory with a (conformal) gauge theory.

- The prime example is the AdS/CFT correspondence

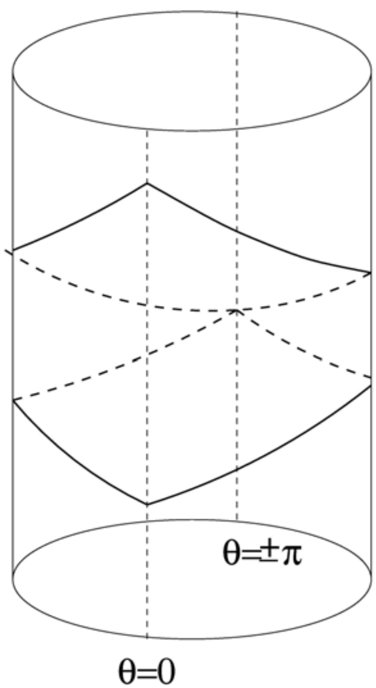
Maldacena 1997

- It states that N=4 four-dimensional SU(N) gauge theory (gauge fields, 4 fermions, 6 scalars) is equivalent to ten-dimensional IIB string theory on AdS$_5 \times S^5$

$$ds^2 = \frac{\ell_{AdS}^2}{r^2}\left[dr^2 + dx^\mu dx_\mu\right] + \ell_{AdS}^2\ (d\Omega_5)^2$$

This space (AdS_5) has infinite volume and a single boundary, at $r = 0$.

• The string theory has as parameters, g_{string}, ℓ_{string}, ℓ_{AdS}. They are related to the gauge theory parameters as



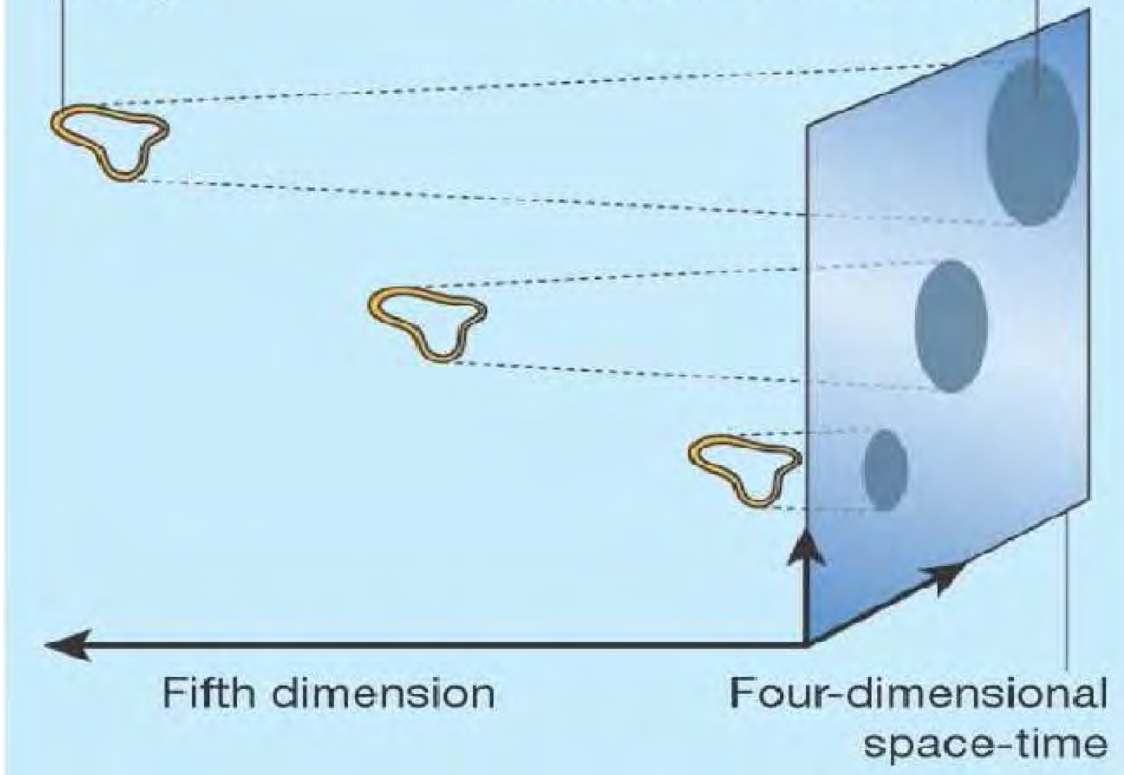
$$g_{YM}^2 = 4\pi \; g_{\text{string}} \quad , \quad \lambda = g_{YM}^2 \; N = \frac{\ell_{AdS}^4}{\ell_{\text{string}}^4}$$

• As $N \to \infty$, $g_{\text{string}} \sim \frac{\lambda}{N} \to 0$.

• As $N \to \infty$, $\lambda \gg 1$ implies that $\ell_{\text{string}} \ll \ell_{AdS}$ and the geometry is very weakly curved. String theory can be approximated by gravity in that regime and is weakly coupled.

• As $N \to \infty$, $\lambda \ll 1$ the gauge theory is weakly coupled, but the string theory is strongly curved.

• The radial coordinate correspond to the RG scale of the dual QFT. The boundary $r \to 0$ corresponds to the UV, while the "center" $r \to \infty$ corresponds to the IR.



• There is one-to-one correspondence between on-shell string states $\Phi(r,x^\mu)$ and gauge-invariant (single-trace) operators $O(x^\mu)$ in the sYM theory

• In the string theory we can compute the "S-matrix" , $S(\phi(x^\mu))$ by studying the response of the system to boundary conditions $\Phi(r=0,x^\mu)=\phi(x^\mu)$

• The correspondence states that this is equivalent to the generating function of correlators of O

$$\langle e^{\int d^4x \ \phi(x) \ O(x)}\rangle \ = \ e^{-S(\phi(x))}$$

Gubser+Klebanov+Polyakov, Witten, 1998

• "String theory is the dynamics of sources of QFT"

The duality at finite temperature

• The finite temperature ground state of the gauge theory corresponds to a different solution in the dual string theory: the AdS-Black-hole solution

E. Witten, 1998

$$ds^2 = \frac{\ell_{AdS}^2}{r^2}\left[\frac{dr^2}{f(r)} + f(r)dt^2 + dx^i dx_i\right] + \ell_{AdS}^2\ (d\Omega_5)^2 \quad , \quad f(r) = 1 - (\pi T)^4 r^4$$

• The horizon is at $r = \frac{1}{\pi T}$

• As the temperature increases, the horizon size increases, reaching the boundary at $T = \infty$.

• The gauge theory entropy is equal to the Bekenstein-Hawking entropy of the black hole.

• The free energy can be obtained from the (appropriately) renormalized on-shell gravitational action.

The gravitational theories

• Select a finite number of "bulk" fields, and their gravitational action.

• Find acceptable regular solutions with appropriate boundary conditions. They correspond to QFT saddle points, describing semiclassical states.

• The solution with lowest free energy is the "vacuum" of the QFT.

• Small fluctuations around the "vacuum solution" describe the spectra of the QFT and the correlation functions of the dual operators.

• Solutions with regular horizons (black branes) correspond to thermal ensembles of the QFT.

The boundary conditions

• Near an AdS_{d+1} boundary $r \to 0$, a massive scalar ϕ dual to the QFT scalar operator O, behaves as

$$\phi(r,x) \simeq \phi_0(x) r^{d-\Delta} + \cdots + \phi_1(x) r^{\Delta} + \cdots \quad , \quad -m^2\ell^2 = \Delta(d-\Delta)$$

Dirichlet bc

• $m^2\ell^2 \geq 0$, $\Delta > d$, irrelevant operators, so $\phi_0 = 0$. $\phi_1(x)$ corresponds to the vev $\phi_1 \sim \langle O \rangle$.

• $\frac{1-d^2}{4} \leq m^2\ell^2 \leq 0$, $\frac{d+1}{2} \leq \Delta \leq d$, relevant operators, ϕ_0 is the "source" ($\int d^dx \phi_0(x) O(x)$), ϕ_1 is the vev.

• $-\frac{d^2}{4} \leq m^2\ell^2 \leq \frac{1-d^2}{4}$, $\frac{d-1}{2} \leq \Delta \leq \frac{d+1}{2}$, relevant operators, $\phi_{0,1}$ can be interchanged, as both solutions are normalizable.

• $-\frac{d^2}{4} \geq m^2\ell^2$ BF bound violated.

• Regularity of the bulk solution reduces the two parameter family to one: the vev is determined by the source, $\phi_1(\phi_0)$.

• There are generalized boundary conditions possible:

• Bulk fields ϕ_i correspond to "single trace" operators, e.g. $Tr[F^2]$. Multiple trace operators, like $(Tr[F^2])^2$ are implemented by generalized boundary conditions

$$\delta S_O = \int d^d x \; W(O) \quad , \quad \phi_0 = W'(\phi_1)$$

• For $W(O) = g_1 \; O \quad \to \quad \phi_0 = g_1$.

• For $W(O) = g_1 \; O + g_2 O^2 \quad \to \quad \phi_0 = g_1 + g_2 \phi_1$.

All of the above generalize to all bulk fields including the metric.

The bulk actions

• Most of what we understand refers to gravitational solutions in $D = d + 1$, that are Poincarínvariant in d dimensions. They depend non-trivially on a spacelike coordinate. They are analogous to the homogeneous and isotropic cosmological solutions.

• Keeping only the metric $g_{\mu\nu} \sim T_{\mu\nu}$ describes the saddle point (AdS_{d+1}) of CFT_d.

$$S_E = M^{d-1} \int d^{d+1}x \sqrt{g} \left[R + \frac{d(d-1)}{\ell^2} \right]$$

• Adding the most relevant (=important) scalar operator $O \sim \phi$,

$$S_{ED} = M^{d-1} \int d^{d+1}x \sqrt{g} \left[R - \frac{1}{2}(\phi)^2 + V(\phi) \right] \quad , \quad V \geq 0$$

• $V'(\phi) = 0$ give AdS solutions $\rightarrow$ CFTs. The general Poincaré invariant solutions are flows between CFTs, modulo a subtle point when $V \to \infty$.

• Adding a conserved current $J_\mu \sim A_\mu$

$$S_{EMD} = M^{d-1} \int d^{d+1}x \sqrt{g} \left[R - \frac{1}{2}(\phi)^2 + V(\phi) - \frac{Z(\phi)}{4} F^2 \right]$$

• A non-conserved current (spontaneous symmetry breaking)

$$S_{EMD} = M^{d-1} \int d^{d+1}x \sqrt{g} \left[R - \frac{1}{2}(\phi)^2 + V(\phi) - \frac{Z(\phi)}{4} F^2 - \frac{W(\phi)}{2} A_\mu^2 \right]$$

Phase transitions

• Phase transition/critical behavior in gravitational systems has been observed in the past, but its appearance remained intriguing.

• Holography explains this behavior in asymptotically AdS contexts by mapping it to QFT phase transitions.

• The problem is set by fixing the asymptotic behavior by fixing all sources as well as Temperature and Chemical potentials (boundary values of gauge fields).

• Then we must find all possible solutions with the same asymptotics behavior and regular horizon.

• The free energy decides which is dominant. When two free energies cross we have phase transitions. (all possible transitions, 1,2,3rd........, and BKT transitions have been observed).

On naked holographic singularities

• If no IR AdS, all Poincaré invariant solutions end up in a naked IR singularity. This happens with solutions where ϕ runs to large values of V.

• In GR naked singularities are proscribed.

• In holographic gravity some may be acceptable. The reason is that their appearance may be due to "coarse graining". They could be resolved by stringy or KK physics.

• We know examples where a higher dimensional regular solution, when dimensionally reduced on a compact manifold to lower dimensions it looks singular. In this case the lower dimensional singularity is "resolved" by adding back the KK modes.

• We also know examples where the singularity is resolved by adding stringy modes, that blow up a tiny horizon of the size of the string length. They will typically involve an infinite amount of non-trivial fields (hair).

• Even singular solutions (black branes) covered with horizons, are "coarse grained". There are concrete arguments based on duality that suggest that the true pure states must have an infinite amount of hair of stringy states. Only exact AdS does not have hair.

• Setting the stringy hair to zero, forces the solution to become singular and develop a horizon (if the entropy is non-trivial and macroscopic). Here pure states of the QFT correspond to regular solutions. Mixed states have horizons.

• We know of exact solutions (with lots of symmetry) that realize these expectations.

Lunin+Mathur (2001)

• The practical problem is that in most cases we do not know the exact string description that will provide a regular solution.

• An important task is to therefore ascertain when naked singularities are acceptable because they will be resolved by effects we have neglected.

• A related questions is: if they are resolvable, when can we calculate reliably without knowing the precise resolution of the naked singularity?

♠ Gubser gave a criterion for "good" (acceptable) naked singularities: They should be limits of solutions with a regular horizon.

Gubser (2000)

♠ It is not known if the Gubser criterion is sufficient. This is an important open problem.

• The second question is when is the physics insensitive to the resolution of the naked singularity?

• A partial answer is to have a well-defined spectral problem for fluctuations around the solution: The second order equations describing all fluctuations are Sturm-Liouville problems (no extra boundary conditions needed at the singularity).

Gursoy+E.K.+Nitti (2008)

• A related condition: The singularity is "repulsive" (like the Liouville wall). It has an overlap with the previous criterion. It involves the calculation of "Wilson loops"

Gursoy+E.K.+Nitti (2008)

• It is not known whether the previous list is complete.

Solutions in Einstein dilaton gravity

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Gursoy+Kiritsis+Mazzanti+Nitti (2009)

$$S_{ED} = M^{d-1}\int d^{d+1}x\sqrt{g}\left[R-\frac{1}{2}(\phi)^2+V(\phi)\right] \quad , \quad V\geq 0$$

• Take V to have a single finite (UV) critical point at $\phi = 0$, and as $\phi \to \infty$,

$$V \simeq e^{\delta\phi} + \cdots$$

We take $p + 1 = 4$.

• All "regular", Poincarínvariant solutions with positive ϕ are asymptotically AdS near the boundary, and are conformal to AdS (with a possible naked singularity) in the interior,

$$ds^2 \simeq r^{\frac{2}{\delta^2-1}}\,(dr^2 - dt^2 + dx^i dx^i)$$

• $0 \leq |\delta| < 1$. There is no naked singularity (the space is asymptotically flat in the interior). There is a continuous spectrum/no mass gap for laplacian. There are black-brane solutions for any $T > 0$.

• $1 < |\delta| < \sqrt{3}$. There is a naked singularity in the interior. It satisfies the Gubser bound. The Laplacian has a discrete spectrum/mass gap. There is a $T_{min} > 0$ for black branes and there are two branches like in global AdS.

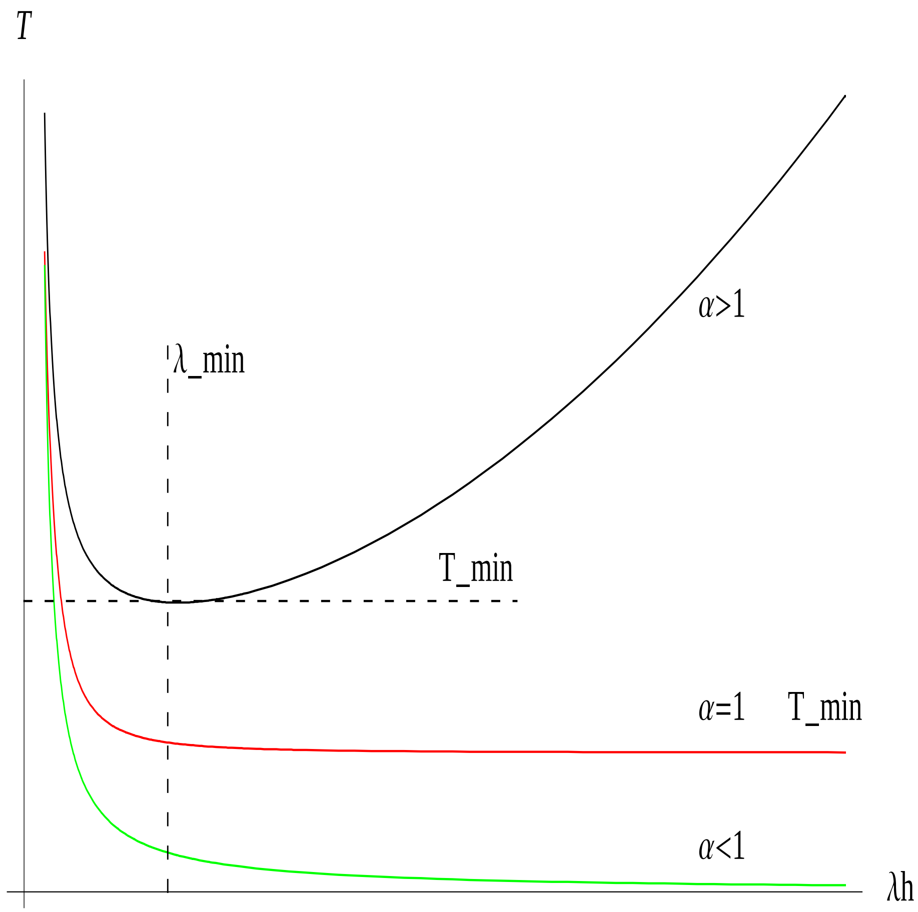
$1 < |\delta| < \sqrt{\frac{5}{3}}$. The spin-2 and spin-0 spectral problem is reliable without resolution.

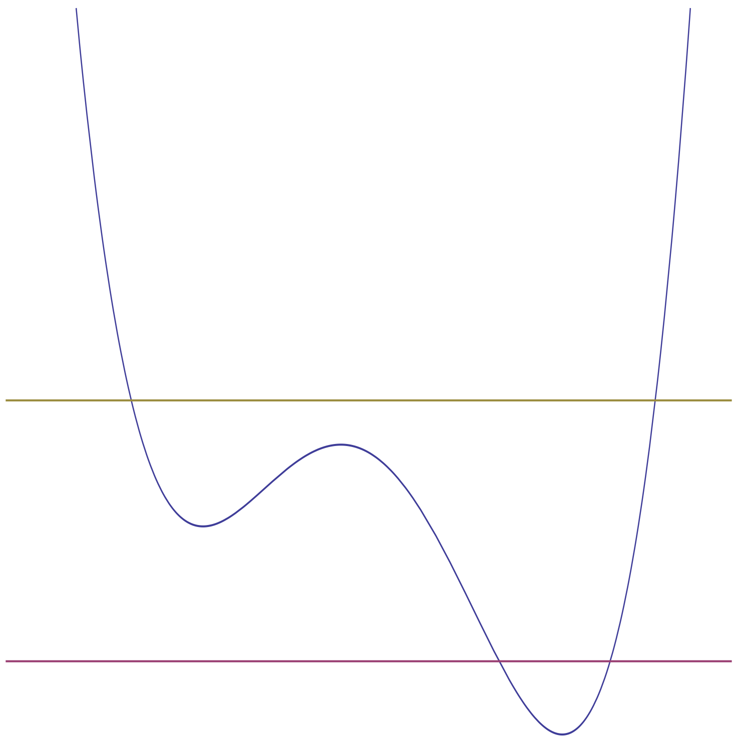
• $|\delta| \geq \sqrt{3}$. Gubser bound violated, singularity$\rightarrow$unacceptable.

The crossover value here is $|\delta| = 1$. For all other $\delta \neq 1$, corrections like $V = e^{-\delta\phi}\phi^k + e^{-\delta'\phi}\phi^{k'} + ...$ give subleading corrections.

• $1 < |\delta| < \sqrt{3}$. In this case, the small BH is unstable and thermodynamically irrelevant. The complete story at finite T depends on the subleading terms in the potential (aka the UV completion).

• There is a first order phase transition at T_c to a large BH.





• For more complicated potentials multiple phase transitions are possible.

Gursoy+Kiritsis+Mazzanti+Nitti (2009), Alanen+Kajantie+Tuominen (2010)

• $|\delta| = 1$. This is the "marginal" case. It has a good singularity, a continuous spectrum and a gap. A lot of the physics of finite temperature transitions depends on subleading terms in the potential:

♠ If $V = e^{\phi}\left[1 + C\ e^{-\frac{2\phi}{n-1}} + \cdots\right]$, then at $T = T_{min} = T_c$ there is an n-th order continuous transition.

♠ If $V = e^{\phi}\left[1 + C/\phi^k + \cdots\right]$, then at $T = T_{min} = T_c$ there is a generalized KT phase transition

Gursoy (2010)

♠ If $V = e^{\phi}\ \phi^P$, with $P < 0$ this behaves as in $|\delta| < 1$. When $P > 0$ like $|\delta| > 1$.

The spectra depend importantly on P, when $P > 0$.

In particular, $P = \frac{1}{2}$ is very much like what we expect in 4D large-N YM $(m_n^2 \sim n)$.

Regularity of "vacuum" solutions in ED theory

- Which one parameter family of the Poincaré invariant solutions is "regular"?

- The solutions can be parameterized in terms of a fake superpotential (d+1=5 here)

$$V = \frac{64}{27}W^2 - \frac{4}{3}\,W'^2 \quad , \quad W \geq \frac{3}{8}\sqrt{3V}$$

Then

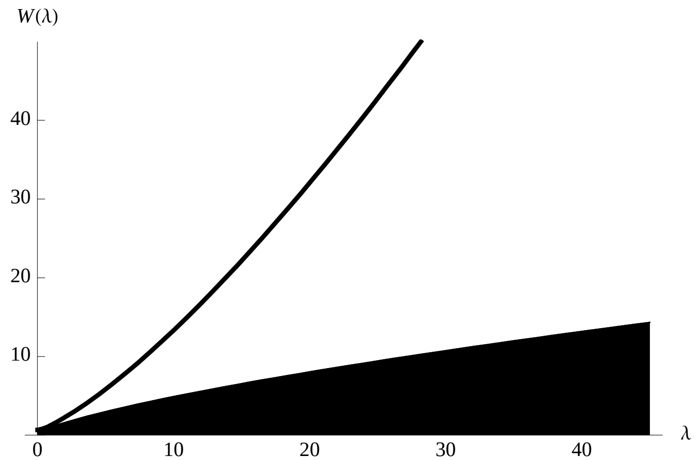
$$\phi' = \frac{dW}{d\phi} \quad , \quad A' = W(\phi)$$

The crucial parameter resides in the solution to the diff. equation above. There are three types of solutions for $W(\phi)$:

Gursoy+E.K.+Mazzanti+Nitti

1. Generic Solutions (with a "bad" IR singularity)

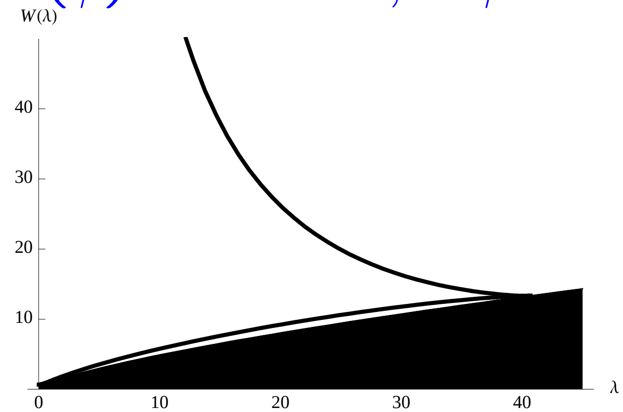
$$W(\phi) \sim e^{\frac{4}{3}\phi} \quad , \quad \phi \to \infty$$



$$\lambda \equiv e^{\phi}$$

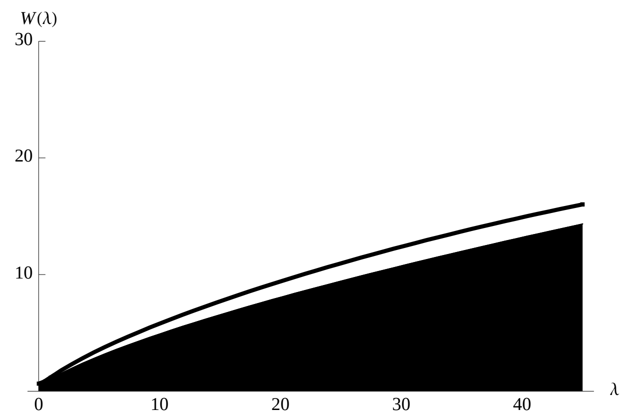
2. Bouncing Solutions (bad IR singularity)

$$W(\phi) \sim e^{-\frac{4}{3}\phi} \quad , \quad \phi \to \infty$$



3. One special "regular" solution.

$$W(\phi) \sim W_\infty e^{\frac{\delta\phi}{2}} \quad , \quad \phi \to \infty \quad , \quad W_\infty = \sqrt{\frac{27 V_0}{4\left(16 - \frac{9\delta^2}{4}\right)}}$$



Good+repulsive IR singularity if $\delta < \frac{8\sqrt{2}}{3}$

Classification of Extremal geometries

• The analysis of several cases seems to suggest that in all systems studied so far, the $T \to 0$ asymptotics involve non-trivial scale invariant holographic geometries (singular, flat or AdS).

• This conclusion remains the same when charge densities (gauge fields) are present.

• The scaling geometries are generalized AdS or Lifshitz geometries with hyperscaling violations.

Charmousis+Gouteraux+Kiritsis+kim+Meyer (2010), Gouteraux+Kiritsis(2011), Huisje+Sachdev+Swingle (2011)

$$ds^2 = r^\theta \left[b_0 \frac{dr^2}{r^2} - \frac{dt^2}{r^{2z}} + \frac{dx^i dx^i}{r^2} \right]$$

• This is valid both in broken and unbroken symmetry phases.

• How general is this? Can we obtain a classification of extremal asymptotics?

gravity with two scalars: V-QCD

Jarvinnen+E.K.

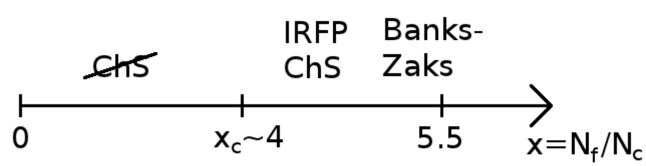
• The theory contains a metric and two scalars, a real one, λ and a complex one T. There is also a U(1) symmetry under which T is charged.

Fixed points of the potential:

UV: $(\lambda = 0, T = 0)$, $\Delta_\lambda = 4, \Delta_T = 3$, unbroken U(1) symmetry.

IR: $(\lambda = \lambda_*, T = 0)$ (non-trivial CFT), unbroken U(1) symmetry or

$(\lambda = \infty, T = \infty)$, broken U(1) symmetry and a different (free) CFT of the Goldstone boson. (YM with massless pions)



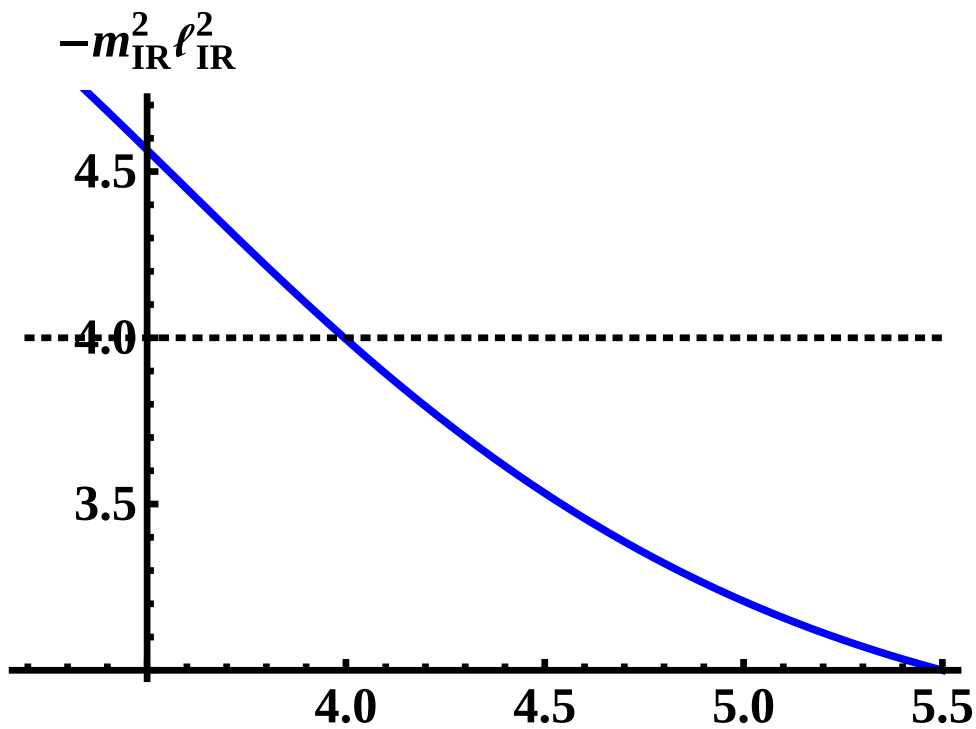
Condensate dimension at the IR fixed point

• By expanding the DBI action we obtain the IR tachyon mass at the IR fixed point $\lambda = \lambda_*$ which gives the chiral condensate dimension:

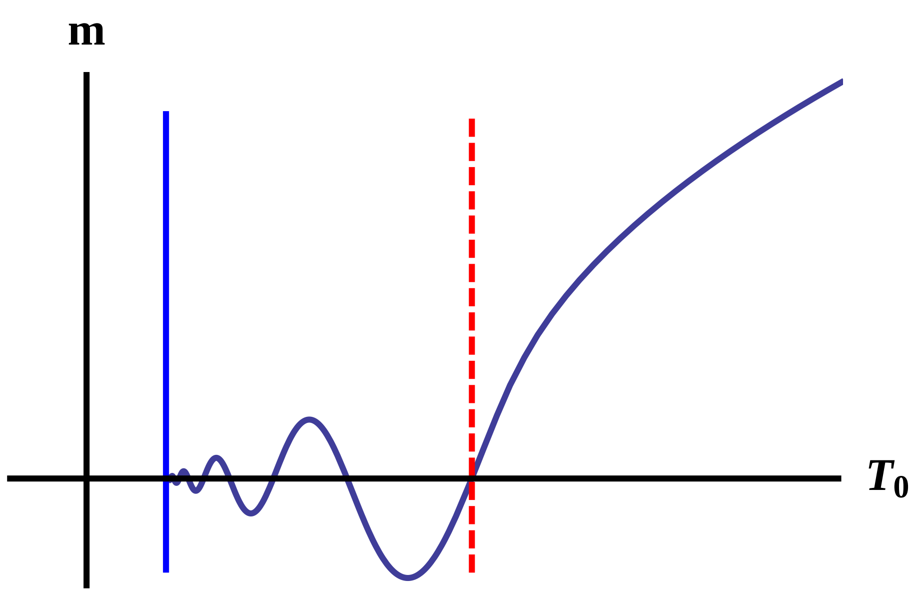
$$-m_{\rm IR}^2 \ell_{\rm IR}^2 = \Delta_{\rm IR}(4 - \Delta_{\rm IR})$$

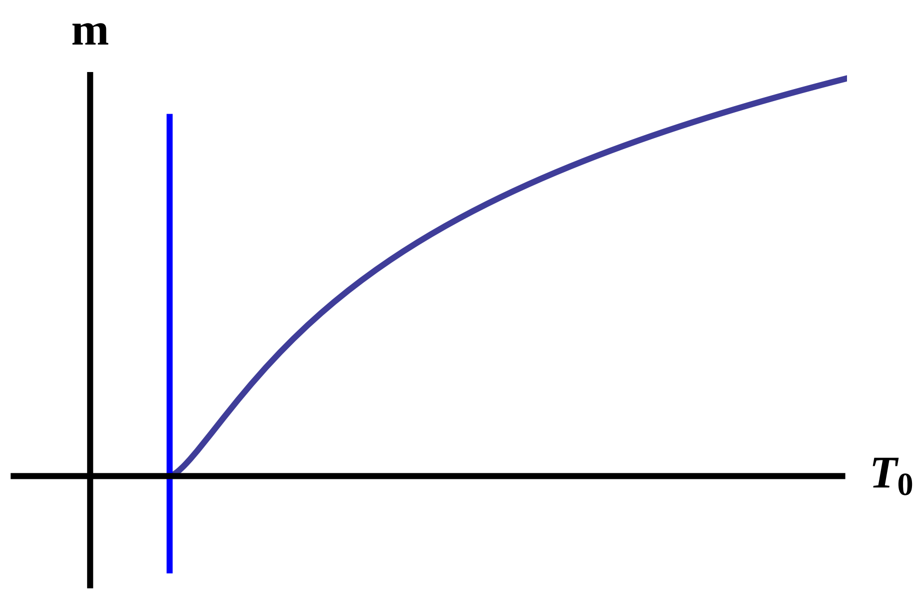
• Must reach the Breitenlohner-Freedman (BF) bound (horizontal line) at some x_c.

• x_c marks the *conformal phase transition*



The symmetry breaking regime





• In the symmetry breaking region there is an infinite number of saddle points.

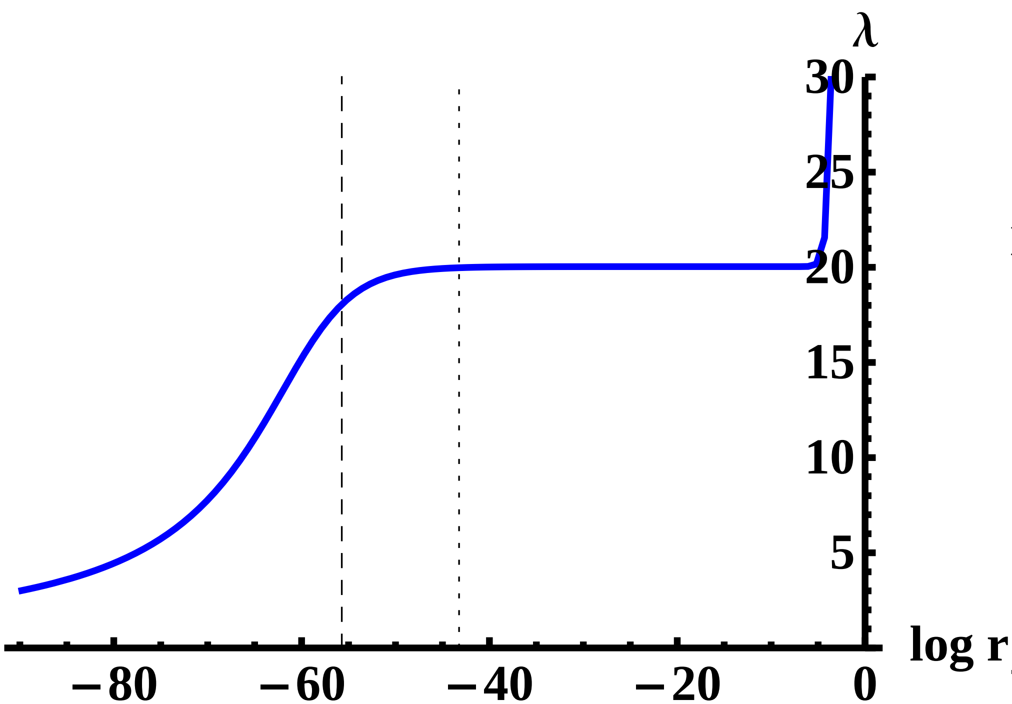
• Their Free energies are ordered

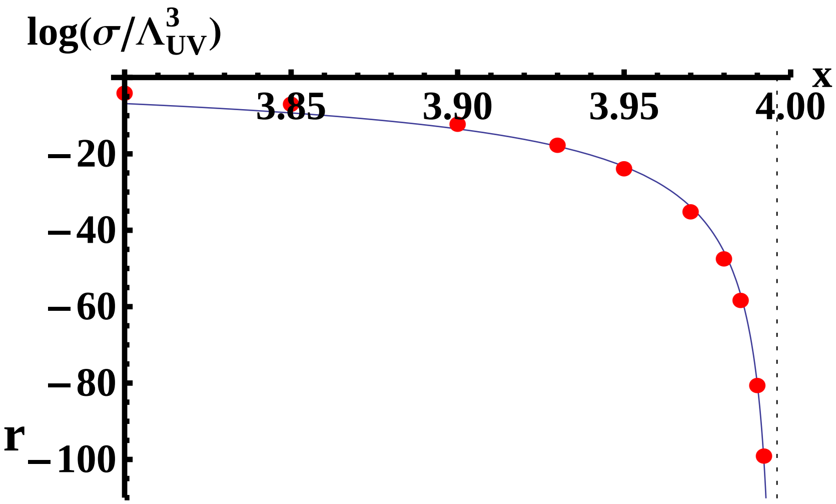
$$F_0 < F_1 < F_2 < \cdots < \quad F \quad {}_{T=0}$$

BKT scaling

• In the symmetry region we have BKT scaling for all symmetry breaking scales

$$\sigma \sim \frac{1}{r_{\rm UV}^3} \exp\left(-\frac{2K}{\sqrt{x_c - x}}\right) \; .$$

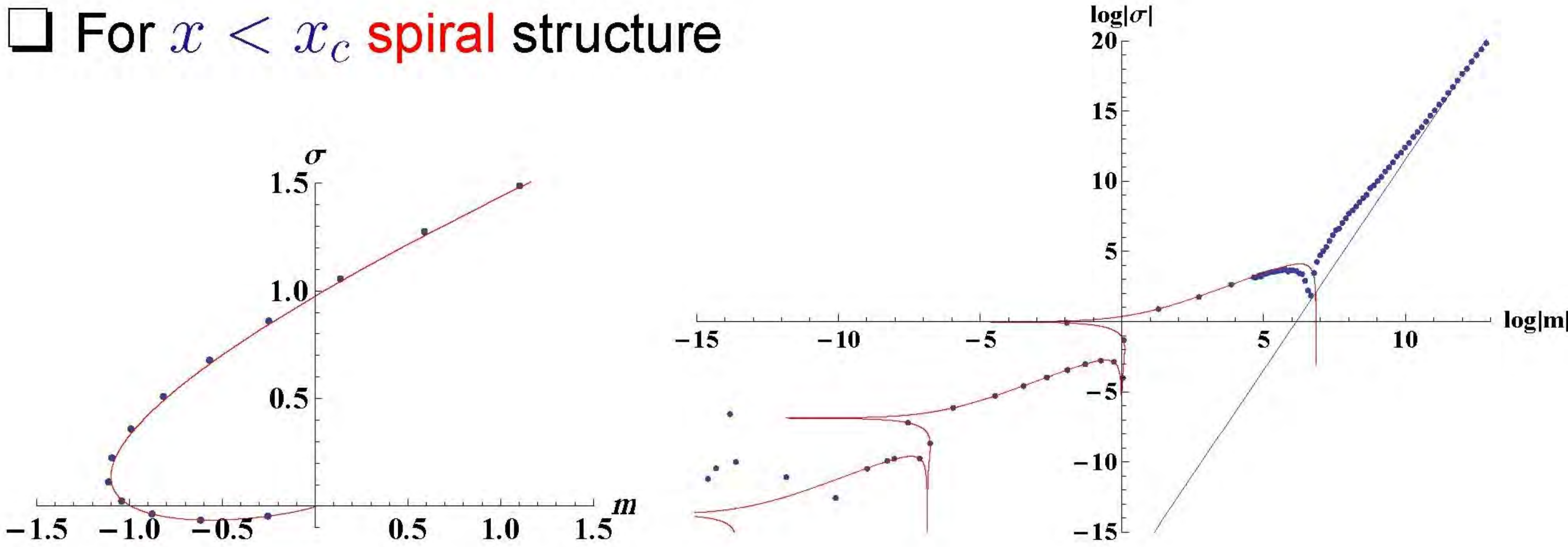




Efimov spiral

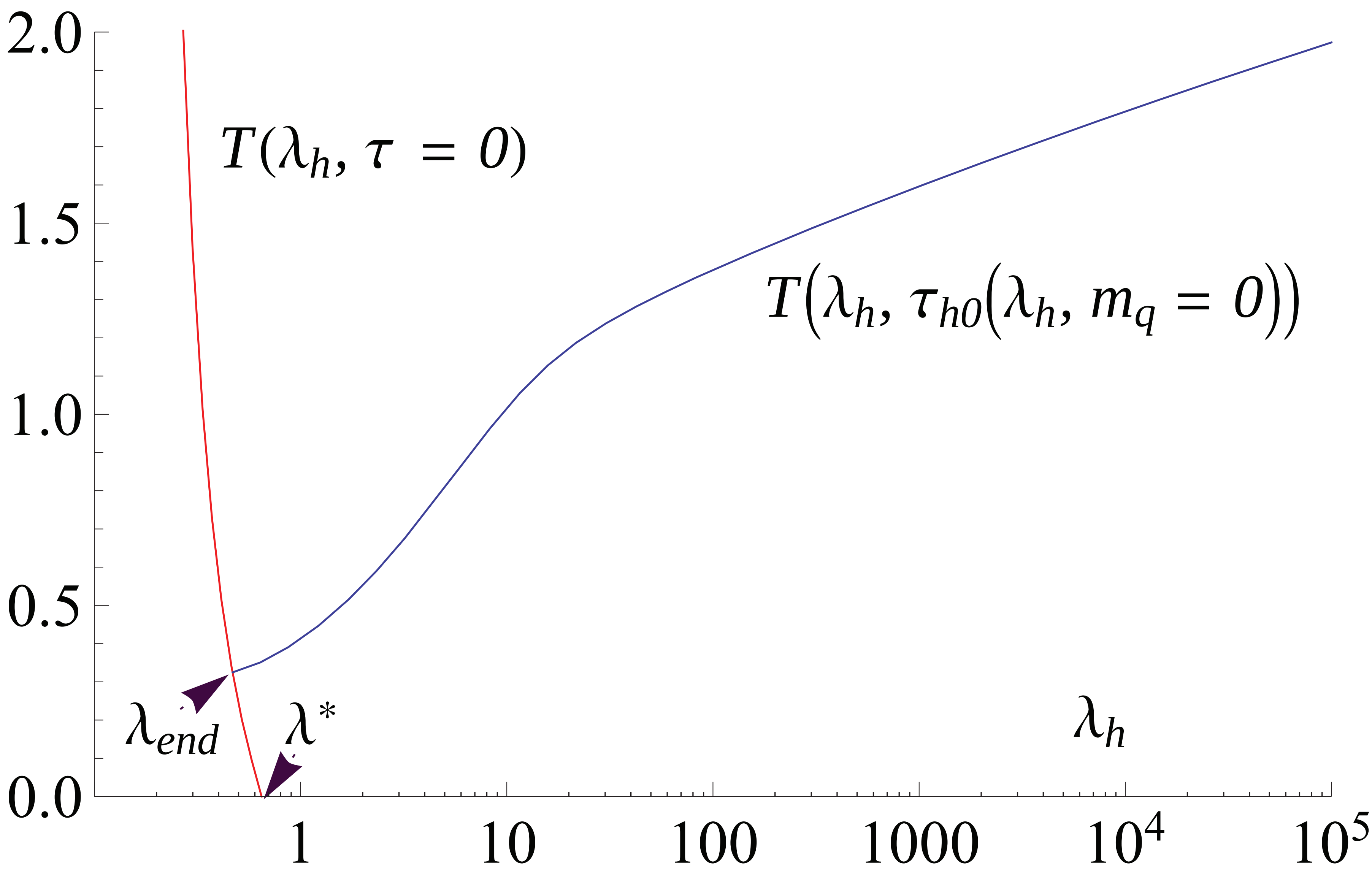
Ongoing work: $\sigma(m)$ dependence

❑ For $x < x_c$ spiral structure

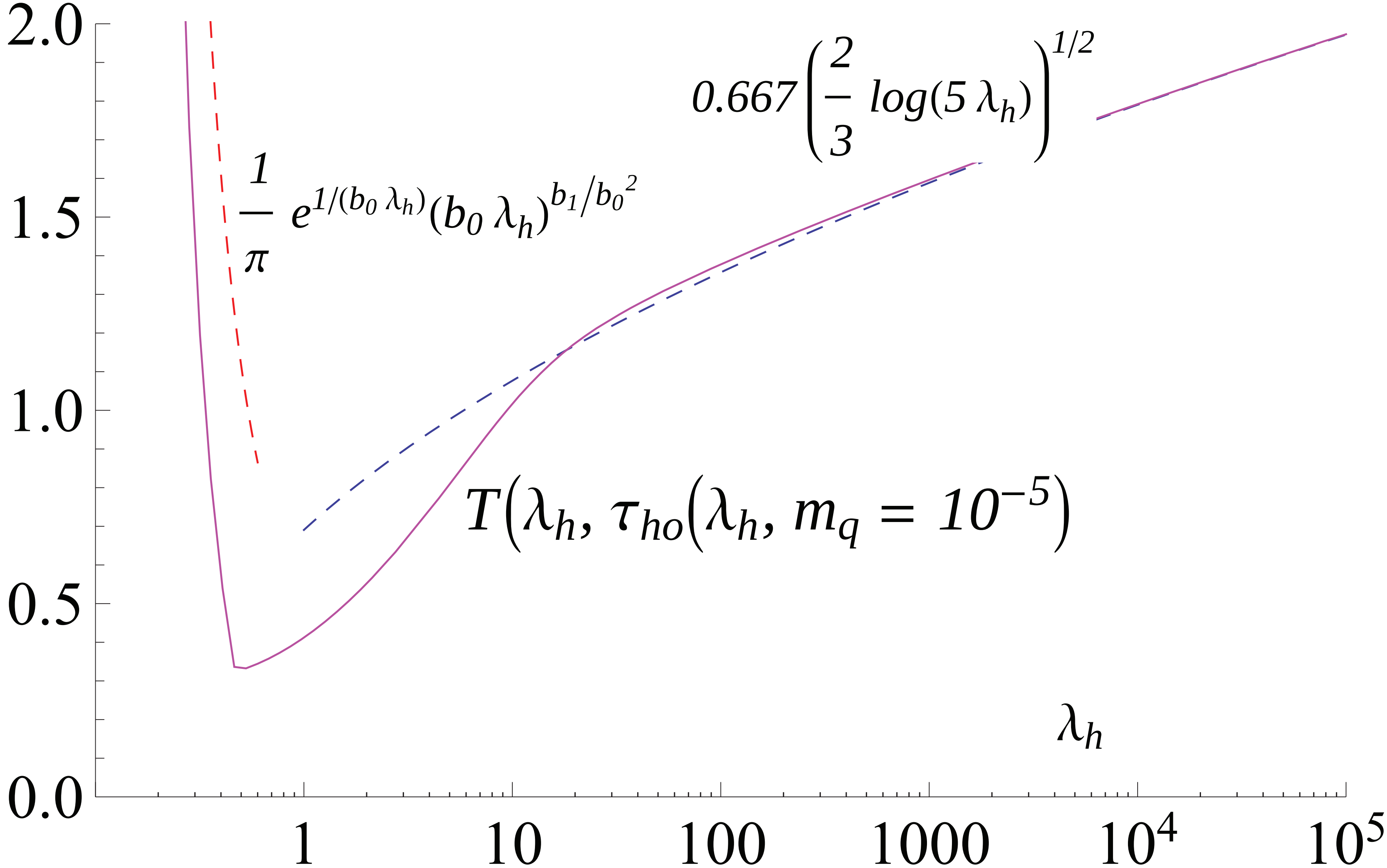


• This suggests that the presence of double trace deformations can alter the ground state of the system and make the second Effimov vacuum be the ground state.

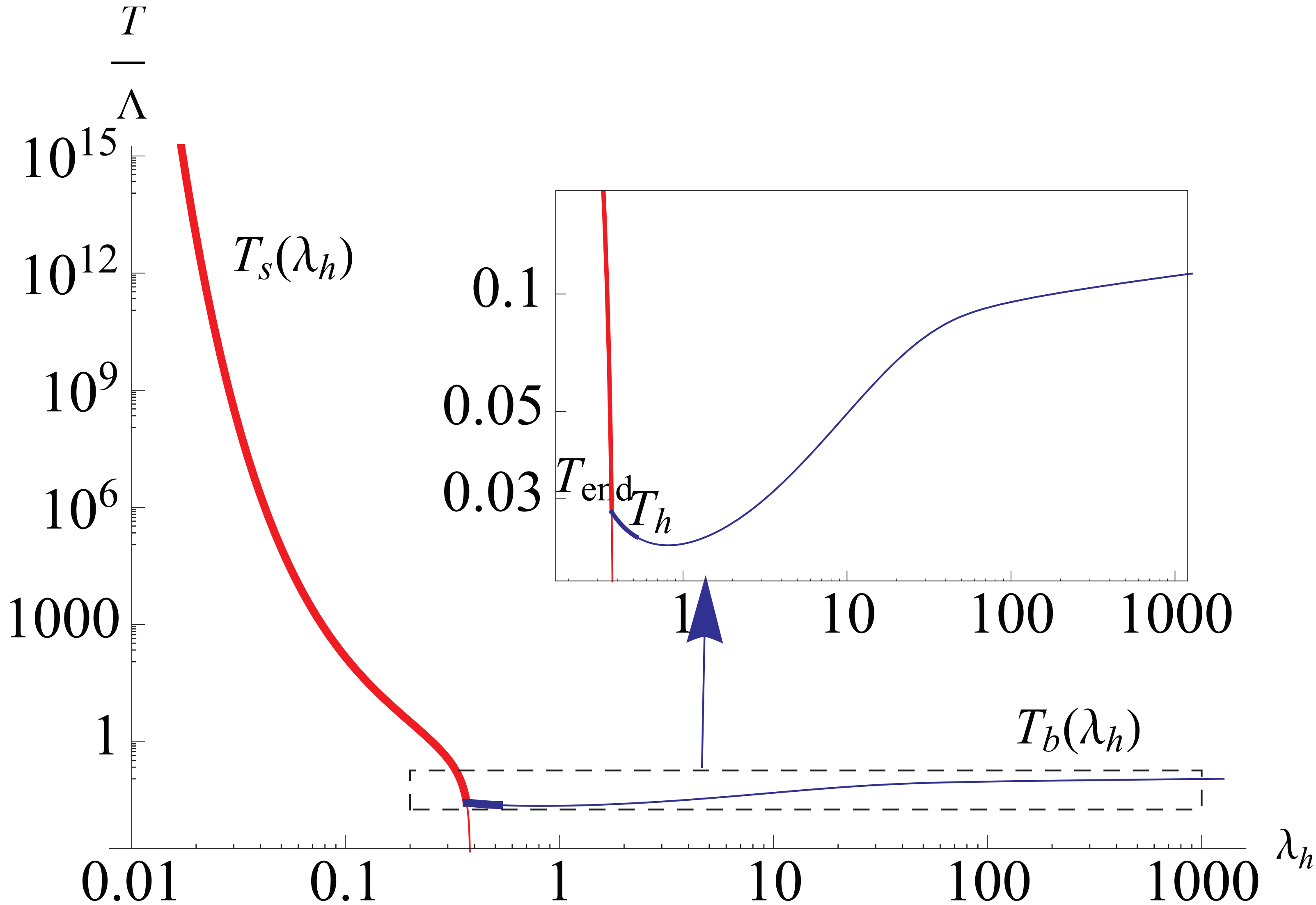
Finite temperature



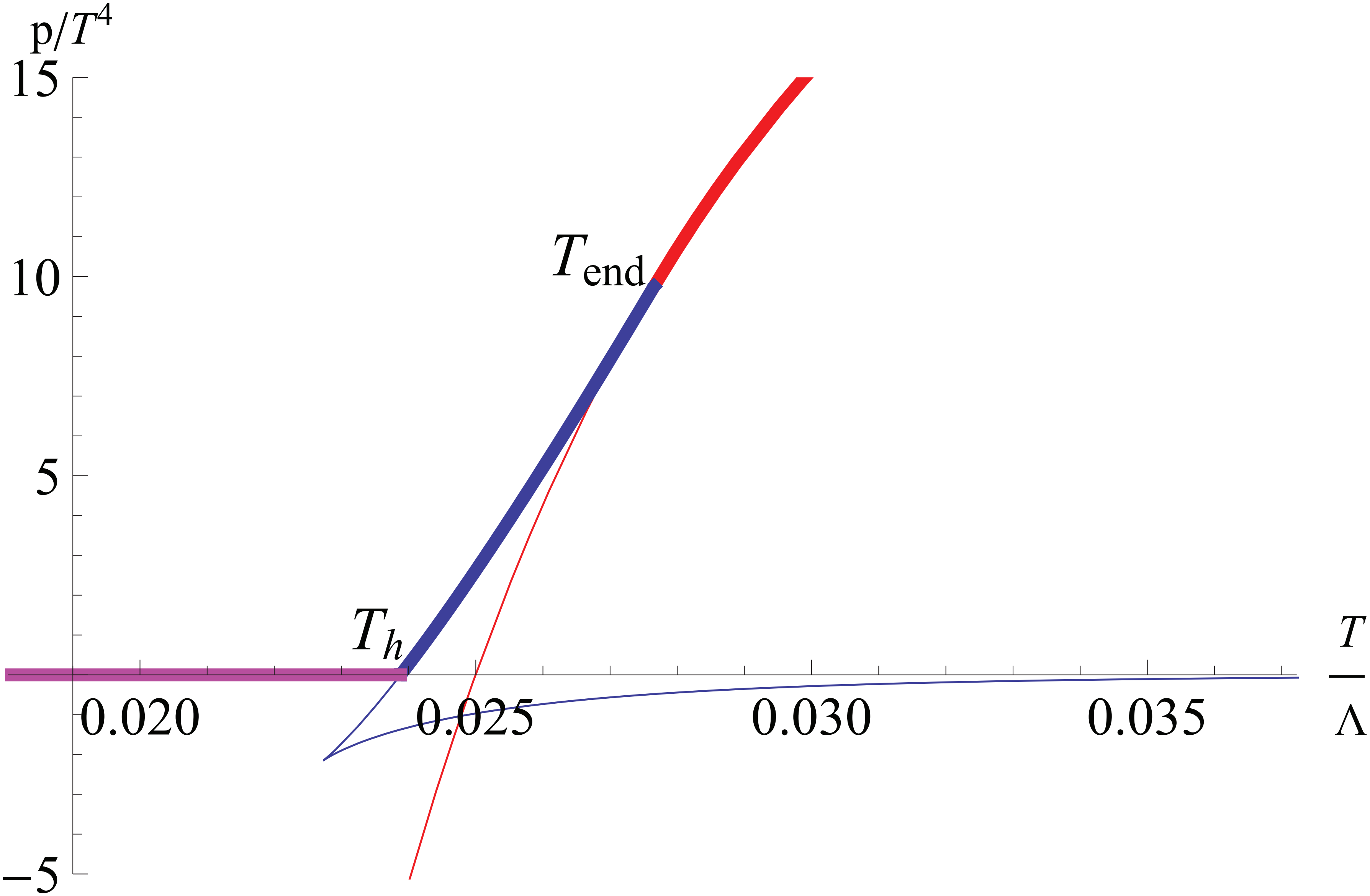
The temperature as a function of λ_h for solutions for Pot II at $x_f = 3\, W_0 = 12/11$, for zero mass



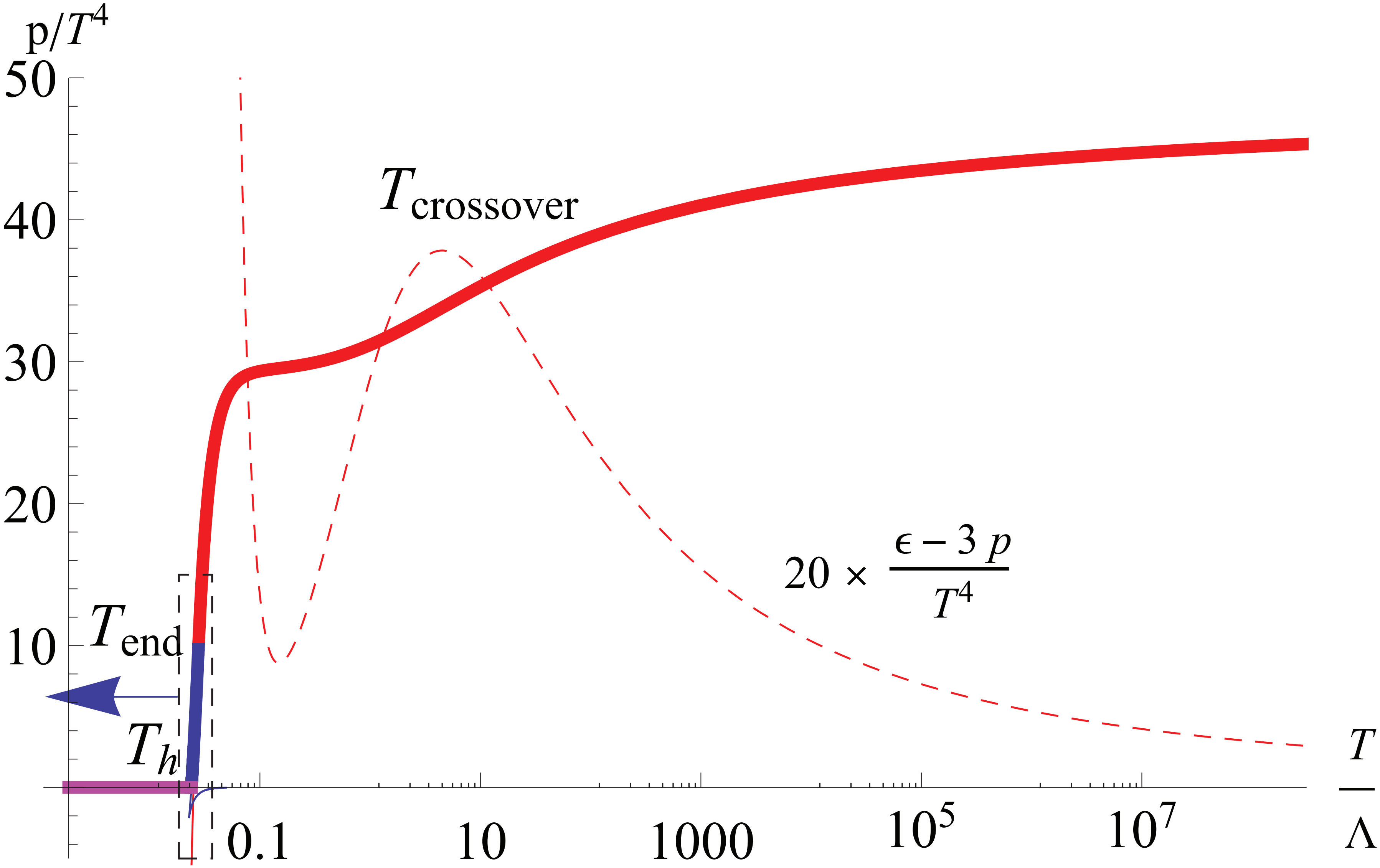
The temperature as a function of λ_h for solutions for Pot II at $x_f = 3\,W_0 = 12/11$, and very small mass . The asymptotic limits are also shown for $m_q = 10^{-5}$, in the range of the figure the UV limit is not yet accurate.



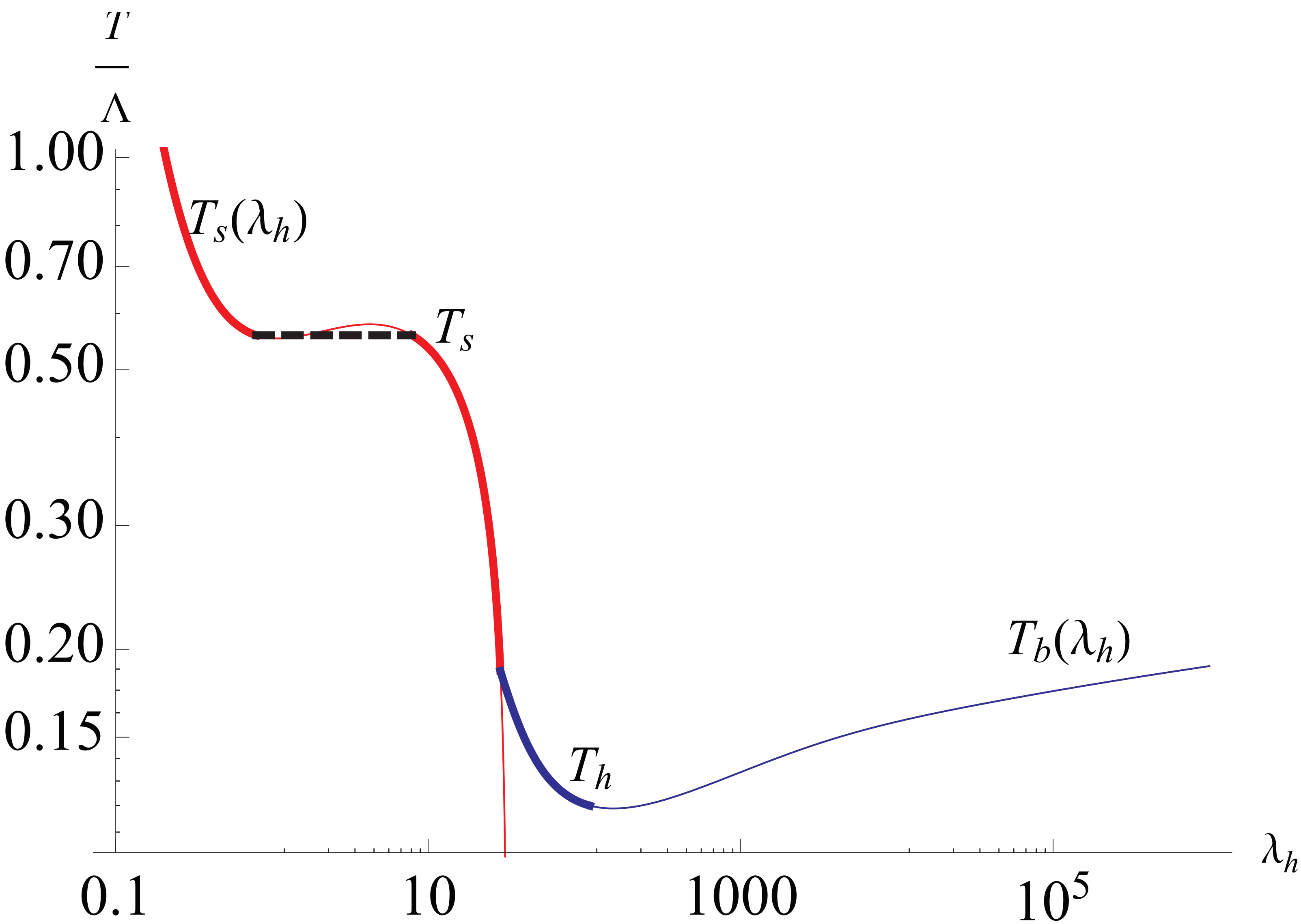
Examples of the T_{end}, T_h and $T_{\text{crossover}}$ transitions in potential II with Stefan-Boltzmann -normalization of $\mathcal{L}_{\text{UV}}$ and with $x_f = 3$. Here: The temperature $T(\lambda_h)$. The curving of $T_s(\lambda_h)$ at $\lambda_h \sim 0.2$, $T \sim 2$ is related to the crossover transition. The inset shows the minimum of $T_b(\lambda_h)$, which causes p_b to be positive between T_h and T_{end}.



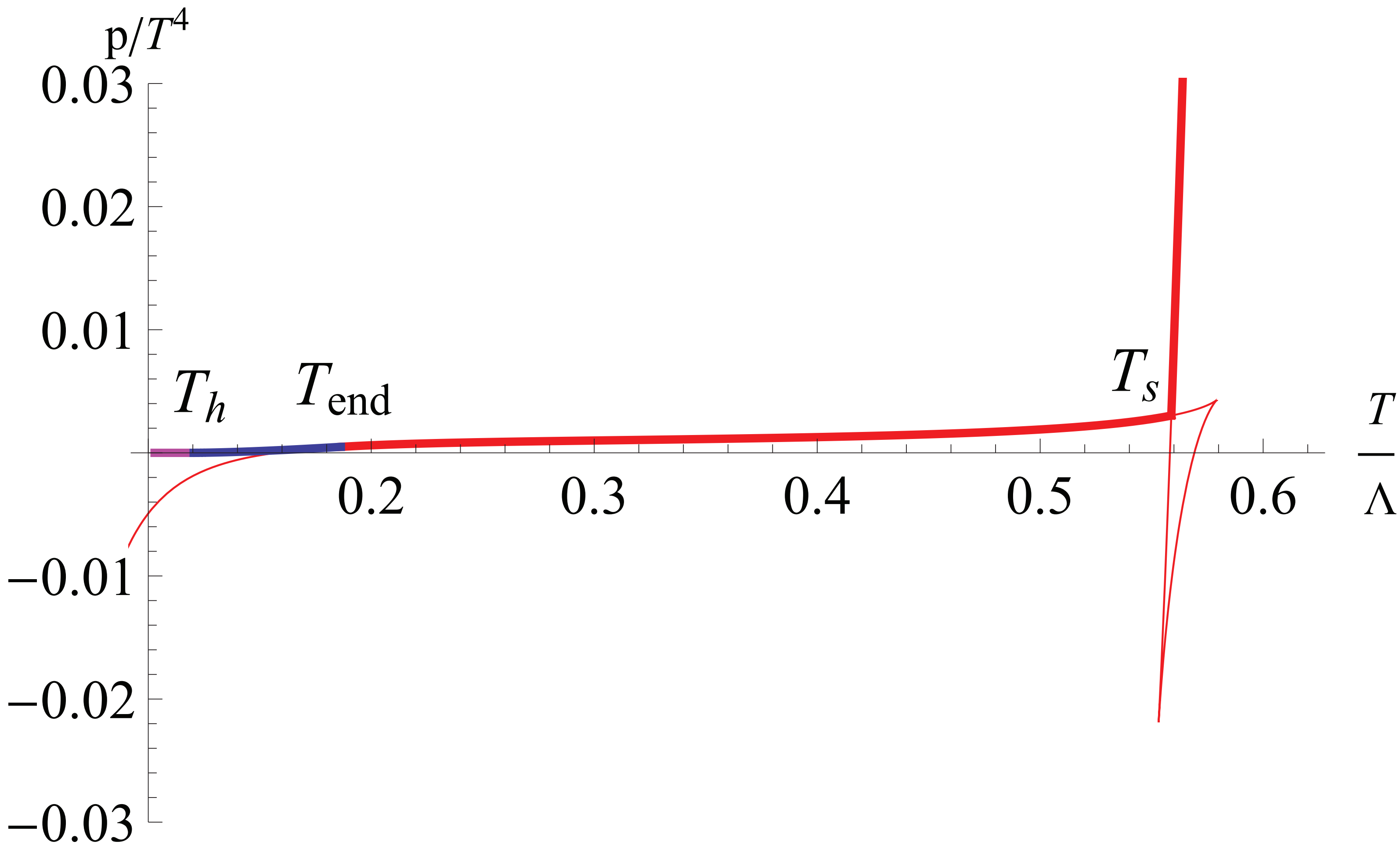
p/T^4 in a close-up around the region of the T_h and T_{end} -transitions.



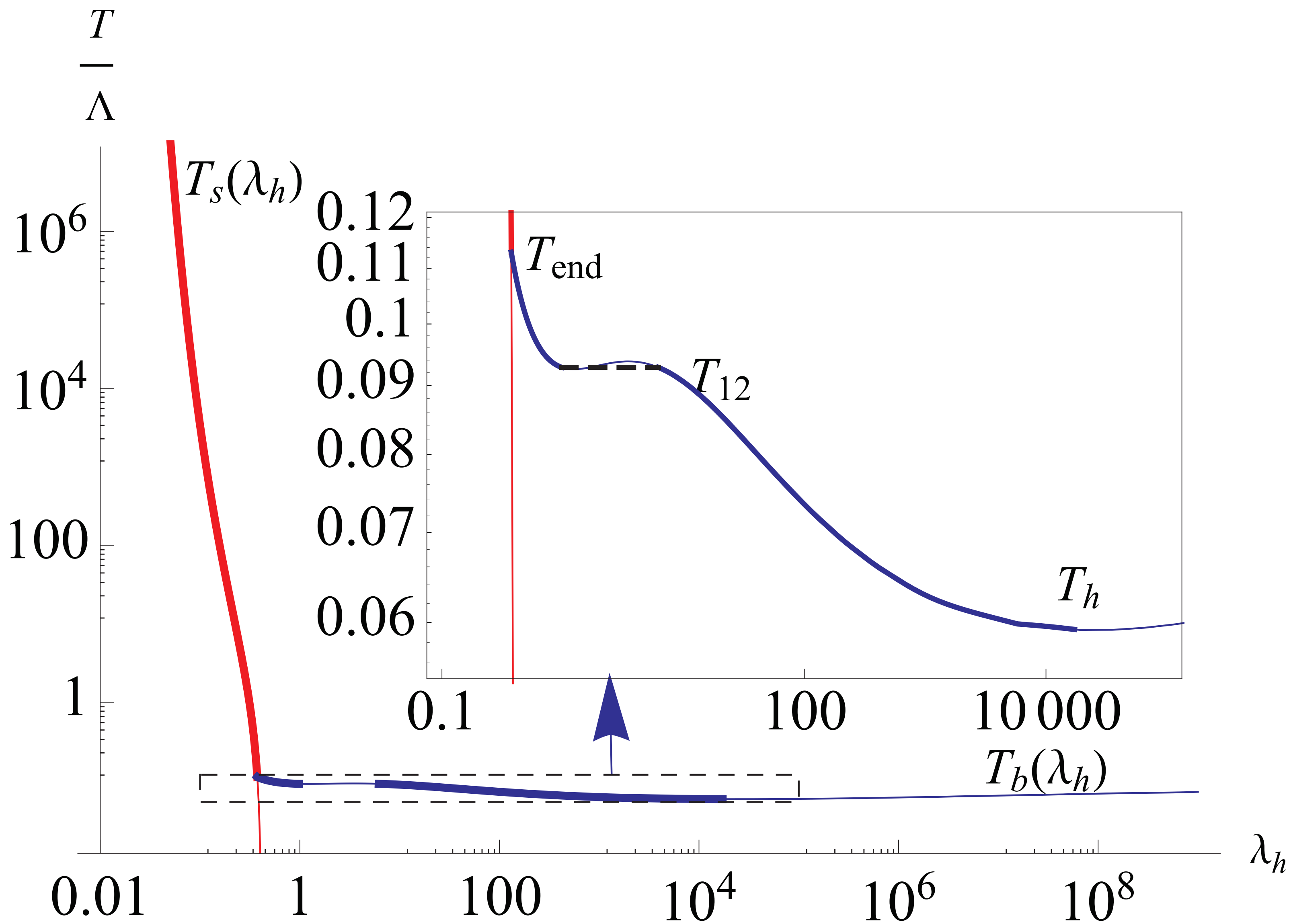
An overview of the pressure in the same case, also showing the interaction measure, which's peak determines the position of $T_{\mathrm{crossover}}$.



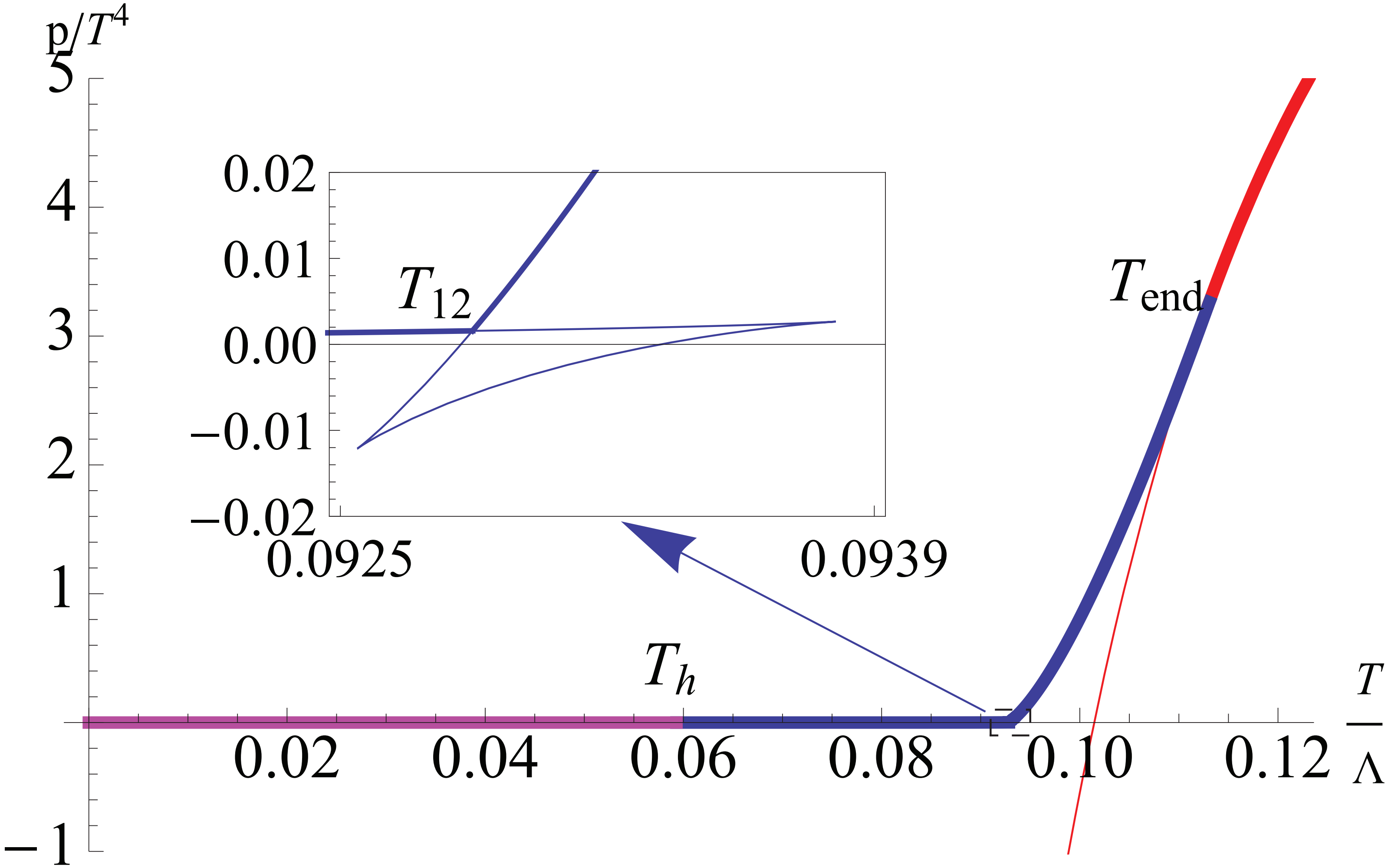
An example of the T_S transition in potential I with $W_0 = 24/11$ and with $x_f = 3$. The local maximum and minimum which generate the first order T_S -transition.



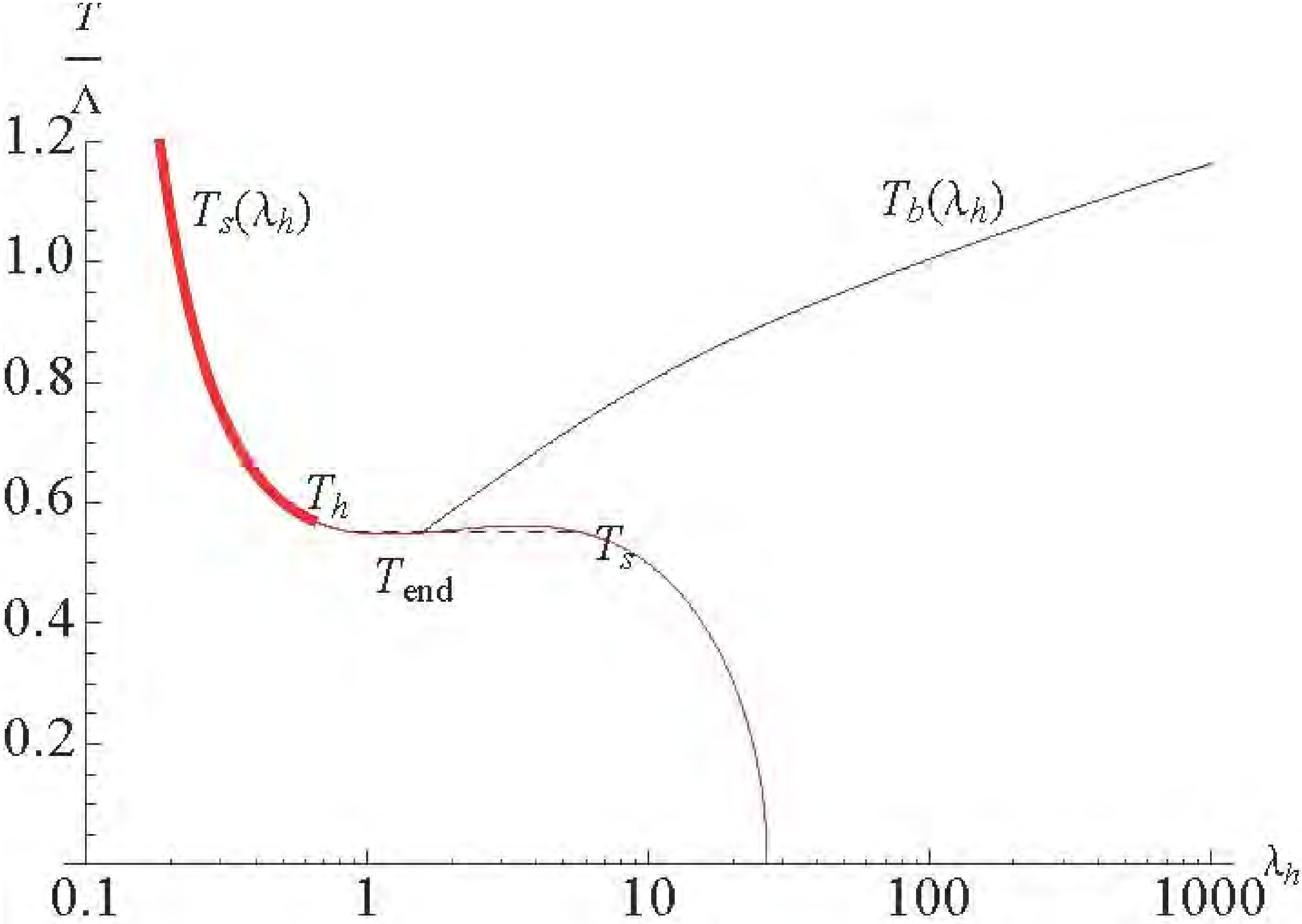
An example of the $T_{\rm S}$ transition in potential I with $W_0 = 24/11$ and with $x_f = 3$. $p(T)/T^4$ in the region around which the first order $T_{\rm S}$ transition takes place, extending to smaller T in order to show the relation to the T_h and $T_{\rm end}$ transitions.



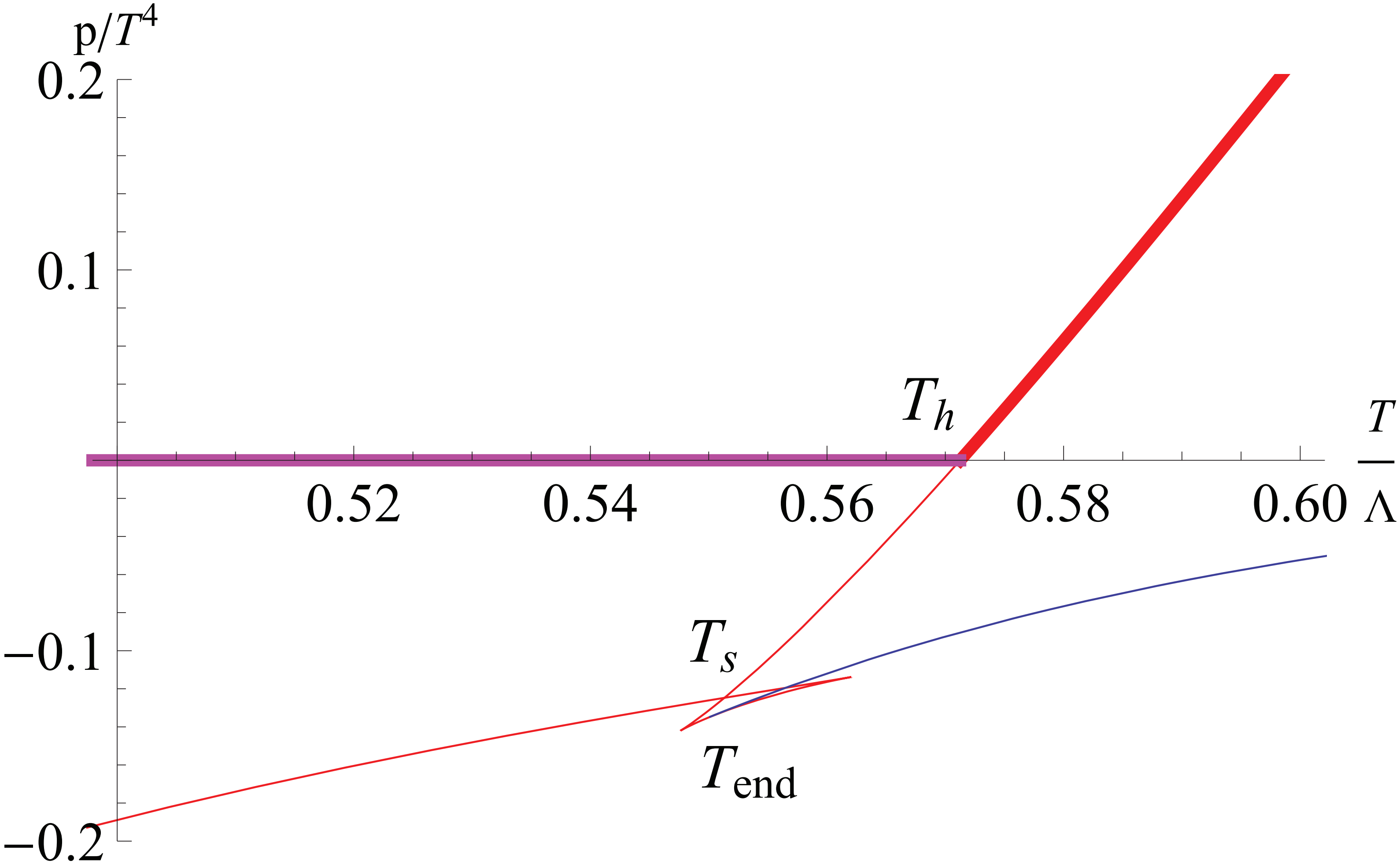
An example of the T_{12} transition in potential I with $W_0 = 12/11$ and with $x_f = 3.5$. The overall structure of $T(\lambda_h)$, with an inset showing the maximum and minimum in more detail.



An example of the T_{12} transition in potential I with $W_0 = 12/11$ and with $x_f = 3.5$. A close-up of $p(T)/T^4$ in the region where the T_{12} -transition happens, with an inset showing further detail.

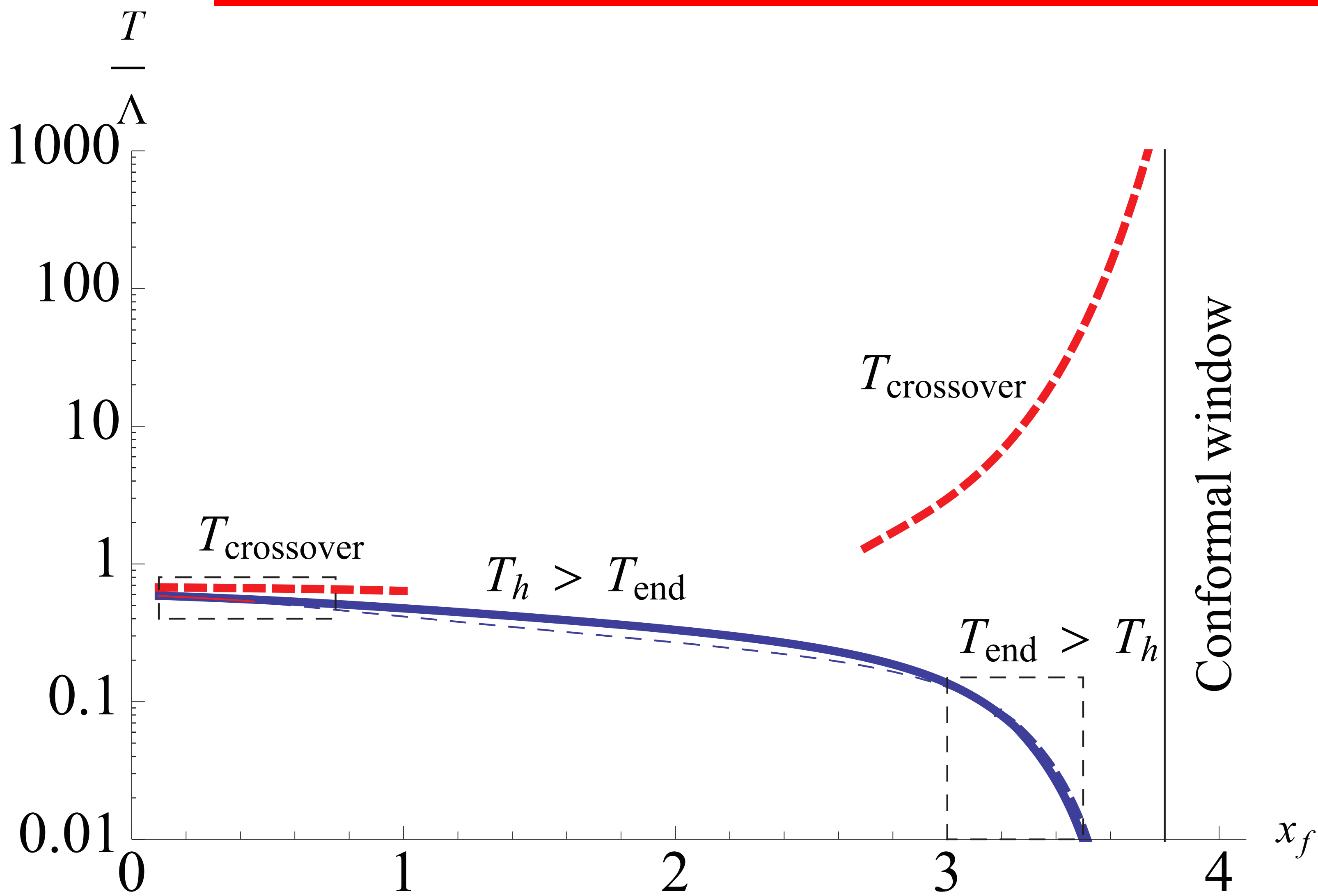


An example of a configuration where all but the crossover and hadronisation transitions $T_{\text{crossover}}$, T_h, are in the thermodynamically unstable region, in the initial stages of the approach to the IHQCD limit. The potential is II with $W_0 = 12/11$ and with $x_f = 0.4$ *Left*:The temperature $T(\lambda_h)$. Note that everything to the right of the T_h transition is in the unstable phase.

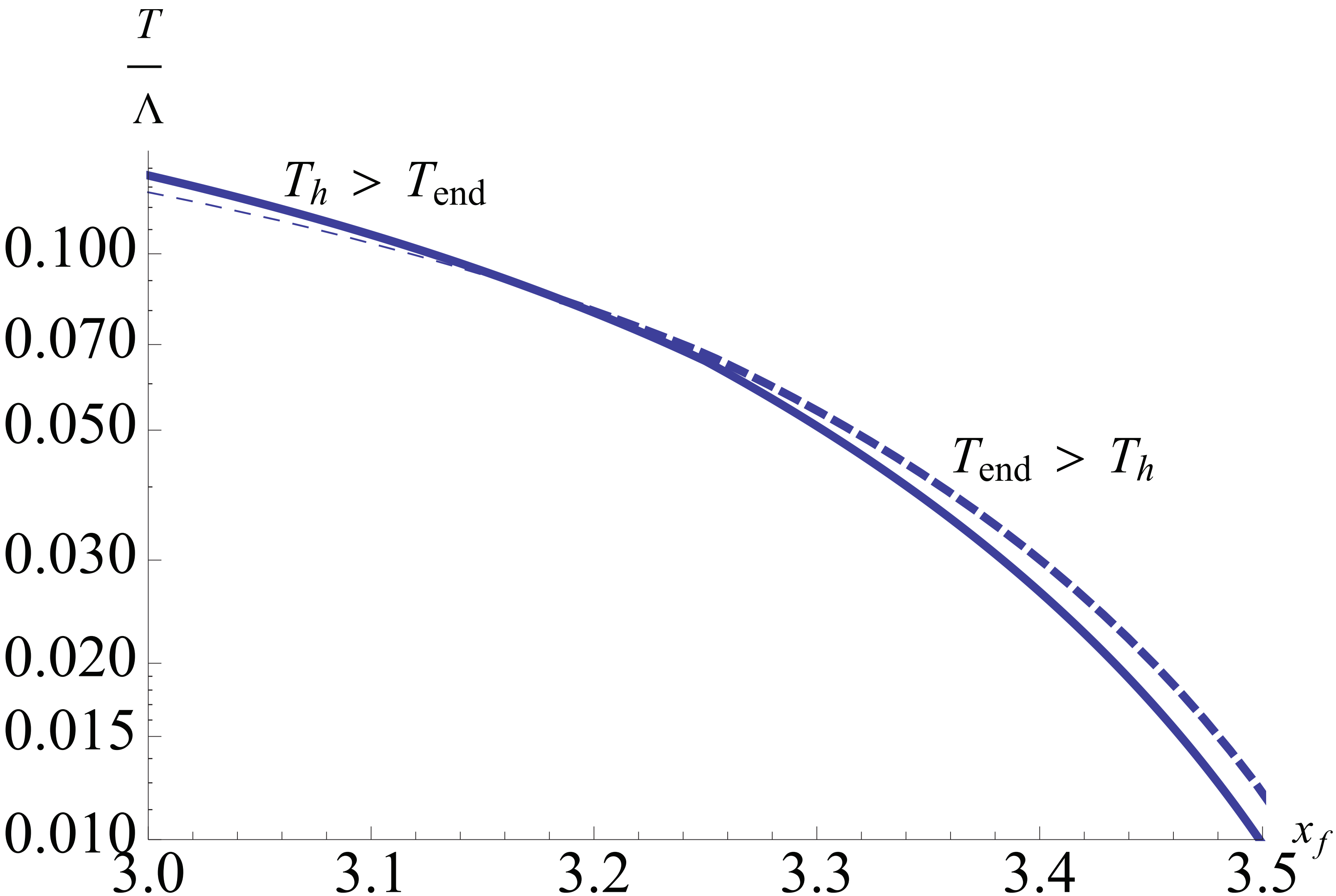


An example of a configuration where all but the crossover and hadronisation transitions $T_{\text{crossover}}$, T_h, are in the thermodynamically unstable region, in the initial stages of the approach to the IHQCD limit. $p(T)/T^4$ in the region where the T_h transition and the unstable T_{end} and T_{s} -transitions happen.

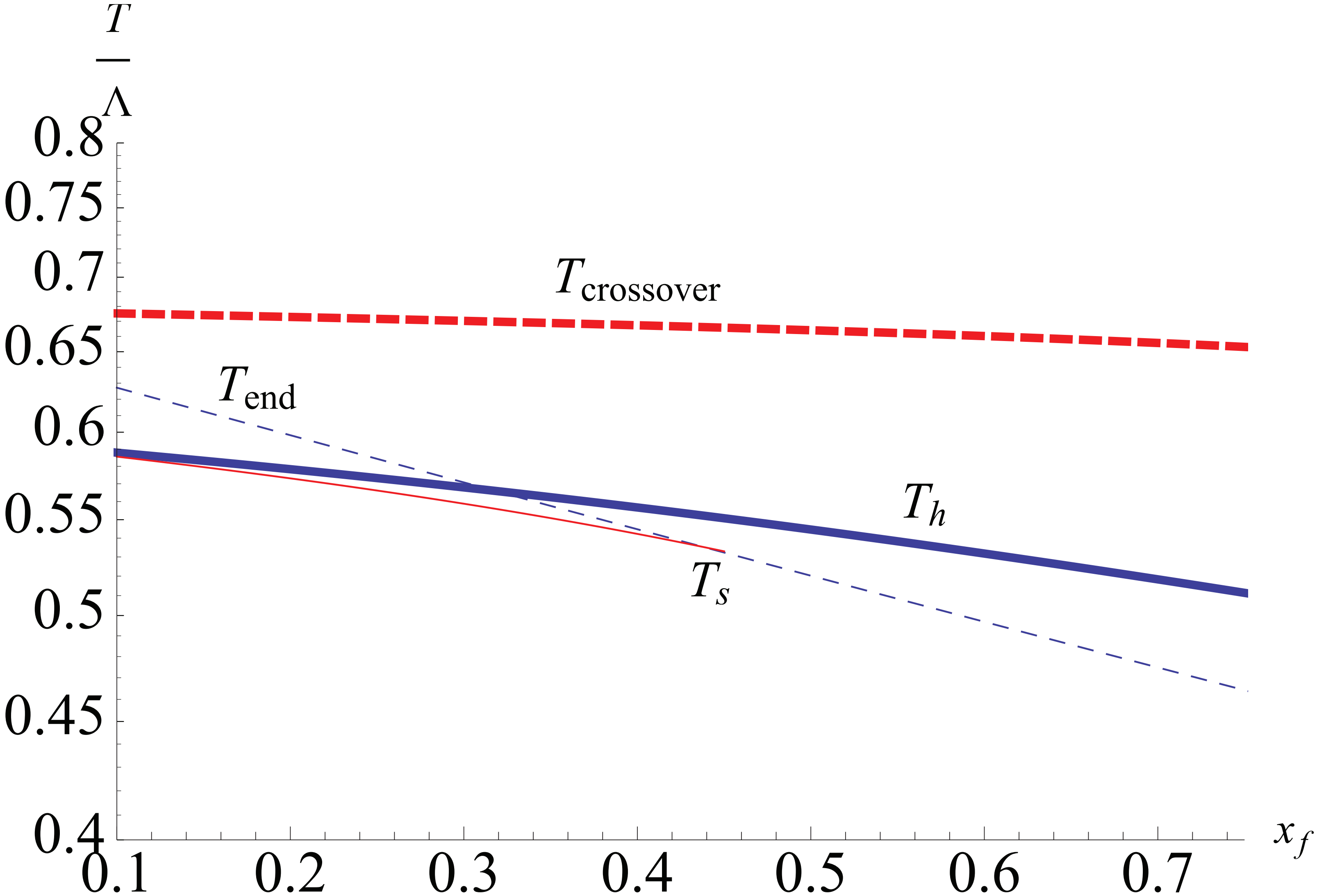
The phase diagram for potential II_2.



The chirally symmetric crossover transition $T_{\text{crossover}}$ is everywhere the highest temperature stable transition, except between $x_f \sim 1$ to $x_f \sim 2.7$, where the interaction measure does not have a maximum and the crossover does not therefore exist.



Above $x_f = 3.19$, the next transition is the second order T_{end}, which goes from the chirally symmetric high-T phase to the chirally broken low-T phase. This is very quickly followed by the T_h transition to the thermal gas solution. Below $x_f = 3.19$ the T_h transition happens first, and therefore T_{end} is in the unstable branch of the solution.



There is further structure at small x_f. a close-up of the small-x_f region. At $x_f \sim 0.4$, the first order T_{S} transition appears in the unstable branch just slightly below T_h. This transition nonetheless develops into the YM -transition at the $x_f \to 0$ -limit. T_{end} crosses above the T_h transition, but it is also in the unstable branch. A close-up of the $x_f \sim 3.2$ -region, where the T_{end} -transition crosses into the unstable branch.

PDEs

• So far the problems addressed involved ODEs.

• There are very interesting problems that involve PDEs. They seem to be a bit more complex from similar problems in asymptotically flat spaces. Some of them are:

• The dynamics of domain or phase walls.

• The collision of energy blobs at high energy. It is dual to the analogue of heavy ion collisions. Some results have been obtained in simplified cases using Penrose's trapped surface analysis.

Kovchegov+Taliotis, Shuryak+Lin, Gubser+Yarom, Kiritsis+Taliotis

• The effect of spatially inhomogeneous boundary conditions on the solutions.

• The instability of translationally invariant solutions in the presence of CP-odd couplings.

ETC....

Outlook

• The holographic duality provides a new set of problems, and a new perspective for gravity.

• Most of these problems can be phrased also in the language of QFT.

• Some of them are known whereas others are new.

• Most of them are unsolved.

• Their solution is expected to provide non-trivial insights both for strongly coupled QFT and gravitational physics.

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THANK YOU

The holographic effective potential

Niarchos+E.K

• In QFT a very useful tool is the quantum effective potential: it is the Legendre transform of the source action, and is a function of the expectation values of fields.

It is a valuable tool in the investigation of dynamical symmetry breaking and the study of phase transitions.

• The analogous concept in holography is in principle computable, but has not been used so far (but for a few exceptions).

• I will outline the formalism in a model class of theories: EMD

$$S = M_P^{p-1}\int d^{p+1}x\sqrt{g}\left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - Z(\phi)F^2\right] + \text{boundary terms}$$

$$ds^2 = e^{2A(u)}\left(-f(u)dt^2 + dx^i dx^i\right) + \frac{du^2}{f(u)}\ , \quad \mathbf{A} = A_t(u)dt\ , \quad \phi = \phi(u)$$

• We will

(a) Find the classical solution with temperature T, charge density ρ, and scalar source $\phi = \phi_0 = constant$ in (t, x^i).

(b) Evaluate the on-shell action, $S_{on-shell}(\phi_0)$.

(c) Legendre transform in ϕ_0 to obtain the effective potential $V_{eff}(\phi_c; T, \rho, M)$ as a function of the classical field $\phi_c = \frac{\partial S_{on-shell}(\phi_0)}{\partial \phi_0}$ and the RG scale $M = e^{A_0}$, at which all field variables are defined.

• The key step here is to introduce the "superpotential" $W(\phi)$ that will be related both to the effective potential and the holographic β-function.

Gursoy+E.K.+Nitti

$$\frac{d\phi}{du} = \frac{dW(\phi)}{d\phi} \quad , \quad \frac{dA}{du} = -\frac{W(\phi)}{2(p-1)} \quad \Rightarrow$$

$$\Rightarrow \quad \frac{d\phi}{dA} = \frac{d\phi}{d\log M} = -2(p-1)\ \partial_\phi \log W = \beta(\phi)$$

• Note that the ϕ equation is solvabe with a single initial condition: $\phi(A_0) \equiv \phi_0$. The vev is hidden in the determination of W.

• The equations of motion for the unknown functions: $W(\phi), A(\phi), f(\phi)$ become

$$\frac{\mathcal{R}'}{\mathcal{R}} = \frac{W}{W'}\ , \quad \mathcal{R} \equiv e^{-2(p-1)A} \tag{1a}$$

$$W'(W'f')' - \frac{pWW'}{2(p-1)}f' = \frac{\rho^2 \mathcal{R}}{Z} \tag{1b}$$

$$\left(\frac{pW^2}{2(p-1)} - W'^2\right) f - WW'f' = 2V - \frac{\rho^2 \mathcal{R}}{Z} \tag{1c}$$

The second order equation can be integrated to a first order one:

$$f' = \frac{e^{-dA}}{W'}\left[D + \rho^2 \int_{\phi_0}^{\phi} \frac{d\chi}{e^{(d-2)A(\chi)} Z(\chi) W'(\chi)}\right]$$

$$D = -4\pi e^{(d-1)A_0}\, T\ S - \rho^2 \int_{\phi_0}^{\phi_h} \frac{d\tilde{\phi}}{e^{(d-2)A} Z W'}$$

- The three constants of integration amount to $T, \langle\phi\rangle$ and the RG scale $M = e^{A_0}$.

- $\langle\phi\rangle$ is tuned to the value that makes the bulk solution "regular".

- We now calculate the Free Energy (on-shell action):

$$\mathcal{F} = S_{on-shell} = M_P^{p-1}\beta V_{p-1}\ e^{pA_0}\ \left(-W + \dot{f}\right)_{u=u_0}$$

and from the equations we obtain $Z(\phi_0, T, \rho, A_0)$.

$$Z = \frac{\mathcal{F}}{M_P^{p-1}\beta V_{p-1}} = -e^{pA_0}W(\phi_0) - 4\pi e^{(p-1)A_0}\ T\ S + \rho^2 \int_{\phi_h}^{\phi_0} \frac{d\tilde{\phi}}{e^{(p-2)A} Z W'}$$

- Z is the single-trace effective action for the source ϕ_0. The full effective action contains possible multitrace deformations:

$$Z_{total} = Z + Z_{\text{multi-trace}} \quad , \quad Z_{\text{multi-trace}} = \sum_{n=2}^{\infty} \frac{g_n}{N^{n-2}}\ \phi_0^n$$

- The Legendre transform of Z with respect to ϕ_0 is the effective potential, $V_{eff}(\phi_c;\ T, \rho, A_0)$.

- The effective potential, is evaluated at an RG scale $M = e^{A_0}$.

- ϕ_c depends implicitly on A_0 as determined by the bulk flow equations.

- RG invariance $(T = \rho = 0)$:

$$\frac{d}{dA_0} Z = 0$$

- In the scaling region around an IR or UV fixed point, V_{eff} can be obtained by a perturbative calculation. It is in general non-polynomial in ϕ_c and provides a generalization of the LG ansatz.
- From scaling:

$$\phi_r \equiv e^{-(p-\Delta)A_0}\phi_0 \quad , \quad \hat{T} \equiv e^{-A_0}T \quad , \quad \hat{\rho} \equiv e^{(2-p)A_0}\rho$$

$$Z = \phi_r^{\frac{p}{p-\Delta}} \zeta(\phi_r^{\frac{1}{p-\Delta}}, \hat{T}, \hat{\rho}) \quad , \quad \lim_{A_0 \to \infty} \zeta = \text{constant}$$

- Transition temperatures can be calculated directly via perturbation theory if they occur in the scaling region.

The effective action

• We can go beyond the effective potential, to the first terms in the effective action

$$S_{source} = \int d^4x\sqrt{g}\left[U(\phi)R - \frac{1}{2}Z(\phi)(\partial\phi)^2 + V(\phi) + \cdots\right]$$

• V was calculated already

• U can be calculated by turning on constant spacial curvature

$$U(\phi_0) = -\int_{\infty}^{\phi_0} \frac{d\phi}{W'} e^{-\frac{1}{4}\int_{\phi_0}^{\phi}\frac{W}{W'}d\phi}$$

at zero temperature and density.

• Calculating $Z(\phi)$ is more complicated.

• Such effective actions are very useful both in condensed matter (generalizations of the LG framework) and cosmology (inflaton as a strongly coupled bound state)

Effective Holographic Theory Program

The strategy advocated in

Charmousis+Gouteraux+Kim+E.K.+Meyer

is:

1. Select the operators expected to be important for the dynamics

2. Write an effective (gravitational) holographic action that captures the (IR) dynamics by parametrizing the IR asymptotics of interactions .

3. Find the scaling solutions describing extremal saddle points. Built the $T \to 0$ bh solutions around them

4. Study the physics around each acceptable saddle point.

This strategy started bearing fruit as it dealt with

• Einstein-Maxwell-Dilaton theories with the most general AdS and Lifshitz asymptotics.

Charmousis+Gouteraux+Kim+E.K.+Meyer, Gouteraux+E.K.

• Einstein-Maxwell-Dilaton theories with also massive asymptotics and non-abelian (Bianchi) scaling symmetries.

Iizuka+Kachru+Kundu+Narayan+Sircar+Trivedi

• Einstein-Maxwell theories with CP-couplings and a magnetic field.

Donos+Gauntlett

• Einstein-Maxwell-Dilaton+axion theories with broken rotational symmetry.

Iizuka+Maeda

• Einstein-Maxwell-Scalar theories in the symmetry broken regime (to be described later in this talk).

Gouteraux+E.K.

• Einstein-two-scalar theories (special classes to be described later in this talk)

Jarvinnen+E.K.

• The bulk metric $g_{\mu\nu} \leftrightarrow T_{\mu\nu}$ is always sourced in any theory. In CFTs it captures all the dynamics of the stress tensor and the solution is AdS_{p+1}.

• In a theory with a conserved U(1) charge, a gauge field is also necessary, $A_\mu \leftrightarrow J_\mu$. If only $g_{\mu\nu}, A_\mu$ are important then we have an AdS-Einstein-Maxwell theory with saddle point solution=AdS-RN.

• The thermodynamics and CM physics of AdS-RN has been analyzed in detail in the last few years, revealing rich physical phenomena
Chamblin+Emparan+Johnson+Myers (1999), Hartnoll+Herzog (2008), Bak+Rey (2009),Cubrovic+Schalm+Zaanen (2009), Faulkner+Liu+McGreevy+Vegh (2009)

1. Emergent AdS_2 scaling symmetry in the IR at finite density

2. Interesting fermionic correlators

and also

3. Is unstable (in N=4) to both neutral and charged scalar perturbations
Gubser+Pufu (2008), Hartnoll+Herzog+Horowitz (2008)

4. Has a non-zero (large) entropy at $T = 0$.

Einstein-Scalar-U(1) theory

- To go beyond RN, we must include the most important (relevant) scalar operator in the IR.

- The most general 2d action (after field redefinitions) is

$$S = \int d^{p+1}x\sqrt{g}\left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - Z(\phi)F^2\right]$$

- It involves two arbitrary functions of ϕ.

Consider first the zero density case:

- There are two types of critical points.

♠ Standard (AdS) critical points: $V'(\phi_*) = 0$ for finite ϕ_*. This is a standard IR or UV fixed point at zero density (depending whether $V''(\phi_*)$ is positive or negative).

♠ "decompactication" asymptotics, $\phi_* \to \pm\infty$. These correspond to geometric "singularities" (sometimes decompactification) in string theory.

These also lead to scale invariant saddle points despite the fact that the extremal solutions have a nontrivial running for ϕ. To find the leading physics at extremality it is enough to parametrize

$$V(\phi) \sim e^{-\delta\phi} \quad , \quad Z(\phi) \sim e^{\gamma\phi} \quad , \quad \phi \to \pm\infty$$

• γ,δ capture the leading physics except if $|\delta| = \sqrt{\frac{2}{p-2}}$.

Finite Density scaling

The fate of zero density Quantum Critical asymptotics, at finite density is as follows:

♠ Standard (AdS) critical points: There is a new density dependent "effective potential" for ϕ

Goldstein+Iizuka+Kachru+Prakash+Trivedi+Westphal

$$V_{eff} = V(\phi) - \frac{q^2}{Z(\phi)}$$

and generically there is a new fixed point at ϕ_{**} at a special density q_*.

$$V'(\phi_{**}) = q_*^2 \frac{Z'(\phi_{**})}{Z^2(\phi_{**})} \quad , \quad V(\phi_{**}) = \frac{2q_*^2}{Z(\phi_{**})}$$

E.K.+Meyer

• If $Z'(\phi_*) = 0$ then $\phi_* = \phi_{**}$. This is the generalization of Reissner-AdS case with the usual IR AdS_2 geometry. In the near IR region, the AdS-RN bh is a solution.

• $Z'(\phi_*) \neq 0$. There is a new QC point at a special value of the density. The metric is $AdS_2 \times R^n$.

Scaling IR asymptotics

• In the IR-AdS region, the IR-extremal metrics are AdS_{p+1} at zero density and AdS_2 at finite density.

• In the case of runaway $\phi \to \pm\infty$ QC points, with $V \sim e^{-\delta\phi}$, $Z \sim e^{\gamma\phi}$, the extremal metrics are general scaling metrics of the form

$$ds^2 = \frac{dr^2}{r^2} + \frac{-dt^2 + dx^i dx^i}{r^{2a}}$$

at zero density and

$$ds^2 = \frac{dr^2}{r^2} - \frac{dt^2}{r^{2a}} + \frac{dx^i dx^i}{r^{2b}}$$

at finite density.

• Their near-extremal asymptotics (small temperatures) are also simply constructed.

• In several cases, the extremal metrics are solutions to the full equations. (as with exponential potentials)

The hidden scale invariance

Gouteraux+E.K.

- At zero density:

$$ds^2 = \frac{dr^2}{f} + \frac{(-f dt^2 + dx \cdot dx)}{r^{-\frac{4}{(p-1)\delta^2}}} \quad , \quad f = 1 - \left(\frac{r_0}{r}\right)^{\frac{2p}{(p-1)\delta^2}-1} \quad , \quad e^{\delta\phi} \sim r^2$$

Changing variables

$$w = r^{1-\frac{2}{(p-1)\delta}}$$

$$ds^2 = e^{2\chi(r)}\left[\frac{dw^2}{w^2 f(w)} + \frac{-f(w)dt^2 + dx \cdot dx}{w^2}\right] \quad , \quad e^{2\chi} \sim r^2 \sim e^{\delta\phi} \sim \frac{1}{V(\phi)}$$

- This is conformal to the AdS-Schwarzschild black hole.

- It is a scaling solution that violates hyperscaling.

- Such solutions can be obtained by dimensional reduction from a higher dimensional theory without a scalar.

• When $\delta^2 < \frac{2}{p-1}$ this is the dimensional reduction of an AdS_{p+1+n} solution on T^n with

$$\delta^2 = \sqrt{\frac{1}{1+\frac{p-1}{n}}} \cdot \frac{2}{p-1} \quad \leq \frac{2}{p-1}$$

Gubser+Nellore, Skenderis+Taylor

• This explains the continuous spectrum and absence of mass gap for $\delta^2 < \frac{2}{p-1}$.

• Therefore, the theory is quantum critical in the IR, despite the non-trivial potential.

• The singularity is resolved by the KK-modes (oxydation). The IR scale becomes the AdS scale in the higher dimensions.

• Different δ can be obtained by extending to real $n > 0$.

• The crossover value $\delta^2 = \frac{2}{p-1}$ is obtained when $n \to \infty$.

• Dimensional Reduction of AdS_{p+1+n} solution on S^n

Gouteraux+E.K.

$$\delta^2 = \frac{2}{p-1} + \frac{2}{n} \quad \geq \frac{2}{p-1}$$

and a naturally discrete spectrum and mass gap.

• Violation of the Gubser bound: $n \leq 1$. Marginal case: $n \to \infty$.

• The theory is again quantum critical in the IR,

Scaling and hyperscaling at finite density

• The extremal solutions for all (γ,δ) are simple powers, and therefore scaling.
• The metric can always be written as

Gouteraux+E.K.

$$ds^2 = e^{\chi}\; d\hat{s}^2 \quad , \quad e^{\chi} \sim e^{\delta\phi} \quad , \quad d\hat{s}^2 = -\frac{dt^2}{w^{2z}} + \frac{dw^2 + dx^i dx^i}{w^2}$$

with

$$z = \frac{(\gamma-\delta)(\gamma+(2p-3)\delta)+2(p-1)}{(\gamma-\delta)(\gamma+(p-2)\delta)}$$

• They are conformal to Lifshitz or AdS solutions.

$$x^i \to \lambda x^i \quad , \quad w \to \lambda w \quad , \quad t \to \lambda^z\, t \quad , \quad ds^2 \to \lambda^{\theta}\, ds^2 \quad , \quad \theta = \frac{2(p-1)\delta}{\gamma+(p-2)\delta}.$$

• θ, the hyperscaling exponent, is set by the scaling of the inverse scalar potential, and controls the violation of hyperscaling.

• They can be also written in a different frame as

Huisje+Sachdev+Swingle

$$ds^2 = \frac{dr^2}{r^{4\frac{\theta-1}{\theta-2}}} - \frac{dt^2}{r^{2\frac{\theta-2z}{\theta-2}}} + \frac{dx^i dx^i}{r^2}$$

with scaling transformations

$$x^i \to \lambda\ x^i \quad , \quad r \to \lambda^{1-\frac{\theta}{2}}\ r \quad , \quad t \to \lambda^z\ t \quad , \quad ds^2 \to \lambda^\theta\ ds^2$$

• Most of these can be lifted to solutions in higher dimensions with generalized scaling symmetry (Boosted AdS black-holes or black AdS q-branes).

• They represent the most general critical behavior at zero temperature, generalizing the AdS and Lifshitz geometries.

• Note that at $\gamma + (p-2)\delta = 0$ we obtain an $AdS_2 \times R^2$ geometry at extremality but with $S = 0$.

• Like the zero density case, they are dimensional reductions of regular or Lifshitz higher-dimensional solutions.

• The higher-dimensional theories are of the following types:

$$S=\int d^{p+q+1}x\sqrt{G}\,[R+2\Lambda]\,.$$

reduced along a torus with a boost.

$$S=\frac{1}{16\pi G_D}\int \mathrm{d}^{p+q+1}x\,\sqrt{-g}\left[R-\frac{1}{2(n+2)!}G^2_{[n+2]}\right].$$

reduced on a sphere.

$$S=\frac{1}{16\pi G_D}\int \mathrm{d}^{p+q+1}x\,\sqrt{-g}\left[R-\frac{1}{2(q+2)!}G^2_{[q+2]}+2\Lambda\right],$$

reduced on a torus.

• The spectra (continuous vs discreet) follow from the curvature of the internal space.

• The thermodynamic variables have the natural scaling of the higher-dimensional theory.

The higher-dimensional picture:

1. Explains the near T=0 scaling behavior.

2. Explains the qualitative difference between EHTs with $C_p < 0$ and $C_p > 0$. In the neutral case it explains the crossover value, δ_c.

3. Provides and alternative view of the Gubser bound.

4. Provides one possible resolution of the zero temperature naked singularity of the original solution.

5. Gives a direct and efficient way to compute the scaling transport coefficients by dimensionally reducing scale invariant hydrodynamics.

Gouteraux+(Smolic)^2+Skenderis+Taylor

(Lifshitz) Scaling in the broken-symmetry phase

• The minimal description of the broken symmetry phase contains the metric, a gauge field and a complex scalar ($D_\mu\Psi = \partial_\mu\Psi + iqA_\mu\Psi$)

$$S = M^2 \int d^4x\sqrt{-g}\left[R - \frac{G(|\Psi|)}{2}|D\Psi|^2 + \tilde{V}(|\Psi|) - \frac{\tilde{Z}(|\Psi|)}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

• In the broken phase, Ψ is non-trivial, $\Psi = \chi\ e^{i\theta}$. Choose the gauge $\theta = 0$ and change variables $\chi \to \phi$ so that the kinetic term of ϕ is properly normalized

$$S = M^2 \int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{W(\phi)}{2}A_\mu A^\mu\right]$$

• Again the interesting IR behavior appears if V,W,Z have extrema, or decompactification (exponential) behavior.

• It can be shown, that both at finite ϕ, or runaway ϕ with exponential IR asymptotics for V, Z, W, we obtain generalized Lifshitz scaling in the IR geometry.

• The Lifshitz exponent z depends non-trivially on the IR asymptotics of the EHT.

Gouteraux+E.K

Constant scalar

- $\phi = \phi_*$ is constant and

$$ds^2 = B_0 \frac{dr^2}{r^2} + \frac{dx^2 + dy^2}{r^2} - \frac{dt^2}{r^{2z}} \quad , \quad A_t = Q\ r^{-z}$$

$$Q^2 = \frac{2(z-1)}{zZ(\phi_*)} \quad , \quad B_0 = 2z \frac{Z(\phi_*)}{W(\phi_*)}$$

with the Lifshitz exponent z satisfying

$$z^2 + \left(1 - \frac{2V(\phi_*)Z(\phi_*)}{W(\phi_*)}\right) z + 4 = 0$$

and

$$\frac{V'_*}{V_*} + \frac{2(z-1)}{z^2+z+4} \frac{W'_*}{W_*} + \frac{z(z-1)}{z^2+z+4} \frac{Z'_*}{Z_*} = 0$$

- This has non-trivial real solutions unless

$$-\frac{3}{2} \leq \frac{V(\phi_*)Z(\phi_*)}{W(\phi_*)} \leq \frac{5}{2}$$

- When $\frac{V(\phi_*)Z(\phi_*)}{W(\phi_*)} = 3$ we obtain $z = 1$ namely AdS.

Running scalar

- A decompactification case

$$V(\phi) = V_0 e^{-\delta\phi} \quad , \quad Z(\phi) = Z_0 \; e^{\gamma\phi} \quad , \quad W(\phi) = W_0 e^{(\gamma-\delta)\phi}$$

- We also obtain a Lifshitz geometry in the IR with

$$z = \frac{\epsilon(\epsilon - 2\gamma)x + 2(\epsilon^2 - \epsilon\gamma - 2)}{(\epsilon^2 + 2\gamma^2 - 4\epsilon\gamma - 2)x + 2\epsilon(\epsilon - \gamma)}$$

with

$$(4-\epsilon^2+4\epsilon\gamma)x^2+\left(2 - 2\epsilon^2 + 2\gamma^2 + (-4 + \epsilon^2 - 4\epsilon\gamma)\frac{V_0 Z_0}{W_0}\right)x+\left(4 + 2\epsilon^2\frac{V_0 Z_0}{W_0}\right) = 0$$

- In the rest of the cases we obtain, Lifshitz geometries or generalized Lifhitz geometries (with hyperscaling violation).

BKT scaling, II

We can derive

$$\Delta_{\rm IR}(4-\Delta_{\rm IR}) = -m_{\rm IR}^2 \ell_{\rm IR}^2 = G(\lambda_*, x) \ ,$$

where

$$G(\lambda, x) \equiv \frac{24 a(\lambda)}{h(\lambda)(V_g(\lambda) - x V_{f0}(\lambda))} \ .$$

and by matching behaviors

$$\sigma \sim \frac{1}{r_{\rm UV}^3} \exp\left(-\frac{2K}{\sqrt{\lambda_* - \lambda_c}}\right) \sim \frac{1}{r_{\rm UV}^3} \exp\left(-\frac{2\hat{K}}{\sqrt{x_c - x}}\right) \ .$$

x_c and λ_c are defined by $G(\lambda_*(x_c), x_c) = 4$ and $G(\lambda_c, x) = 4$, respectively, so that $\lambda_* = \lambda_c$ at $x = x_c$. we obtain

$$K = \frac{\pi}{\sqrt{\frac{\partial}{\partial \lambda} G(\lambda_c, x)}} \ ; \qquad \hat{K} = \frac{\pi}{\sqrt{-\frac{d}{dx} G(\lambda_*(x), x)\Big|_{x=x_c}}} \ .$$

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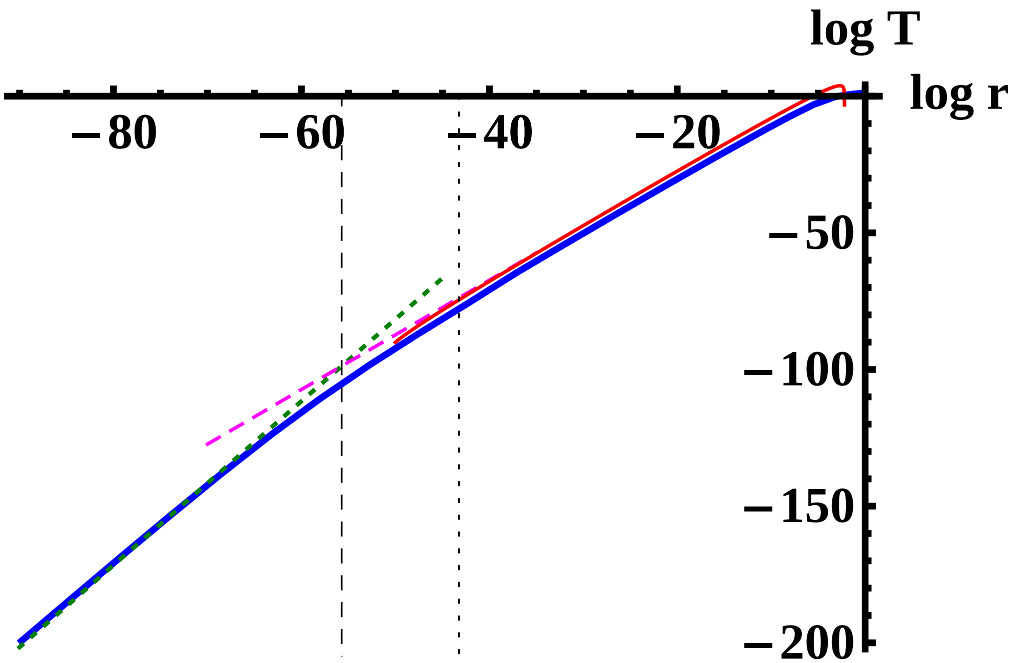
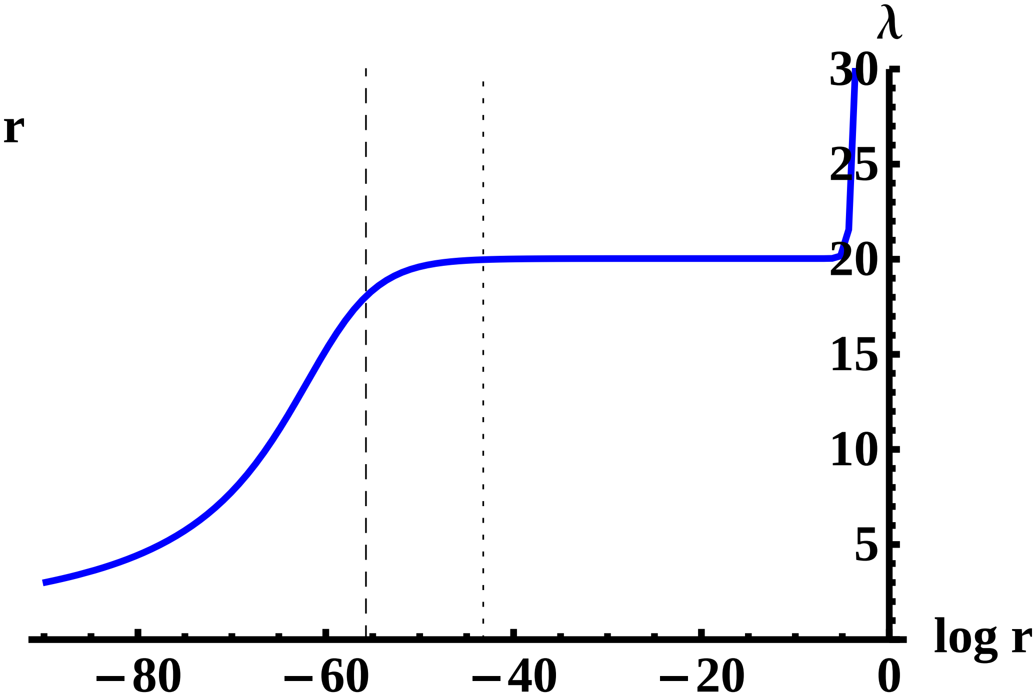
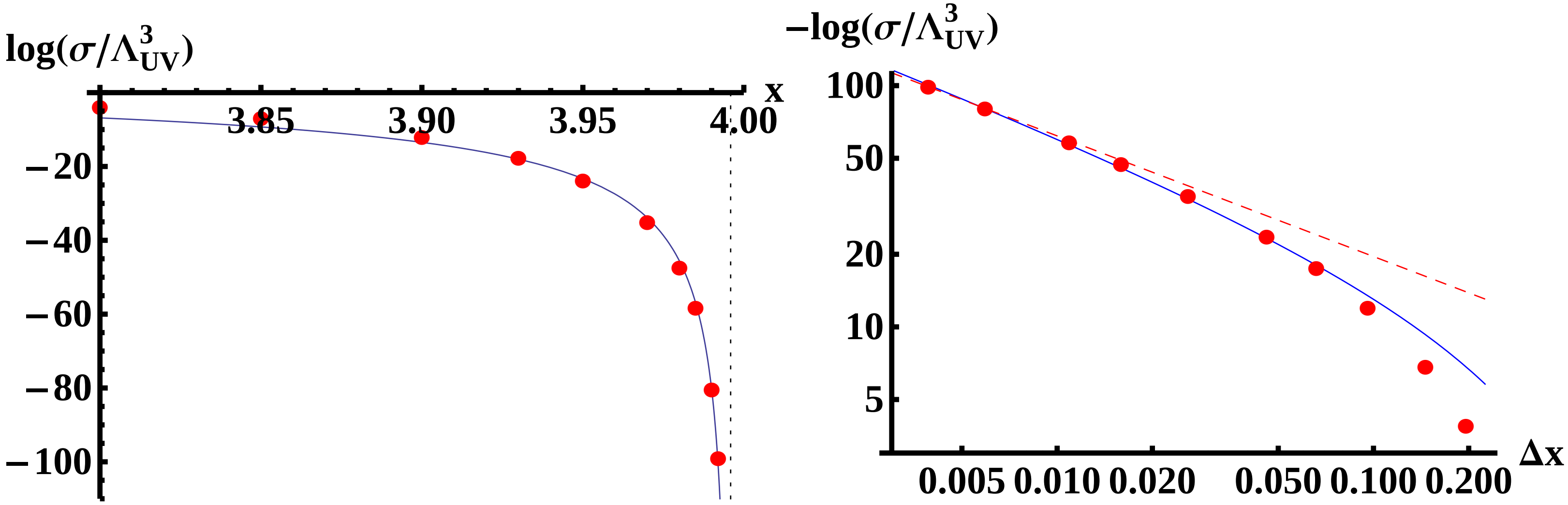
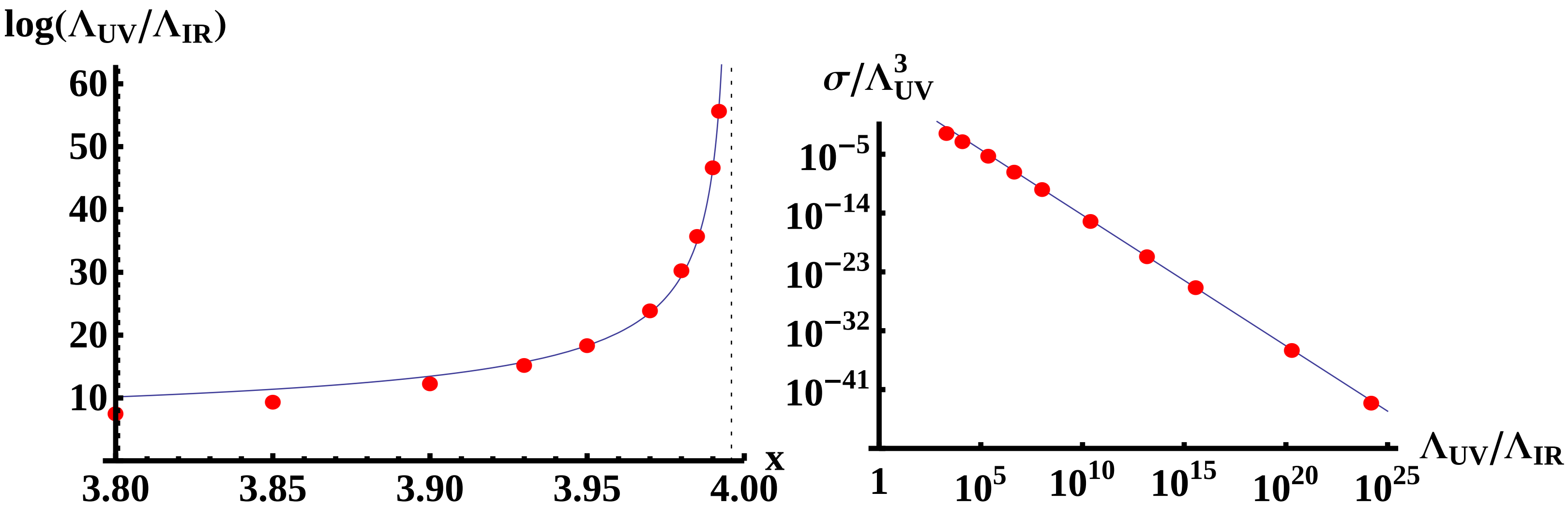




The tachyon $\log T$ (left) and the coupling λ (right) as functions of $\log r$ for an extreme walking background with $x = 3.992$. The thin lines on the left hand plot are the approximations used to derive the BKT scaling.



Left: $\log(\sigma/\Lambda^3)$ as a function of x (dots), compared to a BKT scaling fit (solid line). The vertical dotted line lies at $x = x_c$. Right: the same curve on log-log scale, using $\Delta x = x_c - x$.



Left: $\log(\Lambda_{UV}/\Lambda_{IR})$ as a function of x (dots), compared to a BKT scaling fit (solid line). Right: σ/Λ^3 plotted against $\Lambda_{UV}/\Lambda_{IR}$ on log-log scale.

Effective potential and phase transitions

Iqbal+Liu+Mezei+Si, Jensen, Faulkner+Horowitz+Roberts

• In the scaling region we obtain

$$V_{eff}(\alpha) = -C\alpha^{\frac{d}{\Delta_-}} - (2d-1)\left(\frac{4\pi T}{d}\right)^d - \frac{(2d-1)\Delta_-(d-2\Delta_-)}{4d}\left(\frac{4\pi T}{d}\right)^{d-2\Delta_-}\alpha^2 + \ldots$$

• In the presence of a double-trace deformation on the field theory side

$$\delta\mathcal{L} \sim g\ \mathcal{O}^2$$

the effective potential at zero temperature becomes

$$V_{eff}(\alpha)\Big|_{T=0} \simeq g\alpha^2 - C\alpha^{\frac{d}{\Delta_-}}$$

• a stable symmetry-breaking vacuum exists with vev

$$\alpha \simeq \left(\frac{2g\Delta_-}{dC}\right)^{\frac{\Delta_-}{d-2\Delta_-}}$$

• Adding temperature in the presence of the double-trace deformation we obtain the effective potential

$$V_{eff}(\alpha) \simeq -C\alpha^{\frac{d}{\Delta_-}} - ET^d + g_{eff}\alpha^2 + \ldots$$

where g_{eff} is the temperature-shifted effective double-trace coupling

$$g_{eff} = g + GT^{d-2\Delta_-}$$

• The normal vacuum becomes unstable when $g_{eff} < 0$. The critical temperature T_c that separates the stable from the unstable regime is obtained :

$$g_{eff} = 0 \quad \Leftrightarrow \quad T_c \simeq \left(-\frac{g}{G}\right)^{\frac{1}{d-2\Delta_-}}$$

• At finite density:

$$g_c(\rho) = \frac{2d-1}{d}\rho^2 C_1 A_1^{\frac{d-2}{\Delta_-}}(\rho)\left(C_2 A_1^2(\rho) + \frac{d-2}{\Delta_-}A_2(\rho)\right)$$

• $A_{1,2}, C_{1,2}$ can be determined analytically

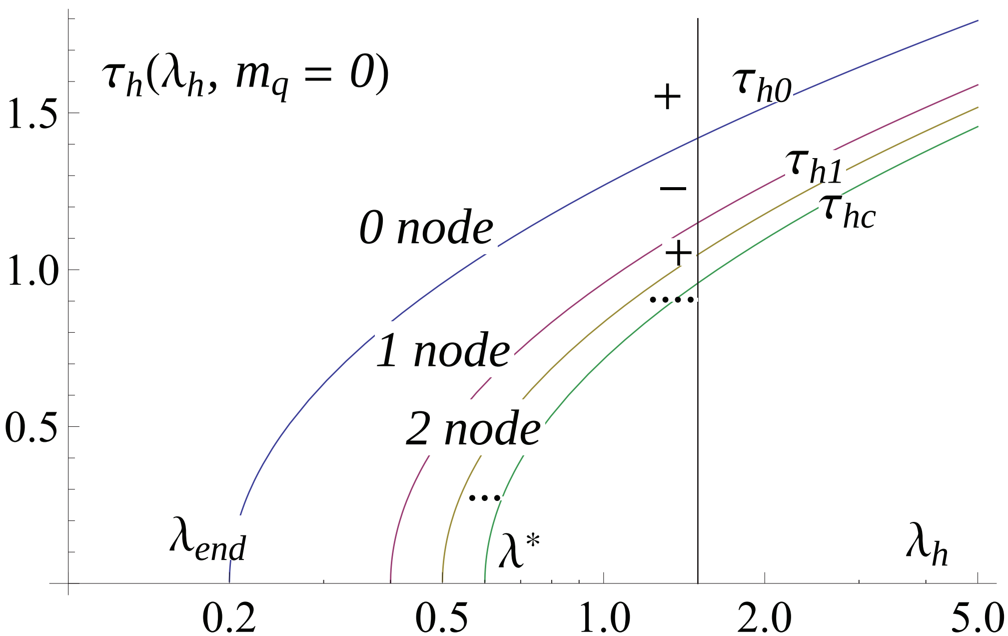
E.K.+Niarchos

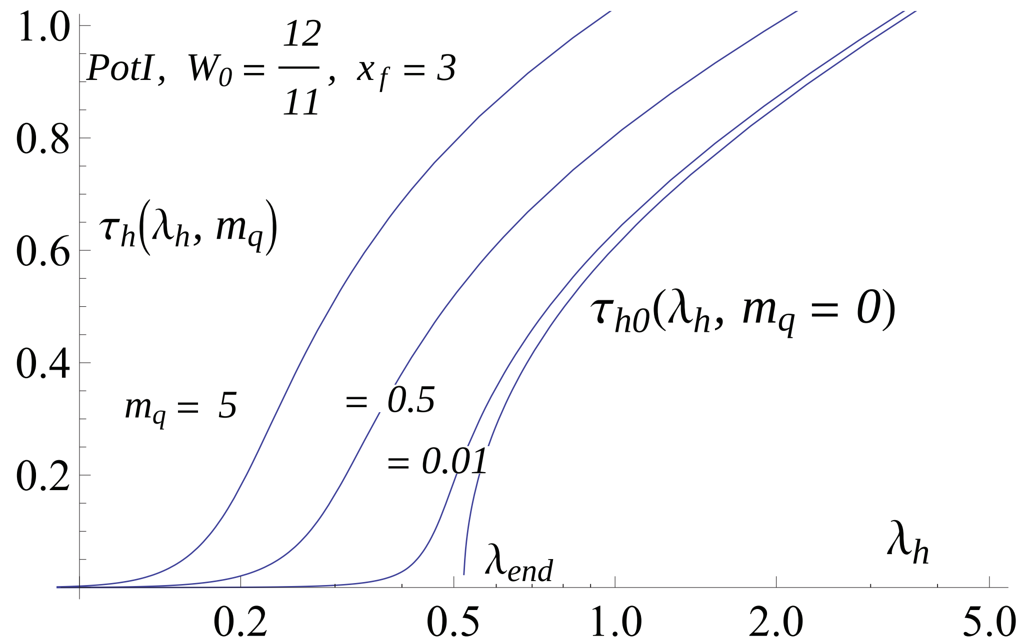
• In the vicinity of the quantum critical point we observe the following scaling of the vev

$$\langle \mathcal{O} \rangle \sim (g_c - g)^{\frac{\Delta_-}{d-2\Delta_-}}$$

RETURN

Horizon values of τ, λ





Detailed plan of the presentation

- The holographic effective potential. 60 minutes
- The effective action 62 minutes
- BKT scaling, II 64 minutes
- Effective Potential and phase transitions 66 minutes
- Horizon values of λ, τ 68 minutes