Condensed Matter Physics Meets High Energy Physics
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On universality classes in strongly coupled doped systems

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Bibliography

Based on ongoing work with:

C. Charmousis, B. Gouteraux (Orsay), B. S. Kim and R. Meyer (Crete)

and previous work

U. Gürsoy, E.K. and F. Nitti, arXiv:0707.1324 [hep-th]

. arXiv:0707.1349 [hep-th]

U. Gürsoy, E.K. L. Mazzanti and F. Nitti, arXiv:0804.0899 [hep-th]

Independent work along similar lines by

M. Cadoni, G. D'Apolonio and P. Pani, http://arxiv.org/abs/0912.3520

The holographic setup

• Holography is providing a gravitational/string theory language for large-N strongly coupled theories.

• There are very few theories that we can control well. Many more that we can control partly.

• Our intuition and "model building" is currently developing.

• An important goal is the analogue of developing "effective holographic theories" (EHT). Unlike the low-energy expansion, they rely on a "gap" in the range of anomalous dimensions.

• Although not always justified, they can be a good "phenomenological laboratory" for strong coupling phenomena admitting a semiclassical description. (two complementary intuitions coming from level-truncation in tachyon condensation studies and ... QCD sum rules).

• As in EFT, the rules of EHTs are slow to be uncovered.

• Condensed matter physicists: patience please!

Einstein-Dilaton-U(1) theory

$g_{\mu\nu} \quad \rightarrow \quad T_{\mu\nu}$ Stress-energy tensor

$A_\mu \quad \rightarrow \quad J_\mu$ conserved current.

ϕ Most important scalar operator that "drives" the interactions.

A familiar example from HEP is QCD

$\phi \rightarrow Tr[F^2]$ and J_μ is the baryon number current.

In many cases this separation of dynamics is pertinent: "glue"+charge.

A generic large-wavelength action (up to two derivatives) is

$$S = \int d^{p+1}x \left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4}F^2 \right] \tag{1}$$

Einstein-Dilaton theory

• The theory with no charge degrees of freedom has been studied extensively lately, as it seem to be very close to the dynamics of large-N YM.

Choosing $p = 4$, and choosing a monotonic potential with

$$V(\phi = -\infty) = \frac{12}{\ell^2} \quad , \quad V(\phi \to \infty) \quad \sim \quad e^{\sqrt{\frac{3}{8}}Q\phi}$$

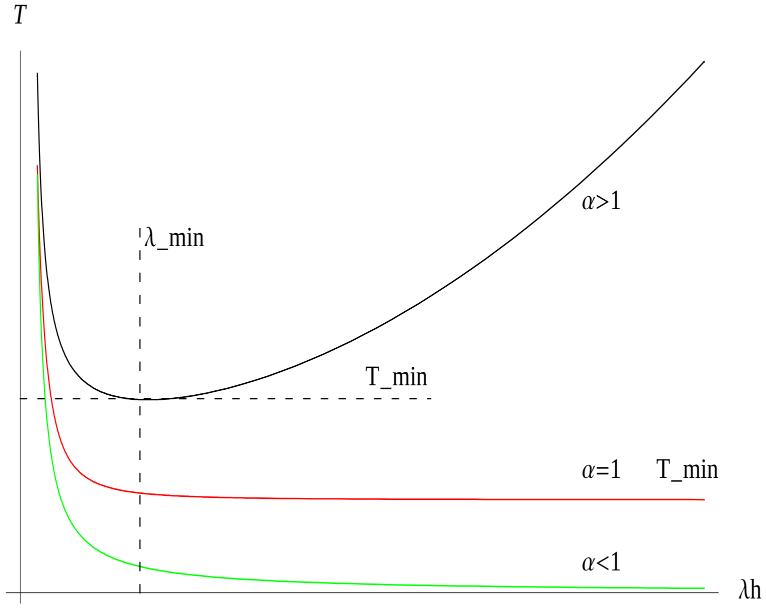
the theory has confinement* of "color", a mass gap, discrete spectrum and a "good" (repulsive) IR singularity if

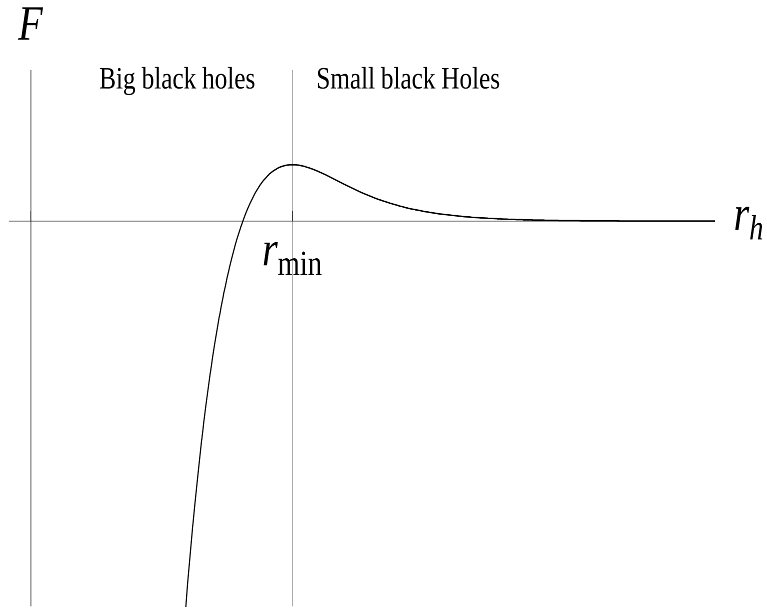
$$\frac{4}{3} < Q < \frac{4\sqrt{2}}{3}$$

For larger values the singularity is "bad". For smaller values the spectrum is continuous.

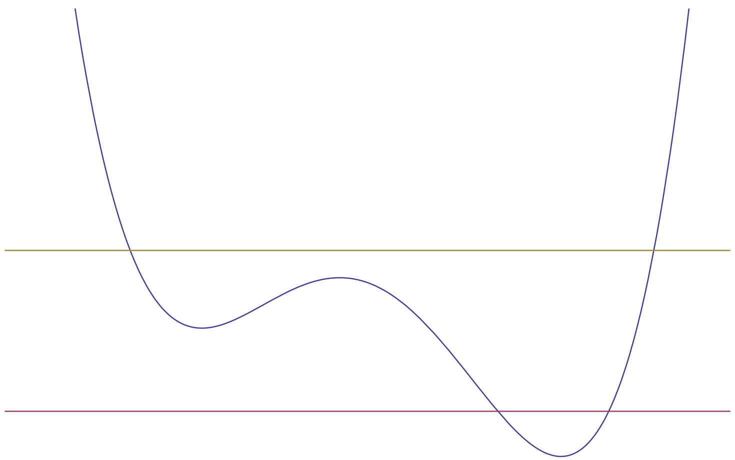
Gursoy+E.K.+Mazzanti+Nitti

• "confinement" is correlated with the existence of a first order "deconfining" phase transition to a black-hole phase.





Small black hole branch has $T \to \infty$ as $r_h \to 0$, where it becomes a naked (but "good" singularity)



A singularity is "good" when

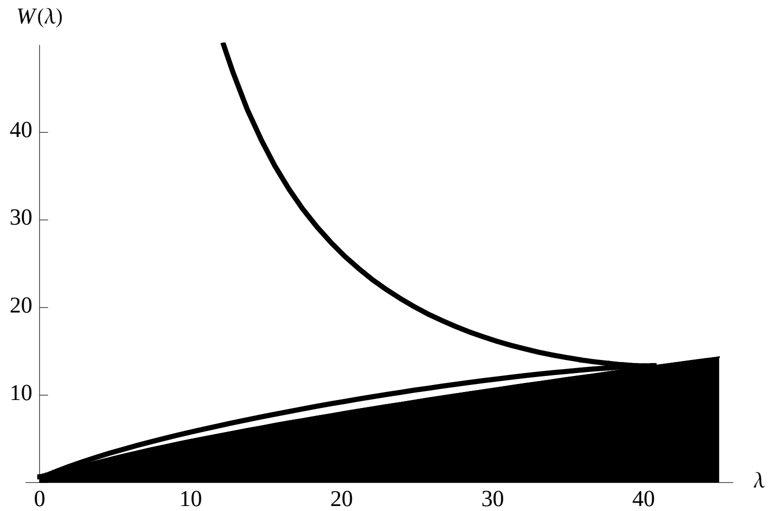
• The second order equations describing all fluctuations are Sturm-Liouville problems (no extra boundary conditions needed at the singularity).

• The singularity is "repulsive" (like the Liouville wall)

Gursoy+E.K.+Nitti

• The singularity can smoothly be cloaked by a horizon.

Gubser



"Doping"

When $F_{\mu\nu} \neq 0$ new dynamics is in order

• Generically the charge can self-interact strongly

• It can have non-trivial back-reaction on the graviton and scalar.

$$S = \int d^{p+1}x \left[R - \frac{4}{3}(\partial\phi)^2 + V(\phi) - Z(\phi)\sqrt{\det(g_{\mu\nu} + F_{\mu\nu})} \right] \quad , \quad Z(\phi) = \frac{1}{g(\phi)^2}$$

• The "probe limit": charge carriers feel a strong force from "glue", but their influence on the glue vacuum is small.

$$V(\phi) >> \frac{1}{g^2(\phi)}\sqrt{1 + \left(\frac{Q\ g^2(\phi)}{\tilde{S}(\phi)}\right)^2}$$

Hartnoll+Polchinski+Silverstein+Tong

• Otherwise the charge back-reaction on the glue is important.

• The "Maxwell" limit: charge self-interactions are unimportant

$$Q\ g^2(\phi) \ll \tilde{S}(\phi)$$

• In this case you expand the DBI action to consider

$$S = \int d^{p+1}x \left[R - \frac{4}{3}(\partial\phi)^2 + V(\phi) - \frac{1}{g^2(\phi)} + \frac{1}{4g^2(\phi)}F^2 \right]$$

• There are several situations that $g^2(\phi) \to \infty$: This is generic in a class of problems involving brane-antibrane annihilation

Sen

• This is what is expected to happen during chiral symmetry breaking in QCD.

Sakai+Sugimoto, Casero+E.K.+Paredes

Einstein-Maxwell-Liouville gravity

$$S = \int d^{p+1}x \left[R - \frac{4}{3}(\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4} F^2 \right]$$

$$V = V_0 \; e^{\mathbf{b}\phi} \quad , \quad Z(\phi) = e^{\mathbf{a}\phi}$$

- We are interested in finding general solutions, describing backreacting dopped systems (neglecting charge "self-interactions").

- The simplest solutions to start from are "scaling" solutions.

- They are trustworthy in places that V becomes large.

- They provide "universality classes" of IR behavior at or near extremality.

- They can be completed to asymptotically AdS solutions in UV regime when $V \to 0$. (Caveat: depending on the AdS completion, new semiclassical solutions may be introduced)

Generic scaling solutions

- We work in the domain wall frame

$$ds^2 = e^{2A}(-e^g dt^2 + dx^i dx_i) + e^{-g} dr^2 \quad , \quad A = A_t(r)\ dt$$

- There is a "$Noether$" charge

Gubser+Rocha

$$\mathcal{Q} = e^{3A} g' e^g - e^A Z A_t A_t' = e^{3A} g' e^g - q A_t \quad , \quad \frac{\partial \mathcal{Q}}{\partial r} = 0$$

and $\mathcal{Q} = 0$ **at extremality.**

Searching for scaling solutions

$$F_{rt} = \frac{q}{Z(\phi) e^{(p-3)A(r)}} \quad , \quad e^A = r^{\frac{4 c_p^2}{3(p-1)}} , \quad e^{\Phi} = e^{\Phi_0}\ r^{c_p}$$

$$f = e^g = f_0\ r^{c_2 + c_f} \left[1 - \left(\frac{r_0}{r}\right)^{c_f}\right] \quad , \quad c_f = 2 - c_2 - b c_p$$

- The generic solutions have

$$c_2^{(1)} = 1 - \frac{4p\ c_p^2}{3(p-1)} \quad , \quad c_2^{(2)} = 0$$

$$c_p^{(1)} = -\frac{3}{8}(a+b) \quad , \quad c_p^{(2)} = 0$$

$$e^{\Phi_0} = \left[\pm\frac{q^2}{2V_0(c_2+c_f)}\left(2+c_p\left(b+\frac{8}{3}c_p\right)\right)\right]^{\frac{1}{a+b}} \quad , \quad f_0 = \frac{2V_0 e^{b\Phi_0}}{c_f\left[2+c_p\left(b+\frac{8}{3}c_p\right)\right]} .$$

- There is a relation between the scalar and gauge charge

- The "special" scaling solutions

$$c_p = -\frac{3}{8}(a+b) \quad , \quad c_2 = 2 + bc_p \quad , \quad c_f = 1 - c_2 - \frac{4p\ c_p^2}{3(p-1)}$$

$$e^{\Phi_0} = \left[\pm\frac{q^2}{2V_0}\left(1+\frac{8}{3}\frac{c_p^2}{c_2}\right)\right]^{\frac{1}{a+b}} \quad , \quad f_0 = \frac{2V_0 e^{b\Phi_0}}{c_f\left(c_2+\frac{8}{3}c_p^2\right) {r_0}^{c_f}} .$$

Special cases found by Mann, Gubser+Rocha, possibly others. Goldstein et al. found solutions with V = constant

• "Physicality" conditions

• At $r \to \infty$, $e^A \to \infty$, so we have a boundary.

• Scalar curvature invariants should be regular at the UV boundary(maybe optional and remain to be investigated)

• $V(\phi) \to 0$ at the boundary (so the solutions can be completed to as. AdS solutions)

• Stability $C_V > 0, C_q > 0$

• S at extremality vanishes.

All of the above select some ranges of the parameters.

The relevant thermodynamic functions are simple scaling functions

$$\mathcal{F}, E, T, \mu \sim T^a \; q^b$$

• No phase diagram can be drawn as we do not know the full set of solutions with the same asymptotics. (but we can find them numerically with some effort)

Charged solutions with $\gamma\delta = 1$

$$V(\phi) = V_0\ e^{-\delta\phi} \quad , \quad Z(\phi) = e^{\gamma\phi} \quad , \quad \gamma\delta = 1$$

The equations can be solved exactly and the general solution by found.

$$\delta^2 \leq 3$$

Charmousis+Goutereaux+Soda

$$ds^2 = -\frac{e^{\frac{\phi-\phi_0}{\delta}}}{r^{3-\delta^2}} U(r) dt^2 + \frac{2(3-\delta^2) e^{\delta\phi} r^{1-\delta^2}}{V_0} \frac{dr^2}{U(r)} + r^2 \left[1 - \left(\frac{r^-}{r}\right)^{3-\delta^2}\right]^{\frac{2(\delta^2-1)^2}{(3-\delta^2)(1+\delta^2)}} \left(dx^2 + dy^2\right)$$

$$e^{\phi} = e^{\phi_0} r^{2\delta} \left[1 - \left(\frac{r^-}{r}\right)^{3-\delta^2}\right]^{\frac{4\delta(\delta^2-1)}{(3-\delta^2)(1+\delta^2)}} \quad , \quad \mathcal{A} = \left(\mu - \sqrt{\frac{4\delta^2}{1+\delta^2}} \frac{q\ e^{-\frac{\phi_0}{2\delta}}}{r^{3-\delta^2}}\right) dt$$

$$U(r) = r^{3-\delta^2} - 2m + q^2 r^{\delta^2-3} \quad , \quad \left(r^{\pm}\right)^{3-\delta^2} = m \pm \sqrt{m^2 - q^2} \quad , \quad \mu = \frac{2|\delta| e^{-\frac{\phi_0}{2\delta}}}{\sqrt{1+\delta^2}} \frac{q}{r_+^{3-\delta^2}} = \mu_0 \frac{q}{r_+^{3-\delta^2}}$$

- Regular bh solution $0 < r^- < r^+$, becomes extremal at $m = q$
- "Equation of state"

$$T = \left(\frac{Q}{\mu} e^{-\frac{\phi_0}{\delta}}\right)^{\frac{1-\delta^2}{3-\delta^2}} \left(1 - \frac{\mu}{\mu_0}\right)^{\frac{-3\delta^4+6\delta^2+1}{(3-\delta^2)(1+\delta^2)}}$$

• At q=0, we have a (singular) extremal solution (S=0) and a regular BH. The extremal solution is "good" according to Gubser, but we must investigate the T=0 spectrum.

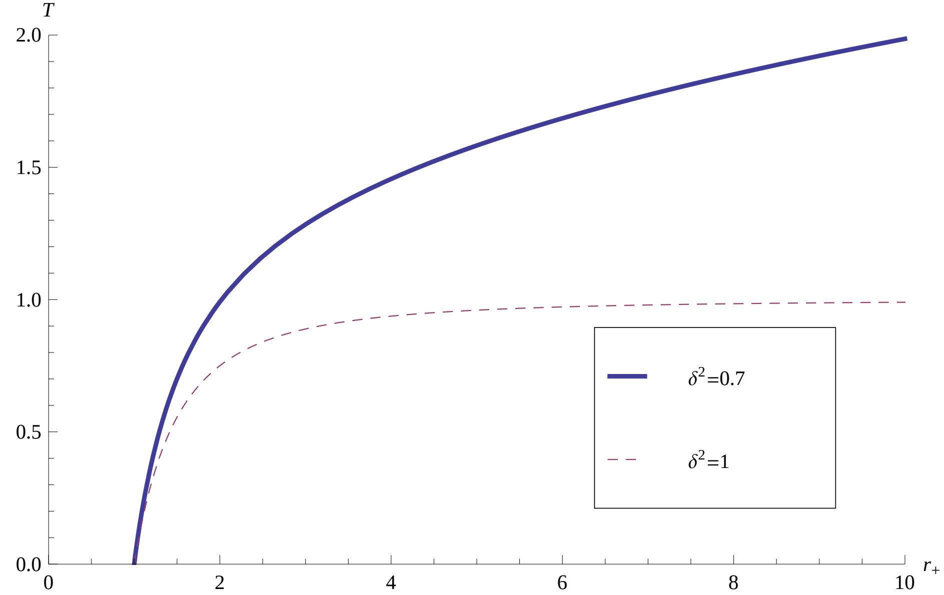
• At q=0, there is a second order phase transition at $T = 0$ from the extremal to the BH solution.

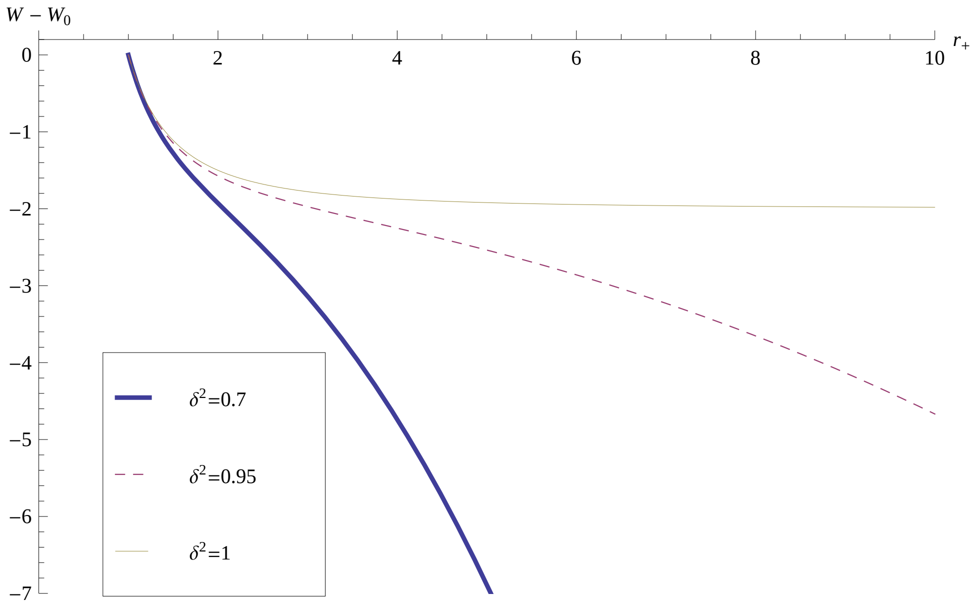
• At $q \neq 0$, we have the extremal solution (S=0) as well as 1 or two regular BH solutions (liquid state). The extremal solution is regular for $1 > \delta^2$ and singular (good à la Gubser) otherwise.

• The solutions can be corrected to asymptotically AdS in the UV as $V(\phi) \to 0$. The DBI action can also linearized everywhere except arbitrarily near the boundary , and near the singularity of the extremal solutions for $\delta^2 > 1$.

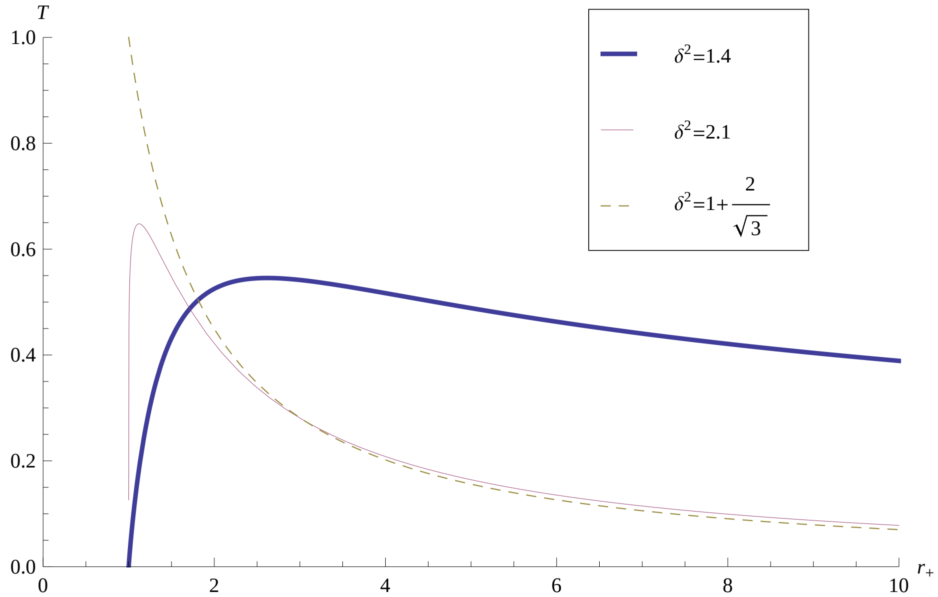
• Near the singularity the DBI action can be treated as a probe, and this completes the phase diagram.

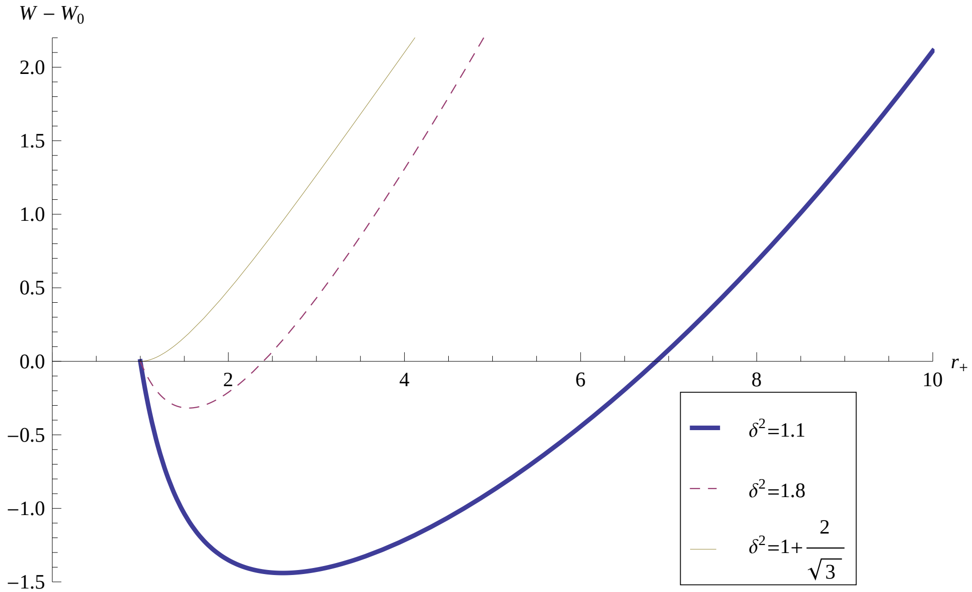
- Three distinct classes of dynamics: $\delta^2 \in [0,1] \cup [1, 1+\frac{2}{\sqrt{3}}] \cup [1+\frac{2}{\sqrt{3}}, 3)$



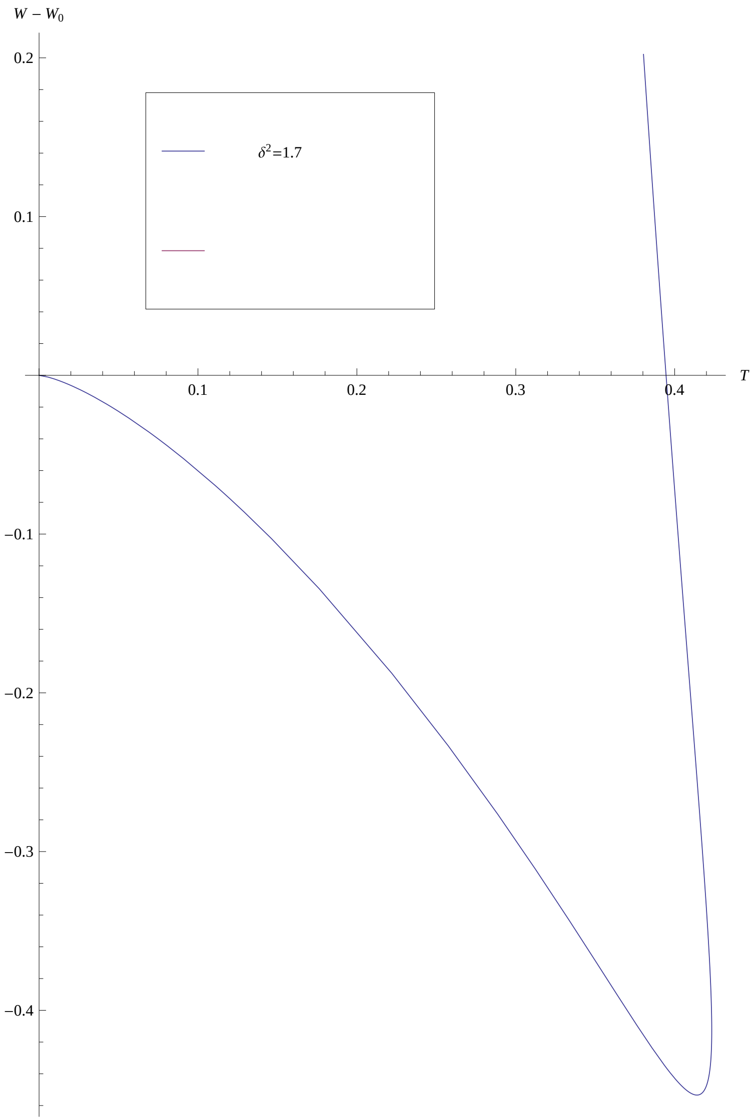


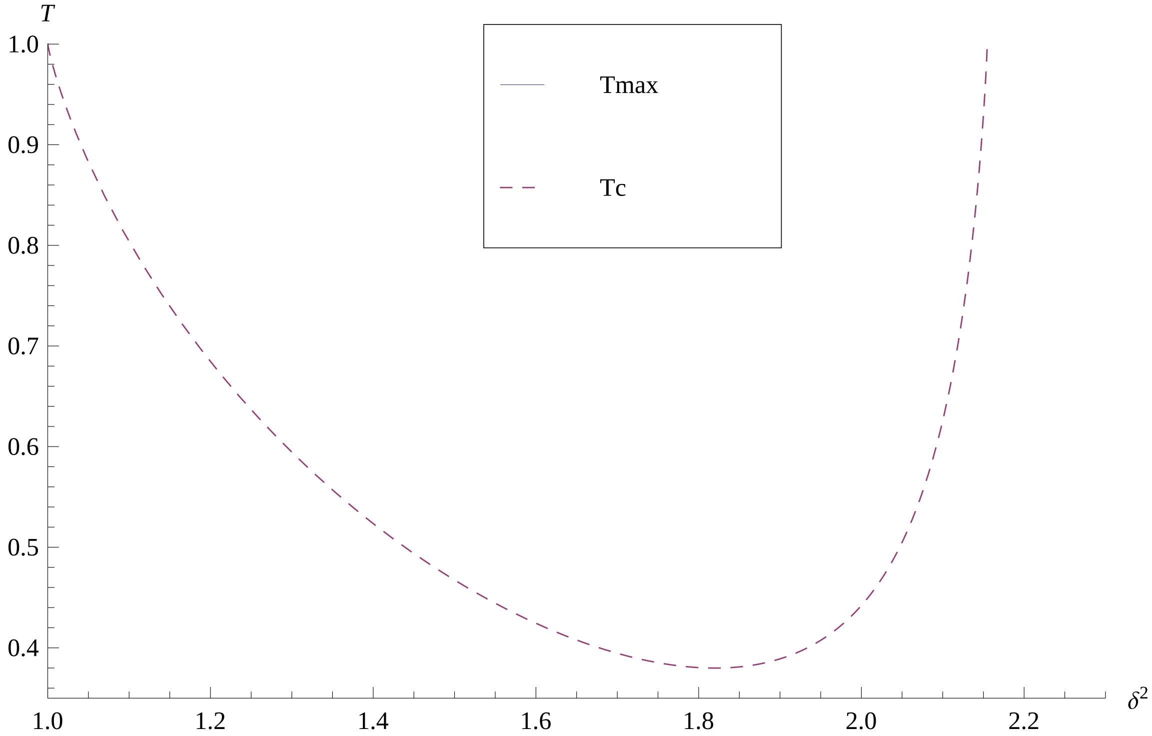
BH always dominates

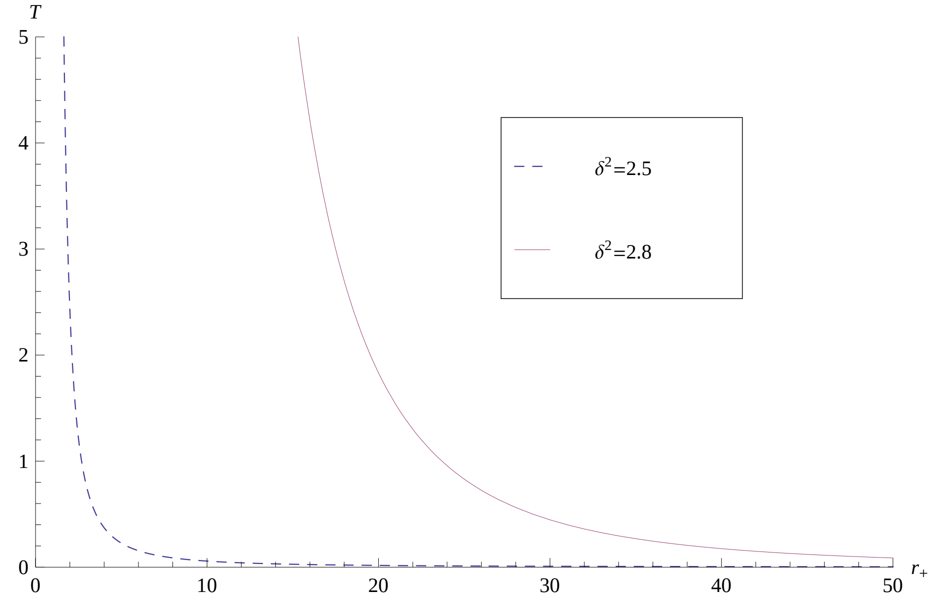


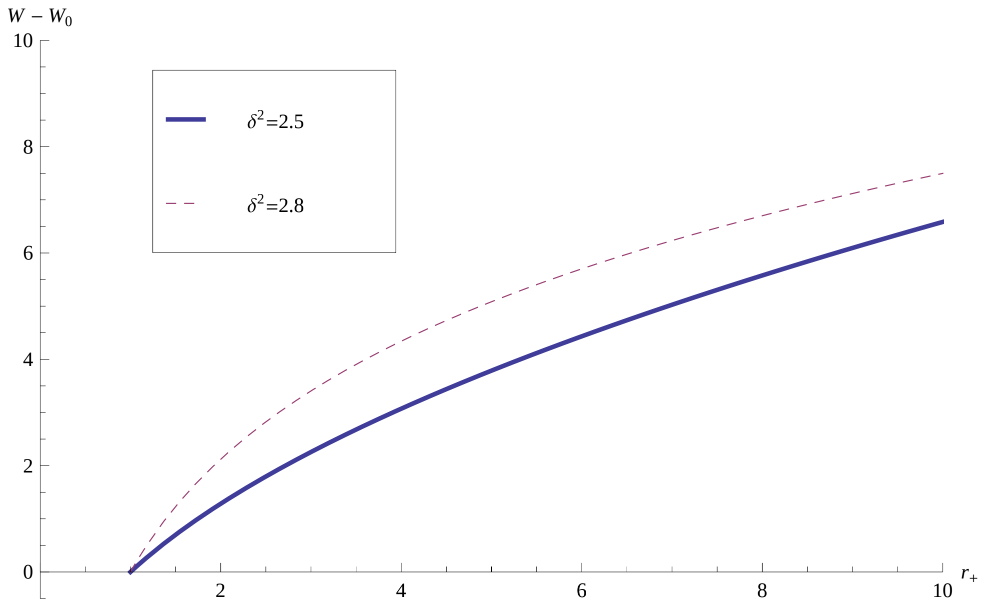


There is a non-trivial phase transition from "small BH" $\rightarrow$ extremal-thermal-solution.


$W - W_0$
0.2
0.1
−0.1
−0.2
−0.3
−0.4
0.1
0.2
0.3
0.4
T
δ^2=1.7


T
1.0
0.9
0.8
0.7
0.6
0.5
0.4
1.0
1.2
1.4
1.6
1.8
2.0
2.2
δ^2
Tmax
Tc





The extremal solution always dominates We used $\frac{Q\sqrt{1+\delta^2}}{2\delta}e^{-\frac{\phi_0}{2\delta}} = 1$

In all cases the winner is stable, the looser is unstable: (C_V, χ)

• The range $1+\frac{2}{\sqrt{3}}$ we believe is unphysical.

• In $1<\delta^2<1+\frac{2}{\sqrt{3}}$ we have the inverse situation from the Hawking-Page transition:

♠ There is a T_{max}

♠ The small BH dominates at $0<T<T_c<T_{max}$

♠ The transition seems to be second order.

A calculation of the AC conductivity at extremality gives

$$\sigma\sim\omega^k \quad , \quad k=\sqrt{1+8(3-\delta^2)}-1 \quad , \quad 0\leq k\leq 4$$

Charged solutions with $\gamma = \delta$

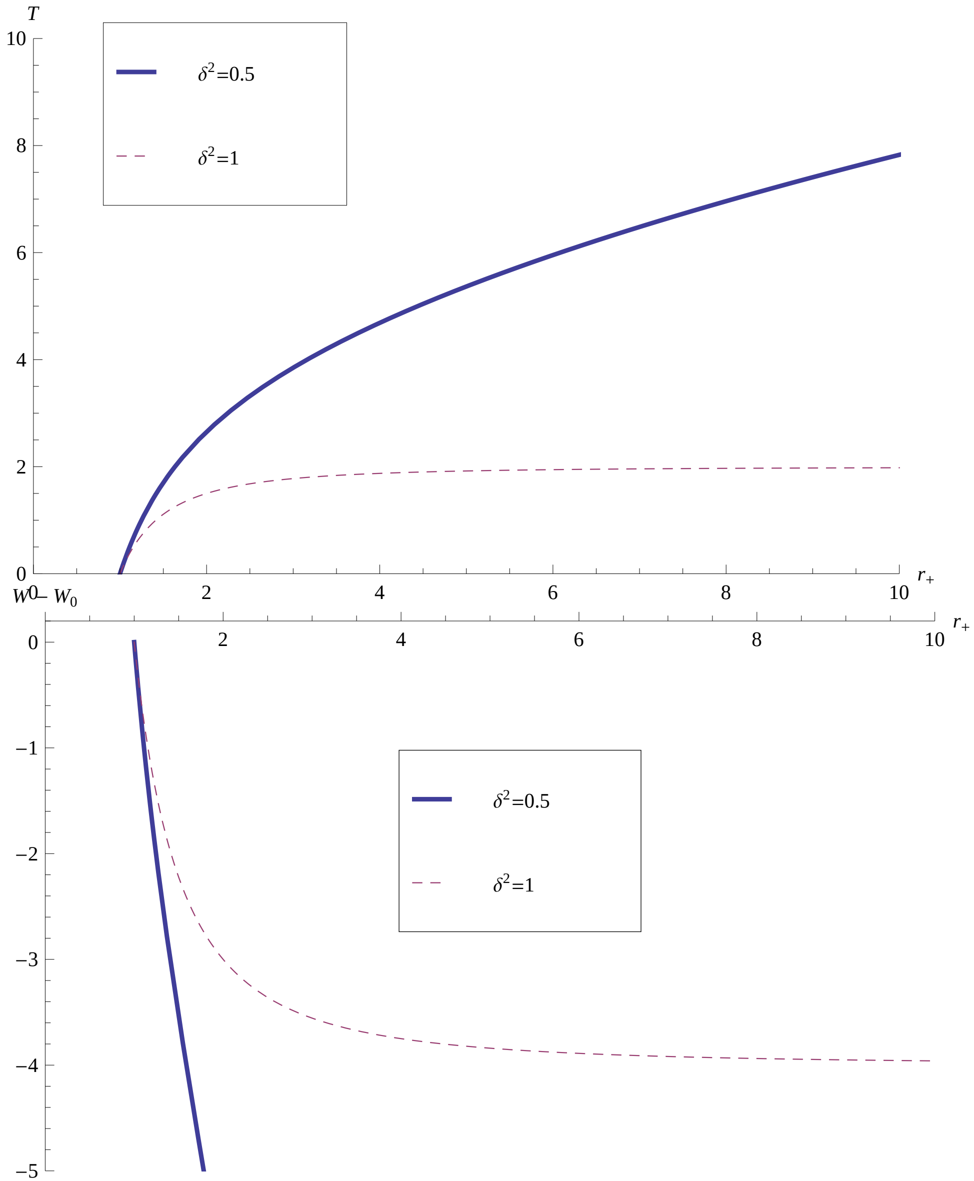
$$ds^2 = -U(r)dt^2 + \frac{(3-\delta^2)^2}{V_0} e^{\delta\phi} \frac{dr^2}{U(r)} + r^2\left(dx^2 + dy^2\right),$$

$$e^\phi = e^{\phi_0} r^{2\delta} \quad , \quad \mathcal{A} = \left(\Phi - \sqrt{|1-\delta^2|}\frac{q}{r^{1+\delta^2}} e^{-\frac{\delta}{2}\phi_0}\right) dt\,,$$

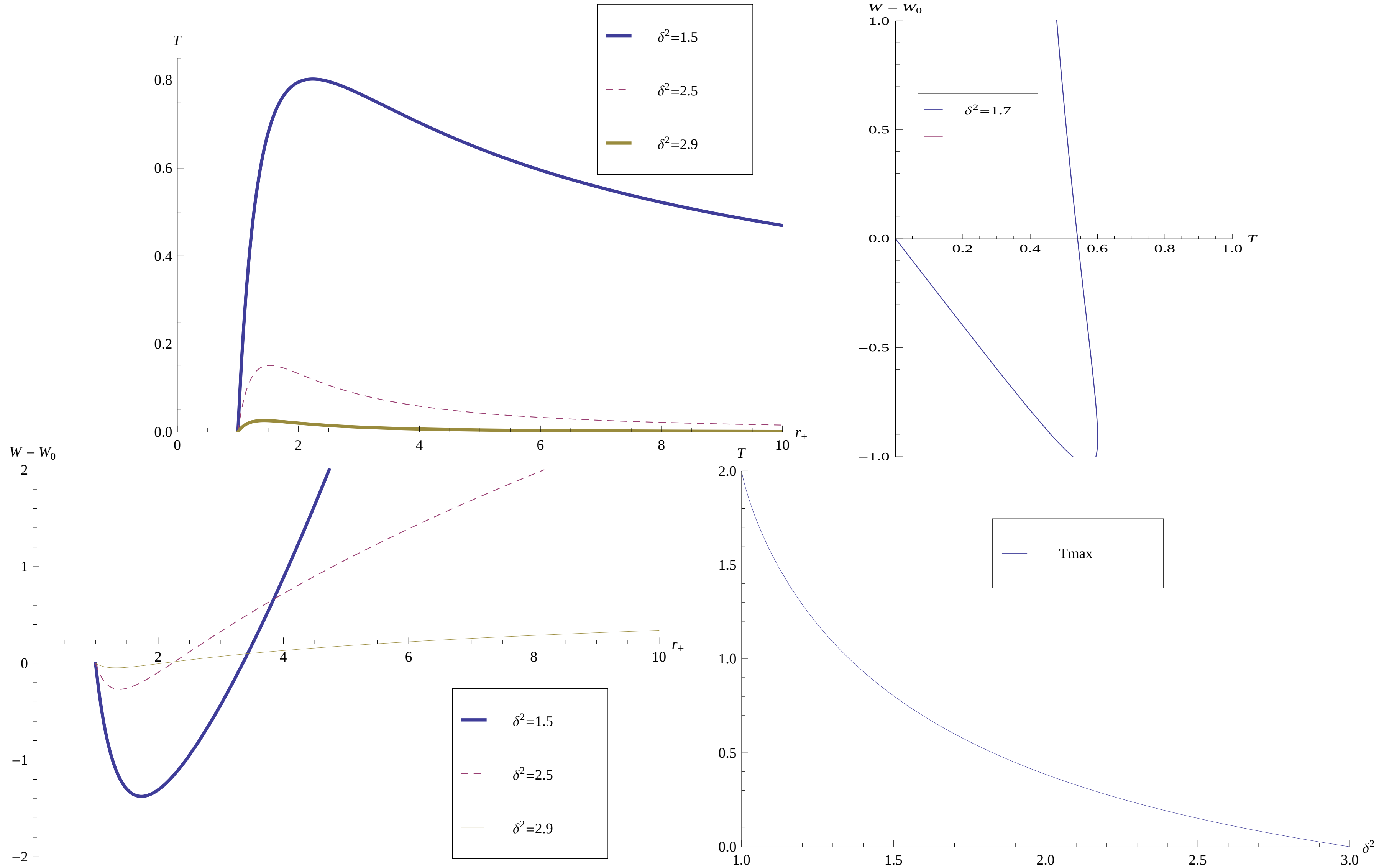
$$U(r) = \frac{3-\delta^2}{2} r^2 - 2mr^{\delta^2-1} + \frac{q^2(1-\delta^4)}{4r^2}\,.$$

- Two regimes

$$0 \le \delta^2 \le 1 \qquad\qquad 1 \le \delta^2 \le 3$$



BH always dominate



BH dominates at low temperatures up to the phase transition

Strong charge interaction limit

• This is opposite of the weak coupling limit that gives the Maxwell theory.

• In this case the gauge field, and therefore the charge density is independent of q because of the properties of the DBI action.

• Therefore there is a maximum charge density attainable. This is UNLIKE the linear Maxwell theory

• There is a generic $AdS_2\times T^2$ BH solution for any γ,δ.

• Another simple solution exists for $\gamma=\pm 1$.

$$e^A=r^{\frac{1}{4}}\quad,\quad e^{\Phi}=r^{\mp\frac{3}{4}}\quad,\quad f=\frac{2V_0}{5}(u^{\frac{5}{4}}-u_0^{\frac{5}{4}})\quad,\quad A_t=\frac{4}{5}(u^{\frac{5}{4}}-u_0^{\frac{5}{4}})$$

$$Q=\frac{4}{5}\quad,\quad \mu=-\frac{4}{5}\left(\frac{8\pi T}{V_0}\right)^{\frac{5}{2}}\quad,\quad S=\frac{1}{4G}\left(\frac{8\pi T}{V_0}\right)^{\frac{3}{2}}$$

This is consisted with the strong coupling approximation in the IR, for $\delta\leq 1$ or $\delta\geq -1$.

Outlook

• We analyzed charge coupled to energy and a scalar operator, with non-trivial back-reaction.

• We have found many scaling solutions. The ones that pass the physical tests will represent universality classes.

• For some cases we found all relevant charged solutions and found a non-trivial and unusual phase structure.

• Further analysis is needed in order to elucidate the viability and nature of these solutions.

• Apply more general techniques to find other classes of solutions.

• Attempt matching to CM systems.

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THANK YOU

Classification of zero temperature solutions

For any positive+monotonic potential $V(\lambda)$, $\lambda \equiv e^{\phi}$ with the asymptotics :

$$V(\lambda) = V_0 + V_1\lambda + V_2\lambda^2 + \ldots \quad V_0 > 0, \qquad \lambda \to 0$$

$$V(\lambda) = V_\infty \lambda^{2Q} (\log\lambda)^P, \quad V_\infty > 0, \qquad \lambda \to \infty$$

the zero-temperature superpotential equation has three types of solutions, that we name the *Generic*, the *Special*, and the *Bouncing* types:

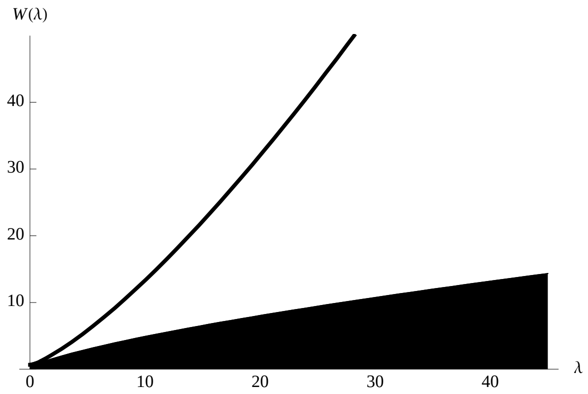
• A continuous one-parameter family that has a fixed power-law expansion near $\lambda = 0$, and reaches the asymptotic large-λ region where it grows as

$$W \simeq C_b\, \lambda^{4/3} \qquad \lambda \to \infty \quad , \quad C_b > 0$$

These solutions lead to backgrounds with "bad" (i.e. non-screened) singularities at finite r_0,

$$b(r) \sim (r_0 - r)^{1/3}, \qquad \lambda(r) \sim (r_0 - r)^{-1/2}$$

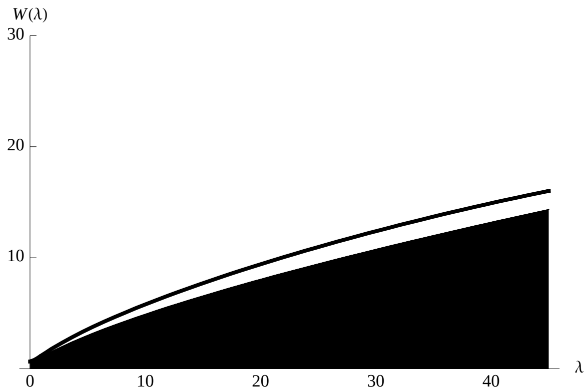
We call this solution *generic*.



• A unique solution, which also reaches the large-λ region, but slower:

$$W(\lambda) \sim W_\infty \lambda^Q (\log\lambda)^{P/2}, \qquad W_\infty = \sqrt{\frac{27V_\infty}{4(16-9Q^2)}}$$

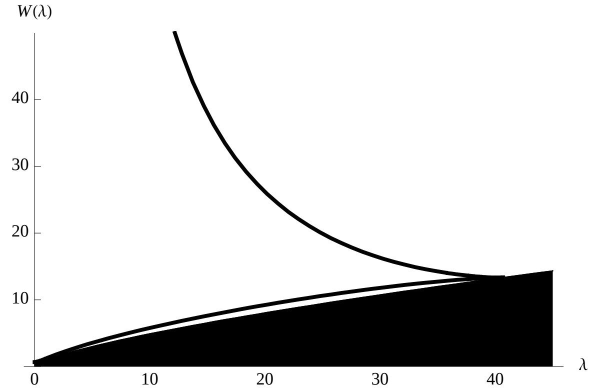
This leads to a repulsive singularity, provided $Q < 2\sqrt{2}/3$ [**?**]. We call this the *special* solution.



• A second continuous one-parameter family where $W(\lambda)$ does not reach the asymptotic region. These solutions have two branches that both reach $\lambda = 0$ (one in the UV, the other in the IR) and merge at a point λ_* where $W(\lambda_*) = \sqrt{27V(\lambda_*)/64}$. The IR branch is again a "bad" singularity at a finite value r_0, where $W \sim \lambda^{-4/3}$, and

$$b(r) \sim (r_0 - r)^{1/3}, \qquad \lambda(r) \sim (r_0 - r)^{1/2}.$$

We call this solution *bouncing*.



The special solution marks the boundary between the generic solutions, that reach the asymptotic large-λ region as $\lambda^{4/3}$ and the bouncing ones, that don't reach it.

If $Q > 4/3$, only bouncing solutions exist.

In all types of solutions the UV corresponds to the region $\lambda \to 0$ on the W_+ branch. There the behavior of W_+ is universal: a power series in λ with *fixed* coefficients, plus a subleading non-analytic piece which depends on an arbitrary integration constant C_w:

$$W = \sum_{i=1}^{\infty} W_i \lambda^i + C_w \lambda^{16/9} e^{-\frac{16W_0}{9W_1}\frac{1}{\lambda}} [1 + O(\lambda)]$$

All the power series coefficients W_i are completely determined by the coefficients in the small λ expansion of $V(\lambda)$, the first few being:

$$W_0 = \frac{\sqrt{27V_0}}{8}, \quad W_1 = \frac{V_1}{16}\sqrt{\frac{27}{V0}}, \quad W_2 = \frac{\sqrt{27}(64V_0V_2 - 7V1^2)}{1024V_0^{3/2}}$$

RETURN

Detailed plan of the presentation

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