Institut d'Ete, ENS
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Thermodynamics and transport in QCD-like holographic theories

Elias Kiritsis

University of Crete (on leave from APC, Paris)

Collaborators

My Collaborators

- Umut Gursoy (Utrecht)
- Liuba Mazzanti (Ecole Polytechnique→ Santiago de Compostella)
- George Michalogiorgakis (Ecole Polytechnique → Purdue)
- Fransesco Nitti (APC, Paris)

Plan of the presentation

- Introduction
- 5D Einstein dilaton gravity with a potential as an approximation to holographic YM
- Comparison with YM Thermodynamics from lattice
- Viscosity
- Drag Force and jet-quenching parameters.
- Langevin dynamics of heavy quarks in gluon plasma.
- Outlook

Introduction

• QCD is strongly coupled in the IR. We have no analytical tools to solve the theory at strong coupling

• Lattice numerical techniques can be used to proceed in this direction but they have limitations, both conceptual and practical

• Every experimental datum of the theory contains at least a part that depends on strong coupling physics.

• Even "clean" high energy experiments depend on initial state data (structure functions) that are measured and final state data (hadronization) that is a hybrid of theory+phenomenological models+data fitting.

• Phenomenological models have been key in understanding aspects of strong coupling dynamics in the confined phase of QCD (T=0) : chiral models, instanton gas, Swinger-Dyson equations, string slinkys (Lund Monte Carlo) etc .

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• In the last ten years we have a large flux of data, some of them from the deconfined phase of QCD.

• Although signals of collective behavior were seen at CERN, the most convincing set of data emerged from RHIC experiments.

• They broadly suggest that during a heavy ion collision, thermalization occurs almost instantaneously, and the plasma formed is strongly coupled.

• This suggests that there is a dynamic evolution in the deconfined case and strong coupling $\Rightarrow$ well beyond the reach of lattice techniques.

Introduction, II

What can we use to derive the QCD physics at $T > T_c$ and in a changing medium?

• Lattice techniques are conceptually unable as they are Euclidean (but practical approaches exist).

• Perturbative QCD techniques are not reliable because we are at strong coupling

• Phenomenological techniques based on hybrid, hadronic+pQCD techniques have similarly "large systematic errors"

♠ Can experimental results be a substitute for the theory? Can we use them to find out what is the correct modeling?

• Not all the way: there are many uncertainties in transforming the raw data into a dynamical model of evolution of the QGP:

♠ The initial state and its interactions are not fully understood (Glauber vs CGC)

♠ The Thermalization mechanism is not well understood.

♠ The evolution of the "thermalized plasma" is subject to diffusion and entropy generation.

♠ The hadronization mechanism has uncertainties.

All generate systematics in our understanding, that add up quadratically

Therefore: any other theoretical technique for computing in QCD in the appropriate regime should be welcome.

The large-N_c expansion in QCD

• The generalization of QCD to N_c colors, has an extra parameter: the theory simplifies in a sense when $N_c \to \infty$.

t 'Hooft 1974

• It has the structure of a string theory, with $g_s \sim \frac{1}{N_c}$. When $N_c = \infty$ the theory contains an infinite number of particles with finite masses and no interactions. The "string" is the "flux tube" of confined color flux that binds quarks and glue together.

• Therefore, at $N_c = \infty$ the theory is "free".

• The particles are color singlets (glueballs, mesons and baryons).

• It is therefore a good starting point for a perturbative expansion in $\frac{1}{N_c}$.

• There is always the usual coupling constant: $\lambda \equiv a_s N_c$.

• it turns out that $N_c = 3$ is not that far from $N_c = \infty$

Alas, even the leading order in QCD (classical at large N_c) is not easy to compute.

• If $\lambda << 1$ we compute in perturbation theory

• This is not the case in QCD at low energy.

♠ ENTER: the AdS/CFT correspondence $\rightarrow$

N=4 sYM being equivalent to IIB strings on $AdS_5 \times S^5$.

Holography and QCD

• AdS/CFT has provided so far controlled/computable examples of confinement, chiral symmetry breaking and hadron spectra of concrete gauge theories, but:

♠ Uncontrolable (soft) string dynamics in the UV if we incorporate asymptotic freedom

♠ The KK problem in non-critical examples with IR QCD-like physics

• Several semi-realistic models were developed:

(a)The Witten Black-D4/M5 model. Rather easy to compute with: its IR physics is QCD-like, but is higher-dimensional in UV.

Witten 1998

(b) The Sakai-Sugimoto Model. Build on the D4-background. Implements a geometrical version of chiral symmetry breaking. It is rather good for mesons, and baryons. It lacks several meson modes and explicit quark masses due to higher-d structure.

Sakai+Sugimoto 2004

• Several phenomenological models (bottom-up) have been developed also with a varying degree of success.

♠ Bottom-up= Write a gravity+else theory, without worrying if it is embedable in a string theory.

(c) AdS/QCD: AdS_5 with a IR cutoff.

Polchinski+Strassler 2001

Some qualitatively correct properties. Best as a background for meson physics

Erlich+Katz+Son+Stephanov 2005, DaRold+Pomarol 2005

(d) Soft wall AdS/QCD. It is reasonably good for mesons but useless (inconsistent) for glue.

Karch+Katz+Son+Stephanov 2006

(e) Improved Holographic QCD. Gravity plus dilaton with a potential.

Gursoy+Kiritsis 2007 , Gursoy+Kiritsis+Nitti 2007 , Gursoy+Kiritsis+Mazzanti+Nitti 2008
Gubser+Nellore 2008 , De Wolfe+Rosen 2009 , Nellore+Chermam 2009

Improved Holographic QCD

• We will write down a model that captures the holographic behavior of SU(N_c) YM in four dimensions, and in particular the breaking of conformal invariance:

• The basic fields will be $g_{\mu\nu}, \phi$. We can neglect the rest

• The action is

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right] \quad , \qquad \lambda = N_c \, e^{\phi}$$

• The β-function $\beta(\lambda)$ is in one-to-one relation to the dilaton potential.

• At $\lambda \to 0$ the potential is dictated by YM perturbation theory. For $\lambda \to \infty$ it is dictated by general principles of confinement etc. $V(\lambda) \sim \lambda^{\frac{4}{3}} \sqrt{\log \lambda}$

• It provides confinement, a mass gap, and a discrete spectrum of glueballs.

General phase structure

• For a general monotonic potential (with no minimum) the following are true :

i. *There exists a phase transition at finite $T = T_c$, if and only if the zero-T theory confines.*

ii. *This transition is first order for all of the confining geometries, with a single exception (linear dilaton in the IR, continuous spectrum with a gap)*

iv. *All of the non-confining geometries at zero T are always in the black hole phase at finite T. They exhibit a second order phase transition at $T = 0^+$.*

Finite-T Confining Theories

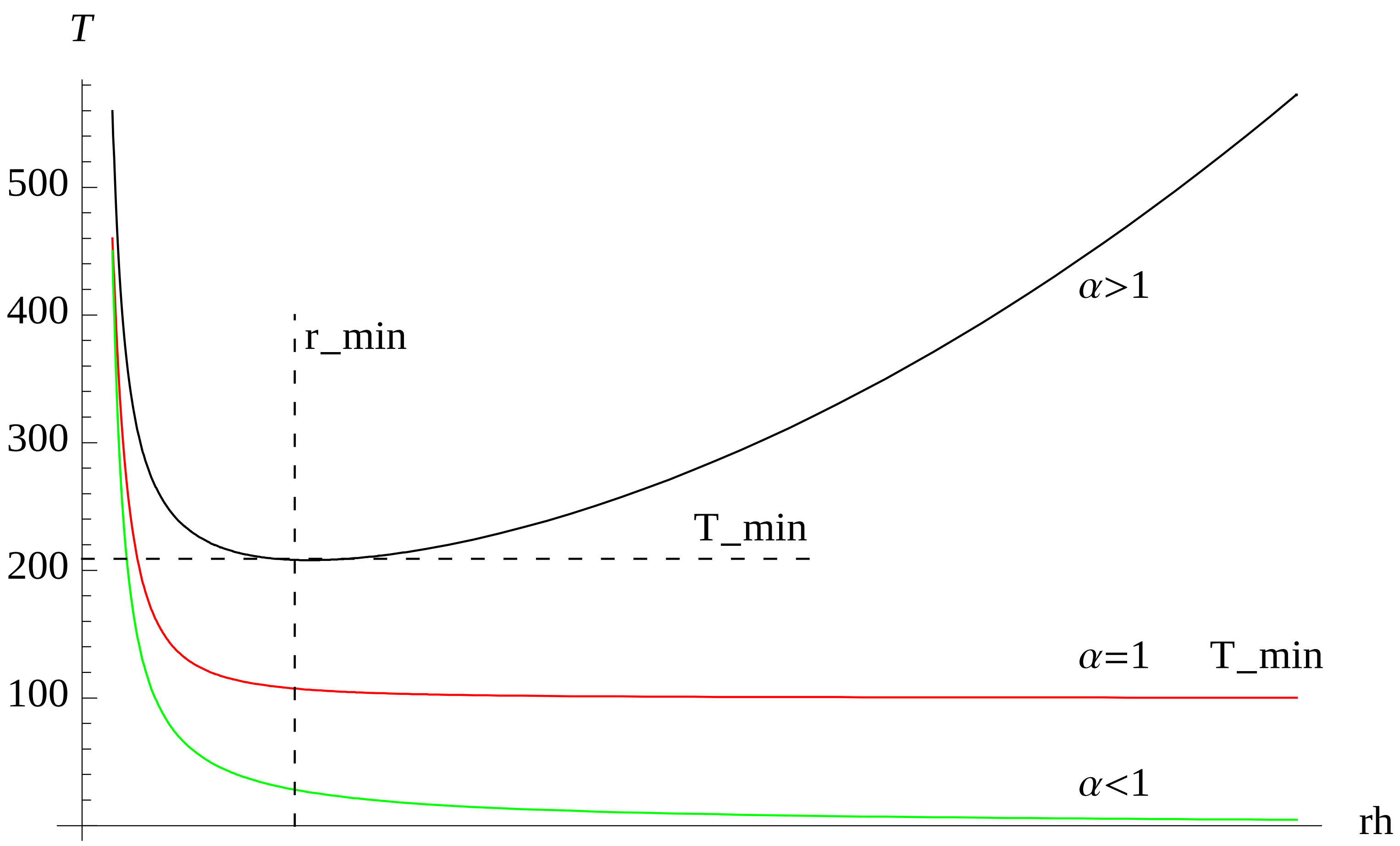
• There is a minimal temperature T_{min} for the existence of Black-hole solutions

• When $T < T_{\rm min}$ only the "thermal vacuum solution" exists: it describes the confined phase at small temperatures.

• For $T > T_{\rm min}$ there are two black-hole solutions with the same temperature but different horizon positions. One is a "large" BH, the other is "small".

• Therefore for $T > T_{\rm min}$ three competing solutions exist. The large BH has the lowest free energy for $T > T_c > T_{min}$. It describes the deconfined "Glasma" phase.

Temperature versus horizon position



We plot the relation $T(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is only one branch of black-hole solutions.

The free energy

• The free energy is calculated from the action as a boundary term for both the black-holes and the thermal vacuum solution. They are all UV divergent but their differences are finite.

$$\frac{\mathcal{F}}{M_p^3 V_3} = 12\mathcal{G}(T) - T\ S(T)$$

• $\mathcal{G}$ is the temperature-depended gluon condensate $\langle Tr[F^2]\rangle_T - \langle Tr[F^2]\rangle_{T=0}$ defined as

$$\lim_{r\to 0}\quad \lambda_T(r) - \lambda_{T=0}(r) = \mathcal{G}(T)\ r^4 + \cdots$$

• It is $\mathcal{G}$ the breaks conformal invariance essentially and leads to a non-trivial deconfining transition (as $S > 0$ always)

• The axion solution must be constant above the phase transition (black-hole). Therefore $\langle F \wedge F\rangle$ vanishes.

The pressure from the lattice at different N

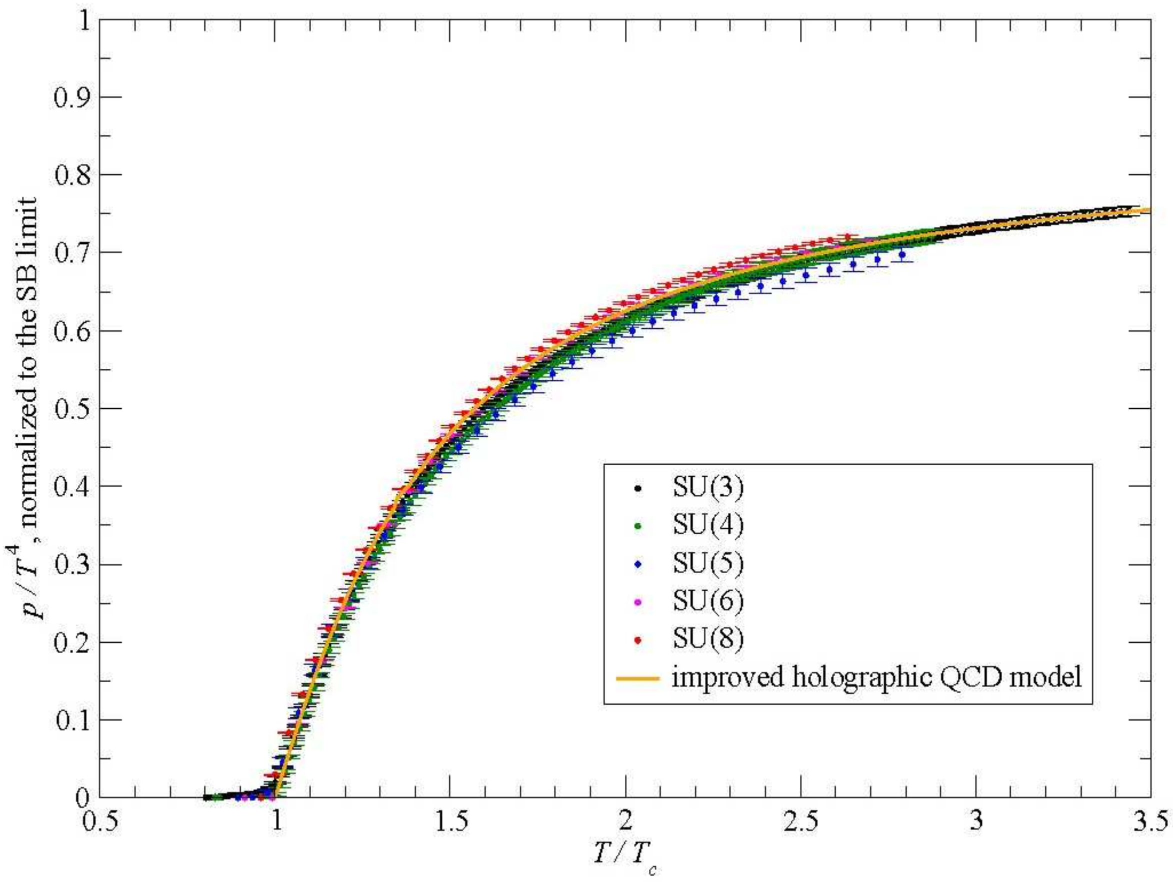


Figure 1: (Color online) The dimensionless ratio p/T^4, normalized to the lattice SB limit $\pi^2(N^2-1)R_I(N_t)/45$, *versus* T/T_c, as obtained from simulations of SU(N) lattice gauge theories on $N_t = 5$ lattices. Errorbars denote statistical uncertainties only. The results corresponding to different gauge groups are denoted by different colors, according to the legend. The yellow solid line denotes the prediction from the improved holographic QCD model from ref. [75] (with a trivial, parameter-free rescaling to our normalization).

Marco Panero arXiv: 0907.3719

The entropy from the lattice at different N

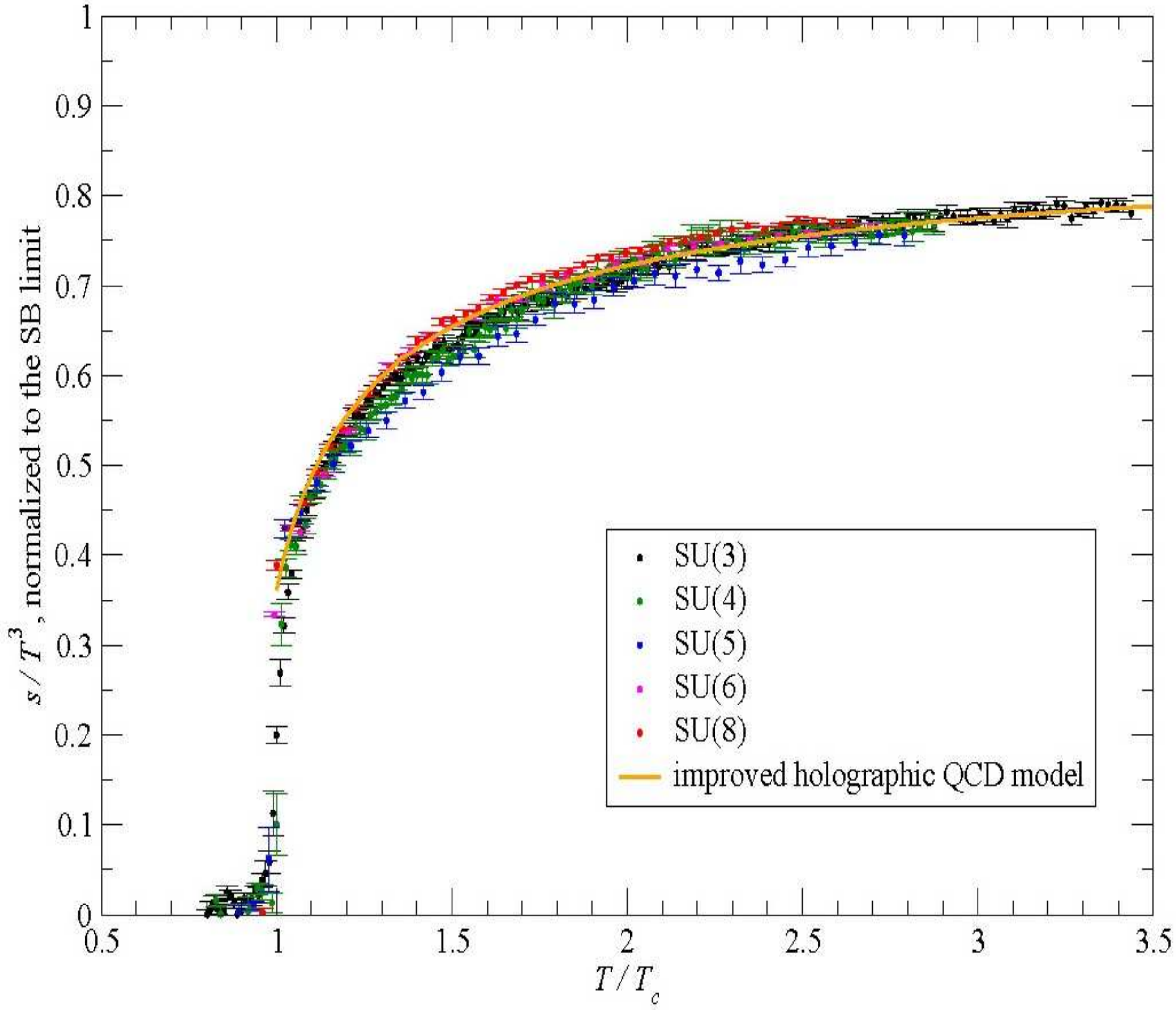


Figure 4: (Color online) Same as in fig. 1, but for the s/T^3 ratio, normalized to the SB limit.

Marco Panero arXiv: 0907.3719

The trace from the lattice at different N

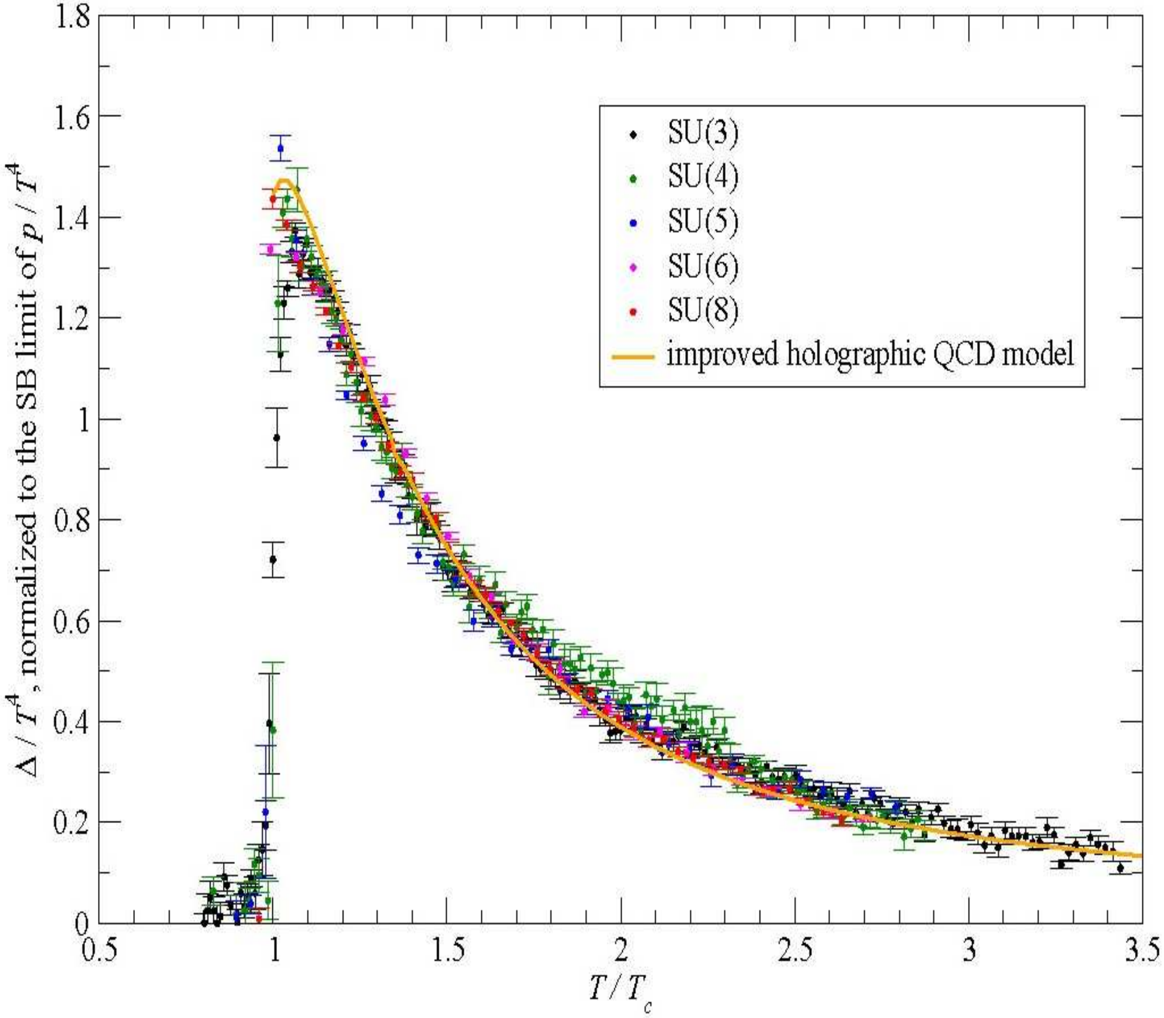
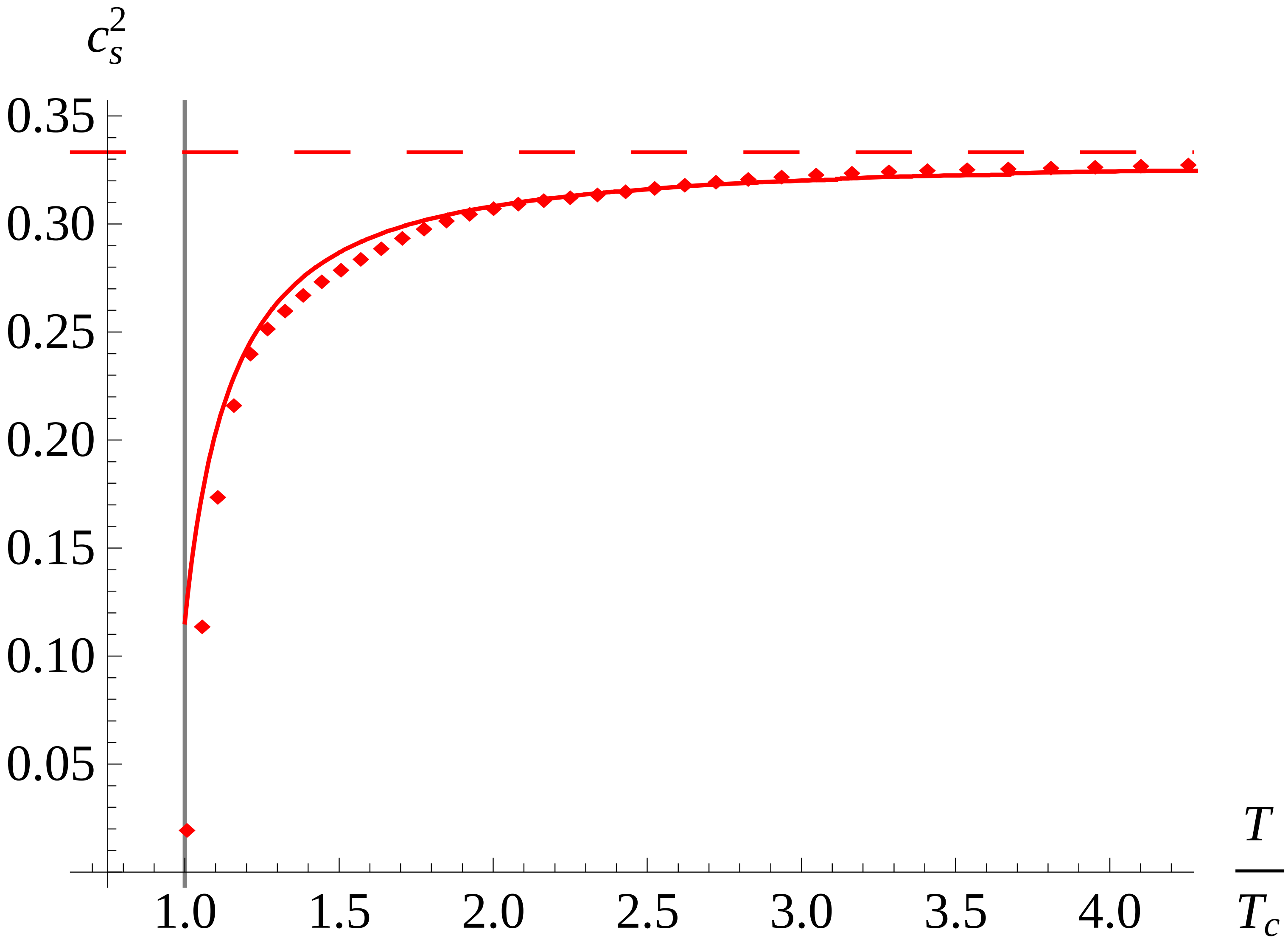


Figure 2: (Color online) Same as in fig. 1, but for the Δ/T^4 ratio, normalized to the SB limit of p/T^4.

Marco Panero arXiv: 0907.3719

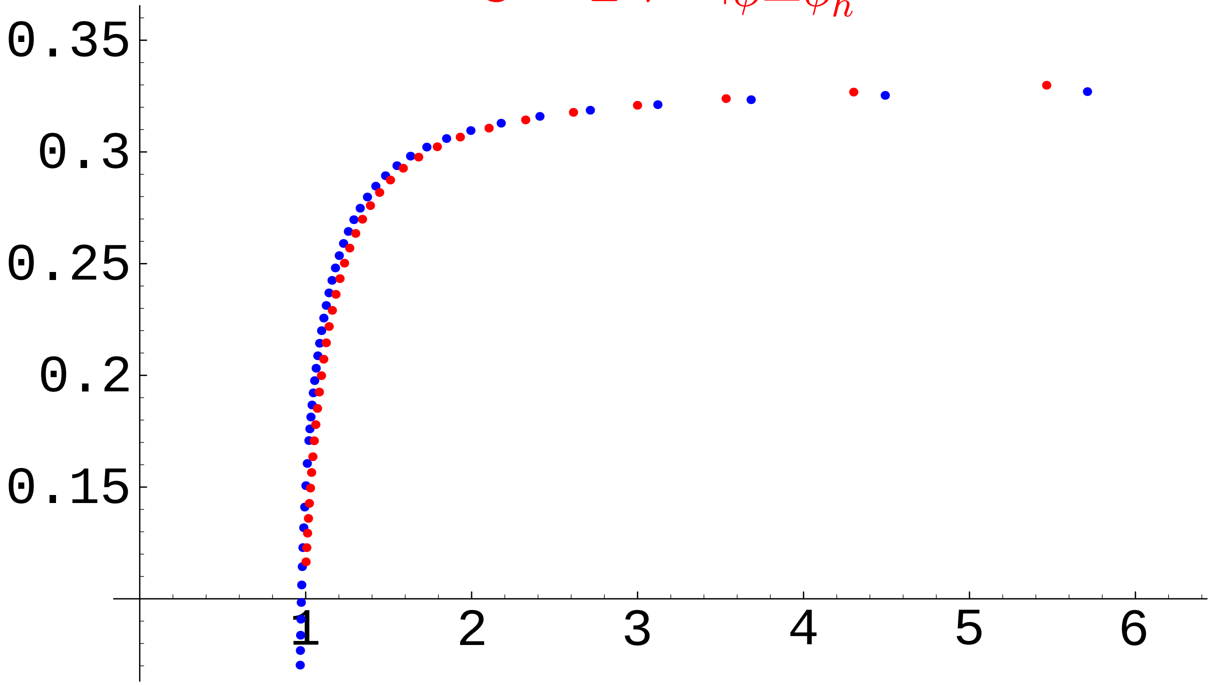
The speed of sound



Comparing to Gubser+Nelore's formula

- Gubser+Nelore proposed the following approximate formula for the speed of sound

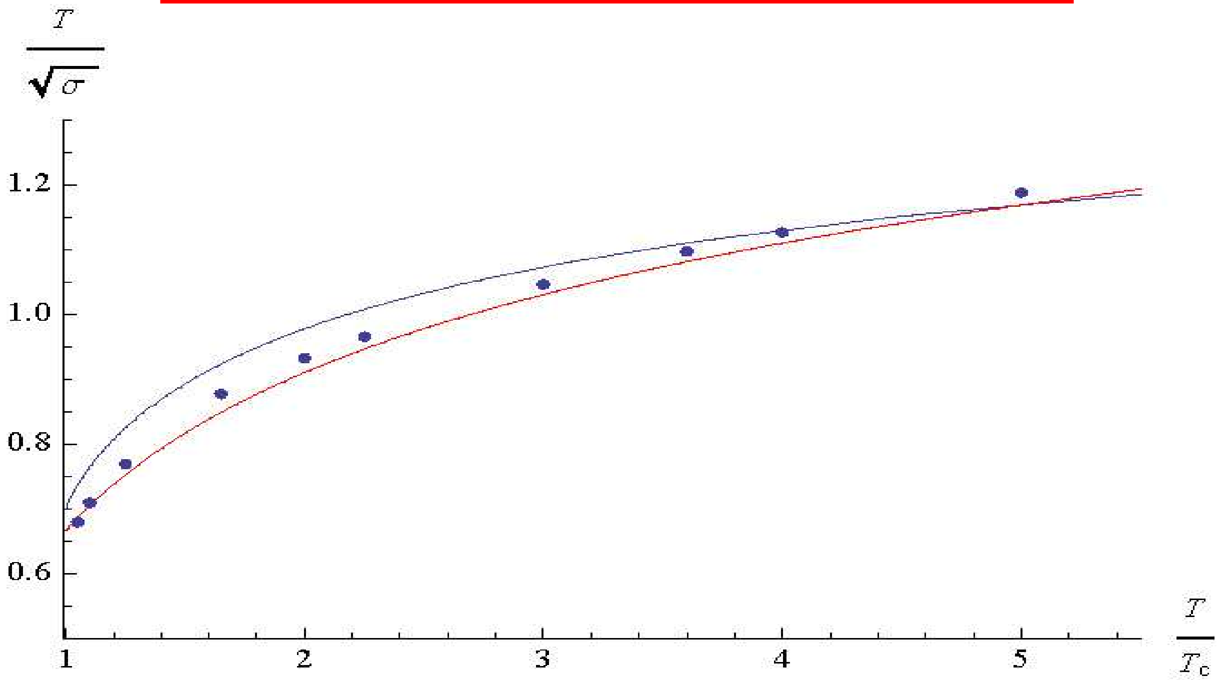
$$c_s^2 \simeq \frac{1}{3} - \frac{1}{2}\frac{V'^2}{V^2}\Big|_{\phi=\phi_h}$$



Gursoy (unpublished) 2009

- Red curve=numerical calculation, Blue curve=Gubser's adiabatic/approximate formula.

Spatial string tension



G. Boyd et al. 1996

- The blue line is the spatial string tension as calculated in Improved hQCD, with no additional fits.

Nitti (unpublished) 2009

- The red line is a semi-phenomenological fit using

$$\frac{T}{\sqrt{\sigma_s}} = 0.51 \ \left[\log\frac{\pi T}{T_c} + \frac{51}{121}\log\left(2\log\frac{\pi T}{T_c}\right)\right]^{\frac{2}{3}}$$

Alanen+Kajantie+Suur-Uski, 2009

Viscosity

- Viscosity (shear and bulk) is related to dissipation and entropy production

$$\frac{\partial s}{\partial t} = \frac{\eta}{T}\left[\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial \cdot v\right]^2 + \frac{\zeta}{T}(\partial \cdot v)^2$$

$$T^{\mu\nu} = (E+p)u^\mu u^\nu + p g^{\mu\nu} + Z^{\mu\alpha}Z^{\nu\beta}\left[\eta\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha) - \frac{2}{3}g_{\alpha\beta}\nabla_\gamma u^\gamma\right) + \zeta g_{\alpha\beta}\nabla_\gamma u^\gamma\right]$$

$$Z^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

- Hydrodynamics is valid as an effective description when relevant length scales $\gg$ mean-free-path:

- Conformal invariance imposes that $\zeta = 0$.

- Viscosity can be calculated from a Kubo-like formula (fluctuation-dissipation)

$$\eta\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{kl}\right)+\zeta\delta_{ij}\delta_{kl}=-\lim_{\omega\to 0}\frac{Im\ G^R_{ij;kl}(\omega)}{\omega}$$

$$G^R_{ij;kl}(\omega)=-i\int d^3x\int dt\ e^{i\omega t}\theta(t)\ \langle 0|[T_{ij}(\vec{x},t),T_{kl}(\vec{0},0)]|0\rangle$$

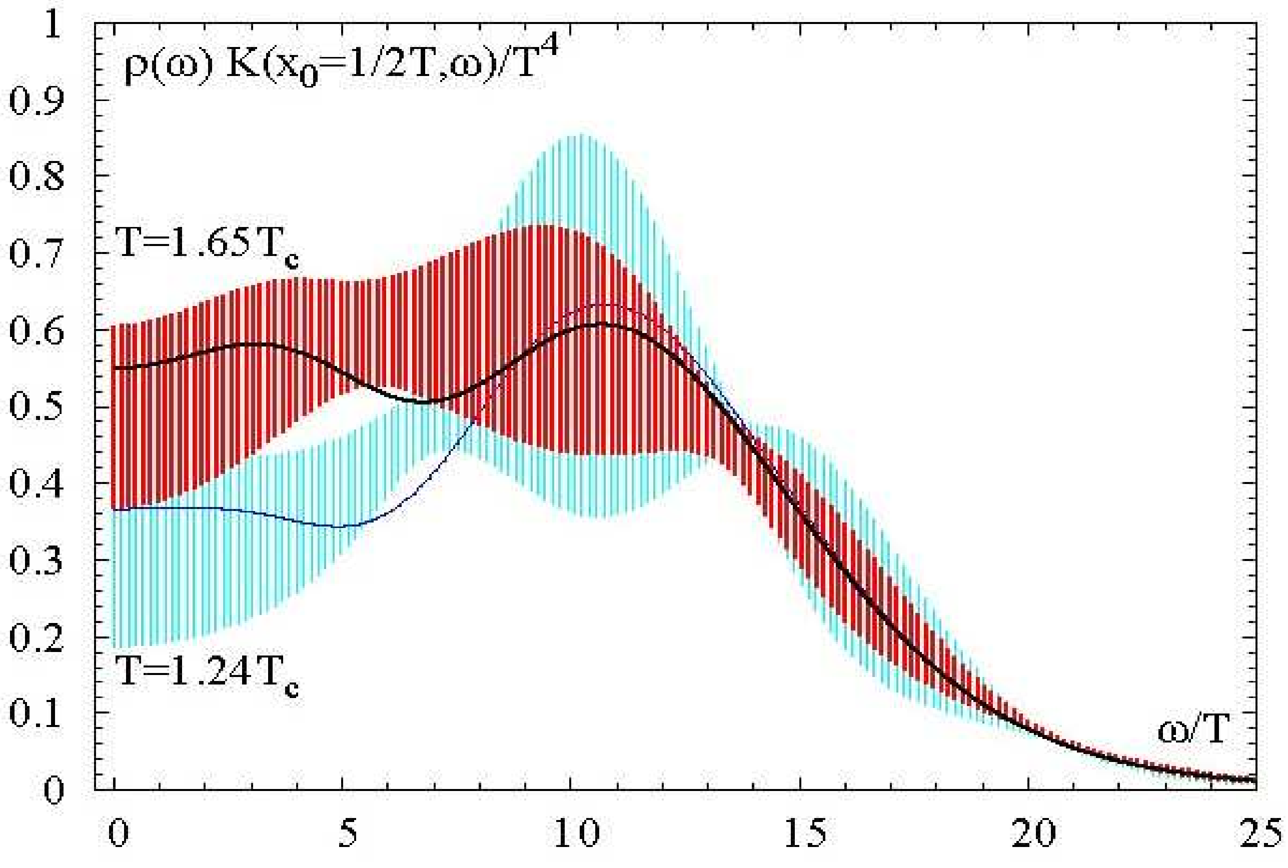
- In all theories with gravity duals ($\lambda\to\infty$) at two-derivative level

$$\frac{\eta}{s}=\frac{1}{4\pi}$$

Policastro+Starinets+Son 2001, Kovtun+Son+Starinets 2003, Buchel+Liu 2003

- In Einstein-dilaton gravity shear viscosity is equal to the universal value.

Shear Viscosity bounds from lattice

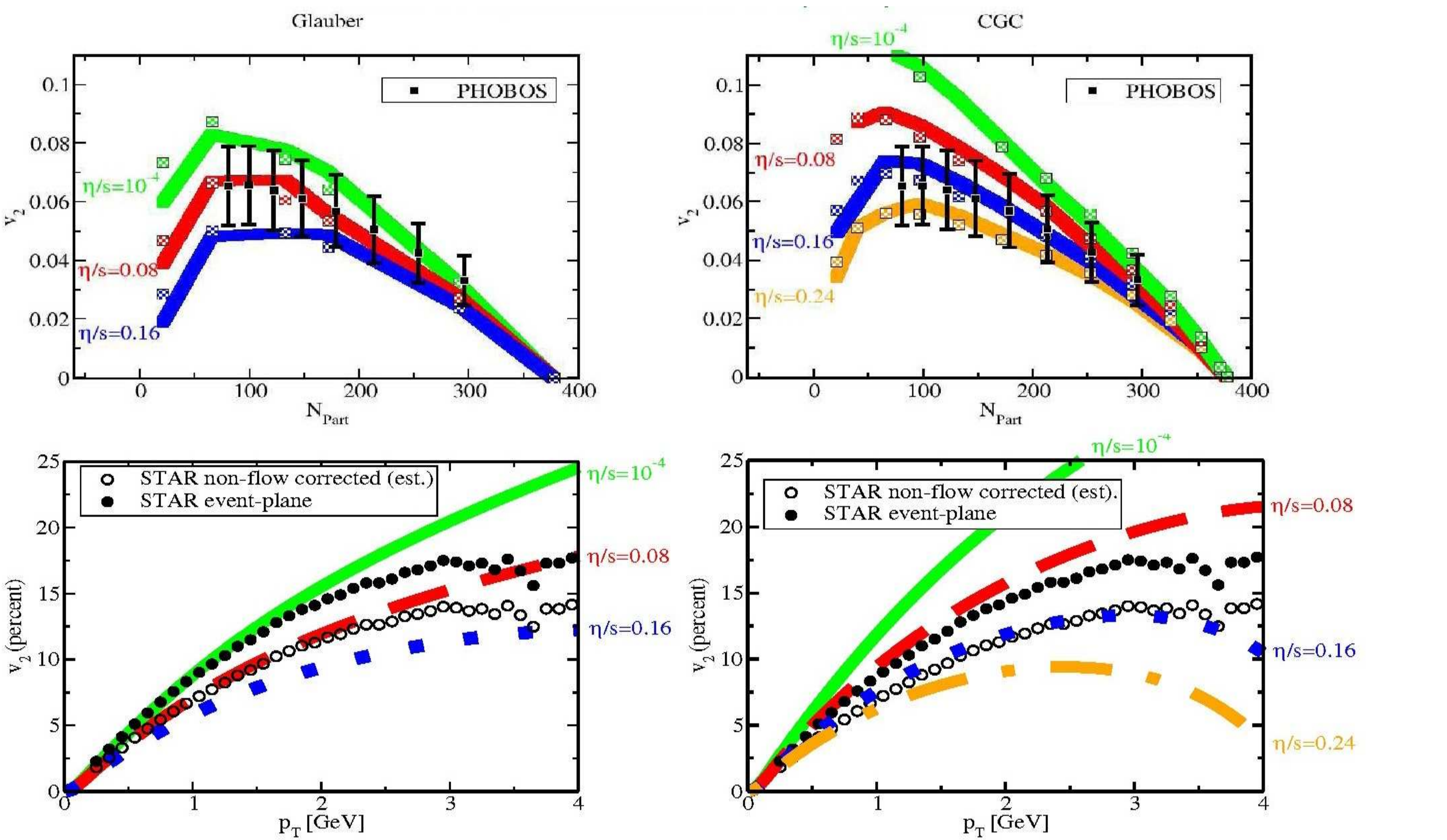


H. Meyer 2007

$$4\pi\frac{\eta}{s} = \begin{cases} 1.68(42), & T = 1.65\ T_c, \\ 1.28(70), & T = 1.24\ T_c. \end{cases}$$

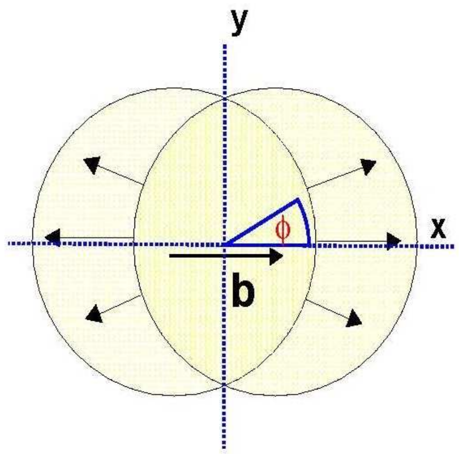
shear viscosity data

- V_2 is the elliptic flow coefficient



Luzum+Romatchke 2008

Elliptic Flow



$$\frac{1}{p_T}\frac{dN}{dp_T d\phi} = \frac{1}{p_T}\frac{dN}{dp_T}\,[1 + v_2(p_T)\cos 2\phi + \cdots]$$

The sum rule method

- Define the (subtracted) spectral density and relate its moment to the Euclidean density

$$\rho(\omega) = -\frac{1}{\pi} Im\ G^R(\omega) \quad , \quad \mathcal{G} \equiv \lim_{\omega \to 0} G^E(\omega) = 2 \int_0^\infty \frac{\rho(u)}{u}\, du$$

Karsch+Kharzeev+Tuchin, 2008, Romatschke+Son 2009

- Using Ward identities we obtain the sum rule

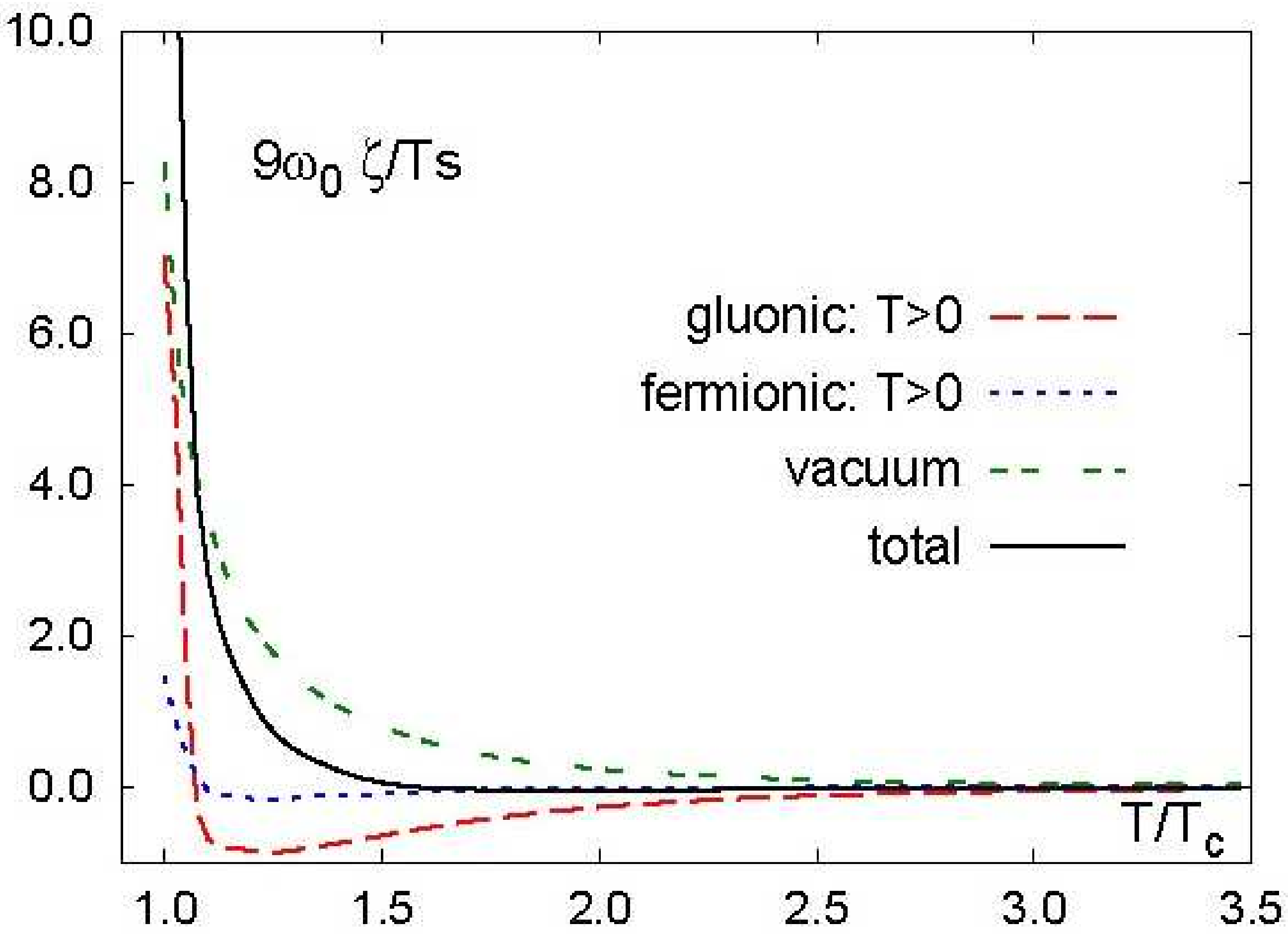
$$\mathcal{G} = \left(T\frac{\partial}{\partial T} - 4\right)(E - 3P + \langle\Theta\rangle_0) + \left(T\frac{\partial}{\partial T} - 2\right)(m\langle\bar{q}q\rangle_T + \langle\Theta_F\rangle_0)$$

with

$$\langle\Theta_F\rangle_0 = m\langle\bar{q}q\rangle \simeq -m_\pi^2\ f_\pi^2 - m_K^2\ f_K^2$$

- Assume a density

$$\frac{\rho(\omega)}{\omega} = \frac{9\zeta(T)}{\pi}\frac{\omega_0(T)^2}{\omega^2 + \omega_0(T)^2}$$

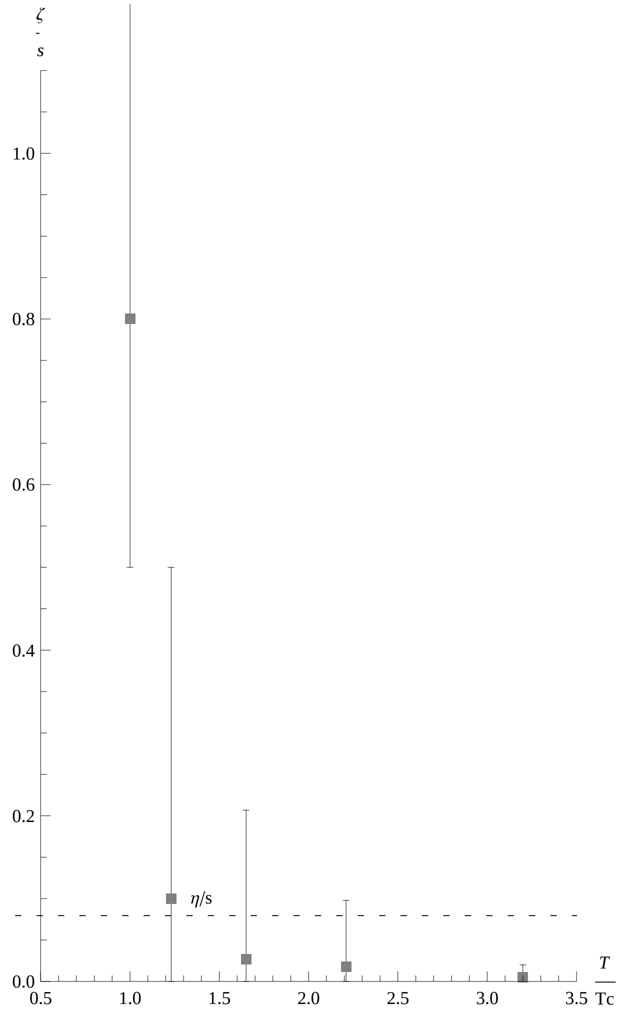


Karsch+Kharzeev+Tuchin, 2008

• A seeming rise near the phase transition but the (temperature-dependent) scale cannot be fixed.

The bulk viscosity in lattice SU(3) YM

H. Meyer 2007



Pure YM only. Error bar are statistical only.

.

• If the lattice result is taken at phase value,

$$\frac{\zeta(T_c)}{s(T_c)} \quad \sim \quad 10\frac{\eta(T_c)}{s(T_c)} \quad = \quad 10\,\frac{1}{4\pi}$$

• Such a large value renders hydrodynamic codes unstable.

Heinz+Song (unpublished)

• At large values of viscosity, cavitation ($p < 0$) happens, signaling a break-down of hydrodynamics.

• This was studied carefully and confirmed very recently

Rajagopal+Tripuraneni 2009

The bulk viscosity in HQCD: theory

- This is harder to calculate.

- Using a parametrization $ds^2 = e^{2A}(f dt^2 + d\vec{x}^2 + \frac{dr^2}{f})$ in a special gauge $\phi = r$ the relevant metric perturbation decouples

Gubser+Nellore+Pufu+Rocha 2008, Gubser+Pufu+Rocha,2008

$$h_{11}'' = -\left(-\frac{1}{3A'} - A' - \frac{f'}{f}\right) h_{11}' + \left(-\frac{\omega^2}{f^2} + \frac{f'}{6fA'} - \frac{f'}{f}A'\right) h_{11}$$

with

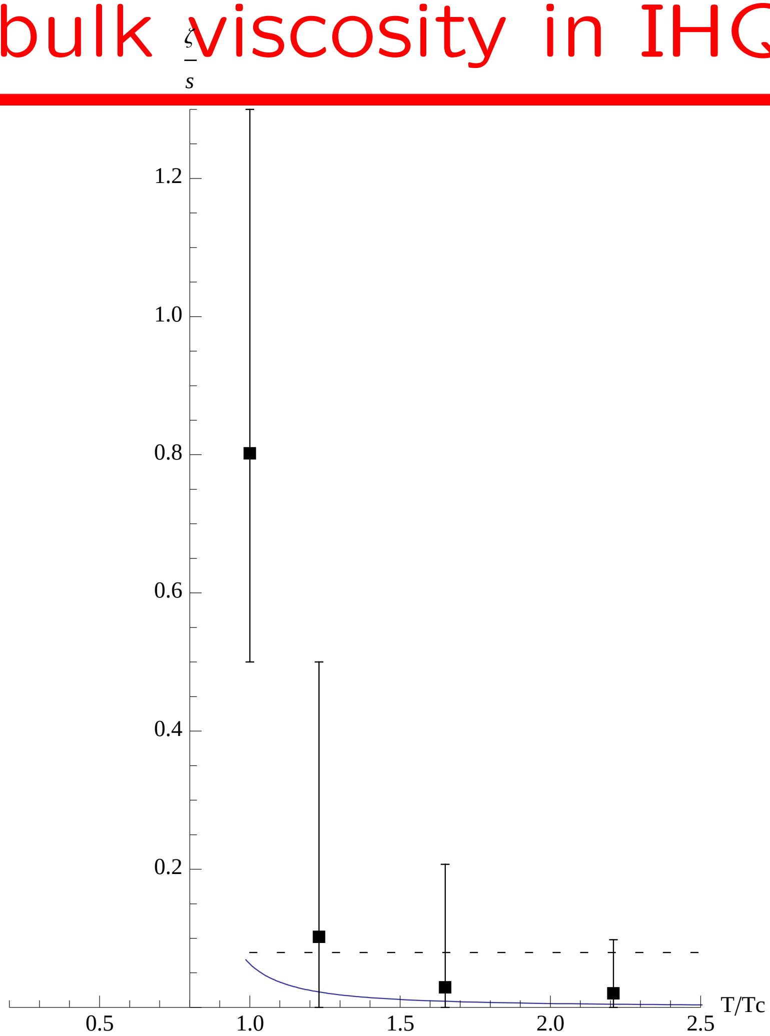
$$h_{11}(0) = 1 \quad , \quad h_{11}(r_h) \simeq C\ e^{i\omega t} \left| \log \frac{\lambda}{\lambda_h} \right|^{-\frac{i\omega}{4\pi T}}$$

The correlator is given by the conserved number of h-quanta

$$Im\ G_R(\omega) = -4M^3 \mathcal{G}(\omega) \quad , \quad \mathcal{G}(\omega) = \frac{e^{3A} f}{4A'^2} |Im[h_{11}^* h_{11}']|$$

$$\frac{\zeta}{s} = \frac{C^2}{4\pi} \left(\frac{V'(\lambda_h)}{V(\lambda_h)}\right)^2$$

The bulk viscosity in IHQCD

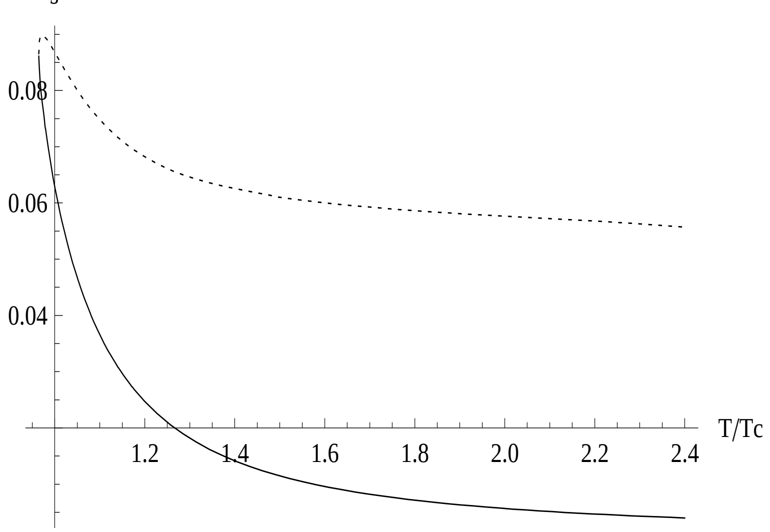


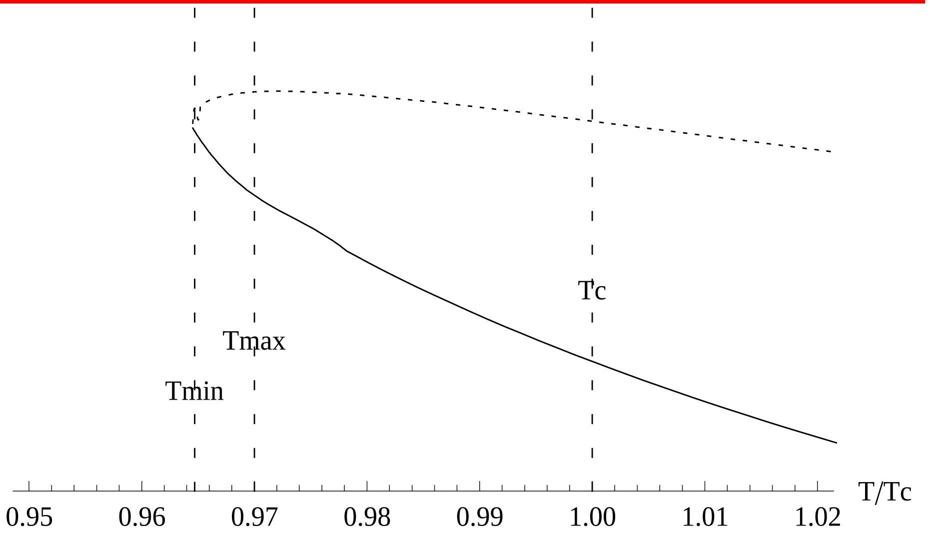
Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

- Pure glue only.

- Calculations with other potentials show robustness *Gubser+Pufu+Rocha 2008, Cherman+Nellore 2009*

The bulk viscosity in the small black hole





Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

- At the turning point the behavior, $C_V \to \infty$ and ζ behaves similar to that observed in the N=2* theory

Buchel+Pagnutti, 2008

- The small black-hole bulk viscosity ratio asymptotes to a constant as $T \to \infty$.

$$\lim_{T\to\infty} \left.\frac{\zeta(T)}{s(T)}\right|_{\text{small}} = \frac{1}{6\pi}$$

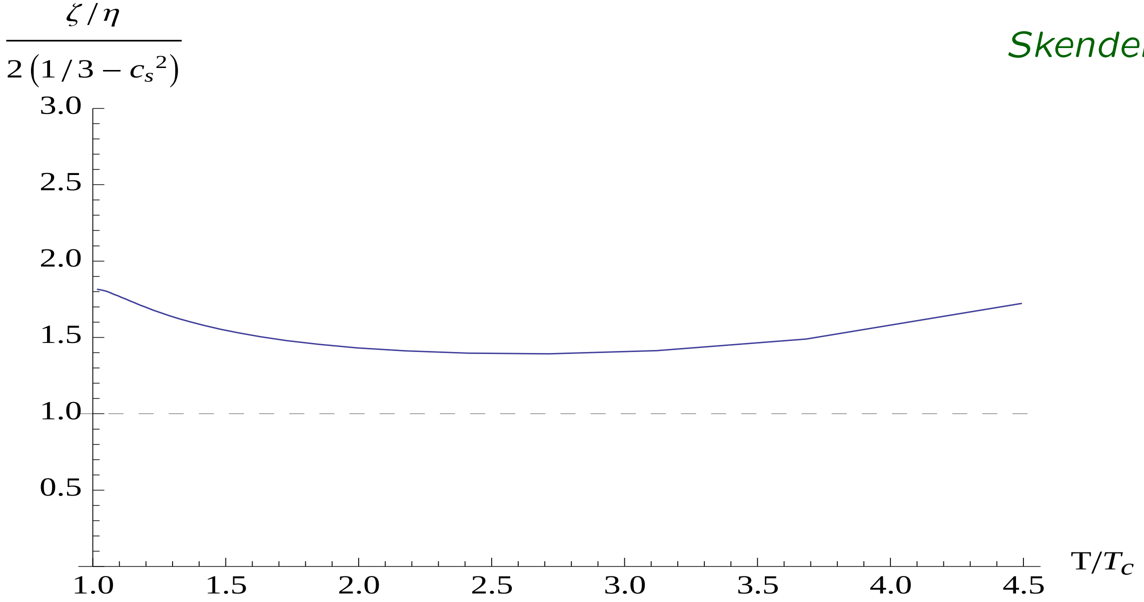
The Buchel parametrization (conjectured bound)

$$\frac{\zeta}{\eta} \geq 2\left(\frac{1}{3} - c_s^2\right)$$

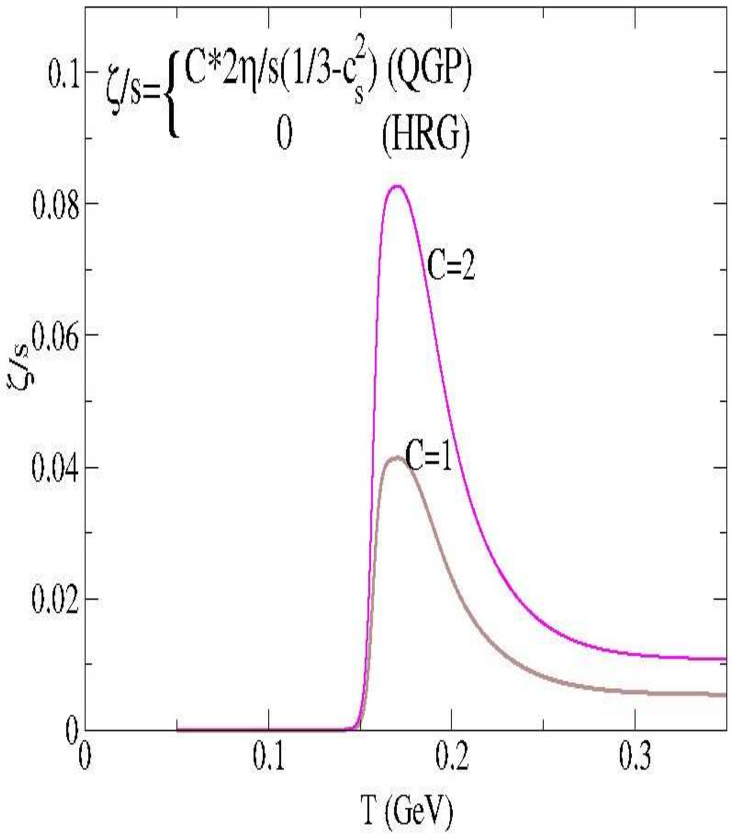
Buchel 2007

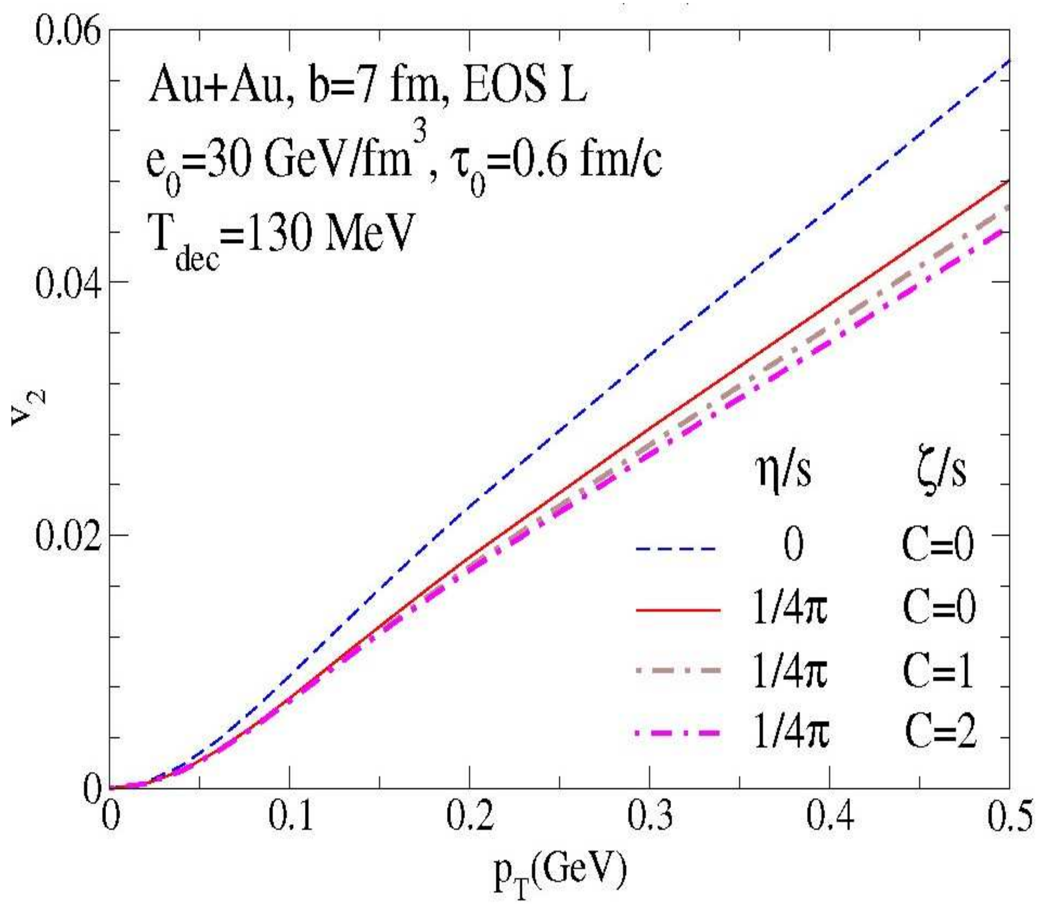
• For Dp branes, equality is a consequence of conformal invariance and dimensional reduction.

Skenderis+Kanitscheider 2009



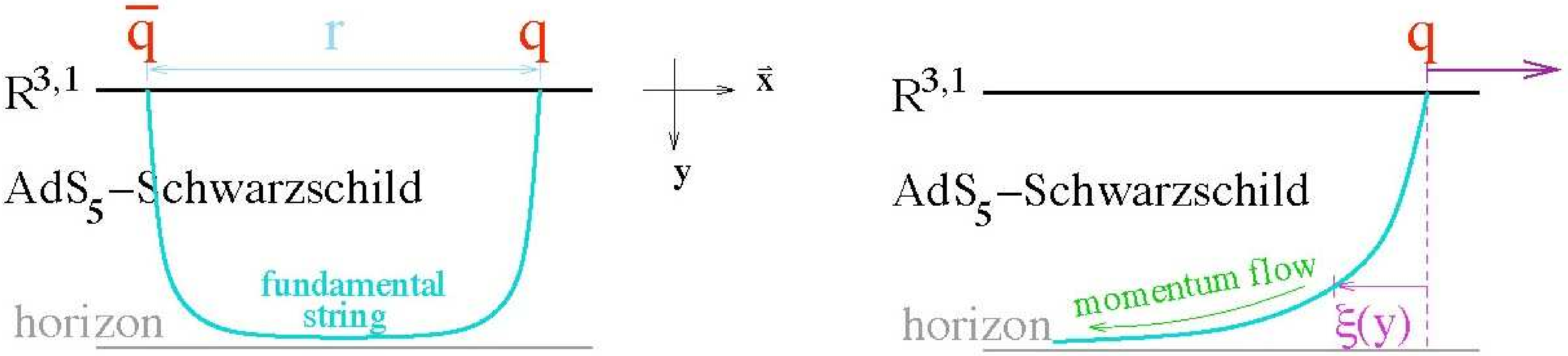
Elliptic Flow vs bulk viscosity





U Heinz+H.Song 2008

Heavy quarks and the drag force



From Gubser's talk at Strings 2008

- We must find a solution to the string equations with

$$x^1 = vt + \xi(r) \quad , \quad x^{2,3} = 0 \quad , \quad \sigma^1 = t \quad , \quad \sigma^2 = r$$

Herzog+Karch+kovtun+Kozcac+Yaffe, Gubser

Casaldelrrey-Solana+Teaney, Liu+Rajagopal+Wiedeman

For a black-hole metric (in string frame)

$$ds^2 = b(r)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x} \cdot d\vec{x} \right]$$

the solution profile is

$$\xi'(r) = \frac{C}{f(r)}\sqrt{\frac{f(r)-v^2}{b^4(r)f(r)-C^2}} \quad , \quad C = vb(r_s)^2 \quad , \quad f(r_s) = v^2$$

• The induced metric on the world-sheet is a 2d black-hole with horizon at the turning point $r = r_s$ ($t = \tau + \zeta(r)$).

$$ds^2 = b^2(r)\left[-(f(r)-v^2)d\tau^2 + \frac{1}{(f(r)-\frac{b^4(r_s)}{b^4(r)}v^2)}dr^2\right]$$

• We can calculate the drag force:

$$F_{\text{drag}} = P_\xi = -\frac{b^2(r_s)\sqrt{f(r_s)}}{2\pi\ell_s^2}$$

• In $\mathcal{N} = 4$ sYM it is given by

$$F_{\text{drag}} = -\frac{\pi}{2}\sqrt{\lambda}\; T^2\frac{v}{\sqrt{1-v^2}} = -\frac{1}{\tau}\frac{p}{M} \quad , \quad \tau = \frac{2M}{\pi\sqrt{\lambda}\; T^2}$$

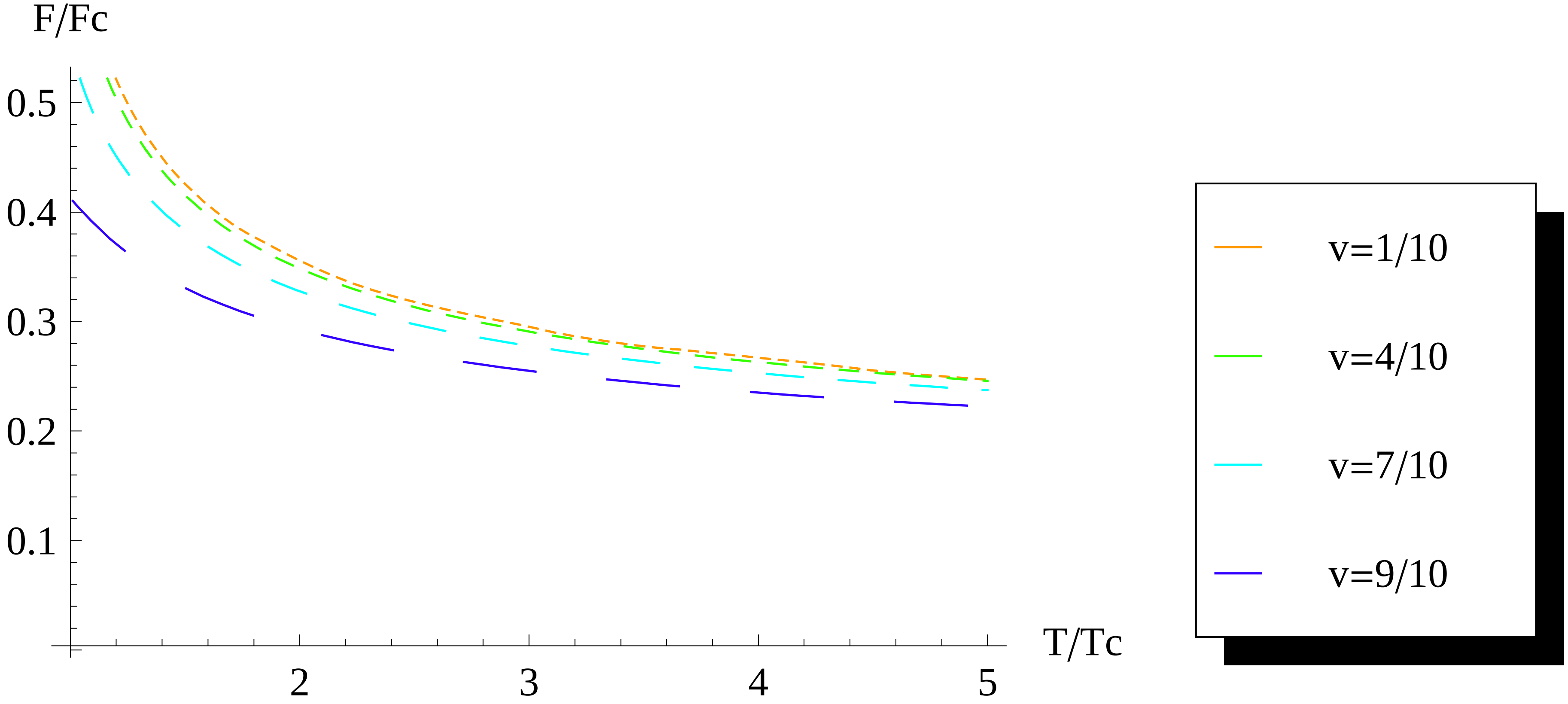
• For non-conformal theories it is a more complicated function of momentum and temperature.

The drag force in IhQCD

Systematic errors:

(a) Flavor description (heavy quark)

(b) Ignore light fermionic degrees of freedom in plasma



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

- F_{conf} calculated with $\lambda = 5.5$

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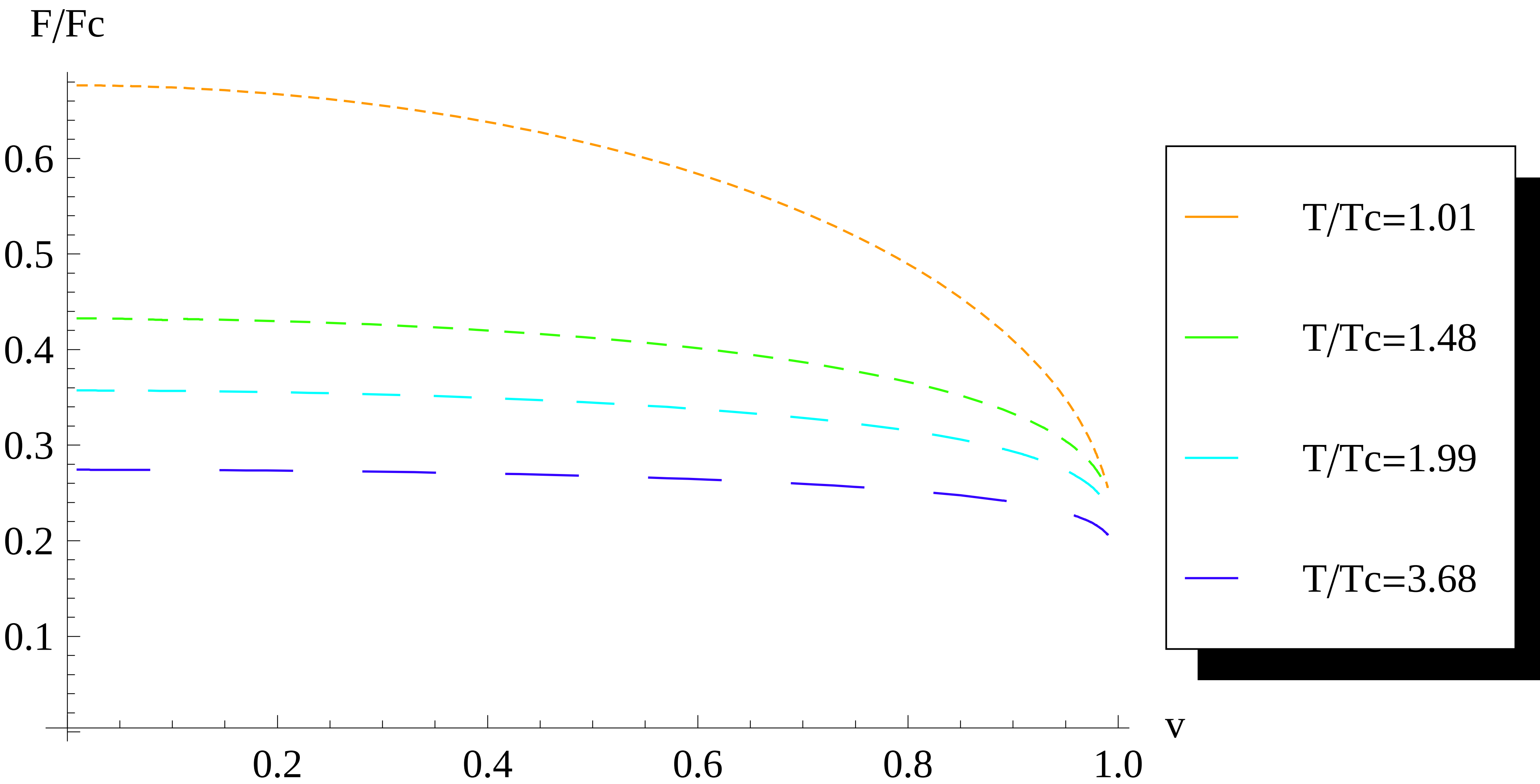
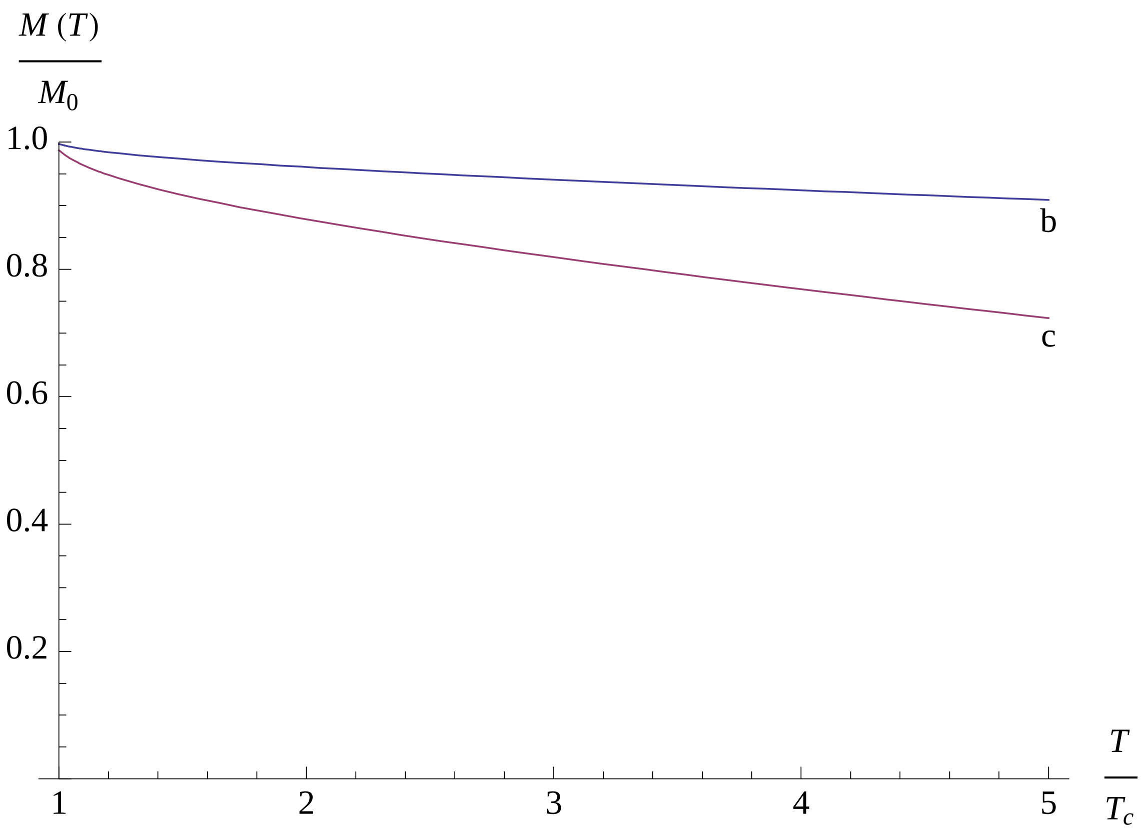


Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

The thermal mass

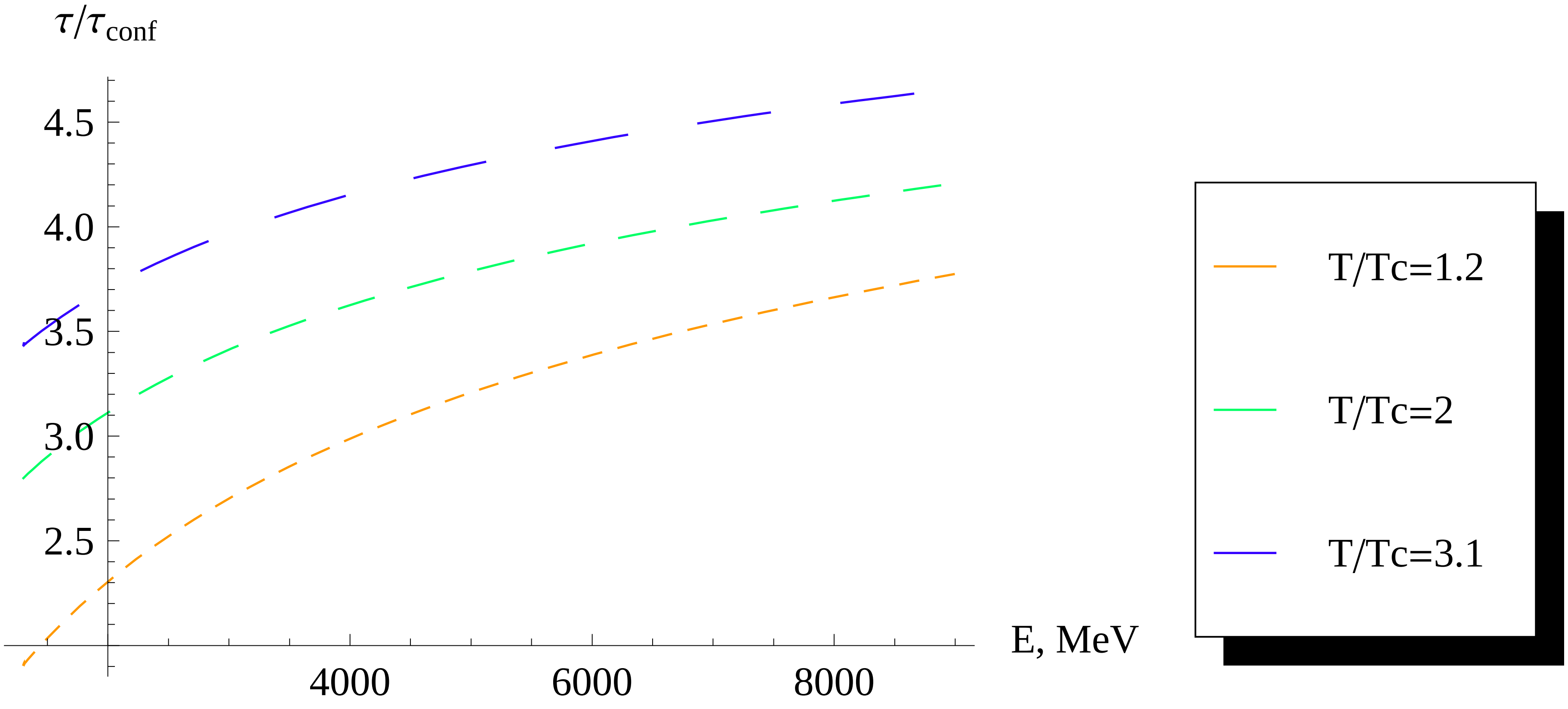


• The mass is defined via a straight string hanging in the bulk

• It is qualitatively in agreement with lattice calculation of the position of the quarkonium resonance shift at finite temperature.

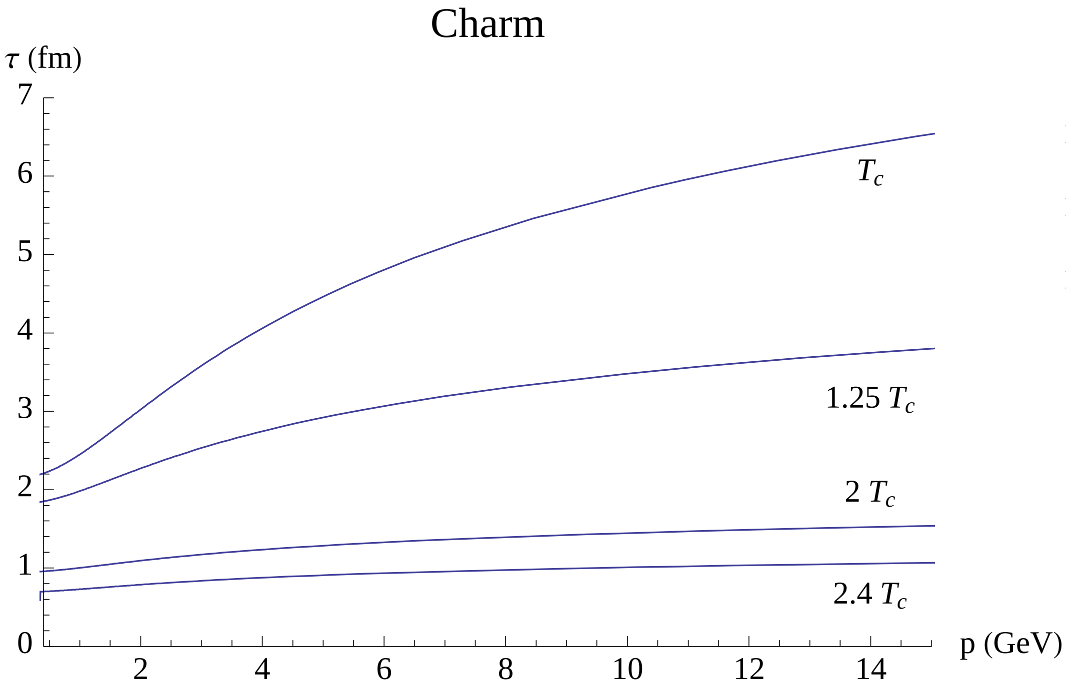
Datta+Karsch+Petreczky+Wetzorke 2004

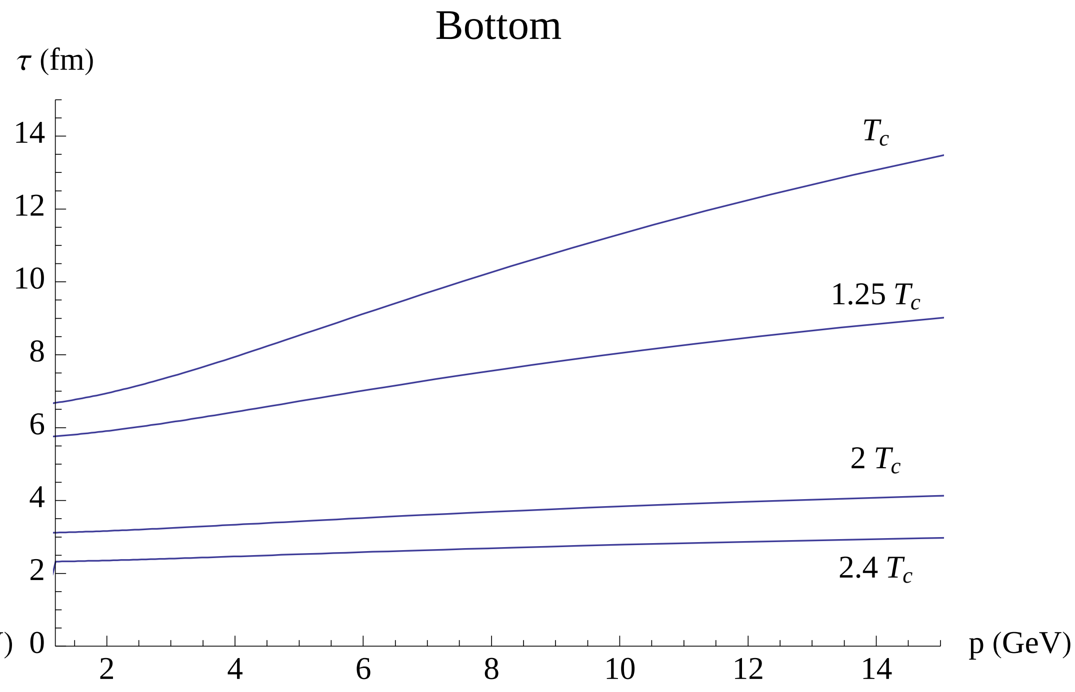
The diffusion time



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

$$\frac{dp}{dt} = -\frac{p}{\tau(p)}$$





Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

	$\gamma = 0.3$	$\gamma = 1$	$\gamma = 3$
τ_c[fm]	22	6.7	2.2
τ_b[fm]	72	21	7.2

thermalized

not thermalized

Akamatsu+Hatsuda+Hirano, 2008

Diffusion times in different schemes

T_{QGP}, MeV	τ_{charm} (fm/c) (direct)	τ_{charm} (fm/c) (energy)	τ_{charm} (fm/c) (entropy)
220	-	3.96	3.64
250	5.67	3.14	2.96
280	4.27	2.56	2.47
310	3.45	2.12	2.08
340	2.88	1.80	1.78
370	2.45	1.54	1.53
400	2.11	1.33	1.34

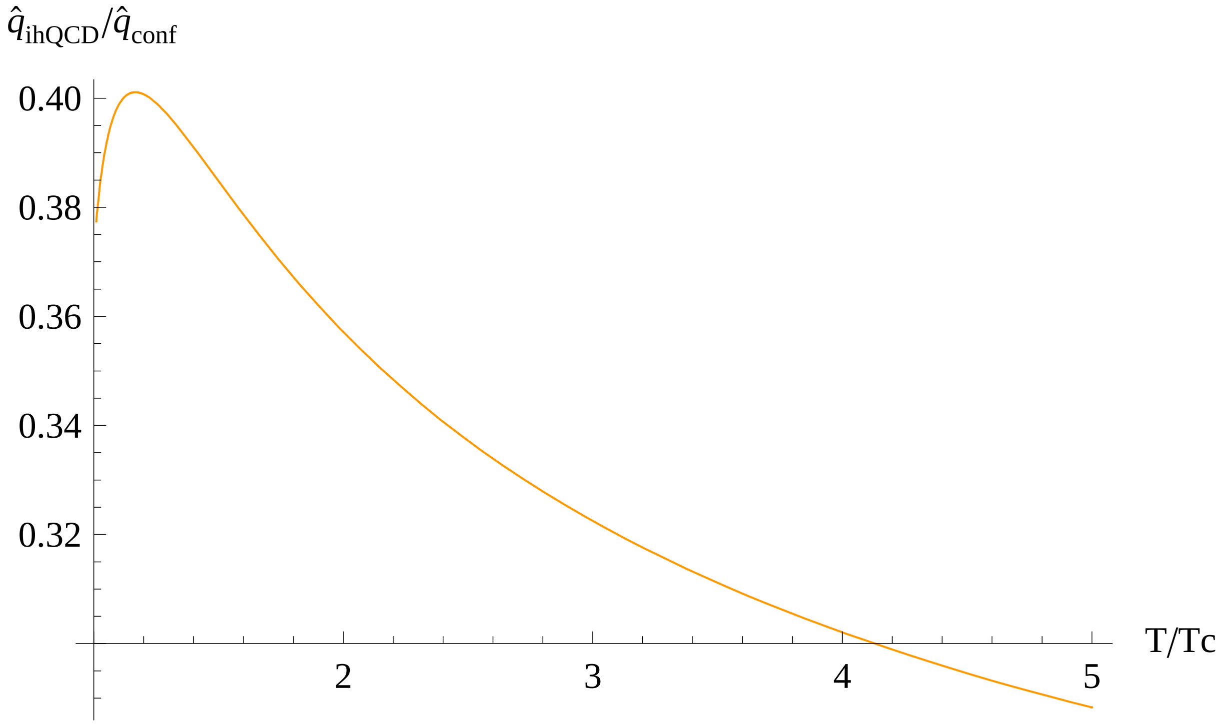
The diffusion times for the charm quark are shown for different temperatures, in the three different schemes. Diffusion times have been evaluated with a quark initial momentum fixed at $p \approx 10\ GeV$.

Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

The jet-quenching parameter

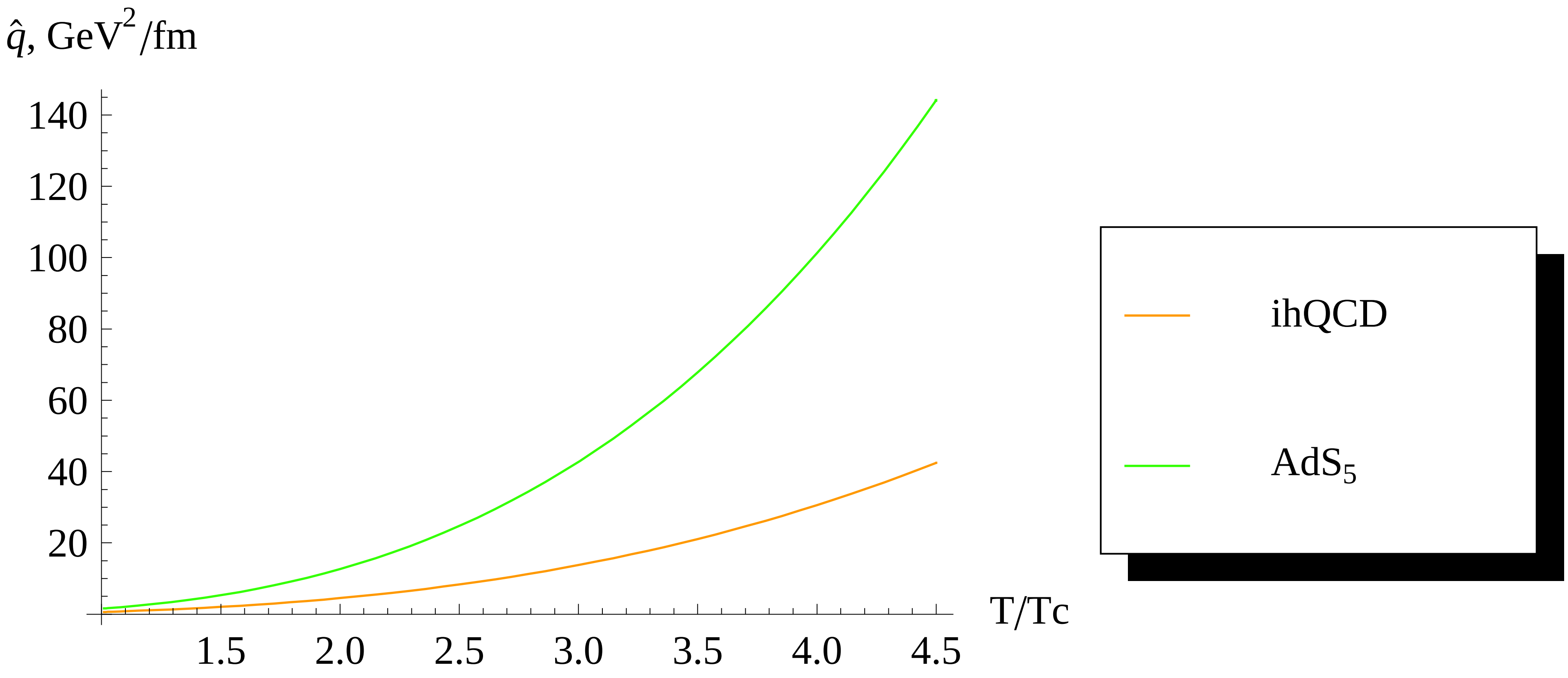
- $\hat{q} \sim \frac{d}{dt} < p^2_{transverse} >$ From the light-like Wilson loop

Liu+Rajagopal+Wiedemann



$$\hat{q}_{\rm conformal} = \frac{\Gamma\left[\frac{3}{4}\right]}{\Gamma\left[\frac{5}{4}\right]} \sqrt{2\lambda}\ \pi^{\frac{3}{2}} T^3 \quad , \quad \lambda = 5.5$$

.



$\hat{q}$, GeV^2/fm
140
120
100
80
60
40
20
1.5
2.0
2.5
3.0
3.5
4.0
4.5
T/Tc
ihQCD
AdS$_5$

Langevin diffusion of heavy quarks

- So far we have described a heavy quark with a classical equation

$$\frac{d\vec{p}}{dt} = \vec{F}_{drag} \simeq \frac{1}{\tau_{dif}}\vec{p}$$

- In a thermal medium we would expect the analogue of Brownian motion for heavy quarks.

- Fluctuations were first studied around the trailing string solution and diffusion coefficients were calculated.

Cassalderey-Solana+Teaney, 2006 Gubser 2006

- A full Langevin-like treatment was derived recently for non-relativistic quarks

Son+Teaney 2009 DeBoer+Hubeny+Rangamani+Shigenori, 2009

- This describes a Langevin process of the form

$$\frac{d\vec{p}}{dt} = \vec{F} + \vec{\xi} \quad , \quad \vec{F} = -\eta\, \vec{p} \quad , \quad \langle \xi^i(t)\xi^j(t')\rangle = \kappa\delta^{ij}\delta(t-t')$$

$\vec{F}$ is the drag force, $\eta = \frac{1}{\tau}$.

- The fully relativistic case was also described recently

Giecold+Iancu+Mueller, 2009

We consider fluctuations around the dragging string solution in the thermal background

$$ds^2 = b^2(r)(\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x}^2) \quad , \quad X^1 = vt + \xi(r) + \delta X^1 \quad , \quad X^{2,3} = \delta X^{2,3}$$

The Nambu-Goto action is expanded as

$$S = S_0 + S_1 + S_2 + \cdots \quad , \quad S_1 = \int d\tau dr \; P^\alpha \; \partial_\alpha \delta X^1 \tag{1}$$

with

$$S_2 = \frac{1}{2\pi\ell_s^2} \int d\tau dr \; \left[\frac{G^{\alpha\beta}}{2} \partial_\alpha \delta X^1 \partial_\beta \delta X^1 + \sum_{i=2}^{3} \frac{\tilde{G}^{\alpha\beta}}{2} \partial_\alpha \delta X^i \partial_\beta \delta X^i \right] \tag{2}$$

with

$$G^{\alpha\beta} = \frac{b^2(r) Z(r)^2}{2} g^{\alpha\beta} \quad , \quad \tilde{G}^{\alpha\beta} = \frac{b(r)^2}{2} g^{\alpha\beta} \quad , \quad Z(r) = \sqrt{1 + f(r)\xi'(r)^2 - \frac{v^2}{f(r)}}$$

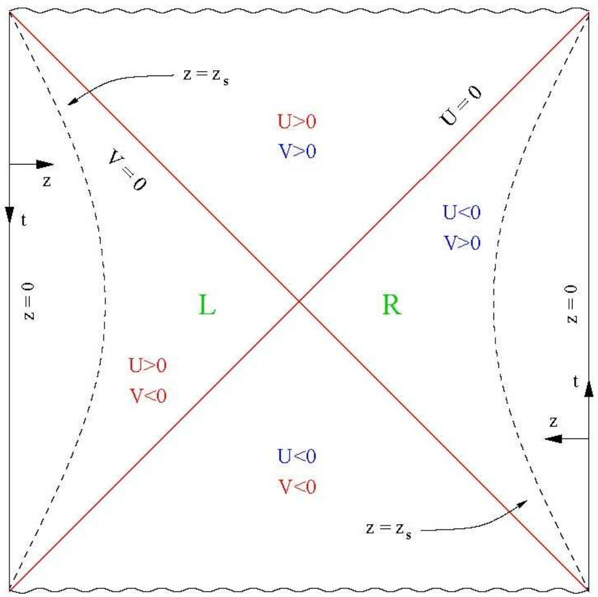
• The fluctuations δX^i satisfy.

$$\partial_\alpha G^{\alpha\beta}\partial_\beta\, \delta X^1 = 0 \quad , \quad \partial_\alpha \tilde{G}^{\alpha\beta}\partial_\beta\, \delta X^{2,3} = 0$$

• The metric in which they are evaluated is of the bh type, but with a different Hawking temperature, T_H. In the CFT case we have $T_H = \sqrt{1-v^2}\; T$

• We double the fields, $\delta X \to \delta X_{L,R}$ and we can define retarded and advanced correlators using the Schwinger-Keldysh formalism as implemented in AdS/CFT

Herzog+Son 2002, Gubser 2006, Skenderis+VanRees 2009



$$S_{\text{boundary}} = \int d\tau_R \left[-P^r \delta X^0_R + \frac{1}{2} \delta X^0_R G^{r\alpha} \partial_\alpha \delta X^0_R \right] - (L \leftrightarrow R)$$

$$= -\int \frac{d\omega}{2\pi}\ \delta X^0_a(-\omega) G^R(\omega) \delta X^0_r(\omega) + \frac{i}{2} \int \frac{d\omega}{2\pi}\ \delta X^0_a(-\omega) G^{sym}(\omega) \delta X^0_a(\omega)$$

with

$$\delta X_r = \frac{1}{2}(\delta X_L + \delta X_R) \quad , \quad \delta X_a = (\delta X_L - \delta X_R)$$

and

$$G_{sym}(\omega) = \frac{1 + e^{\frac{\omega}{T_H}}}{1 - e^{\frac{\omega}{T_H}}}\ G_R(\omega)$$

- We may derive a Langevin equation by starting with

$$Z = \int [D\delta X^0_{L,R}][D\delta X_{L,R}]\ e^{i(S_R - S_L)} = \int [D\delta X^0_{a,r}]\ e^{iS_{\text{boundary}}}$$

and introduce a dummy variable ξ to linearize the quadratic term of the a-fields

$$Z = \int [D\delta X^0_{a,r}][D\xi]\; e^{-\frac{1}{2}\int dtdt' \xi(t) G^{-1}_{sym}(t,t')\xi(t')} \times$$

$$\times \exp\left[-i\int dtdt'\; \delta X^0_a \left[G_R(t,t')\delta X^0_r(t') + \delta(t-t')(P^r - \xi(t'))\right]\right]$$

Integration over $\delta X^0_{a,r}$ gives the Langevin system

$$\int dt'\; G_R(t,t')\delta X^0_r(t') + P^r - \xi(t) = 0 \quad , \quad \langle \xi(t)\xi(t')\rangle = G_{\text{sym}}(t,t')$$

Giecold+Iancu+Mueller, 2009

• For $|t-t'|$ large we can replace the retarded propagator with a (second) time derivative and the symmetric one by a δ-function to finally obtain in the conformal case

$$\frac{dp^i_\perp}{dt} = -\eta\; p^i_\perp + \xi^i_\perp \quad , \quad \langle \xi^i_\perp(t)\xi^j_\perp(t')\rangle = \kappa_\perp \delta^{ij}\delta(t-t') \quad , \quad \kappa_\perp = \frac{\pi\sqrt{\lambda}T^3}{(1-v^2)^{\frac{1}{4}}}$$

$$\frac{dp_\parallel}{dt} = -\eta\; p_\parallel + \xi_\parallel \quad , \quad \langle \xi_\parallel(t)\xi_\parallel(t')\rangle = \kappa_\parallel \delta(t-t') \quad , \quad \kappa_\parallel = \frac{\pi\sqrt{\lambda}T^3}{(1-v^2)^{\frac{5}{4}}}$$

• In the non-relativistic limit the world-sheet horizon and the spacetime horizon coincide. In this case there is a Maxwell equilibrium distribution and the Einstein relation ($\kappa = 2ET\eta$) holds.

Cassalderey-Solana+Teaney, 2006 Gubser 2006
Son+Teaney 2009 DeBoer+Hubeny+Rangamani+Shigenori, 2009

• The diffusion is asymmetric in the relativistic case.

• The Maxwell distribution is not an equilibrium distribution of this system! This resolves previous puzzles of symmetric relativistic Langevin diffusion.

• Curiously, the failure of the Einstein relation was also seen in the heavy-ion data.

Wolchin 1999

• The (conformal) relativistic Langevin equation with symmetric diffusion was applied to data analysis at RHIC, but the Einstein relation was kept.

Akamatsu+Hatsuda+Hirano, 2008

In view of the above a reanalysis is necessary.

• The non-conformal correlators G_R are under study

Further directions

• Evaluation of the Langevin correlator in IhQCD and use as input for langevin MonteCarlo (both CFT and non-conformal)

• Second order transport coefficients (matter of principle)

• Implementation of a more realistic structure for the quarks in QGP: this will involve a more realistic holographic theory of flavor using $D_4\bar{D}_4$ branes

• Holographic calculation of two-point correlators of the stress tensor in the non-conformal (IhQCD) case. Application to lattice extraction techniques via sum rules (that may include fermions)

• Study of early thermalization procedure, as well as entropy production

• The search for other heavy-ion observables that may be calculable using holographic techniques.

.

Thank you for your Patience

Bibliography

• U. Gursoy, E. Kiritsis, G. Michalogiorgakis and F. Nitti,
" Thermal Transport and Drag Force in Improved Holographic QCD"
[ArXiv:0906.1890][hep-ph],.

• U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti,
"Improved Holographic Yang-Mills at Finite Temperature: Comparison with Data."
[ArXiv:0903.2859][hep-th],.

• E. Kiritsis,
" Dissecting the string theory dual of QCD.,"
[ArXiv:0901.1772][hep-th],.

• U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti,
"Thermodynamics of 5D Dilaton-gravity.,"
JHEP **0905** (2009) 033; [ArXiv:0812.0792][hep-th],.

• U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti,
"Deconfinement and Gluon-Plasma Dynamics in Improved Holographic QCD,"
Phys. Rev. Lett. **101**, 181601 (2008) [ArXiv:0804.0899][hep-th],.

• U. Gursoy and E. Kiritsis,
"Exploring improved holographic theories for QCD: Part I,"
JHEP **0802** (2008) 032[ArXiv:0707.1324][hep-th].

• U. Gursoy, E. Kiritsis and F. Nitti,
"Exploring improved holographic theories for QCD: Part II,"
JHEP **0802** (2008) 019[ArXiv:0707.1349][hep-th].

• Elias Kiritsis and F. Nitti
On massless 4D gravitons from asymptotically AdS(5) space-times.
Nucl.Phys.**B772** (2007) 67-102;[arXiv:hep-th/0611344]

• R. Casero, E. Kiritsis and A. Paredes,
"Chiral symmetry breaking as open string tachyon condensation,"
Nucl. Phys. B **787** (2007) 98;[arXiv:hep-th/0702155].

AdS/QCD

♠ A basic phenomenological approach: use a slice of AdS_5, with a UV cutoff, and an IR cutoff.

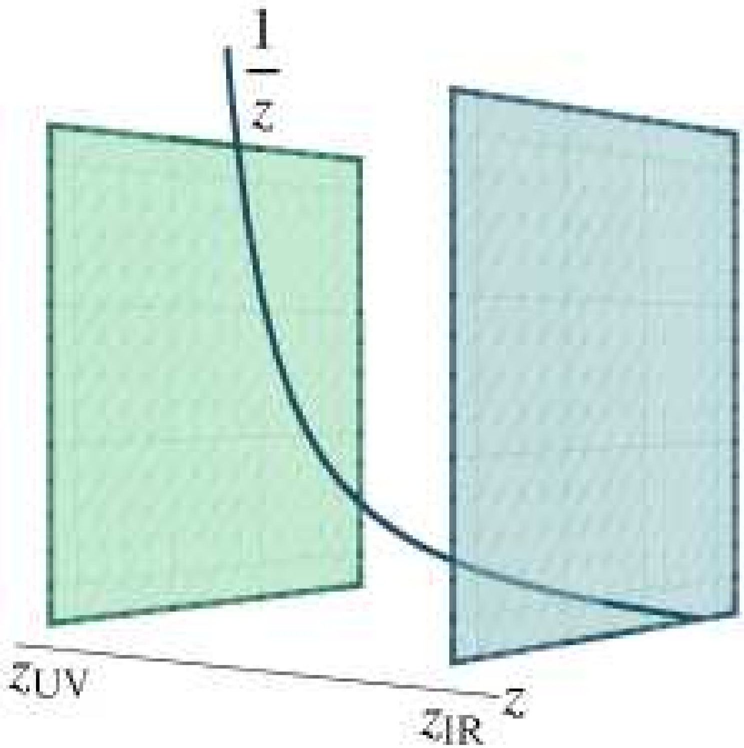
Polchinski+Strassler

♠ It successfully exhibits confinement (trivially via IR cutoff), and power-like behavior in hard scattering amplitudes

♠ It may be equipped with a bifundamental scalar, T, and $U(N_f)_L \times U(N_f)_R$, gauge fields to describe mesons.

Erlich+Katz+Son+Stepanov, DaRold+Pomarol

Chiral symmetry is broken by hand, via IR boundary conditions. The low-lying meson spectrum looks "reasonable".



♠ Shortcomings:

- The glueball spectrum does not fit very well the lattice calculations. It has the wrong asymptotic behavior $m_n^2 \sim n^2$ at large n.

- Magnetic quarks are confined instead of screened.

- Chiral symmetry breaking is input by hand.

- The meson spectrum has also the wrong UV asymptotics $m_n^2 \sim n^2$.

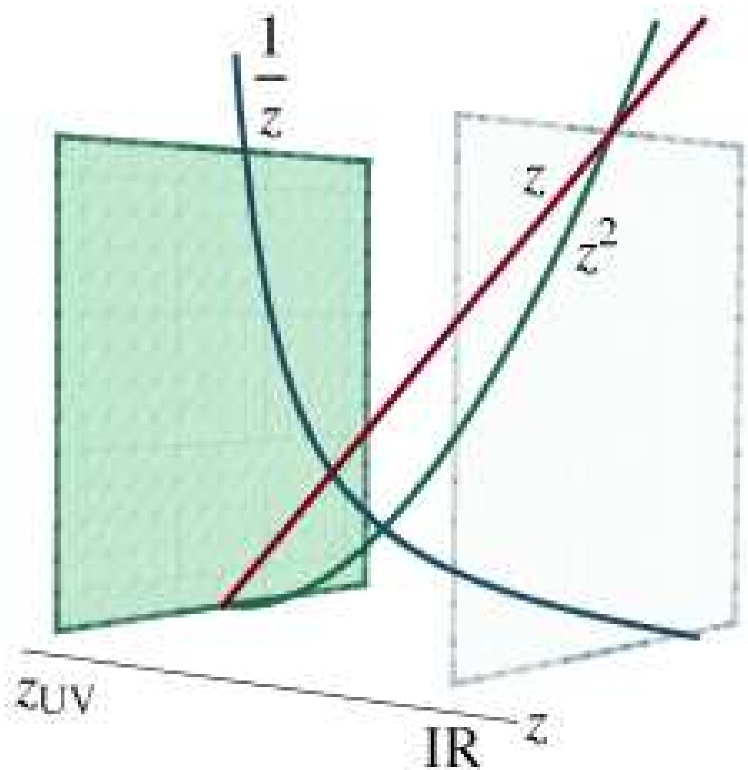
- at finite temperature there is a deconfining transition but the equation of state is trivial (conformal) (e-2p) and the speed of sound is $c_s^2 = \frac{1}{3}$.

The "soft wall"

♠ The asymptotic spectrum can be fixed by introducing a **non-dynamical** dilaton profile $\Phi \sim r^2$ (soft wall)

Karch+Katz+Son+Stephanov

• It is not a solution of equations of motion: the metric is still AdS: Neither $g_{\mu\nu}$ nor Φ solves the equations of motion.



A string theory for QCD:basic expectations

• Pure SU(N_c) d=4 YM at large N_c is expected to be dual to a string theory in 5 dimensions only. Essentially a single adjoint field $\to$ a single extra dimension.

• The theory becomes asymptotically free and conformal at high energy $\to$ we expect the classical saddle point solution to asymptote to AdS_5.

♠ Operators with lowest dimension (or better: lowest bulk masses) are expected to be the only important non-trivial bulk fields in the large-N_c saddle-point

• Scalar YM operators with $\Delta_{UV} > 4 \to m^2 > 0$ fields near the AdS$_5$ boundary $\to$ vanish fast in the UV regime and do not affect correlators of low-dimension operators.

.

• Their dimension may grow large in the IR so they are also irrelevant there. The large ’t Hooft coupling is expected to suppress the effects of such operators.

• This is suggested by the success of low-energy SVZ sum rules as compared to data.

• What are all gauge invariant YM operators of dimension 4 or less?

• They are given by $Tr[F_{\mu\nu}F_{\rho\sigma}]$.
Decomposing into U(4) reps:

$$\left(\begin{array}{|c|}\hline ~\\\hline ~\\\hline\end{array}\otimes\begin{array}{|c|}\hline ~\\\hline ~\\\hline\end{array}\right)_{\text{symmetric}}=\begin{array}{|c|c|}\hline ~&~\\\hline ~&~\\\hline\end{array}\oplus\begin{array}{|c|}\hline ~\\\hline ~\\\hline ~\\\hline ~\\\hline\end{array} \tag{3}$$

We must remove traces to construct the irreducible representations of O(4):

$$\begin{array}{|c|c|}\hline ~&~\\\hline ~&~\\\hline\end{array}=\underline{\begin{array}{|c|c|}\hline ~&~\\\hline ~&~\\\hline\end{array}}\oplus\underline{\begin{array}{|c|c|}\hline ~&~\\\hline\end{array}}\oplus\bullet\quad , \quad \begin{array}{|c|}\hline ~\\\hline ~\\\hline ~\\\hline ~\\\hline\end{array}=\bullet$$

The two singlets are the scalar (dilaton) and pseudoscalar (axion)

$$\phi \leftrightarrow Tr[F^2] \quad , \quad a \leftrightarrow Tr[F \wedge F]$$

The traceless symmetric tensor

$$\square\!\square \quad \to \quad T_{\mu\nu} = Tr\left[F^2_{\mu\nu} - \frac{1}{4} g_{\mu\nu} F^2\right]$$

is the conserved stress tensor dual to a massless graviton in 5d reflecting the translational symmetry of YM.

$$\boxplus \;\; \to \;\; T^4_{\mu\nu;\rho\sigma} = Tr[F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2}(g_{\mu\rho}F^2_{\nu\sigma} - g_{\nu\rho}F^2_{\mu\sigma} - g_{\mu\sigma}F^2_{\nu\rho} + g_{\nu\sigma}F^2_{\mu\rho}) + \frac{1}{6}(g_{\mu\rho}g_{\nu\sigma} - g_{\nu\rho}g_{\mu\sigma})F^2$$

It has 10 independent d.o.f, it is not conserved and it should correspond to a similar massive tensor in 5d. We do not expect it to play an non-trivial role in the large-N_c, YM vacuum also for reasons of Lorentz invariance.

♠ Therefore we will consider

$$T_{\mu\nu} \leftrightarrow g_{\mu\nu}, \; tr[F^2] \leftrightarrow \phi, \; tr[F \wedge F] \leftrightarrow a$$

bosonic string or superstring? I

• The string theory must have no on-shell fermionic states at all because there are no gauge invariant fermionic operators in pure YM. (even with quarks modulo baryons).

• There is a direct argument that the axion, dual to the instanton density $F \wedge F$ must be a RR field (as in $\mathcal{N} = 4$).

• Therefore the string theory must be a 5d-superstring theory resembling the II-0 class.

♠ Another RR field we expect to have is the RR 4-form, as it is necessary to "seed" the D_3 branes responsible for the gauge group.

• It is non-propagating in 5D

• We will see later however that it is responsible for the non-trivial IR structure of the gauge theory vacuum.

Bosonic string or superstring? II

• Consider the axion a dual to $Tr[F\wedge F]$. We can show that it must come from a RR sector.

In large-N$_c$ YM, the proper scaling of couplings is obtained from

$$\mathcal{L}_{YM} = N_c\; Tr\left[\frac{1}{\lambda}F^2 + \frac{\theta}{N_c}F\wedge F\right] \quad , \quad \zeta \equiv \frac{\theta}{N_c} \sim \mathcal{O}(1)$$

It can be shown

$$E_{YM}(\theta) \simeq C_0\; N_c^2 + C_1\theta^2 + C_2\frac{\theta^4}{N_c^2} + \cdots$$

Witten

In the string theory action

$$S \sim \int e^{-2\phi}\left[R + \cdots\right] + (\partial a)^2 + e^{2\phi}(\partial a)^4 + \cdots \quad , \quad e^{\phi} \sim g_{YM}^2 \quad , \quad \lambda \sim N_c e^{\phi}$$

$$\sim \int \frac{N_c^2}{\lambda^2}\left[R + \cdots\right] + (\partial a)^2 + \frac{\lambda^2}{N_c^2}(\partial a)^4 + \cdots \quad , \quad a = \theta[1 + \cdots]$$

RETURN

The minimal effective string theory spectrum

• NS-NS $\rightarrow$ $g_{\mu\nu} \leftrightarrow T_{\mu\nu}$, $B_{\mu\nu} \leftrightarrow Tr[F]^3$, $\phi \leftrightarrow Tr[F^2]$

• RR $\rightarrow$ Spinor$_5$×Spinor$_5$=$F_0 + F_1 + F_2 + (F_3 + F_4 + F_5)$

♠ $F_0 \leftrightarrow F_5 \rightarrow$ C_4, background flux $\rightarrow$ no propagating degrees of freedom.

♠ $F_1 \leftrightarrow F_4 \rightarrow$ $C_3 \leftrightarrow C_0$: C_0 is the axion, C_3 its 5d dual that couples to domain walls separating oblique confinement vacua.

♠ $F_2 \leftrightarrow F_3 \rightarrow$ $C_1 \leftrightarrow C_2$: They are associated with baryon number (as we will see later when we add flavor). C_2 mixes with B_2 because of the C_4 flux, and is massive.

• In an ISO(3,1) invariant vacuum solution, only $g_{\mu\nu}, \phi, C_0 = a$ can be non-trivial.

$$ds^2 = e^{2A(r)}(dr^2 + dx_4^2) \quad , \quad a(r), \phi(r)$$

The relevant “defects”

• $B_{\mu\nu} \rightarrow$ Fundamental string (F_1). This is the QCD (glue) string: fundamental tension $\ell_s^2 \sim \mathcal{O}(1)$

• Its dual $\tilde{B}_\mu \rightarrow NS_0$: Tension is $\mathcal{O}(N_c^2)$. It is an effective magnetic baryon vertex binding N_c magnetic quarks.

• $C_5 \rightarrow D_4$: Space filling flavor branes. They must be introduced in pairs: $D_4 + \bar{D}_4$ for charge neutrality/tadpole cancelation $\rightarrow$ gauge anomaly cancelation in QCD.

• $C_4 \rightarrow D_3$ branes generating the gauge symmetry.

.

• $C_3 \to D_2$ branes : domain walls separating different oblique confinement vacua (where $\theta_{k+1} = \theta_k + 2\pi$). Its tension is $\mathcal{O}(N_c)$

• $C_2 \to D_1$ branes: These are the magnetic strings: (strings attached to magnetic quarks) with tension $\mathcal{O}(N_c)$

• $C_1 \to D_0$ branes. These are the baryon vertices: they bind N_c quarks, and their tension is $\mathcal{O}(N_c)$.
Its instantonic source is the (solitonic) baryon in the string theory.

• $C_0 \to D_{-1}$ branes: These are the Yang-Mills instantons.

The effective action, I

- as $N_c \to \infty$, only string tree-level is dominant.

- Relevant field for the vacuum solution: $g_{\mu\nu}, a, \phi, F_5$.

- The vev of $F_5 \sim N_c\ \epsilon_5$. It appears always in the combination $e^{2\phi}F_5^2 \sim \lambda^2$, with $\lambda \sim N_c\ e^{\phi}$ All higher derivative corrections $(e^{2\phi}F_5^2)^n$ are $\mathcal{O}(1)$.
A non-trivial potential for the dilaton will be generated already at string tree-level.

- This is not the case for all other RR fields: in particular for the axion as $a \sim \mathcal{O}(1)$

$$(\partial a)^2 \sim \mathcal{O}(1) \quad , \quad e^{2\phi}(\partial a)^4 = \frac{\lambda^2}{N_c^2}(\partial a)^4 \sim \mathcal{O}\left(N_c^{-2}\right)$$

Therefore to leading order $\mathcal{O}(N_c^2)$ we can neglect the axion.

The UV regime

- In the far UV, the space should asymptote to AdS$_5$.

- The 't Hooft coupling should behave as $(r \to 0)$

$$\lambda \sim \frac{1}{\log(r\Lambda)} + \cdots \quad \to \quad 0 \quad , \quad r \sim \frac{1}{E}$$

The effective action to leading order in N_c is

$$S_{eff} \sim \int d^5x\sqrt{g}\; e^{-2\phi}\; Z(\; \ell_s^2 R \;,\; \ell_s^2(\partial\phi)^2 \;,\; e^{2\phi}\ell_s^2 F_5^2 \;)$$

Solving the equation of motion of F_5 amounts to replacing

$$e^{2\phi}\; \ell_s^2\; F_5^2 \sim e^{2\phi} N_c^2 \equiv \lambda^2$$

$$S_{eff} \sim N_c^2 \int d^5x\sqrt{g}\; \frac{1}{\lambda^2}\; H(\; \ell_s^2 R \;,\; \ell_s^2(\partial\lambda)^2 \;,\; \lambda^2 \;)$$

• As $r \to 0$

$$\text{Curvature} \to \text{finite} \quad , \quad \Box\phi \sim (\partial\phi)^2 \sim \frac{(\partial\lambda)^2}{\lambda^2} \sim \lambda^2 \sim \frac{1}{\log^2(r\Lambda)} \to 0$$

• For $\lambda \to 0$ the potential in the Einstein frame starts as $V(\lambda) \sim \lambda^{\frac{4}{3}}$ and cannot support the asymptotic AdS_5 solution.

• Therefore asymptotic AdS_5 must arise from curvature corrections:

$$S_{eff} \simeq \int d^5x \, \frac{1}{\lambda^2} \, H\left(\ell_s^2 \; R, 0, 0\right)$$

• Setting $\lambda = 0$ at leading order we can generically get an AdS_5 solution coming from balancing the higher curvature corrections.

INTERESTING QUESTION: Is there a good toy example of string vacuum (CFT) which is not Ricci flat, and is supported only by a metric?

• There is a "good" (but hard to derive the coefficients) perturbative expansion around this asymptotic AdS_5 solution by perturbing inwards :

$$e^A = \frac{\ell}{r}[1 + \delta A(r)] \quad , \quad \lambda = \frac{1}{b_0 \log(r\Lambda)} + \cdots$$

• This turns out to be a regular expansion of the solution in powers of

$$\frac{P_n(\log\log(r\Lambda))}{(log(r\Lambda))^{-n}}$$

• Effectively this can be rearranged as a "perturbative" expansion in $\lambda(r)$. In the case of running coupling, the radial coordinate can be substituted by $\lambda(r)$.

• Using λ as a radial coordinate the solution for the metric can be written

$$E \equiv e^A = \frac{\ell}{r(\lambda)}\left[1 + c_1\lambda + c_2\lambda^2 + \cdots\right] = \ell\,(e^{-\frac{b_0}{\lambda}})\left[1 + c_1'\lambda + c_2'\lambda^2 + \cdots\right] \quad , \quad \lambda \to 0$$

.

Conclusion 1: The asymptotic AdS_5 is stringy, but the rest of the geometry is "perturbative around the asymptotics". We cannot however do computations even if we know the structure.

Conclusion 2: It has been a mystery how can one get free field theory at the boundary. This is automatic here since all non-trivial connected correlators are proportional to positive powers of λ that vanishes in the UV.

The IR regime

• Here the situation is more obscure. The constraints/input will be: confinement, discreteness of the spectrum and mass gap.

• We do expect that $\lambda \to \infty$ (or becomes large) at the IR bottom.

• Intuition from N=4 and other 10d strongly coupled theories suggests that in this regime there should be an (approximate) two-derivative description of the physics.

• The simplest solution with this property is the linear dilaton solution with

$$\lambda \sim e^{Qr} \quad , \quad V(\lambda) \sim \delta c = 10 - D \quad \to \quad \text{constant} \quad , \quad R = 0$$

• This property persists with potentials $V(\lambda) \sim (\log \lambda)^P$. Moreover all such cases have confinement, a mass gap and a discrete spectrum (except the P=0 case).

• At the IR bottom (in the string frame) the scale factor vanishes, and 5D space becomes (asymptotically) flat.

Comments on confining backgrounds

• For all confining backgrounds with $r_0 = \infty$, although the space-time is singular in the Einstein frame, the string frame geometry is asymptotically flat for large r. Therefore only λ grows indefinitely.

• String world-sheets do not probe the strong coupling region, at least classically. The string stays away from the strong coupling region.

• Therefore: singular confining backgrounds have generically the property that the singularity is *repulsive*, i.e. only highly excited states can probe it. This will also be reflected in the analysis of the particle spectrum (to be presented later)

• The confining backgrounds must also screen magnetic color charges. This can be checked by calculating 't Hooft loops using D_1 probes:

♠ All confining backgrounds with $r_0 = \infty$ and most at finite r_0 screen properly

♠ In particular "hard-wall" AdS/QCD confines also the magnetic quarks.

Organizing the vacuum solutions

A useful variable is the phase variable

$$X \equiv \frac{\Phi'}{3A'} = \frac{\beta(\lambda)}{3\lambda} \quad , \quad e^{\Phi} \equiv \lambda$$

and a superpotential

$$W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \Phi}\right)^2 = \left(\frac{3}{4}\right)^3 V(\Phi).$$

with

$$A' = -\frac{4}{9}W \quad , \quad \Phi' = \frac{dW}{d\Phi}$$

$$X = -\frac{3}{4}\frac{d\log W}{d\log\lambda} \quad , \quad \beta(\lambda) = -\frac{9}{4}\lambda\frac{d\log W}{d\log\lambda}$$

♠ The equations have three integration constants: (two for Φ and one for A) One corresponds to the "gluon condensate" in the UV. It must be set to zero otherwise the IR behavior is unacceptable. The other is Λ. The third one is a gauge artifact (corresponds to overall translation in the radial coordinate).

The IR regime

For any asymptotically AdS$_5$ solution ($e^A \sim \frac{\ell}{r}$):

• The scale factor $e^{A(r)}$ is monotonically decreasing

Girardelo+Petrini+Porrati+Zaffaroni
Freedman+Gubser+Pilch+Warner

• Moreover, there are only three possible, mutually exclusive IR asymptotics:

♠ *there is another asymptotic AdS_5 region, at $r \to \infty$, where* $\exp A(r) \sim \ell'/r$, *and* $\ell' \leq \ell$ (equality holds if and only if the space is exactly AdS_5 everywhere);

♠ there is a curvature singularity at some finite value of the radial coordinate, $r = r_0$;

♠ **there is a curvature singularity at $r \to \infty$, where the scale factor vanishes and the space-time shrinks to zero size.**

Wilson-Loops and confinement

• Calculation of the static quark potential using the vev of the Wilson loop calculated via an F-string worldsheet.

Rey+Yee, Maldacena

$$T\ E(L) = S_{minimal}(X)$$

We calculate

$$L = 2\int_0^{r_0} dr \frac{1}{\sqrt{e^{4A_S(r)-4A_S(r_0)} - 1}}.$$

It diverges when e^{A_s} has a minimum (at $r = r_*$). Then

$$E(L) \sim T_f\ e^{2A_S(r_*)}\ L$$

• Confinement $\to$ $A_s(r_*)$ is finite. This is a more general condition that considered before as A_S **is not monotonic in general**.

• Effective string tension

$$T_{\text{string}} = T_f\ e^{2A_S(r_*)}$$

General criterion for confinement

• **the geometric version**:
A geometry that shrinks to zero size in the IR is dual to a confining 4D theory if and only if the Einstein metric in conformal coordinates vanishes as (or faster than) e^{-Cr} as $r \to \infty$, for some $C > 0$.

• It is understood here that a metric vanishing at finite $r = r_0$ also satisfies the above condition.

♠ **the superpotential**
A 5D background is dual to a confining theory if the superpotential grows as (or faster than)

$$W \sim (\log \lambda)^{P/2} \lambda^{2/3} \quad \text{as} \quad \lambda \to \infty \quad , \quad P \geq 0$$

♠ **the β-function** A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda\to\infty} \left(\frac{\beta(\lambda)}{3\lambda} + \frac{1}{2} \right) \log \lambda = K, \qquad -\infty \leq K \leq 0$$

(No explicit reference to any coordinate system) Linear trajectories correspond to $K = -\frac{3}{16}$

Classification of confining superpotentials

Classification of confining superpotentials $W(\lambda)$ as $\lambda \to \infty$ in IR:

$$W(\lambda) \sim (\log\lambda)^{\frac{P}{2}}\, \lambda^Q \quad , \quad \lambda \sim E^{-\frac{9}{4}Q}\left(\log\frac{1}{E}\right)^{\frac{P}{2Q}}, \qquad E \to 0.$$

- $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

$$e^A(r) \sim \begin{cases} (r_0 - r)^{\frac{4}{9Q^2-4}} & Q > \frac{2}{3} \\ \exp\left[-\frac{C}{(r_0-r)^{1/(P-1)}}\right] & Q = \frac{2}{3} \end{cases}$$

• $Q = 2/3$, and $0 \le P < 1$ leads to confinement and a singularity at $r = \infty$ The scale factor e^A vanishes there as

$$e^A(r) \sim \exp[-Cr^{1/(1-P)}].$$

• $Q = 2/3, P = 1$ leads to confinement but the singularity may be at a finite or infinite value of r depending on subleading asymptotics of the superpotential.

♠ If $Q < 2\sqrt{2}/3$, no *ad hoc* boundary conditions are needed to determine the glueball spectrum $\to$ One-to-one correspondence with the β-function This is unlike standard AdS/QCD and other approaches.

• when $Q > 2\sqrt{2}/3$, the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

Confining β-functions

A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda\to\infty}\left(\frac{\beta(\lambda)}{3\lambda}+\frac{1}{2}\right)\log\lambda = K, \qquad -\infty \le K \le 0$$

(No explicit reference to any coordinate system). Linear trajectories correspond to $K = -\frac{3}{16}$

- We can determine the geometry if we specify K:
- $K = -\infty$: the scale factor goes to zero at some finite r_0, not faster than a power-law.
- $-\infty < K < -3/8$: the scale factor goes to zero at some finite r_0 faster than any power-law.
- $-3/8 < K < 0$: the scale factor goes to zero as $r \to \infty$ faster than $e^{-Cr^{1+\epsilon}}$ for some $\epsilon > 0$.
- $K = 0$: the scale factor goes to zero as $r \to \infty$ as e^{-Cr} (or faster), but slower than $e^{-Cr^{1+\epsilon}}$ for any $\epsilon > 0$.

The borderline case, $K = -3/8$, is certainly confining (by continuity), but whether or not the singularity is at finite r depends on the subleading terms.

Particle Spectra: generalities

- Linearized equation:

$$\ddot{\xi} + 2\dot{B}\dot{\xi} + \Box_4 \xi = 0 \quad , \quad \xi(r,x) = \xi(r)\xi^{(4)}(x), \qquad \Box \xi^{(4)}(x) = m^2 \xi^{(4)}(x)$$

- Can be mapped to Schrodinger problem

$$-\frac{d^2}{dr^2}\psi + V(r)\psi = m^2\psi \quad , \quad V(r) = \frac{d^2B}{dr^2} + \left(\frac{dB}{dr}\right)^2 \quad , \quad \xi(r) = e^{-B(r)}\psi(r)$$

- Mass gap and discrete spectrum visible from the asymptotics of the potential.

- Large n asymptotics of masses obtained from WKB

$$n\pi = \int_{r_1}^{r_2} \sqrt{m^2 - V(r)}\; dr$$

- Spectrum depends only on initial condition for λ ($\sim \Lambda_{QCD}$) and an overall energy scale (e^A) that must be fixed.

- scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log\frac{\beta(\lambda)^2}{9\lambda^2}$$

- tensor glueballs

$$B(r) = \frac{3}{2}A(r)$$

- pseudo-scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log Z(\lambda)$$

- Universality of asymptotics

$$\frac{m^2_{n\to\infty}(0^{++})}{m^2_{n\to\infty}(2^{++})} \to 1 \quad , \quad \frac{m^2_{n\to\infty}(0^{+-})}{m^2_{n\to\infty}(0^{++})} = \frac{1}{4}(d-2)^2$$

predicts $d = 4$ via

$$\frac{m^2}{2\pi\sigma_a} = 2n + J + c,$$

Adding flavor

• To add N_f quarks q_L^I and antiquarks $q_R^{\bar{I}}$ we must add (in 5d) space-filling N_f D_4 and N_f $\bar{D}_4$ branes.
(tadpole cancellation=gauge anomaly cancellation)

• The q_L^I should be the "zero modes" of the $D_3 - D_4$ strings while $q_R^{\bar{I}}$ are the "zero modes" of the $D_3 - \bar{D}_4$

• The low-lying fields on the D_4 branes ($D_4 - D_4$ strings) are U$(N_f)_L$ gauge fields A_μ^L. The low-lying fields on the $\bar{D}_4$ branes ($\bar{D}_4 - \bar{D}_4$ strings) are U$(N_f)_R$ gauge fields A_μ^R. They are dual to the J_L^μ and J_μ^R

$$\delta S_A \sim \bar{q}_L^I \ \gamma^\mu \ (A_\mu^L)^{IJ} \ q_L^J + \bar{q}_R^{\bar{I}} \ \gamma^\mu \ (A_\mu^R)^{\bar{I}\bar{J}} \ q_R^{\bar{J}} = Tr[J_L^\mu \ A_\mu^L + J_R^\mu \ A_\mu^R]$$

• There are also the low lying fields of the ($D_4 - \bar{D}_4$ strings), essentially the string-theory "tachyon" $T_{I\bar{J}}$ transforming as $(N_f, \bar{N}_f)$ under the chiral symmetry $U(N_f)_L \times U(N_f)_R$. It is dual to the quark mass terms

$$\delta S_T \sim \bar{q}_L^I \ T_{I\bar{J}} \ q_R^{\bar{J}} + \text{complex} \quad \text{congugate}$$

• The interactions on the flavor branes are weak, so that $A_\mu^{L,R}, T$ are as sources for the quarks.

• Integrating out the quarks, generates an effective action $S_{flavor}(A_\mu^{L,R}, T)$, so that $A_\mu^{L,R}, T$ can be thought as effective $q\bar{q}$ composites, that is : mesons

• On the string theory side: integrating out $D_3 - D_4$ and $D_3 - \bar{D}_4$ strings gives rise to the DBI action for the $D_4 - \bar{D}_4$ branes in the D_3 background:

$$S_{flavor}(A_\mu^{L,R}, T) \quad \longleftrightarrow \quad S_{DBI}(A_\mu^{L,R}, T) \quad \text{holographically}$$

• In the "vacuum" only T can have a non-trivial profile: $T^{I\bar{J}}(r)$. Near the AdS_5 boundary ($r \to 0$)

$$T^{I\bar{J}}(r) = M_{I\bar{J}}\, r + \cdots + \langle \bar{q}_L^I\, q_R^{\bar{J}} \rangle r^3 + \cdots$$

Casero+Kiritsis+Paredes

• A typical solution is T vanishing in the UV and $T \to \infty$ in the IR. At the point $r = r_*$ where $T = \infty$, the D_4 and $\bar{D}_4$ branes "fuse". The true vacuum is a brane that enters folds on itself and goes back to the boundary. A non-zero T breaks chiral symmetry.

• A GOR relation is satisfied (for an asymptotic AdS$_5$ space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\,\langle \bar{q}q \rangle \quad , \quad m_q \to 0$$

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

• Fluctuations around the T solution for $T, A_\mu^{L,R}$ give the spectra (and interactions) of various meson trajectories.

• Studying the spectrum of highly excited mesons, we find the expected property of linear confinement: $m_n^2 \sim n$.

• The detailed spectrum of mesons remains to be worked out

Quarks ($N_f \ll N_c$) and mesons

• Flavor is introduced by N_f $D_4 + \bar{D}_4$ branes pairs inside the bulk background. Their back-reaction on the bulk geometry is suppressed by N_f/N_c.

• The important world-volume fields are

$$T_{ij} \leftrightarrow \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \quad , \quad A^{ij\,L,R}_\mu \leftrightarrow \bar{q}^i_a \frac{1\pm\gamma^5}{2}\gamma^\mu q^j_a$$

Generating the $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

• The UV mass matrix m_{ij} corresponds to the source term of the Tachyon field. It breaks the chiral (gauge) symmetry. The normalizable mode corresponds to the vev $\langle \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \rangle$.

• We show that the expectation value of the tachyon is non-zero and $T \sim \mathbf{1}$, breaking chiral symmetry $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$. The anomaly plays an important role in this (holographic Coleman-Witten)

• The fact that the tachyon diverges in the IR (fusing D with $\bar{D}$) constraints the UV asymptotics and determines the quark condensate $\langle \bar{q}q \rangle$ in terms of m_q. A GOR relation is satisfied (for an asymptotic AdS$_5$ space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2} \langle \bar{q}q \rangle \quad , \quad m_q \to 0$$

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

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• Studying the spectrum of highly excited mesons, we find the expected property of linear confinement: $m_n^2 \sim n$.

• The detailed spectrum of mesons remains to be worked out

Tachyon dynamics

- In the vacuum the gauge fields vanish and $T \sim \mathbf{1}$. Only DBI survives

$$S[\tau] = T_{D_4} \int dr d^4x \, \frac{e^{4A_s(r)}}{\lambda} \, V(\tau) \, \sqrt{e^{2A_s(r)} + \dot{\tau}(r)^2} \quad , \quad V(\tau) = e^{-\frac{\mu^2}{2}\tau^2}$$

- We obtain the nonlinear field equation:

$$\ddot{\tau} + \left(3\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right)\dot{\tau} + e^{2A_S}\mu^2\tau + e^{-2A_S}\left[4\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right]\dot{\tau}^3 + \mu^2\tau \, \dot{\tau}^2 = 0.$$

- In the UV we expect

$$\tau = m_q \, r + \sigma \, r^3 + \cdots \quad , \quad \mu^2\ell^2 = 3$$

- We expect that the tachyon must diverge before or at $r = r_0$. We find that indeed it does `at the singularity`. For the $r_0 = \infty$ backgrounds

$$\tau \sim \exp\left[\frac{2}{a} \, \frac{R}{\ell^2} \, r\right] \qquad \text{as} \qquad r \to \infty$$

• Generically the solutions have spurious singularities: $\tau(r_*)$ stays finite but its derivatives diverges as:

$$\tau \sim \tau_* + \gamma\sqrt{r_* - r}.$$

The condition that they are absent determines σ as a function of m_q.

• The easiest spectrum to analyze is that of vector mesons. We find ($r_0 = \infty$)

$$\Lambda_{glueballs} = \frac{1}{R}, \qquad \Lambda_{mesons} = \frac{3}{\ell}\left(\frac{\alpha\ell^2}{2R^2}\right)^{(\alpha-1)/2} \propto \frac{1}{R}\left(\frac{\ell}{R}\right)^{\alpha-2}.$$

This suggests that $\alpha = 2$. preferred also from the glue sector.

The axion background

- The axion solution can be interpreted as a "running" θ-angle

- This is in accordance with the absence of UV divergences and Seiberg-Witten type solutions.

- The axion action is down by $1/N_c^2$

$$S_{axion} = -\frac{M_p^3}{2}\int d^5x\sqrt{g}\ Z(\lambda)\ (\partial a)^2$$

$$\lim_{\lambda\to 0} Z(\lambda) = Z_0\left[1 + c_1\lambda + c_2\lambda^2 + \cdots\right] \quad , \quad \lim_{\lambda\to\infty} Z(\lambda) = c_a\lambda^4 + \cdots$$

- The equation of motion is

$$\ddot{a} + \left(3\dot{A} + \frac{\dot{Z}(\lambda)}{Z(\lambda)}\right)\dot{a} = 0 \quad \to \quad \dot{a} = \frac{C\ e^{-3A}}{Z(\lambda)}$$

- The full solution is

$$a(r) = \theta_{UV} + 2\pi k + C\ \int_0^r dr\frac{e^{-3A}}{Z(\lambda)} \quad , \quad C = \langle Tr[F\wedge F]\rangle$$

- $a(r)$ is a running effective θ-angle. Its running is non-perturbative,

$$a(r) \sim r^4 \sim e^{-\frac{4}{b_0\lambda}}$$

- The vacuum energy is

$$E(\theta_{UV}) = -\frac{M^3}{2N_c^2}\int d^5x\sqrt{g}\ Z(\lambda)\ (\partial a)^2 = -\frac{M^3}{2N_c^2}Ca(r)\ \Big|_{r=0}^{r=r_0}$$

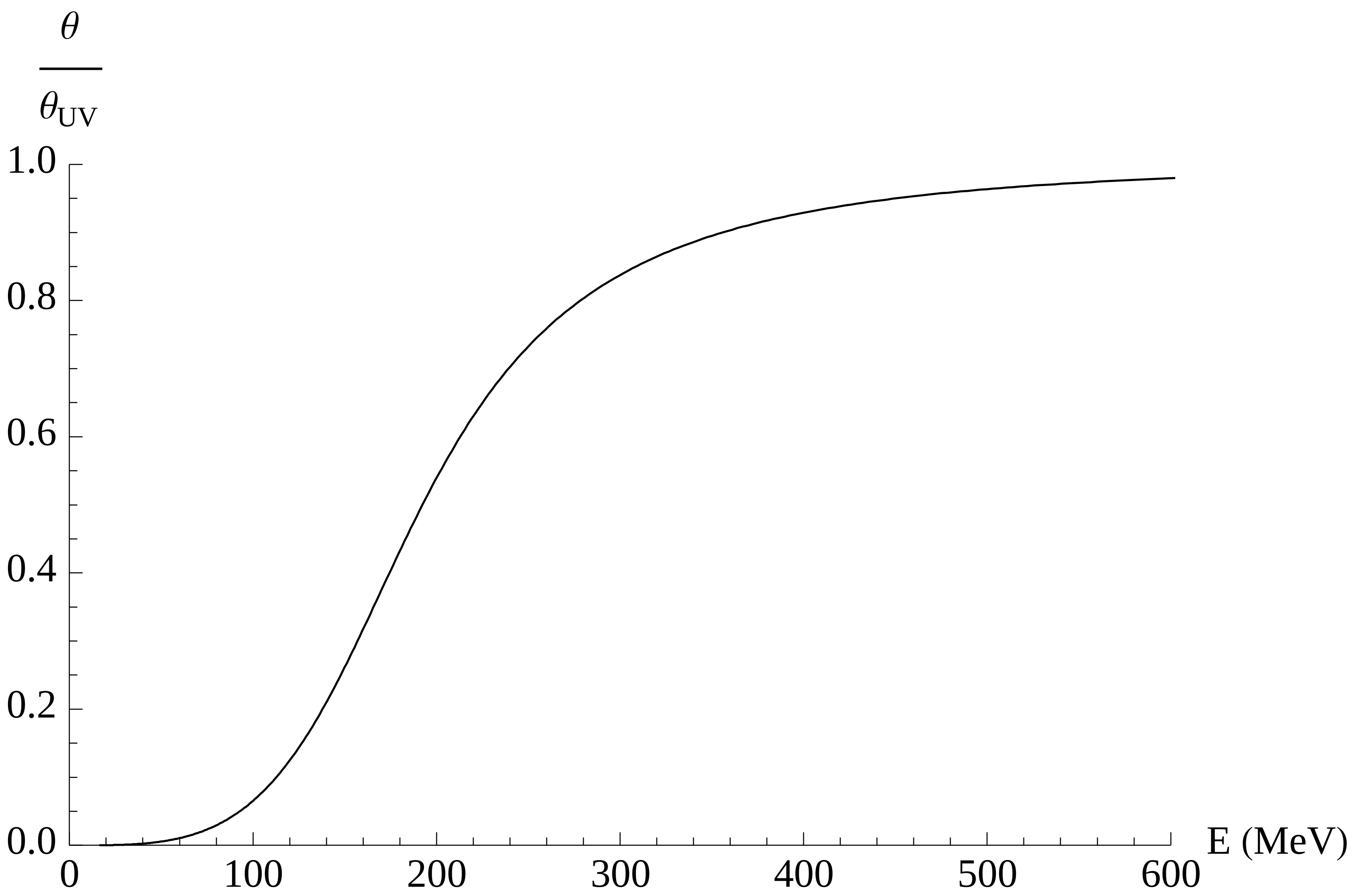
- Consistency requires to impose that $a(r_0) = 0$. This determines C and

$$E(\theta_{UV}) = \frac{M^3}{2}\text{Min}_k\ \frac{(\theta_{UV}+2\pi k)^2}{\int_0^{r_0}\frac{dr}{e^{3A}Z(\lambda)}}$$

$$\frac{a(r)}{\theta_{UV}+2\pi k} = \frac{\int_r^{r_0}\frac{dr}{e^{3A}Z(\lambda)}}{\int_0^{r_0}\frac{dr}{e^{3A}Z(\lambda)}}$$

- The topological susceptibility is given by

$$E(\theta) = \frac{1}{2}\chi\ \theta^2 + \mathcal{O}(\theta^4) \quad , \quad \chi = \frac{M_p^3}{\int_0^{r_0}\frac{dr}{e^{3A}Z(\lambda)}}$$



We take: $Z(\lambda) = Z_0(1 + c_a\lambda^4)$

An assessment of IR asymptotics

- We define the superpotential W as

$$V(\lambda) = \frac{4}{3}\lambda^2 \left(\frac{dW}{d\lambda}\right)^2 + \frac{64}{27} W^2$$

- We parameterize the UV ($\lambda \to 0$) and IR asymptotics ($\lambda \to \infty$) as

$$V(\lambda) = \frac{12}{\ell^2}[1 + \mathcal{O}(\lambda)] \quad , \quad V(\lambda) \sim V_\infty \lambda^Q (\log\lambda)^P$$

- All confining solutions have an IR singularity.

There are three types of solution for W:

- The "Good type" (single solution)

$$W(\lambda) \sim (\log\lambda)^{\frac{P}{2}} \ \lambda^{\frac{Q}{2}}$$

It leads to a "good" IR singularity, confinement, a mass gap, discrete spectrum of glueballs and screening of magnetic charges if

$$\frac{8}{3} > Q > \frac{4}{3} \quad or \quad Q = \frac{4}{3} \quad and \quad P > 0$$

.

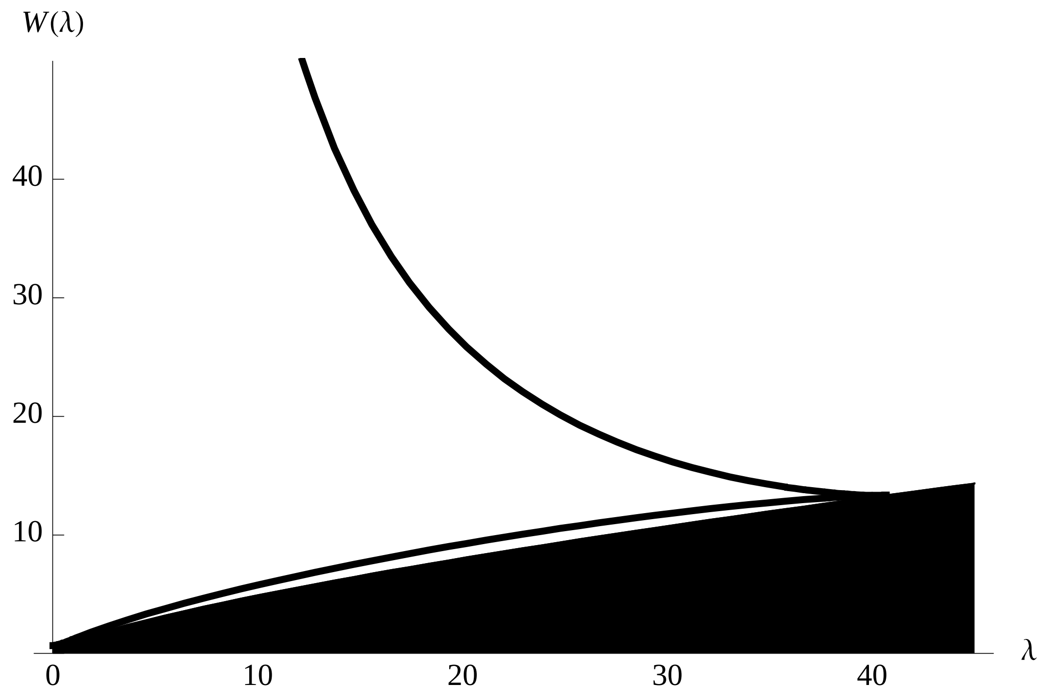
• The asymptotic spectrum of glueballs is linear if $Q = \frac{4}{3}$ and $P = \frac{1}{2}$.

• The Bad type. This is a one parameter family of solutions with

$$W(\lambda) \sim \lambda^{\frac{4}{3}}$$

It has a bad IR singularity.

♠ The Ugly type. This is a one parameter family of solutions. In such solutions there are two branches but they never reach the IR $\lambda \to \infty$. Instead λ goes back to zero



Selecting the IR asymptotics

The $Q = 4/3$, $0 \leq P < 1$ solutions have a singularity at $r = \infty$. They are compatible with

- Confinement (it happens non-trivially: a minimum in the string frame scale factor)
- Mass gap+discrete spectrum (except P=0)
- good singularity
- $R \to 0$ justifying the original assumption. More precisely: the string frame metric becomes flat at the IR .

♠ It is interesting that the lower endpoint: P=0 corresponds to linear dilaton and flat space (string frame). It is confining with a mass gap but continuous spectrum.

- For linear asymptotic trajectories for fluctuations (glueballs) we must choose $P = 1/2$

$$V(\lambda) =\sim \lambda^{\frac{4}{3}}\sqrt{\log\lambda} + \text{subleading} \qquad \text{as} \qquad \lambda \to \infty$$

Concrete potential

- The superpotential chosen is

$$W = (3 + 2b_0\lambda)^{2/3}\left[18 + \left(2b_0^2 + 3b_1\right)\log(1+\lambda^2)\right]^{4/3},$$

with corresponding potential

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3+2b_0\lambda} - \frac{6(2b_0^2+3b_1^2)\lambda^3}{(1+\lambda^2)\left(18+\left(2b_0^2+3b_1^2\right)\log(1+\lambda^2)\right)}$$

which is everywhere regular and has the correct UV and IR asymptotics.

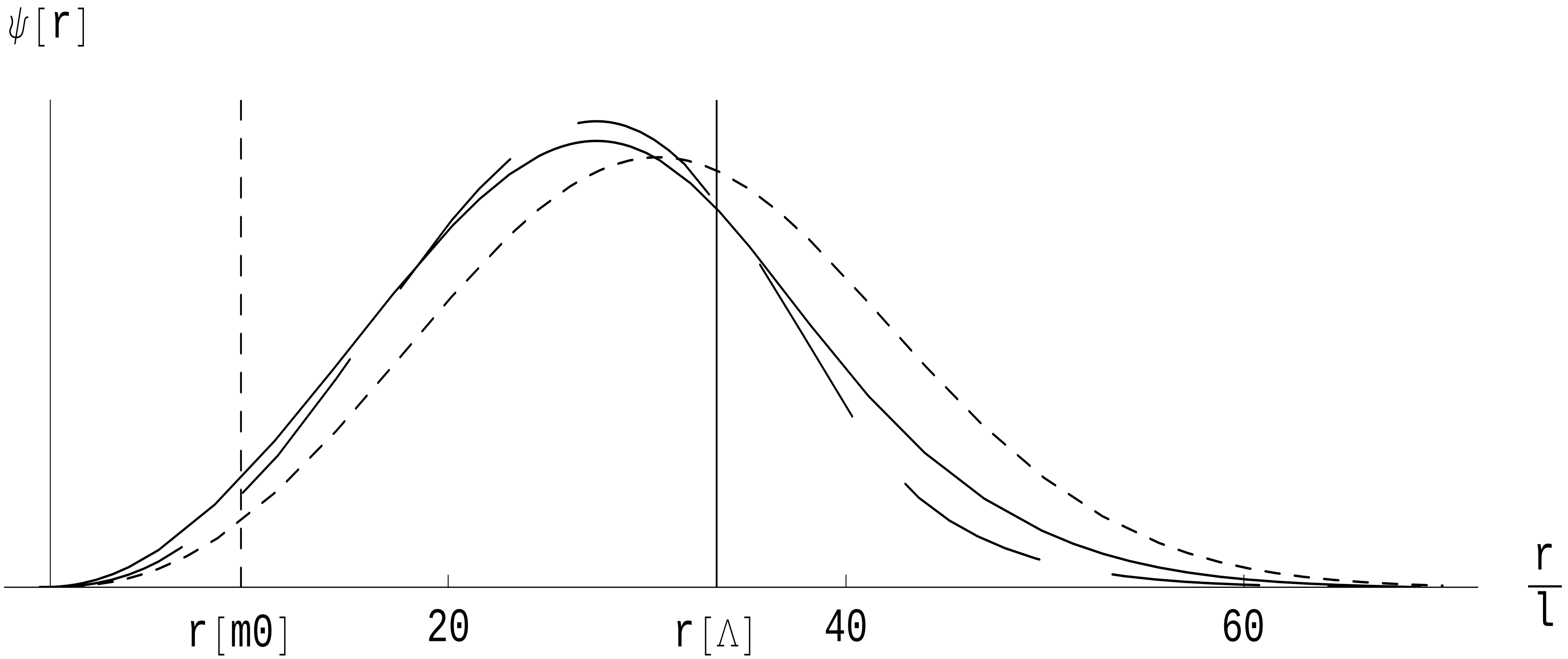
- b_0 is a free parameter and b_1/b_0^2 is taken from the QCD β-function

The fit to glueball lattice data

J^{PC}	Ref I (MeV)	Our model (MeV)	Mismatch	$N_c \to \infty$	Mismatch
0^{++}	**1475 (4%)**	**1475**	0	**1475**	0
2^{++}	2150 (5%)	2055	4%	2153 (10%)	5%
0^{-+}	**2250 (4%)**	**2243**	0		
0^{++*}	**2755 (4%)**	**2753**	0	2814 (12%)	2%
2^{++*}	2880 (5%)	2991	4%		
0^{-+*}	3370 (4%)	3288	2%		
0^{++**}	3370 (4%)	3561	5%		
0^{++***}	3990 (5%)	4253	6%		

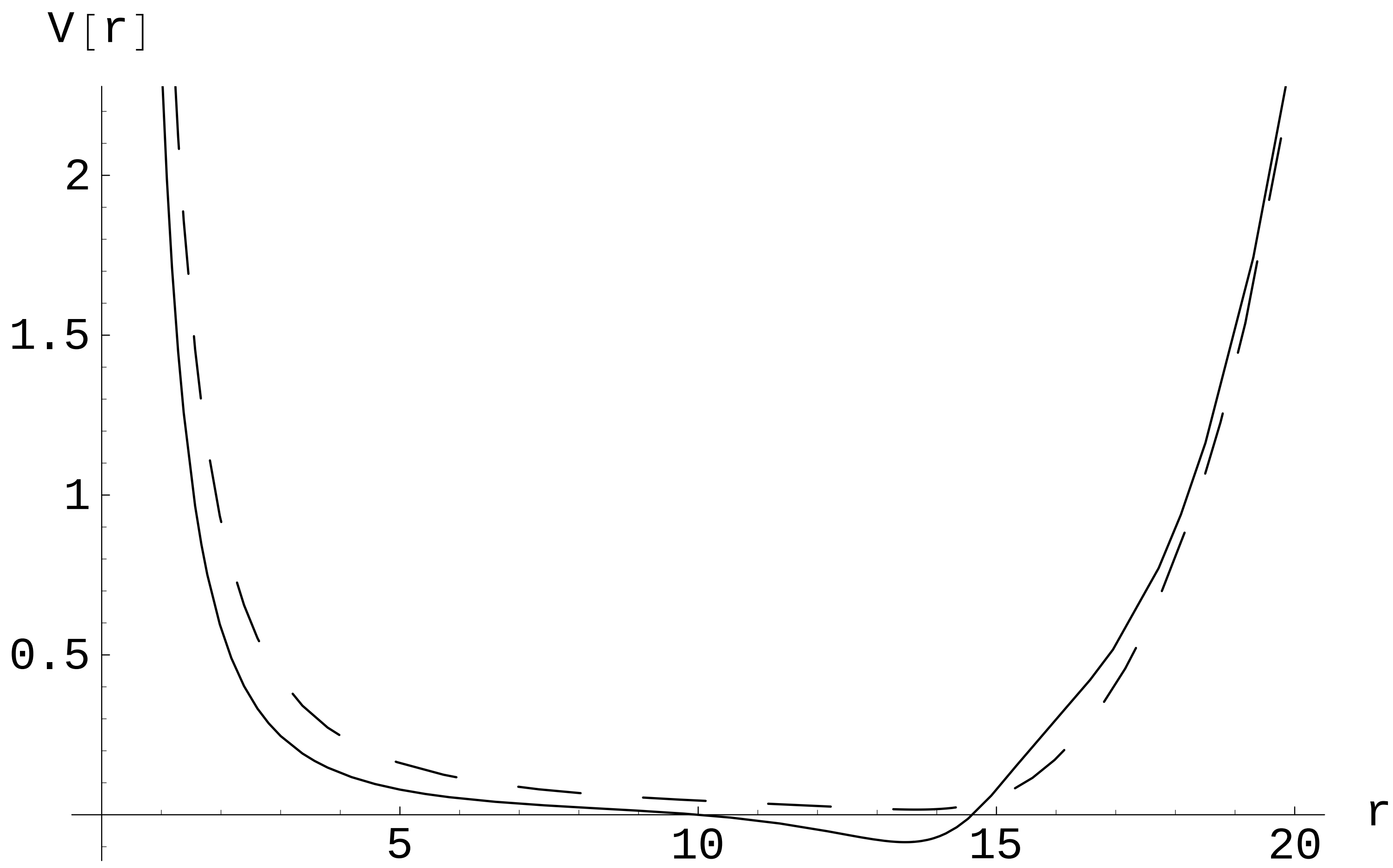
Comparison between the glueball spectra in Ref. I and in our model. The states we use as input in our fit are marked in red. The parenthesis in the lattice data indicate the percent accuracy.

The glueball wavefunctions



Normalized wave-function profiles for the ground states of the 0^{++} (solid line) ,0^{-+} (dashed line), and 2^{++} (dotted line) towers, as a function of the radial conformal coordinate. The vertical lines represent the position corresponding to $E = m_{0^{++}}$ and $E = \Lambda_p$.

Comparison of scalar and tensor potential



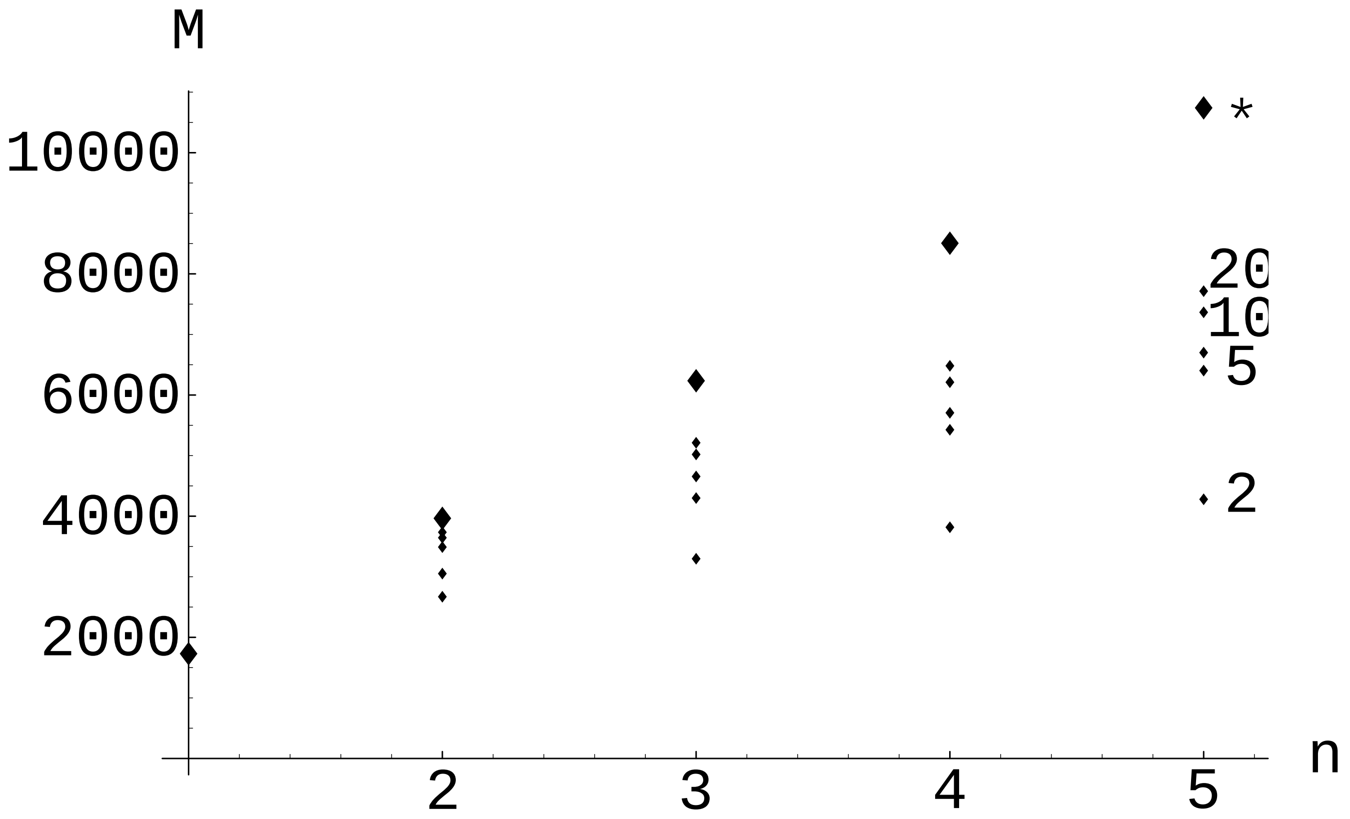
Effective Schrödinger potentials for scalar (solid line) and tensor (dashed line) glueballs. The units are chosen such that $\ell = 0.5$.

The lattice glueball data

J^{++}	Ref. I $(m/\sqrt{\sigma})$	Ref. I (MeV)	Ref. II (mr_0)	Ref. II (MeV)	$N_c \to \infty (m/\sqrt{\sigma})$
0	3.347(68)	1475(30)(65)	4.16(11)(4)	1710(50)(80)	3.37(15)
0^*	6.26(16)	2755(70)(120)	6.50(44)(7)	2670(180)(130)	6.43(50)
0^{**}	7.65(23)	3370(100)(150)	NA	NA	NA
0^{***}	9.06(49)	3990(210)(180)	NA	NA	NA
2	4.916(91)	2150(30)(100)	5.83(5)(6)	2390(30)(120)	4.93(30)
2^*	6.48(22)	2880(100)(130)	NA	NA	NA
R_{20}	1.46(5)	1.46(5)	1.40(5)	1.40(5)	1.46(11)
R_{00}	1.87(8)	1.87(8)	1.56(15)	1.56(15)	1.90(17)

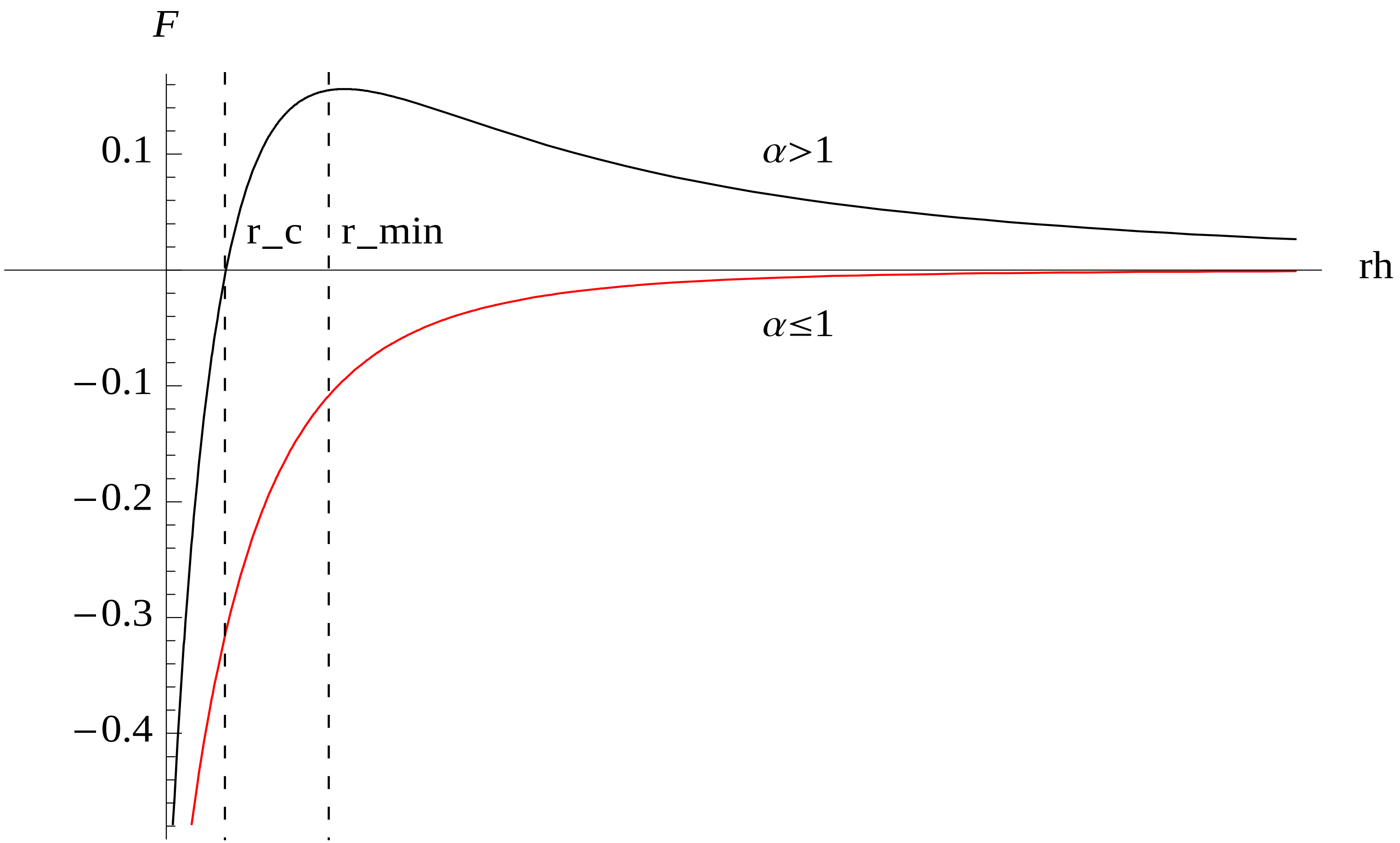
Available lattice data for the scalar and the tensor glueballs. Ref. I =H. B. Meyer, [arXiv:hep-lat/0508002]. and Ref. II = C. J. Morningstar and M. J. Peardon, [arXiv:hep-lat/9901004] + Y. Chen *et al.*, [arXiv:hep-lat/0510074]. The first error corresponds to the statistical error from the the continuum extrapolation. The second error in Ref.I is due to the uncertainty in the string tension $\sqrt{\sigma}$. (Note that this does not affect the mass ratios). The second error in the Ref. II is the estimated uncertainty from the anisotropy. In the last column we present the available large N_c estimates according to B. Lucini and M. Teper, [arXiv:hep-lat/0103027]. The parenthesis in this column shows the total possible error followed by the estimations in the same reference.

α-dependence of scalar spectrum



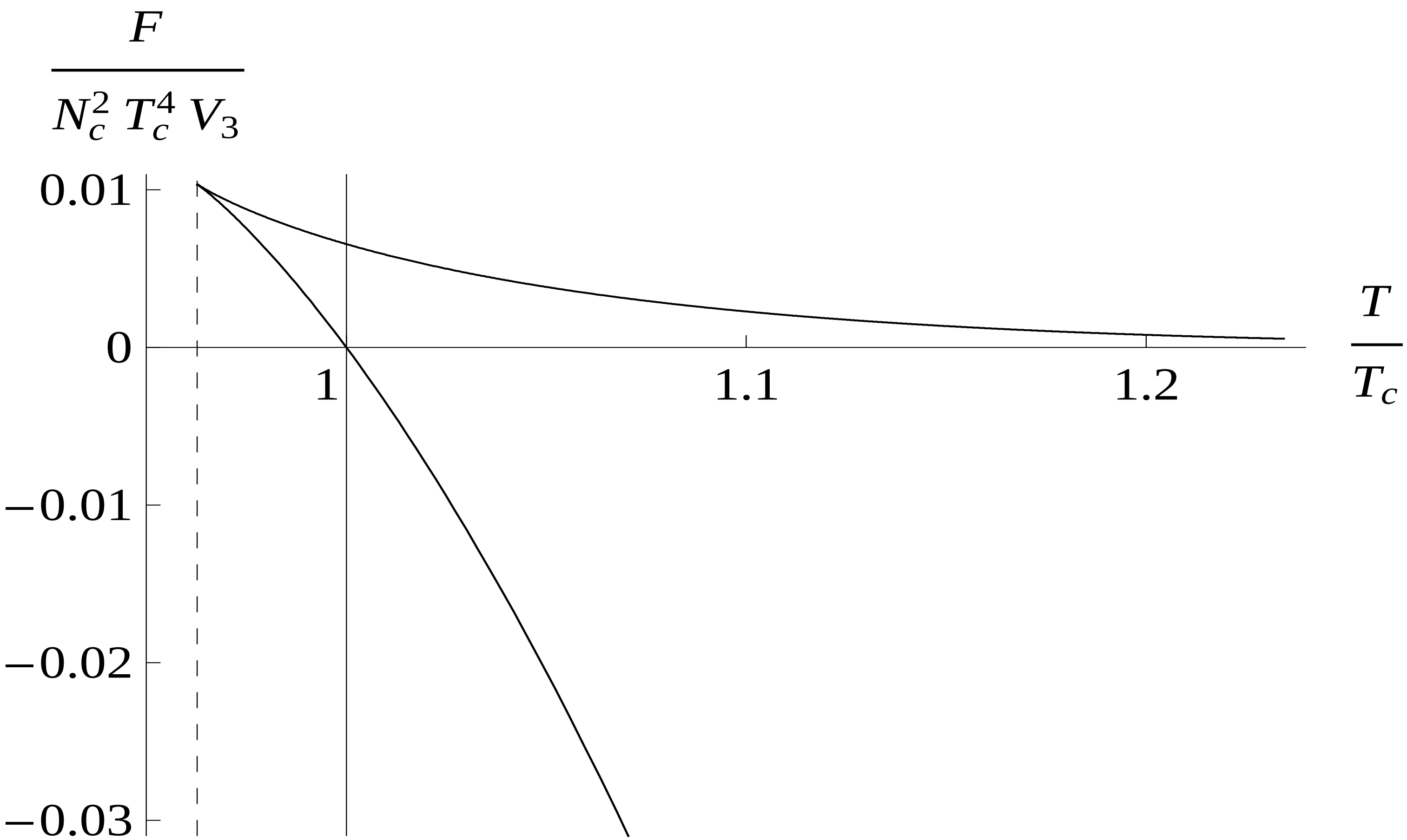
The 0^{++} spectra for varying values of α that are shown at the right end of the plot. The symbol $*$ denotes the AdS/QCD result.

Free energy versus horizon position



We plot the relation $\mathcal{F}(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is no first order phase transition .

The transition in the free energy



- G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lutgemeier and B. Petersson, *"Thermodynamics of SU(3) Lattice Gauge Theory,"* Nucl. Phys. B **469**, 419 (1996) [arXiv:hep-lat/9602007].

- B. Lucini, M. Teper and U. Wenger, *"Properties of the deconfining phase transition in SU(N) gauge theories,"* JHEP **0502**, 033 (2005) [arXiv:hep-lat/0502003];
  "SU(N) gauge theories in four dimensions: Exploring the approach to $N = \infty$," JHEP **0106**, 050 (2001) [arXiv:hep-lat/0103027].

- Y. Chen *et al.*, *"Glueball spectrum and matrix elements on anisotropic lattices,"* Phys. Rev. D **73** (2006) 014516 [arXiv:hep-lat/0510074].

- L. Del Debbio, L. Giusti and C. Pica, *"Topological susceptibility in the SU(3) gauge theory,"* Phys. Rev. Lett. **94**, 032003 (2005) [arXiv:hep-th/0407052].

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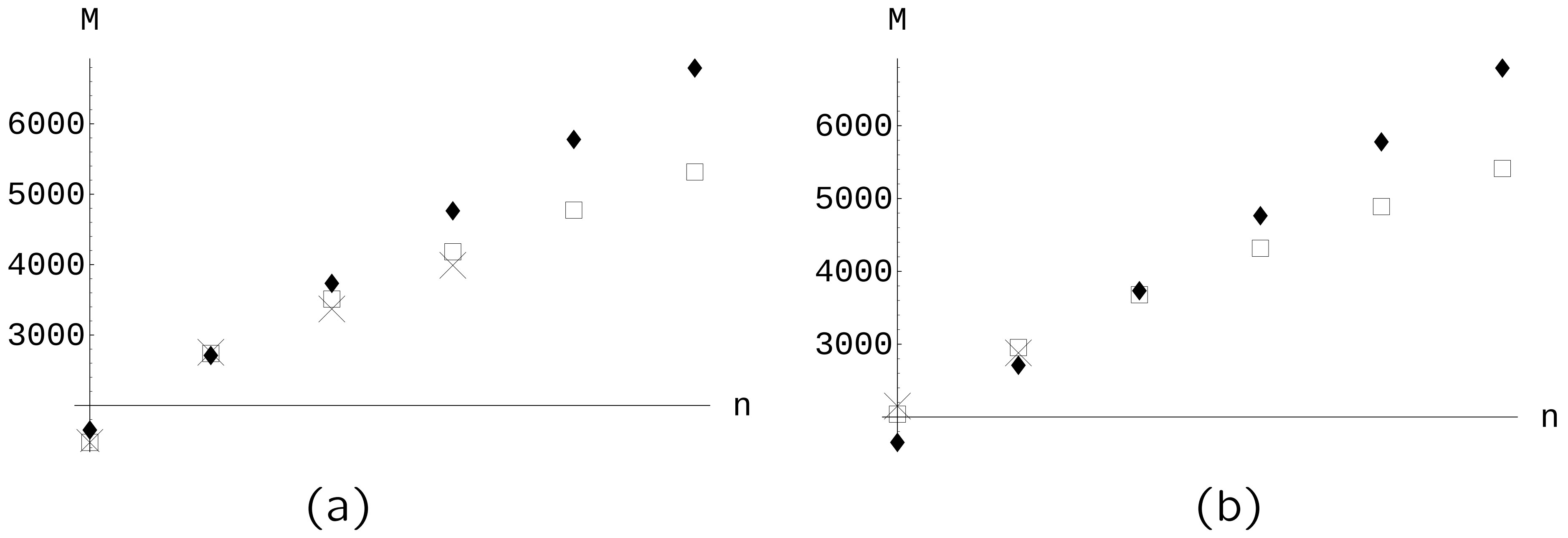
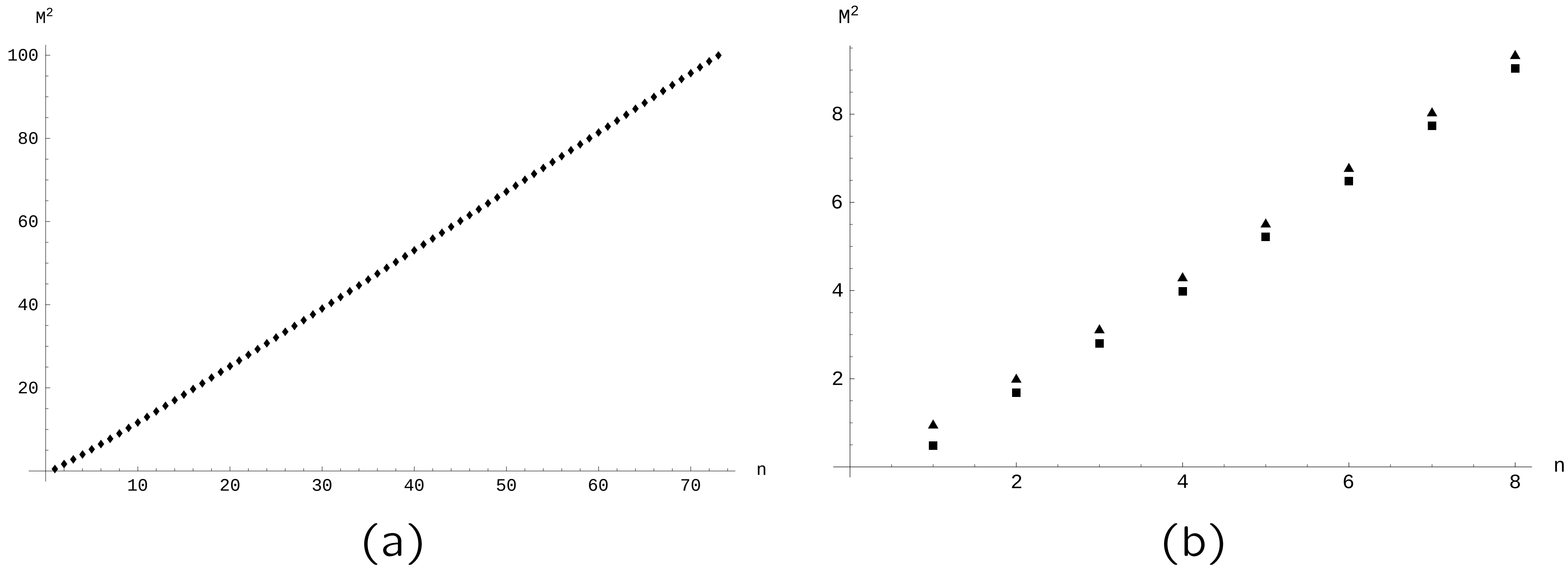
Comparison with lattice data



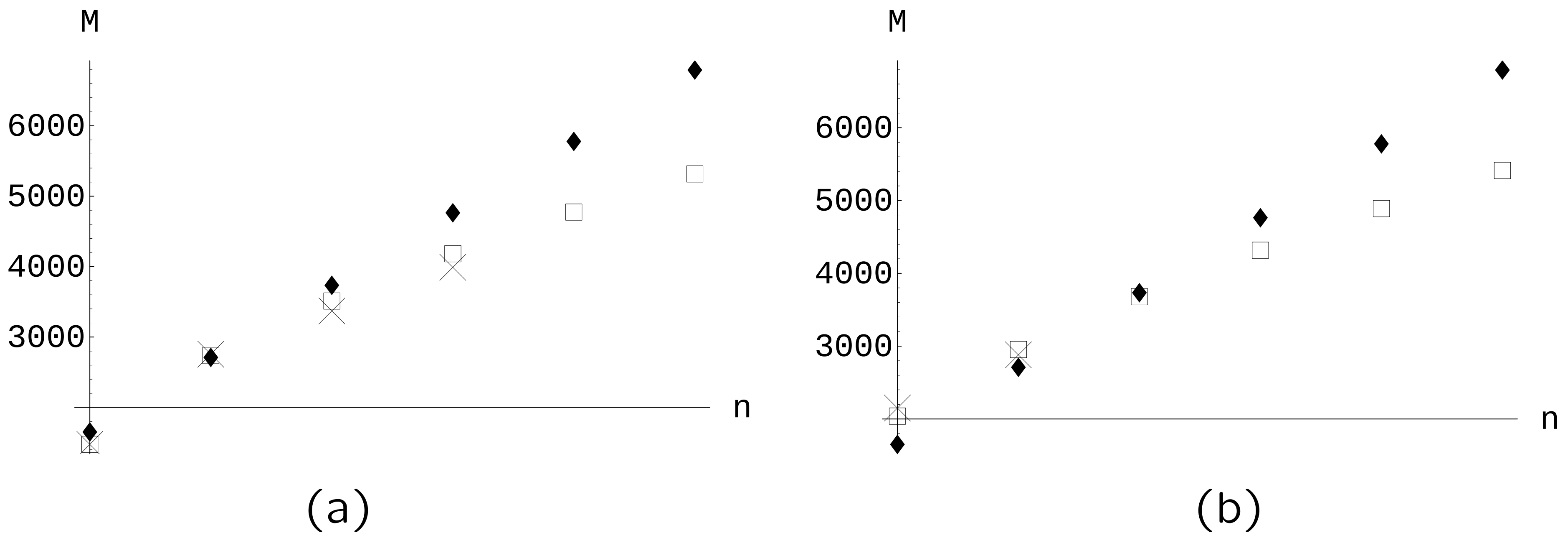
Linearity of the glueball spectrum



(a) Linear pattern in the spectrum for the first 40 0^{++} glueball states. M^2 is shown units of $0.015\ell^{-2}$.

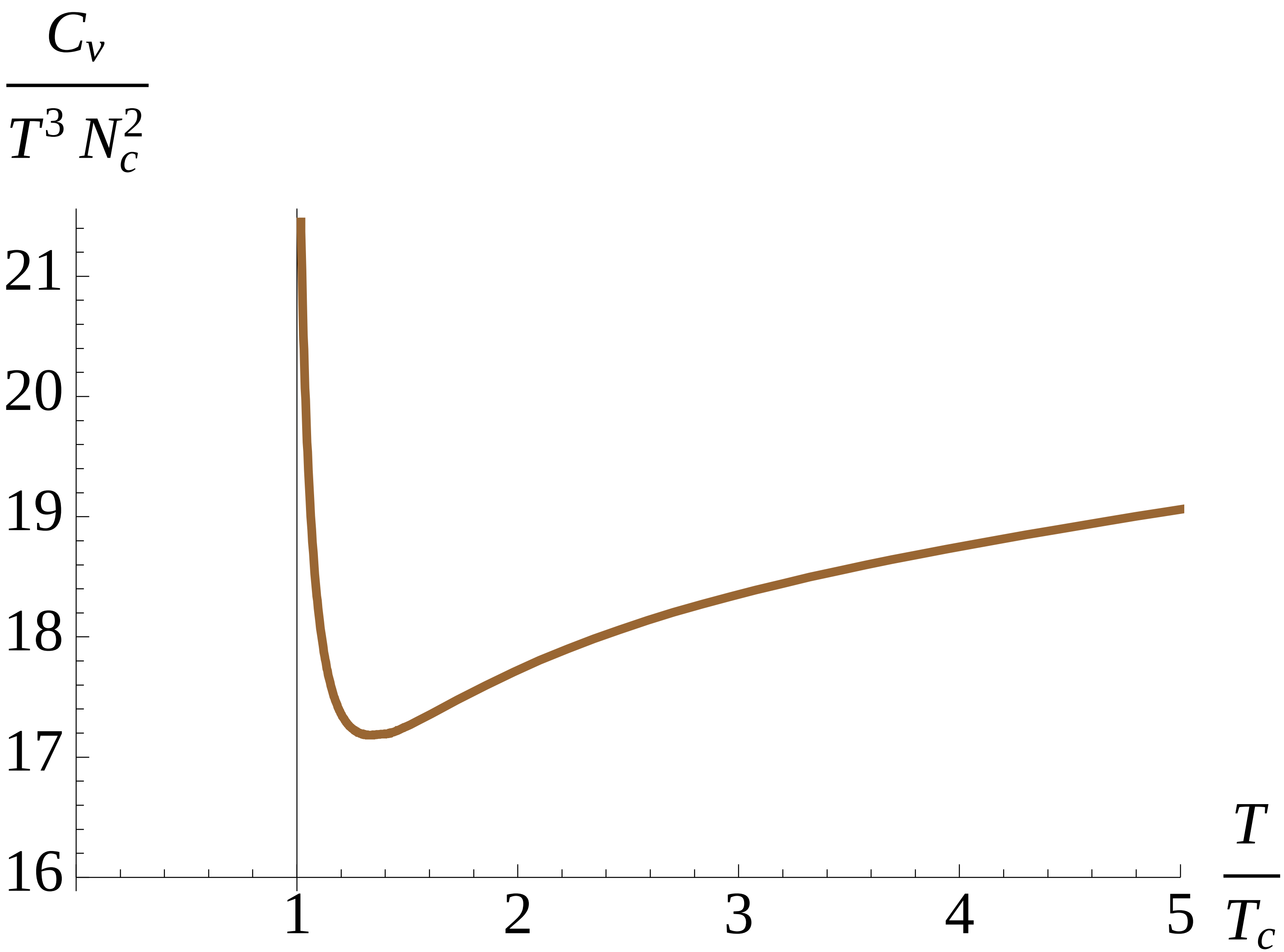
(b) The first 8 0^{++} (squares) and the 2^{++} (triangles) glueballs. These spectra are obtained in the background I with $b_0 = 4.2, \lambda_0 = 0.05$.

Comparison with lattice data (Meyer)



Comparison of glueball spectra from our model with $b_0 = 4.2, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. I (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. I.

The specific heat



The sum rule method (details)

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt\int d^3x\ e^{i\omega t}\langle[T_{ii}(\vec{x},t),T_{jj}(\vec{0},0)]\rangle$$

We use

$$\langle[\int d^3x T_{00}(\vec{x},0),O]\rangle_{equ} = \langle[H,O]\rangle_{equ} = i\langle\frac{\partial O}{\partial t}\rangle_{equ} = 0$$

and rewrite

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt\int d^3x\ e^{i\omega t}\langle[\Theta(\vec{x},t),\Theta(\vec{0},0)]\rangle \quad , \quad \Theta = T_\mu{}^\mu$$

$$\zeta = \lim_{\omega\to 0}\frac{1}{9\omega}\int dt\int d^3x\ e^{i\omega t}\ iG^R(x) = \lim_{\omega\to 0}\frac{1}{9\omega}\ iG^R(\omega) = -\lim_{\omega\to 0}\frac{1}{9\omega}Im\ G^R(\omega)$$

We now use

$$\Theta = m\bar{q}q + \frac{\beta(g)}{2g}Tr[F^2] = \Theta_F + \Theta_G$$

We also use

$$\langle[\Theta,O]\rangle = \left(T\frac{\partial}{\partial T} - d\right)\langle O\rangle$$

RETURN

Parameters

• We have 3 initial conditions in the system of graviton-dilaton equations:

♠ One is fixed by picking the branch that corresponds asymptotically to $\lambda \sim \frac{1}{\log(r\Lambda)}$

♠ The other fixes $\Lambda \to \Lambda_{QCD}$.

♠ The third is a gauge artifact as it corresponds to a choice of the origin of the radial coordinate.

• We parameterize the potential as

$$V(\lambda) = \frac{12}{\ell^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\left[\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)\right]^{1/2}\right\},$$

• We fix the one and two loop β-function coefficients:

$$V_0 = \frac{8}{9}b_0 \quad , \quad V_2 = b_0^4\left(\frac{23 + 36b_1/b_0^2}{81V_1^2}\right)^2, \quad \frac{b_1}{b_0^2} = \frac{51}{121}.$$

and remain with two leftover arbitrary (phenomenological) coefficients.

• We also have the Planck scale M_p

Asking for correct $T \to \infty$ thermodynamics (free gas) fixes

$$(M_p \ell)^3 = \frac{1}{45\pi^2} \quad , \quad M_{\text{physical}} = M_p N_c^{\frac{2}{3}} = \left(\frac{8}{45\pi^2 \ell^3}\right)^{\frac{1}{3}} \simeq 4.6 \text{ GeV}$$

• The fundamental string scale. It can be fixed by comparing with lattice string tension

$$\sigma = \frac{b^2(r_*)\lambda^{4/3}(r_*)}{2\pi\ell_s^2},$$

$\ell/\ell_s \sim \mathcal{O}(1)$.

• ℓ is not really a parameter as it can be rescaled into a redefinition of λ.

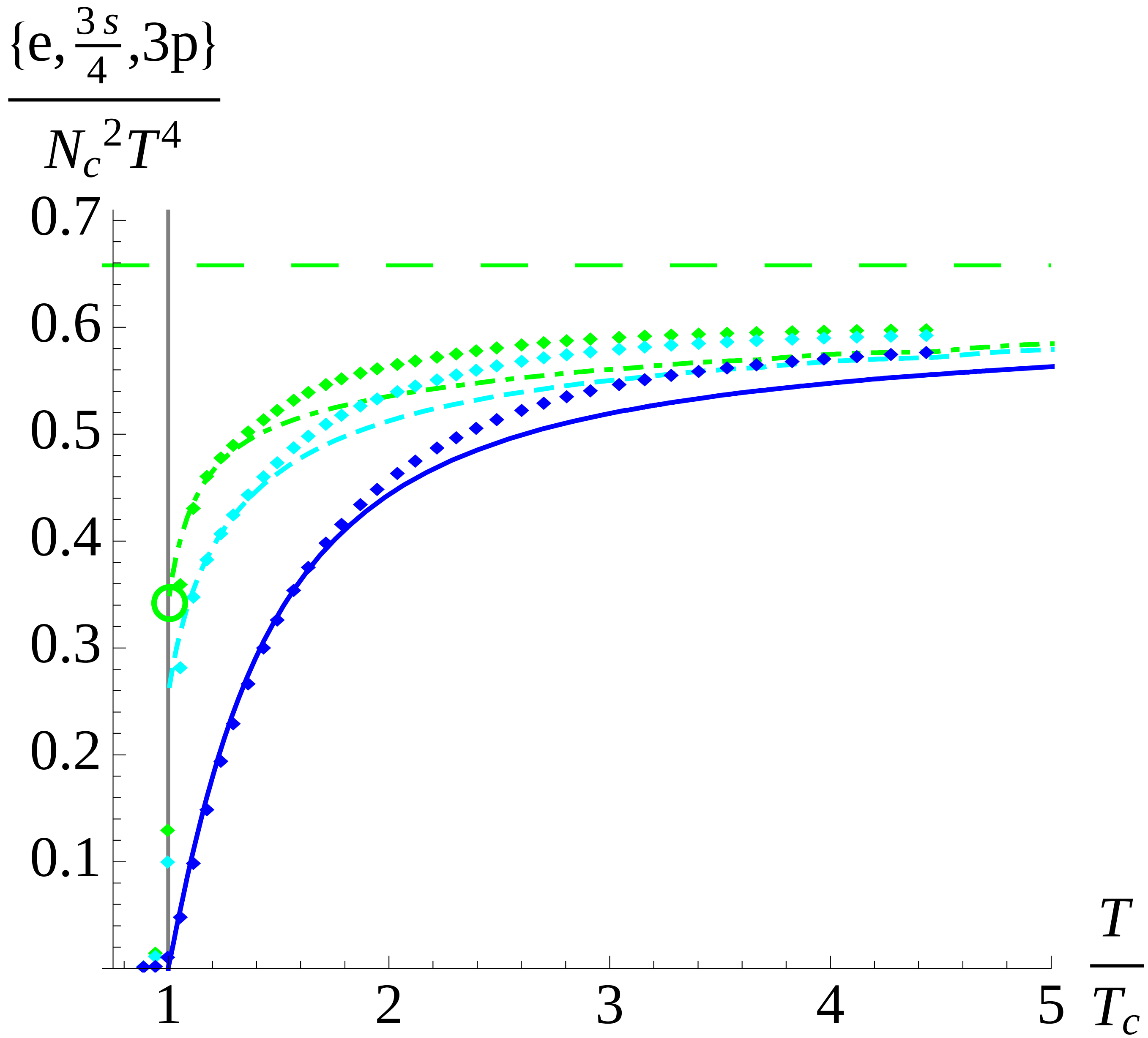
♠ In the CP-odd sector (axion) there are two more parameters:

$$Z(\lambda) = Z_0(1 + c_a\lambda^4)$$

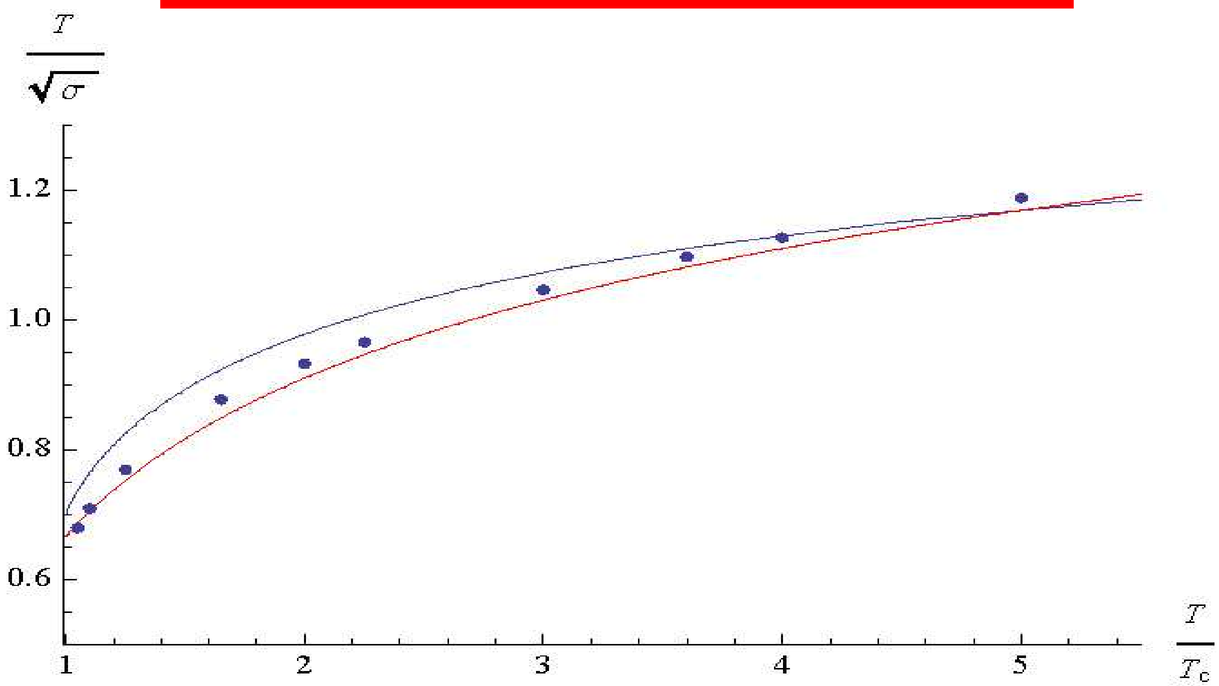
Fit and comparison

	IhQCD	lattice $N_c = 3$	lattice $N_c \to \infty$	Parameter
$[p/(N_c^2 T^4)]_{T=2T_c}$	**1.2**	**1.2**	-	$V1 = 14$
$L_h/(N_c^2 T_c^4)$	**0.31**	0.28 (Karsch)	**0.31** (Teper+Lucini)	$V3 = 170$
$[p/(N_c^2 T^4)]_{T\to+\infty}$	$\pi^2/45$	$\pi^2/45$	$\pi^2/45$	$M_p \ell = [45\pi^2]^{-1/3}$
$m_{0^{++}}/\sqrt{\sigma}$	**3.37**	3.56 (Chen)	**3.37** (Teper+Lucini)	$\ell_s/\ell = 0.92$
$m_{0^{-+}}/m_{0^{++}}$	**1.49**	**1.49** (Chen)	-	$c_a = 0.26$
χ	$(191 MeV)^4$	$(191 MeV)^4$ (DelDebbio)	-	$Z_0 = 133$
$T_c/m_{0^{++}}$	0.167	-	0.177(7)	
$m_{0^{*++}}/m_{0^{++}}$	1.61	1.56(11)	1.90(17)	
$m_{2^{++}}/m_{0^{++}}$	1.36	1.40(4)	1.46(11)	
$m_{0^{*-+}}/m_{0^{++}}$	2.10	2.12(10)	-	

Thermodynamic variables



Spatial string tension



G. Boyd et al. 1996

• The blue line is the spatial string tension as calculated in Improved hQCD, with no additional fits.

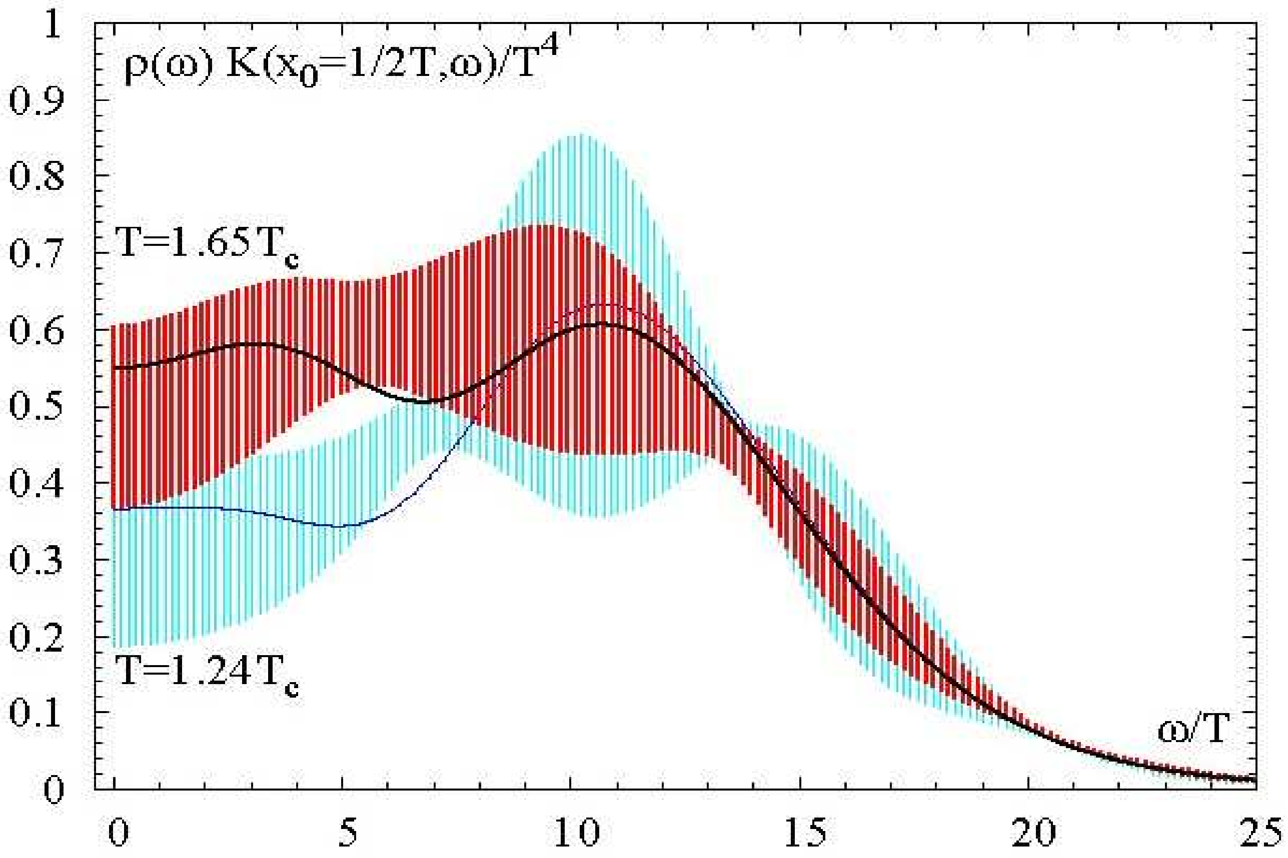
Nitti (unpublished) 2009

• The red line is a semi-phenomenological fit using

$$\frac{T}{\sqrt{\sigma_s}} = 0.51 \ \left[\log\frac{\pi T}{T_c} + \frac{51}{121}\log\left(2\log\frac{\pi T}{T_c}\right)\right]^{\frac{2}{3}}$$

Alanen+Kajantie+Suur-Uski, 2009

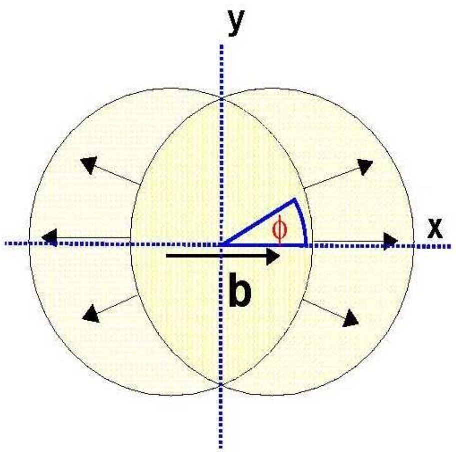
Shear Viscosity bounds from lattice



H. Meyer 2007

$$4\pi\frac{\eta}{s} = \begin{cases} 1.68(42), & T = 1.65\ T_c, \\ 1.28(70), & T = 1.24\ T_c. \end{cases}$$

Elliptic Flow



$$\frac{1}{p_T}\frac{dN}{dp_T d\phi} = \frac{1}{p_T}\frac{dN}{dp_T}\left[1 + v_2(p_T)\cos 2\phi + \cdots\right]$$

The sum rule method

- Define the (subtracted) spectral density and relate its moment to the Euclidean density

$$\rho(\omega) = -\frac{1}{\pi} Im\ G^R(\omega) \quad , \quad \mathcal{G} \equiv \lim_{\omega \to 0} G^E(\omega) = 2\int_0^\infty \frac{\rho(u)}{u}\, du$$

Karsch+Kharzeev+Tuchin, 2008, Romatschke+Son 2009

- Using Ward identities we obtain the sum rule

$$\mathcal{G} = \left(T\frac{\partial}{\partial T} - 4\right)(E - 3P + \langle\Theta\rangle_0) + \left(T\frac{\partial}{\partial T} - 2\right)(m\langle\bar{q}q\rangle_T + \langle\Theta_F\rangle_0)$$

with

$$\langle\Theta_F\rangle_0 = m\langle\bar{q}q\rangle \simeq -m_\pi^2\ f_\pi^2 - m_K^2\ f_K^2$$

- Assume a density

$$\frac{\rho(\omega)}{\omega} = \frac{9\zeta}{\pi}\frac{{\omega_0}^2}{\omega^2 + {\omega_0}^2}$$

The bulk viscosity: theory

- This is harder to calculate.

- Using a parametrization $ds^2 = e^{2A}(f dt^2 + d\vec{x}^2 + \frac{dr^2}{f})$ in a special gauge $\phi = r$ the relevant metric perturbation decouples

Gubser+Nellore+Pufu+Rocha 2008, Gubser+Pufu+Rocha,2008

$$h''_{11} = -\left(-\frac{1}{3A'} - A' - \frac{f'}{f}\right) h'_{11} + \left(-\frac{\omega^2}{f^2} + \frac{f'}{6fA'} - \frac{f'}{f}A'\right) h_{11}$$

with

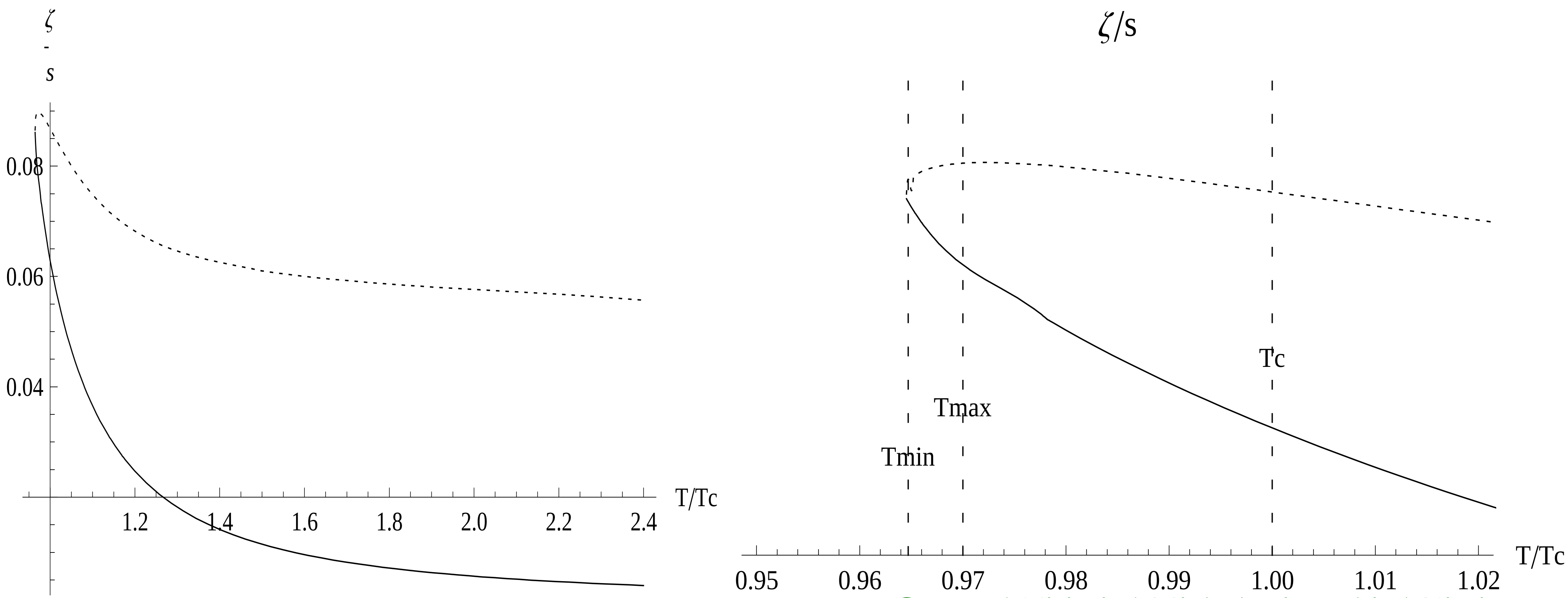
$$h_{11}(0) = 1 \quad , \quad h_{11}(r_h) \simeq C\; e^{i\omega t} \left| \log \frac{\lambda}{\lambda_h} \right|^{-\frac{i\omega}{4\pi T}}$$

The correlator is given by the conserved number of h-quanta

$$Im\; G_R(\omega) = -4M^3 \mathcal{G}(\omega) \quad , \quad \mathcal{G}(\omega) = \frac{e^{3A} f}{4A'^2} |Im[h^*_{11} h'_{11}]|$$

$$\frac{\zeta}{s} = \frac{C^2}{4\pi} \left(\frac{V'(\lambda_h)}{V(\lambda_h)}\right)^2$$

The bulk viscosity in the small black hole



Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

• At the turning point the behavior, $C_V \to \infty$ and ζ behaves similar to that observed in the N=2* theory

Buchel+Pagnutti, 2008

• The small black-hole bulk viscosity ratio asymptotes to a constant as $T \to \infty$.

High-T asymptotics of transport coefficients

• In CFTs perturbed with a relevant operator the speed of sound is bounded above as

$$c_s^2 = \frac{1}{3} - \frac{(4-\Delta)(4-2\Delta)\Gamma\left[\frac{\Delta}{4}\right]^4 \tan(\pi\Delta/4)}{18\pi\ \Gamma\left[\frac{\Delta}{2}-1\right]^2}(\pi\ell T)^{2(\Delta-4)} + \mathcal{O}\left(T^{3(\Delta-4)}\right)$$

Cherman+Cohen+Nellore 2009, Hohler+Stephanov, 2009

• The same is true $\left(c_s^2 \simeq \frac{1}{3} - \mathcal{O}\left(\frac{1}{\log^2 T}\right)\right)$ in the logarithmic case $\Delta = 4$.

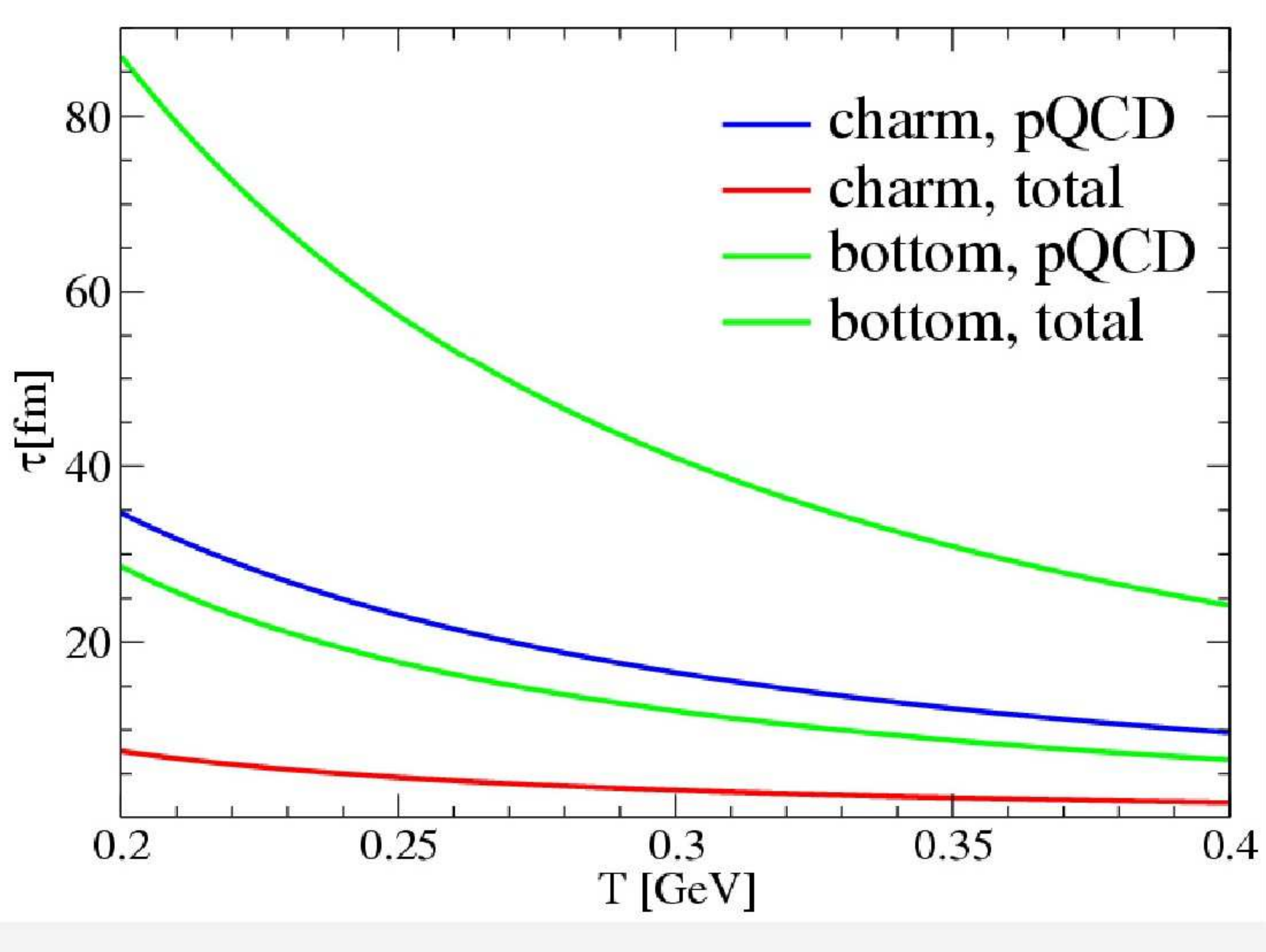
Gursoy+Kiritsis+Mazzanti+Nitti,2008

• In general for single relevant perturbations, if $\xi_i \in \left(\frac{\zeta}{s}, v_s^2, 2\pi T D, \frac{\sigma}{\pi T}, \frac{\Xi}{2\pi^2 T^2}\right)$ as $T \to \infty$

$$\xi_i(T) = \xi_i^{CFT} + \mathcal{C}_i(\Delta)\ T^{-2(4-\Delta)} + \mathcal{O}\left(T^{3(\Delta-4)}\right)$$

Cherman+Nellore 2009

Rapp results



• In qualitative agreement with Rapp et al. (talk at Quark Matter 2009) with a different method of calculation.

Langevin diffusion of heavy quarks (details)

• In a thermal medium we would expect the analogue of Brownian motion for heavy quarks.

• Fluctuations were first studied around the trailing string solution and diffusion coefficients were calculated.

Cassalderey-Solana+Teaney, 2006 Gubser 2006

• A full Langevin-like treatment was derived recently for non-relativistic quarks

Son+Teaney 2009 DeBoer+Hubeny+Rangamani+Shigenori, 2009

• This describes a Langevin process of the form

$$\frac{d\vec{p}}{dt} = \vec{F} + \vec{\xi} \quad , \quad \vec{F} = -\eta\ \vec{p} \quad , \quad \langle \xi^i(t)\xi^j(t')\rangle = \kappa\delta^{ij}\delta(t-t')$$

$\vec{F}$ is the drag force, $\eta = \frac{1}{\tau}$.

• The fully relativistic case was also described recently

Giecold+Iancu+Mueller, 2009

We consider fluctuations around the dragging string solution in the thermal background

$$ds^2 = b^2(r)\left(\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x}^2\right) \quad , \quad X^1 = vt + \xi(r) + \delta X^1 \quad , \quad X^{2,3} = \delta X^{2,3}$$

The Nambu-Goto action is expanded as

$$S = S_0 + S_1 + S_2 + \cdots \quad , \quad S_1 = \int d\tau dr\ P^\alpha\ \partial_\alpha \delta X^1 \tag{4}$$

with

$$S_2 = \frac{1}{2\pi\ell_s^2} \int d\tau dr\ \left[\frac{G^{\alpha\beta}}{2}\partial_\alpha \delta X^1 \partial_\beta \delta X^1 + \sum_{i=2}^{3} \frac{\tilde{G}^{\alpha\beta}}{2}\partial_\alpha \delta X^i \partial_\beta \delta X^i\right] \tag{5}$$

with

$$G^{\alpha\beta} = \frac{b^2(r)Z(r)^2}{2}\, g^{\alpha\beta} \quad , \quad \tilde{G}^{\alpha\beta} = \frac{b(r)^2}{2}\, g^{\alpha\beta} \quad , \quad Z(r) = \sqrt{1 + f(r)\xi'(r)^2 - \frac{v^2}{f(r)}}$$

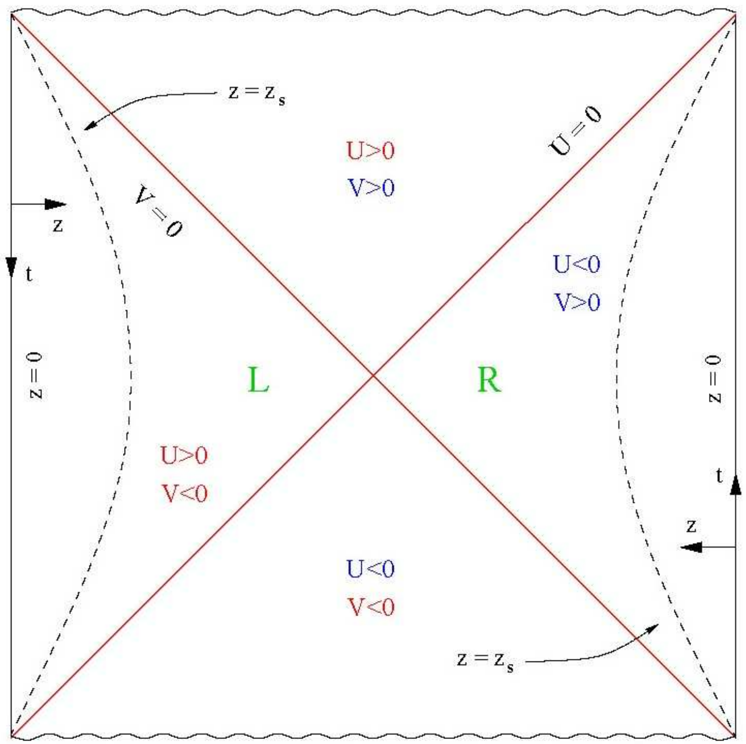
- The fluctuations δX^i satisfy.

$$\partial_\alpha G^{\alpha\beta}\partial_\beta\ \delta X^1 = 0 \quad , \quad \partial_\alpha \tilde{G}^{\alpha\beta}\partial_\beta\ \delta X^{2,3} = 0$$

• The metric in which they are evaluated is of the bh type, but with a different Hawking temperature, T_H. In the CFT case we have $T_H = \sqrt{1-v^2}\ T$

• We double the fields, $\delta X \to \delta X_{L,R}$ and we can define retarded and advanced correlators using the Schwinger-Keldysh formalism as implemented in AdS/CFT

Herzog+Son 2002, Gubser 2006, Skenderis+VanRees 2009



$$S_{\text{boundary}} = \int d\tau_R \left[-P^r \delta X^0_R + \frac{1}{2} \delta X^0_R G^{r\alpha} \partial_\alpha \delta X^0_R \right] - (L \leftrightarrow R)$$

$$= -\int \frac{d\omega}{2\pi}\ \delta X^0_a(-\omega) G^R(\omega) \delta X^0_r(\omega) + \frac{i}{2} \int \frac{d\omega}{2\pi}\ \delta X^0_a(-\omega) G^{sym}(\omega) \delta X^0_a(\omega)$$

with

$$\delta X_r = \frac{1}{2}(\delta X_L + \delta X_R) \quad , \quad \delta X_a = (\delta X_L - \delta X_R)$$

and

$$G_{sym}(\omega) = \frac{1 + e^{\frac{\omega}{T_H}}}{1 - e^{\frac{\omega}{T_H}}}\ G_R(\omega)$$

- We may derive a Langevin equation by starting with

$$Z = \int [D\delta X^0_{L,R}][D\delta X_{L,R}]\ e^{i(S_R - S_L)} = \int [D\delta X^0_{a,r}]\ e^{iS_{\text{boundary}}}$$

and introduce a dummy variable ξ to linearize the quadratic term of the a-fields

$$Z = \int [D\delta X^0_{a,r}][D\xi]\ e^{-\frac{1}{2}\int dtdt'\xi(t)G^{-1}_{sym}(t,t')\xi(t')}\times$$

$$\times \exp\left[-i\int dtdt'\ \delta X^0_a \left[G_R(t,t')\delta X^0_r(t') + \delta(t-t')(P^r - \xi(t'))\right]\right]$$

Integration over $\delta X^0_{a,r}$ gives the Langevin system

$$\int dt'\ G_R(t,t')\delta X^0_r(t') + P^r - \xi(t) = 0 \quad , \quad \langle \xi(t)\xi(t')\rangle = G_{\mathrm{sym}}(t,t')$$

Giecold+Iancu+Mueller, 2009

• For $|t-t'|$ large we can replace the retarded propagator with a (second) time derivative and the symmetric one by a δ-function to finally obtain in the conformal case

$$\frac{dp^i_\perp}{dt} = -\eta\ p^i_\perp + \xi^i_\perp \quad , \quad \langle \xi^i_\perp(t)\xi^j_\perp(t')\rangle = \kappa_\perp \delta^{ij}\delta(t-t') \quad , \quad \kappa_\perp = \frac{\pi\sqrt{\lambda}T^3}{(1-v^2)^{\frac{1}{4}}}$$

$$\frac{dp_\parallel}{dt} = -\eta\ p_\parallel + \xi_\parallel \quad , \quad \langle \xi_\parallel(t)\xi_\parallel(t')\rangle = \kappa_\parallel \delta(t-t') \quad , \quad \kappa_\parallel = \frac{\pi\sqrt{\lambda}T^3}{(1-v^2)^{\frac{5}{4}}}$$

• In the non-relativistic limit the world-sheet horizon and the spacetime horizon coincide. In this case there is a Maxwell equilibrium distribution and the Einstein relation ($\kappa = 2ET\eta$) holds.

Cassalderey-Solana+Teaney, 2006 Gubser 2006

Son+Teaney 2009 DeBoer+Hubeny+Rangamani+Shigenori, 2009

• The diffusion is asymmetric in the relativistic case. There is no thermal equilibrium distribution. This resolves previous puzzles of symmetric relativistic Langevin diffusion.

• The failure of the Einstein relation was also seen in the heavy-ion data.

Wolchin 1999

• The (conformal) relativistic Langevin equation with symmetric diffusion was applied to data analysis at RHIC, but the Einstein relation was kept.

Akamatsu+Hatsuda+Hirano, 2008

In view of the above a re-analysis seems necessary.

The sum rule method (details)

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt\int d^3x\; e^{i\omega t}\langle[T_{ii}(\vec{x},t),T_{jj}(\vec{0},0)]\rangle$$

We use

$$\langle[\int d^3x T_{00}(\vec{x},0),O]\rangle_{equ} = \langle[H,O]\rangle_{equ} = i\langle\frac{\partial O}{\partial t}\rangle_{equ} = 0$$

and rewrite

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega}\int_0^\infty dt\int d^3x\; e^{i\omega t}\langle[\Theta(\vec{x},t),\Theta(\vec{0},0)]\rangle \quad , \quad \Theta = T_\mu{}^\mu$$

$$\zeta = \lim_{\omega\to 0}\frac{1}{9\omega}\int dt\int d^3x\; e^{i\omega t}\; iG^R(x) = \lim_{\omega\to 0}\frac{1}{9\omega}\; iG^R(\omega) = -\lim_{\omega\to 0}\frac{1}{9\omega}Im\; G^R(\omega)$$

We now use

$$\Theta = m\bar{q}q + \frac{\beta(g)}{2g}Tr[F^2] = \Theta_F + \Theta_G$$

We also use

$$\langle[\Theta,O]\rangle = \left(T\frac{\partial}{\partial T} - d\right)\langle O\rangle$$

RETURN

Diffusion times in different schemes (more)

T_{QGP}, MeV	τ_{charm} (fm/c) (direct)	τ_{charm} (fm/c) (energy)	τ_{charm} (fm/c) (entropy)
220	-	3.96	3.64
250	5.67	3.14	2.96
280	4.27	2.56	2.47
310	3.45	2.12	2.08
340	2.88	1.80	1.78
370	2.45	1.54	1.53
400	2.11	1.33	1.34

The diffusion times for the charm quark are shown for different temperatures, in the three different schemes. Diffusion times have been evaluated with a quark initial momentum fixed at $p \approx 10\ GeV$.

Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

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$T_{QGP}(MeV)$	τ_{bottom} (fm/c) (direct)	τ_{bottom} (fm/c) (energy)	τ_{bottom} (fm/c) (entropy)
220	-	8.90	8.36
250	11.39	7.46	7.12
280	10.11	6.32	6.14
310	8.62	5.40	5.32
340	7.50	4.70	4.65
370	6.63	4.10	4.09
400	5.78	3.61	3.63

Diffusion times for the bottom quark are shown for different temperatures, in the three different schemes. Diffusion times have been evaluated with a quark initial momentum fixed at $p \approx 10\ GeV$.

Gursoy+Kiritsis+Michalogiorgakis+Nitti, 2009

$\widehat{q}$ at different schemes

T_{QGP}, MeV	$\hat{q}$ (GeV^2/fm) (direct)	$\hat{q}$ (GeV^2/fm) (energy)	$\hat{q}$ (GeV^2/fm) (entropy)
220	-	0.89	1.01
250	0.53	1.21	1.32
280	0.79	1.64	1.73
310	1.07	2.14	2.21
340	1.39	2.73	2.77
370	1.76	3.37	3.42
400	2.18	4.20	4.15

- $\widehat{q}$ computed in the three different comparison "schemes", No cutoff

T_{QGP}, MeV	$\hat{q}_{charm}$ (GeV^2/fm) (direct)	$\hat{q}_{charm}$ (GeV^2/fm) (energy)	$\hat{q}_{charm}$ (GeV^2/fm) (entropy)
220	-	1.34	1.53
250	0.78	1.86	2.04
280	1.17	2.59	2.75
310	1.65	3.50	3.63
340	2.18	4.61	4.70
370	2.81	5.88	5.98
400	3.56	7.63	7.51

• $\hat{q}$ computed in the three different comparison "schemes" with a cutoff at the mass of the charm.

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T_{QGP}, MeV	$\hat{q}_{bottom}$ (GeV^2/fm) (direct)	$\hat{q}_{bottom}$ (GeV^2/fm) (energy)	$\hat{q}_{bottom}$ (GeV^2/fm) (entropy)
220	-	1.01	1.14
250	0.59	1.37	1.50
280	0.88	1.87	1.98
310	1.23	2.47	2.56
340	1.59	3.18	3.23
370	2.02	3.95	4.00
400	2.51	4.97	4.90

• $\hat{q}$ computed in the three different comparison "schemes" with a cutoff at the mass of the bottom. This has little effect as it is much higher than the temperatures involved.

Detailed plan of the presentation

- Title page 1 minutes
- Collaborators 2 minutes
- Plan of the presentation 3 minutes
- Introduction 4 minutes
- Introduction, II 6 minutes
- The large-N_c expansion in QCD 8 minutes
- Holography and QCD 11 minutes
- Improved Holographic QCD 13 minutes

FINITE TEMPERATURE

- The general phase structure 14 minutes
- Finite-T confining theories 16 minutes
- Temperature versus horizon position 18 minutes
- The free energy 20 minutes
- The pressure 21 minutes