ESF Exploratory Workshop
Porto, 11 September 2009

Improved AdS/QCD

Elias Kiritsis

University of Crete (on leave from APC, Paris)

Collaborators

My Collaborators

- Umut Gursoy (Utrecht)
- Liuba Mazzanti (Ecole Polytechnique$\longrightarrow$ Santiago de Compostella)
- George Michalogiorgakis (Ecole Polytechnique $\longrightarrow$ Purdue)
- Fransesco Nitti (APC, Paris)

Introduction

• So far none of the methods we have to attack QCD is bullet-proof and universal.

• pQCD works at high energy only.

• Lattice works in Euclidean space , with real probabilities and with not too many final states.

• Models like chiral perturbation theory have a limited region of validity.

• Theoretical techniques like the large-N approximation and the associated AdS/CFT correspondence assume large N and really solve other theories (like N=4 SYM).

• Data coming from colliders or heavy ion collisions need an underlying theoretical model in order to be converted to statements about QCD. To the largest extend such theoretical models are largely phenomenological.

.

♠ If we are to understand QCD in the near future it is via a tandem of different techniques that complement each other.

• Here I will focus on phenomenological models based on holographic/stringy ideas.

• My focus will be mostly the physics in the deconfined phase, explored recently in heavy ion collisions.

• The goal will be to assemble intuition both from string theory and QCD in order to construct a model that describes well strongly coupled physics in the deconfined (as well as confined) phase.

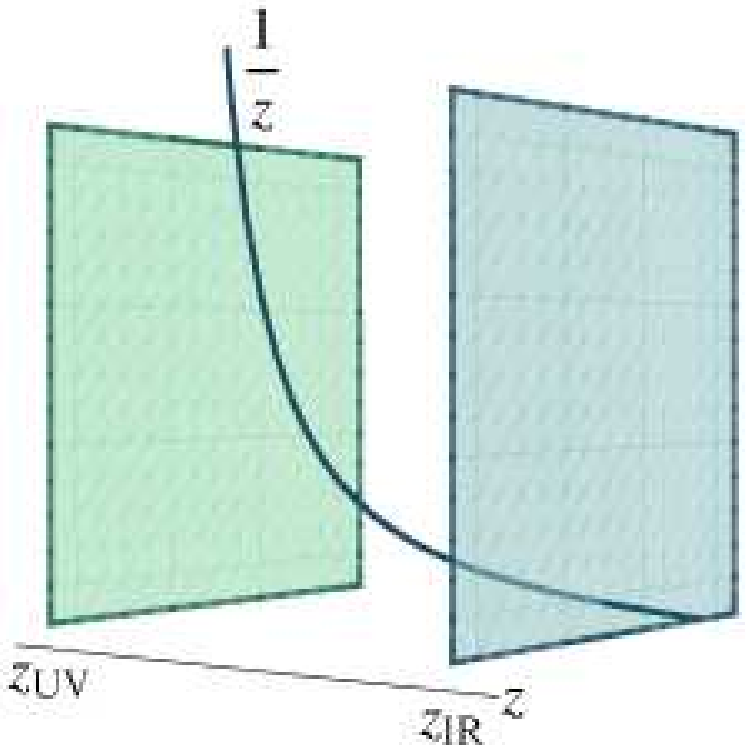
AdS/QCD

♠ A basic phenomenological approach: use a slice of AdS_5, with a UV cutoff, and an IR cutoff.
Polchinski+Strassler

♠ It successfully exhibits confinement (trivially via IR cutoff), and power-like behavior in hard scattering amplitudes

♠ It may be equipped with a bifundamental scalar, T, and $U(N_f)_L \times U(N_f)_R$, gauge fields to describe mesons.
Erlich+Katz+Son+Stepanov, DaRold+Pomarol

Chiral symmetry is broken by hand, via IR boundary conditions. The low-lying meson spectrum looks "reasonable".



♠ Shortcomings:

- The glueball spectrum does not fit very well the lattice calculations. Its asymptotic behavior is $m_n^2 \sim n^2$ instead of linear at large n.

- Magnetic quarks are confined instead of screened.

- Chiral symmetry breaking is input by hand.

- The meson spectrum has also the wrong UV asymptotics $m_n^2 \sim n^2$.

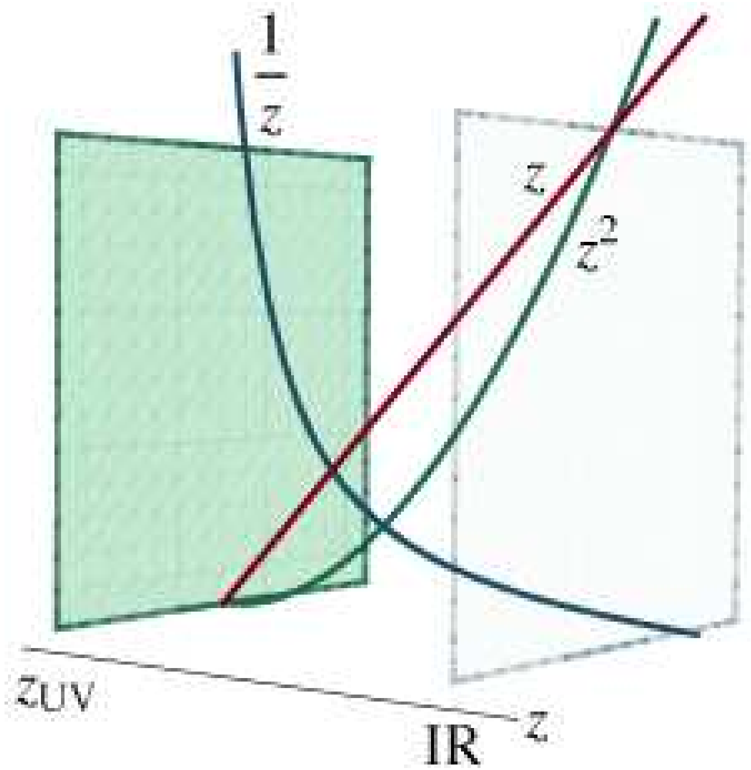
- At finite temperature there is a deconfining transition but the equation of state is trivial (conformal) (e=3p) and the speed of sound is $c_s^2 = \frac{1}{3}$.

The "soft wall"

♠ The asymptotic spectrum can be fixed by introducing a **non-dynamical** dilaton profile $\Phi \sim r^2$ (soft wall)

Karch+Katz+Son+Stephanov

• It is not a solution of equations of motion: the metric is still AdS: Neither $g_{\mu\nu}$ nor Φ solves the equations of motion.



.

• It is consistent to use it as background in the meson sector.

• Chiral symmetry breaking is still input by hand

• It is an "inconsistent" background in the glue sector: Spectra cannot be calculated nor finite temperature dynamics.

Improved Holographic QCD

• We would like to construct a holographic model that captures the holographic behavior of SU(N_c) YM in four dimensions, and in particular the breaking of conformal invariance.

• We will have to add to the metric, a scalar field ϕ (the dilaton) that is dual to $tr[F^2]$. Any other fields will be neglected from the vacuum structure (the axion may not if the θ angle is non-trivial but its contribution is subleading in N.)

• Although the physics in the UV is expected to be stringy, we will approximate it appropriately via a two derivative truncation. A phenomenological model with many derivatives involved is not useful.

• The action is

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right] \quad , \quad \lambda = N_c \, e^{\phi}$$

The UV region

Choose a monotonic potential with UV asymptotics (no minima).

$$\lim_{\lambda\to 0} V(\lambda) = \frac{12}{\ell^2}\left(1+\sum_{n=1}^{\infty} c_n\lambda^n\right) = \frac{12}{\ell^2}\left(1+c_1\lambda+c_2\lambda^2+\cdots\right)$$

- The Poincaré invariant ansatz is

$$ds^2 = b(r)^2(dr^2+dx^\mu dx_\mu) \quad , \quad \lambda \to \lambda(r)$$

- The small λ asymptotics generate the UV expansion around AdS_5:

$$\frac{1}{\lambda} = -b_0 \log(r\Lambda) + \cdots \quad , \quad b \equiv e^A = \frac{\ell}{r}\left[1+\frac{2}{9\log(r\Lambda)}+\cdots\right]$$

- There is a 1-1 correspondence between the holographic "YM" β-function, $\beta(\lambda)$ and W defined as $\left(\frac{3}{4}\right)^3 V(\lambda) = W^2 - \left(\frac{3}{4}\right)^2\left(\frac{\partial W}{\partial \log\lambda}\right)^2$:

$$\frac{d\lambda}{d\log E} = \beta(\lambda) = -\frac{9}{4}\lambda^2\,\frac{d\log W(\lambda)}{d\lambda} = -b_0\lambda^2 + b_1\lambda^3 + \cdots$$

with $E = e^A$

The IR asymptotics

• The solutions in the IR either asymptote to AdS_5 (extremum of V) or have a singularity.

• We demand that the singularity is "good" (à la Gubser) and "repulsive".

• We also demand confinement, a mass gap and a discrete spectrum.

♠ Parametrize

$$V(\lambda) \sim \lambda^Q \, (\log \lambda)^P \qquad as \qquad \lambda \to \infty$$

• There is confinement, discrete spectrum and a mass gap for $Q \geq \frac{4}{3}$

• The IR singularity is "good" if $Q < \frac{8}{3}$

• There are linear trajectories when $Q = 4/3$ and $P = 1/2$. This is the case the is closer to YM

General phase structure

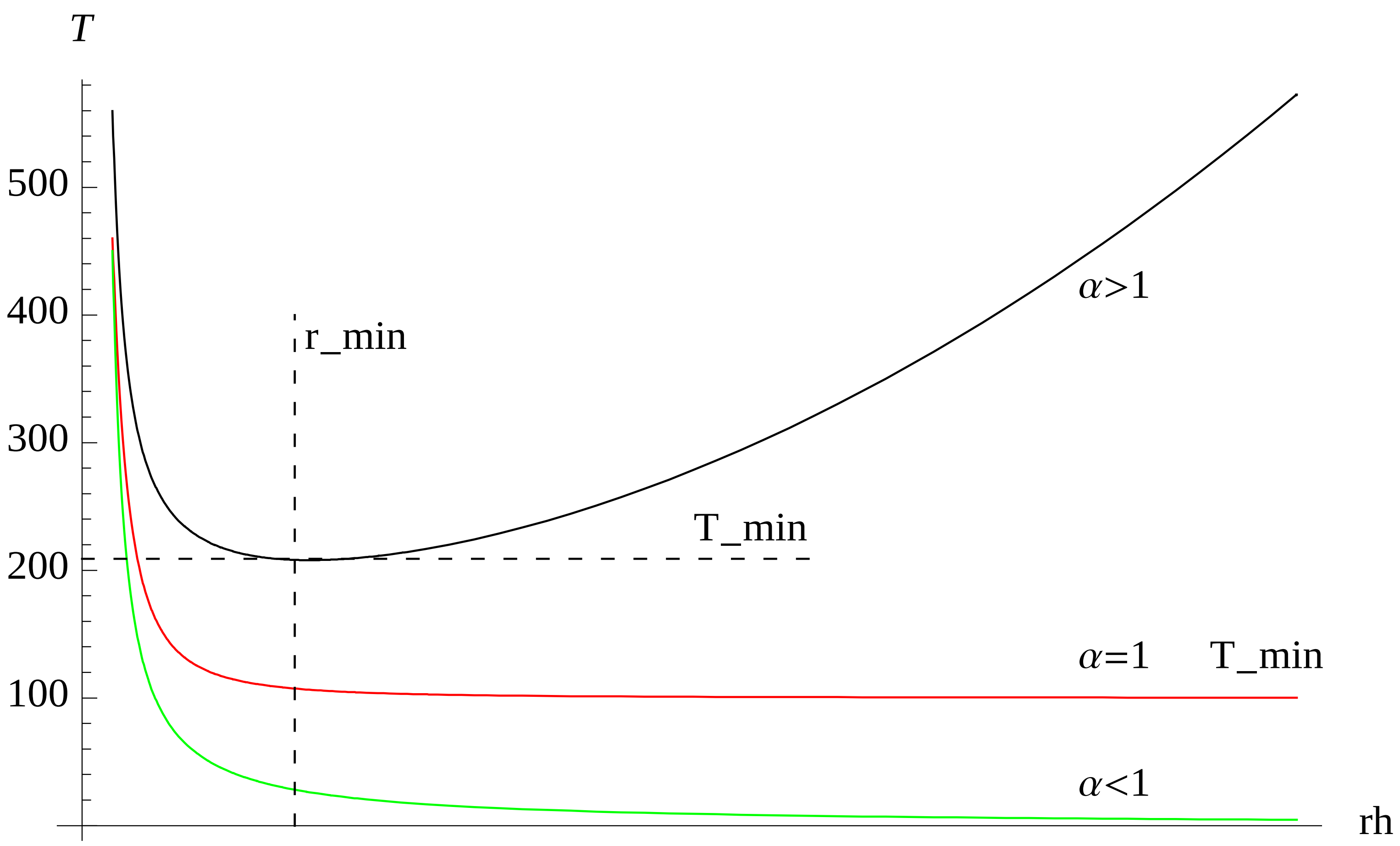
• For a general monotonic potential (with no minimum) the following are true :

i. *There exists a phase transition at finite $T = T_c$, if and only if the zero-T theory confines.*

ii. *This transition is first order for all of the confining geometries, with a single exception (linear dilaton in the IR, continuous spectrum with a gap)*

iv. *All of the non-confining geometries at zero T are always in the black hole phase at finite T. They exhibit a second order phase transition at $T = 0^+$.*

Temperature versus horizon position



We plot the relation $T(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is only one branch of black-hole solutions.

The pressure from the lattice at different N

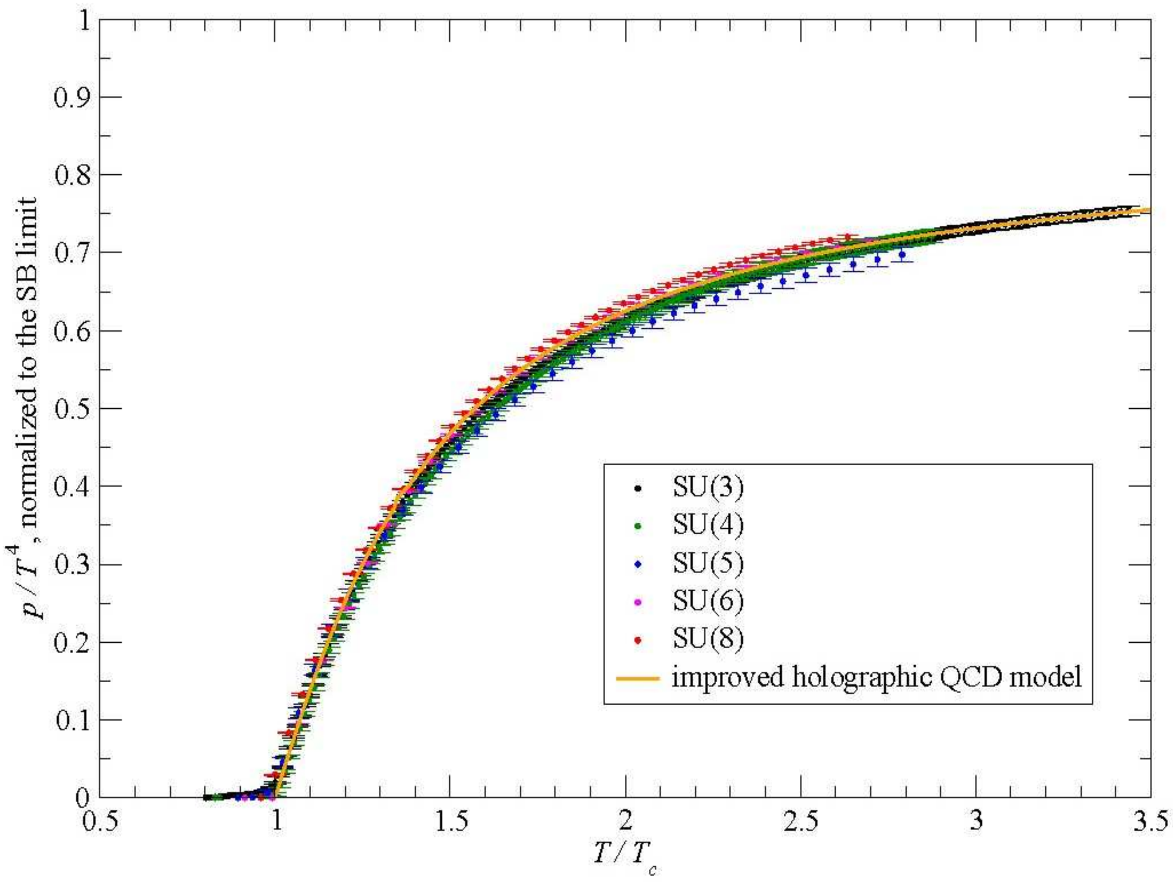


Figure 1: (Color online) The dimensionless ratio p/T^4, normalized to the lattice SB limit $\pi^2(N^2-1)R_I(N_t)/45$, *versus* T/T_c, as obtained from simulations of SU(N) lattice gauge theories on $N_t = 5$ lattices. Errorbars denote statistical uncertainties only. The results corresponding to different gauge groups are denoted by different colors, according to the legend. The yellow solid line denotes the prediction from the improved holographic QCD model from ref. [75] (with a trivial, parameter-free rescaling to our normalization).

Marco Panero arXiv: 0907.3719

The entropy from the lattice at different N

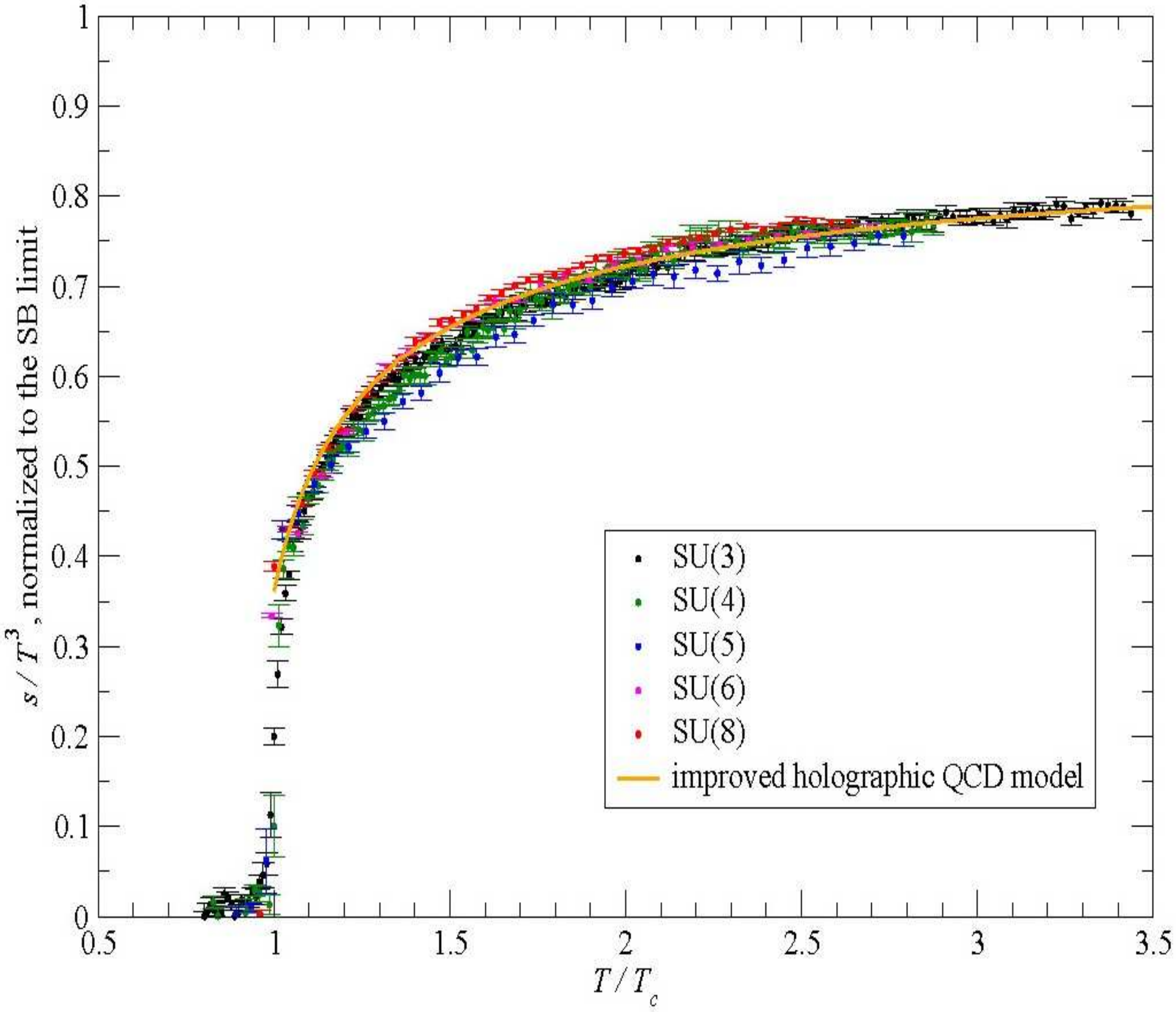


Figure 4: (Color online) Same as in fig. 1, but for the s/T^3 ratio, normalized to the SB limit.

Marco Panero arXiv: 0907.3719

The trace from the lattice at different N

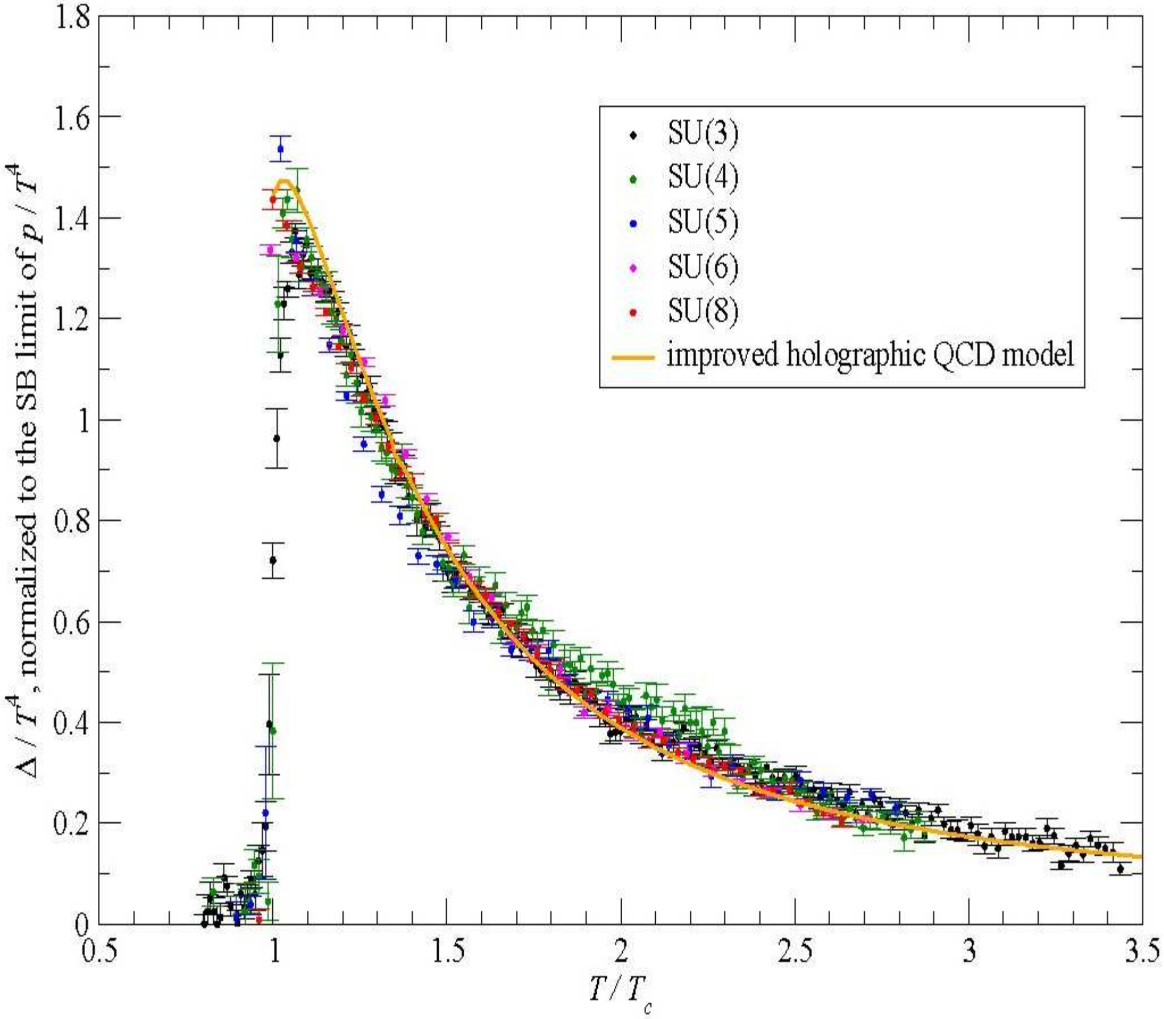
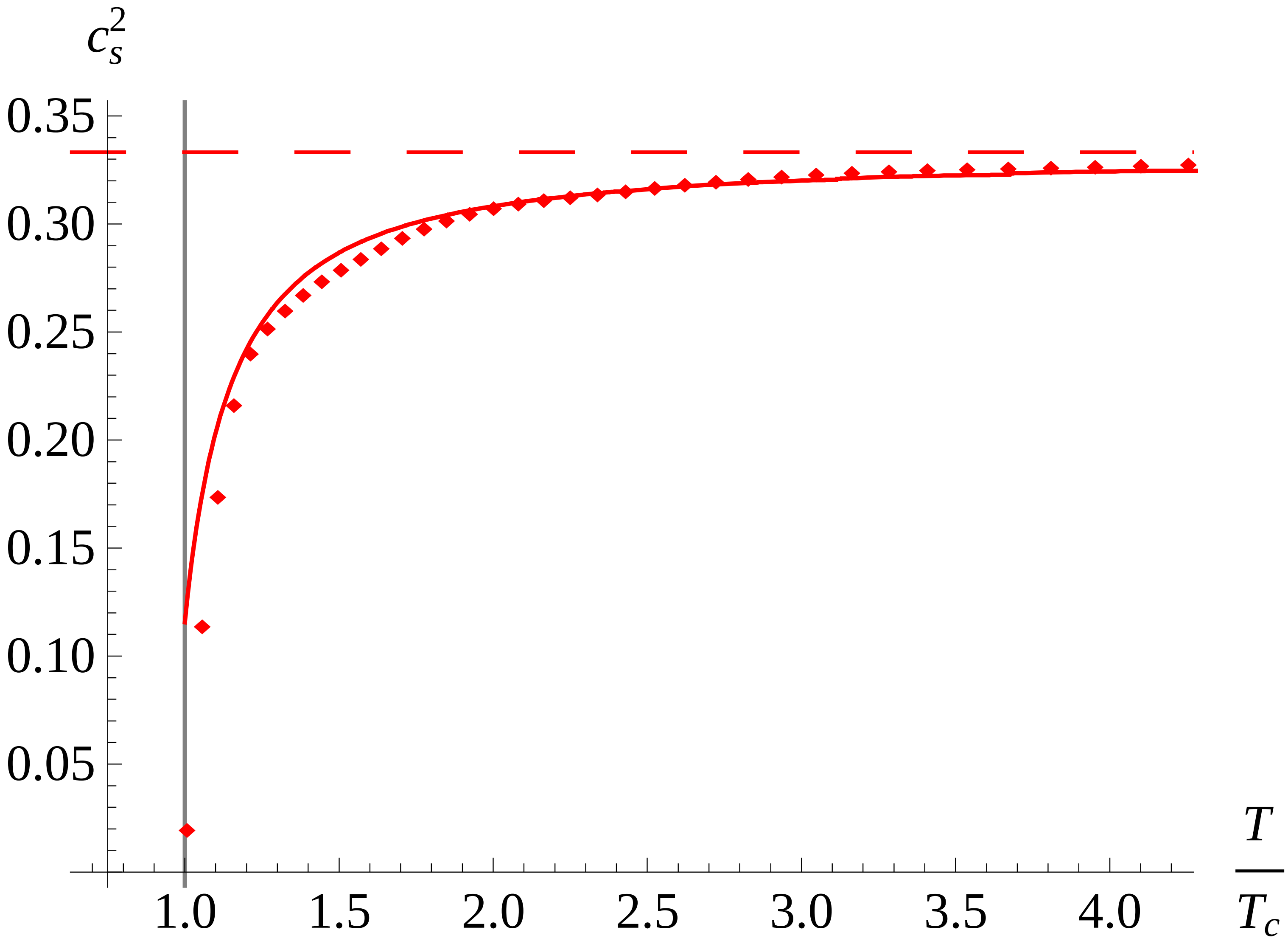


Figure 2: (Color online) Same as in fig. 1, but for the Δ/T^4 ratio, normalized to the SB limit of p/T^4.

Marco Panero arXiv: 0907.3719

The speed of sound



Viscosity

- Viscosity (shear and bulk) is related to dissipation and entropy production

$$\frac{\partial s}{\partial t} = \frac{\eta}{T}\left[\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial\cdot v\right]^2 + \frac{\zeta}{T}(\partial\cdot v)^2$$

- Conformal invariance imposes that $\zeta = 0$.

- Viscosity can be calculated from a Kubo-like formula (fluctuation-dissipation)

$$\eta\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right) + \zeta\delta_{ij}\delta_{kl} = -\lim_{\omega\to 0}\frac{Im\ G^R_{ij;kl}(\omega)}{\omega}$$

$$G^R_{ij;kl}(\omega) = -i\int d^3x\int dt\ e^{i\omega t}\theta(t)\ \langle 0|[T_{ij}(\vec{x},t), T_{kl}(\vec{0},0)]|0\rangle$$

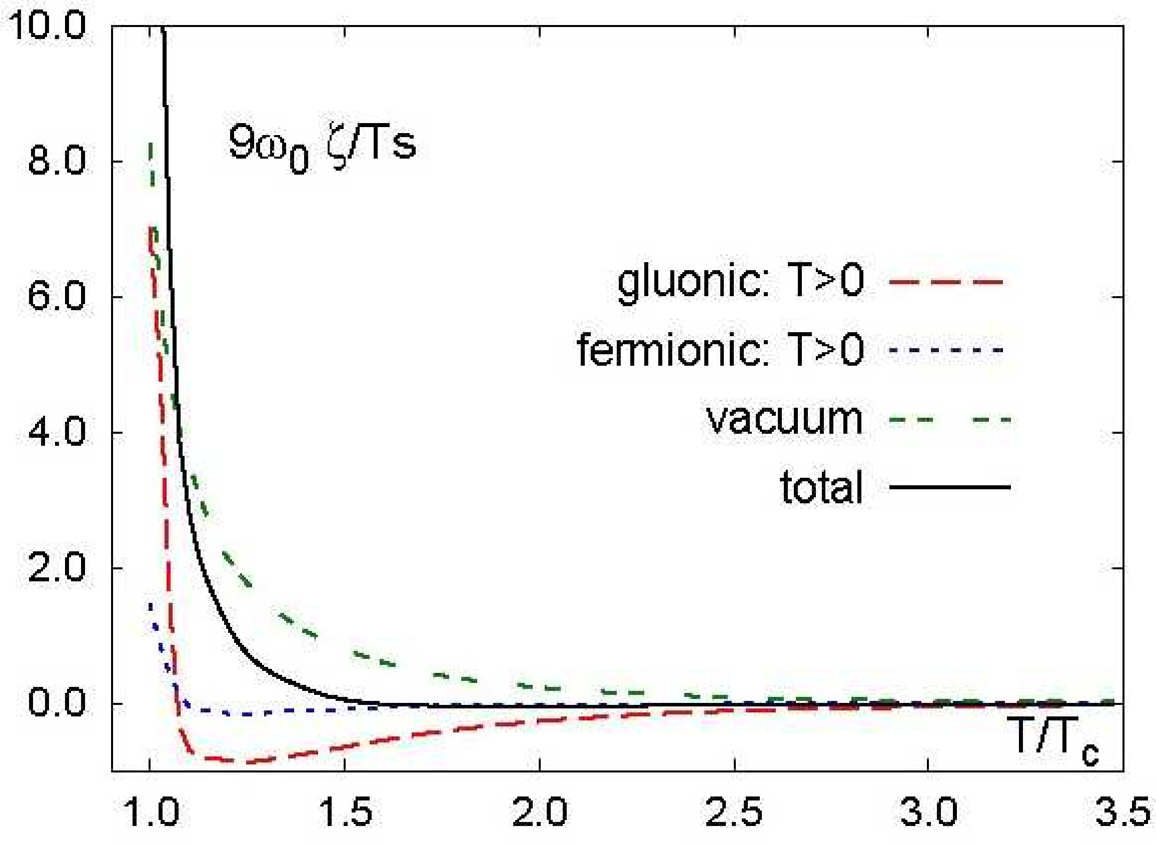
- In all theories with gravity duals ($\lambda\to\infty$) at two-derivative level

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Policastro+Starinets+Son 2001, Kovtun+Son+Starinets 2003, Buchel+Liu 2003

- In Einstein-dilaton gravity shear viscosity is equal to the universal value.

The sum rule method

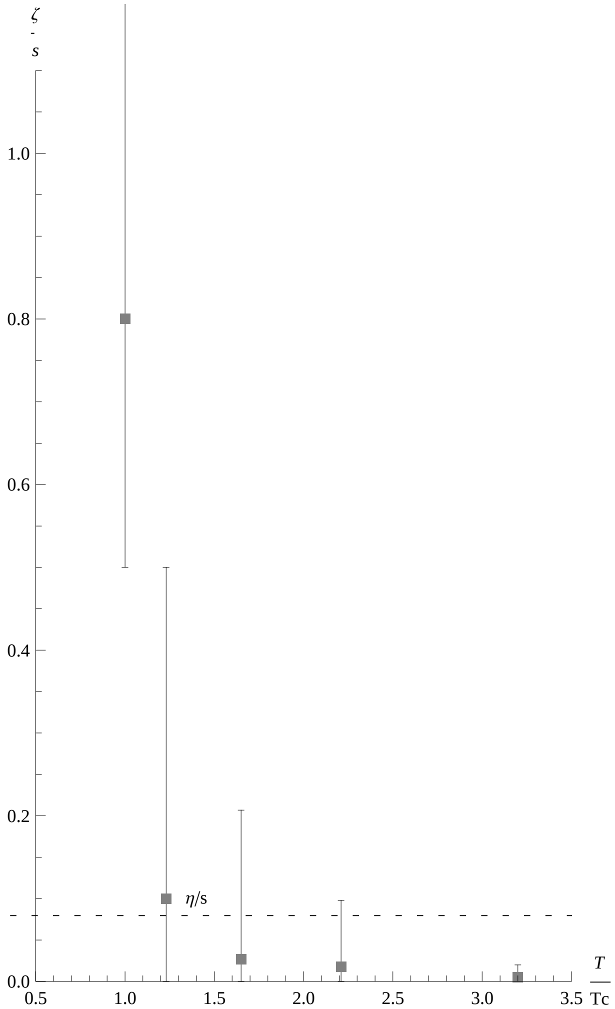


Karsch+Kharzeev+Tuchin, 2008

• A potential rise near the phase transition but the (temperature-dependent) scale cannot be fixed.

The bulk viscosity in lattice SU(3) YM

H. Meyer 2007



Pure YM only. Error bar are statistical only.

.

- If the lattice result is taken at phase value,

$$\frac{\zeta(T_c)}{s(T_c)} \quad \sim \quad 10\frac{\eta(T_c)}{s(T_c)} \quad = \quad 10\ \frac{1}{4\pi}$$

- Such a large value renders hydrodynamic codes unstable.

Heinz+Song (unpublished)

- At large values of viscosity, cavitation ($p < 0$) happens, signaling a break-down of hydrodynamics.

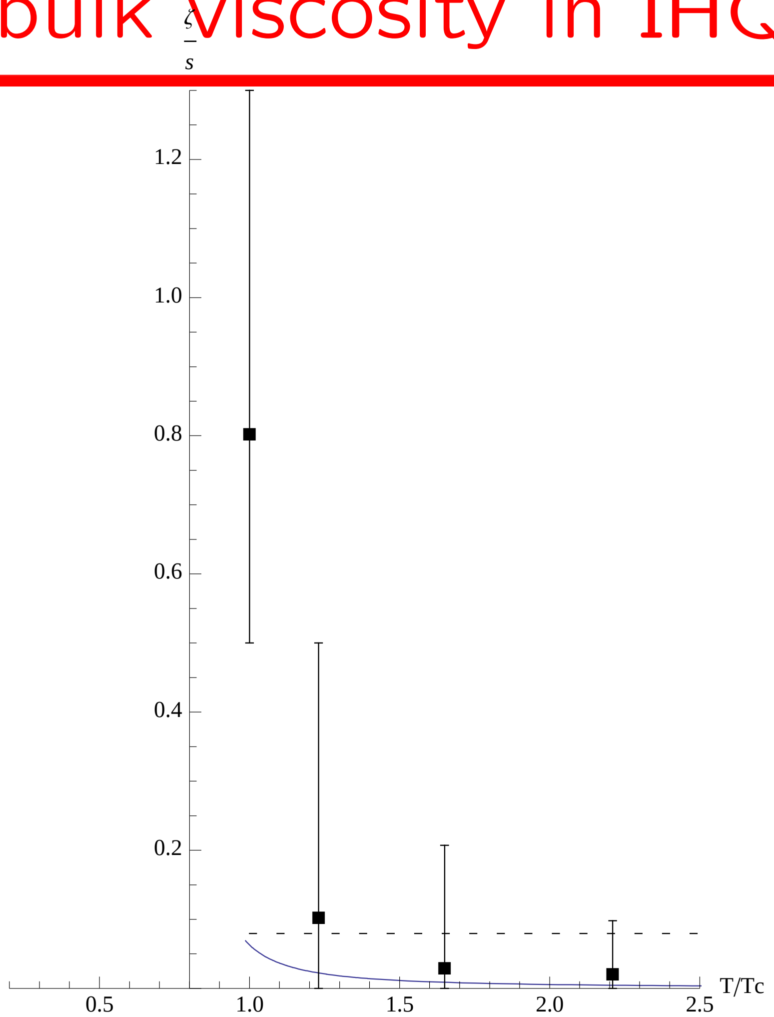
- This was studied carefully and confirmed very recently

Rajagopal+Tripuraneni 2009

- Both the lattice and sum rule values were disputed later.

Moore+Saremi

The bulk viscosity in IHQCD



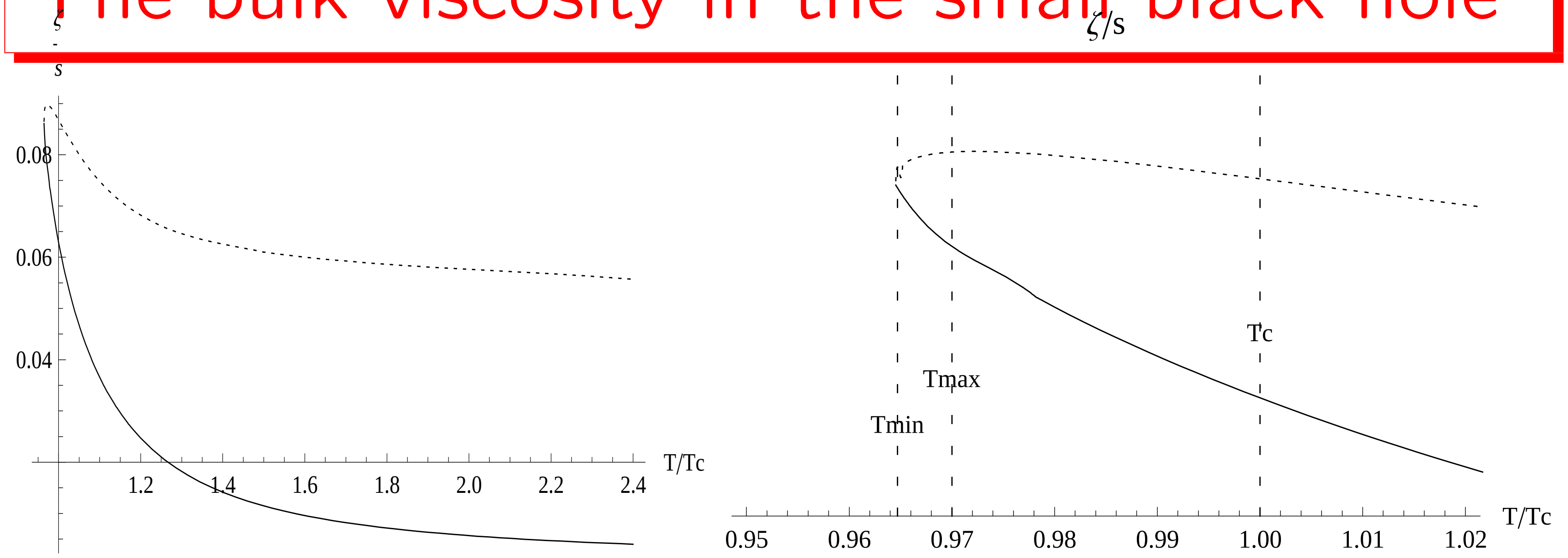
Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

- Pure glue only.

- Calculations with other potentials show robustness

Gubser+Pufu+Rocha 2008, Cherman+Nellore 2009

The bulk viscosity in the small black hole



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

- The small black-hole bulk viscosity ratio asymptotes to a constant as $T \to \infty$.

$$\lim_{T\to\infty} \left.\frac{\zeta(T)}{s(T)}\right|_{\text{small}} = \frac{1}{6\pi}$$

From

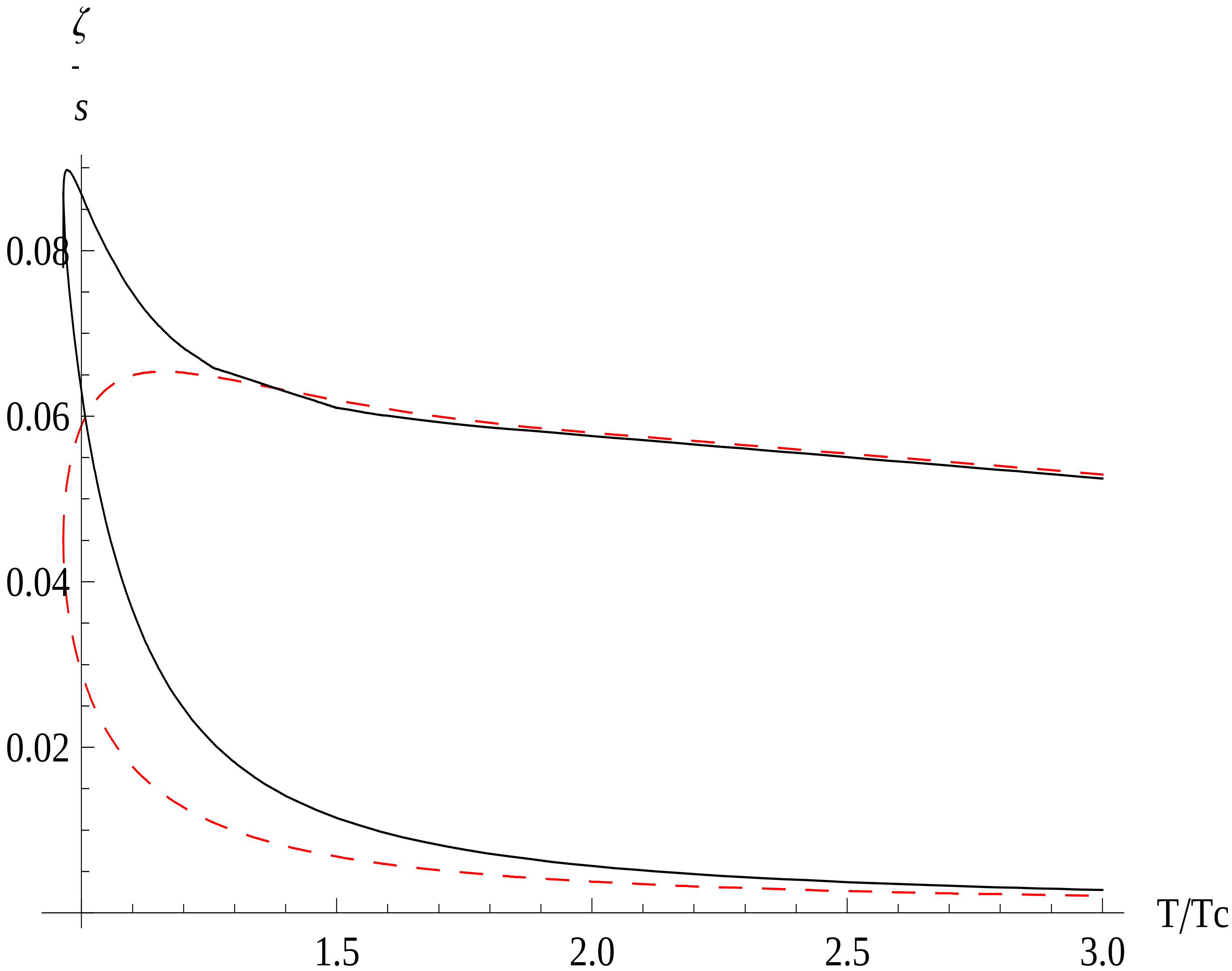
$$\frac{\zeta}{s} = \frac{3}{32\pi}\left(\frac{V'(\Phi_h)}{V(\Phi_h)}\right)^2 |c_b|^2 \quad , \quad h_{11} \to c_b(\phi_h - \phi)^{-\frac{i\omega}{4\pi T}}$$

- For a potential $V \sim \lambda^Q$ then $\lim_{T\to\infty} \left.\frac{\zeta(T)}{s(T)}\right|_{\text{small}} = \frac{3Q^2}{32\pi}$

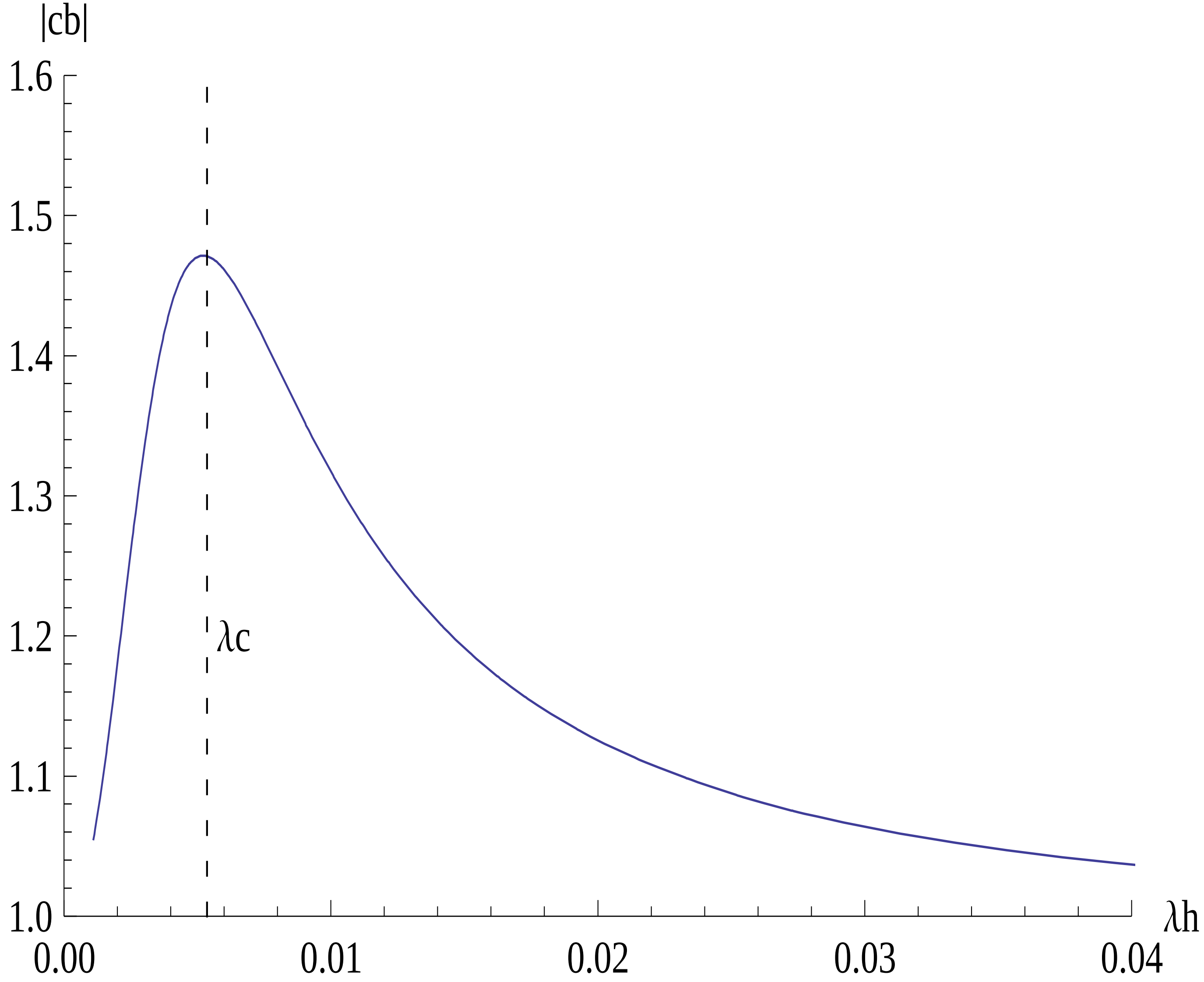
- This puts a upper and lower bound coming from confinement

$$\frac{1}{6\pi} \leq \left.\frac{\zeta}{s}\right|_{\text{small,asymptotic}} \leq \frac{1}{3\pi}$$

- Using the adiabatic approximation this gives the order of magnitude on ζ/s near T_c



$\frac{\zeta}{s}$
0.08
0.06
0.04
0.02
1.5
2.0
2.5
3.0
T/Tc


|cb|
1.6
1.5
1.4
1.3
1.2
1.1
1.0
λc
λh
0.00
0.01
0.02
0.03
0.04

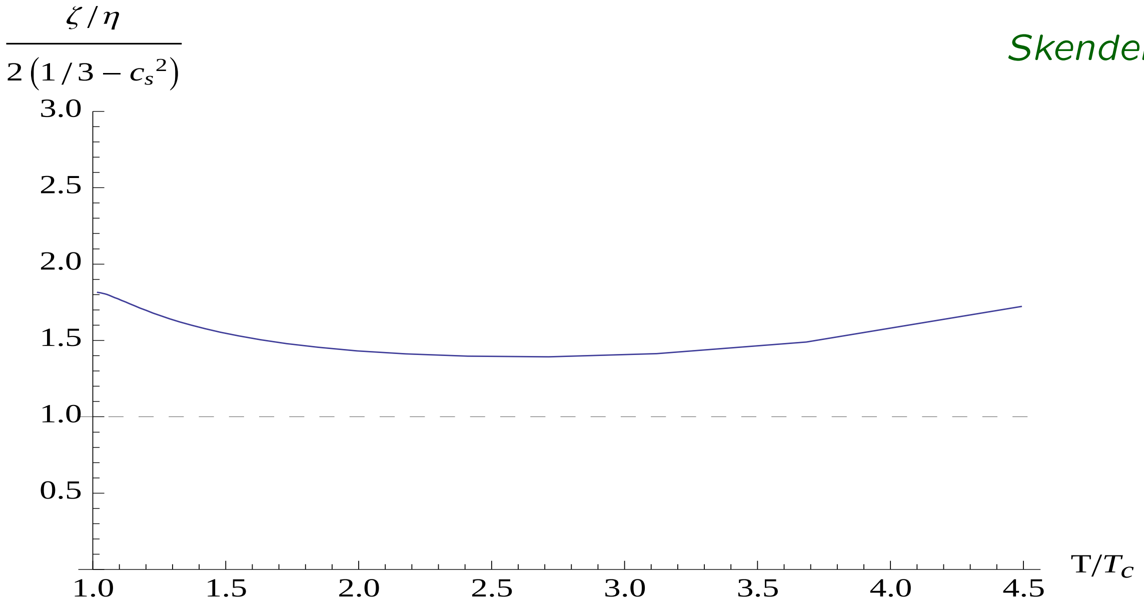
The Buchel parametrization (conjectured bound)

$$\frac{\zeta}{\eta} \geq 2\left(\frac{1}{3} - c_s^2\right)$$

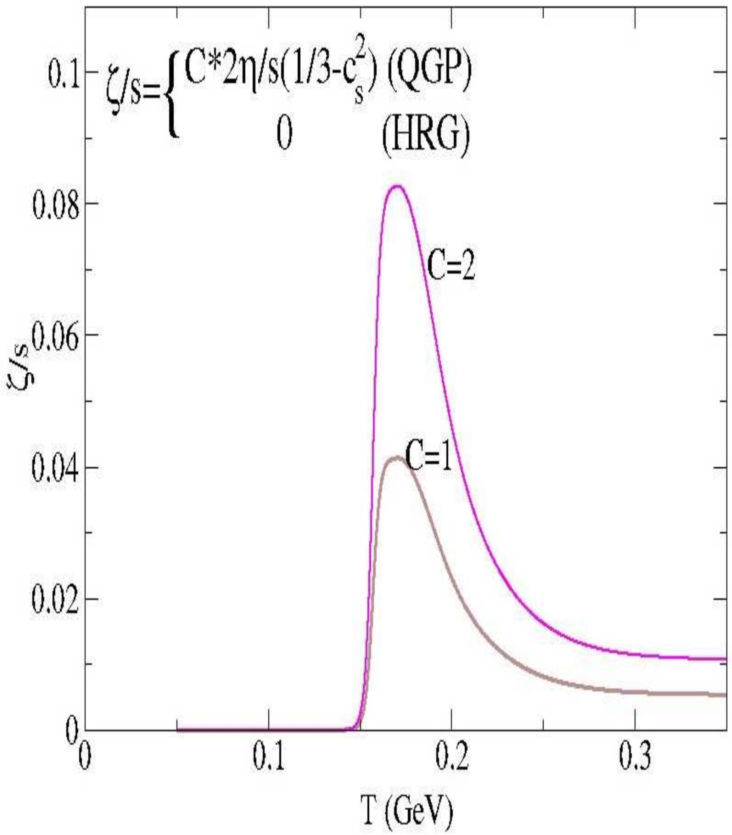
Buchel 2007

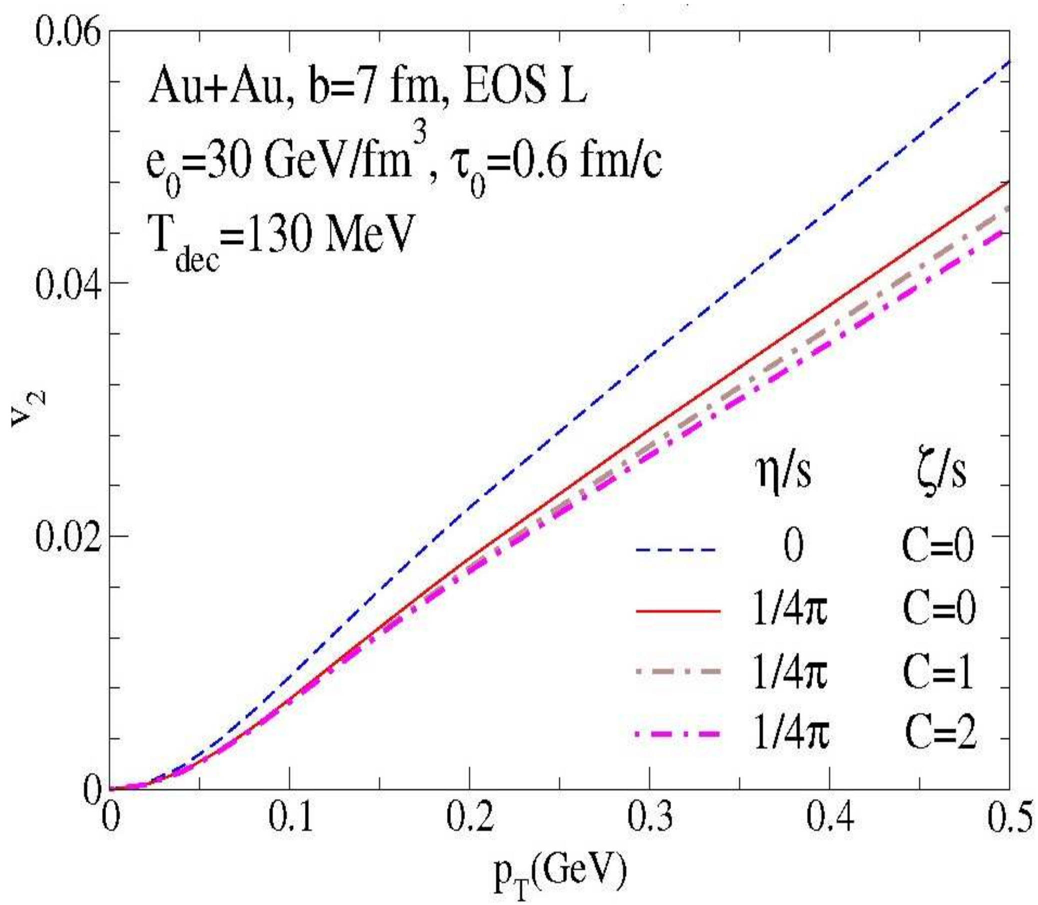
• For Dp branes, equality is a consequence of conformal invariance and dimensional reduction.

Skenderis+Kanitscheider 2009



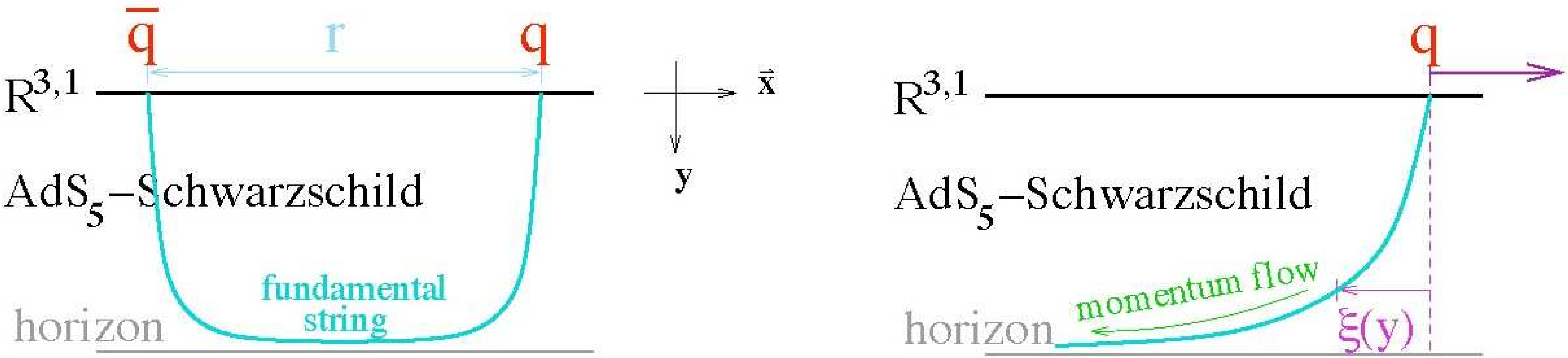
Elliptic Flow vs bulk viscosity





U Heinz+H.Song 2008

Heavy quarks and the drag force



From Gubser's talk at Strings 2008

- The induced metric on the world-sheet is a 2d black-hole with horizon at the turning point $r = r_s$ $(t = \tau + \zeta(r))$.

$$ds^2 = b^2(r)\left[-(f(r)-v^2)d\tau^2 + \frac{1}{(f(r)-\frac{b^4(r_s)}{b^4(r)}v^2)}dr^2\right]$$

- We can calculate the drag force:

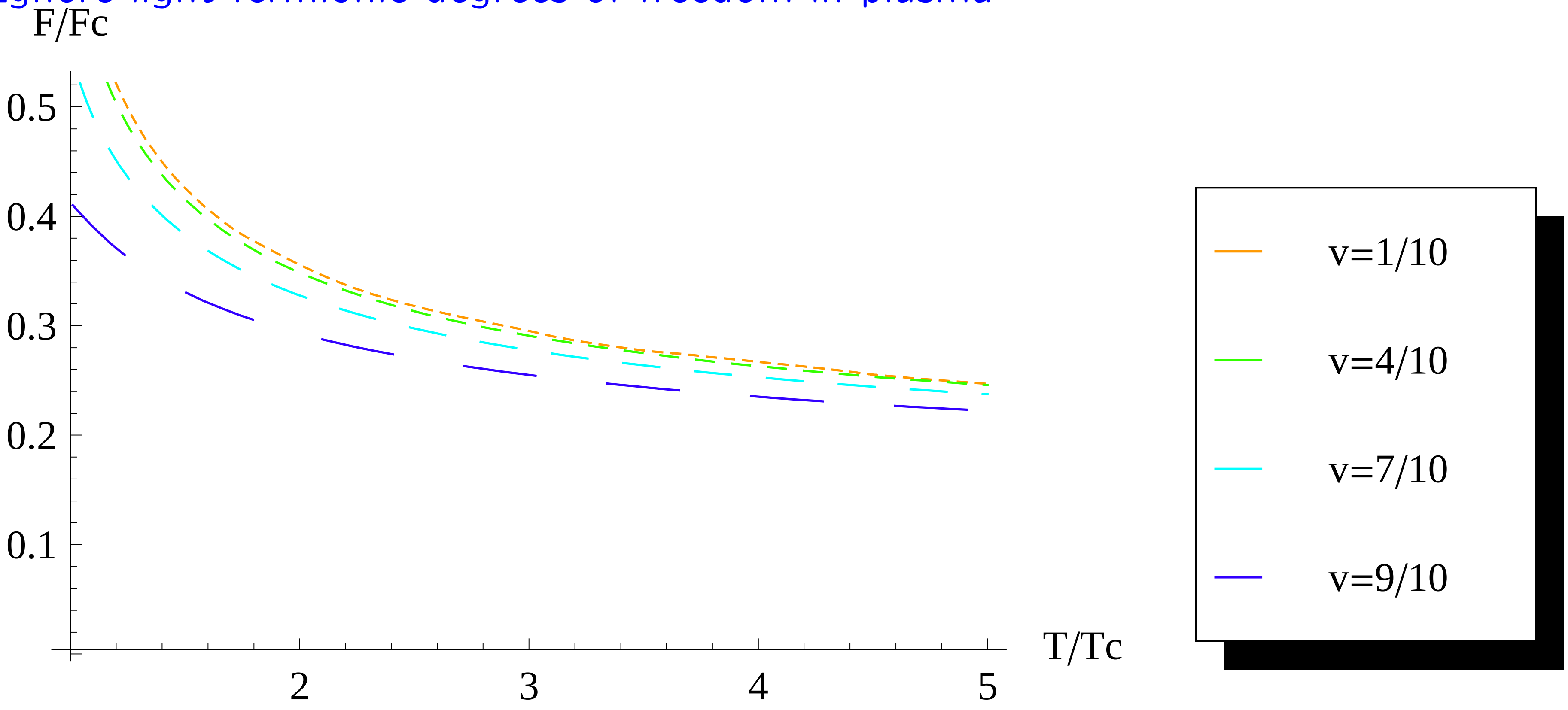
$$F_{\text{drag}} = P_\xi = -\frac{b^2(r_s)\sqrt{f(r_s)}}{2\pi\ell_s^2}$$

The drag force in IhQCD

Systematic errors:

(a) Flavor description (heavy quark)

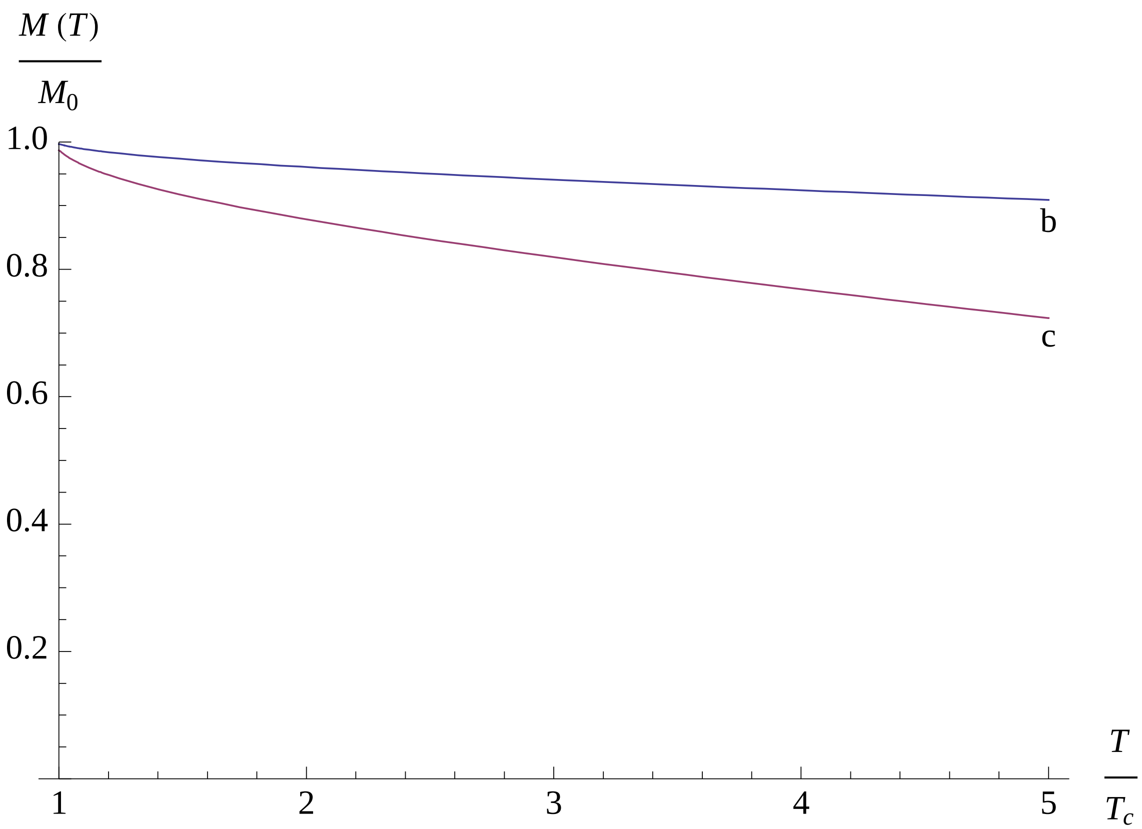
(b) Ignore light fermionic degrees of freedom in plasma



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

- F_{conf} calculated with $\lambda = 5.5$

The thermal mass

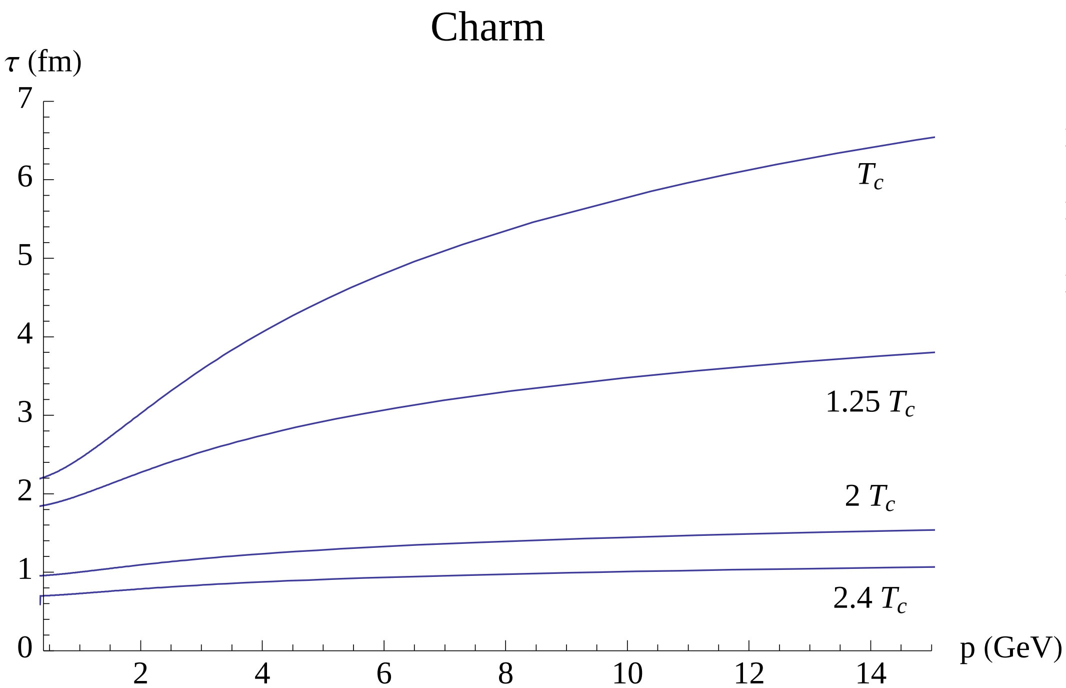


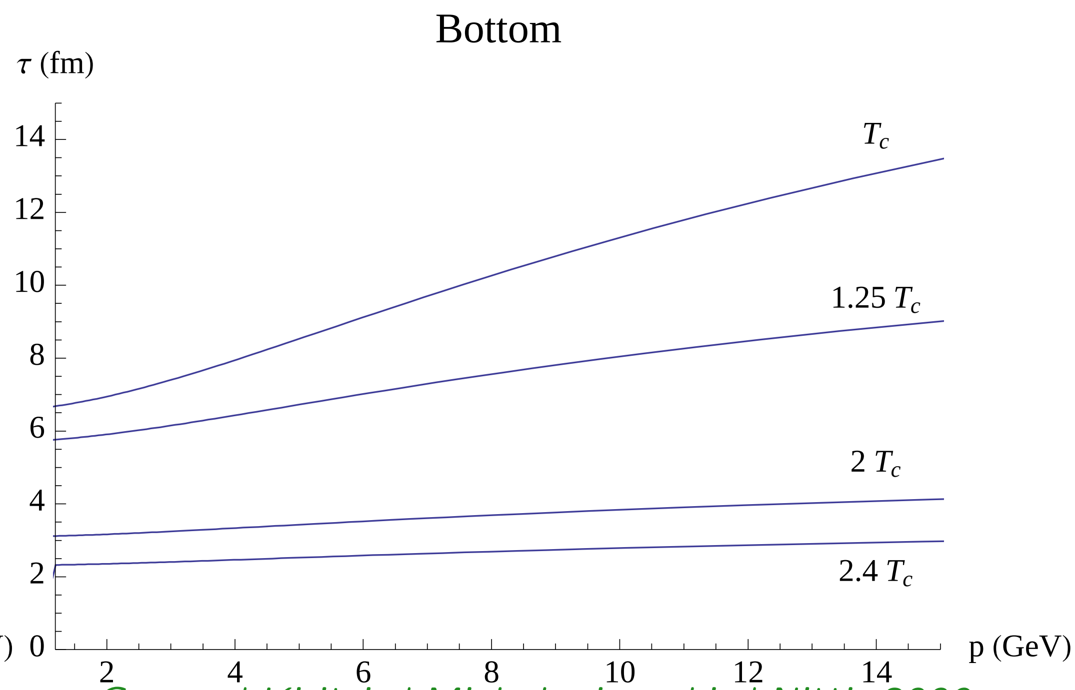
- The mass is defined via a straight string hanging in the bulk

- It is qualitatively in agreement with lattice calculation of the position of the quarkonium resonance shift at finite temperature.

Datta+Karsch+Petreczky+Wetzorke 2004

The diffusion time





Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

	$\gamma = 0.3$	$\gamma = 1$	$\gamma = 3$
τ_c[fm]	22	6.7	2.2
τ_b[fm]	72	21	7.2

thermalized

not thermalized

Akamatsu+Hatsuda+Hirano, 2008

Outlook

• Improved AdS/QCD models can address well several issues in QCD

(a)Static properties and spectra in YM

(b) Deconfinement, static and dynamical properties at finite temperature (at least up to T $5-10T_c$) Here we can calculate, second order transport coefficients, stress tensor correlators, study the correlators of string fluctuations to obtain the parameters of Langevin dynamics of heavy quarks, study the initial thermalization process.

(c) The flavor sector can also be engineered, to provide chiral symmetry breaking and the meson spectra and interactions.

• It does not do well (expectedly) :

(a) For various UV effects (curvature dependent conformal anomaly, UV asymptotics of shear-viscosity, details of UV asymptotics of correlators).

• Will need an upgrade (N_f N_c) to describe the nontrivial phase structure at finite chemical potential. and the CFL phase.

Thank you for your Patience

Bibliography

• U. Gursoy, E. Kiritsis, G. Michalogiorgakis and F. Nitti,
" Thermal Transport and Drag Force in Improved Holographic QCD"
[ArXiv:0906.1890][hep-ph],.

• U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti,
"Improved Holographic Yang-Mills at Finite Temperature: Comparison with Data."
Nucl.Phys.B820:148-177,2009. [ArXiv:0903.2859][hep-th],.

• E. Kiritsis,
" Dissecting the string theory dual of QCD.,"
Fortsch.Phys.57:396-417,2009. [ArXiv:0901.1772][hep-th],.

• U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti,
"Thermodynamics of 5D Dilaton-gravity.,"
JHEP **0905** (2009) 033; [ArXiv:0812.0792][hep-th],.

• U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti,
"Deconfinement and Gluon-Plasma Dynamics in Improved Holographic QCD,"
Phys. Rev. Lett. **101**, 181601 (2008) [ArXiv:0804.0899][hep-th],.

• U. Gursoy and E. Kiritsis,
"Exploring improved holographic theories for QCD: Part I,"
JHEP **0802** (2008) 032[ArXiv:0707.1324][hep-th].

• U. Gursoy, E. Kiritsis and F. Nitti,
"Exploring improved holographic theories for QCD: Part II,"
JHEP **0802** (2008) 019[ArXiv:0707.1349][hep-th].

• Elias Kiritsis and F. Nitti
On massless 4D gravitons from asymptotically AdS(5) space-times.
Nucl.Phys.**B772** (2007) 67-102;[arXiv:hep-th/0611344]

• R. Casero, E. Kiritsis and A. Paredes,
"Chiral symmetry breaking as open string tachyon condensation,"
Nucl. Phys. B **787** (2007) 98;[arXiv:hep-th/0702155].

A string theory for QCD:basic expectations

• Pure SU(N_c) d=4 YM at large N_c is expected to be dual to a string theory in 5 dimensions only. Essentially a single adjoint field $\to$ a single extra dimension.

• The theory becomes asymptotically free and conformal at high energy $\to$ we expect the classical saddle point solution to asymptote to AdS_5.

♠ Operators with lowest dimension (or better: lowest bulk masses) are expected to be the only important non-trivial bulk fields in the large-N_c saddle-point

• Scalar YM operators with $\Delta_{UV} > 4 \to m^2 > 0$ fields near the AdS$_5$ boundary $\to$ vanish fast in the UV regime and do not affect correlators of low-dimension operators.

.

• Their dimension may grow large in the IR so they are also irrelevant there. The large ’t Hooft coupling is expected to suppress the effects of such operators.

• This is suggested by the success of low-energy SVZ sum rules as compared to data.

• What are all gauge invariant YM operators of dimension 4 or less?

• They are given by $Tr[F_{\mu\nu}F_{\rho\sigma}]$.
Decomposing into U(4) reps:

$$(\ydiagram{1,1} \otimes \ydiagram{1,1})_{\text{symmetric}} = \ydiagram{2,2} \oplus \ydiagram{1,1,1,1} \tag{1}$$

We must remove traces to construct the irreducible representations of O(4):

$$\ydiagram{2,2} = \underline{\ydiagram{2,2}} \oplus \underline{\ydiagram{2}} \oplus \bullet \quad , \quad \ydiagram{1,1,1,1} = \bullet$$

The two singlets are the scalar (dilaton) and pseudoscalar (axion)

$$\phi \leftrightarrow Tr[F^2] \quad , \quad a \leftrightarrow Tr[F \wedge F]$$

The traceless symmetric tensor

$$\square\square \quad \to \quad T_{\mu\nu} = Tr\left[F^2_{\mu\nu} - \frac{1}{4} g_{\mu\nu} F^2\right]$$

is the conserved stress tensor dual to a massless graviton in 5d reflecting the translational symmetry of YM.

$$\begin{smallmatrix}\square&\square\\\square&\square\end{smallmatrix} \;\to\; T^4_{\mu\nu;\rho\sigma} = Tr[F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2}(g_{\mu\rho}F^2_{\nu\sigma} - g_{\nu\rho}F^2_{\mu\sigma} - g_{\mu\sigma}F^2_{\nu\rho} + g_{\nu\sigma}F^2_{\mu\rho}) + \frac{1}{6}(g_{\mu\rho}g_{\nu\sigma} - g_{\nu\rho}g_{\mu\sigma})F^2$$

It has 10 independent d.o.f, it is not conserved and it should correspond to a similar massive tensor in 5d. We do not expect it to play an non-trivial role in the large-N_c, YM vacuum also for reasons of Lorentz invariance.

♠ Therefore we will consider

$$T_{\mu\nu} \leftrightarrow g_{\mu\nu}, \; tr[F^2] \leftrightarrow \phi, \; tr[F \wedge F] \leftrightarrow a$$

bosonic string or superstring? I

• The string theory must have no on-shell fermionic states at all because there are no gauge invariant fermionic operators in pure YM. (even with quarks modulo baryons).

• There is a direct argument that the axion, dual to the instanton density $F \wedge F$ must be a RR field (as in $\mathcal{N} = 4$).

• Therefore the string theory must be a 5d-superstring theory resembling the II-0 class.

♠ Another RR field we expect to have is the RR 4-form, as it is necessary to "seed" the D_3 branes responsible for the gauge group.

• It is non-propagating in 5D

• We will see later however that it is responsible for the non-trivial IR structure of the gauge theory vacuum.

Bosonic string or superstring? II

• Consider the axion a dual to $Tr[F \wedge F]$. We can show that it must come from a RR sector.

In large-N$_c$ YM, the proper scaling of couplings is obtained from

$$\mathcal{L}_{YM} = N_c \; Tr\left[\frac{1}{\lambda}F^2 + \frac{\theta}{N_c} F \wedge F\right] \quad , \quad \zeta \equiv \frac{\theta}{N_c} \sim \mathcal{O}(1)$$

It can be shown

$$E_{YM}(\theta) \simeq C_0 \; N_c^2 + C_1 \theta^2 + C_2 \frac{\theta^4}{N_c^2} + \cdots$$

Witten

In the string theory action

$$S \sim \int e^{-2\phi} \, [R + \cdots] + (\partial a)^2 + e^{2\phi} (\partial a)^4 + \cdots \quad , \quad e^{\phi} \sim g_{YM}^2 \quad , \quad \lambda \sim N_c e^{\phi}$$

$$\sim \int \frac{N_c^2}{\lambda^2} \, [R + \cdots] + (\partial a)^2 + \frac{\lambda^2}{N_c^2} (\partial a)^4 + \cdots \quad , \quad a = \theta[1 + \cdots]$$

RETURN

The minimal effective string theory spectrum

- NS-NS $\quad\rightarrow\qquad g_{\mu\nu}\leftrightarrow T_{\mu\nu}\quad,\quad B_{\mu\nu}\leftrightarrow Tr[F]^3\quad,\quad \phi\leftrightarrow Tr[F^2]$

- RR $\quad\rightarrow\qquad$ Spinor$_5\times$Spinor$_5$$=F_0+F_1+F_2+(F_3+F_4+F_5)$

♠ $F_0\leftrightarrow F_5\rightarrow\quad C_4$, background flux $\rightarrow$ no propagating degrees of freedom.

♠ $F_1\leftrightarrow F_4\rightarrow\quad C_3\leftrightarrow C_0$: C_0 is the axion, C_3 its 5d dual that couples to domain walls separating oblique confinement vacua.

♠ $F_2\leftrightarrow F_3\rightarrow\quad C_1\leftrightarrow C_2$: They are associated with baryon number (as we will see later when we add flavor). C_2 mixes with B_2 because of the C_4 flux, and is massive.

- In an ISO(3,1) invariant vacuum solution, only $g_{\mu\nu},\phi,C_0=a$ can be non-trivial.

$$ds^2=e^{2A(r)}(dr^2+dx_4^2)\quad,\quad a(r),\phi(r)$$

The relevant "defects"

- $B_{\mu\nu} \to$ Fundamental string (F_1). This is the QCD (glue) string: fundamental tension $\ell_s^2 \sim \mathcal{O}(1)$

- Its dual $\tilde{B}_\mu \to NS_0$: Tension is $\mathcal{O}(N_c^2)$. It is an effective magnetic baryon vertex binding N_c magnetic quarks.

- $C_5 \;\; \to D_4$: Space filling flavor branes. They must be introduced in pairs: $D_4 + \bar{D}_4$ for charge neutrality/tadpole cancelation $\to$ gauge anomaly cancelation in QCD.

- $C_4 \to D_3$ branes generating the gauge symmetry.

.

• $C_3 \to D_2$ branes : domain walls separating different oblique confinement vacua (where $\theta_{k+1} = \theta_k + 2\pi$). Its tension is $\mathcal{O}(N_c)$

• $C_2 \to D_1$ branes: These are the magnetic strings: (strings attached to magnetic quarks) with tension $\mathcal{O}(N_c)$

• $C_1 \to D_0$ branes. These are the baryon vertices: they bind N_c quarks, and their tension is $\mathcal{O}(N_c)$.
Its instantonic source is the (solitonic) baryon in the string theory.

• $C_0 \to D_{-1}$ branes: These are the Yang-Mills instantons.

The effective action, I

- as $N_c \to \infty$, only string tree-level is dominant.

- Relevant field for the vacuum solution: $g_{\mu\nu}, a, \phi, F_5$.

- The vev of $F_5 \sim N_c\ \epsilon_5$. It appears always in the combination $e^{2\phi}F_5^2 \sim \lambda^2$, with $\lambda \sim N_c\ e^{\phi}$ All higher derivative corrections $(e^{2\phi}F_5^2)^n$ are $\mathcal{O}(1)$.
A non-trivial potential for the dilaton will be generated already at string tree-level.

- This is not the case for all other RR fields: in particular for the axion as $a \sim \mathcal{O}(1)$

$$(\partial a)^2 \sim \mathcal{O}(1) \quad , \quad e^{2\phi}(\partial a)^4 = \frac{\lambda^2}{N_c^2}(\partial a)^4 \sim \mathcal{O}\left(N_c^{-2}\right)$$

Therefore to leading order $\mathcal{O}(N_c^2)$ we can neglect the axion.

The UV regime

- In the far UV, the space should asymptote to AdS$_5$.

- The 't Hooft coupling should behave as ($r \to 0$)

$$\lambda \sim \frac{1}{\log(r\Lambda)} + \cdots \quad \to \quad 0 \quad , \quad r \sim \frac{1}{E}$$

The effective action to leading order in N_c is

$$S_{eff} \sim \int d^5x \sqrt{g}\; e^{-2\phi}\; Z(\; \ell_s^2 R \;,\; \ell_s^2 (\partial\phi)^2 \;,\; e^{2\phi} \ell_s^2 F_5^2 \;)$$

Solving the equation of motion of F_5 amounts to replacing

$$e^{2\phi}\; \ell_s^2\; F_5^2 \sim e^{2\phi} N_c^2 \equiv \lambda^2$$

$$S_{eff} \sim N_c^2 \int d^5x \sqrt{g}\; \frac{1}{\lambda^2}\; H(\; \ell_s^2 R \;,\; \ell_s^2 (\partial\lambda)^2 \;,\; \lambda^2 \;)$$

• As $r \to 0$

$$\text{Curvature} \to \text{finite} \quad , \quad \Box\phi \sim (\partial\phi)^2 \sim \frac{(\partial\lambda)^2}{\lambda^2} \sim \lambda^2 \sim \frac{1}{\log^2(r\Lambda)} \to 0$$

• For $\lambda \to 0$ the potential in the Einstein frame starts as $V(\lambda) \sim \lambda^{\frac{4}{3}}$ and cannot support the asymptotic AdS_5 solution.

• Therefore asymptotic AdS_5 must arise from curvature corrections:

$$S_{eff} \simeq \int d^5x \, \frac{1}{\lambda^2} \, H\left(\ell_s^2 \, R, 0, 0\right)$$

• Setting $\lambda = 0$ at leading order we can generically get an AdS_5 solution coming from balancing the higher curvature corrections.

INTERESTING QUESTION: Is there a good toy example of string vacuum (CFT) which is not Ricci flat, and is supported only by a metric?

• There is a "good" (but hard to derive the coefficients) perturbative expansion around this asymptotic AdS_5 solution by perturbing inwards :

$$e^A = \frac{\ell}{r}[1+\delta A(r)] \quad , \quad \lambda = \frac{1}{b_0 \log(r\Lambda)} + \cdots$$

• This turns out to be a regular expansion of the solution in powers of

$$\frac{P_n(\log\log(r\Lambda))}{(log(r\Lambda))^{-n}}$$

• Effectively this can be rearranged as a "perturbative" expansion in $\lambda(r)$. In the case of running coupling, the radial coordinate can be substituted by $\lambda(r)$.

• Using λ as a radial coordinate the solution for the metric can be written

$$E \equiv e^A = \frac{\ell}{r(\lambda)}\left[1 + c_1\lambda + c_2\lambda^2 + \cdots\right] = \ell\,(e^{-\frac{b_0}{\lambda}})\left[1 + c_1'\lambda + c_2'\lambda^2 + \cdots\right] \quad , \quad \lambda \to 0$$

.

Conclusion 1: The asymptotic AdS_5 is stringy, but the rest of the geometry is "perturbative around the asymptotics". We cannot however do computations even if we know the structure.

Conclusion 2: It has been a mystery how can one get free field theory at the boundary. This is automatic here since all non-trivial connected correlators are proportional to positive powers of λ that vanishes in the UV.

The IR regime

• Here the situation is more obscure. The constraints/input will be: confinement, discreteness of the spectrum and mass gap.

• We do expect that $\lambda \to \infty$ (or becomes large) at the IR bottom.

• Intuition from N=4 and other 10d strongly coupled theories suggests that in this regime there should be an (approximate) two-derivative description of the physics.

• The simplest solution with this property is the linear dilaton solution with

$$\lambda \sim e^{Qr} \quad , \quad V(\lambda) \sim \delta c = 10 - D \quad \to \quad \text{constant} \quad , \quad R = 0$$

• This property persists with potentials $V(\lambda) \sim (\log \lambda)^P$. Moreover all such cases have confinement, a mass gap and a discrete spectrum (except the P=0 case).

• At the IR bottom (in the string frame) the scale factor vanishes, and 5D space becomes (asymptotically) flat.

Comments on confining backgrounds

• For all confining backgrounds with $r_0 = \infty$, although the space-time is singular in the Einstein frame, the string frame geometry is asymptotically flat for large r. Therefore only λ grows indefinitely.

• String world-sheets do not probe the strong coupling region, at least classically. The string stays away from the strong coupling region.

• Therefore: singular confining backgrounds have generically the property that the singularity is *repulsive*, i.e. only highly excited states can probe it. This will also be reflected in the analysis of the particle spectrum (to be presented later)

• The confining backgrounds must also screen magnetic color charges. This can be checked by calculating 't Hooft loops using D_1 probes:

♠ All confining backgrounds with $r_0 = \infty$ and most at finite r_0 screen properly

♠ In particular "hard-wall" AdS/QCD confines also the magnetic quarks.

Organizing the vacuum solutions

A useful variable is the phase variable

$$X \equiv \frac{\Phi'}{3A'} = \frac{\beta(\lambda)}{3\lambda} \quad , \quad e^{\Phi} \equiv \lambda$$

and a superpotential

$$W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \Phi}\right)^2 = \left(\frac{3}{4}\right)^3 V(\Phi).$$

with

$$A' = -\frac{4}{9}W \quad , \quad \Phi' = \frac{dW}{d\Phi}$$

$$X = -\frac{3}{4}\frac{d\log W}{d\log\lambda} \quad , \quad \beta(\lambda) = -\frac{9}{4}\lambda\frac{d\log W}{d\log\lambda}$$

♠ The equations have three integration constants: (two for Φ and one for A) One corresponds to the "gluon condensate" in the UV. It must be set to zero otherwise the IR behavior is unacceptable. The other is Λ. The third one is a gauge artifact (corresponds to overall translation in the radial coordinate).

The IR regime

For any asymptotically AdS$_5$ solution ($e^A \sim \frac{\ell}{r}$):

• The scale factor $e^{A(r)}$ is monotonically decreasing

Girardelo+Petrini+Porrati+Zaffaroni
Freedman+Gubser+Pilch+Warner

• Moreover, there are only three possible, mutually exclusive IR asymptotics:

♠ *there is another asymptotic AdS_5 region, at $r \to \infty$, where* $\exp A(r) \sim \ell'/r$, *and* $\ell' \le \ell$ (equality holds if and only if the space is exactly AdS_5 everywhere);

♠ there is a curvature singularity at some finite value of the radial coordinate, $r = r_0$;

♠ **there is a curvature singularity at $r \to \infty$, where the scale factor vanishes and the space-time shrinks to zero size.**

Wilson-Loops and confinement

• Calculation of the static quark potential using the vev of the Wilson loop calculated via an F-string worldsheet.

Rey+Yee, Maldacena

$$T\ E(L) = S_{minimal}(X)$$

We calculate

$$L = 2\int_0^{r_0} dr \frac{1}{\sqrt{e^{4A_S(r)-4A_S(r_0)} - 1}}.$$

It diverges when e^{A_s} has a minimum (at $r = r_*$). Then

$$E(L) \sim T_f\ e^{2A_S(r_*)}\ L$$

• Confinement $\rightarrow$ $A_s(r_*)$ is finite. This is a more general condition that considered before as A_S `is not monotonic in general`.

• Effective string tension

$$T_{\text{string}} = T_f\ e^{2A_S(r_*)}$$

General criterion for confinement

• **the geometric version**:
A geometry that shrinks to zero size in the IR is dual to a confining 4D theory if and only if the Einstein metric in conformal coordinates vanishes as (or faster than) e^{-Cr} as $r \to \infty$, for some $C > 0$.

• It is understood here that a metric vanishing at finite $r = r_0$ also satisfies the above condition.

♠ **the superpotential**
A 5D background is dual to a confining theory if the superpotential grows as (or faster than)

$$W \sim (\log \lambda)^{P/2} \lambda^{2/3} \quad \text{as} \quad \lambda \to \infty \quad , \quad P \geq 0$$

♠ **the β-function** A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda \to \infty} \left(\frac{\beta(\lambda)}{3\lambda} + \frac{1}{2} \right) \log \lambda = K, \qquad -\infty \leq K \leq 0$$

(No explicit reference to any coordinate system) Linear trajectories correspond to $K = -\frac{3}{16}$

Classification of confining superpotentials

Classification of confining superpotentials $W(\lambda)$ as $\lambda \to \infty$ in IR:

$$W(\lambda) \sim (\log\lambda)^{\frac{P}{2}}\ \lambda^{Q} \quad , \quad \lambda \sim E^{-\frac{9}{4}Q}\left(\log\frac{1}{E}\right)^{\frac{P}{2Q}}, \qquad E \to 0.$$

• $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

$$e^{A}(r) \sim \begin{cases} (r_0 - r)^{\frac{4}{9Q^2-4}} & Q > \frac{2}{3} \\ \exp\left[-\frac{C}{(r_0-r)^{1/(P-1)}}\right] & Q = \frac{2}{3} \end{cases}$$

• $Q = 2/3$, and $0 \leq P < 1$ leads to confinement and a singularity at $r = \infty$ The scale factor e^A vanishes there as

$$e^{A}(r) \sim \exp[-Cr^{1/(1-P)}].$$

• $Q = 2/3, P = 1$ leads to confinement but the singularity may be at a finite or infinite value of r depending on subleading asymptotics of the superpotential.

♠ If $Q < 2\sqrt{2}/3$, no *ad hoc* boundary conditions are needed to determine the glueball spectrum $\to$ One-to-one correspondence with the β-function This is unlike standard AdS/QCD and other approaches.

• when $Q > 2\sqrt{2}/3$, the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

Confining β-functions

A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda\to\infty}\left(\frac{\beta(\lambda)}{3\lambda}+\frac{1}{2}\right)\log\lambda = K, \qquad -\infty \le K \le 0$$

(No explicit reference to any coordinate system). Linear trajectories correspond to $K = -\frac{3}{16}$

- We can determine the geometry if we specify K:
- $K = -\infty$: the scale factor goes to zero at some finite r_0, not faster than a power-law.

- $-\infty < K < -3/8$: the scale factor goes to zero at some finite r_0 faster than any power-law.

- $-3/8 < K < 0$: the scale factor goes to zero as $r \to \infty$ faster than $e^{-Cr^{1+\epsilon}}$ for some $\epsilon > 0$.

- $K = 0$: the scale factor goes to zero as $r \to \infty$ as e^{-Cr} (or faster), but slower than $e^{-Cr^{1+\epsilon}}$ for any $\epsilon > 0$.

The borderline case, $K = -3/8$, is certainly confining (by continuity), but whether or not the singularity is at finite r depends on the subleading terms.

Particle Spectra: generalities

- Linearized equation:

$$\ddot{\xi} + 2\dot{B}\dot{\xi} + \Box_4 \xi = 0 \quad , \quad \xi(r,x) = \xi(r)\xi^{(4)}(x), \qquad \Box\xi^{(4)}(x) = m^2 \xi^{(4)}(x)$$

- Can be mapped to Schrodinger problem

$$-\frac{d^2}{dr^2}\psi + V(r)\psi = m^2\psi \quad , \quad V(r) = \frac{d^2B}{dr^2} + \left(\frac{dB}{dr}\right)^2 \quad , \quad \xi(r) = e^{-B(r)}\psi(r)$$

- Mass gap and discrete spectrum visible from the asymptotics of the potential.

- Large n asymptotics of masses obtained from WKB

$$n\pi = \int_{r_1}^{r_2} \sqrt{m^2 - V(r)}\, dr$$

- Spectrum depends only on initial condition for λ ($\sim \Lambda_{QCD}$) and an overall energy scale (e^A) that must be fixed.

• scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log\frac{\beta(\lambda)^2}{9\lambda^2}$$

• tensor glueballs

$$B(r) = \frac{3}{2}A(r)$$

• pseudo-scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log Z(\lambda)$$

• Universality of asymptotics

$$\frac{m^2_{n\to\infty}(0^{++})}{m^2_{n\to\infty}(2^{++})} \to 1 \quad , \quad \frac{m^2_{n\to\infty}(0^{+-})}{m^2_{n\to\infty}(0^{++})} = \frac{1}{4}(d-2)^2$$

predicts $d = 4$ via

$$\frac{m^2}{2\pi\sigma_a} = 2n + J + c,$$

The free energy

• The free energy is calculated from the action as a boundary term for both the black-holes and the thermal vacuum solution. They are all UV divergent but their differences are finite.

$$\frac{\mathcal{F}}{M_p^3 V_3} = 12\mathcal{G}(T) - T\ S(T)$$

• $\mathcal{G}$ is the temperature-depended gluon condensate $\langle Tr[F^2]\rangle_T - \langle Tr[F^2]\rangle_{T=0}$ defined as

$$\lim_{r\to 0}\quad \lambda_T(r) - \lambda_{T=0}(r) = \mathcal{G}(T)\ r^4 + \cdots$$

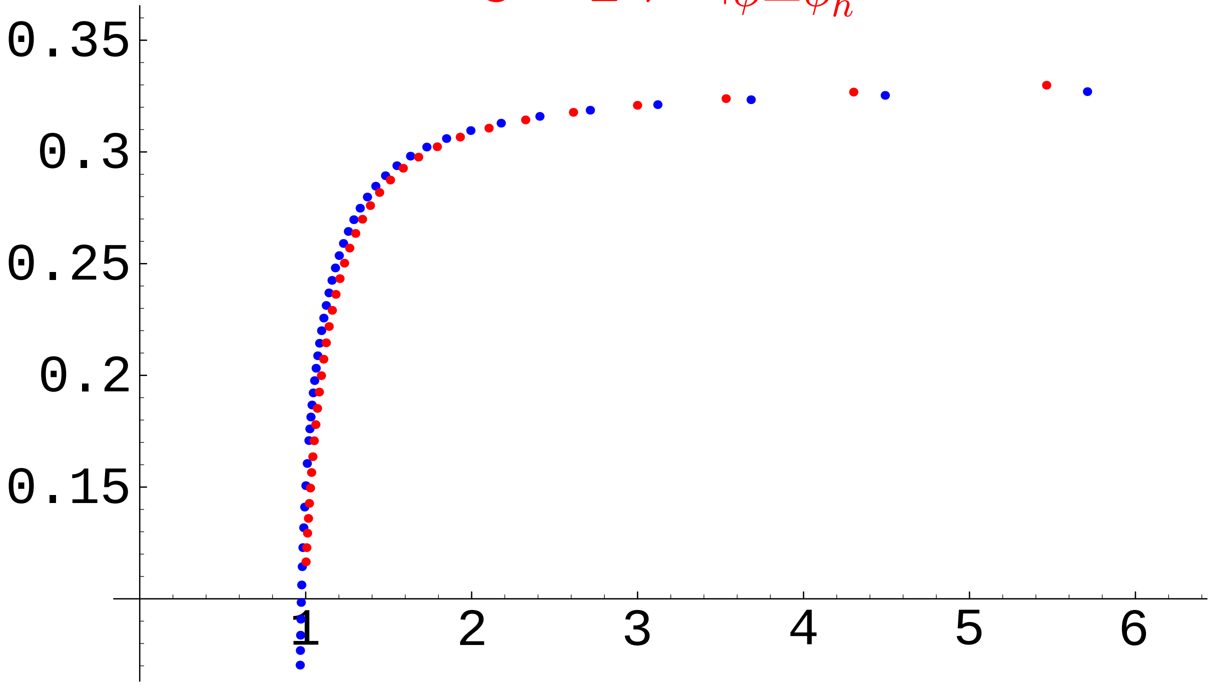
• It is $\mathcal{G}$ the breaks conformal invariance essentially and leads to a non-trivial deconfining transition (as $S > 0$ always)

• The axion solution must be constant above the phase transition (black-hole). Therefore $\langle F \wedge F\rangle$ vanishes.

Comparing to Gubser+Nelore's formula

- Gubser+Nelore proposed the following approximate formula for the speed of sound

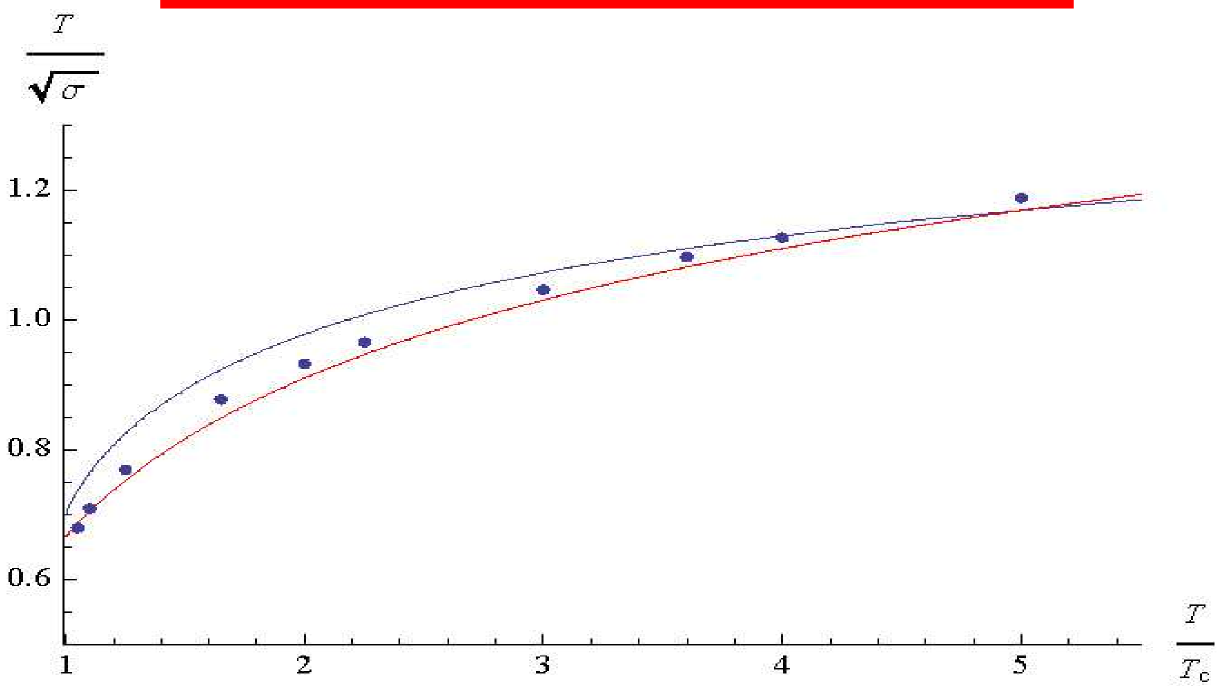
$$c_s^2 \simeq \frac{1}{3} - \frac{1}{2}\frac{V'^2}{V^2}\Big|_{\phi=\phi_h}$$



Gursoy (unpublished) 2009

- Red curve=numerical calculation, Blue curve=Gubser's adiabatic/approximate formula.

Spatial string tension



G. Boyd et al. 1996

• The blue line is the spatial string tension as calculated in Improved hQCD, with no additional fits.

Nitti (unpublished) 2009

• The red line is a semi-phenomenological fit using

$$\frac{T}{\sqrt{\sigma_s}} = 0.51 \ \left[\log\frac{\pi T}{T_c} + \frac{51}{121}\log\left(2\log\frac{\pi T}{T_c}\right)\right]^{\frac{2}{3}}$$

Alanen+Kajantie+Suur-Uski, 2009

The sum rule method, II

• Define the (subtracted) spectral density and relate its moment to the Euclidean density

$$\rho(\omega) = -\frac{1}{\pi} Im\ G^R(\omega) \quad , \quad \mathcal{G} \equiv \lim_{\omega \to 0} G^E(\omega) = 2 \int_0^\infty \frac{\rho(u)}{u}\, du$$

Karsch+Kharzeev+Tuchin, 2008, Romatschke+Son 2009

• Using Ward identities we obtain the sum rule

$$\mathcal{G} = \left(T\frac{\partial}{\partial T} - 4\right)(E - 3P + \langle\Theta\rangle_0) + \left(T\frac{\partial}{\partial T} - 2\right)(m\langle\bar{q}q\rangle_T + \langle\Theta_F\rangle_0)$$

with

$$\langle\Theta_F\rangle_0 = m\langle\bar{q}q\rangle \simeq -m_\pi^2\ f_\pi^2 - m_K^2\ f_K^2$$

• Assume a density

$$\frac{\rho(\omega)}{\omega} = \frac{9\zeta(T)}{\pi}\frac{\omega_0(T)^2}{\omega^2 + \omega_0(T)^2}$$

The bulk viscosity in HQCD: theory

- This is harder to calculate.

- Using a parametrization $ds^2 = e^{2A}(f dt^2 + d\vec{x}^2 + \frac{dr^2}{f})$ in a special gauge $\phi = r$ the relevant metric perturbation decouples

Gubser+Nellore+Pufu+Rocha 2008, Gubser+Pufu+Rocha,2008

$$h''_{11} = -\left(-\frac{1}{3A'} - A' - \frac{f'}{f}\right) h'_{11} + \left(-\frac{\omega^2}{f^2} + \frac{f'}{6fA'} - \frac{f'}{f}A'\right) h_{11}$$

with

$$h_{11}(0) = 1 \quad , \quad h_{11}(r_h) \simeq C\ e^{i\omega t}\left|\log\frac{\lambda}{\lambda_h}\right|^{-\frac{i\omega}{4\pi T}}$$

The correlator is given by the conserved number of h-quanta

$$Im\ G_R(\omega) = -4M^3\mathcal{G}(\omega) \quad , \quad \mathcal{G}(\omega) = \frac{e^{3A}f}{4A'^2}|Im[h^*_{11}h'_{11}]|$$

$$\frac{\zeta}{s} = \frac{C^2}{4\pi}\ \left(\frac{V'(\lambda_h)}{V(\lambda_h)}\right)^2$$

Adding flavor

• To add N_f quarks q_L^I and antiquarks $q_R^{\bar{I}}$ we must add (in 5d) space-filling N_f D_4 and N_f $\bar{D}_4$ branes.
(tadpole cancellation=gauge anomaly cancellation)

• The q_L^I should be the "zero modes" of the $D_3 - D_4$ strings while $q_R^{\bar{I}}$ are the "zero modes" of the $D_3 - \bar{D}_4$

• The low-lying fields on the D_4 branes ($D_4 - D_4$ strings) are U$(N_f)_L$ gauge fields A_μ^L. The low-lying fields on the $\bar{D}_4$ branes ($\bar{D}_4 - \bar{D}_4$ strings) are U$(N_f)_R$ gauge fields A_μ^R. They are dual to the J_L^μ and J_μ^R

$$\delta S_A \sim \bar{q}_L^I \; \gamma^\mu \; (A_\mu^L)^{IJ} \; q_L^J + \bar{q}_R^{\bar{I}} \; \gamma^\mu \; (A_\mu^R)^{\bar{I}\bar{J}} \; q_R^{\bar{J}} = Tr[J_L^\mu \; A_\mu^L + J_R^\mu \; A_\mu^R]$$

• There are also the low lying fields of the ($D_4 - \bar{D}_4$ strings), essentially the string-theory "tachyon" $T_{I\bar{J}}$ transforming as $(N_f, \bar{N}_f)$ under the chiral symmetry $U(N_f)_L \times U(N_f)_R$. It is dual to the quark mass terms

$$\delta S_T \sim \bar{q}_L^I \; T_{I\bar{J}} \; q_R^{\bar{J}} + \text{complex} \quad \text{congugate}$$

• The interactions on the flavor branes are weak, so that $A_\mu^{L,R}, T$ are as sources for the quarks.

• Integrating out the quarks, generates an effective action $S_{flavor}(A_\mu^{L,R}, T)$, so that $A_\mu^{L,R}, T$ can be thought as effective $q\bar{q}$ composites, that is : mesons

• On the string theory side: integrating out $D_3 - D_4$ and $D_3 - \bar{D}_4$ strings gives rise to the DBI action for the $D_4 - \bar{D}_4$ branes in the D_3 background:

$$S_{flavor}(A_\mu^{L,R}, T) \quad \longleftrightarrow \quad S_{DBI}(A_\mu^{L,R}, T) \quad \text{holographically}$$

• In the "vacuum" only T can have a non-trivial profile: $T^{I\bar{J}}(r)$. Near the AdS_5 boundary ($r \to 0$)

$$T^{I\bar{J}}(r) = M_{I\bar{J}}\, r + \cdots + \langle \bar{q}_L^I \; q_R^{\bar{J}} \rangle r^3 + \cdots$$

Casero+Kiritsis+Paredes

• A typical solution is T vanishing in the UV and $T \to \infty$ in the IR. At the point $r = r_*$ where $T = \infty$, the D_4 and $\bar{D}_4$ branes "fuse". The true vacuum is a brane that enters folds on itself and goes back to the boundary. A non-zero T breaks chiral symmetry.

• A GOR relation is satisfied (for an asymptotic AdS$_5$ space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\,\langle \bar{q}q\rangle \quad , \quad m_q \to 0$$

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

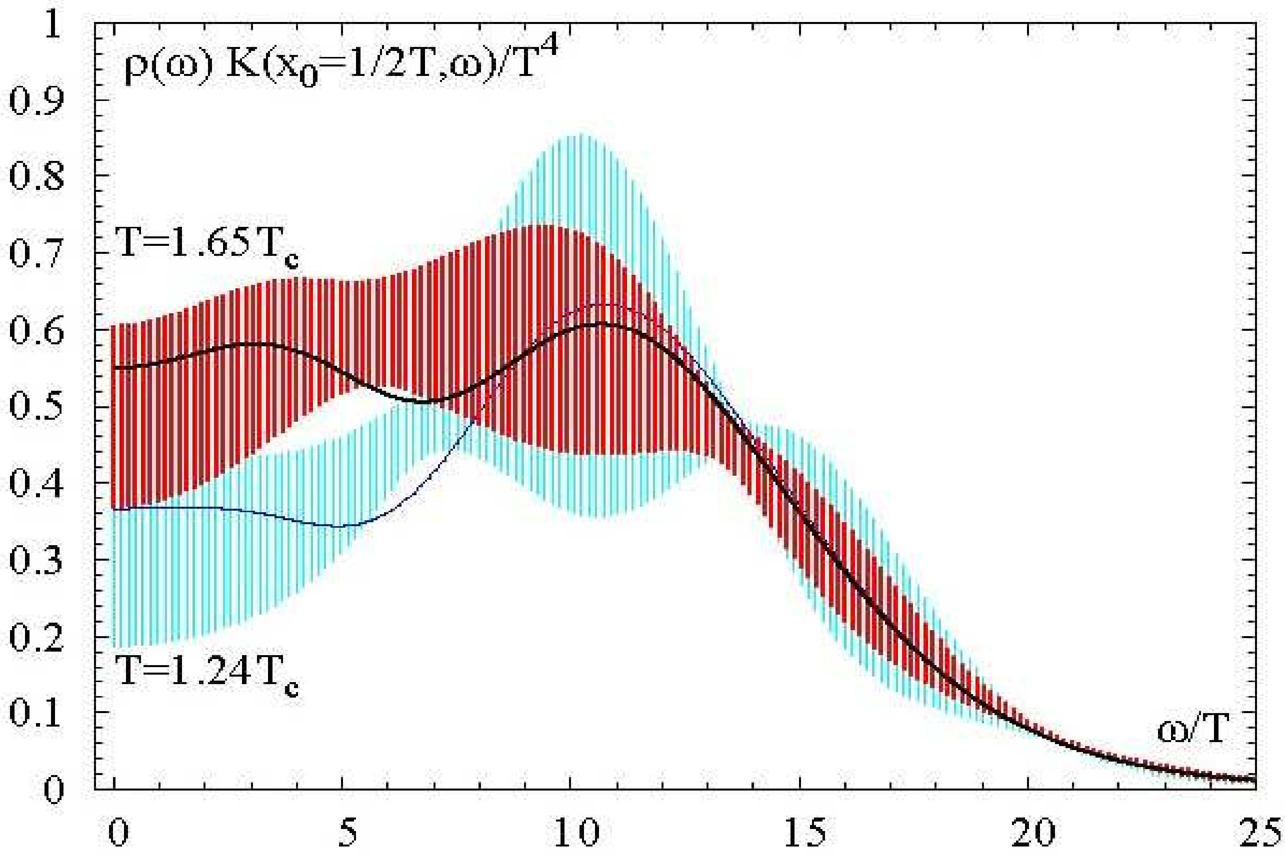
• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

• Fluctuations around the T solution for $T, A_\mu^{L,R}$ give the spectra (and interactions) of various meson trajectories.

• Studying the spectrum of highly excited mesons, we find the expected property of linear confinement: $m_n^2 \sim n$.

• The detailed spectrum of mesons remains to be worked out

Shear Viscosity bounds from lattice

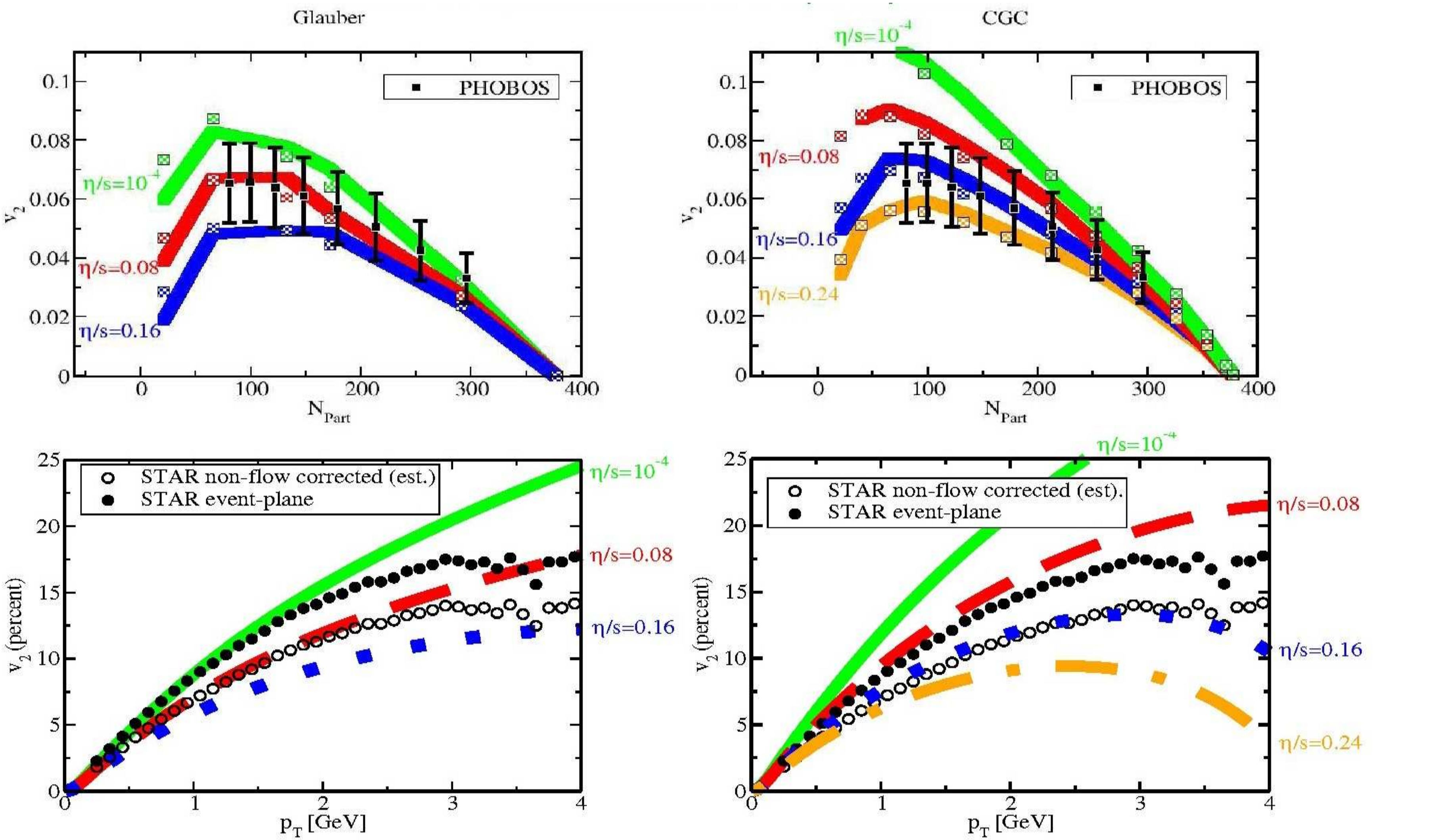


H. Meyer 2007

$$4\pi\frac{\eta}{s} = \begin{cases} 1.68(42), & T = 1.65\ T_c, \\ 1.28(70), & T = 1.24\ T_c. \end{cases}$$

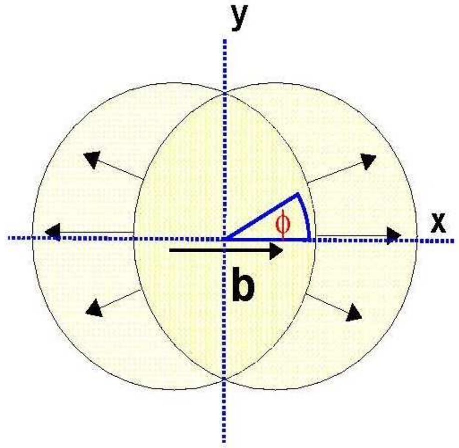
shear viscosity data

- V_2 is the elliptic flow coefficient



Luzum+Romatchke 2008

Elliptic Flow



$$\frac{1}{p_T}\frac{dN}{dp_T d\phi} = \frac{1}{p_T}\frac{dN}{dp_T}\left[1 + v_2(p_T)\cos 2\phi + \cdots\right]$$

Quarks ($N_f \ll N_c$) and mesons

• Flavor is introduced by N_f $D_4 + \bar{D}_4$ branes pairs inside the bulk background. Their back-reaction on the bulk geometry is suppressed by N_f/N_c.

• The important world-volume fields are

$$T_{ij} \leftrightarrow \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \quad , \quad A^{ij\,L,R}_\mu \leftrightarrow \bar{q}^i_a \frac{1 \pm \gamma^5}{2} \gamma^\mu q^j_a$$

Generating the $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

• The UV mass matrix m_{ij} corresponds to the source term of the Tachyon field. It breaks the chiral (gauge) symmetry. The normalizable mode corresponds to the vev $\langle \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \rangle$.

• We show that the expectation value of the tachyon is non-zero and $T \sim \mathbf{1}$, breaking chiral symmetry $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$. The anomaly plays an important role in this (holographic Coleman-Witten)

• The fact that the tachyon diverges in the IR (fusing D with $\bar{D}$) constraints the UV asymptotics and determines the quark condensate $\langle \bar{q}q \rangle$ in terms of m_q. A GOR relation is satisfied (for an asymptotic AdS$_5$ space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\ \langle \bar{q}q \rangle \quad , \quad m_q \to 0$$

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

• Studying the spectrum of highly excited mesons, we find the expected property of linear confinement: $m_n^2 \sim n$.

• The detailed spectrum of mesons remains to be worked out

Tachyon dynamics

- In the vacuum the gauge fields vanish and $T \sim 1$. Only DBI survives

$$S[\tau] = T_{D_4} \int dr d^4x \, \frac{e^{4A_s(r)}}{\lambda} \, V(\tau) \, \sqrt{e^{2A_s(r)} + \dot{\tau}(r)^2} \quad , \quad V(\tau) = e^{-\frac{\mu^2}{2}\tau^2}$$

- We obtain the nonlinear field equation:

$$\ddot{\tau} + \left(3\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right)\dot{\tau} + e^{2A_S}\mu^2\tau + e^{-2A_S}\left[4\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right]\dot{\tau}^3 + \mu^2\tau \; \dot{\tau}^2 = 0.$$

- In the UV we expect

$$\tau = m_q \; r + \sigma \; r^3 + \cdots \quad , \quad \mu^2\ell^2 = 3$$

- We expect that the tachyon must diverge before or at $r = r_0$. We find that indeed it does `at the singularity`. For the $r_0 = \infty$ backgrounds

$$\tau \sim \exp\left[\frac{2}{a} \, \frac{R}{\ell^2} \, r\right] \qquad \text{as} \qquad r \to \infty$$

• Generically the solutions have spurious singularities: $\tau(r_*)$ stays finite but its derivatives diverges as:

$$\tau \sim \tau_* + \gamma\sqrt{r_* - r}.$$

The condition that they are absent determines σ as a function of m_q.

• The easiest spectrum to analyze is that of vector mesons. We find $(r_0 = \infty)$

$$\Lambda_{glueballs} = \frac{1}{R}, \qquad \Lambda_{mesons} = \frac{3}{\ell}\left(\frac{\alpha\ell^2}{2R^2}\right)^{(\alpha-1)/2} \propto \frac{1}{R}\left(\frac{\ell}{R}\right)^{\alpha-2}.$$

This suggests that $\alpha = 2$. preferred also from the glue sector.

The axion background

- The axion solution can be interpreted as a "running" θ-angle

- This is in accordance with the absence of UV divergences and Seiberg-Witten type solutions.

- The axion action is down by $1/N_c^2$

$$S_{axion} = -\frac{M_p^3}{2}\int d^5x\sqrt{g}\ Z(\lambda)\ (\partial a)^2$$

$$\lim_{\lambda\to 0} Z(\lambda) = Z_0\left[1 + c_1\lambda + c_2\lambda^2 + \cdots\right] \quad , \quad \lim_{\lambda\to\infty} Z(\lambda) = c_a\lambda^4 + \cdots$$

- The equation of motion is

$$\ddot{a} + \left(3\dot{A} + \frac{\dot{Z}(\lambda)}{Z(\lambda)}\right)\dot{a} = 0 \quad \to \quad \dot{a} = \frac{C\ e^{-3A}}{Z(\lambda)}$$

- The full solution is

$$a(r) = \theta_{UV} + 2\pi k + C\ \int_0^r dr\frac{e^{-3A}}{Z(\lambda)} \quad , \quad C = \langle Tr[F\wedge F]\rangle$$

- $a(r)$ is a running effective θ-angle. Its running is non-perturbative,

$$a(r) \sim r^4 \sim e^{-\frac{4}{b_0 \lambda}}$$

- The vacuum energy is

$$E(\theta_{UV}) = -\frac{M^3}{2N_c^2} \int d^5x \sqrt{g}\ Z(\lambda)\ (\partial a)^2 = -\frac{M^3}{2N_c^2} C a(r) \Big|_{r=0}^{r=r_0}$$

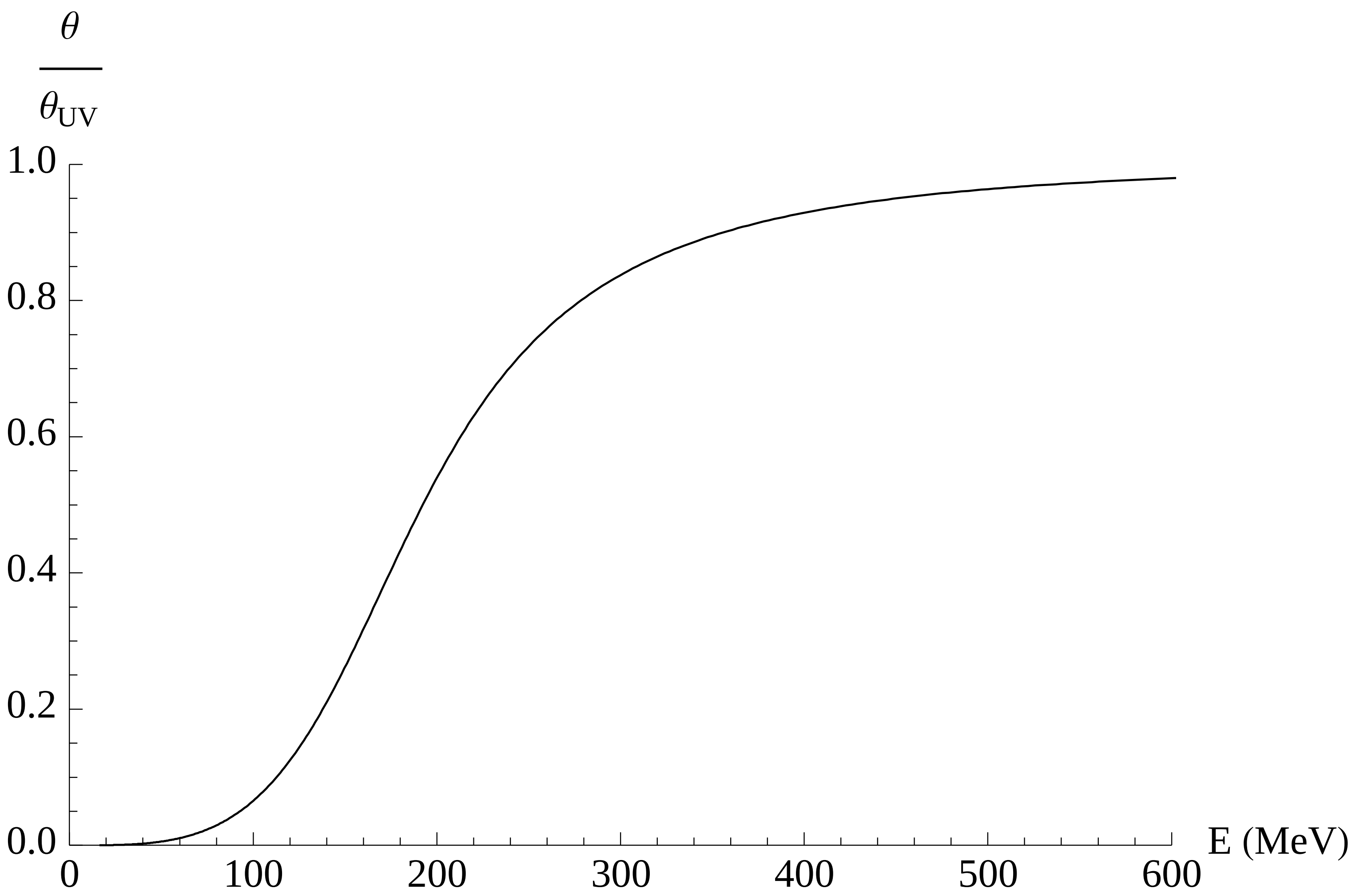
- Consistency requires to impose that $a(r_0) = 0$. This determines C and

$$E(\theta_{UV}) = \frac{M^3}{2} \text{Min}_k \ \frac{(\theta_{UV} + 2\pi k)^2}{\int_0^{r_0} \frac{dr}{e^{3A} Z(\lambda)}}$$

$$\frac{a(r)}{\theta_{UV} + 2\pi k} = \frac{\int_r^{r_0} \frac{dr}{e^{3A} Z(\lambda)}}{\int_0^{r_0} \frac{dr}{e^{3A} Z(\lambda)}}$$

- The topological susceptibility is given by

$$E(\theta) = \frac{1}{2} \chi\ \theta^2 + \mathcal{O}(\theta^4) \quad , \quad \chi = \frac{M_p^3}{\int_0^{r_0} \frac{dr}{e^{3A} Z(\lambda)}}$$



We take: $Z(\lambda) = Z_0(1 + c_a \lambda^4)$

An assessment of IR asymptotics

- We define the superpotential W as

$$V(\lambda) = \frac{4}{3}\lambda^2 \left(\frac{dW}{d\lambda}\right)^2 + \frac{64}{27}W^2$$

- We parameterize the UV ($\lambda \to 0$) and IR asymptotics ($\lambda \to \infty$) as

$$V(\lambda) = \frac{12}{\ell^2}\left[1 + \mathcal{O}(\lambda)\right] \quad , \quad V(\lambda) \sim V_\infty \lambda^Q (\log \lambda)^P$$

- All confining solutions have an IR singularity.

There are three types of solution for W:

- The "Good type" (single solution)

$$W(\lambda) \sim (\log \lambda)^{\frac{P}{2}} \ \lambda^{\frac{Q}{2}}$$

It leads to a "good" IR singularity, confinement, a mass gap, discrete spectrum of glueballs and screening of magnetic charges if

$$\frac{8}{3} > Q > \frac{4}{3} \quad or \quad Q = \frac{4}{3} \quad and \quad P > 0$$

.

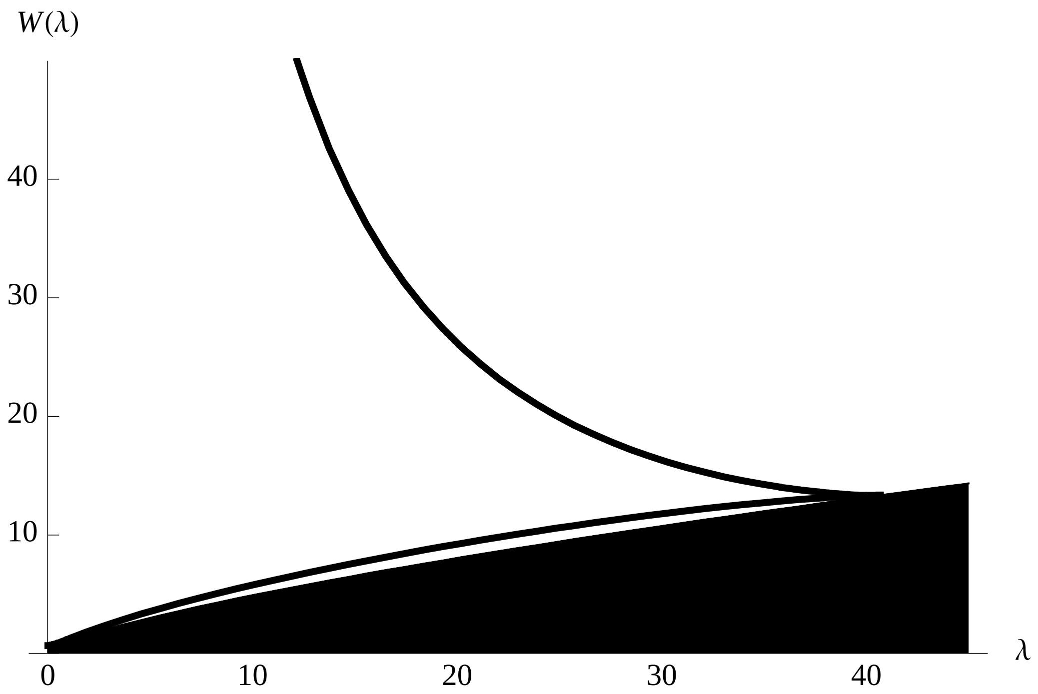
- The asymptotic spectrum of glueballs is linear if $Q = \frac{4}{3}$ and $P = \frac{1}{2}$.

- The Bad type. This is a one parameter family of solutions with

$$W(\lambda) \sim \lambda^{\frac{4}{3}}$$

It has a bad IR singularity.

♠ The Ugly type. This is a one parameter family of solutions. In such solutions there are two branches but they never reach the IR $\lambda \to \infty$. Instead λ goes back to zero



Selecting the IR asymptotics

The $Q = 4/3$, $0 \leq P < 1$ solutions have a singularity at $r = \infty$. They are compatible with

- Confinement (it happens non-trivially: a minimum in the string frame scale factor)
- Mass gap+discrete spectrum (except P=0)
- good singularity
- $R \to 0$ justifying the original assumption. More precisely: the string frame metric becomes flat at the IR .

♠ It is interesting that the lower endpoint: P=0 corresponds to linear dilaton and flat space (string frame). It is confining with a mass gap but continuous spectrum.

- For linear asymptotic trajectories for fluctuations (glueballs) we must choose $P = 1/2$

$$V(\lambda) =\sim \lambda^{\frac{4}{3}}\sqrt{\log\lambda} + \text{subleading} \qquad \text{as} \qquad \lambda \to \infty$$

Concrete potential

- The superpotential chosen is

$$W = (3+2b_0\lambda)^{2/3}\left[18+\left(2b_0^2+3b_1\right)\log(1+\lambda^2)\right]^{4/3},$$

with corresponding potential

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3+2b_0\lambda} - \frac{6(2b_0^2+3b_1^2)\lambda^3}{(1+\lambda^2)\left(18+\left(2b_0^2+3b_1^2\right)\log(1+\lambda^2)\right)}$$

which is everywhere regular and has the correct UV and IR asymptotics.

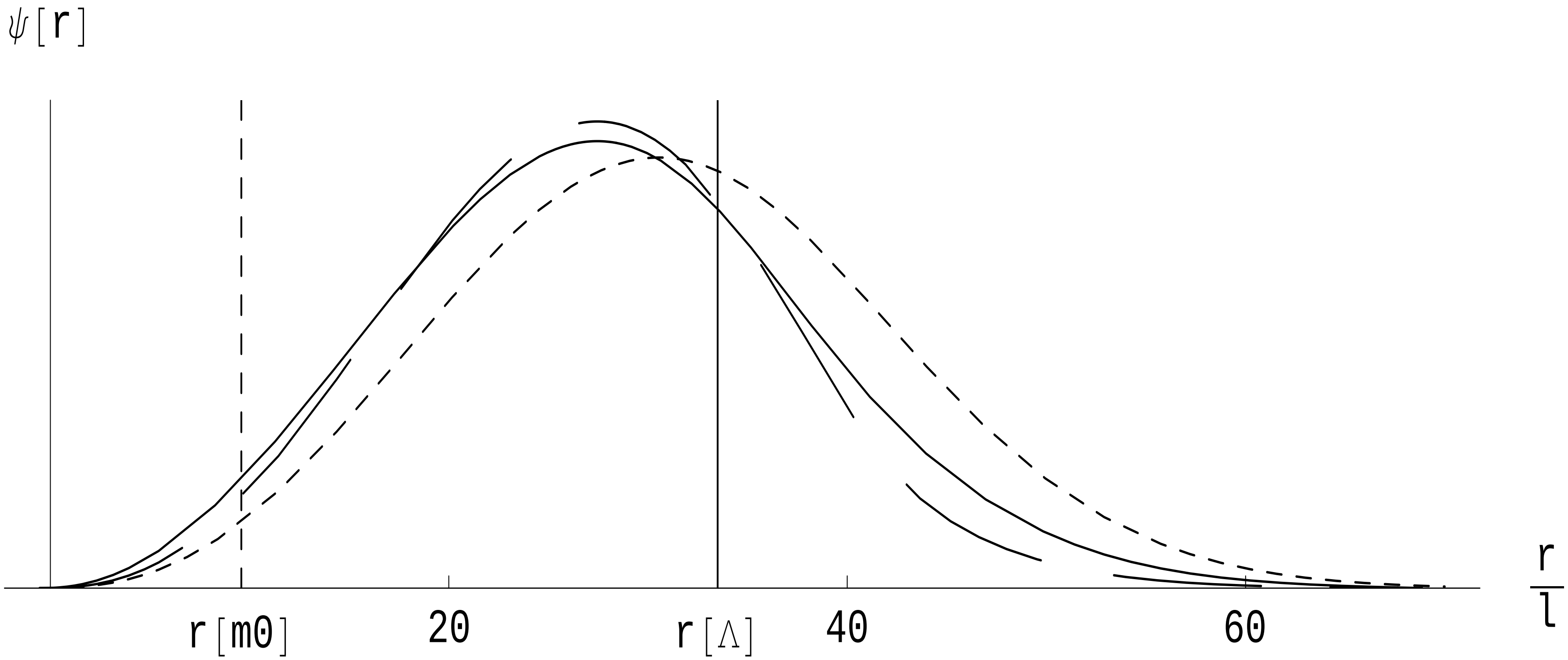
- b_0 is a free parameter and b_1/b_0^2 is taken from the QCD β-function

The fit to glueball lattice data

J^{PC}	Ref I (MeV)	Our model (MeV)	Mismatch	$N_c \to \infty$	Mismatch
0^{++}	**1475 (4%)**	**1475**	0	**1475**	0
2^{++}	2150 (5%)	2055	4%	2153 (10%)	5%
0^{-+}	**2250 (4%)**	**2243**	0		
0^{++*}	**2755 (4%)**	**2753**	0	2814 (12%)	2%
2^{++*}	2880 (5%)	2991	4%		
0^{-+*}	3370 (4%)	3288	2%		
0^{++**}	3370 (4%)	3561	5%		
0^{++***}	3990 (5%)	4253	6%		

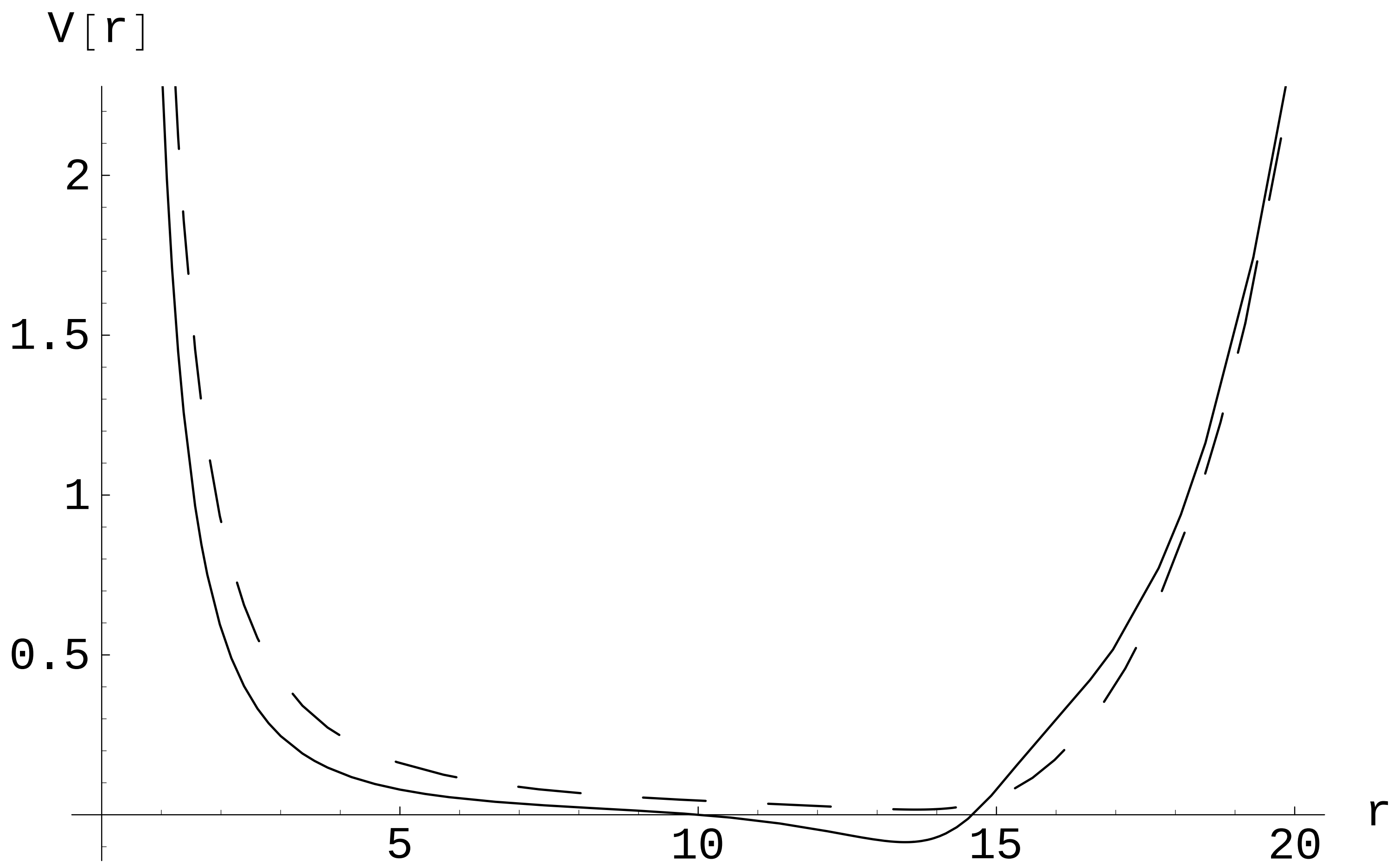
Comparison between the glueball spectra in Ref. I and in our model. The states we use as input in our fit are marked in red. The parenthesis in the lattice data indicate the percent accuracy.

The glueball wavefunctions



Normalized wave-function profiles for the ground states of the 0^{++} (solid line) ,0^{-+} (dashed line), and 2^{++} (dotted line) towers, as a function of the radial conformal coordinate. The vertical lines represent the position corresponding to $E = m_{0^{++}}$ and $E = \Lambda_p$.

Comparison of scalar and tensor potential



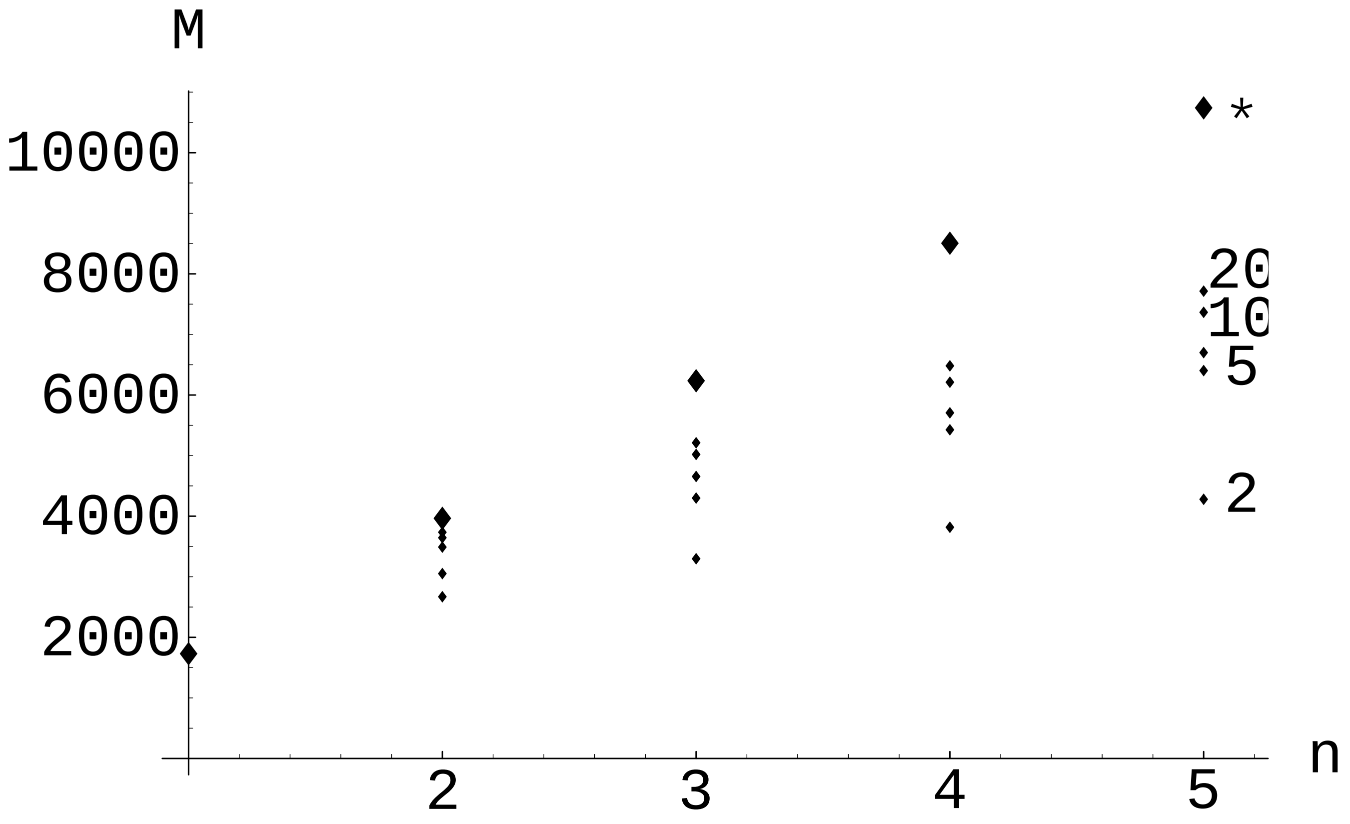
Effective Schrödinger potentials for scalar (solid line) and tensor (dashed line) glueballs. The units are chosen such that $\ell = 0.5$.

The lattice glueball data

J^{++}	Ref. I $(m/\sqrt{\sigma})$	Ref. I (MeV)	Ref. II (mr_0)	Ref. II (MeV)	$N_c \to \infty (m/\sqrt{\sigma})$
0	3.347(68)	1475(30)(65)	4.16(11)(4)	1710(50)(80)	3.37(15)
0^*	6.26(16)	2755(70)(120)	6.50(44)(7)	2670(180)(130)	6.43(50)
0^{**}	7.65(23)	3370(100)(150)	NA	NA	NA
0^{***}	9.06(49)	3990(210)(180)	NA	NA	NA
2	4.916(91)	2150(30)(100)	5.83(5)(6)	2390(30)(120)	4.93(30)
2^*	6.48(22)	2880(100)(130)	NA	NA	NA
R_{20}	1.46(5)	1.46(5)	1.40(5)	1.40(5)	1.46(11)
R_{00}	1.87(8)	1.87(8)	1.56(15)	1.56(15)	1.90(17)

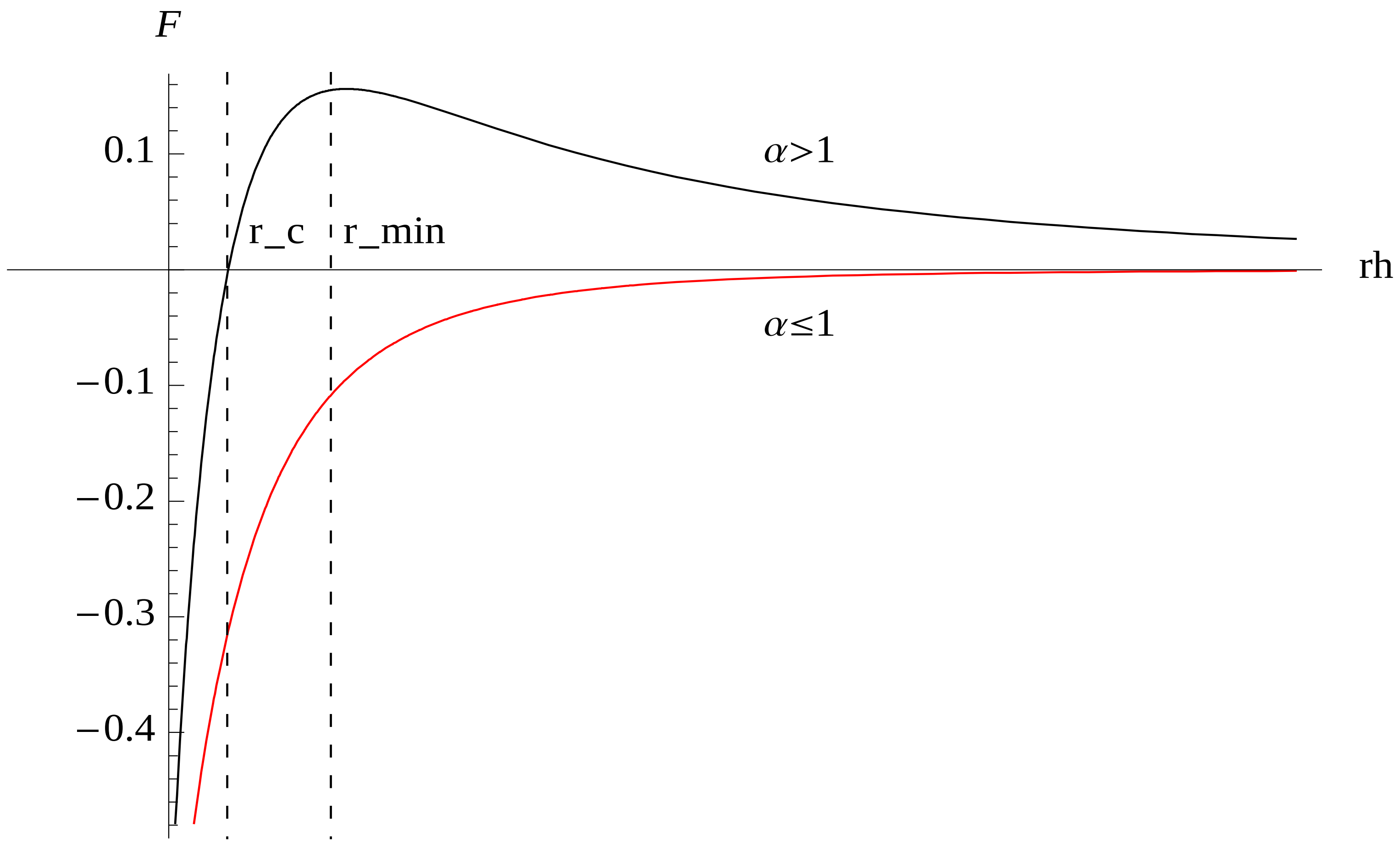
Available lattice data for the scalar and the tensor glueballs. Ref. I =H. B. Meyer, [arXiv:hep-lat/0508002]. and Ref. II = C. J. Morningstar and M. J. Peardon, [arXiv:hep-lat/9901004] + Y. Chen *et al.*, [arXiv:hep-lat/0510074]. The first error corresponds to the statistical error from the the continuum extrapolation. The second error in Ref.I is due to the uncertainty in the string tension $\sqrt{\sigma}$. (Note that this does not affect the mass ratios). The second error in the Ref. II is the estimated uncertainty from the anisotropy. In the last column we present the available large N_c estimates according to B. Lucini and M. Teper, [arXiv:hep-lat/0103027]. The parenthesis in this column shows the total possible error followed by the estimations in the same reference.

α-dependence of scalar spectrum



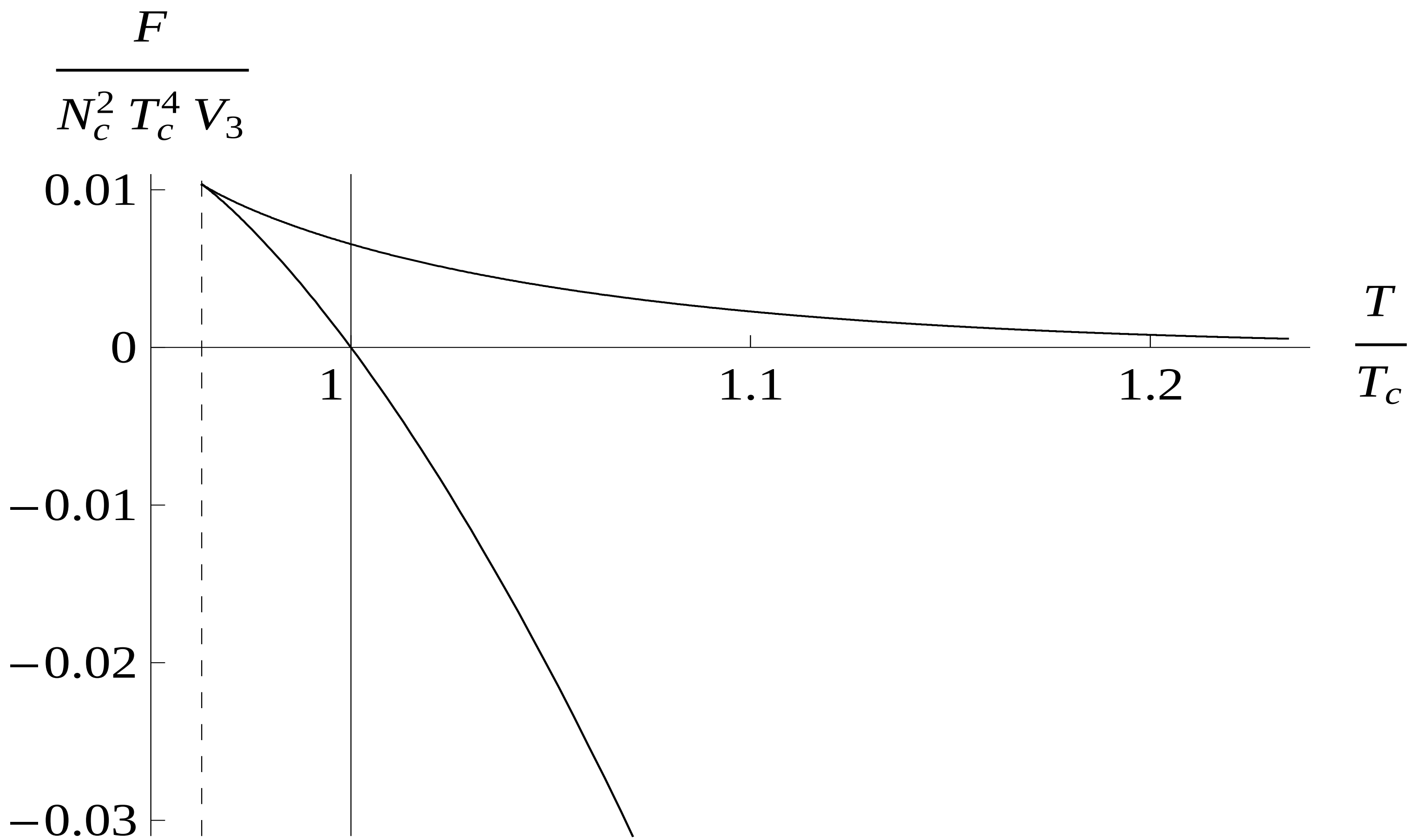
The 0^{++} spectra for varying values of α that are shown at the right end of the plot. The symbol $*$ denotes the AdS/QCD result.

Free energy versus horizon position



We plot the relation $\mathcal{F}(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is no first order phase transition .

The transition in the free energy



- G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lutgemeier and B. Petersson, *"Thermodynamics of SU(3) Lattice Gauge Theory,"* Nucl. Phys. B **469**, 419 (1996) [arXiv:hep-lat/9602007].

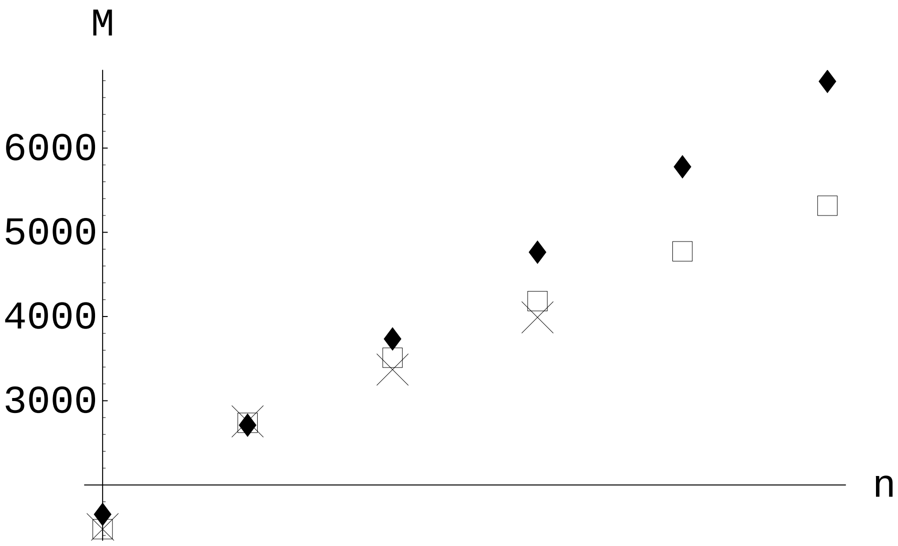
- B. Lucini, M. Teper and U. Wenger, *"Properties of the deconfining phase transition in SU(N) gauge theories,"* JHEP **0502**, 033 (2005) [arXiv:hep-lat/0502003];
"SU(N) gauge theories in four dimensions: Exploring the approach to $N = \infty$," JHEP **0106**, 050 (2001) [arXiv:hep-lat/0103027].

- Y. Chen *et al.*, *"Glueball spectrum and matrix elements on anisotropic lattices,"* Phys. Rev. D **73** (2006) 014516 [arXiv:hep-lat/0510074].

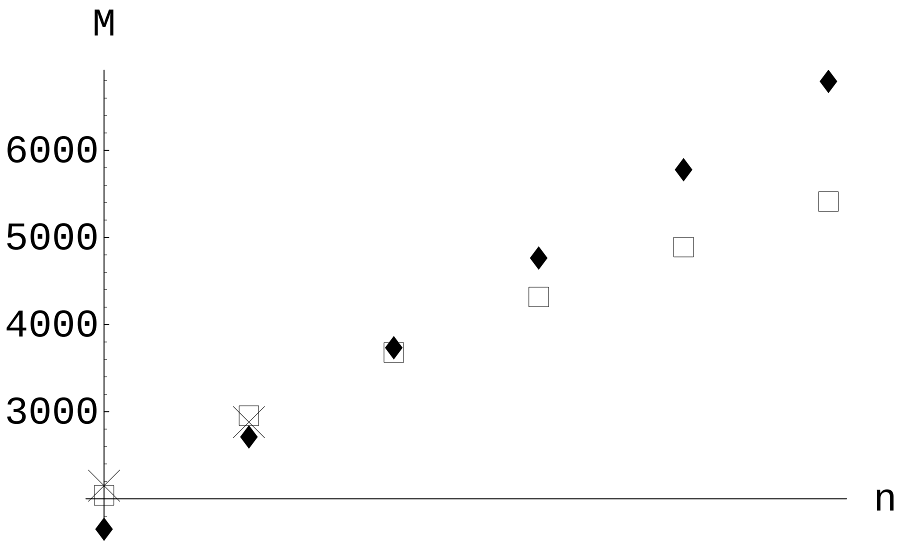
- L. Del Debbio, L. Giusti and C. Pica, *"Topological susceptibility in the SU(3) gauge theory,"* Phys. Rev. Lett. **94**, 032003 (2005) [arXiv:hep-th/0407052].

RETURN

Comparison with lattice data

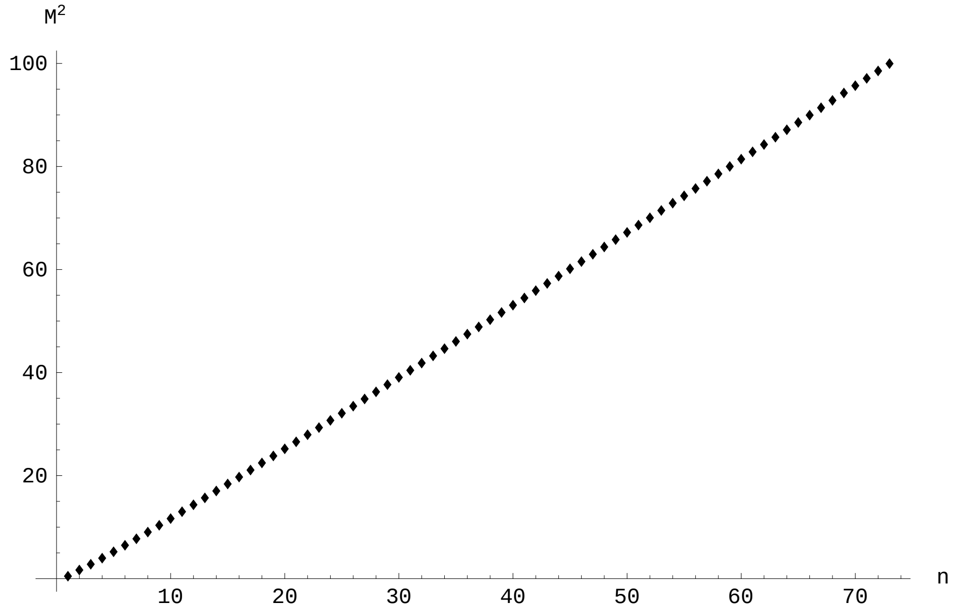


(a)

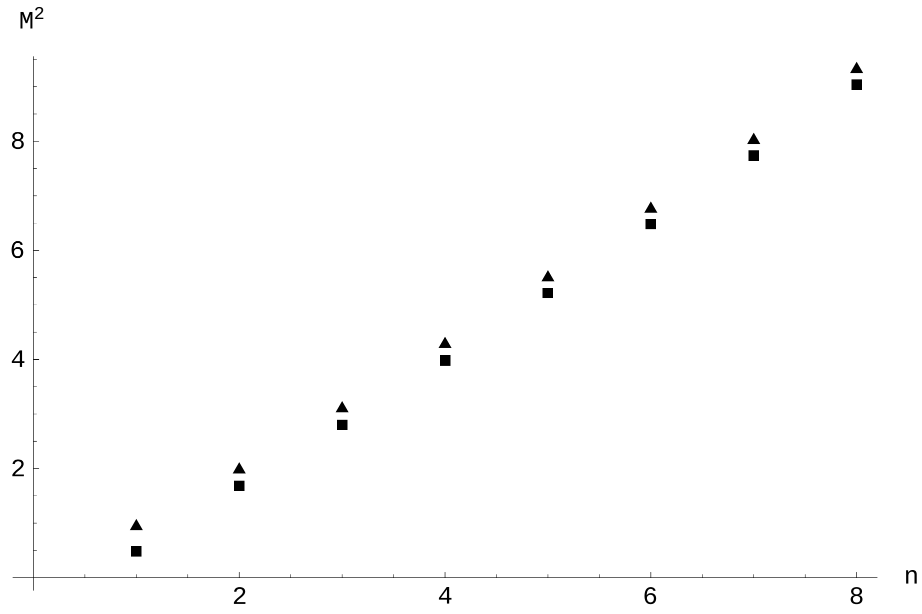


(b)

Linearity of the glueball spectrum



(a)

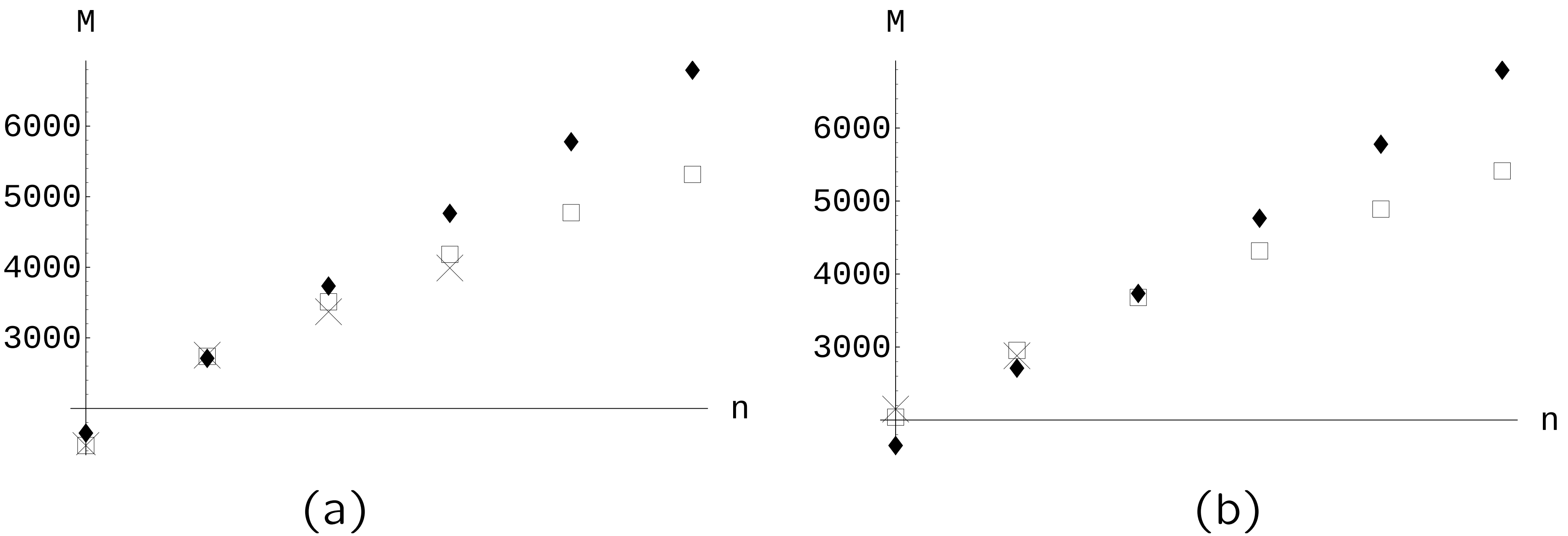


(b)

(a) Linear pattern in the spectrum for the first 40 0^{++} glueball states. M^2 is shown units of $0.015\ell^{-2}$.

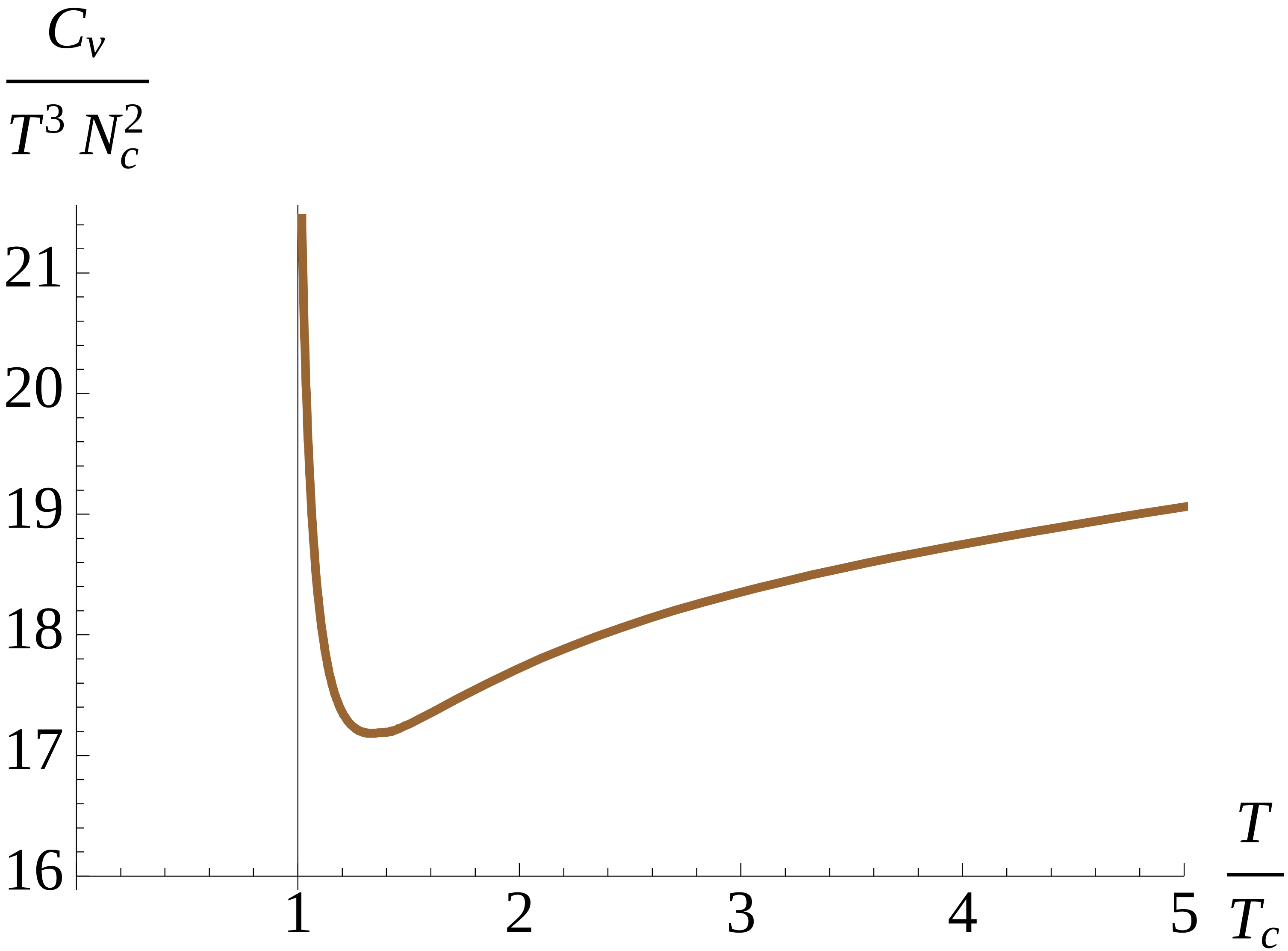
(b) The first 8 0^{++} (squares) and the 2^{++} (triangles) glueballs. These spectra are obtained in the background I with $b_0 = 4.2, \lambda_0 = 0.05$.

Comparison with lattice data (Meyer)



Comparison of glueball spectra from our model with $b_0 = 4.2, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. I (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. I.

The specific heat



The sum rule method (details)

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt\int d^3x\ e^{i\omega t}\langle[T_{ii}(\vec{x},t),T_{jj}(\vec{0},0)]\rangle$$

We use

$$\langle[\int d^3x T_{00}(\vec{x},0),O]\rangle_{equ} = \langle[H,O]\rangle_{equ} = i\langle\frac{\partial O}{\partial t}\rangle_{equ} = 0$$

and rewrite

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt\int d^3x\ e^{i\omega t}\langle[\Theta(\vec{x},t),\Theta(\vec{0},0)]\rangle \quad , \quad \Theta = T_\mu{}^\mu$$

$$\zeta = \lim_{\omega\to 0}\frac{1}{9\omega}\int dt\int d^3x\ e^{i\omega t}\ iG^R(x) = \lim_{\omega\to 0}\frac{1}{9\omega}\ iG^R(\omega) = -\lim_{\omega\to 0}\frac{1}{9\omega}Im\ G^R(\omega)$$

We now use

$$\Theta = m\bar{q}q + \frac{\beta(g)}{2g}Tr[F^2] = \Theta_F + \Theta_G$$

We also use

$$\langle[\Theta,O]\rangle = \left(T\frac{\partial}{\partial T} - d\right)\langle O\rangle$$

RETURN

Parameters

- We have 3 initial conditions in the system of graviton-dilaton equations:

♠ One is fixed by picking the branch that corresponds asymptotically to $\lambda \sim \frac{1}{\log(r\Lambda)}$

♠ The other fixes $\Lambda \to \Lambda_{QCD}$.

♠ The third is a gauge artifact as it corresponds to a choice of the origin of the radial coordinate.

- We parameterize the potential as

$$V(\lambda) = \frac{12}{\ell^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\left[\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)\right]^{1/2}\right\},$$

- We fix the one and two loop β-function coefficients:

$$V_0 = \frac{8}{9}b_0 \quad , \quad V_2 = b_0^4\left(\frac{23 + 36b_1/b_0^2}{81V_1^2}\right)^2, \quad \frac{b_1}{b_0^2} = \frac{51}{121}.$$

and remain with two leftover arbitrary (phenomenological) coefficients.

• We also have the Planck scale M_p

Asking for correct $T \to \infty$ thermodynamics (free gas) fixes

$$(M_p \ell)^3 = \frac{1}{45\pi^2} \quad , \quad M_{\text{physical}} = M_p N_c^{\frac{2}{3}} = \left(\frac{8}{45\pi^2 \ell^3}\right)^{\frac{1}{3}} \simeq 4.6 \text{ GeV}$$

• The fundamental string scale. It can be fixed by comparing with lattice string tension

$$\sigma = \frac{b^2(r_*)\lambda^{4/3}(r_*)}{2\pi\ell_s^2},$$

$\ell/\ell_s \sim \mathcal{O}(1)$.

• ℓ is not really a parameter as it can be rescaled into a redefinition of λ.

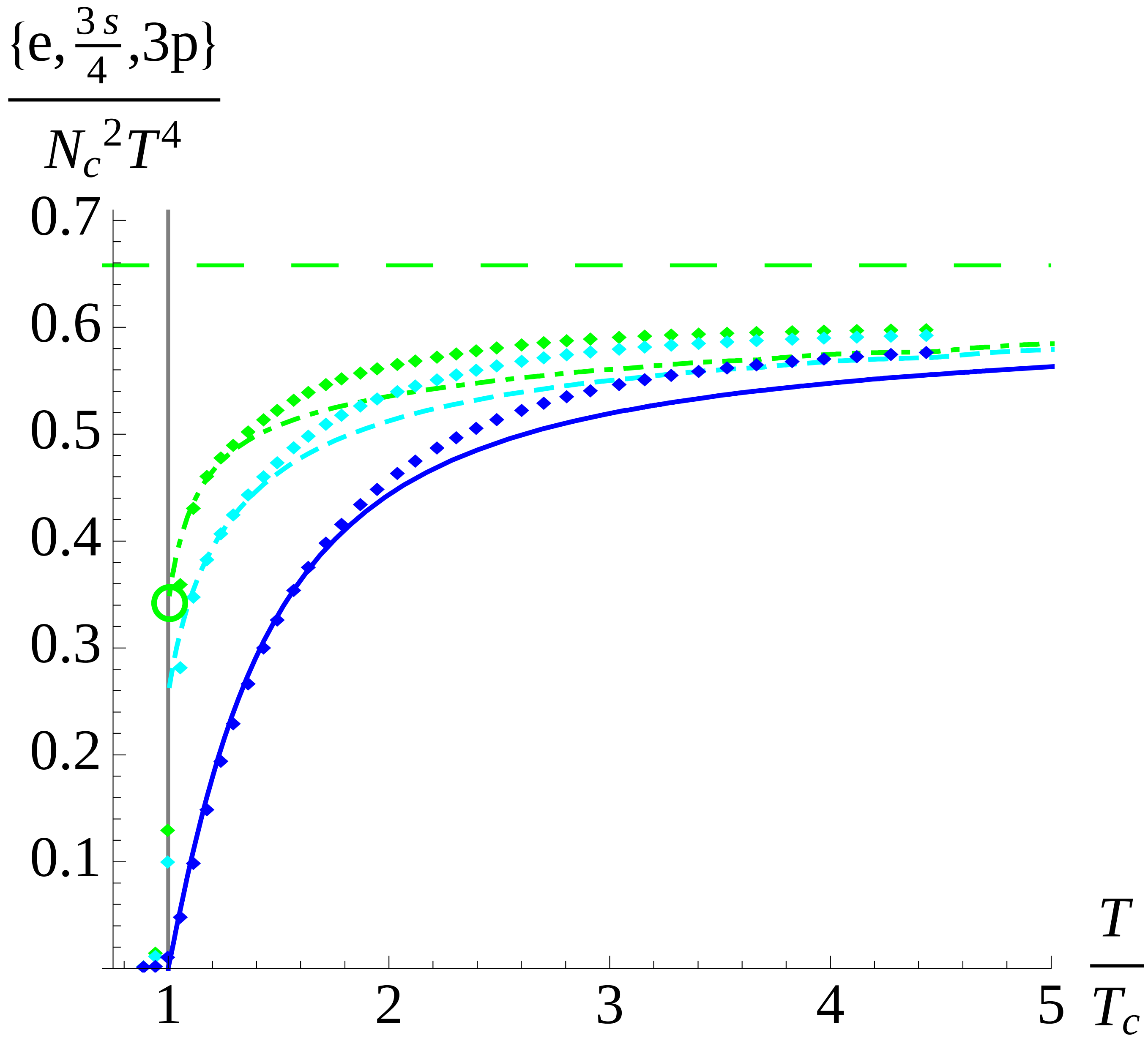
♠ In the CP-odd sector (axion) there are two more parameters:

$$Z(\lambda) = Z_0(1 + c_a\lambda^4)$$

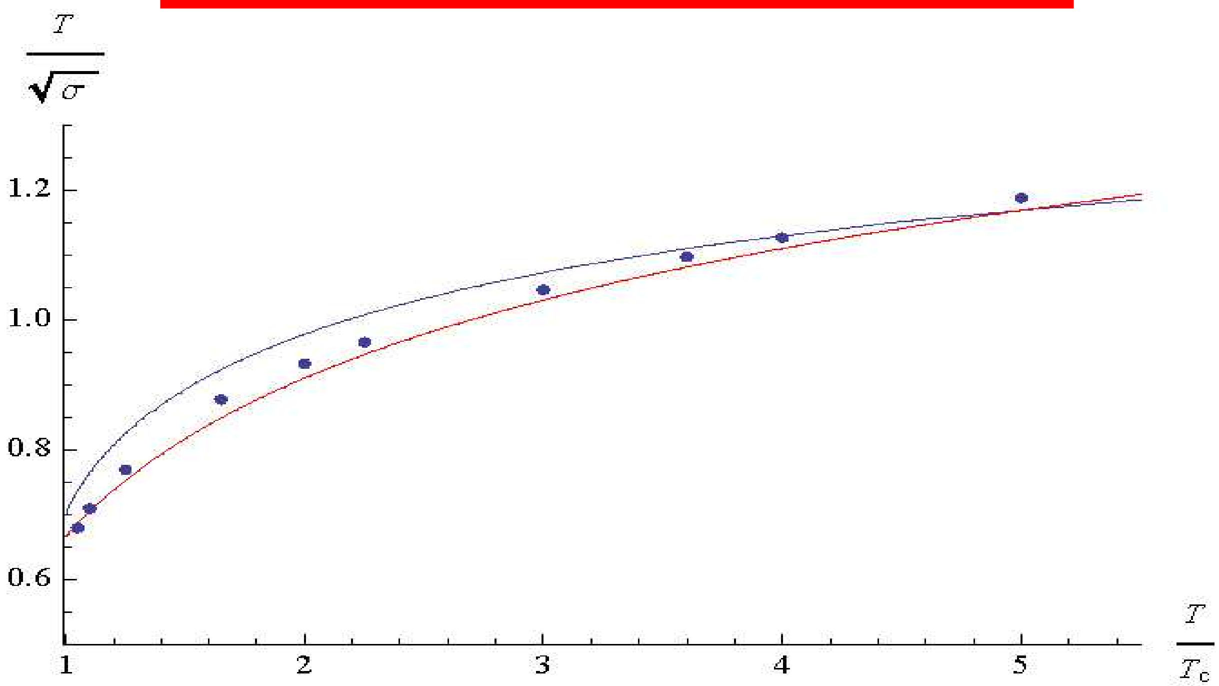
Fit and comparison

	IhQCD	lattice $N_c = 3$	lattice $N_c \to \infty$	Parameter
$[p/(N_c^2 T^4)]_{T=2T_c}$	**1.2**	**1.2**	-	$V1 = 14$
$L_h/(N_c^2 T_c^4)$	**0.31**	0.28 (Karsch)	**0.31** (Teper+Lucini)	$V3 = 170$
$[p/(N_c^2 T^4)]_{T\to+\infty}$	$\pi^2/45$	$\pi^2/45$	$\pi^2/45$	$M_p \ell = [45\pi^2]^{-1/3}$
$m_{0^{++}}/\sqrt{\sigma}$	**3.37**	3.56 (Chen)	**3.37** (Teper+Lucini)	$\ell_s/\ell = 0.92$
$m_{0^{-+}}/m_{0^{++}}$	**1.49**	**1.49** (Chen)	-	$c_a = 0.26$
χ	$(191 MeV)^4$	$(191 MeV)^4$ (DelDebbio)	-	$Z_0 = 133$
$T_c/m_{0^{++}}$	0.167	-	0.177(7)	
$m_{0^{*++}}/m_{0^{++}}$	1.61	1.56(11)	1.90(17)	
$m_{2^{++}}/m_{0^{++}}$	1.36	1.40(4)	1.46(11)	
$m_{0^{*-+}}/m_{0^{++}}$	2.10	2.12(10)	-	

Thermodynamic variables



Spatial string tension



G. Boyd et al. 1996

• The blue line is the spatial string tension as calculated in Improved hQCD, with no additional fits.

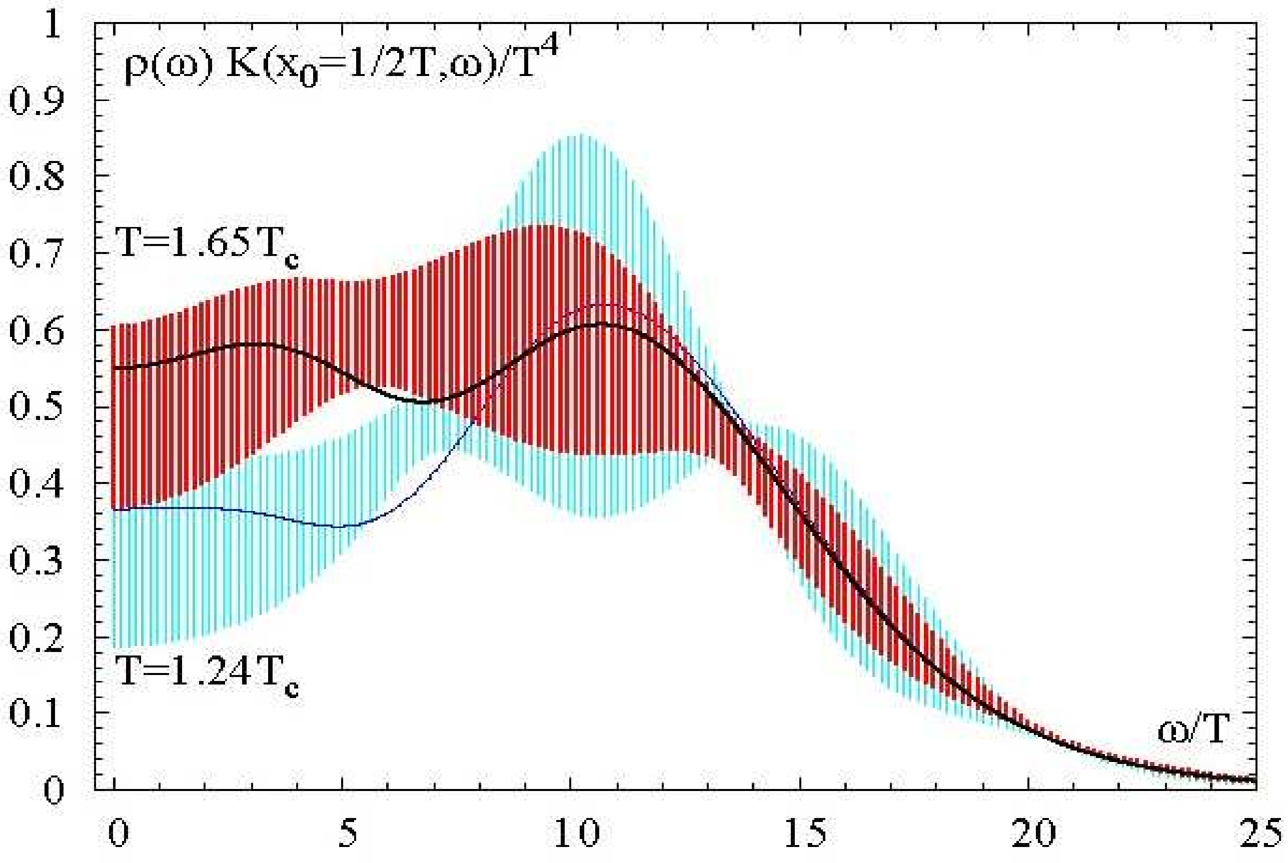
Nitti (unpublished) 2009

• The red line is a semi-phenomenological fit using

$$\frac{T}{\sqrt{\sigma_s}} = 0.51 \left[\log\frac{\pi T}{T_c} + \frac{51}{121}\log\left(2\log\frac{\pi T}{T_c}\right)\right]^{\frac{2}{3}}$$

Alanen+Kajantie+Suur-Uski, 2009

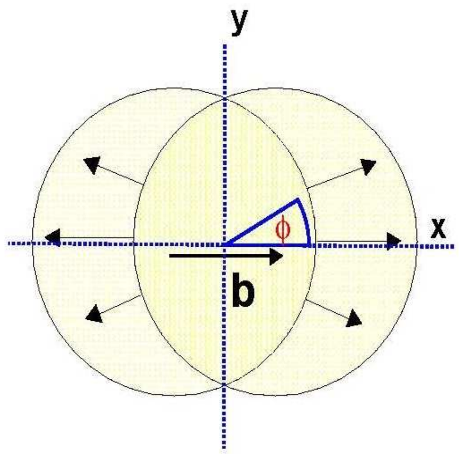
Shear Viscosity bounds from lattice



H. Meyer 2007

$$4\pi\frac{\eta}{s} = \begin{cases} 1.68(42), & T = 1.65\ T_c, \\ 1.28(70), & T = 1.24\ T_c. \end{cases}$$

Elliptic Flow



$$\frac{1}{p_T}\frac{dN}{dp_T d\phi} = \frac{1}{p_T}\frac{dN}{dp_T}\left[1 + v_2(p_T)\cos 2\phi + \cdots\right]$$

The sum rule method

• Define the (subtracted) spectral density and relate its moment to the Euclidean density

$$\rho(\omega) = -\frac{1}{\pi} Im\ G^R(\omega) \quad , \quad \mathcal{G} \equiv \lim_{\omega\to 0} G^E(\omega) = 2\int_0^\infty \frac{\rho(u)}{u}\ du$$

Karsch+Kharzeev+Tuchin, 2008, Romatschke+Son 2009

• Using Ward identities we obtain the sum rule

$$\mathcal{G} = \left(T\frac{\partial}{\partial T} - 4\right)(E - 3P + \langle\Theta\rangle_0) + \left(T\frac{\partial}{\partial T} - 2\right)(m\langle\bar{q}q\rangle_T + \langle\Theta_F\rangle_0)$$

with

$$\langle\Theta_F\rangle_0 = m\langle\bar{q}q\rangle \simeq -m_\pi^2\ f_\pi^2 - m_K^2\ f_K^2$$

• Assume a density

$$\frac{\rho(\omega)}{\omega} = \frac{9\zeta}{\pi}\frac{\omega_0{}^2}{\omega^2 + \omega_0{}^2}$$

The bulk viscosity: theory

- This is harder to calculate.

- Using a parametrization $ds^2 = e^{2A}(f dt^2 + d\vec{x}^2 + \frac{dr^2}{f})$ in a special gauge $\phi = r$ the relevant metric perturbation decouples

Gubser+Nellore+Pufu+Rocha 2008, Gubser+Pufu+Rocha,2008

$$h''_{11} = -\left(-\frac{1}{3A'} - A' - \frac{f'}{f}\right) h'_{11} + \left(-\frac{\omega^2}{f^2} + \frac{f'}{6fA'} - \frac{f'}{f}A'\right) h_{11}$$

with

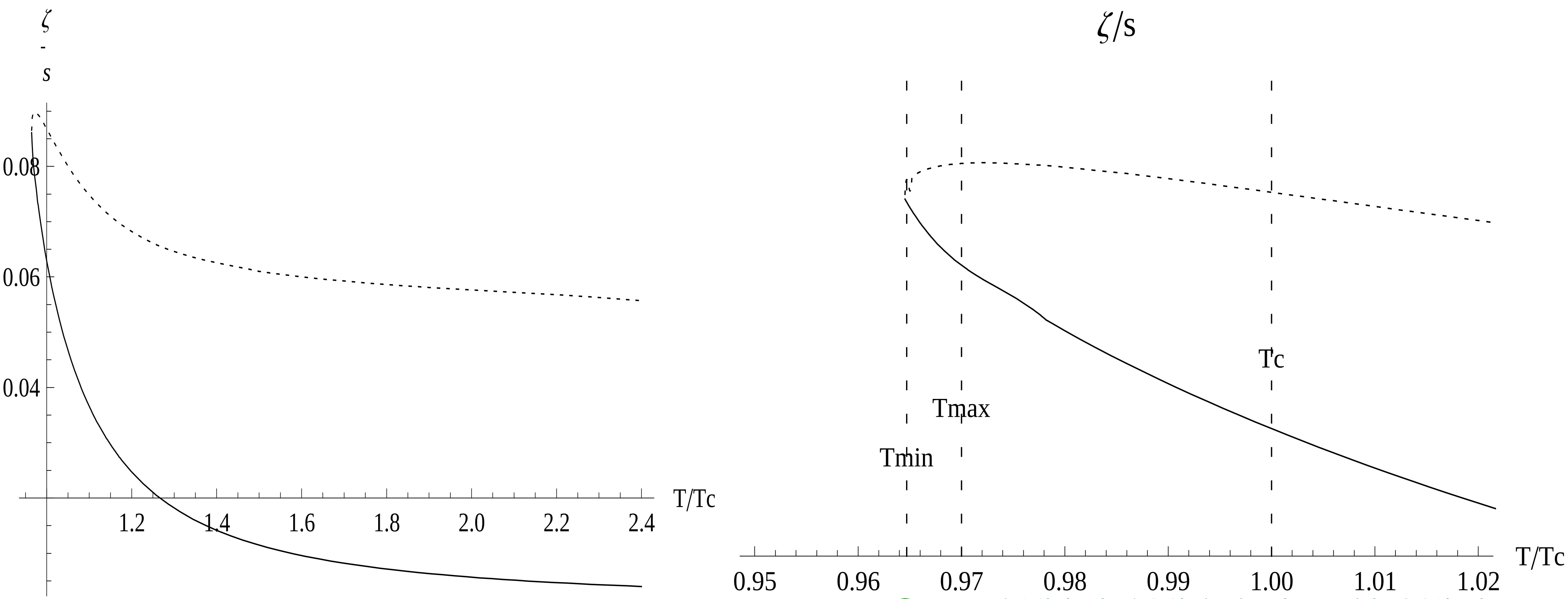
$$h_{11}(0) = 1 \quad , \quad h_{11}(r_h) \simeq C\ e^{i\omega t}\left|\log\frac{\lambda}{\lambda_h}\right|^{-\frac{i\omega}{4\pi T}}$$

The correlator is given by the conserved number of h-quanta

$$Im\ G_R(\omega) = -4M^3\mathcal{G}(\omega) \quad , \quad \mathcal{G}(\omega) = \frac{e^{3A}f}{4A'^2}|Im[h^*_{11}h'_{11}]|$$

$$\frac{\zeta}{s} = \frac{C^2}{4\pi}\left(\frac{V'(\lambda_h)}{V(\lambda_h)}\right)^2$$

The bulk viscosity in the small black hole



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

• At the turning point the behavior, $C_V \to \infty$ and ζ behaves similar to that observed in the N=2* theory

Buchel+Pagnutti, 2008

• The small black-hole bulk viscosity ratio asymptotes to a constant as $T \to \infty$.

High-T asymptotics of transport coefficients

• In CFTs perturbed with a relevant operator the speed of sound is bounded above as

$$c_s^2 = \frac{1}{3} - \frac{(4-\Delta)(4-2\Delta)\Gamma\left[\frac{\Delta}{4}\right]^4 \tan(\pi\Delta/4)}{18\pi\ \Gamma\left[\frac{\Delta}{2}-1\right]^2}(\pi\ell T)^{2(\Delta-4)} + \mathcal{O}\left(T^{3(\Delta-4)}\right)$$

Cherman+Cohen+Nellore 2009, Hohler+Stephanov, 2009

• The same is true $\left(c_s^2 \simeq \frac{1}{3} - \mathcal{O}\left(\frac{1}{\log^2 T}\right)\right)$ in the logarithmic case $\Delta = 4$.

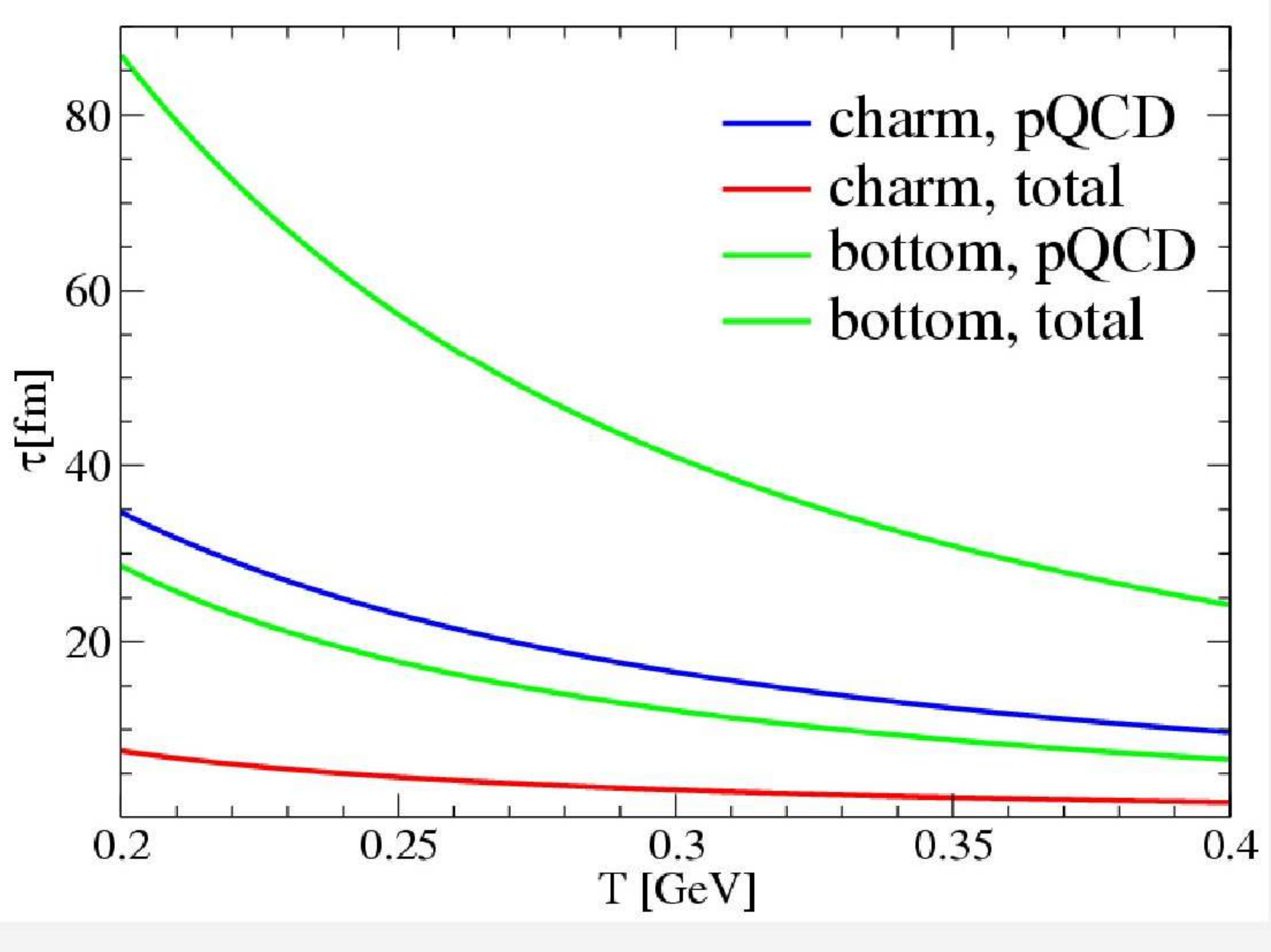
Gursoy+Kiritsis+Mazzanti+Nitti,2008

• In general for single relevant perturbations, if $\xi_i \in \left(\frac{\zeta}{s}, v_s^2, 2\pi T D, \frac{\sigma}{\pi T}, \frac{\Xi}{2\pi^2 T^2}\right)$ as $T \to \infty$

$$\xi_i(T) = \xi_i^{CFT} + \mathcal{C}_i(\Delta)\ T^{-2(4-\Delta)} + \mathcal{O}\left(T^{3(\Delta-4)}\right)$$

Cherman+Nellore 2009

Rapp results



• In qualitative agreement with Rapp et al. (talk at Quark Matter 2009) with a different method of calculation.

Langevin diffusion of heavy quarks (details)

• In a thermal medium we would expect the analogue of Brownian motion for heavy quarks.

• Fluctuations were first studied around the trailing string solution and diffusion coefficients were calculated.

Cassalderey-Solana+Teaney, 2006 Gubser 2006

• A full Langevin-like treatment was derived recently for non-relativistic quarks

Son+Teaney 2009 DeBoer+Hubeny+Rangamani+Shigenori, 2009

• This describes a Langevin process of the form

$$\frac{d\vec{p}}{dt} = \vec{F} + \vec{\xi} \quad , \quad \vec{F} = -\eta\ \vec{p} \quad , \quad \langle \xi^i(t)\xi^j(t')\rangle = \kappa\delta^{ij}\delta(t-t')$$

$\vec{F}$ is the drag force, $\eta = \frac{1}{\tau}$.

• The fully relativistic case was also described recently

Giecold+Iancu+Mueller, 2009

We consider fluctuations around the dragging string solution in the thermal background

$$ds^2 = b^2(r)\left(\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x}^2\right) \quad , \quad X^1 = vt + \xi(r) + \delta X^1 \quad , \quad X^{2,3} = \delta X^{2,3}$$

The Nambu-Goto action is expanded as

$$S = S_0 + S_1 + S_2 + \cdots \quad , \quad S_1 = \int d\tau dr \; P^\alpha \; \partial_\alpha \delta X^1 \tag{2}$$

with

$$S_2 = \frac{1}{2\pi\ell_s^2}\int d\tau dr \; \left[\frac{G^{\alpha\beta}}{2}\partial_\alpha \delta X^1 \partial_\beta \delta X^1 + \sum_{i=2}^{3} \frac{\tilde{G}^{\alpha\beta}}{2}\partial_\alpha \delta X^i \partial_\beta \delta X^i\right] \tag{3}$$

with

$$G^{\alpha\beta} = \frac{b^2(r)Z(r)^2}{2}\, g^{\alpha\beta} \quad , \quad \tilde{G}^{\alpha\beta} = \frac{b(r)^2}{2}\, g^{\alpha\beta} \quad , \quad Z(r) = \sqrt{1 + f(r)\xi'(r)^2 - \frac{v^2}{f(r)}}$$

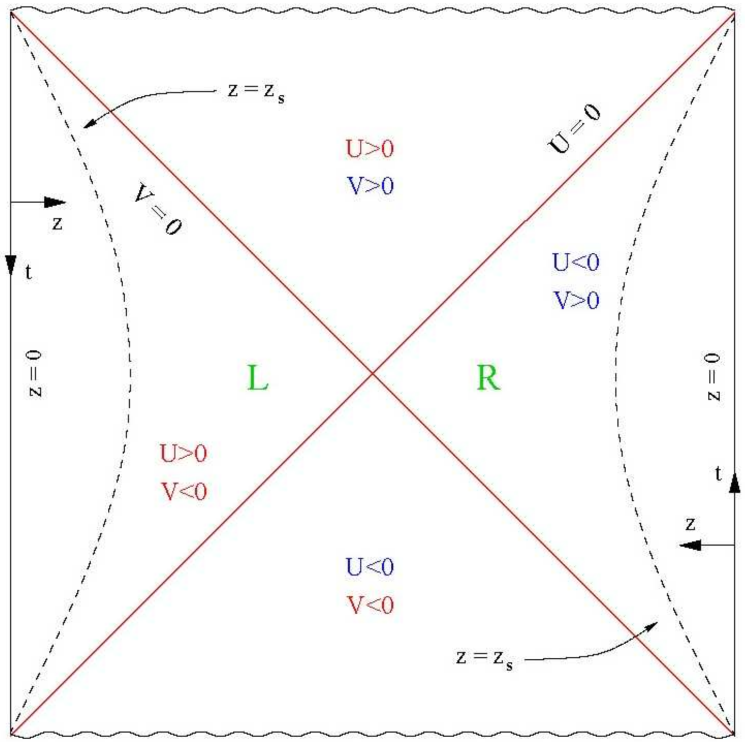
- The fluctuations δX^i satisfy.

$$\partial_\alpha G^{\alpha\beta}\partial_\beta \; \delta X^1 = 0 \quad , \quad \partial_\alpha \tilde{G}^{\alpha\beta}\partial_\beta \; \delta X^{2,3} = 0$$

• The metric in which they are evaluated is of the bh type, but with a different Hawking temperature, T_H. In the CFT case we have $T_H = \sqrt{1-v^2}\, T$

• We double the fields, $\delta X \to \delta X_{L,R}$ and we can define retarded and advanced correlators using the Schwinger-Keldysh formalism as implemented in AdS/CFT

Herzog+Son 2002, Gubser 2006, Skenderis+VanRees 2009



$$S_{\text{boundary}} = \int d\tau_R \left[-P^r \delta X^0_R + \frac{1}{2} \delta X^0_R G^{r\alpha} \partial_\alpha \delta X^0_R \right] - (L \leftrightarrow R)$$

$$= -\int \frac{d\omega}{2\pi}\ \delta X^0_a(-\omega) G^R(\omega) \delta X^0_r(\omega) + \frac{i}{2} \int \frac{d\omega}{2\pi}\ \delta X^0_a(-\omega) G^{sym}(\omega) \delta X^0_a(\omega)$$

with

$$\delta X_r = \frac{1}{2}(\delta X_L + \delta X_R) \quad , \quad \delta X_a = (\delta X_L - \delta X_R)$$

and

$$G_{sym}(\omega) = \frac{1 + e^{\frac{\omega}{T_H}}}{1 - e^{\frac{\omega}{T_H}}}\ G_R(\omega)$$

- We may derive a Langevin equation by starting with

$$Z = \int [D\delta X^0_{L,R}][D\delta X_{L,R}]\ e^{i(S_R - S_L)} = \int [D\delta X^0_{a,r}]\ e^{iS_{\text{boundary}}}$$

and introduce a dummy variable ξ to linearize the quadratic term of the a-fields

$$Z = \int [D\delta X^0_{a,r}][D\xi] \; e^{-\frac{1}{2}\int dtdt'\xi(t)G^{-1}_{sym}(t,t')\xi(t')} \times$$

$$\times \exp\left[-i\int dtdt' \;\delta X^0_a \left[G_R(t,t')\delta X^0_r(t') + \delta(t-t')(P^r - \xi(t'))\right]\right]$$

Integration over $\delta X^0_{a,r}$ gives the Langevin system

$$\int dt' \; G_R(t,t')\delta X^0_r(t') + P^r - \xi(t) = 0 \quad , \quad \langle \xi(t)\xi(t')\rangle = G_{\mathrm{sym}}(t,t')$$

Giecold+Iancu+Mueller, 2009

• For $|t-t'|$ large we can replace the retarded propagator with a (second) time derivative and the symmetric one by a δ-function to finally obtain in the conformal case

$$\frac{dp^i_\perp}{dt} = -\eta \; p^i_\perp + \xi^i_\perp \quad , \quad \langle \xi^i_\perp(t)\xi^j_\perp(t')\rangle = \kappa_\perp \delta^{ij}\delta(t-t') \quad , \quad \kappa_\perp = \frac{\pi\sqrt{\lambda}T^3}{(1-v^2)^{\frac{1}{4}}}$$

$$\frac{dp_\parallel}{dt} = -\eta \; p_\parallel + \xi_\parallel \quad , \quad \langle \xi_\parallel(t)\xi_\parallel(t')\rangle = \kappa_\parallel \delta(t-t') \quad , \quad \kappa_\parallel = \frac{\pi\sqrt{\lambda}T^3}{(1-v^2)^{\frac{5}{4}}}$$

• In the non-relativistic limit the world-sheet horizon and the spacetime horizon coincide. In this case there is a Maxwell equilibrium distribution and the Einstein relation ($\kappa = 2ET\eta$) holds.

Cassalderey-Solana+Teaney, 2006 Gubser 2006

Son+Teaney 2009 DeBoer+Hubeny+Rangamani+Shigenori, 2009

• The diffusion is asymmetric in the relativistic case. There is no thermal equilibrium distribution. This resolves previous puzzles of symmetric relativistic Langevin diffusion.

• The failure of the Einstein relation was also seen in the heavy-ion data.

Wolchin 1999

• The (conformal) relativistic Langevin equation with symmetric diffusion was applied to data analysis at RHIC, but the Einstein relation was kept.

Akamatsu+Hatsuda+Hirano, 2008

In view of the above a re-analysis seems necessary.

The sum rule method (details)

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt\int d^3x\ e^{i\omega t}\langle[T_{ii}(\vec{x},t),T_{jj}(\vec{0},0)]\rangle$$

We use

$$\langle[\int d^3x T_{00}(\vec{x},0),O]\rangle_{equ} = \langle[H,O]\rangle_{equ} = i\langle\frac{\partial O}{\partial t}\rangle_{equ} = 0$$

and rewrite

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt\int d^3x\ e^{i\omega t}\langle[\Theta(\vec{x},t),\Theta(\vec{0},0)]\rangle \quad , \quad \Theta = T_\mu{}^\mu$$

$$\zeta = \lim_{\omega\to 0}\frac{1}{9\omega}\int dt\int d^3x\ e^{i\omega t}\ iG^R(x) = \lim_{\omega\to 0}\frac{1}{9\omega}\ iG^R(\omega) = -\lim_{\omega\to 0}\frac{1}{9\omega}Im\ G^R(\omega)$$

We now use

$$\Theta = m\bar{q}q + \frac{\beta(g)}{2g}Tr[F^2] = \Theta_F + \Theta_G$$

We also use

$$\langle[\Theta,O]\rangle = \left(T\frac{\partial}{\partial T} - d\right)\langle O\rangle$$

RETURN

Diffusion times in different schemes (more)

T_{QGP}, MeV	τ_{charm} (fm/c) (direct)	τ_{charm} (fm/c) (energy)	τ_{charm} (fm/c) (entropy)
220	-	3.96	3.64
250	5.67	3.14	2.96
280	4.27	2.56	2.47
310	3.45	2.12	2.08
340	2.88	1.80	1.78
370	2.45	1.54	1.53
400	2.11	1.33	1.34

The diffusion times for the charm quark are shown for different temperatures, in the three different schemes. Diffusion times have been evaluated with a quark initial momentum fixed at $p \approx 10\ GeV$.

Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

.

$T_{QGP}(MeV)$	τ_{bottom} (fm/c) (direct)	τ_{bottom} (fm/c) (energy)	τ_{bottom} (fm/c) (entropy)
220	-	8.90	8.36
250	11.39	7.46	7.12
280	10.11	6.32	6.14
310	8.62	5.40	5.32
340	7.50	4.70	4.65
370	6.63	4.10	4.09
400	5.78	3.61	3.63

Diffusion times for the bottom quark are shown for different temperatures, in the three different schemes. Diffusion times have been evaluated with a quark initial momentum fixed at $p \approx 10\ GeV$.

Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

$\widehat{q}$ at different schemes

T_{QGP}, MeV	$\hat{q}$ (GeV^2/fm) (direct)	$\hat{q}$ (GeV^2/fm) (energy)	$\hat{q}$ (GeV^2/fm) (entropy)
220	-	0.89	1.01
250	0.53	1.21	1.32
280	0.79	1.64	1.73
310	1.07	2.14	2.21
340	1.39	2.73	2.77
370	1.76	3.37	3.42
400	2.18	4.20	4.15

- $\widehat{q}$ computed in the three different comparison "schemes", No cutoff

T_{QGP}, MeV	$\hat{q}_{charm}$ (GeV^2/fm) (direct)	$\hat{q}_{charm}$ (GeV^2/fm) (energy)	$\hat{q}_{charm}$ (GeV^2/fm) (entropy)
220	-	1.34	1.53
250	0.78	1.86	2.04
280	1.17	2.59	2.75
310	1.65	3.50	3.63
340	2.18	4.61	4.70
370	2.81	5.88	5.98
400	3.56	7.63	7.51

• $\hat{q}$ computed in the three different comparison "schemes" with a cutoff at the mass of the charm.

.

T_{QGP}, MeV	$\hat{q}_{bottom}$ (GeV^2/fm) (direct)	$\hat{q}_{bottom}$ (GeV^2/fm) (energy)	$\hat{q}_{bottom}$ (GeV^2/fm) (entropy)
220	-	1.01	1.14
250	0.59	1.37	1.50
280	0.88	1.87	1.98
310	1.23	2.47	2.56
340	1.59	3.18	3.23
370	2.02	3.95	4.00
400	2.51	4.97	4.90

• $\hat{q}$ computed in the three different comparison "schemes" with a cutoff at the mass of the bottom. This has little effect as it is much higher than the temperatures involved.

Diffusion times in different schemes

T_{QGP}, MeV	τ_{charm} (fm/c) (direct)	τ_{charm} (fm/c) (energy)	τ_{charm} (fm/c) (entropy)
220	-	3.96	3.64
250	5.67	3.14	2.96
280	4.27	2.56	2.47
310	3.45	2.12	2.08
340	2.88	1.80	1.78
370	2.45	1.54	1.53
400	2.11	1.33	1.34

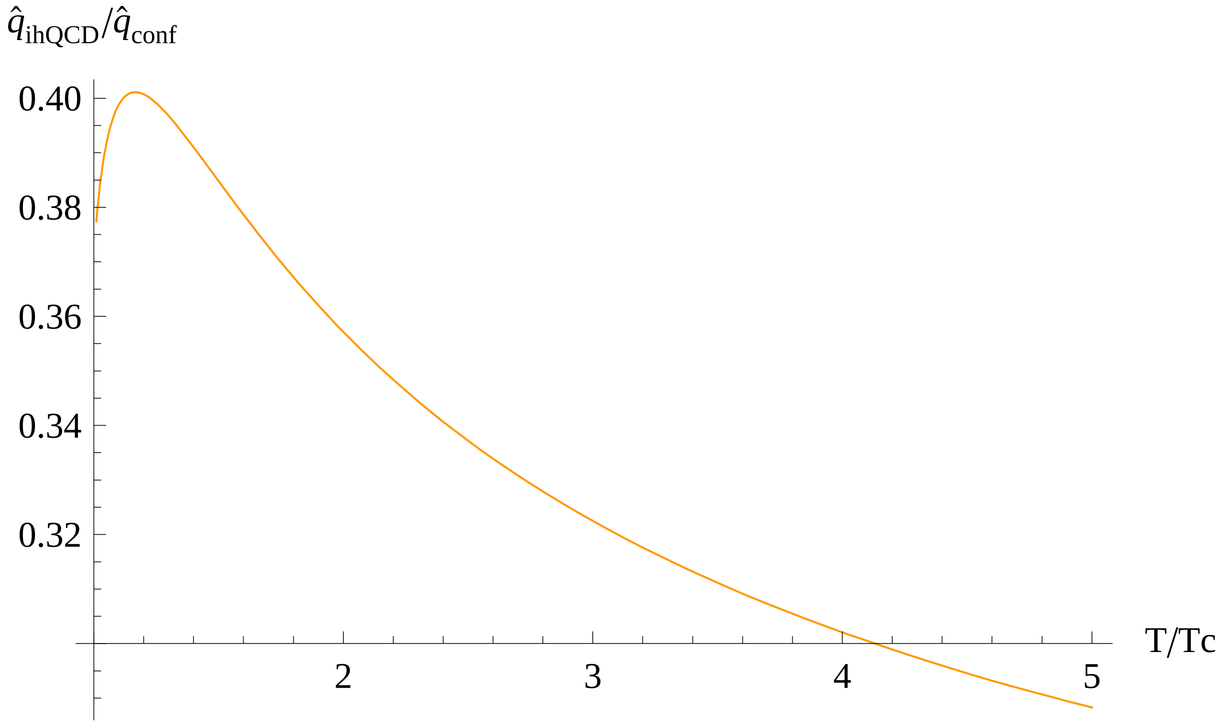
The diffusion times for the charm quark are shown for different temperatures, in the three different schemes. Diffusion times have been evaluated with a quark initial momentum fixed at $p \approx 10\ GeV$.

Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

The jet-quenching parameter

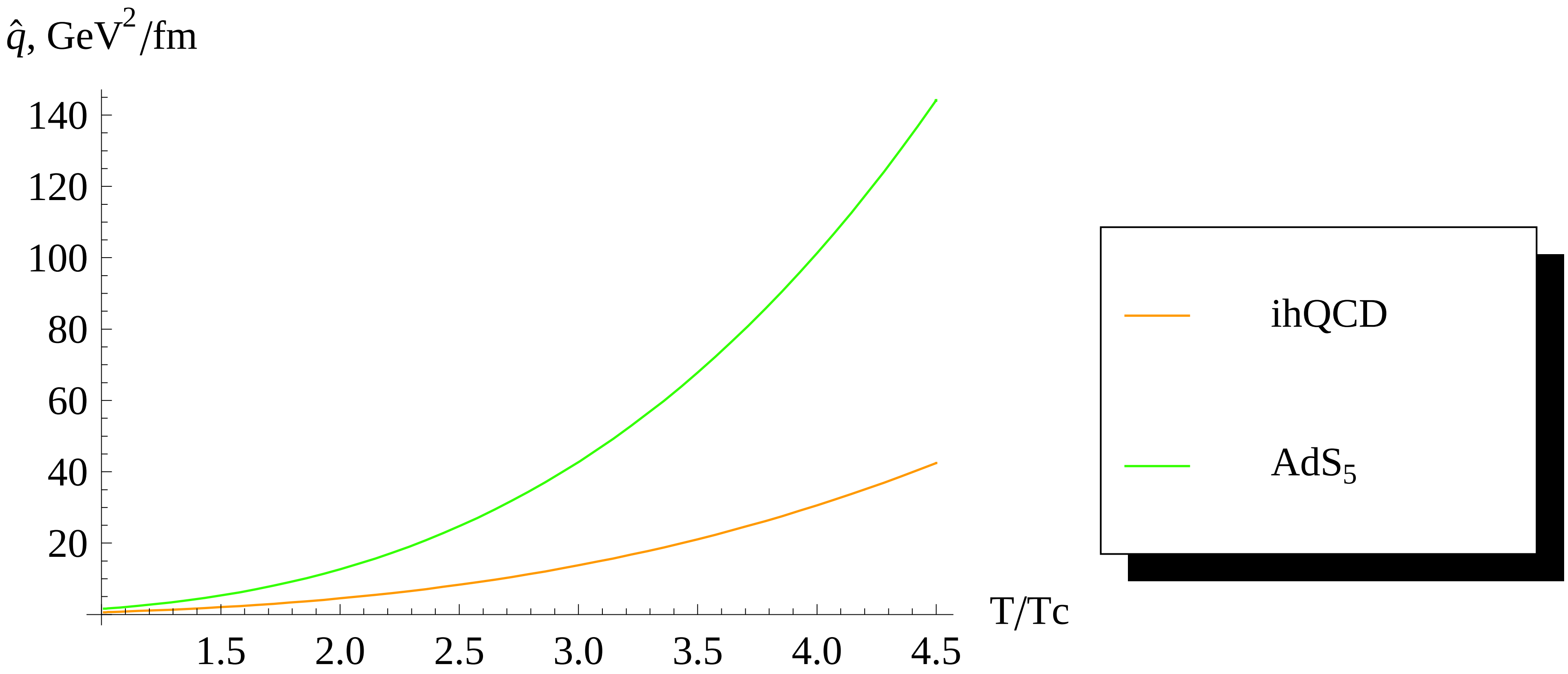
- $\hat{q} \sim \frac{d}{dt} < p^2_{transverse} >$ From the light-like Wilson loop

Liu+Rajagopal+Wiedemann



$$\hat{q}_{\text{conformal}} = \frac{\Gamma\left[\frac{3}{4}\right]}{\Gamma\left[\frac{5}{4}\right]} \sqrt{2\lambda}\ \pi^{\frac{3}{2}} T^3 \quad , \quad \lambda = 5.5$$

.


$\hat{q}$, GeV^2/fm
140
120
100
80
60
40
20
1.5
2.0
2.5
3.0
3.5
4.0
4.5
T/Tc
ihQCD
AdS$_5$

Langevin diffusion of heavy quarks

- So far we have described a heavy quark with a classical equation

$$\frac{d\vec{p}}{dt} = \vec{F}_{drag} \simeq \frac{1}{\tau_{dif}} \vec{p}$$

- In a thermal medium we would expect the analogue of Brownian motion for heavy quarks.

- Fluctuations were first studied around the trailing string solution and diffusion coefficients were calculated.

Cassalderey-Solana+Teaney, 2006 Gubser 2006

- A full Langevin-like treatment was derived recently for non-relativistic quarks

Son+Teaney 2009 DeBoer+Hubeny+Rangamani+Shigenori, 2009

- This describes a Langevin process of the form

$$\frac{d\vec{p}}{dt} = \vec{F} + \vec{\xi} \quad , \quad \vec{F} = -\eta\, \vec{p} \quad , \quad \langle \xi^i(t) \xi^j(t') \rangle = \kappa \delta^{ij} \delta(t - t')$$

$\vec{F}$ is the drag force, $\eta = \frac{1}{\tau}$.

- The fully relativistic case was also described recently

Giecold+Iancu+Mueller, 2009

We consider fluctuations around the dragging string solution in the thermal background

$$ds^2 = b^2(r)\left(\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x}^2\right) \quad , \quad X^1 = vt + \xi(r) + \delta X^1 \quad , \quad X^{2,3} = \delta X^{2,3}$$

The Nambu-Goto action is expanded as

$$S = S_0 + S_1 + S_2 + \cdots \quad , \quad S_1 = \int d\tau dr \; P^\alpha \; \partial_\alpha \delta X^1 \tag{4}$$

with

$$S_2 = \frac{1}{2\pi\ell_s^2} \int d\tau dr \; \left[\frac{G^{\alpha\beta}}{2} \partial_\alpha \delta X^1 \partial_\beta \delta X^1 + \sum_{i=2}^{3} \frac{\tilde{G}^{\alpha\beta}}{2} \partial_\alpha \delta X^i \partial_\beta \delta X^i\right] \tag{5}$$

with

$$G^{\alpha\beta} = \frac{b^2(r)Z(r)^2}{2} g^{\alpha\beta} \quad , \quad \tilde{G}^{\alpha\beta} = \frac{b(r)^2}{2} g^{\alpha\beta} \quad , \quad Z(r) = \sqrt{1 + f(r)\xi'(r)^2 - \frac{v^2}{f(r)}}$$

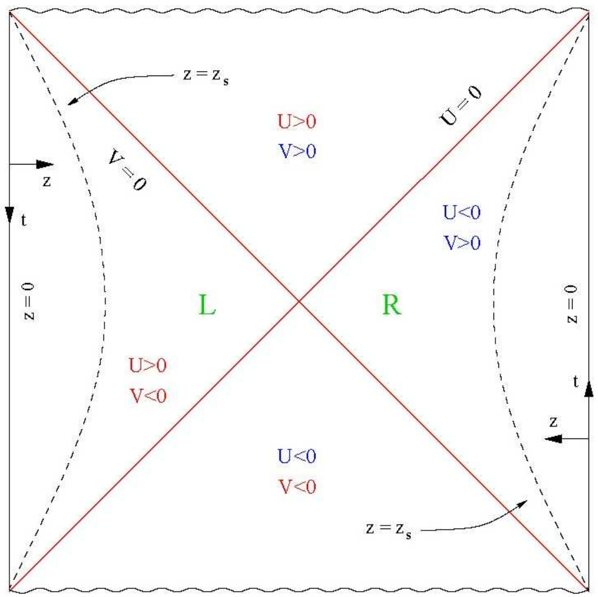
• The fluctuations δX^i satisfy.

$$\partial_\alpha G^{\alpha\beta}\partial_\beta\ \delta X^1 = 0 \quad , \quad \partial_\alpha \tilde{G}^{\alpha\beta}\partial_\beta\ \delta X^{2,3} = 0$$

• The metric in which they are evaluated is of the bh type, but with a different Hawking temperature, T_H. In the CFT case we have $T_H = \sqrt{1-v^2}\ T$

• We double the fields, $\delta X \to \delta X_{L,R}$ and we can define retarded and advanced correlators using the Schwinger-Keldysh formalism as implemented in AdS/CFT

Herzog+Son 2002, Gubser 2006, Skenderis+VanRees 2009



$$S_{\text{boundary}} = \int d\tau_R \left[-P^r \delta X^0_R + \frac{1}{2} \delta X^0_R G^{r\alpha} \partial_\alpha \delta X^0_R \right] - (L \leftrightarrow R)$$

$$= -\int \frac{d\omega}{2\pi}\ \delta X^0_a(-\omega) G^R(\omega) \delta X^0_r(\omega) + \frac{i}{2} \int \frac{d\omega}{2\pi}\ \delta X^0_a(-\omega) G^{sym}(\omega) \delta X^0_a(\omega)$$

with

$$\delta X_r = \frac{1}{2}(\delta X_L + \delta X_R) \quad , \quad \delta X_a = (\delta X_L - \delta X_R)$$

and

$$G_{sym}(\omega) = \frac{1 + e^{\frac{\omega}{T_H}}}{1 - e^{\frac{\omega}{T_H}}}\ G_R(\omega)$$

- We may derive a Langevin equation by starting with

$$Z = \int [D\delta X^0_{L,R}][D\delta X_{L,R}]\ e^{i(S_R - S_L)} = \int [D\delta X^0_{a,r}]\ e^{iS_{\text{boundary}}}$$

and introduce a dummy variable ξ to linearize the quadratic term of the a-fields

$$Z = \int [D\delta X^0_{a,r}][D\xi]\ e^{-\frac{1}{2}\int dtdt'\xi(t)G^{-1}_{sym}(t,t')\xi(t')} \times$$

$$\times \exp\left[-i\int dtdt'\ \delta X^0_a \left[G_R(t,t')\delta X^0_r(t') + \delta(t-t')(P^r - \xi(t'))\right]\right]$$

Integration over $\delta X^0_{a,r}$ gives the Langevin system

$$\int dt'\ G_R(t,t')\delta X^0_r(t') + P^r - \xi(t) = 0 \quad , \quad \langle \xi(t)\xi(t')\rangle = G_{\text{sym}}(t,t')$$

Giecold+Iancu+Mueller, 2009

• For $|t-t'|$ large we can replace the retarded propagator with a (second) time derivative and the symmetric one by a δ-function to finally obtain in the conformal case

$$\frac{dp^i_\perp}{dt} = -\eta\ p^i_\perp + \xi^i_\perp \quad , \quad \langle \xi^i_\perp(t)\xi^j_\perp(t')\rangle = \kappa_\perp \delta^{ij}\delta(t-t') \quad , \quad \kappa_\perp = \frac{\pi\sqrt{\lambda}T^3}{(1-v^2)^{\frac{1}{4}}}$$

$$\frac{dp_\parallel}{dt} = -\eta\ p_\parallel + \xi_\parallel \quad , \quad \langle \xi_\parallel(t)\xi_\parallel(t')\rangle = \kappa_\parallel \delta(t-t') \quad , \quad \kappa_\parallel = \frac{\pi\sqrt{\lambda}T^3}{(1-v^2)^{\frac{5}{4}}}$$

• In the non-relativistic limit the world-sheet horizon and the spacetime horizon coincide. In this case there is a Maxwell equilibrium distribution and the Einstein relation ($\kappa = 2ET\eta$) holds.

Cassalderey-Solana+Teaney, 2006 Gubser 2006
Son+Teaney 2009 DeBoer+Hubeny+Rangamani+Shigenori, 2009

• The diffusion is asymmetric in the relativistic case.

• The Maxwell distribution is not an equilibrium distribution of this system! This resolves previous puzzles of symmetric relativistic Langevin diffusion.

• Curiously, the failure of the Einstein relation was also seen in the heavy-ion data.

Wolchin 1999

• The (conformal) relativistic Langevin equation with symmetric diffusion was applied to data analysis at RHIC, but the Einstein relation was kept.

Akamatsu+Hatsuda+Hirano, 2008

In view of the above a reanalysis is necessary.

• The non-conformal correlators G_R are under study

Detailed plan of the presentation

- Title page 1 minutes
- Collaborators 2 minutes
- Introduction 3 minutes
- AdS/QCD 5 minutes
- The “soft wall” 7 minutes
- Improved Holographic QCD 10 minutes
- The UV region 13 minutes
- The IR asymptotics 16 minutes

FINITE TEMPERATURE

- The general phase structure 17 minutes
- Temperature versus horizon position 19 minutes
- The pressure 20 minutes
- The entropy 21 minutes

- Bulk viscosity: theory 150 minutes
- The bulk viscosity in the small black hole 151 minutes
- High-T asymptotics of transport coefficients 152 minutes
- Rapp results 153 minutes
- Langevin diffusion of heavy quarks (details) 161 minutes
- The sum rule method (details) 163 minutes
- Diffusion times in different schemes 166 minutes
- $\hat{q}$ in different schemes 168 minutes
- The diffusion time in different schemes 170 minutes
- The jet-quenching parameter 172 minutes
- Langevin diffusion of heavy quarks 184 minutes