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Lorentz Violation, String Theory, Dissipation and Holography

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Plan

- Introduction: Brief History of Lorentz Invariance
- Important questions
- Parametrizing Lorentz Violation: LV Effective Field Theory
- Coupling to standard gravity
- Varying speed of light in string/brane theories
- The Holographic point of view.
- Lorentz Violation, dissipation and high energy instabilities.
- Outlook

Introduction and History

• Lorentz invariance is one of the most important pillars of modern day physics, experimentally unchallenged for more than a hundred years.

• It has been tested to unprecedent accuracy reaching 10^{-21} in several contexts, and down to 10^{-29} recently in the neutron sector.

Romalis et al. 2011

• Its reign coincided with the demise of “aether”. Michelson and Morley and Einstein (1905) eventually banished aether as a physical theory.

• Few physicists realized that a new kind of ”aether” made it back into physics after 1915, with the general theory of relativity: the gravitational field.

• Only rarely, a few theorists have questioned the universal applicability of Lorentz invariance. The most prominent idea was due to H. Nielsen and collaborators, that sought to explain LI as an emergent IR symmetry with inconclusive results.

Nielsen et al. 1984

• Related ideas were applied to other low-energy symmetries, prompting the term anti-grand-unification.

Iliopoulos+Nanopoulos+Tomaras, 1985

• Kostelecky and Samuel argued that in open string theory, unusual tachyon vevs can trigger vevs of tensors and therefore LV. We know now that this cannot happen as such in tachyon condensation. This triggered a long term effort to parametrize LV QFTs.

Kostelecky+Samuel, 1989

• A new series of investigations saw the light in the 90's prompted by vague claims of "quantum gravity, spacetime foams and deformed Lorentz invariance". Their solid outcome were parametrizations of LV dispersion relations and comparison with (astrophysical) data.

Coleman+Glashow, 1997, Amelino-Camelia+Ellis+Mavromatos+Sarkar, 1997

• It was noted in late 90's that the velocity of light on probe D-branes in string theory can be variable, and smaller from the speed of light in the bulk, being affected by the bulk gravitational fields.

Kiritsis, 1999

• It was suggested that it may be used to solve the horizon problem in cosmology.

Kiritsis, 1999, Chung+Freese, 1999

• It was also suggested that LV is generic in string theory orientifolds and the idea was brought to its natural conclusion giving rise to "Mirage Cosmology" in string theory.

Kehagias+Kiritsis, 1999, Kraus, 1999

• Varying speed of light theories using branes were constructed by stabilizing "universe" branes like planets in the solar system, around black branes.

Kiritsis, 1999, Alexander 1999, Burgess+Martineau+Quevedo+Rabadan, 2003

• It was shown in general that the speed of light on branes is always smaller or equal to that of the bulk,

Gibbons+Herdeiro, 2000

• Many years later, Gubser constructed a braneworld solution with varying speed of light, but carrying no entropy (no bulk horizon). In previous examples the varying speed of light was triggered by the gravitational field in the bulk being that of a black brane.

Gubser, 2008

• Such solutions appear today in holographic backgrounds describing strongly coupled holographic systems at finite density.

Charmousis+Gouteraux+Kim+Kiritsis+Meyer, 2010

• It was pointed out that strong violations of LI in the UV can allow for a resolution of the hierarchy problem, modifying significantly the power counting of interactions.

Anselmi+Halat, 2007

• A similar idea applied to gravity gave rise to a power-counting renormalizable class of LV theories of gravity, with Lifshitz asymptotics.

Hořava, 2009

• This class of (non-relativistic) gravity theories solves several of the problems we need inflation for, automatically.

Kiritsis+Kofinas, 2009, Mukohyama, 2009

• The original Hořava theory suffers from several issues, including strong coupling and phenomenological problems. Phenomenologically acceptable alternatives were proposed.

Blas+Pujolas+Sibiryakov, 2010

Questions on Lorentz Invariance

♠ Is there ANY motivation for the study of LV QFT?

YES because:

♠ It is the main theoretical tool in contexts where LI is broken by energy and charge densities. The whole of condensed matter physics is in this class.

♠ The same issue can appear in more exotic contexts: e.g. quantum matter in the gravitational fields of a black hole and the strange phenomena it entails.

♠ It is important to understand the logistics of interplay between quantum effects and symmetry breaking (LV) effects. This might give models for the spontaneous breaking, and guide experimental tests.

♠ Last must most important: as I will argue, LV is one of the most efficient windows to new IR-sensitive physics.

Questions

• Is Lorentz invariance an accident of low energies? If it is, what is the dependence of the breaking scale with energy?

• Under what conditions the breaking of LI is "natural" (almost insensitive to the scale of LV physics) in the standard sense of Quantum field theory?

• Can LI be spontaneously (dynamically) broken?

• What is the general connection between LV and dissipation?. In the simplest known cases dissipation is related to Cerenkov radiation.

• How is LV compatible with general coordinate invariance and gravity as we know it? Is gravity coupled to a LV QFT a consistent theory?

• How are we supposed to change coordinate systems in a LV theory?

♠ One might think that LV in QFT and gravity are completely decoupled subjects. We will argue they are intimately related.

Lorentz Violation in QFT

- Parametrizing LV in QFT is a simple exercise

- It amounts to provide **sources** for **all** tensor operators:

$$L = V(\phi) + b^\mu \partial_\mu \phi + c^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + d^{\mu\nu\rho} \partial_\mu \phi \partial_\nu \phi \partial_\rho \phi + \cdots$$

$$V(\phi) = a_0 + a_1 \phi + a_2 \phi^2 + a_3 \phi^3 + a_4 \phi^4 + \cdots$$

- An interesting dynamical function is the correlator-generating-functional

$$\exp\left[W(b^\mu, c^{\mu\nu}, d^{m\nu\rho}, \cdots)\right] = \int \mathcal{D}\phi \; e^{-\int d^4x \;\; L(\phi, b, c, d, \cdots)}$$

- All correlation functions of the quantum field ϕ can be obtained via functional differentiation of the functional W.

- If we keep only LI constant sources these are the (bare) couplings of quantum field theory.

- For constant generic $b^\mu, c^{\mu\nu}, d^{\mu\nu\rho}$ the theory breaks LI.
When $b^\mu = d^{\mu\nu\rho} = 0, c^{\mu\nu} \sim \eta^{\mu\nu}$ the theory is Lorentz Invariant.

• In particular $\frac{c^{ii}}{c^{00}}$ is the speed of "ϕ-light", c^2_{eff}. Different light speeds for different species of particles are typical signals of Lorentz violation.

• When $b^\mu, c^{\mu\nu}, d^{\mu\nu\rho}$ are functions of the spacetime point, translational invariance also breaks.

• If the theory is in a non-trivial metric then

$$c^{\mu\nu} = \sqrt{-g}\ g^{\mu\nu}$$

• LI is spontaneously broken for non-trivial $g_{\mu\nu}$

♠ Consider the existence of a conserved current J^μ (particle number). The Lorentz violating coupling

$$\delta S = \int d^4x\ B_\mu J^\mu$$

can be interpreted as
(a) a general source for particle number
(b) B_0 is the chemical potential for particle number.

• The LV couplings can be systematically parametrized using the previous idea. The various effective LV theories going under different names (like LV SM) are based on this.

Colladay+Kostlecky, 1998

What is the "view" of such couplings in the standard model:

• Kinetic tensors translate into the spacetime metric.

• θ-angles translate to pseudoscalars (axions), like $\theta F \wedge F$.

• Chemical potentials translate into graviphotons.

• Gauge, scalar and Yukawa coulings are scalars (moduli fields)

• String theory suggests that all couplings of QFT are dynamical fields in a "gravitational sector".

• We will argue this is general in any theory that contains gravity.

We can make a rather general claim motivated by string theory and the AdS/CFT correspondence:

• The sources of QFT are in 1-1 correspondence with QFT operators ($T_{\mu\nu}$ has a source $g_{\mu\nu}$, J^{μ} has a source A_{μ} etc).

• The dynamics of sources of QFT is a classical generalization of gravitational physics (it contains among others a single massless metric $g_{\mu\nu}$.

• There is an argument in open string theory indicating that consistency at one-loop implies that the sources must be dynamical.

• This leads to the radical statement

Gravity/string theory is the dynamics of sources of QFT.

• This is explicitly realized in holography.

In general, the collection of an infinite number of sources for a QFT matches (multi)string states in a string theory. If the QFT is at at large N, the string theory is semiclassical (loops suppressed). If it is also strongly coupled then the string is stiff and the theory is approximated by a gravitational theory.

• Holography states that the effective action for sources of a QFT, is a gravitational theory in higher dimensions.

• In this language, LV is the existence of non-trivial tensor backgrounds in the gravitational sector (vectors, 2-tensors, etc).

There are two ways LV can appear in this context:

♠ "Environmental LV": Energy and charges generate gravitational and other classical fields, and they break LI.

♠ "Dynamical LV": The effective potential for tensor operators in QFT has non-trivial minima, that break LI. It is unlikely this can happen at weak coupling, but it might at strong coupling. Holography provides a formalism to compute such effective potentials

Kiritsis+Niarchos ongoing work

In both cases there are Lorentz-Violating vevs.

Coupling LV to gravity

• There is an obvious puzzle that generalizes similar statements for other symmetries.

• If a global symmetry is broken in QFT, how can it couple to a "gauge theory" where this symmetry is "gauged"?

• This is precisely the issue if we want to couple a LV QFT To gravity: gravity gauges Lorentz and translation invariance.

• It is well known from the examples of standard gauge symmetry, and supergravity, that such consistent couplings exist, (above two dimensions) only if the breaking of the symmetry in the QFT is spontaneous (non-trivial vevs).

• In plain words: you cannot couple a gauge theory to a theory where the relevant current is NOT conserved (it may be broken only spontaneously)

• Equivalently, the QFT is invariant under the symmetry if the symmetry violating couplings, transform under the symmetry, so that the symmetry is formally restored.

• A simple example of a U(1) symmetry:

$$L = |\partial\psi|^2 + Re[g\psi^2] \quad , \quad \psi \to \psi\ e^{i\epsilon} \quad , \quad g \to g\ e^{-2i\epsilon}$$

is U(1) invariant if g transforms appropriately.

• It can be promoted to a new scalar field $g \to ge^{-2ia}$

$$L' = |\partial\psi|^2 + Re[g\psi^2 e^{-2ia}] - \frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}(\partial_\mu a + A_\mu)^2$$

$$A_\mu \to A_\mu - \partial_\mu\epsilon \quad , \quad a \to a + \epsilon \quad , \quad \psi \to \psi\ e^{i\epsilon}$$

• Note the appearance of the Stuckelberg coupling: the general procedure for coupling consistently is a generalization of the Stuckelberg procedure: promote the symmetry parameters to dynamical fields.

• Gravity can be thought of as a gauge theory of translations, or Lorentz transformations (or both) depending on the formalism.

• Therefore a LV theory can consistently couple to gravity iff the breaking is spontaneous/environmental.

• Translated in plain words: a LV QFT is a QFT coupled to a generalized gravitational sector (with possibly many tensor fields), some of which have non-trivial vevs/classical values.

• For example it is known that non-commutativity of space-time in gravitational theories is equivalent to non-trivial background fields.

Seiber+Witten, Andriot+Hohm+Larfors+Lüst+Patalong

• Note, that in this language, the theory of fluctuating fields, both QFT and gravitational, is fully general coordinate(+Lorentz) invariant. Only the presence of background fields break the symmetries.

• In particular, the gravitational interaction is general-coordinate invariant and <u>not</u> of the Hořava type.

• However, HL theories can be written as Einstein-Aether theories that are fully diff-covariant

Sfetsos

Changing Frames

- A puzzle that confuses the issues is: how do you go from one inertial frame to another if you have broken Lorentz Invariance.

- There are several contradictory answers in the literature.

- Our previous conclusion settles the issue: The full Lorentz invariant theory coupled to the gravitational sector is fully general coordinate invariant.

- Therefore one changes coordinates as always. Einstein is intact.

- In the process, the "Lorentz violating couplings" change.

Lorentz-Violation:recap

• We have shown (under very weak assumptions) that any Lorentz violation in QFT is associated with background fields of a generalized gravitational sector.

• This result implies that "Lorentz violation $\simeq$ fifth forces"

• An unknown new force, (mediated by $\phi, A_\mu,\ g_{\mu\nu}$ etc) should couple to some SM particles. Therefore, they generate a classical background that in return will provide LV interactions.

• Can Lorentz violation originate from a known force?

• Yes if the knowledge of the dynamics of the known force is incomplete. Example: the gravitational force if the dynamics is modified in the IR.

• Knowledge of sources, determines the gravitational field, and this determines the LV in a given process.

• If in some regime the GR dynamics is modified, then part of the LV will be "unexplained".

♠ Searching for Lorentz Violation is a very efficient way of searching for new long-range forces or modifications in known ones.

LV on the brane

• We will examine here a brane-world as a simple example of varying speed of light and Lorentz-Violation.

• A brane-world is a brane (submanifold) embedded in a higher-dimensional bulk.

• There are fields/particles living on the brane (matter, the SM etc) and fields propagating in the bulk (like the graviton and some other fields).

• We will therefore couple the QFT that lives on the brane-world to a non-trivial gravitational field in the bulk to generate LV.

• A natural setup to do this is to embed brane-worlds in bulk LV metrics.

Kiritsis 1999

• This is an example where a modification of the law of gravitation (it is becoming higher dimensional here) will be interpreted a LV and a varying speed of the light on the brane world.

• The concrete example is a (5-dimensional) black-3 brane metric

$$ds^2 = e^{2A(r)}\left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx^i\right]$$

with $f(r)$ the "blackness" function: $0 \le f \le c_{UV}^2$.

• c_{UV} is the UV speed of light and also the speed of gravitons in the bulk.

• Embedding a D3 brane at $r = r_*$, with a gauge interaction (a photon) on it, we obtain the effective action

$$S_{\text{brane}} \sim -\frac{\sqrt{-det(\hat{g})}}{4}\hat{g}^{\mu\mu'}\hat{g}^{\nu\nu'}F_{\mu\nu}F_{\mu'\nu'} + \cdots$$

with the embedding metric

$$\hat{g}_{ab} = G_{\mu\nu}\,\frac{\partial X^\mu}{\partial \xi^a}\,\frac{\partial X^\nu}{\partial \xi^b}$$

$$d\hat{s}^2 = e^{2A(r_*)}\left[-f(r_*)dt^2 + dx^i dx^i\right]$$

• The effective speed of light is

$$c_{eff}^2 \;=\; f(r_*) \;\le\; c_{UV}^2$$

• It is a general property of gravity that f is monotonic and decreases towards a horizon.

• f vanishes at the bulk horizon, $f(r_h) = 0$. There the world-volume theory approaches the "Carolean limit": $c_{eff} \to 0$.

• In more complicated embeddings (like the brane moving with constant velocity in an internal dimension), the effective speed of light on the brane depends on more data, both the bulk fields and the motion parameters:

$$c_{eff}^2 = f(r_*) - V^2$$

• The world-volume theory has of a space-filling brane has a new horizon at r_w such that

$$c_{eff}^2 = f(r_w) - V^2 = 0$$

.

• The speed of light breaks LI.

The holographic dual view

• The previous setup corresponds to a holographic large-N theory Θ_{Large} with many degrees of freedom (that generates the bulk black-brane metric) coupled to a (weakly coupled) theory Θ_{small} (the theory on the brane).

• The presence of the blackness factor, implies a non-trivial uniform energy density in the dual large-N QFT Θ_{Large}.

• The presence of a bulk horizon implies that such a state is thermal.

• It is the non-trivial energy density that triggers LV.

• The bare action of the Θ_{small} theory, corresponds to the brane being placed at the UV boundary $r \to \infty$ of the geometry.

• As usual in holography, the radial scale stands for the RG scale $r \sim E$ of the QFT.

• The induced action of the brane placed at an intermediate radial position, r_* corresponds to the effective action of theory Θ_{small}, at energy $E = r_*$, after integrating out the quantum corrections of the theory Θ.

$$c^2_{eff} = f(r_*) = f(E)$$

gives the RG dependence of the speed of light of Θ' due to the quantum effects of Θ. It decreases down to zero.

The most general scaling IR behavior is

$$c_{eff} \quad \sim \qquad E^{\frac{2(z-1)}{2z-\theta}}$$

• z is the Lifshitz exponent, and θ the hyperscaling violation exponent:

$$ds^2 = r^\theta \left[-\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx^i dx^i}{r^2} \right]$$

$$x^i \to \lambda \ x^i \quad , \quad t \to \lambda^z \ t \quad , \quad r \to \lambda \ r \quad , \quad ds \to \lambda^{-\theta} \ ds$$

Gouteraux+Kiritsis

It is constant on AdS (z=1).

Braneworlds, Strongly coupled QFTs and c: recap

• Braneworlds provide a natural setup for varying speed of light.

• Most string theory constructions of the standard model use this paradigm (SM is composed of some branes in a 10d bulk)

• Generic bulk fields produced different speeds of light on different SM branes (and always smaller than speed of gravitons in bulk)

• The speed of light decreases with energy, $c \sim E^a$ while flowing to LV geometries.

• The relevant power law depends on the details of the theory.

• All of the above have an alternative interpetation of a SM interacting with a large-N theory in a LV state.

Dispersion and energy loss

• Cerenkov radiation is a generic phenomenon in the presence of superluminal particles. It is present whether the different speeds are fundamental or effective.

• It is generated by three point vertices of light/massless particles that allow decays, as the relevant speeds of light are not equal.

• This is a mechanism that was studied at weak coupling. It is an important constraint on experimental tests of superluminality.

Cohen+GLashow, Nelson+Moore, Moore+Stoica

• The loss rate depends on the relevant interactions. $\frac{dE}{dx} \sim E^2$ for photons or $\frac{dE}{dx} \sim E^3$ for neutrinos.

• Are there strong coupling analogs of the Cerenkov energy loss?

• We will present a class of examples that are generic in strongly coupled theories with holographic duals.

• The energy loss acts on the strongly coupled particles (the "quarks"). They are living on the "braneworld".

• This type of Cerenkov energy loss exists only iff the LI is violated It is present even in the absence of horizons.

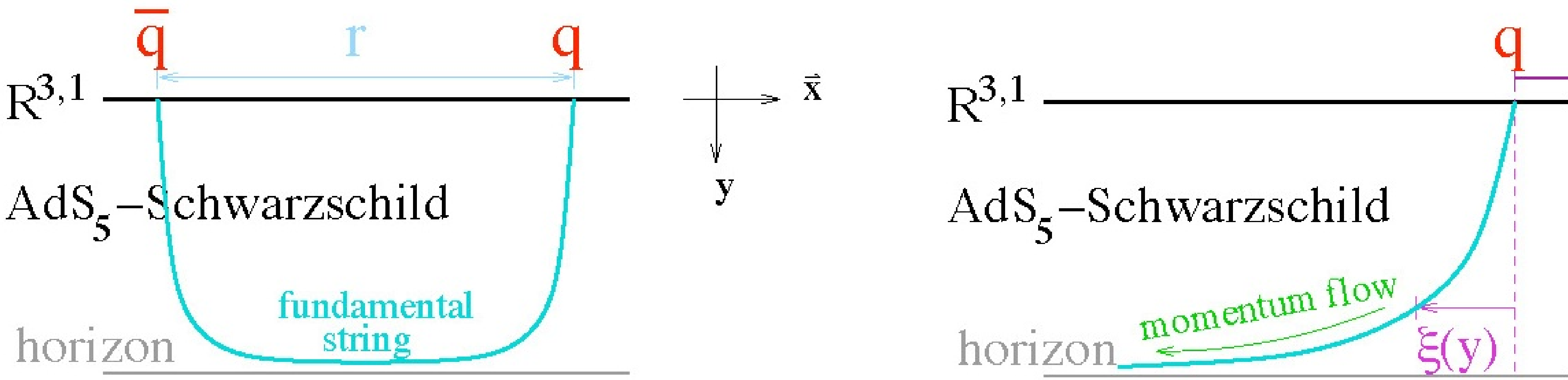
LV Energy loss at strong coupling

- A heavy "quark" has a string attached behind. We will drag this string with velocity v inside the bulk gravitational metric.

- We must find a solution to the string equations with

$$x^1 = vt + \xi(r) \quad , \quad x^{2,3} = 0 \quad , \quad \sigma^1 = t \quad , \quad \sigma^2 = r$$

Herzog+Karch+kovtun+Kozcac+Yaffe, 2006 Gubser, 2006
Casaldelrrey-Solana+Teaney, 2006, Gursoy+Kiritsis+Mazzanti+Nitti, 2010



The string dynamics is governed by the Nambu-Goto action:

$$S_{\text{string}} = \frac{1}{4\pi\ell_s^2}\int d^2\sigma\ \sqrt{\hat{g}} = \text{Tension} \times \text{Area}$$

For a LV metric (in string frame)

$$ds^2 = b(r)^2\left[\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x}\cdot d\vec{x}\right]$$

the solution profile is

$$\xi'(r) = \frac{C}{f(r)}\sqrt{\frac{f(r)-v^2}{b^4(r)f(r)-C^2}} \quad , \quad C = vb(r_s)^2 \quad , \quad f(r_s) = v^2$$

• The induced metric on the world-sheet is a 2d black-hole with horizon at the turning point $r = r_s$

$$ds^2 = b^2(r)\left[-(f(r)-v^2)d\tau^2 + \frac{1}{(f(r) - \frac{b^4(r_s)}{b^4(r)}v^2)}dr^2\right]$$

• We can calculate the drag force:

$$F_{\rm drag} = \frac{dp}{dt} = -\frac{b^2(r_s)\sqrt{f(r_s)}}{2\pi\ell_s^2}$$

• In the AdS-black hole, corresponding to scale invariant theories the result is universal.

$$\frac{dE}{dt} \sim \frac{1}{M\tau}E \quad ,$$

• For a generalized Lifshitz metric $ds^2 = u^\theta\left[-\frac{dt^2}{u^{2z}} + \frac{du^2+dx^i dx^i}{u^2}\right]$. we obtain

$$M\frac{dv}{dt} = F_{\rm drag} \sim r_s^{\frac{\theta-z-1}{2-z}} \sim v^{\frac{z+1-\theta}{(z-1)}}$$

• There is also Cerenkov emission of on-the-brane particles. This is sub-leading at large N_c.

LV Energy loss: Interpretation

- All portions of the trailing string move with the same velocity, v.

- The local "velocity of light" at r, is given by $c_{eff}^2 = \sqrt{f(r)}$ and varies from the standard speed of light down to zero.

- The portion of the string below the world sheet horizon at $r = r_s$ is locally superluminal.

- It therefore dissipates energy (that in this example is provided by the source).

- This is a strong coupling analogue of Cerenkov radiation.

Fluctuations and Langevin Evolution

- Particles are subject to dispersion and energy loss, in the presence of LV both at strong coupling and weak coupling.

- In some cases such processes can be thermal, while in other cases they are not.

Hashimoto+Iizuka+Oka, 2010, Kiritsis+Pavlopoulos, 2011

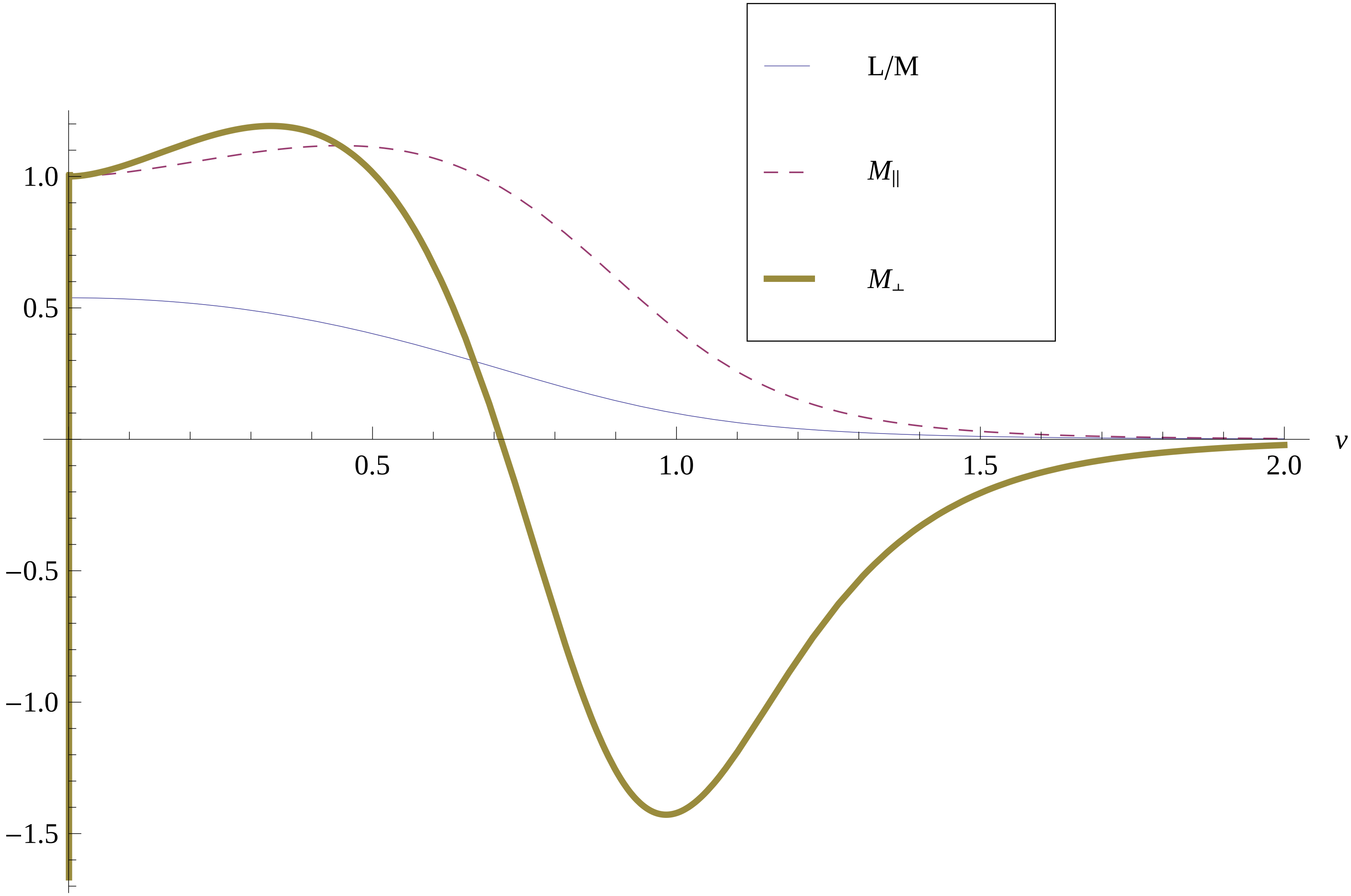
- Thermal cases are associated to world-volume horizons.

- Fluctuations are important observables like in the case of Brownian motion (provided that the noise correlation time is much shorter than the diffusion time).

- Fluctuations generate a Langevin diffusion of superluminal particles.

- The diffusion coefficients are being calculated at strong coupling.

- There is an instability of all high-energy Lifshitz regimes in theories with holographic duals.



A simultaneous plot of $\frac{L}{M}$, $M_{||}$ and $M_{\perp}$ at $z = 1.3$.

Outlook

• LV in QFT is intimately connected to gravity.

• All LV protocols are generated with non-trivial background fields (metric etc).

• LV is equivalent to the presence of unknown forces (fifth forces) or arise from incomplete knowledge of known forces.

• Therefore, experimental search for LV is an important window to new IR-sensitive physics, and is not so exotic as some might think.

• A generic feature of LV is Energy dissipation. Lifshitz scaling theories are unstable in the UV.

• Open Question:Is spontaneous LV possible in a QFT vacuum?

Non-relativistic (Lifshitz) scaling symmetries

• In non-relativistic theories (emerging from condensed matter) there are scale invariant fixed points with non-relativistic symmetries:

• The simplest example is the Lifshitz free fixed-point. The general scale invariance is

$$t \to \lambda^z \; t \quad , \quad x^i \to \lambda \; x^i$$

• A simple free field realization with $z = n \geq 1$.

$$L = \dot{\phi}^2 - \phi \; \Box^n \phi + \cdots \quad , \quad z = n$$

• This is realizable experimentally in condensed matter.

• Note also that for $z > 1$, $c \to \infty$.

• It can be a bona-fide UV fixed point that will define non-trivial RG flows.

• The theory has improved UV behavior as

$$\langle \phi(t, \vec{x}) \phi(0, \vec{0}) \rangle \quad \sim \quad \int dE d^3p \; \frac{i \; e^{iEt + i\vec{p}\cdot\vec{x}}}{E^2 - (\vec{p})^{2z}}$$

• This can be seen from simple dimensional analysis

$$S = \int dt\ d^{d-1}x \left[\dot{\phi}^2 - \phi\ \Box^z \phi + \cdots \right] \quad , \quad [\phi] = \frac{d-z-1}{2}$$

• In $d = 4$, if $z = 3$, all ϕ^n are renormalizable terms. If $z = 2$, up to ϕ^{10} are renormalizable.

• For fermions, $[\psi] = \frac{d-1}{2}$, and if $z = 2$, the Nambu-Jona-Lasinio type of terms are renormalizable (marginal)

$$S_{NJL} \sim \int g_{ijkl}\ \ \bar{\psi}^i \bar{\psi}^j \psi^k \psi^l$$

and could perform EW symmetry breaking without fundamental scalars (no hierarchy problem).

• In the Liftshitz theories the speed of light term in the action $\sim c^2\ \phi \Box \phi$ is relevant (like a mass term) and becomes important at low energies.

Hořava(-Lifshitz) gravity

• This is a gravitational theory that is (a) non-relativistic (b) has Lifshitz symmetry in the UV.

Hořava, 2009

• Start from the ADM decomposition of the metric

$$ds^2 = -N^2 \ dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

• The kinetic terms are given in terms of the extrinsic curvature. and the extrinsic curvature

$$K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

$$S_K = \frac{2}{\kappa^2} \int dt d^3x \sqrt{g} N \ \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

• The rest of the action does not contain time derivatives on g_{ij}.

• The key point is that if in the UV z=3, the theory is power-counting renormalizable.

$$t \to \lambda^3 \ t \quad , \quad x^i \to \lambda \ x^i$$

$$[N] = 0 \quad , \quad [g_{ij}] = 0 \quad , \quad [N_i] = 2$$

• The "potential" is

$$V = \int dt d^3x \; \sqrt{g} N \; V(g_{ij})$$

• For renormalizability it should contain up to six derivatives. The six-derivative terms are classically-scale invariant. Terms with a lower number of derivatives are "relevant".

$$\nabla_i R_{jk} \nabla^i R^{jk} \quad , \quad \nabla_i R_{jk} \nabla^j R^{ik} \quad , \quad R \Box R \quad , \quad R_{ij} \Box R^{ij}$$

modify already the propagator while

$$R^3 \quad , \quad R R_{ij} R^{ij} \quad , \quad R_{ij} R^i{}_k R^{jk}$$

provide scale invariant interactions.

• The (local) invariance of the theory is

$$t \to h^0(t) \quad , \quad x^i \to h^i(t, x^j)$$

• Renormalizability has not yet been tested at the loop level.

Lorentz Violation and Hořava-Lifshitz gravity

• According to our previous conclusion, there is no place for Hořava gravity coupled to LV QFT.

• There is a hidden assumption in our previous arguments: the QFT is defined by perturbations of standard CFTs with full conformal symmetry.

• If we change "frame" and start from scaling QFTs with Lifshitz-symmetries then the gravity they will couple to will be (a generalized) Hořava-Lifshitz gravity.

• A quick way to see this is to check that the renormalization countertrms of holographic Lifshitz theories generate Hořava-Lifshitz gravity.

Griffin+Horava+Melby-Thompson

• This in retrospect says that HL gravity can be written as standard general coordinate invariant gravity coupled to more fields that have vevs. Indeed, this is known to be true.

Sfetsos

• The two cases on the QFT side seem to be related with a basis rearrangement of operators, and the UV scaling symmetry.

Finite (hidden) charge density

• There are other "sources" of LV in such metrics. An important one, in theories Θ with a conserved charge, is the presence of finite charge density.

• It was shown in general that at zero temperature, there are critical (scale invariant) solutions with a generalized Lifshitz symmetry

Charmousis+Gouteraux+Kim+Kiritsis+Meyer, 2010, Gouteraux+Kiritsis, 2011

$$ds^2 = u^{\theta}\left[-\frac{dt^2}{u^{2z}} + \frac{du^2 + dx^i dx^i}{u^2}\right] \quad , \quad t \to \lambda^z t \quad , \quad u \to \lambda u \quad , \quad x^i \to \lambda x^i$$

with dynamical exponent z. Putting the metric in the standard form

$$ds^2 \sim r^{\frac{\theta-2}{2-z}}\left[-f(r)dt^2 + \frac{dr^2}{f(r)} + dx^i dx^i\right] \quad , \quad f(r) \sim r^{2\frac{1-z}{2-z}}$$

We also relate the radial position to energy

$$E^2 = r^{\frac{\theta-2}{2-z}} \quad \to \quad r = E^{\frac{2(2-z)}{2-\theta}}$$

$$c_{eff} \sim \sqrt{f(r)} \sim r^{\frac{1-z}{2-z}} \sim E^{\frac{2(1-z)}{2-\theta}}$$

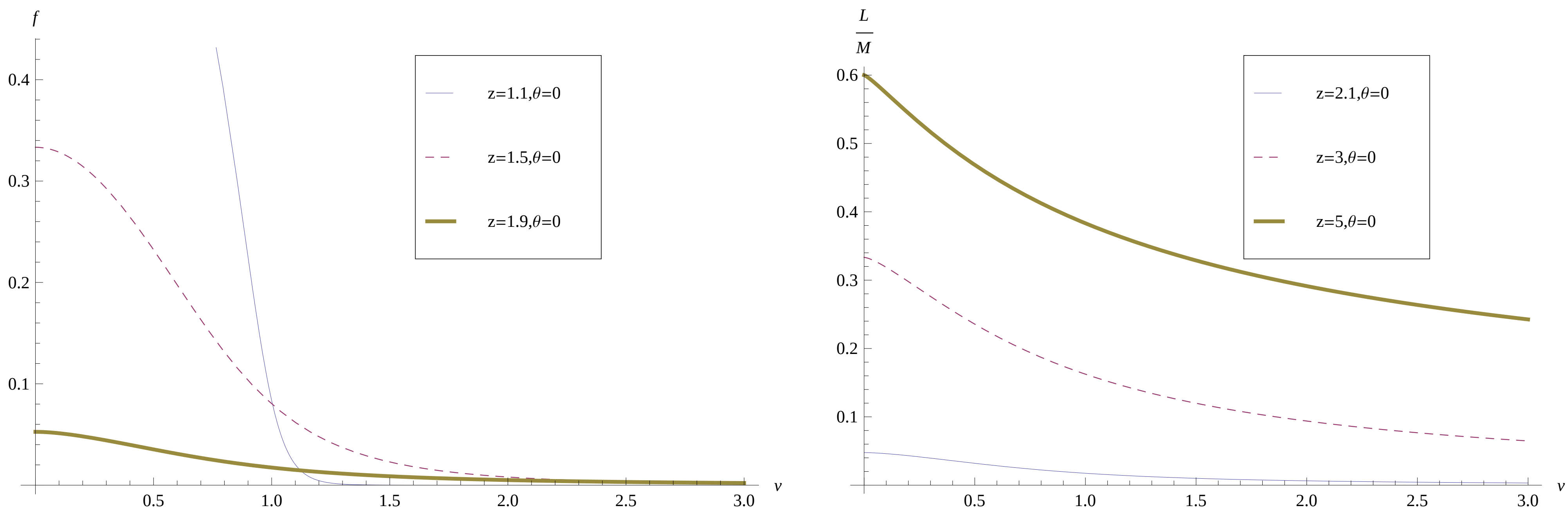
- This is the more general scaling behavior (with "hyperscale violation" if $\theta \neq 0$).

- Since $z \geq 1$, $c_{eff} \to \infty$ when $E \to \infty$ and vanishes at $E \to 0$.

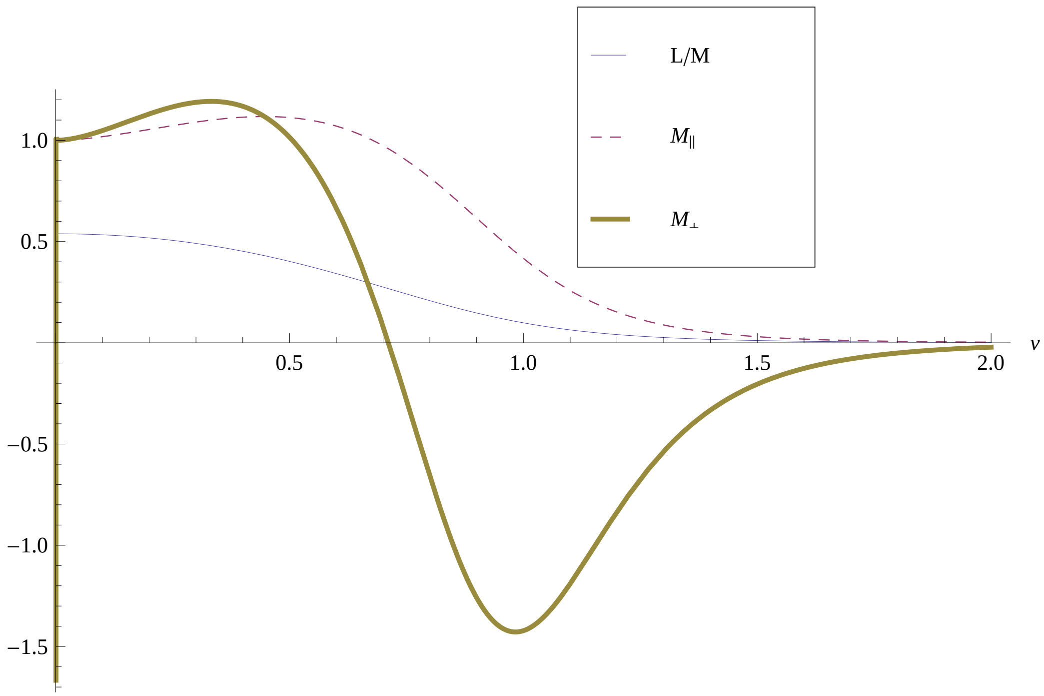
- The black brane solution has finite entropy, as the horizon area is finite.

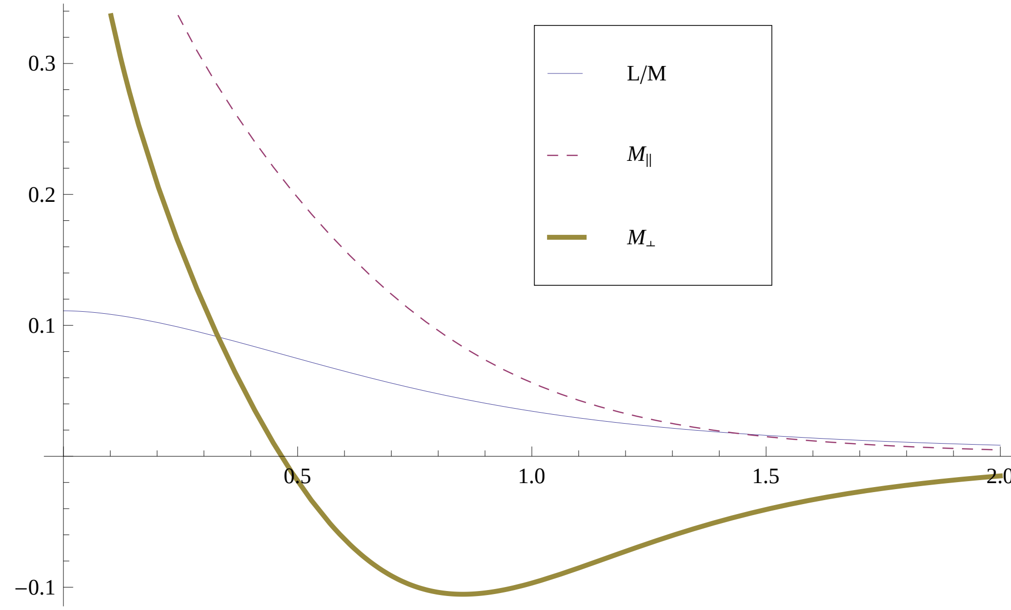
- The charge-density Lifshitz solutions have no entropy.

Lifshitz diffusion

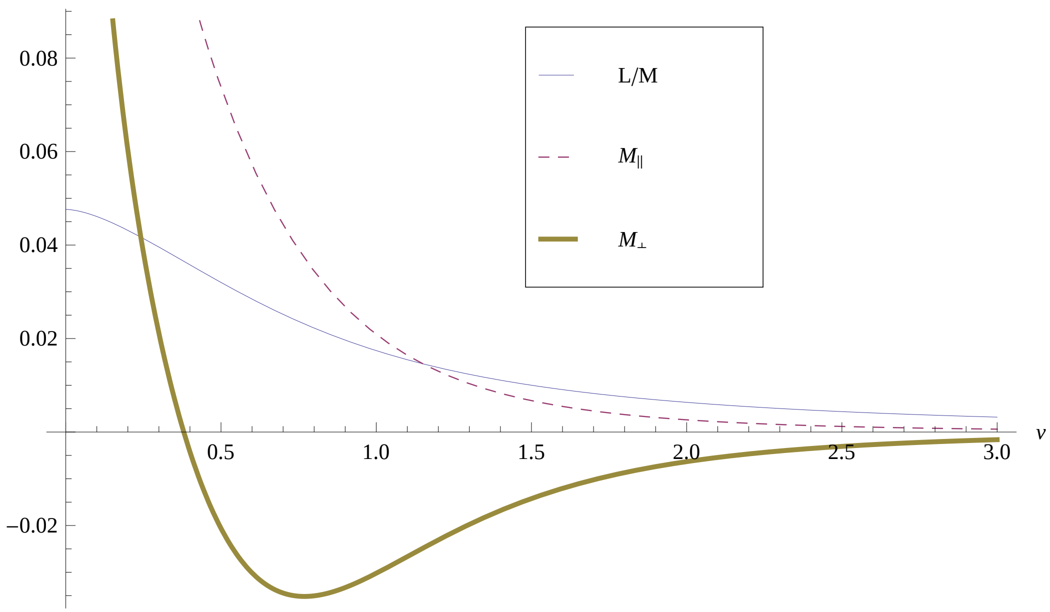


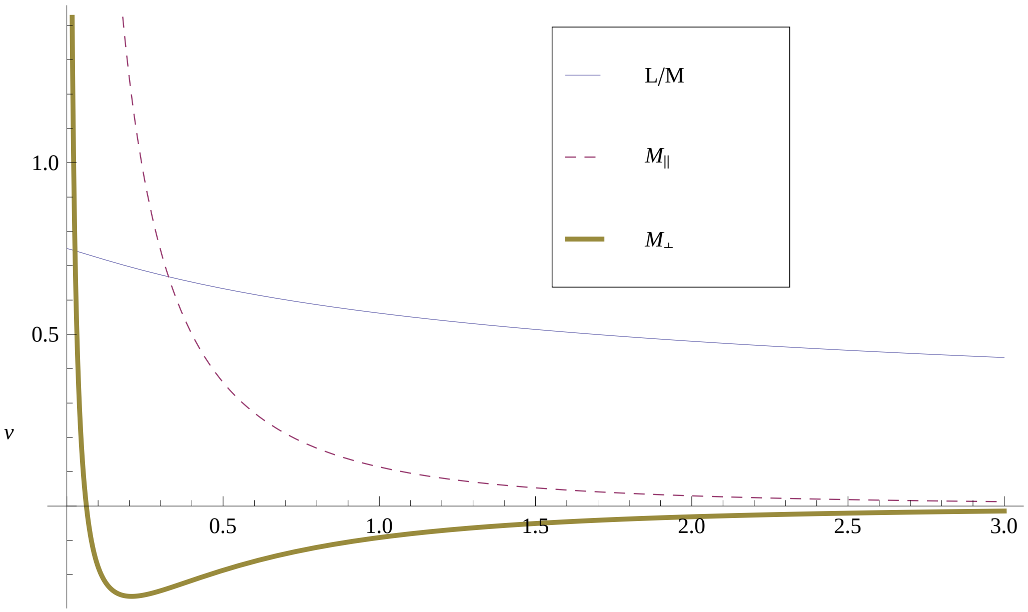
Left: A plot of the function $\frac{L}{M} = v^{\frac{z-\theta}{z-2}} F[z, \theta; v^{-\frac{\theta-2}{z-2}}]$ as function of $1 < z < 2$ at $\theta = 0$. Right: the same function for $z > 2$ at $\theta = 0$.





Left:A simultaneous plot of $\frac{L}{M}$, $M_{||}$ and $M_\perp$ at $z = 1.3$. Right: the same plot for $z = 1.8$.





Left:A simultaneous plot of $\frac{L}{M}$, $M_{||}$ and $M_{\perp}$ at $z = 2.1$. Right: the same plot for $z = 8$.

Detailed plan of the presentation

- Title page 1 minutes
- Plan 2 minutes
- Introduction and History 7 minutes
- Questions on Lorentz Invariance 10 minutes
- LV in QFT 19 minutes
- Coupling LV to gravity. 24 minutes
- Changing Frames 25 minutes
- Lorentz-Violation:recap 27 minutes
- LV on the brane 33 minutes
- The holographic dual view 37 minutes

- Braneworlds, Strongly coupled QFTs and c: recap 38 minutes
- Dispersion and energy loss 39 minutes
- LV Energy loss at strong coupling 45 minutes
- LV Energy loss: Interpretation 46 minutes
- Fluctuations 47 minutes
- Outlook 48 minutes

- Non-relativistic Lifshitz symmetries 50 minutes
- Hořava(-Lifshitz) gravity 52 minutes
- Lorentz Violation and Hořava-Lifshitz gravity 55 minutes
- Finite (hidden) charge density 58 minutes
- Lifshitz difussion 61 minutes