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Gravity and Cosmology in the Hořava-Lifshitz context

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Introduction

• Gravity is the oldest known but least understood force.

• The biggest puzzles today (dark energy and the cosmological constant problem) have gravity as their weakest link.

• The major clash seems to be between gravity and the quantum theory. Both issues are summarized in: "What is quantum (gravity+matter)".

• There are not many candidates on the table:
(a) String theory provides a perturbative theory of quantum gravity valid well below the Planck scale. It sheds no light so far on the puzzles mentioned above although it has given positive hints related to microstates of the black holes and an underlying gauge theory description of gravitational phenomena.

(b) Canonical quantum gravity is not yet at the point it can be called a theory.

• A proposal by Hořava on a field-theoretic quantum theory of gravity by abandoning full diffeomorphism invariance has been scrutinized in the last year.

The Hořava-Lifshitz idea

• By breaking full diffeomorphism invariance a power-counting renormalizable UV theory theory can be constructed. Such a theory contains terms up to cubic in spatial curvatures.

• The theory seems to contain Einstein gravity in the IR.

• It provides a natural alternative to inflation without the need of important fine tuning.

• The projectable version of the theory may also provide an alternative to dark matter.

However:

• The power counting renormalizability is not enough for a UV complete description. The RG pattern of classically marginal couplings in the UV is important and unknown. The IR fixed points of some of the couplings is also crucial for the viability of the theory as an alternative to standard gravity.

• The breaking of diffeomorphism invariance indicates the presence of an extra scalar mode in the non-projectable theory. Such a mode can become often strongly coupled. This can become troublesome for observations or a place where the semiclassical description breaks down.

• The canonical structure of the non-projectable theory theory is unusual and very much depends on the semiclassical asymptotics.

• The breaking of Lorentz invariance is communicated to matter. There is no natural mechanism known for reinstating Lorentz invariance in the IR. Current limits in $\delta c/c$ are very stringent especially for Neutrinos.

• There are important puzzles that are raised when one considers black holes and their relatives: the presence of horizons can be energy and particle dependent, and the usual thermodynamic picture including the notion of microstates becomes ambiguous at least and problematic generically.

The plan

• Our approach is "realistic". In non-linear bosonic theories what is most important is the physics near semiclassical configurations that may be relevant for our universe. There are two basic classes for such solutions: FRW and static nearly spherically symmetric solutions.

• Review of some aspects of HL cosmology

• Analysis of recent work on static spherically symmetric solutions both for the standard and modified theory

• Dispersive geodesics

• Outlook and open problems

Hořava-Lifshitz Gravity

• Start from the ADM decomposition of the metric

$$ds^2 = -N^2\ dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \quad , \quad N_i = g_{ij}N^j.$$

• The kinetic terms are given by

$$S_K = \frac{2}{\kappa^2}\int dt d^3x \sqrt{g} N\ \left(K_{ij}K^{ij} - \lambda K^2\right)$$

in terms of the extrinsic curvature

$$K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

• λ is a dimensionless coupling that breaks full diff invariance. General Relativity has $\lambda = 1$

• For renormalizability we would like to impose that $z = 3$, so that the spatial metric components are dimensionless

$$t \to b^3\ t \quad , \quad x^i \to b\ x^i \quad , \quad [N] = 0 \quad , \quad [g_{ij}] = 0 \quad , \quad [N_i] = 2 \quad , \quad [w] = 0$$

• The "potential" is

$$V = \int dtd^3x \ \sqrt{g}N \ V(g_{ij})$$

• For renormalizability it should contain up to six derivatives. The six-derivative terms are classically-scale invariant. Terms with a lower number of derivatives are "relevant".

$$\nabla_i R_{jk} \nabla^i R^{jk} \quad , \quad \nabla_i R_{jk} \nabla^j R^{ik} \quad , \quad R \Box R \quad , \quad R_{ij} \Box R^{ij}$$

modify already the propagator while

$$R^3 \quad , \quad R R_{ij} R^{ij} \quad , \quad R_{ij} R^i{}_k R^{jk}$$

provide scale invariant interactions.

• The (local) invariance of the theory is

$$t \to h^0(t) \quad , \quad x^i \to h^i(t, x^j)$$

• The theory can be written in the Stuckelberg form as a theory with full diffeomorphism invariance plus a scalar. Fixing the "gauge" $\phi = t$ gives back the initial formulation. In this form the breaking of diffeomorphism and Lorentz invariance can be ascribed to the "background".

The projectable theory

• There is a different definition leading to the "projectable" theory

Hořava

$$N \to N(t)$$

• In this case $N(t)$ can be gauge-fixed to 1, if non-zero.

• The Hamiltonian constraint is integrated over space: this si gives a global condition.

• The canonical structure of this theory is different.

Detailed Balance

• Hořava postulated an action implementing "detailed balance"

$$V = \frac{\delta W(g_{ij})}{\delta_{ij}} \mathcal{G}_{ij;kl} \frac{\delta W(g_{kl})}{\delta_{kl}}$$

• W is an invariant functional in 3d

• Motivation stems from simplicity (reduction of coupling constants, and special renormalizability properties)

• At the marginal level

$$W = \frac{1}{w^2} \int \omega_3(\Gamma) + \mu \int d^3x \, [R - 2\Lambda_W]$$

where

$$[w] = 0 \quad , \quad [\mu] = [-1] \quad , \quad [\Lambda_W] = -2$$

• The full action (obeying detailed balance) is :

.

$$S=\int dt d^3x\sqrt{g}N\Big[\alpha(K_{ij}K^{ij}-\lambda K^2)+\beta C_{ij}C^{ij}+\gamma\mathcal{E}^{ijk}R_{il}\nabla_j R^l{}_k+$$

$$+\zeta R_{ij}R^{ij}+\eta R^2+\xi R+\sigma\Big],$$

$$\alpha=\frac{2}{\kappa^2}\quad,\quad\beta=-\frac{\kappa^2}{2w^4}\quad,\quad\gamma=\frac{\kappa^2\mu}{2w^2}\quad,\quad\zeta=-\frac{\kappa^2\mu^2}{8}$$

$$\eta=\frac{\kappa^2\mu^2}{8(1-3\lambda)}\frac{1-4\lambda}{4}\quad,\quad\xi=\frac{\kappa^2\mu^2}{8(1-3\lambda)}\Lambda_W\quad,\quad\sigma=\frac{\kappa^2\mu^2}{8(1-3\lambda)}(-3\Lambda_W^2).$$

$$[\kappa]=0\quad,\quad[w]=0\quad,\quad[\lambda]=0\quad,\quad[\mu]=-1\quad,\quad[\Lambda_W]=-3$$

• It was realized early on that the theory with detailed balance is not a good starting point if matching to observable gravity is the goal

Kiritsis+Kofinas, Nastase, Sotiriou+Visser+Weinfurter

• Dropping detailed balance, all 8 parameters above can be considered independent.

HL Cosmology: generalities

• As $c \to \infty$ in the UV, we expect that there is no horizon problem in the HL theory.

Kiritsis+Kofinas

• Spatial curvature effects are enhanced, to $\frac{1}{a^6}$ making the flatness problem milder.

Kiritsis+Kofinas

• The fact that there are both R^2 and R^3 terms with the sign of R^2 not seriously constrained allows for bouncing cosmologies.

Calcagni, Kiritsis+Kofinas, Brandenberger, Gao+Wang+Xue+Brandenberger

• The theory contains higher derivatives but in a controlable/bounded fashion. They may be relevant in resolving singularities

• The UV theory is scale invariant: therefore it can generate a scale invariant spectrum of cosmological perturbations without the need for acceleration.

Mukohyama, Kofinas+Kiritsis, Gao+Wang+Brandenberger+Riotto

Cosmological backgrounds

- We make a cosmological ansatz

$$N = 1 \quad , \quad N_i = 0 \quad , \quad g_{ij} = a^2(t)\gamma_{ij}$$

- The Friedman equations are

$$3\alpha(3\lambda - 1)H^2 = \rho - \sigma - \frac{6k\xi}{a^2} - \frac{12k^2(\zeta + 3\eta)}{a^4} + \frac{\theta\ k^3}{a^6}$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

- The "cosmological" Planck scale is distinct from the gravitational one.

- The k^2/a^4 although generated by curvature, resembles mirage/dark radiation. It can generate a bouncing cosmology provided there is non-relativistic matter

Calcagni, Kiritsis+Kofinas

- It was argued that in the contracting phase before the bounce scale-invariant perturbations can be generated

Brandenberger

Scale-Invariant Cosmological Perturbations

• Homogeneous cosmology sees very little of the structure of the classical theory and in particular the scale-invariant part.

• The perturbations see the full structure of the theory

• Tensor perturbations satisfy

Takahashi+Soda

$$\frac{\partial^2}{\partial\eta^2}v^A_{\vec{k}} + \left[(k^A_{eff})^2 - \frac{2}{\eta^2}\right]v^A_{\vec{k}} = 0$$

$$(k^A_{eff})^2 = c^2k^2\left[1 + \frac{(1-3\lambda)}{\Lambda_W c^2}H^2(ck\eta)^2(1+\rho^A\frac{2H}{w^2\mu c}ck\eta)^2\right]$$

and are polarization dependent if the CP-odd term $R\nabla R$ appears in the action.

• The non-trivial polarization may have an observable size depending on the value of the couplings.

• Scalar perturbations are also interesting. We assume a spatially flat universe and the scalar field in Fourier space: the quadratic action is

$$S = \int d^3k \int dt \ a^3 \left[|\dot{\Phi}|^2 + \frac{1}{a^6}\left(-\ell^4 k^6 + y_2 \ell^2 k^4 - y_1 k^2 - m^2\right)|\Phi|^2 \right],$$

• ℓ is a length scale characteristic of the UV behavior of the scalar theory, and $y_{1,2}$ are dimensionless coefficients. In particular, y_1 is the square of the speed of light in the scalar theory.

• The fluctuations $\delta\Phi$ satisfy

$$\delta\ddot{\Phi} + 3H\delta\dot{\Phi} + \frac{\ell^4 k^6 - y_2 \ell^2 k^4 + y_1 k^2 + m^2}{a^6}\delta\Phi = 0.$$

• At high energy the dispersion relation is

$$E^2 \simeq \ell^4 \frac{k^6}{a^6}$$

• Typically, a fluctuation mode oscillates if $E >> H$, while it is frozen in the opposite limit $E << H$.

• Here:

$$\frac{E^2}{H^2} \simeq \frac{\ell^4 \ k^6}{H^2 a^6}$$

• If $H^2 a^6$ is an increasing function of time this will freeze the oscillations eventually. This is a key feature of standard inflation.

• From the cosmological equations we find that this is satisfied for all matter with $w < 1$, including curvature.

$$H^2 \; a^6 \quad \sim \quad \frac{a^6}{a^{3(1+w)}} \quad \sim \quad a^{3(1-w)}$$

(in normal cosmology, we have $H^2 a^2 \quad \sim a^{-(1+3w)}$ instead and $w < -\frac{1}{3}$).

• At freezout, $H^2 a^6 = \ell^4 k^6$ and

$$\frac{k}{a} \sim \frac{H^{\frac{1}{3}}}{\ell^{\frac{2}{3}}} \quad \to \quad H\lambda_{phys} = H\frac{a}{k} \sim (H\ell)^{\frac{2}{3}} \gg 1$$

Therefore, they produce super-horizon scales if this happens in the early (HL) era.

• The solution is

$$\delta\Phi(t, \vec{k}) = \frac{1}{(2\pi)^3\sqrt{2\kappa}} e^{-i\kappa \int \frac{dt}{a^3} + i\vec{k}\cdot\vec{x}} \quad , \quad \kappa \equiv \sqrt{\ell^4 k^6 - y_2 \ell^2 k^4 + y_1 k^2 + m^2}.$$

It is exact in the HL era! It freezes when $\frac{k^3\ell^2}{a^3} \simeq H$.

- The power spectrum is

$$\langle \delta\Phi(t,\vec{k})\delta\Phi(t,\vec{k}')\rangle = \frac{(2\pi)^3}{2\kappa}\delta(\vec{k}+\vec{k}') \equiv (2\pi)^3\delta(\vec{k}+\vec{k}')\frac{2\pi^2}{k^3}\mathcal{P}_{\delta\Phi}$$

From where we obtain as $k\ell >> 1$ a scale invariant spectrum,

Mukohyama, Kiritsis+Kofinas

$$\sqrt{\mathcal{P}_{\delta\Phi}} = \frac{1}{2\pi\ell}$$

- The corrections to this relation come from the relevant corrections (powers in k) as well as logarithmic UV renormalization

- The scalar will remain frozen until the universe cools, then it becomes relativistic and may decay to other particles.

- No exit problem exists: a mechanism of transferring the perturbations to observable matter is needed.

- There is no strong coupling problems at the linearized level near cosmological backgrounds. Strong coupling might develop however at the non-linear level.

Gao+Pang+Brandenberger+Riotto

Spherically symmetric, static, (star) solutions

• To fully study the theory we need to know the backgrounds with high symmetry. The reason is theefold:

(a) Such solutions are good approximations to realistic situations,

(b) They are the starting point of perturbation theory that captures the physics of such configurations

(c) Are important for conceptual issues (eg. black holes).

• Such backgrounds are FRW backgrounds and spherically symmetric ones.

• An important set of backgrounds in our neighborhood and else-where concerns gravitational sources that are (almost) static and (almost) spherically symmetric.

• In standard GR, such solutions are described, up to diffeos by a single function $f(r)$

$$ds^2 = -f(r)\ dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_k^2$$

• In the HL theory the most general ansatz

$$ds^2 = -(\hat{N}(r)^2 - N_r(r)^2)f(r)dt^2 + 2N_r(r)drdt + \frac{dr^2}{f(r)} + r^2 d\Omega_k^2,$$

involves three functions, $f(r), N_r(r), \hat{N}(r)$.

• As the full set of diffeomorphisms are not symmetries, different coordinate systems may correspond to different solutions. In particular, zeros of f, are singularities in HL gravity.

• Such solutions in this static coordinate system correspond to stars, in a region of the radial coordinate.

• Black holes with regular horizons can be described in other systems of coordinates, for example Eddington-Finkelstein coordinates:

$$ds^2 = N(r)dv^2 - 2B(r)dvdr + r^2 d\Omega_k^2$$

We will first analyze general solutions with zero shift $N_r = 0$ for the generalized action

$$S = \int dt d^3x \sqrt{g} N \Big[\alpha (K_{ij}K^{ij} - \lambda K^2) + \beta C_{ij}C^{ij} + \gamma \mathcal{E}^{ijk} R_{il} \nabla_j R^l{}_k +$$

$$+ \zeta R_{ij}R^{ij} + \eta R^2 + \xi R + \sigma \Big],$$

As the ansatz

$$ds^2 = -\hat{N}(r)^2 f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_k^2,$$

is conformally invariant in 3d, the $\alpha, \beta, \gamma, \lambda$ terms do not contribute to the equations of motion (we set c=1).

Equations of motion

$$(3\zeta+8\eta)r^2f'^2+4r(f-k)\left[(\zeta+4\eta)f'-\xi r\right]-4\xi r^3f'+4(\zeta+2\eta)(f-k)^2+2\sigma r^4=0.$$

$$\hat{A}_{11}(\ln\hat{N})'+\hat{B}_{11}=0,$$

$$\hat{A}_{11}=r[(3\zeta+8\eta)rf'+2(\zeta+4\eta)(f-k)-2\xi r^2] \quad , \quad \hat{B}_{11}=(3\zeta+8\eta)\left[r^2f''-2(f-k)\right].$$

• This system can be integrated fully, and has a rich set of solutions that depend importantly on the couplings.

• In most of the solutions we will find, we have recognizable large distance asymptotics of the form

$$f(r)=k-\frac{\Lambda_{eff}}{3}r^2-\frac{2GM}{r}+\mathcal{O}(r^{-4}) \quad , \quad N^2=\hat{N}^2f=k-\frac{\Lambda_{eff}}{3}r^2-\frac{2G\tilde{M}}{r}+\mathcal{O}(r^{-4}).$$

$$\Lambda_{eff}=-\frac{3}{4(\zeta+3\eta)}\left[\xi\pm\sqrt{\xi^2-\frac{4\sigma}{3}(\zeta+3\eta)}\right]$$

In all cases with such asymptotics M and $\tilde{M}$ are the same.

• In special cases (including detailed balance) the asymptotics are different ("resonant").

Special cases

• $\zeta + 3\eta = 0$, $\zeta\eta \neq 0$.

$$f(r) = k + \frac{\sigma}{6\xi}r^2 + \frac{\xi}{\eta}r^2 y(r) \quad , \quad (\sqrt{1-3y} - \epsilon)\; e^{\epsilon\sqrt{1-3y}} = \left(\frac{r_o}{r}\right)^3 \quad , \quad \epsilon = \pm 1$$

The $\epsilon = 1$ branch has standard asymptotics. The $\epsilon = -1$ branch is non-standard

$$N^2 \simeq k + \frac{\sigma}{6\xi}r^2 + \frac{\xi r^2}{3\eta}\left[\left(\log\left(\frac{r^3}{r_o^3}\right) - \log\log\left(\frac{r^3}{r_o^3}\right)\right)^2 - 1\right] - \frac{2G\tilde{M}(r)}{r} + \mathcal{O}(r^{-4})$$

$$2G\tilde{M}(r) = \frac{r_o^3}{\log^2(r^3/r_o^3)}\left[\frac{\xi}{3\eta}\left(\log\left(\frac{r^3}{r_o^3}\right) - \log\log\left(\frac{r^3}{r_o^3}\right)\right)^2 - 1 + \frac{\sigma}{6\xi}\right] + \cdots.$$

• It behaves as there are log corrections to the cosmological constant and mass parameter.

• The Detailed-balance case with $\lambda = \infty$ is a singular limit in this class, but the solution in that case is simpler.

• $3\zeta+8\eta=0$, $\zeta\cdot\eta\neq 0$

$$f(r)=k+\frac{\xi}{4(\zeta+3\eta)}r^2+g(r).$$

$$(3\zeta+8\eta)r^2g'^2+4(\zeta+4\eta)rgg'+4(\zeta+2\eta)g^2+\frac{1}{2}\Big(4\sigma-\frac{3\xi^2}{\zeta+3\eta}\Big)r^4=0$$

that in this case simplifies to

$$2(\zeta+4\eta)r(g^2)'+4(\zeta+2\eta)g^2+\frac{1}{2}\Big(4\sigma-\frac{3\xi^2}{\zeta+3\eta}\Big)r^4=0\to g^2=r\left[c+\frac{2}{3\zeta}\Big(\sigma+\frac{6\xi^2}{\zeta}\Big)r^3\right],$$

and $\hat{N}=1$.

• Special cases of this solution were found before

Lu+Mei+Pope,Kehagias+Sfetsos,Park

• Although this solution has generically standard asymptotics, in the detailed balance case it behaves as

$$f(r)=k-\Lambda_W r^2+\sqrt{cr}\,,$$

The generic solutions

- $(\zeta+3\eta)(3\zeta+8\eta)\neq 0$

$$f(r)=k+\frac{\xi}{4(\zeta+3\eta)}r^2+g(r) \quad , \quad r^2g'^2+(C-4)rgg'+\frac{1}{2}(8-C)g^2-Br^4=0$$

$$A=\frac{8\zeta(\zeta+3\eta)}{(3\zeta+8\eta)^2}\ ,\ B=\frac{1}{3\zeta+8\eta}\left(\frac{3\xi^2}{2(\zeta+3\eta)}-2\sigma\right)\ ,\ C=\frac{16(\zeta+3\eta)}{3\zeta+8\eta}, A=\frac{C(6-C)}{4}$$

- There are three cases,($B\neq 0$ and $A\geq 0$, $A<0$) and $B=0$.
- When $A\geq 0$

$$\left(\frac{r}{r_o}\right)^3\left|\sqrt{1-\frac{A}{B}\frac{g^2}{r^4}}-\epsilon\frac{C}{2\sqrt{B}}\frac{g}{r^2}\right|=\exp\left[\frac{2\epsilon\sqrt{A}}{C}\ \text{arcsin}\left(\sqrt{\frac{A}{B}}\frac{g}{r^2}\right)\right] \quad , \quad \epsilon=\pm 1$$

- There is a similar solution at $A<0$, which apart from standard large distance asymptotics has also a non-standard one

$$f(r)\simeq k+\frac{1}{4(\zeta+3\eta)}\xi r^2+\frac{(2\sqrt{|A|})^{\frac{2\sqrt{|A|}}{|C|-2\sqrt{|A|}}}\ r^2}{\left(\sqrt{|A|}+\frac{|C|}{2}\right)^{\frac{|C|}{|C|-2\sqrt{|A|}}}}\ \left(\frac{r_o}{r}\right)^{\frac{3|C|}{|C|-2\sqrt{|A|}}}+\cdots$$

$$\hat{N}(r) \sim \left(\frac{r_o}{r}\right)^{-3\frac{|C|+2\sqrt{|A|}}{|C|-2\sqrt{|A|}}}.$$

• Such non-standard asymptotics exists in other modified gravity theories as well.

• When $B = 0$ the solution exists only if $A \leq 0$

$$g(r) = c_1\, r^{2-\frac{C}{2}+\varepsilon\sqrt{|A|}} \quad , \quad \hat{N}(r) = \hat{N}_o\, r^{\frac{C}{2}-2\varepsilon\sqrt{|A|}+2\frac{|A|}{C}} \quad , \quad \varepsilon = \pm 1$$

• This is the case that corresponds to the detailed-balance theory with

$$A_{DB} = 2\frac{1-3\lambda}{(1-\lambda)^2} \quad , \quad B_{DB} = 0 \quad , \quad C_{DB} = \frac{4}{1-\lambda}$$

and the solution is the one found by Lu, Mei and Pope

$$g(r) \sim r^{(2\lambda+\varepsilon\sqrt{6\lambda-2})/(\lambda-1)} \quad , \quad \hat{N}(r) \sim r^{-(1+3\lambda+2\varepsilon\sqrt{6\lambda-2})/(\lambda-1)},$$

Solutions in the modified HL theory

• Blas, Pujolas and Sibiryakov proposed a modified theory to avoid the strong-coupling problems. The simplest version of this theory is

$$S = \int dt d^3x \sqrt{g} N \Big[\alpha (K_{ij}K^{ij} - \lambda K^2) + \beta C_{ij}C^{ij} + \xi R + a_1 (a_i a^i) \Big] \quad , \quad a_i \equiv \frac{\partial_i N}{N}$$

and we will look for spherically symmetric solutions with zero shift of the form

$$ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

• Defining the dimensionless variable $b = \frac{4\xi}{a_1}$, the linearized stability of the theory constraints $0 < \frac{a_1}{\xi} < 2$ or $b > 2$.

Blas+Pujolas+Sibiryakov

• The nonlinear equations can be solved exactly and there are the following types of solutions:

• In the stable range, $b > 2 \to a < 0$, we find an asymptotically flat solution with positive "mass". The asymptotic expansion of this solution is

$$f = 1 - \frac{2GM}{r} - \frac{(2GM)^2}{2br^2} - \frac{(2GM)^3}{4br^3} + \mathcal{O}(r^{-4})$$

$$N^2 = 1 - \frac{2GM}{r} + \frac{2(GM)^3}{3br^3} + \mathcal{O}(r^{-4})$$

• There is another class of solutions with non-trivial asymptotic behavior

$$f \sim r^{b-2+\sqrt{b(b-2)}} \quad , \quad N \sim r^{-\frac{1}{2}\sqrt{\frac{b}{b-2}}}$$

as $r \to \infty$. For such solutions, the four-dimensional scalar curvature is singular as $r \to \infty$.

• In the borderline case $b = 2$, we find solutions with $f(r)$ arbitrary and

$$\frac{rN'}{N} = -1 + \frac{\epsilon}{\sqrt{f}} \quad , \quad \epsilon^2 = 1$$

This is in tune with the degeneracy of the spatial derivative action of the scalar mode in this case shown by Blas et al.

• In the unstable range, $0 < b < 2$, we also find an asymptotically flat solution with positive "mass". Its large distance expansion is similar to the stable case. There are no exotic asymptotics in this case.

• Constraints from PPN parameters constraint $-a \gtrsim 10^7$.

Blas+Pujolas+Sibiryakov

• The solutions to this theory are in one-to-one correspondence to hypersurface orthogonal solutions of the Einstein-Ether theory.

Blas+Pujolas+Sibiryakov, Jacobson

Adding the cosmological constant

$$f = 1 + \frac{b}{a} + \frac{c}{ab} r^2 + g(r) \quad , \quad r^2 g'^2 + 2a(rg' + g)g + (2b + cr^2)g = 0$$

$$a_1 = w + 2\xi \; , \; a = \frac{2w}{(w+2\xi)} \; , \; b = \frac{4\xi}{(w+2\xi)} \quad , \quad c = \frac{4\sigma}{(w+2\xi)} \quad , \quad a + b = 2$$

- The equation for the large distance asymptotics can be solved exactly
- The possible asymptotics are :

1. for $0 < |a| < \frac{2}{3}$, the only possible large-distance asymptotics is $g \sim r^2$ with subleading terms that are powers of $r^{\frac{3|a|-2}{|a|-2}}$.

2. for $\frac{2}{3} < |a| < 2$ the only large-distance asymptotics possible is $g \sim r^{2+\frac{1}{a_-}}$ with

$$a_- = \frac{\sqrt{|a|(2-a)} - (|a|-2)}{6|a|-4} \simeq \frac{1}{2|a|} + \mathcal{O}(|a|^{-2})$$

.

3. finally for $|a| > 2$ both of the previous two asymptotics are possible. This is the relevant range physically.

- The regular solutions have an expansion that is given by

$$f = 2 - \frac{2\xi}{2\xi - a_1} + \frac{(\xi - a_1)^2}{\xi(3\xi - 2a_1)(2\xi - a_1)}\sigma r^2 + \frac{C}{r^{\frac{a-2}{a+2}}} + \cdots \quad , \quad a = 2 - 4\frac{\xi}{a_1}$$

- The power of the Newtonian tail is now $1 + \frac{a_1}{\xi} \simeq 1 + \mathcal{O}(10^{-7})$.

Dispersive geodesics

• Particles with non-standard dispersion relations, do not follow the usual geodesics in the gravitational field.

Capasso+Polychronakos, Suyama, Rama,Kiritsis+Kofinas

• Start from the metric in ADM form

$$G_{00} = -N^2 + N_i g^{ij} N_j \quad , \quad G_{0i} = N_i \quad , \quad G_{ij} = g_{ij} \quad , \quad \det[G] = \det[g] N^2$$

• We also consider scalar matter for simplicity (at quadratic) level.

$$S_{\rm nr} = \int d^3x dt \sqrt{g} N \left[-\frac{1}{N^2} (\dot{\Phi} - N^i \partial_i \Phi)^2 - \Phi F[\Box] \Phi \right] \quad , \quad \Box = g^{ij} \nabla_i \nabla_j$$

• We are going to derive now the geodesic equations using the geometric optics approximation to the full equations of motion.

• To pass to point-like trajectories, we replace $i\partial_t \to p_0$, $i\partial_i \to p_i$, and neglect metric derivatives to obtain the equivalent Hamiltonian ("zero energy") constraint

$$H = -\frac{(p_0 - N^i p_i)^2}{N^2} + F[\zeta] = 0 \quad , \quad \zeta = g^{ij} p_i p_j$$

This is implemented with a Lagrange multiplier e, to obtain the world-line action as

$$S_{wl} = \int_0^1 d\tau \left[p_0 \dot{t} + p_i \dot{x}^i + \frac{e}{2} H \right]$$

where τ is the affine time of the path and dot stands for ∂_τ. e is a one-dimensional einbein.

• The action can be brought to the form

$$S_{wl} = \frac{1}{2} \int_0^1 d\tau \left[\frac{N^2}{e} \dot{t}^2 + e(F(\zeta) - 2\zeta F'(\zeta)) \right]$$

where ζ should be thought as a function of the metric, $\dot{x}^i$, and the einbein as given from

$$\zeta (F'(\zeta))^2 = \frac{g_{ij}(\dot{x}^i + N^i \dot{t})(\dot{x}^j + N^j \dot{t})}{e^2} \equiv \frac{\xi}{e^2}$$

• This action reduces to the standard relativistic action for $F(\zeta) = \zeta + m^2$.

$$S_{wl} = \frac{1}{2}\int_0^1 d\tau \left[\frac{N^2\dot{t}^2 - g_{ij}(\dot{x}^i + N^i\dot{t})(\dot{x}^j + N^j\dot{t})}{e} + em^2\right]$$

• We now consider a particle with a non-standard dispersion relation $F(\zeta) = \zeta^n$ (corresponds to a dispersion relation $(p^0)^2 - (\vec{p}^2)^n = 0$) a radial geodesic and a spherical symmetric metric to obtain

$$S = \frac{1}{2}\int_0^1 d\tau \left[\frac{N^2\dot{t}^2}{e} + (1-2n)e^{-\frac{1}{2n-1}}\left(\frac{\dot{r}}{2n\sqrt{f}}\right)^{\frac{2n}{2n-1}}\right]$$

$$e = \left(\frac{2N}{E}\right)^{\frac{2n-1}{n}}\frac{\dot{r}}{2n\sqrt{f}} \quad , \quad \dot{t} = \frac{2}{E}\left(\frac{2N}{E}\right)^{-\frac{1}{n}}\frac{\dot{r}}{2n\sqrt{f}}$$

$$\frac{dr}{dt} = nE\sqrt{f}\left(\frac{2N}{E}\right)^{\frac{1}{n}} \sim f^{\frac{1}{2}}\ (N^2)^{\frac{1}{2n}}$$

• At weak gravitational fields $G_N \to \frac{1}{2}\left(1+\frac{1}{n}\right) G_N < G_N$ and the effective gravitational interaction is weaker.

• For any $n > 1$ the geodesic is regular at the zeros of f, N.

• This is in accordance with the intuitive notion.

• If at r_* f has an a-th order zero and N^2 a b-th order zero, then the geodesic is singular if

$$a + \frac{b}{n} \geq 2.$$

• Extremal horizons are visible in HL gravity.

Outlook and Open problems

• What is the RG flow of marginal couplings? Is there asymptotic freedom in the UV? Is $\lambda = 1$ an IR fixed point?

• Is the theory really renormalizable?

• Does the extra scalar degree of freedom creates further problems for the theory

• Why Lorentz invariance in the matter sector is such a good symmetry.

• What is the physics and thermodynamics of BH in such gravity theories and how the associated puzzles are resolved?

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Thank you.

Equations of motion

The equation obtained by varying N is

$$-\alpha\left(K_{ij}K^{ij}-\lambda K^2\right)+\beta C_{ij}C^{ij}+\gamma\mathcal{E}^{ijk}R_{il}\nabla_j R^l{}_k+\zeta R_{ij}R^{ij}+\eta R^2+\xi R+\sigma=J_N,$$

with

$$J_N=-\mathcal{L}_{\text{matter}}-N\frac{\delta\mathcal{L}_{\text{matter}}}{\delta N}$$

The equation obtained by varying N_i is

$$2\alpha(\nabla_j K^{ji}-\lambda\nabla^i K)+N\frac{\delta\mathcal{L}_{\text{matter}}}{\delta N_i}=0.$$

The equation obtained varying g_{ij} is

$$\frac{1}{2}\Big[(\mathcal{E}^{mk\ell}Q_{mi})_{;kj\ell}+(\mathcal{E}^{mk\ell}Q_m^n)_{;kin}g_{j\ell}-(\mathcal{E}^{mk\ell}Q_{mi})_{;kn}{}^{;n}g_{j\ell}-(\mathcal{E}^{mk\ell}Q_{mi})_{;k}R_{j\ell}$$

$$-(\mathcal{E}^{mk\ell}Q_{mi}R_k^n)_{;n}g_{j\ell}+(\mathcal{E}^{mk\ell}Q_m^nR_{ki})_{;n}g_{j\ell}+\frac{1}{2}(\mathcal{E}^{mk\ell}R^n{}_{pk\ell}Q_m^p)_{;n}g_{ij}-Q_{k\ell}C^{k\ell}g_{ij}+$$

$$\mathcal{E}^{mk\ell}Q_{mi}R_{j\ell;k}\Big]+\Box[N(2\eta R+\xi)]g_{ij}+N(2\eta R+\xi)R_{ij}+2N(\zeta R_{ik}R_j^k-\beta C_{ik}C_j^k)$$

$$-[N(2\eta R+\xi)]_{;ij}+\Box[N(\zeta R_{ij}+\frac{\gamma}{2}C_{ij})]-2[N(\zeta R_{ik}+\frac{\gamma}{2}C_{ik})]_{;j}{}^{;k}+[N(\zeta R^{k\ell}+\frac{\gamma}{2}C^{k\ell})]_{;k}$$

$$-\frac{N}{2}(\beta C_{k\ell}C^{k\ell}+\gamma R_{k\ell}C^{k\ell}+\zeta R_{k\ell}R^{k\ell}+\eta R^2+\xi R+\sigma)g_{ij}+2\alpha N(K_{ik}K_j^k-\lambda KK_{ij})$$

$$-\frac{\alpha N}{2}(K_{k\ell}K^{k\ell}-\lambda K^2)g_{ij}+\frac{\alpha}{\sqrt{g}}g_{ik}g_{j\ell}\frac{\partial}{\partial t}[\sqrt{g}(K^{k\ell}-\lambda Kg^{k\ell})]+\alpha[(K_{ik}-\lambda Kg_{ik})N_j]^{;k}$$

$$+\alpha[(K_{jk}-\lambda Kg_{jk})N_i]^{;k}-\alpha[(K_{ij}-\lambda Kg_{ij})N_k]^{;k}+(i\leftrightarrow j)=-2N\frac{\delta\mathcal{L}_{\text{matter}}}{\delta g^{ij}},$$

where

$$Q_{ij}\equiv N(\gamma R_{ij}+2\beta C_{ij}).$$

The Lifshitz scaling symmetry

• Scale invariance is another central principle, that although typically broken in nature, it is powerful enough to organize whole regions of parameters in fundamental theories. (All perturbative theories we use are in this class.)

$$t \to b\ t \quad , \quad x^i \to b\ x^i$$

• Lorentz invariance implies an isotropic scaling. Poincaré invariance and locality together with scale invariance implies conformal invariance.

• In low energy+condensed matter systems, non-relativistic dynamics emerges naturally.

• Sometimes dynamical criticality emerges and scale invariance is non-relativistic

$$t \to b^z\ t \quad , \quad x^i \to b\ x^i$$

z is a dynamical critical exponent.

• A typical example of such a scaling appears in the Lifshitz critical theory

The Lifshitz (free) field theory

$$S_L = \int dtd^2x \left[\frac{1}{2}\dot{\Phi}^2 - \gamma(\Box\Phi)^2\right] \quad , \quad \Box = \sum_{i=1}^{2} \partial_i\partial_i$$

with $z = 2$:

$$[t] = 2 \quad , \quad [x^i] = 1 \quad , \quad [\Phi] = 0 \quad , \quad [\gamma] = 0$$

• It appears as a tri-critical point in a theory, with normal, BCS and striped phases by tuning

$$S_g = \int dtd^2x \left[\frac{1}{2}\dot{\Phi}^2 - \alpha\ \Phi\Box\Phi - \gamma\ (\Box\Phi)^2 + \cdots\right]$$

by tuning $\alpha \to 0$

• Although this is a 3d theory, its has 2d properties: in particular any polynomial in Φ is classically marginal and the propagator is logarithmically divergent in the IR

$$\frac{\partial}{\partial|\Delta\vec{x}|}\langle\Phi(t_1,\vec{x}_1)\Phi(t_2,\vec{x}_2)\rangle = \frac{1 - e^{-\frac{|\Delta\vec{x}|^2}{4\Delta t}}}{|\Delta\vec{x}|}$$

• The Lifshitz theory with $z > 1$ has at least one obvious relevant operator, namely $(\partial\Phi)^2$ which drives the theory to a Lorentz invariant theory in the IR,

$$S_{full} = \int dt d^2x \left[\frac{1}{2}\dot{\Phi}^2 + m^2 \; \Phi\Box\Phi - \gamma \; (\Box\Phi)^2 + \cdots \right] \to \int dt d^2x \left[\frac{1}{2}\dot{\Phi}^2 + \frac{m^2}{2} \; \Phi\Box\Phi \right]$$

with $m = c$.

• Lorentz invariance is not always guaranteed in the IR. For several Lifshitz scalars the individual speed of light could be different.

• In theories with dynamical critical exponent $z > 1$ the lower critical dimension is raised. 3+1 dimensional gravity can become marginal if $z = 3$

Hořava

• So far holographic flows have been found where a $z > 1$ theory holographically flows to a $z = 1$ theory.

Kachru+Liu+Mullingan

• Very recently an opposite holographic flow was found in the D3-D7 system from $z = 1$ to $z > 1$.

Azeyanagi+Li+Takayanagi

• It seems that always $z \geq 1$

Higher derivative Gravity

• The use of higher derivative couplings to improve gravity's UV behavior is not new.

• It is known that $R + R^2$ gravity is asymptotically free with propagator

$$\frac{1}{k^2 - \frac{(k^2)^2}{M_p^2}} = \frac{1}{k^2} - \frac{1}{k^2 - M_p^2}$$

Tomboulis

• It has however ghosts

• The idea is to combine:

1. broken Lorentz invariance to avoid ghosts (by including higher spatial derivatives but no time derivatives)

2. anisotropic scaling to make the theory scale invariant in the UV.

Hořava

The Cotton tensor

• There is a special scale invariant term that is also conformal $C_{ij}C^{ij}$ with C_{ij} the Cotton-tensor

$$C^{ij} = \frac{\epsilon^{ikl}}{\sqrt{g}}\nabla_k\left(R^j{}_l - \frac{1}{4}R\delta^j{}_l\right)$$

• In 3d it is the unique tensor that satisfies

$$C_{ij} = C_{ji} \quad , \quad C_i{}^i = 0 \quad , \quad \nabla^i\, C_{ij} = 0 \quad ,$$

and is conformal

$$g_{ij} \to e^{2\phi(x)}\, g_{ij} \quad , \quad C^{ij} \to e^{5\phi(x)}\, C^{ij}$$

• It is the analogue of the Weyl tensor in 3d.

• It can be obtained by the variation of the 3d gravitational CS action

$$S = \int \omega_3(\Gamma) \quad , \quad \omega_3(\Gamma) = Tr[\Gamma \wedge d\Gamma + \frac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma]$$

• Adding it to the gravitational potential provides a source of CP violation in gravity, that may have measurable consequences

Takahashi+Soda

Generalized Lifshitz QFTs (vectors)

- For (abelian) vectors, $[A_0]=-2\quad,\quad [A_i]=0$

$$S_{nr}=-\frac{1}{4g^2}\int d^3xdt\sqrt{g}N\left[-\frac{2}{N^2}g^{ij}(F_{0i}-N^kF_{ki})(F_{0j}-N^lF_{lj})-\right.$$

$$\left.-\frac{M^2}{N^2}(A_0-N^iA_i)(A_0-N^jA_j)+G[A_i]\right],$$

$$F_{0i}=\partial_tA_i-\partial_iA_0\quad,\quad F_{ij}=\partial_iA_j-\partial_jA_i$$

Define the magnetic field

$$B_i=\frac{1}{2}\frac{\epsilon_i{}^{jk}}{\sqrt{g}}F_{jk}\quad,\quad F_{ij}=\frac{\epsilon_{ij}{}^k}{\sqrt{g}}B_k\quad,\quad \nabla^iB_i=0.$$

$$G=a_0+a_1\zeta_1+a_2\zeta_1^2+a_3\zeta_1^3+a_4\zeta_2+a_5\zeta_1\zeta_2+a_6\zeta_3+a_7\zeta_4,$$

$$\zeta_1=B_iB^i\quad,\quad \zeta_2=\nabla_iB_j\nabla^iB^j\quad,\quad \zeta_3=\nabla_iB_j\nabla^iB^k\nabla^jB_k\quad,\quad \zeta_4=\nabla_i\nabla_jB_k\nabla^i\nabla$$

- a_i are arbitrary functions of A_iA^i.

- The "stress-tensor" is no-longer traceless $\to$ $w\neq\frac{1}{3}$.

Generalized Lifshitz QFTs (scalar matter)

- For scalars($[\Phi]=0$)

$$S_{\rm nr}=\int d^3x dt\sqrt{g}N\left[\frac{1}{N^2}(\dot\Phi-N^i\partial_i\Phi)^2+F[\xi_1,\xi_2,\cdots,\Phi]\right]\quad,\quad \xi_n=\Phi\Box^n\Phi$$

where the (renormalizable) potential F is (modulo spatial derivatives of the lapse)

$$F[\xi_n,\Phi]=F_0(\Phi)+F_1(\Phi)\ \xi_1+F_{11}(\Phi)\xi_1^2+F_{111}(\Phi)\xi_1^3+$$

$$+F_2(\Phi)\xi_2+F_{21}(\Phi)\xi_2\xi_1+F_3(\Phi)\xi_3.$$

and dispersion relation

$$\frac{E^2}{F_1(0)}=\frac{1}{4}\frac{F_0''(0)}{F_1(0)}+(\vec k^2)-\frac{F_2(0)}{F_1(0)}(\vec k^2)^2+\frac{F_3(0)}{F_1(0)}(\vec k^2)^3$$

- Like in gravity the speed of light is infinite in the UV $(E^2\sim k^6)$.

- In the IR it is $c^2=F_1(0)$. It is not a priori equal to that of gravity.

The IR action

• In the IR the most relevant terms are

$$S=\int dt d^3x\sqrt{g}N\left[\alpha(K_{ij}K^{ij}-\lambda K^2)+\xi R+\sigma\right],$$

• In order for this to reproduce Einstein gravity this must have $\lambda \simeq 1$ in the IR to a good degree of accuracy.

• Defining $x^0 = ct$, choosing $\lambda = 1$ and

$$c=\sqrt{\frac{\xi}{\alpha}} \quad , \quad 16\pi G_N=\frac{1}{\sqrt{\alpha\xi}} \quad , \quad \Lambda_E=-\frac{\sigma}{2\xi}, \quad , \quad [c]=-2 \quad , \quad [x^0]=1$$

the action is that of Einstein

$$S_E=\frac{1}{16\pi G_N}\int d^4x\sqrt{\tilde{g}}\left[K_{ij}K^{ij}-K^2+R-2\Lambda_E\right]=\frac{1}{16\pi G_N}\int d^4x\sqrt{\tilde{g}}\left[\tilde{R}-2\Lambda_E\right].$$

• Diff invariance is restored in the IR.

• However, the low energy asymptotics can be misleading as they ignore the presence of non-trivial backgrounds.

Propagating degrees of freedom

- The UV (free) fixed point is obtain by taking $\alpha, \beta \to 0$.

- We expand in small perturbations

$$g_{ij} \simeq \delta_{ij} + h_{ij} \quad , \quad N \simeq 1 + n \quad , \quad N_i \simeq n_i$$

- n drops out at quadratic level and we fix the gauge

$$n_i = 0 \quad , \quad H_{ij} \equiv h_{ij} - \lambda \delta_{ij} h \quad , \quad \partial_i H_{ij} = 0$$

- Separating the trace and traceless part

$$H_{ij} = \hat{H}_{ij} + \frac{1}{2}\left(\delta_{ij} - \frac{\partial_i \partial_j}{\partial^2}\right) H \quad , \quad S_{Kin} \sim \int dt d^3x \left[(\dot{\hat{H}}_{ij})^2 + \frac{1-\lambda}{2(1-3\lambda)}(\dot{H})^2\right]$$

- The potential coming from the Cotton tensor is

$$V \sim \int dt d^3x \ \hat{H}_{ij} \Box^3 \hat{H}_{ij}$$

• $\widehat{H}_{ij}$ is a massless spin-two graviton. It has a dispersion relation of the form

$$\frac{E^2}{c^2} = \frac{(\vec{k}^2)^3}{M^4} \pm \frac{(\vec{k}^2)^2}{m^2} + \vec{k}^2$$

• H is an extra degree of freedom. Near flat space it has a potential containing $(\vec{k}^2)^2$ and $(\vec{k}^2)$ terms but no $(\vec{k}^2)^3$ term.

• It is at the heart of the strong coupling problems of the theory

Charmousis+Niz+Padilla+Saffin, Blas+Pujolas+Sibiryakov

• It has been recently argued that the addition of terms involving spatial derivatives of the lapse, N, may alleviate the string coupling problems.

Blas+Pujolas+Sibiryakov

Detailed plan of the presentation