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Cosmology and Hořava-Lifshitz Gravity

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Introduction

• Lorentz invariance has emerged as fundamental symmetry in particle physics since the early 1900's

• Although attempts have been made to violate it (especially recently), it remain in the minds of physicists as a central principle of physics.

• We should not forget that we believe in Lorentz invariance because experiment says so.

It is a direct corollary of the constancy of the speed of light in all inertial frames.

• There seems to be no fundamental reason that prohibits Lorentz-invariance violations at higher energies

The Lifshitz scaling symmetry

• Scale invariance is another central principle, that although typically broken in nature, it is powerful enough to organize whole regions of parameters in fundamental theories. (All perturbative theories we use are in this class.)

$$t \to b\ t \quad , \quad x^i \to b\ x^i$$

• Lorentz invariance implies an isotropic scaling. Poincaré invariance and locality together with scale invariance implies conformal invariance.

• In low energy+condensed matter systems, non-relativistic dynamics emerges naturally.

• Sometimes dynamical criticality emerges and scale invariance is non-relativistic

$$t \to b^z\ t \quad , \quad x^i \to b\ x^i$$

z is a dynamical critical exponent.

• A typical example of such a scaling appears in the Lifshitz critical theory

The Lifshitz (free) field theory

$$S_L = \int dt d^2x \left[\frac{1}{2}\dot{\Phi}^2 - \gamma(\Box\Phi)^2\right] \quad , \quad \Box = \sum_{i=1}^{2} \partial_i \partial_i$$

with $z = 2$:

$$[t] = 2 \quad , \quad [x^i] = 1 \quad , \quad [\Phi] = 0 \quad , \quad [\gamma] = 0$$

• It appears as a tri-critical point in a theory, with normal, BCS and striped phases by tuning

$$S_g = \int dt d^2x \left[\frac{1}{2}\dot{\Phi}^2 - \alpha\ \Phi\Box\Phi - \gamma\ (\Box\Phi)^2 + \cdots\right]$$

by tuning $\alpha \to 0$

• Although this is a 3d theory, its has 2d properties: in particular any polynomial in Φ is classically marginal and the propagator is logarithmically divergent in the IR

$$\frac{\partial}{\partial|\Delta\vec{x}|}\langle\Phi(t_1,\vec{x}_1)\Phi(t_2,\vec{x}_2)\rangle = \frac{1 - e^{-\frac{|\Delta\vec{x}|^2}{4\Delta t}}}{|\Delta\vec{x}|}$$

• The Lifshitz theory with $z > 1$ has at least one obvious relevant operator, namely $(\partial\Phi)^2$ which drives the theory to a Lorentz invariant theory in the IR,

$$S_{full} = \int dtd^2x \left[\frac{1}{2}\dot{\Phi}^2 + m^2 \ \Phi\Box\Phi - \gamma \ (\Box\Phi)^2 + \cdots\right] \to \int dtd^2x \left[\frac{1}{2}\dot{\Phi}^2 + \frac{m^2}{2} \ \Phi\Box\Phi\right]$$

with $m = c$.

• Lorentz invariance is not always guaranteed in the IR. For several Lifshitz scalars the individual speed of light could be different.

• In theories with dynamical critical exponent $z > 1$ the lower critical dimension is raised. 3+1 dimensional gravity can become marginal if $z = 3$

Hořava

• So far holographic flows have been found where a $z > 1$ theory holographically flows to a $z = 1$ theory.

Kachru+Liu+Mullingan

• Very recently an opposite holographic flow was found in the D3-D7 system from $z = 1$ to $z > 1$.

Azeyanagi+Li+Takayanagi

• It seems that always $z \geq 1$

Higher derivative Gravity

• The use of higher derivative couplings to improve gravity's UV behavior is not new.

• It is known that $R + R^2$ gravity is asymptotically free with propagator

$$\frac{1}{k^2 - \frac{(k^2)^2}{M_p^2}} = \frac{1}{k^2} - \frac{1}{k^2 - M_p^2}$$

Tomboulis

• It has however ghosts

• The idea is to combine:

1. broken Lorentz invariance to avoid ghosts (by including higher spatial derivatives but no time derivatives)

2. anisotropic scaling to make the theory scale invariant in the UV.

Hořava

Hořava-Lifshitz Gravity

- Start from the ADM decomposition of the metric

$$ds^2 = -N^2\ dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \quad , \quad N_i = g_{ij}N^j.$$

- The kinetic terms are given by

$$S_K = \frac{2}{\kappa^2}\int dt d^3x \sqrt{g} N\ K_{ij}\ \mathcal{G}^{ij;kl}\ K_{kl} = \frac{2}{\kappa^2}\int dt d^3x \sqrt{g} N\ \left(K_{ij}K^{ij} - \lambda K^2\right)$$

in terms of the supermetric

$$\mathcal{G}^{ij;kl} = \frac{1}{2}\left(g^{ik}g^{jl} + g^{il}g^{jk}\right) - \lambda g^{ij}g^{kl} \quad , \quad \mathcal{G}_{ij;kl} = \frac{1}{2}\left(g_{ik}g_{jl} + g_{il}g_{jk}\right) + \frac{\lambda}{1-3\lambda}g_{ij}g_{kl}$$

and the extrinsic curvature

$$K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

- λ is a dimensionless coupling that breaks full diff invariance

- General Relativity has $\lambda = 1$

• For renormalizability we would like to impose that $z = 3$, so that the spatial metric components are dimensionless

$$t \to b^3\ t \quad , \quad x^i \to b\ x^i$$

$$[N] = 0 \quad , \quad [g_{ij}] = 0 \quad , \quad [N_i] = 2 \quad , \quad [w] = 0$$

• The "potential" is

$$V = \int dt d^3x\ \sqrt{g} N\ V(g_{ij})$$

• For renormalizability it should contain up to six derivatives. The six-derivative terms are classically-scale invariant. Terms with a lower number of derivatives are "relevant".

$$\nabla_i R_{jk} \nabla^i R^{jk} \quad , \quad \nabla_i R_{jk} \nabla^j R^{ik} \quad , \quad R \Box R \quad , \quad R_{ij} \Box R^{ij}$$

modify already the propagator while

$$R^3 \quad , \quad R R_{ij} R^{ij} \quad , \quad R_{ij} R^i{}_k R^{jk}$$

provide scale invariant interactions.

• The (local) invariance of the theory is

$$t \to h^0(t) \quad , \quad x^i \to h^i(t, x^j)$$

The Cotton tensor

• There is a special scale invariant term that is also conformal $C_{ij}C^{ij}$ with C_{ij} the Cotton-tensor

$$C^{ij} = \frac{\epsilon^{ikl}}{\sqrt{g}} \nabla_k \left(R^j{}_l - \frac{1}{4} R \delta^j{}_l \right)$$

• In 3d it is the unique tensor that satisfies

$$C_{ij} = C_{ji} \quad , \quad C_i{}^i = 0 \quad , \quad \nabla^i \, C_{ij} = 0 \quad ,$$

and is conformal

$$g_{ij} \to e^{2\phi(x)} \, g_{ij} \quad , \quad C^{ij} \to e^{5\phi(x)} \, C^{ij}$$

• It is the analogue of the Weyl tensor in 3d.

• It can be obtained by the variation of the 3d gravitational CS action

$$S = \int \omega_3(\Gamma) \quad , \quad \omega_3(\Gamma) = Tr[\Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma]$$

• Adding it to the gravitational potential provides a source of CP violation in gravity, that may have measurable consequences

Takahashi+Soda

Detailed Balance

• There are potentials that have special properties:

$$V = \frac{\delta W(g_{ij})}{\delta_{ij}} \mathcal{G}_{ij;kl} \frac{\delta W(g_{kl})}{\delta_{kl}}$$

• W is an invariant functional in 3d

• This property is known as "Detailed Balance"

• In condensed matter physics it is known that such potentials have non-trivial RG properties.

• It is an important ingredient of stochastic quantization *Parisi+Wu*

• Hořava postulated at the marginal level

$$W = \frac{1}{w^2} \int \omega_3(\Gamma) + \mu \int d^3x \, [R - 2\Lambda_W]$$

where

$$[w] = 0 \quad , \quad [\mu] = [-1] \quad , \quad [\Lambda_W] = -2$$

• The full action (obeying detailed balance) is :

.

$$S=\int dt d^3x\sqrt{g}N\Big[\alpha(K_{ij}K^{ij}-\lambda K^2)+\beta C_{ij}C^{ij}+\gamma\mathcal{E}^{ijk}R_{il}\nabla_j R^l{}_k+$$

$$+\zeta R_{ij}R^{ij}+\eta R^2+\xi R+\sigma\Big],$$

$$\alpha=\frac{2}{\kappa^2}\quad,\quad\beta=-\frac{\kappa^2}{2w^4}\quad,\quad\gamma=\frac{\kappa^2\mu}{2w^2}\quad,\quad\zeta=-\frac{\kappa^2\mu^2}{8}$$

$$\eta=\frac{\kappa^2\mu^2}{8(1-3\lambda)}\frac{1-4\lambda}{4}\quad,\quad\xi=\frac{\kappa^2\mu^2}{8(1-3\lambda)}\Lambda_W\quad,\quad\sigma=\frac{\kappa^2\mu^2}{8(1-3\lambda)}(-3\Lambda_W^2).$$

$$[\kappa]=0\quad,\quad[w]=0\quad,\quad[\lambda]=0\quad,\quad[\mu]=-1\quad,\quad[\Lambda_W]=-3$$

• Dropping detailed balance, all 8 parameters above can be considered independent.

• There is a special value of $\lambda=\frac{1}{3}$. For this value, the marginal part of the action is Weyl invariant.

• The theory has a spectral dimension (Hořava)

$$d_s\equiv-2\frac{dP(\sigma)}{d\log\sigma}=2$$

The IR action

• In the IR the most relevant terms are

$$S=\int dtd^3x\sqrt{g}N\left[\alpha(K_{ij}K^{ij}-\lambda K^2)+\xi R+\sigma\right],$$

• In order for this to reproduce Einstein gravity this must have $\lambda \simeq 1$ in the IR to a good degree of accuracy.

• Defining $x^0 = ct$, choosing $\lambda = 1$ and

$$c=\sqrt{\frac{\xi}{\alpha}} \quad , \quad 16\pi G_N = \frac{1}{\sqrt{\alpha\xi}} \quad , \quad \Lambda_E = -\frac{\sigma}{2\xi}, \quad , \quad [c]=-2 \quad , \quad [x^0]=1$$

the action is that of Einstein

$$S_E = \frac{1}{16\pi G_N}\int d^4x\sqrt{\tilde{g}}\left[K_{ij}K^{ij} - K^2 + R - 2\Lambda_E\right] = \frac{1}{16\pi G_N}\int d^4x\sqrt{\tilde{g}}\left[\tilde{R} - 2\Lambda_E\right].$$

• Diff invariance is restored in the IR.

• Similar ideas about gauge invariance and Lorentz invariance have been explored earlier

Nielsen+Chanda+Ninomiya, Iliopoulos+Nanopoulos+Tomaras

Propagating degrees of freedom

- The UV fixed point is obtain by taking $\kappa, w \to 0$ with $\gamma = \frac{\kappa}{w}$ and λ fixed.

- Expanding

$$g_{ij} \simeq \delta_{ij} + w h_{ij} \quad , \quad N \simeq 1 + w\ n \quad , \quad N_i \simeq w\ n_i$$

- n drops out at quadratic level and we fix the gauge

$$n_i = 0 \quad , \quad H_{ij} \equiv h_{ij} - \lambda \delta_{ij} h \quad , \quad \partial_i H_{ij} = 0$$

- Separating the trace and traceless part

$$H_{ij} = \hat{H}_{ij} + \frac{1}{2}\left(\delta_{ij} - \frac{\partial_i \partial_j}{\partial^2}\right) H$$

we obtain

$$S_{Kin} \sim \int dt d^3x \left[(\dot{\hat{H}}_{ij})^2 + \frac{1-\lambda}{2(1-3\lambda)} (\dot{H})^2 \right]$$

• The potential coming from the Cotton tensor is

$$V \sim \int dt d^3x \ \hat{H}_{ij} \Box^3 \hat{H}_{ij}$$

• $\hat{H}_{ij}$ is a massless spin-two graviton. It has a dispersion relation of the form

$$\frac{E^2}{c^2} = \frac{(\vec{k}^2)^3}{M^4} \pm \frac{(\vec{k}^2)^2}{m^2} + \vec{k}^2$$

• Note that the $(\vec{k}^2)^2$ term that is coming from the R^2 terms can have either sign. The only constraint is that there should be no poles in the Euclidean propagator: $m^2 < 2M^2$.

• The speed of light is infinite in the UV and finite in the IR.

• This leads to a static gravitational potential that is

$$V \sim \frac{1}{4\pi r} \quad r \to \infty \qquad , \qquad V \sim -M^3 r^2 \quad r \to 0$$

.

• H is an extra degree of freedom. It has a potential containing $(\vec{k}^2)^2$ and $(\vec{k}^2)$ terms but no $(\vec{k}^2)^3$ term. It might generate non-renormalizability problems.

• It should decouple at $\lambda = \frac{1}{3}$ (Weyl symmetry) and at $\lambda = 1$ (diffeomorphism invariance)

• An important question which is crucial for the fate of this theory is the RG running of λ.

• If $\lambda = 1$ is a stable fixed point, then things are fine. Otherwise there seems to be insurmountable trouble

Generalized Lifshitz QFTs (scalar matter)

- For scalars($[\Phi]=0$)

$$S_{\rm nr}=\int d^3x dt\sqrt{g}N\left[\frac{1}{N^2}(\dot\Phi-N^i\partial_i\Phi)^2+F[\xi_1,\xi_2,\cdots,\Phi]\right]\quad,\quad \xi_n=\Phi\Box^n\Phi$$

where the (renormalizable) potential F is

$$F[\xi_n,\Phi]=F_0(\Phi)+F_1(\Phi)\ \xi_1+F_{11}(\Phi)\xi_1^2+F_{111}(\Phi)\xi_1^3+$$

$$+F_2(\Phi)\xi_2+F_{21}(\Phi)\xi_2\xi_1+F_3(\Phi)\xi_3.$$

and dispersion relation

$$\frac{E^2}{F_1(0)}=\frac{1}{4}\frac{F_0''(0)}{F_1(0)}+(\vec{k}^2)-\frac{F_2(0)}{F_1(0)}(\vec{k}^2)^2+\frac{F_3(0)}{F_1(0)}(\vec{k}^2)^3$$

- Like in gravity the speed of light is infinite in the UV $(E^2\sim k^6)$.

- In the IR it is $c^2=F_1(0)$. It is not a priori equal to that of gravity.

Generalized Lifshitz QFTs (vectors)

- For (abelian) vectors, $[A_0]=-2 \quad , \quad [A_i]=0$

$$S_{nr}=-\frac{1}{4g^2}\int d^3x dt\sqrt{g}N\left[-\frac{2}{N^2}g^{ij}(F_{0i}-N^kF_{ki})(F_{0j}-N^lF_{lj})-\right.$$

$$\left.-\frac{M^2}{N^2}(A_0-N^iA_i)(A_0-N^jA_j)+G[A_i]\right],$$

$$F_{0i}=\partial_t A_i-\partial_i A_0 \quad , \quad F_{ij}=\partial_i A_j-\partial_j A_i$$

Define the magnetic field

$$B_i=\frac{1}{2}\frac{\epsilon_i{}^{jk}}{\sqrt{g}}F_{jk} \quad , \quad F_{ij}=\frac{\epsilon_{ij}{}^k}{\sqrt{g}}B_k \quad , \quad \nabla^i B_i=0.$$

$$G=a_0+a_1\zeta_1+a_2\zeta_1^2+a_3\zeta_1^3+a_4\zeta_2+a_5\zeta_1\zeta_2+a_6\zeta_3+a_7\zeta_4,$$

$$\zeta_1=B_iB^i \quad , \quad \zeta_2=\nabla_iB_j\nabla^iB^j \quad , \quad \zeta_3=\nabla_iB_j\nabla^iB^k\nabla^jB_k \quad , \quad \zeta_4=\nabla_i\nabla_jB_k\nabla^i\nabla^jB^k,$$

- a_i are arbitrary functions of A_iA^i.

- The "stress-tensor" is no-longer traceless $\rightarrow$ $w\neq\frac{1}{3}$.

Cosmology: General expectations

• As $c \to \infty$ in the UV, it is plausible that no inflation is needed to solve the horizon problem.

• The theory contains higher derivatives but in a controlable fashion. They may be relevant in resolving singularities

• The UV theory is scale invariant: therefore it can generate a scale invariant spectrum of cosmological perturbations without the need for acceleration.

• Spatial curvature effects are enhanced, to $\frac{1}{a^4}$ or $\frac{1}{a^6}$ making the flatness problem milder.

Cosmological backgrounds

- We make a cosmological ansatz

$$N(t) \quad , \quad N_i = 0 \quad , \quad g_{ij} = a^2(t)\gamma_{ij}$$

We can fix the gauge $N = 1$

- The Friedman equations are

$$3\alpha(3\lambda - 1)H^2 = \rho - \sigma - \frac{6k\xi}{a^2} - \frac{12k^2(\zeta + 3\eta)}{a^4}$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

$$\rho = -\frac{1}{\sqrt{g}}\frac{\delta S_M}{\delta N} \quad , \quad p = -\frac{2}{3N\sqrt{g}}g^{ij}\frac{\delta S_M}{\delta g^{ij}}.$$

- if we add an R^3 term in the action, there will also be a term $\sim \frac{k^3}{a^6}$

- If $\lambda < \frac{1}{3}$, then all positivity properties are reversed.

• The k^2/a^4 although generated by curvature, resembles mirage/dark radiation. It is due to the R^2 terms as in the case of holography

Kiritsis

• It can generate a bouncing cosmology provided there is non-relativistic matter

Calcagni, Kiritsis+Kofinas

• It was argued that in the contracting phase before the bounce scale invariant perturbations can be generated

Brandenberger

• For non-relativistic scalars, we obtain the same pressure and density as in relativistic ones (spatial derivatives do not contribute)

• This seems not to be the case for vectors.

On cosmological perturbations

• Homogeneous cosmology sees very little of the structure of the classical theory and in particular the scale-invariant part.

• The perturbations see the full structure of the theory

• tensor perturbations satisfy

Takahashi+Soda

$$\frac{\partial^2}{\partial\eta^2}v^A_{\vec{k}} + \left[(k^A_{eff})^2 - \frac{2}{\eta^2}\right] v^A_{\vec{k}} = 0$$

$$(k^A_{eff})^2 = c^2k^2\left[1 + \frac{(1-3\lambda)}{\Lambda_W c^2}H^2(ck\eta)^2(1+\rho^A\frac{2H}{w^2\mu c}ck\eta)^2\right]$$

• The non-trivial polarization may be observable depending on the value of the couplings.

• Scalar perturbations are also interesting. We assume a spatially flat universe and the scalar field in Fourier space: the quadratic action is

$$S = \int d^3k \int dt \; a^3 \left[|\dot{\Phi}|^2 + \frac{1}{a^6} \left(-\ell^4 k^6 + y_2 \ell^2 k^4 - y_1 k^2 - m^2 \right) |\Phi|^2 \right],$$

• ℓ is a length scale characteristic of the UV behavior of the scalar theory, and $y_{1,2}$ are dimensionless coefficients. In particular, y_1 is the square of the speed of light in the scalar theory.

• The fluctuations $\delta\Phi$ satisfy

$$\delta\ddot{\Phi} + 3H\delta\dot{\Phi} + \frac{\ell^4 k^6 - y_2 \ell^2 k^4 + y_1 k^2 + m^2}{a^6} \delta\Phi = 0.$$

• At high energy the dispersion relation is

$$E^2 \simeq \ell^4 \frac{k^6}{a^6}$$

• Typically, a fluctuation mode oscillates if $E >> H$, while it is frozen in the opposite limit $E << H$.

• Here:

$$\frac{E^2}{H^2} \simeq \frac{\ell^4 \; k^6}{H^2 a^6}$$

• If $H^2 a^6$ is an increasing function of time this will freeze the oscillations eventually.

• From the cosmological equations we find that this is satisfied for all matter with $w < 1$, including curvature. (in normal cosmology, we have $H^2 a^2$ instead and $w < -\frac{1}{3}$).

• At freezout,

$$\frac{k}{a} \sim \frac{H^{\frac{1}{3}}}{\ell^{\frac{2}{3}}} \quad \to \quad H\lambda_{phys} \sim (H\ell)^{\frac{2}{3}} \gg 1$$

• The solution is

$$\delta\Phi(t,\vec{k}) = \frac{1}{(2\pi)^3\sqrt{2\kappa}} e^{-i\kappa\int \frac{dt}{a^3} + i\vec{k}\cdot\vec{x}} \quad , \quad \kappa \equiv \sqrt{\ell^4 k^6 - y_2 \ell^2 k^4 + y_1 k^2 + m^2}.$$

$$\langle \delta\Phi(t,\vec{k})\delta\Phi(t,\vec{k}')\rangle = \frac{(2\pi)^3}{2\kappa}\delta(\vec{k}+\vec{k}') \equiv (2\pi)^3\delta(\vec{k}+\vec{k}')\frac{2\pi^2}{k^3}\mathcal{P}_{\delta\Phi}$$

From where we obtain as $k\ell >> 1$ a scale invariant spectrum,

Mukohyama, Kiritsis+Kofinas

$$\sqrt{\mathcal{P}_{\delta\Phi}} = \frac{1}{2\pi\ell}$$

- The corrections to this relation come from the relevant corrections as well as logarithmic UV renormalization

- The scalar will remain frozen until the universe cools, then it becomes relativistic and may decay to other particles.

- No graceful exit seems to be needed here.

Outlook and Open problems

- What is the RG flow of marginal couplings? Is $\lambda = 1$ a fixed point?

- Is there asymptotic freedom in the UV? This may be important in order to generate an exponential hierarchy of scales.

- Does this affect how a cosmological constant renormalizes?

- Is the theory really-renormalizable? (there are several subtleties in Lifshitz like theories including non-commutativity of short distance limits, etc)

- Does the extra scalar degree of freedom decouple properly?

- What is the detailed gravitational interaction both static and dynamic?

- What are constraints on the parameters from solar system measurements?

- Is the core of BH different? How is thermodynamics working?

Equations of motion

The equation obtained by varying N is

$$-\alpha\left(K_{ij}K^{ij}-\lambda K^2\right)+\beta C_{ij}C^{ij}+\gamma\mathcal{E}^{ijk}R_{il}\nabla_j R^l{}_k+\zeta R_{ij}R^{ij}+\eta R^2+\xi R+\sigma=J_N,$$

with

$$J_N=-\mathcal{L}_{\text{matter}}-N\frac{\delta\mathcal{L}_{\text{matter}}}{\delta N}$$

The equation obtained by varying N_i is

$$2\alpha(\nabla_j K^{ji}-\lambda\nabla^i K)+N\frac{\delta\mathcal{L}_{\text{matter}}}{\delta N_i}=0.$$

The equation obtained varying g_{ij} is

$$\frac{1}{2}\Big[(\mathcal{E}^{mk\ell}Q_{mi})_{;kj\ell}+(\mathcal{E}^{mk\ell}Q^n_m)_{;kin}g_{j\ell}-(\mathcal{E}^{mk\ell}Q_{mi})_{;kn}{}^{;n}g_{j\ell}-(\mathcal{E}^{mk\ell}Q_{mi})_{;k}R_{j\ell}$$
$$-(\mathcal{E}^{mk\ell}Q_{mi}R^n_k)_{;n}g_{j\ell}+(\mathcal{E}^{mk\ell}Q^n_m R_{ki})_{;n}g_{j\ell}+\frac{1}{2}(\mathcal{E}^{mk\ell}R^n{}_{pk\ell}Q^p_m)_{;n}g_{ij}-Q_{k\ell}C^{k\ell}g_{ij}+$$
$$\mathcal{E}^{mk\ell}Q_{mi}R_{j\ell;k}\Big]+\Box[N(2\eta R+\xi)]g_{ij}+N(2\eta R+\xi)R_{ij}+2N(\zeta R_{ik}R^k_j-\beta C_{ik}C^k_j)$$
$$-[N(2\eta R+\xi)]_{;ij}+\Box[N(\zeta R_{ij}+\frac{\gamma}{2}C_{ij})]-2[N(\zeta R_{ik}+\frac{\gamma}{2}C_{ik})]_{;j}{}^{;k}+[N(\zeta R^{k\ell}+\frac{\gamma}{2}C^{k\ell})]_{;k\ell}g_{ij}$$
$$-\frac{N}{2}(\beta C_{k\ell}C^{k\ell}+\gamma R_{k\ell}C^{k\ell}+\zeta R_{k\ell}R^{k\ell}+\eta R^2+\xi R+\sigma)g_{ij}+2\alpha N(K_{ik}K^k_j-\lambda KK_{ij})$$
$$-\frac{\alpha N}{2}(K_{k\ell}K^{k\ell}-\lambda K^2)g_{ij}+\frac{\alpha}{\sqrt{g}}g_{ik}g_{j\ell}\frac{\partial}{\partial t}[\sqrt{g}(K^{k\ell}-\lambda Kg^{k\ell})]+\alpha[(K_{ik}-\lambda Kg_{ik})N_j]^{;k}$$
$$+\alpha[(K_{jk}-\lambda Kg_{jk})N_i]^{;k}-\alpha[(K_{ij}-\lambda Kg_{ij})N_k]^{;k}+(i\leftrightarrow j)=-2N\frac{\delta\mathcal{L}_{\mathrm{matter}}}{\delta g^{ij}}, \qquad (1)$$

where

$$Q_{ij}\equiv N(\gamma R_{ij}+2\beta C_{ij}).$$

Black hole solutions

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