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Dynamics and Thermodynamics of Improved Holographic QCD

Elias Kiritsis

University of Crete and Ecole Polytechnique

Bibliography

• U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti, *"Deconfinement and Gluon-Plasma Dynamics in Improved Holographic QCD,"* [ArXiv:0804.0899][hep-th].

• Ongoing work with:
U. Gursoy, L. Mazzanti, G. Michalogiorgakis, F. Nitti

Based on previous work

• U. Gursoy and E. Kiritsis, *"Exploring improved holographic theories for QCD: Part I,"* JHEP **0802** (2008) 032 [ArXiv:0707.1324][hep-th].

• U. Gursoy, E. Kiritsis and F. Nitti, *"Exploring improved holographic theories for QCD: Part II,"* JHEP **0802** (2008) 019 [ArXiv:0707.1349][hep-th].

R. Casero, E. Kiritsis and A. Paredes, *"Chiral symmetry breaking as open string tachyon condensation,"* Nucl. Phys. B **787** (2007) 98; [arXiv:hep-th/0702155].

Introduction

• AdS/CFT has provided so far controlled/computable examples of confinement, chiral symmetry breaking and hadron spectra of concrete gauge theories

• Its direct application to QCD is marred by two (related) problems:

♠ The KK problem of critical examples

♠ The strong curvature problem of non-critical (or hierarchically separated critical examples)

• We will follow the non-critical road, try to understand what we expect from the string theory dual to QCD, and motivate a phenomenological approximation (model).

• We will then use the model to compute experimentally interesting non-perturbative quantities like transport coefficients.

What are we after?

• Interactions of hadrons at medium or low energy (little or no help from lattice, partial help from chiral perturbation theory)

• Transport coefficients of the deconfined phase (not computable directly from lattice, crucial for understanding current (RHIC) and future (LHC) heavy-ion data)

• The phase structure and properties of dense matter (not computable from lattice, important for understanding properties of nuclei, and dense nuclear matter, like neutron stars)

• Exploring the strong dynamics of other QCD-like theories, eg.

♠ N=1 super- QCD. (a very interesting toy model and may be relevant for nature)

♠ Technicolor theories

A string theory for QCD:basic expectations

• Pure SU(N_c) d=4 YM at large N_c is expected to be dual to a string theory in 5 dimensions only. Essentially a single adjoint field $\rightarrow$ a single extra dimension.

• The theory becomes asymptotically free and conformal at high energy $\rightarrow$ we expect the classical saddle point solution to asymptote to AdS_5.

♠ Operators with lowest dimension (or better: lowest bulk masses) are expected to be the only important non-trivial bulk fields in the large-N_c saddle-point

• Scalar YM operators with $\Delta_{UV} > 4 \rightarrow m^2 > 0$ fields near the AdS$_5$ boundary $\rightarrow$ vanish fast in the UV regime and do not affect correlators of low-dimension operators.

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• Their dimension may grow large in the IR. Large 't Hooft coupling is expected to supress the effects the growth in the IR

• This is suggested by the success of low-energy SVZ sum rules as compared to data.

♠ Therefore we will consider

$$T_{\mu\nu} \leftrightarrow g_{\mu\nu}, \; tr[F^2] \leftrightarrow \phi, \; tr[F \wedge F] \leftrightarrow a$$

Bosonic string or superstring?

• Consider the axion a dual to $Tr[F \wedge F]$. We can show that it must come from a RR sector.

In large-N$_c$ YM, the proper scaling of couplings is obtained from

$$\mathcal{L}_{YM} = N_c \; Tr\left[\frac{1}{\lambda}F^2 + \frac{\theta}{N_c}F \wedge F\right] \quad , \quad \zeta \equiv \frac{\theta}{N_c} \sim \mathcal{O}(1)$$

It can be shown

$$E_{YM}(\theta) \simeq C_0 \; N_c^2 + C_1\theta^2 + C_2\frac{\theta^4}{N_c^2} + \cdots$$

Witten

In the string theory action

$$S \sim \int e^{-2\phi}\,[R + \cdots] + (\partial a)^2 + e^{2\phi}(\partial a)^4 + \cdots \quad , \quad e^{\phi} \sim g_{YM}^2 \quad , \quad \lambda \sim N_c e^{\phi}$$

$$\sim \int \frac{N_c^2}{\lambda^2}\,[R + \cdots] + (\partial a)^2 + \frac{\lambda^2}{N_c^2}(\partial a)^4 + \cdots \quad , \quad a = \theta[1 + \cdots]$$

bosonic string or superstring? (continued)

• The string theory must have no on-shell fermionic states at all because there are no gauge invariant fermionic operators in pure YM. (even with quarks modulo baryons).

• Therefore the string theory must be a 5d-superstring theory resembling the II-0 class.

♠ Another RR field we expect to have is the RR 4-form, as it is necessary to "seed" the D_3 branes responsible for the gauge group.

• It is non-propagating in 5D

• We will see later however that it is responsible for the non-trivial IR structure of the gauge theory vacuum.

The minimal effective string theory spectrum

- NS-NS $\rightarrow$ $g_{\mu\nu} \leftrightarrow T_{\mu\nu} \quad , \quad B_{\mu\nu} \leftrightarrow Tr[F]^3 \quad , \quad \phi \leftrightarrow Tr[F^2]$

- RR $\rightarrow$ $\text{Spinor}_5 \times \text{Spinor}_5 = F_0 + F_1 + F_2 + (F_3 + F_4 + F_5)$

♠ $F_0 \leftrightarrow F_5 \rightarrow$ C_4, background flux $\rightarrow$ no propagating degrees of freedom.

♠ $F_1 \leftrightarrow F_4 \rightarrow$ $C_3 \leftrightarrow C_0$: C_0 is the axion, C_3 its 5d dual that couples to domain walls separating oblique confinement vacua.

♠ $F_2 \leftrightarrow F_3 \rightarrow$ $C_1 \leftrightarrow C_2$: They are associated with baryon number (as we will see later when we add flavor). Dual operators are a mystery (topological currents?).

- In an ISO(3,1) invariant vacuum solution, only $g_{\mu\nu}, \phi, C_0 = a$ can be non-trivial.

$$ds^2 = e^{2A(r)}(dr^2 + dx_4^2) \quad , \quad a(r), \phi(r)$$

The relevant "defects"

• $B_{\mu\nu} \to$ Fundamental string (F_1). This is the QCD (glue) string: fundamental tension $\ell_s^2 \sim \mathcal{O}(1)$

• Its dual $\tilde{B}_\mu \to NS_0$: Tension is $\mathcal{O}(N_c^2)$. It is an effective magnetic baryon vertex binding N_c magnetic quarks.

• $C_5 \quad \to D_4$: Space filling flavor branes. They must be introduced in pairs: $D_4 + \bar{D}_4$ for charge neutrality/tadpole cancelation $\quad \to \quad$ gauge anomaly cancelation in QCD.

• $C_4 \to D_3$ branes generating the gauge symmetry.

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- $C_3 \to D_2$ branes : domain walls separating different oblique confinement vacua (where $\theta_{k+1} = \theta_k + 2\pi$). Its tension is $\mathcal{O}(N_c)$

- $C_2 \to D_1$ branes: These are the magnetic strings:
(strings attached to magnetic quarks) with tension $\mathcal{O}(N_c)$

- $C_1 \to D_0$ branes. These are the baryon vertices: they bind N_c quarks, and their tension is $\mathcal{O}(N_c)$.
Its instantonic source is the (solitonic) baryon in the string theory.

- $C_0 \to D_{-1}$ branes: These are the Yang-Mills instantons.

The effective action, I

- as $N_c \to \infty$, only string tree-level is dominant.

- Relevant field for the vacuum solution: $g_{\mu\nu}, a, \phi, F_5$.

- The vev of $F_5 \sim N_c \, \epsilon_5$. It appears always in the combination $e^{2\phi} F_5^2 \sim \lambda^2$, with $\lambda \sim N_c \, e^{\phi}$ All higher derivative corrections $(e^{2\phi} F_5^2)^n$ are $\mathcal{O}(1)$.
A non-trivial potential for the dilaton will be generated already at string tree-level.

- This is not the case for all other RR fields: in particular for the axion as $a \sim \mathcal{O}(1)$

$$(\partial a)^2 \sim \mathcal{O}(1) \quad , \quad e^{2\phi} (\partial a)^4 = \frac{\lambda^2}{N_c^2} (\partial a)^4 \sim \mathcal{O}\left(N_c^{-2}\right)$$

Therefore to leading order $\mathcal{O}(N_c^2)$ we can neglect the axion.

The UV regime

- In the far UV, the space should asymptote to AdS$_5$.

- The 't Hooft coupling should behave as $(r \to 0)$

$$\lambda \sim \frac{1}{\log(r\Lambda)} + \cdots \quad \to \quad 0 \quad , \quad r \sim \frac{1}{E}$$

The effective action to leading order in N_c is

$$S_{eff} \sim \int d^5x\sqrt{g}\; e^{-2\phi}\; Z(\; \ell_s^2 R \;,\; \ell_s^2(\partial\phi)^2 \;,\; e^{2\phi}\ell_s^2 F_5^2 \;)$$

Solving the equation of motion of F_5 amounts to replacing

$$e^{2\phi}\; \ell_s^2\; F_5^2 \sim e^{2\phi} N_c^2 \equiv \lambda^2$$

$$S_{eff} \sim N_c^2 \int d^5x\sqrt{g}\; \frac{1}{\lambda^2}\; H(\; \ell_s^2 R \;,\; \ell_s^2(\partial\lambda)^2 \;,\; \lambda^2 \;)$$

- As $r \to 0$

$$\text{Curvature} \to \text{finite} \quad , \quad \Box\phi \sim (\partial\phi)^2 \sim \frac{(\partial\lambda)^2}{\lambda^2} \sim \lambda^2 \sim \frac{1}{\log^2(r\Lambda)} \to 0$$

- For $\lambda \to 0$ the potential in the Einstein frame starts as $V(\lambda) \sim \lambda^{\frac{4}{3}}$ and cannot support the asymptotic AdS_5 solution.

- Therefore asymptotic AdS_5 must arise from curvature corrections:

$$S_{eff} \simeq \int d^5x \, \frac{1}{\lambda^2} \, H\left(\ell_s^2 \; R, 0, 0\right)$$

- Setting $\lambda = 0$ at leading order we can generically get an AdS_5 solution coming from balancing the higher curvature corrections.

$$H(x,0,0) \equiv f(x) \quad , \quad x_* f'(x_*) + \frac{5}{2} f(x_*) = 0 \quad , \quad x_* = \ell_s^2 R = 12 \, \frac{\ell_s^2}{\ell^2}$$

INTERESTING QUESTION: Is there a good toy example of string vacuum (CFT) which is not Ricci flat, and is supported only by a metric?

• There is a "good" (but hard to derive the coefficients) perturbative expansion around this asymptotic AdS_5 solution by perturbing inwards :

$$e^A = \frac{\ell}{r}[1 + \delta A(r)] \quad , \quad \lambda = \frac{1}{b_0 \log(r\Lambda)} + \cdots$$

• This turns out to be a regular expansion of the solution in powers of

$$\frac{P_n(\log\log(r\Lambda))}{(log(r\Lambda))^{-n}}$$

• Effectively this can be rearranged as a "perturbative" expansion in $\lambda(r)$. In the case of running coupling, the radial coordinate can be substituted by $\lambda(r)$.

• Using λ as a radial coordinate the solution for the metric can be written

$$E \equiv e^A = \frac{\ell}{r(\lambda)}\left[1 + c_1\lambda + c_2\lambda^2 + \cdots\right] = \ell\,(e^{-\frac{b_0}{\lambda}})\left[1 + c_1'\lambda + c_2'\lambda^2 + \cdots\right] \quad , \quad \lambda \to 0$$

.

Conclusion: The asymptotic AdS_5 is stringy, but the rest of the geometry is "perturbative around the asymptotics". We cannot however do computations even if we seen the structure.

QUESTION: Can one constrain $H(R, 0, 0)$ by asking the stress tensor correlators asymptote to free correlators near the boundary?

The IR regime

• Here the situation is more obscure. The constraints/input will be: confinement and mass gap.

• We do expect that $\lambda \to \infty$ (or becomes large) at the IR bottom.

• Intuition from N=4 and other 10d strongly coupled theories suggests that in this regime there should be an (approximate) two-derivative description of the physics.

• From the string σ-model we can write

Tseytlin

$$S_{eff} \sim N_c^2 \int d^5x \sqrt{g}\, \frac{1}{\lambda^2} \left[\frac{(\partial\lambda)^2}{\lambda^2} + H(\ \ell_s^2 R\ ,\ \lambda^2\ )\right]$$

• For the theory to reduce to two derivatives, $R \to 0$ in the IR (string frame). Then

$$H(\ \ell_s^2 R\ ,\ \lambda^2\ ) \sim \ell_s^2\ R + V(\lambda) + \mathcal{O}(R^2)$$

• The simplest solution linear dilaton with this property is the linear dilaton solution with

$$\lambda \sim e^{Qr} \quad , \quad V(\lambda) \sim \delta c = 10 - D \quad \to \quad \text{constant} \quad , \quad R = 0$$

• As we shall see this property persists with potentials $V(\lambda) \sim (\log \lambda)^P$. Moreover all such cases have confinement, a mass gap and a discrete spectrum (except the P=0 case).

• At the IR bottom (in the string frame) the scale factor vanishes, and 5D space becomes (asymptotically) flat.

Improved Holographic QCD: a model

The simplification in this model relies on writing down a two-derivative action

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right]$$

with

$$\lim_{\lambda\to 0} V(\lambda) = \frac{12}{\ell^2}\left(1 + \sum_{n=1}^{\infty} c_n \lambda^n\right) \quad , \quad \lim_{\lambda\to\infty} V(\lambda) = \lambda^{\frac{4}{3}}\sqrt{\log\lambda} + \text{subleading}$$

- The small λ asymptotics “simulate” the UV expansion around AdS_5.

- There is a 1-1 correspondence between the YM β-function, $\beta(\lambda)$ and W:

$$\left(\frac{3}{4}\right)^3 V(\lambda) = W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \log\lambda}\right)^2 \quad , \quad \beta(\lambda) = -\frac{9}{4}\lambda^2 \frac{d \log W(\lambda)}{d\lambda}$$

once a choice of energy is made (here $\log E = A_E$).

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Not everything is perfect: There are some shortcomings localized at the UV

- The conformal anomaly is incorrect.

- Shear viscosity ratio is constant and equal to that of N=4 sYM.

(This is not expected to be a serious error in the experimentally interesting $T_c \leq T \leq 4T_c$ range.)

Both of the above need Riemann curvature corrections.

- We shall see that other observables can come out very well both at T=0 and finite T

Parameters

• We have 3 initial conditions in the system of graviton-dilaton equations:

♠ One is fixed by picking the branch that corresponds asymptotically to $\lambda \sim \frac{1}{\log(r\Lambda)}$

♠ The other fixes $\Lambda \to \Lambda_{QCD}$.

♠ The third is a gauge artifact as it corresponds to a choice of the origin of the radial coordinate.

• We also have the Planck scale M_p, and the AdS length, ℓ.
Asking for correct $T \to \infty$ thermodynamics (free gas) fixes

$$(M_p\ell)^3 = \frac{1}{45\pi^2} \quad , \quad M_{\text{physical}} = M_p N_c^{\frac{2}{3}} = \left(\frac{8}{45\pi^2\ell^3}\right)^{\frac{1}{3}} \simeq 4.6 \text{ GeV}$$

ℓ is not a parameter but a unit of length.

.

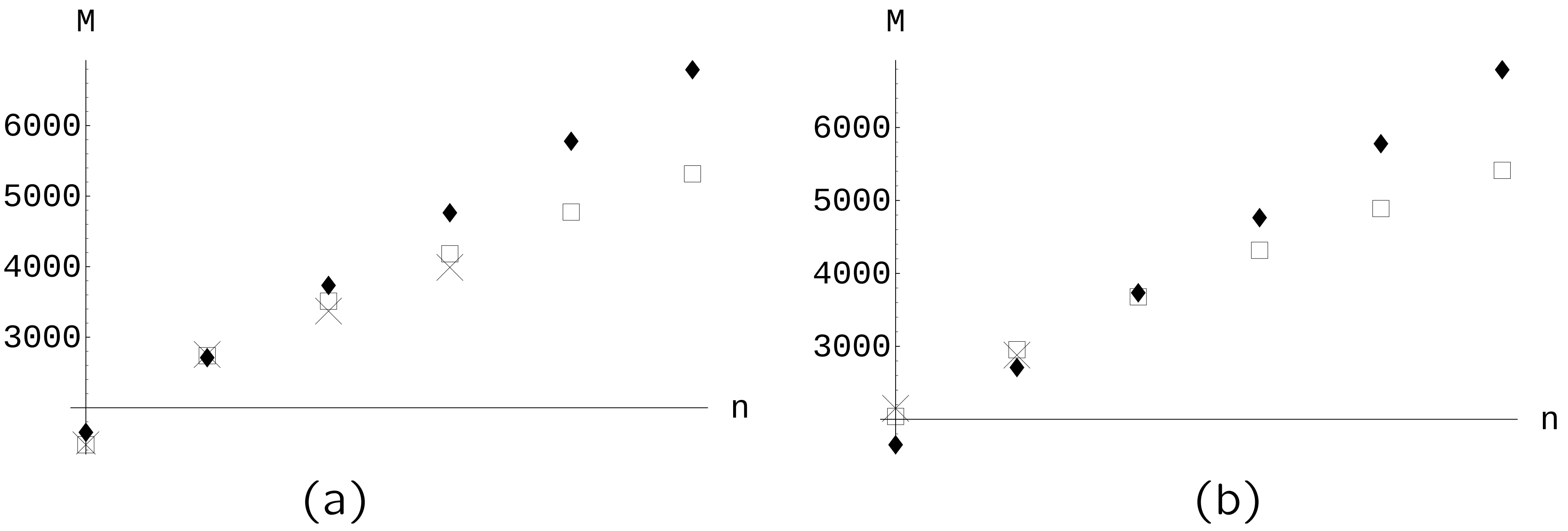
• All dimensionless coefficients of the potential are a priori parameters. However, a simple form is typically chosen for simplicity.

• At $T = 0$ we fit only one parameter:the normalization of the 't Hooft coupling.

• At $T > 0$ we fit one more parameter:the coefficient of the leading strong coupling term.

• We choose a dilaton potential with large-λ asymptotics $V(\lambda) \sim \sqrt{\log \lambda}$ to obtain linear asymptotic glueball spectrum.

Comparison with lattice data



Comparison of glueball spectra from our model with $b_0 = 4.2, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Meyer (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state.

$$\text{We measure} \quad : \frac{\ell_{eff}}{\ell} = 2.62 \quad , \quad \frac{\ell_s}{\ell} \simeq 0.16$$

and post-dict

$$\alpha_s(1.2\ GeV) = 0.34,$$

which is within the error of the quoted experimental value $\alpha_s^{(exp)}(1.2\ GeV) = 0.35 \pm 0.01$

Finite temperature

The theory at finite temperature can be described by:
(1) The "thermal vacuum solution". This is the zero-temperature solution we described so far with time periodically identified with period β.

(2) "black-hole" solutions

$$ds^2 = b(r)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx^i\right], \qquad \lambda = \lambda(r)$$

♠ We need VERY UNUSUAL boundary conditions: The dilaton (scalar) is diverging at the boundary so that $\lambda \sim e^{\phi} \to \frac{1}{\log r} \to 0$

- The boundary AdS is NOT at a minimum of the potential.

- No such type of solutions have been analyzed so far in the literature.

General phase structure

- For a general potential (with no minimum) the following can be shown :

i. *There exists a phase transition at finite $T = T_c$, if and only if the zero-T theory confines.*

ii. *This transition is of the first order for all of the confining geometries, with a single exception described in iii:*

iii. *In the limit confining geometry $b_0(r) \to e^{-Cr}$, $\lambda_0 \to e^{\frac{3}{2}Cr}$, (as $r \to \infty$), the phase transition is of the second order and happens at $T = 3C/4\pi$.*
This is the linear dilaton vacuum solution in the IR.

iv. *All of the non-confining geometries at zero T are always in the black hole phase at finite T. They exhibit a second order phase transition at $T = 0^+$.*

Finite-T Confining Theories

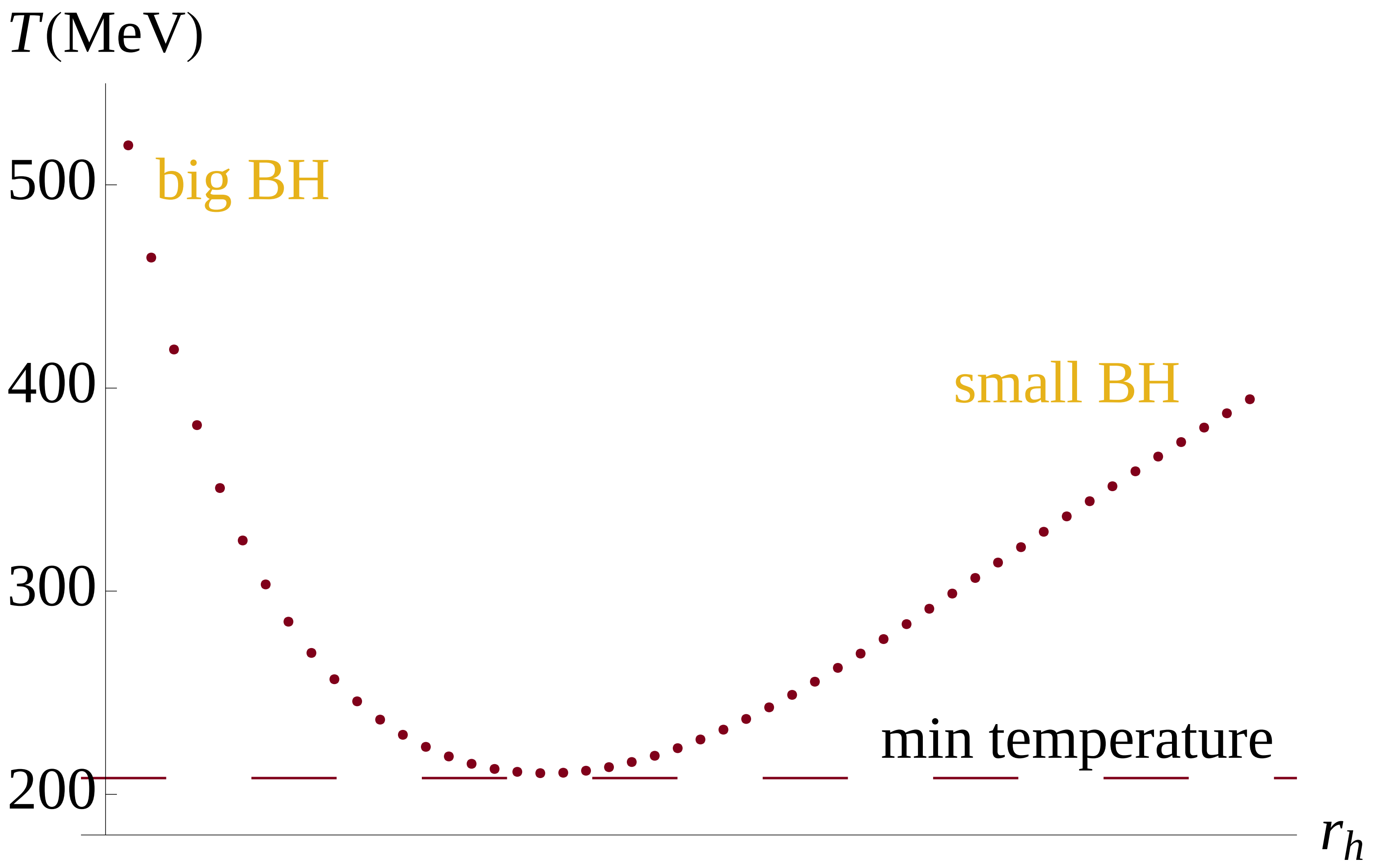
- There is a minimal temperature T_{min} for the existence of Black-hole solutions

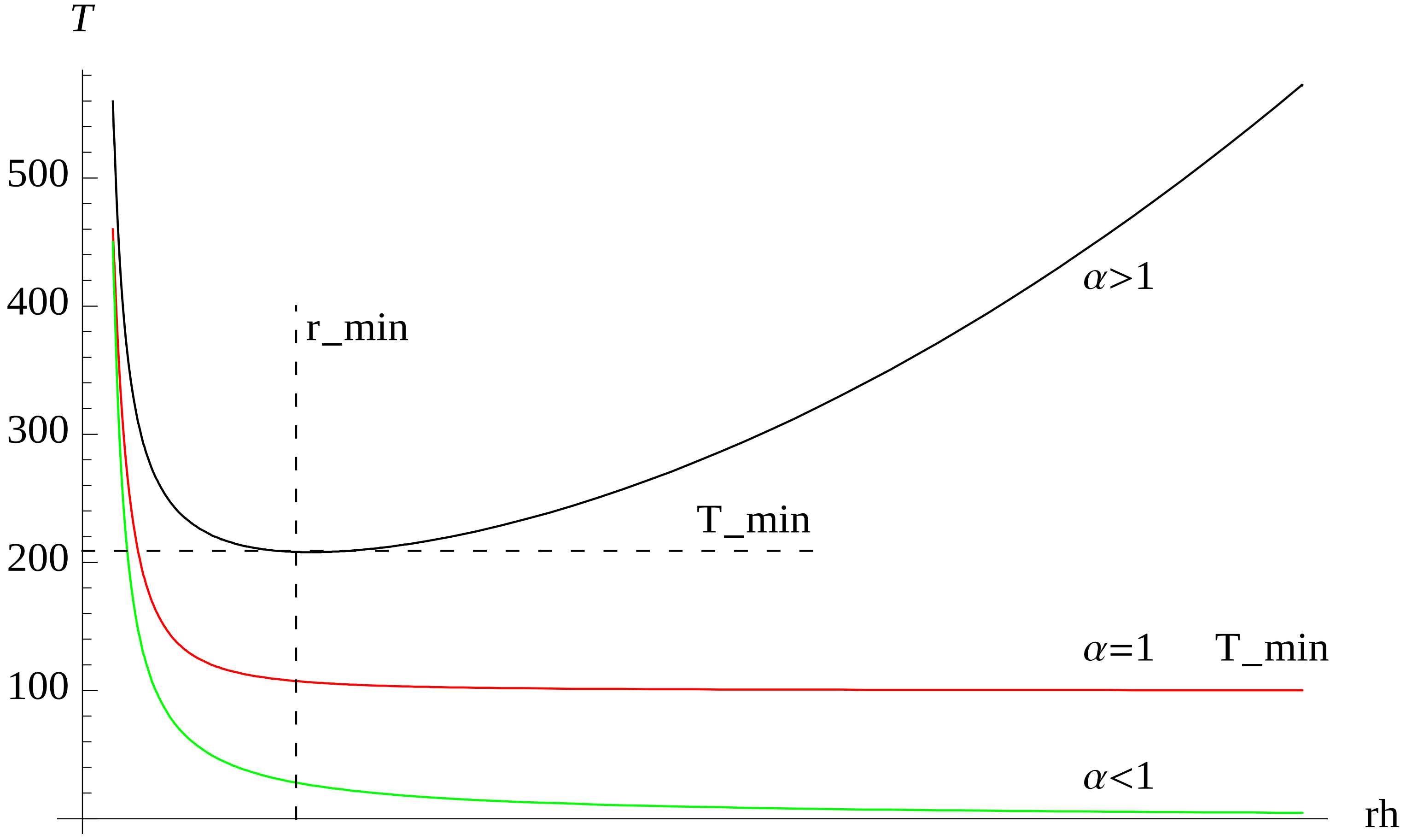
- When $T < T_{\rm min}$ only the "thermal vacuum solution" exists: it describes the confined phase at small temperatures.

- For $T > T_{\rm min}$ there are two black-hole solutions with the same temperature but different horizon positions. One is a "large" BH the other is "small".

- When $T > T_{\rm min}$ three competing solutions exist. The large BH has the lowest free energy for $T > T_c > T_{min}$. It describes the deconfined "Gluon-Glass" phase.

Temperature versus horizon position





We plot the relation $T(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is only one branch of black-hole solutions.

The free energy

• The free energy is calculated from the action as a boundary term for both the black-holes and the thermal vacuum solution. They are all UV divergent but their differences are finite.

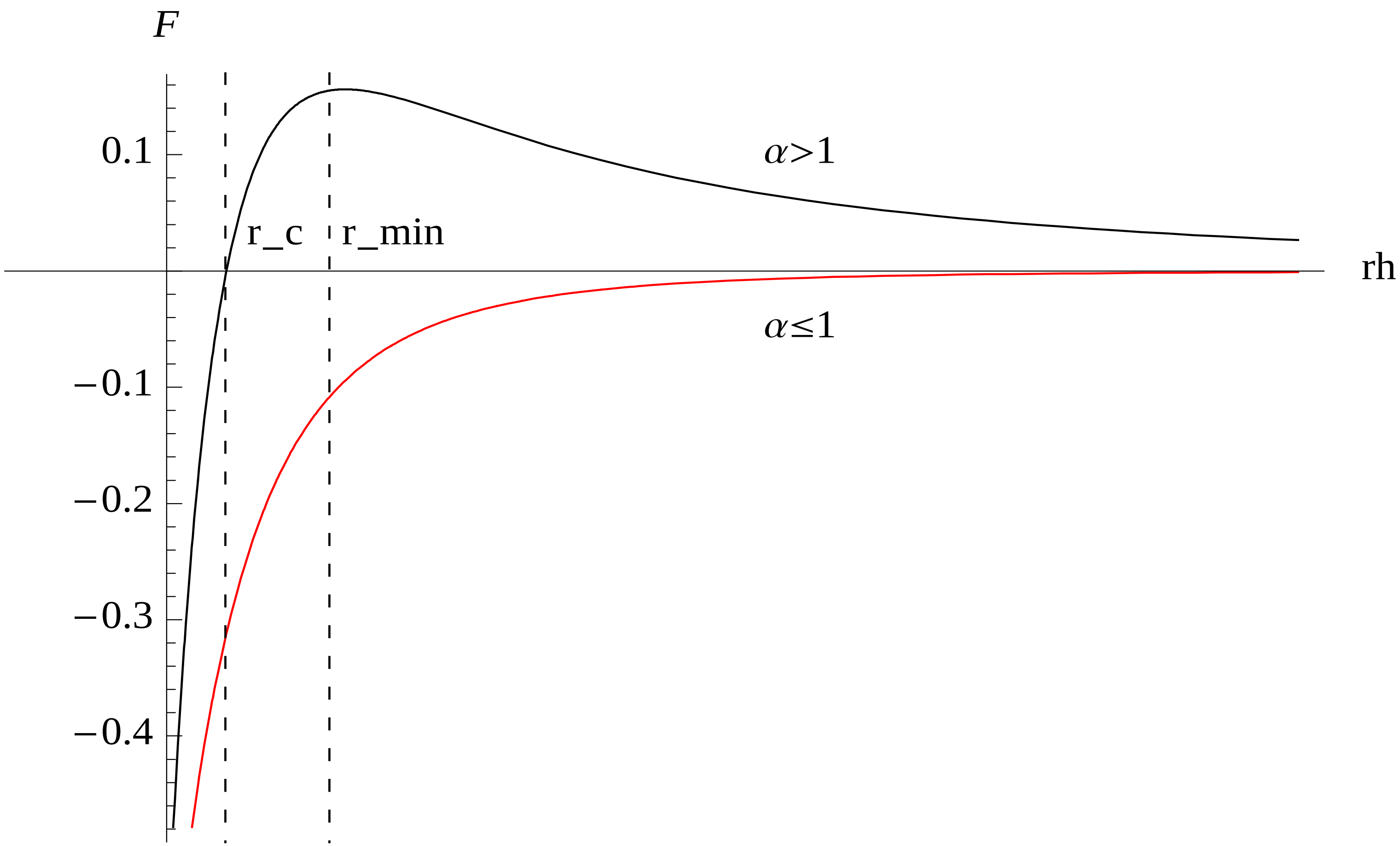
$$\frac{\mathcal{F}}{M_p^3 V_3} = 12\mathcal{G}(T) - T\ S(T)$$

• $\mathcal{G}$ is the temperature-depended gluon condensate $\langle Tr[F^2]\rangle_T - \langle Tr[F^2]\rangle_{T=0}$ defined as

$$\lim_{r\to 0} \quad \lambda_T(r) - \lambda_{T=0}(r) = \mathcal{G}(T)\ r^4 + \cdots$$

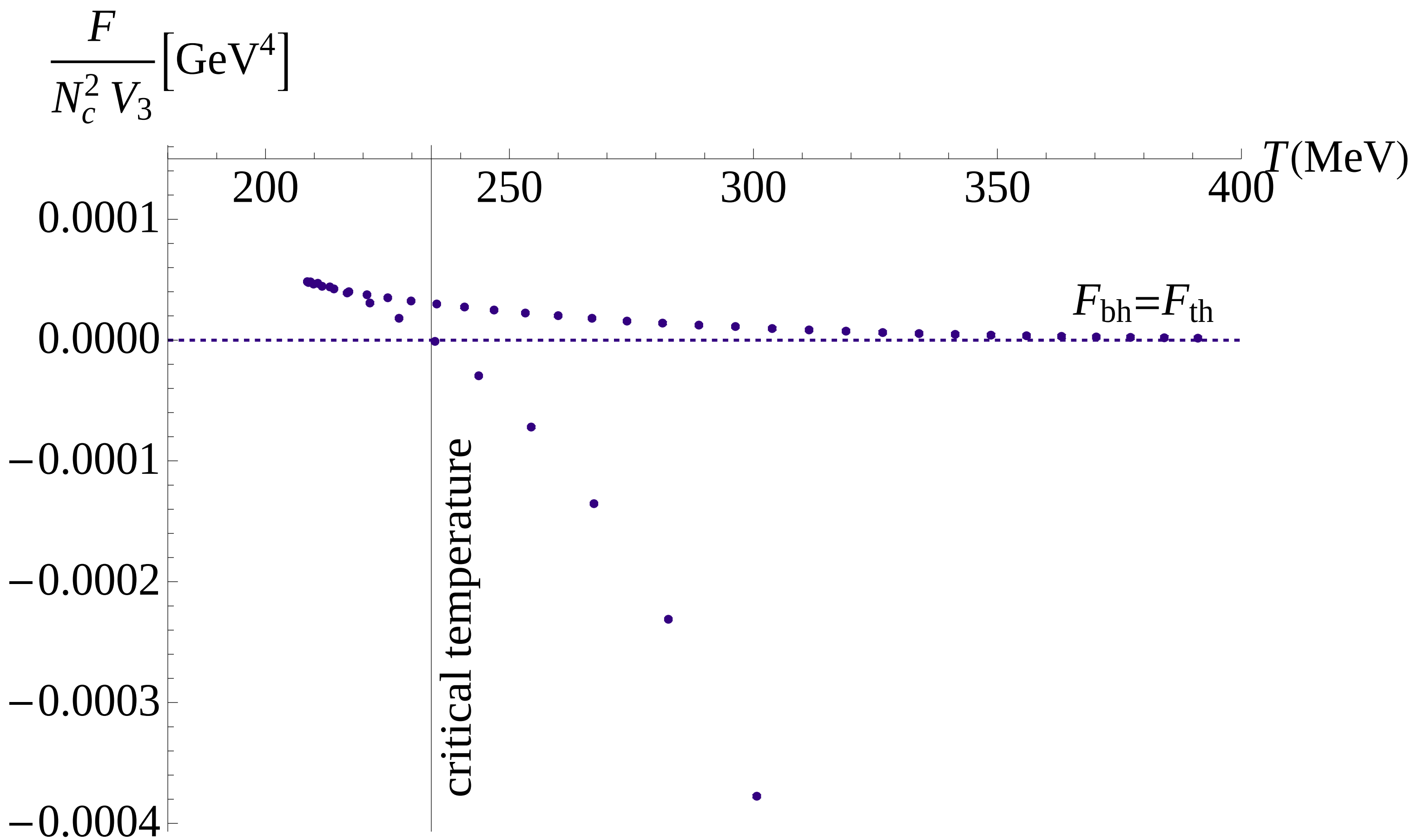
• It is $\mathcal{G}$ the breaks conformal invariance essentially and leads to a non-trivial deconfining transition (as $S > 0$ always)

Free energy versus horizon position



We plot the relation $\mathcal{F}(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is no first order phase transition .

The transition in the free energy



Critical temperature and latent heat

• For the YM potential the minimum temperature for the black-holes is $T_{\text{min}} \simeq 210$ MeV with $\lambda_h \simeq 12$. The critical temperature is

$$T_c \simeq 235 \pm 15 \quad \text{MeV} \quad \text{with} \quad \lambda_h \simeq 8 \quad , \quad \frac{L_h^{\frac{1}{4}}}{T_c} = 0.65\sqrt{N_c}$$

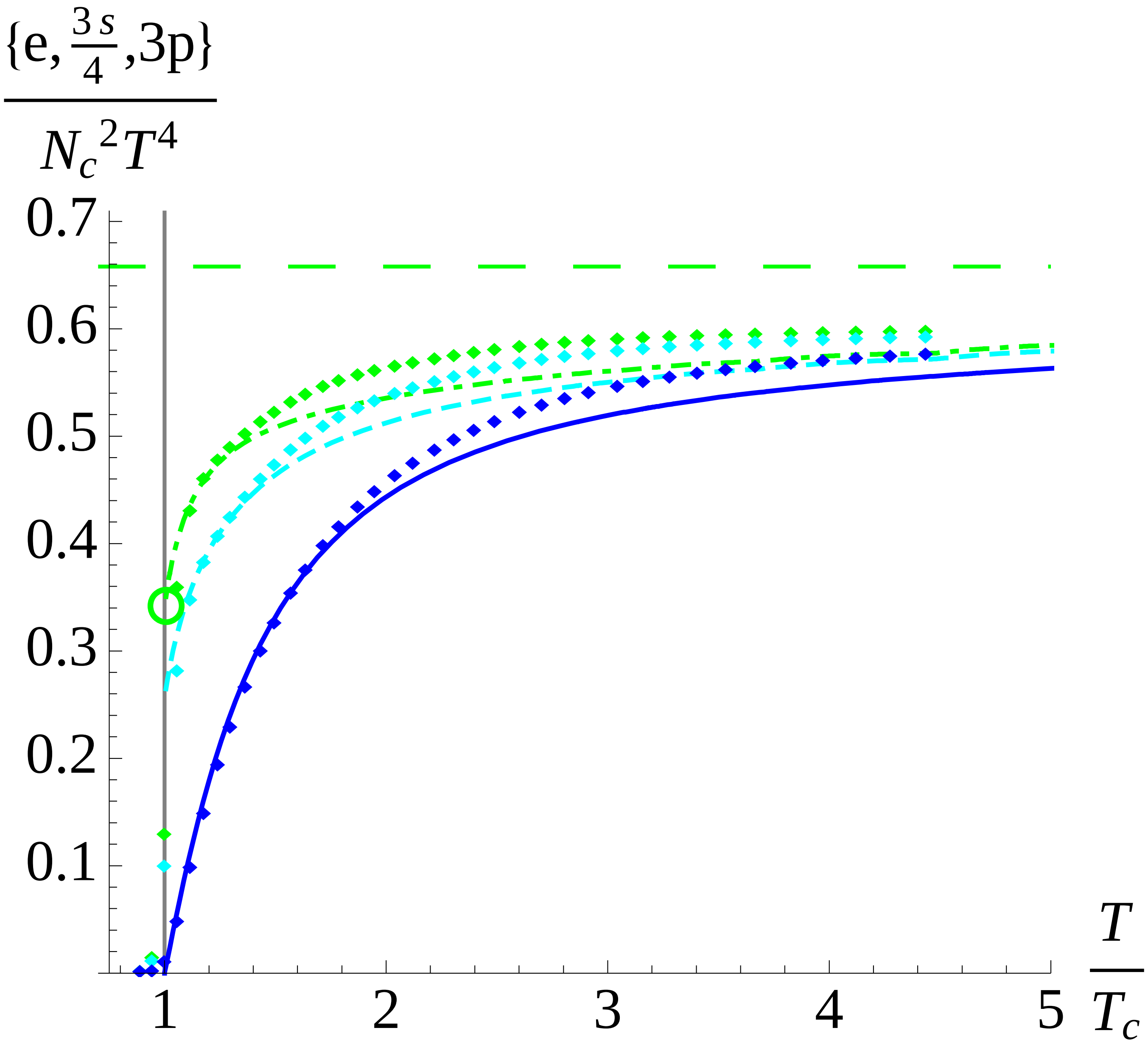
to be compared with 260 ± 11 MeV and $0.77\sqrt{N_c}$

Lucini+Teper, Lucini+Teper+Wenger

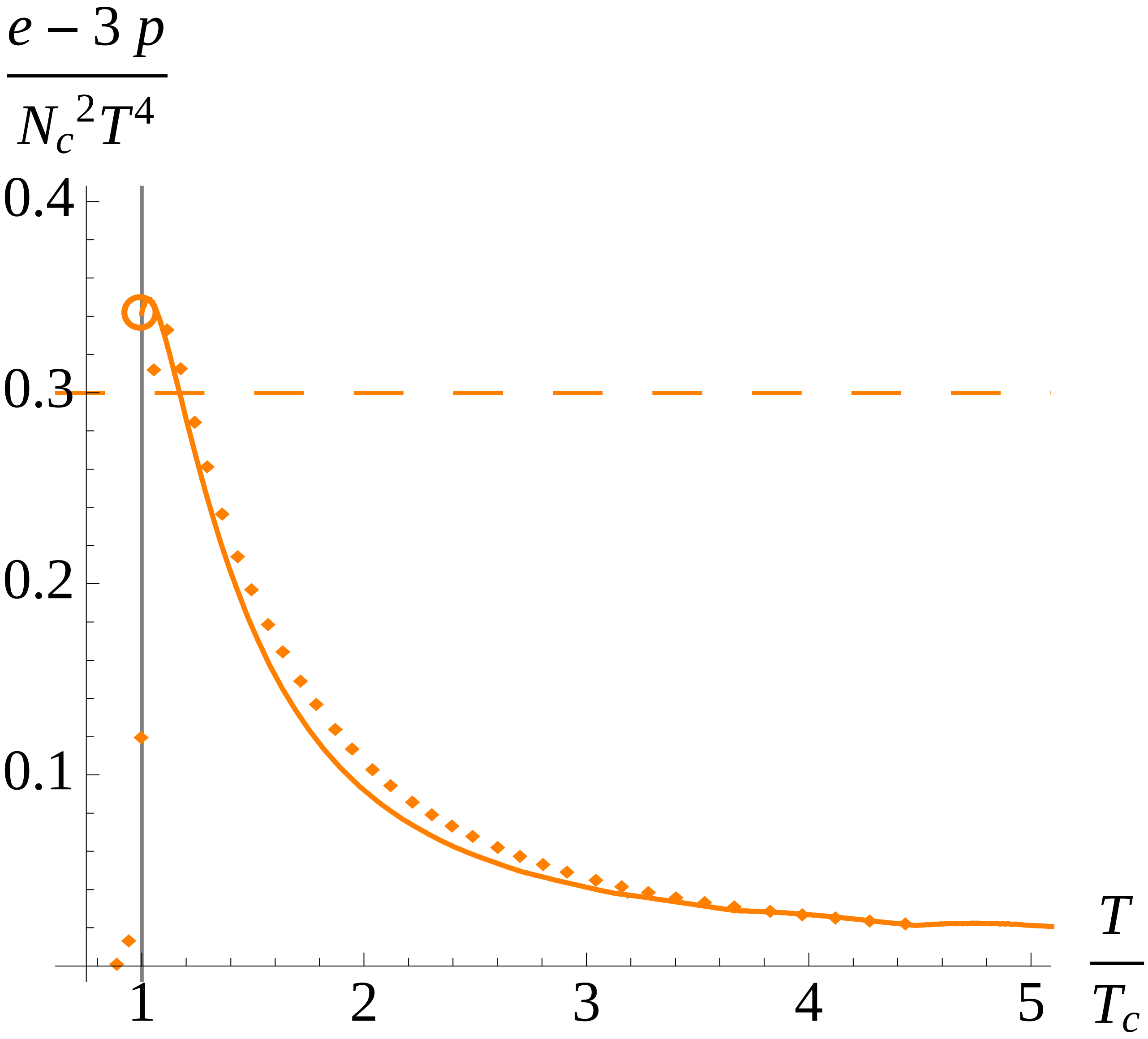
• The specific heat for the QGP solution is positive as it should. For the small black-hole it is negative.

• In the QGP phase, the $q\bar{q}$ potential is screened.

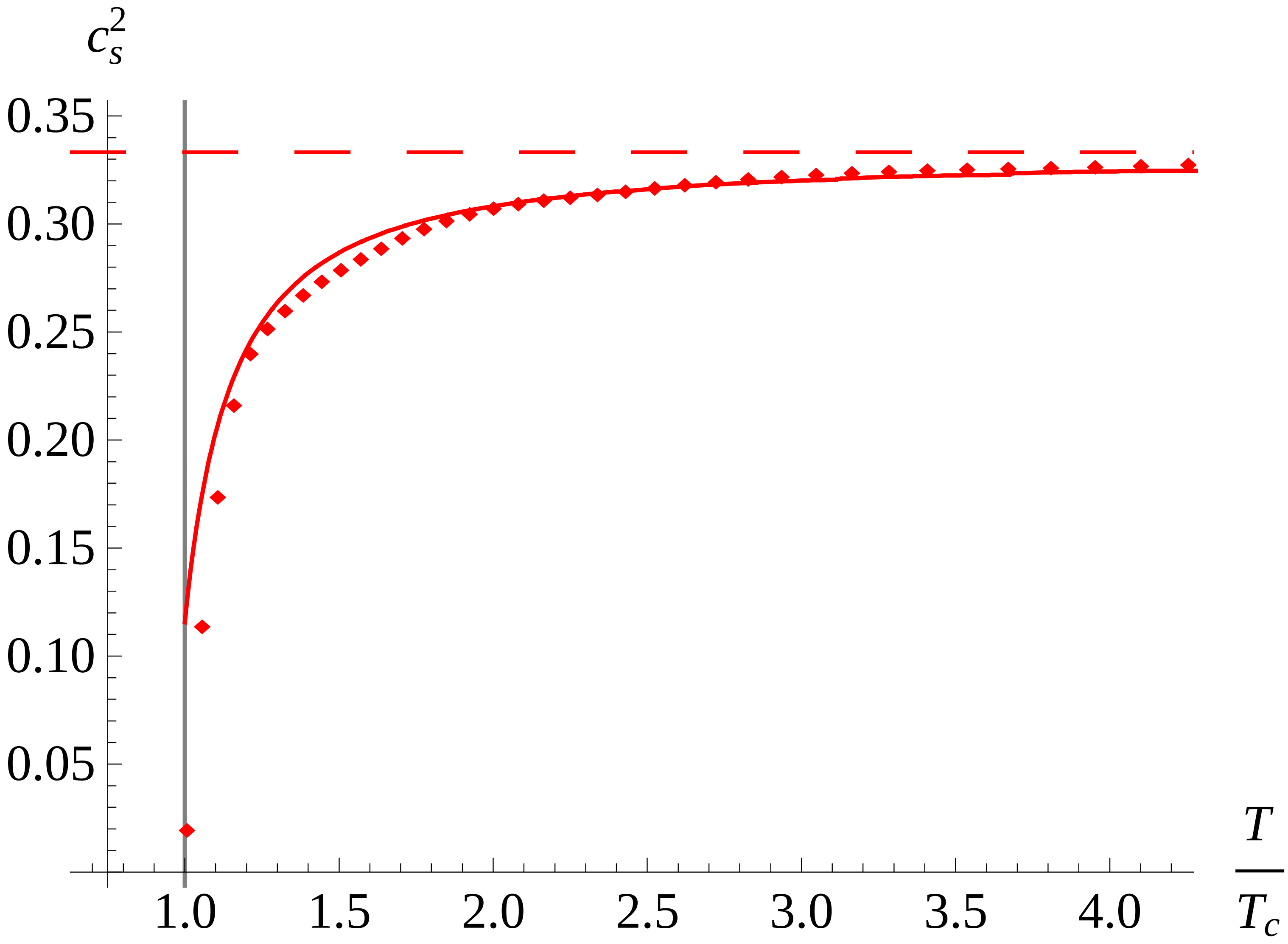
Thermodynamic variables



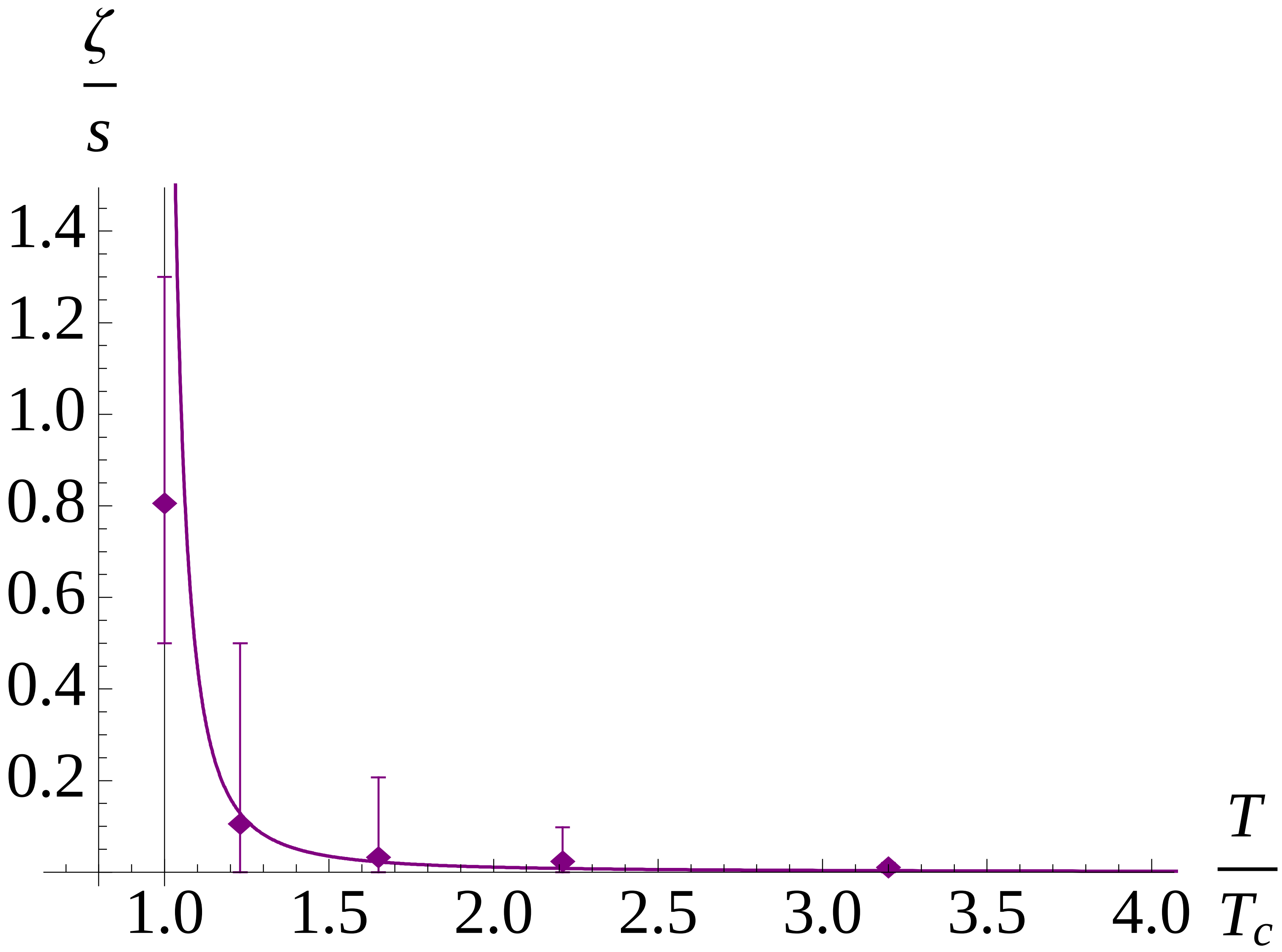
Equation of state



The speed of sound



The bulk viscosity:data



Open problems

THEORETICAL:

• Investigate further the structure of the string dual of QCD. Try to control the UV physics (to which RR flux plays little role).

MORE PRACTICAL:

• Re-Calculate quantities relevant for heavy ion collisions: jet quenching parameter, drag force etc.

• Calculate the finite-temperature Polyakov loops and Debye screening lengths in various symmetry channels.

• Investigate quantitatively the meson sector

• Calculate the phase diagram in the presence of baryon number.

AdS/QCD

♠ A basic phenomenological approach: use a slice of AdS$_5$, with a UV cutoff, and an IR cutoff.

Polchinski+Strassler

♠ It successfully exhibits confinement (trivially via IR cutoff), and power-like behavior in hard scattering amplitudes

♠ It may be equipped with a bifundamental scalar, T, and $U(N_f)_L \times U(N_f)_R$, gauge fields to describe mesons.

Erlich+Katz+Son+Stepanov, DaRold+Pomarol

Chiral symmetry is broken by hand, via IR boundary conditions. The low-lying meson spectrum looks "reasonable".

♠ Shortcomings:

- The glueball spectrum does not fit very well the lattice calculations. It has the wrong asymptotic behavior $m_n^2 \sim n^2$ at large n.
- Magnetic quarks are confined instead of screened.
- Chiral symmetry breaking is input by hand.
- The meson spectrum has also the wrong UV asymptotics $m_n^2 \sim n^2$.

♠ The asymptotic spectrum can be fixed by introducing a **non-dynamical** dilaton profile $\Phi \sim r^2$ (soft wall)

Karch+Katz+Son+Stephanov

The low dimension spectrum

- What are all gauge invariant YM operators of dimension 4 or less?

- They are given by $Tr[F_{\mu\nu}F_{\rho\sigma}]$.

Decomposing into U(4) reps:

$$\left(\ydiagram{1,1} \otimes \ydiagram{1,1}\right)_{\text{symmetric}} = \ydiagram{2,2} \oplus \ydiagram{1,1,1,1} \qquad (1)$$

We must remove traces to construct the irreducible representations of O(4):

$$\ydiagram{2,2} = \underline{\ydiagram{2,2}} \oplus \underline{\ydiagram{2}} \oplus \bullet \quad , \quad \ydiagram{1,1,1,1} = \bullet$$

The two singlets are the scalar (dilaton) and pseudoscalar (axion)

$$\phi \leftrightarrow Tr[F^2] \quad , \quad a \leftrightarrow Tr[F \wedge F]$$

The traceless symmetric tensor

$$\underline{\ydiagram{2}} \quad \to \quad T_{\mu\nu} = Tr\left[F^2_{\mu\nu} - \frac{1}{4} g_{\mu\nu} F^2\right]$$

is the conserved stress tensor dual to a massless graviton in 5d reflecting the translational symmetry of YM.

$$\underline{\ydiagram{2,2}} \quad \to \quad T^4_{\mu\nu;\rho\sigma} = Tr[F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2}(g_{\mu\rho}F^2_{\nu\sigma} - g_{\nu\rho}F^2_{\mu\sigma} - g_{\mu\sigma}F^2_{\nu\rho} + g_{\nu\sigma}F^2_{\mu\rho}) + \frac{1}{6}(g_{\mu\rho}g_{\nu\sigma} - g_{\nu\rho}g_{\mu\sigma})F^2]$$

It has 10 independent d.o.f, it is not conserved and it should correspond to a similar massive tensor in 5d. We do not expect it to play an non-trivial role in the large-N_c, YM vacuum also for reasons of Lorentz invariance.

- Therefore the nontrivial fields are expected to be:

$$g_{\mu\nu}, \phi, a$$

An assessment of IR asymptotics

• As $\lambda \to \infty$ we assume that the potential terms dominate and we parameterize the effective action in the IR as

$$S_{eff} \sim \int \sqrt{g} \left[R + \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) = \right] \quad , \quad V(\lambda) = \frac{4}{3}\lambda^2 \left(\frac{dW}{d\lambda}\right)^2 + \frac{64}{27}W^2$$

Parameterize the IR asymptotics ($\lambda \to \infty$) as

$$W(\lambda) \sim (\log\lambda)^{\frac{P}{2}}\ \lambda^Q$$

• $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

$$e^A(r) \sim \begin{cases} (r_0 - r)^{\frac{4}{9Q^2-4}} & Q > \frac{2}{3} \\ \exp\left[-\frac{C}{(r_0-r)^{1/(P-1)}}\right] & Q = \frac{2}{3} \end{cases}$$

• $Q = 2/3$, and $0 \leq P < 1$ leads to confinement and a singularity at $r = \infty$
The scale factor e^A vanishes there as

$$e^A(r) \sim \exp[-Cr^{1/(1-P)}].$$

The asymptotic spectrum of glueballs is linear only if $P = \frac{1}{2}$

• $Q = 2/3, P = 1$ leads to confinement but the singularity may be at a finite or infinite value of r depending on subleading asymptotics of the superpotential.

♠ If $Q < 2\sqrt{2}/3$, no *ad hoc* boundary conditions are needed to determine the glueball spectrum: the singularity is "good" (repulsive).

♠ when $Q > 2\sqrt{2}/3$, the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

Selecting the IR asymptotics

Only the $Q = 2/3$, $0 \leq P < 1$ is compatible with

- Confinement (it happens non-trivially: a minimum in the string frame scale factor)
- Mass gap+discrete spectrum (except P=0)
- good singularity
- $R \to 0$ partly justifying the original assumption. More precisely: the string frame metric becomes flat at the IR . But $(\partial\phi)^2 \sim V(\lambda)$.

♠ It is interesting that the lower endpoint: P=0 corresponds to linear dilaton and flat space (string frame). It is confining with a mass gap but continuous spectrum.

- For linear asymptotic trajectories for fluctuations (glueballs) we must choose $P = 1/2$

$$V(\lambda) = \lambda^{\frac{4}{3}}\left[1 + c_1\lambda^2 + c_2\lambda^4 + \cdots\right] \sim \lambda^{\frac{4}{3}}\sqrt{\log\lambda} + \text{subleading} \qquad \text{as} \qquad \lambda \to \infty$$

Further α' corrections

There are further dilaton terms generated by the 5-form in:

• The kinetic terms of the graviton and the dilaton $\sim \lambda^{2n}$.

• The kinetic terms on probe D_3 branes that affect the identification of the gauge-coupling constant, $\sim \lambda^{2n+1}$. There is also a multiplicative factor relating g_{YM^2} to e^{ϕ}, (not known). Can be traded for b_0.

• Corrections to the identification of the energy. At $r = 0$, $E = 1/r$. There can be log corrections to our identification $E = e^A$, and these are a power series in $\sim \lambda^{2n}$.

• It is a remarkable fact that all such corrections affect the higher that the first two terms in the β-function (or equivalently the potential), that are known to be non-universal!

the metric is also insensitive to the change of b_0 by changing Λ.

Organizing the vacuum solutions

A useful variable is the phase variable

$$X \equiv \frac{\Phi'}{3A'} = \frac{\beta(\lambda)}{3\lambda} \quad , \quad e^{\Phi} \equiv \lambda$$

and a superpotential

$$W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \Phi}\right)^2 = \left(\frac{3}{4}\right)^3 V(\Phi).$$

with

$$A' = -\frac{4}{9}W \quad , \quad \Phi' = \frac{dW}{d\Phi}$$

$$X = -\frac{3}{4}\frac{d\log W}{d\log\lambda} \quad , \quad \beta(\lambda) = -\frac{9}{4}\lambda\frac{d\log W}{d\log\lambda}$$

♠ The equations have three integration constants: (two for Φ and one for A) One corresponds to the "gluon condensate" in the UV. It must be set to zero otherwise the IR behavior is unacceptable. The other is Λ. The third one is a gauge artifact (corresponds to overall translation in the radial coordinate).

The IR regime

For any asymptotically AdS$_5$ solution ($e^A \sim \frac{\ell}{r}$):

• The scale factor $e^{A(r)}$ is monotonically decreasing

Girardelo+Petrini+Porrati+Zaffaroni
Freedman+Gubser+Pilch+Warner

• Moreover, there are only three possible, mutually exclusive IR asymptotics:

♠ *there is another asymptotic AdS_5 region, at $r \to \infty$, where* $\exp A(r) \sim \ell'/r$, *and* $\ell' \leq \ell$ (equality holds if and only if the space is exactly AdS_5 everywhere);

♠ there is a curvature singularity at some finite value of the radial coordinate, $r = r_0$;

♠ **there is a curvature singularity at $r \to \infty$, where the scale factor vanishes and the space-time shrinks to zero size.**

Wilson-Loops and confinement

• Calculation of the static quark potential using the vev of the Wilson loop calculated via an F-string worldsheet.

Rey+Yee, Maldacena

$$T\ E(L) = S_{minimal}(X)$$

We calculate

$$L = 2\int_0^{r_0} dr \frac{1}{\sqrt{e^{4A_S(r)-4A_S(r_0)}-1}}.$$

It diverges when e^{A_s} has a minimum (at $r = r_*$). Then

$$E(L) \sim T_f\ e^{2A_S(r_*)}\ L$$

• Confinement $\to$ $A_s(r_*)$ is finite. This is a more general condition that considered before as A_S `is not monotonic in general`.

• Effective string tension

$$T_{\text{string}} = T_f\ e^{2A_S(r_*)}$$

General criterion for confinement

• **the geometric version**:
A geometry that shrinks to zero size in the IR is dual to a confining 4D theory if and only if the Einstein metric in conformal coordinates vanishes as (or faster than) e^{-Cr} as $r \to \infty$, for some $C > 0$.

• It is understood here that a metric vanishing at finite $r = r_0$ also satisfies the above condition.

♠ **the superpotential**
A 5D background is dual to a confining theory if the superpotential grows as (or faster than)

$$W \sim (\log \lambda)^{P/2} \lambda^{2/3} \quad \text{as} \quad \lambda \to \infty \quad , \quad P \geq 0$$

♠ **the β-function** A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda \to \infty} \left(\frac{\beta(\lambda)}{3\lambda} + \frac{1}{2} \right) \log \lambda = K, \qquad -\infty \leq K \leq 0$$

(No explicit reference to any coordinate system) Linear trajectories correspond to $K = -\frac{3}{16}$

Classification of confining superpotentials

Classification of confining superpotentials $W(\lambda)$ as $\lambda \to \infty$ in IR:

$$W(\lambda) \sim (\log \lambda)^{\frac{P}{2}}\ \lambda^Q \quad , \quad \lambda \sim E^{-\frac{9}{4}Q}\left(\log \frac{1}{E}\right)^{\frac{P}{2Q}}, \qquad E \to 0.$$

- $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

$$e^A(r) \sim \begin{cases} (r_0 - r)^{\frac{4}{9Q^2-4}} & Q > \frac{2}{3} \\ \exp\left[-\frac{C}{(r_0-r)^{1/(P-1)}}\right] & Q = \frac{2}{3} \end{cases}$$

- $Q = 2/3$, and $0 \le P < 1$ leads to confinement and a singularity at $r = \infty$ The scale factor e^A vanishes there as

$$e^A(r) \sim \exp[-Cr^{1/(1-P)}].$$

- $Q = 2/3, P = 1$ leads to confinement but the singularity may be at a finite or infinite value of r depending on subleading asymptotics of the superpotential.

♠ If $Q < 2\sqrt{2}/3$, no *ad hoc* boundary conditions are needed to determine the glueball spectrum → One-to-one correspondence with the β-function This is unlike standard AdS/QCD and other approaches.

- when $Q > 2\sqrt{2}/3$, the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

Confining β-functions

A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda\to\infty}\left(\frac{\beta(\lambda)}{3\lambda}+\frac{1}{2}\right)\log\lambda = K, \qquad -\infty \le K \le 0$$

(No explicit reference to any coordinate system). Linear trajectories correspond to $K = -\frac{3}{16}$

- We can determine the geometry if we specify K:
- $K = -\infty$: the scale factor goes to zero at some finite r_0, not faster than a power-law.
- $-\infty < K < -3/8$: the scale factor goes to zero at some finite r_0 faster than any power-law.
- $-3/8 < K < 0$: the scale factor goes to zero as $r \to \infty$ faster than $e^{-Cr^{1+\epsilon}}$ for some $\epsilon > 0$.
- $K = 0$: the scale factor goes to zero as $r \to \infty$ as e^{-Cr} (or faster), but slower than $e^{-Cr^{1+\epsilon}}$ for any $\epsilon > 0$.

The borderline case, $K = -3/8$, is certainly confining (by continuity), but whether or not the singularity is at finite r depends on the subleading terms.

Comments on confining backgrounds

• For all confining backgrounds with $r_0 = \infty$, although the space-time is singular in the Einstein frame, the string frame geometry is asymptotically flat for large r. Therefore only λ grows indefinitely.

• String world-sheets do not probe the strong coupling region, at least classically. The string stays away from the strong coupling region.

• Therefore: singular confining backgrounds have generically the property that the singularity is *repulsive*, i.e. only highly excited states can probe it. This will also be reflected in the analysis of the particle spectrum (to be presented later)

• The confining backgrounds must also screen magnetic color charges. This can be checked by calculating 't Hooft loops using D_1 probes:

♠ All confining backgrounds with $r_0 = \infty$ and most at finite r_0 screen properly

♠ In particular "hard-wall" AdS/QCD confines also the magnetic quarks.

Particle Spectra: generalities

- Linearized equation:

$$\ddot{\xi} + 2\dot{B}\dot{\xi} + \Box_4 \xi = 0 \quad , \quad \xi(r,x) = \xi(r)\xi^{(4)}(x), \qquad \Box\xi^{(4)}(x) = m^2\xi^{(4)}(x)$$

- Can be mapped to Schrodinger problem

$$-\frac{d^2}{dr^2}\psi + V(r)\psi = m^2\psi \quad , \quad V(r) = \frac{d^2B}{dr^2} + \left(\frac{dB}{dr}\right)^2 \quad , \quad \xi(r) = e^{-B(r)}\psi(r)$$

- Mass gap and discrete spectrum visible from the asymptotics of the potential.

- Large n asymptotics of masses obtained from WKB

$$n\pi = \int_{r_1}^{r_2} \sqrt{m^2 - V(r)}\; dr$$

- Spectrum depends only on initial condition for λ $(\sim \Lambda_{QCD})$ and an overall energy scale (e^A) that must be fixed.

- scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log\frac{\beta(\lambda)^2}{9\lambda^2}$$

- tensor glueballs

$$B(r) = \frac{3}{2}A(r)$$

- pseudo-scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log Z(\lambda)$$

- Universality of asymptotics

$$\frac{m^2_{n\to\infty}(0^{++})}{m^2_{n\to\infty}(2^{++})} \to 1 \quad , \quad \frac{m^2_{n\to\infty}(0^{+-})}{m^2_{n\to\infty}(0^{++})} = \frac{1}{4}(d-2)^2$$

predicts $d = 4$ via

$$\frac{m^2}{2\pi\sigma_a} = 2n + J + c,$$

The axion background

• The kinetic term of the axion is suppressed by $1/N_c^2$. (it is an angle in the gauge theory, it is RR in string theory)

$$\ddot{a} + \left(3\dot{A} + \frac{\dot{Z}(\lambda)}{Z(\lambda)}\right)\dot{a} = 0 \quad \to \quad \dot{a} = \frac{C\ e^{-3A}}{Z(\lambda)}$$

It can be interpreted as the flow equation of the effective θ-angle.

• The full solution is

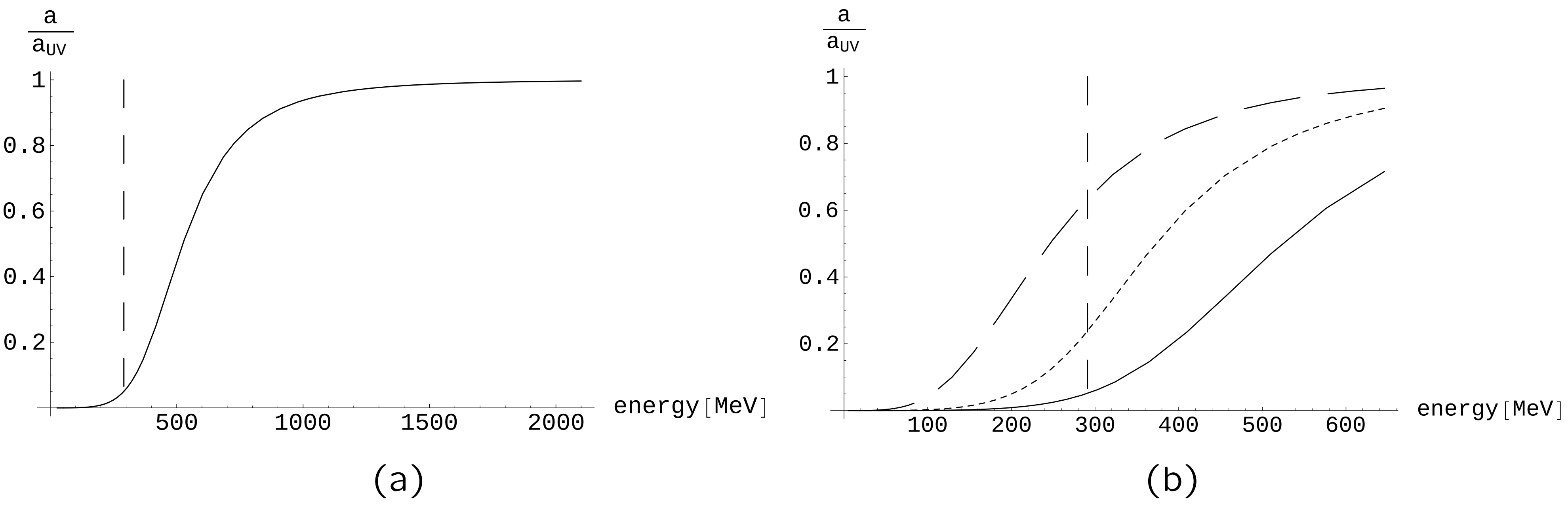
$$a(r) = \theta_{UV} + 2\pi k + C\ \int_0^r r\frac{e^{-3A}}{Z(\lambda)} \quad , \quad C = \langle Tr[F \wedge F]\rangle$$

• The vacuum energy is

$$E(\theta_{UV}) = \frac{M^3}{2N_c^2}\int d^5x\sqrt{g}Z(\lambda)(\partial a)^2 = \frac{M^3}{2N_c^2}Ca(r)\ \Big|_{r=0}^{r=r_0}$$

• Consistency requires to impose that $a(r_0) = 0$. This determines C and

$$E(\theta_{UV}) = -\frac{M^3}{2}\text{Min}_k\ \frac{(\theta_{UV} + 2\pi k)^2}{\int_0^{r_0}\frac{dr}{e^{3A}Z(\lambda)}} \quad , \quad \frac{a(r)}{\theta_{UV} + 2\pi k} = \frac{\int_r^{r_0}\frac{dr}{e^{3A}Z(\lambda)}}{\int_0^{r_0}\frac{dr}{e^{3A}Z(\lambda)}}$$



(a) An example of the axion profile (normalized to one in the UV) as a function of energy, in one of the explicit cases we treat numerically. The energy scale is in MeV, and it is normalized to match the mass of the lowest scalar glueball from lattice data, $m_0 = 1475 MeV$. The axion kinetic function is taken as $Z(\lambda) = Z_a(1 + c_a\lambda^4)$, with $c_a = 100$ (the masses do not depend on the value of Z_a). The vertical dashed line corresponds to $\Lambda_p \equiv \frac{1}{\ell}\frac{\exp\left[A(\lambda_0) - \frac{1}{b_0\lambda_0}\right]}{(b_0\lambda_0)^{b1/b_0^2}}$. In this particular case $\Lambda = 290 MeV$.

(b)A detail showing the different axion profiles for different values of c_a. The values are $c_a = 0.1$ (dashed line), $c_a = 10$ (dotted line) and $c_a = 100$ (solid line).

Quarks ($N_f \ll N_c$) and mesons

• Flavor is introduced by N_f $D_4 + \bar{D}_4$ branes pairs inside the bulk background. Their back-reaction on the bulk geometry is suppressed by N_f/N_c.

• The important world-volume fields are

$$T_{ij} \leftrightarrow \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \quad , \quad A^{ij\,L,R}_\mu \leftrightarrow \bar{q}^i_a \frac{1\pm\gamma^5}{2} \gamma^\mu q^j_a$$

Generating the $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

• The UV mass matrix m_{ij} corresponds to the source term of the Tachyon field. It breaks the chiral (gauge) symmetry. The normalizable mode corresponds to the vev $\langle \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \rangle$.

• We show that the expectation value of the tachyon is non-zero and $T \sim \mathbf{1}$, breaking chiral symmetry $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$. The anomaly plays an important role in this (holographic Coleman-Witten)

• The fact that the tachyon diverges in the IR (fusing D with $\bar{D}$) constraints the UV asymptotics and determines the quark condensate $\langle\bar{q}q\rangle$ in terms of m_q. A GOR relation is satisfied (for an asymptotic AdS_5 space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\ \langle\bar{q}q\rangle \quad , \quad m_q \to 0$$

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

• Studying the spectrum of highly excited mesons, we find the expected property of linear confinement: $m_n^2 \sim n$.

• The detailed spectrum of mesons remains to be worked out

Tachyon dynamics

• In the vacuum the gauge fields vanish and $T \sim 1$. Only DBI survives

$$S[\tau] = T_{D_4} \int dr d^4x \, \frac{e^{4A_s(r)}}{\lambda} \, V(\tau) \, \sqrt{e^{2A_s(r)} + \dot{\tau}(r)^2} \quad , \quad V(\tau) = e^{-\frac{\mu^2}{2}\tau^2}$$

• We obtain the nonlinear field equation:

$$\ddot{\tau} + \left(3\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right)\dot{\tau} + e^{2A_S}\mu^2\tau + e^{-2A_S}\left[4\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right]\dot{\tau}^3 + \mu^2\tau \; \dot{\tau}^2 = 0.$$

• In the UV we expect

$$\tau = m_q \; r + \sigma \; r^3 + \cdots \quad , \quad \mu^2\ell^2 = 3$$

• We expect that the tachyon must diverge before or at $r = r_0$. We find that indeed it does `at the singularity`. For the $r_0 = \infty$ backgrounds

$$\tau \sim \exp\left[\frac{2}{a} \, \frac{R}{\ell^2} \; r\right] \qquad \text{as} \qquad r \to \infty$$

• Generically the solutions have spurious singularities: $\tau(r_*)$ stays finite but its derivatives diverges as:

$$\tau \sim \tau_* + \gamma\sqrt{r_* - r}.$$

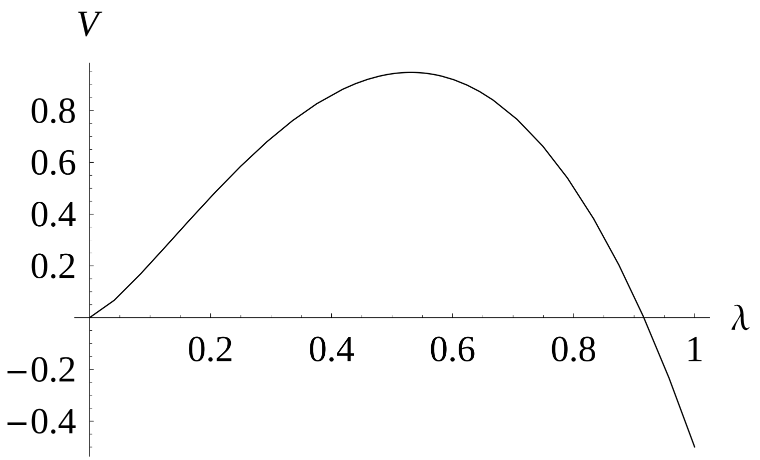
The condition that they are absent determines σ as a function of m_q.

• The easiest spectrum to analyze is that of vector mesons. We find $(r_0 = \infty)$

$$\Lambda_{glueballs} = \frac{1}{R}, \qquad \Lambda_{mesons} = \frac{3}{\ell}\left(\frac{\alpha\ell^2}{2R^2}\right)^{(\alpha-1)/2} \propto \frac{1}{R}\left(\frac{\ell}{R}\right)^{\alpha-2}.$$

This suggests that $\alpha = 2$. preferred also from the glue sector.

Fluctuations around the AdS$_5$ extremum



- In QCD we expect that

$$\frac{1}{\lambda} = \frac{1}{N_c e^{\phi}} \sim \frac{1}{\log r} \quad , \quad ds^2 \sim \frac{1}{r^2}(dr^2 + dx_\mu dx^\mu) \qquad \text{as} \qquad r \to 0$$

- Any potential with $V(\lambda) \sim \lambda^a$ when $\lambda \ll 1$ gives a power different that of AdS$_5$

- There is an AdS$_5$ minimum at a finite value λ_*. This cannot be the UV of QCD as dimensions do not match.

Near an AdS extremum

$$V = \frac{12}{\ell^2} - \frac{16\xi}{3\ell^2}\phi^2 + \mathcal{O}(\phi^3) \quad , \quad \frac{18}{\ell}\delta A' = \delta\phi'^2 - \frac{4}{\ell^2}\phi^2 = \mathcal{O}(\delta\phi^2) \quad , \quad \delta\phi'' - \frac{4}{\ell}\delta\phi' - \frac{4\xi}{\ell^2}\delta\phi = 0$$

where $\phi << 1$. The general solution of the second equation is

$$\delta\phi = C_+ e^{\frac{(2+2\sqrt{1+\xi})u}{\ell}} + C_- e^{\frac{(2-2\sqrt{1+\xi})u}{\ell}}$$

For the potential in question

$$V(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2}\left[5 - \frac{N_c^2}{2}e^{2\phi} - N_f\ e^{\phi}\right] \quad , \quad \lambda_0 \equiv N_c e^{\phi_0} = \frac{-7x + \sqrt{49x^2+400}}{10} \quad , \quad x \equiv \frac{N_f}{N_c}$$

$$\xi = \frac{5}{4}\left[\frac{400 + 49x^2 - 7x\sqrt{49x^2+400}}{100 + 7x^2 - x\sqrt{49x^2+400}}\right] \quad , \quad \frac{\ell_s^2}{\ell^2} = e^{\frac{4}{3}\phi_0}\left[\frac{100 + 7x^2 - x\sqrt{49x^2+400}}{400}\right]$$

The associated dimension is $\Delta = 2 + 2\sqrt{1+\xi}$ and satisfies

$$2 + 3\sqrt{2} < \Delta < 2 + 2\sqrt{6} \qquad \text{or equivalently} \qquad 6.24 < \Delta < 6.90$$

It corresponds to an irrelevant operator. It is most probably relevant for the Banks-Zaks fixed points.

Bigazzi+Casero+Cotrone+Kiritsis+Paredes

RETURN

Concrete potential

- The superpotential chosen is

$$W = (3 + 2b_0\lambda)^{2/3}\left[18 + \left(2b_0^2 + 3b_1\right)\log(1+\lambda^2)\right]^{4/3},$$

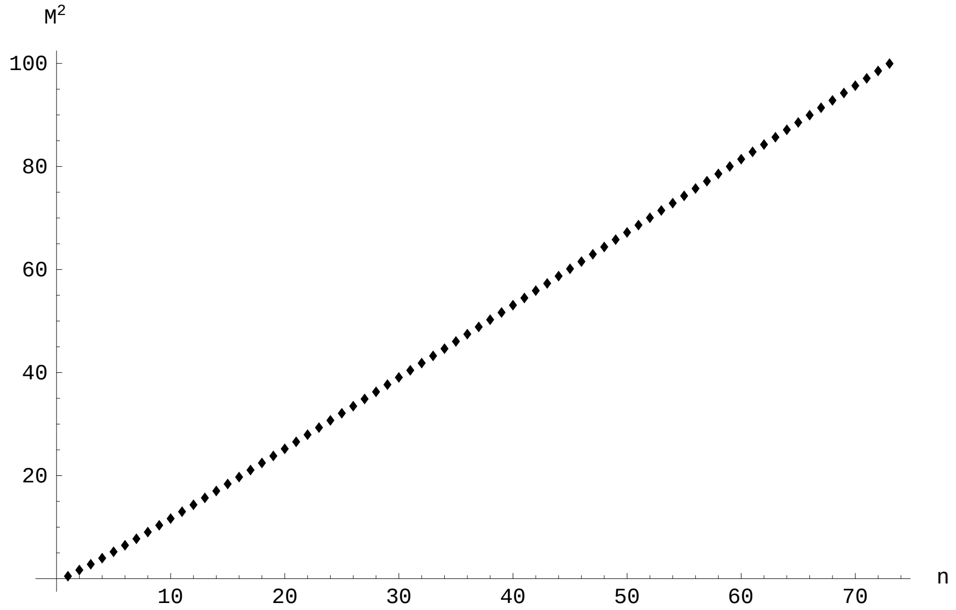
with corresponding potential

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3+2b_0\lambda} - \frac{6(2b_0^2+3b_1^2)\lambda^3}{(1+\lambda^2)\left(18+\left(2b_0^2+3b_1^2\right)\log(1+\lambda^2)\right)}$$

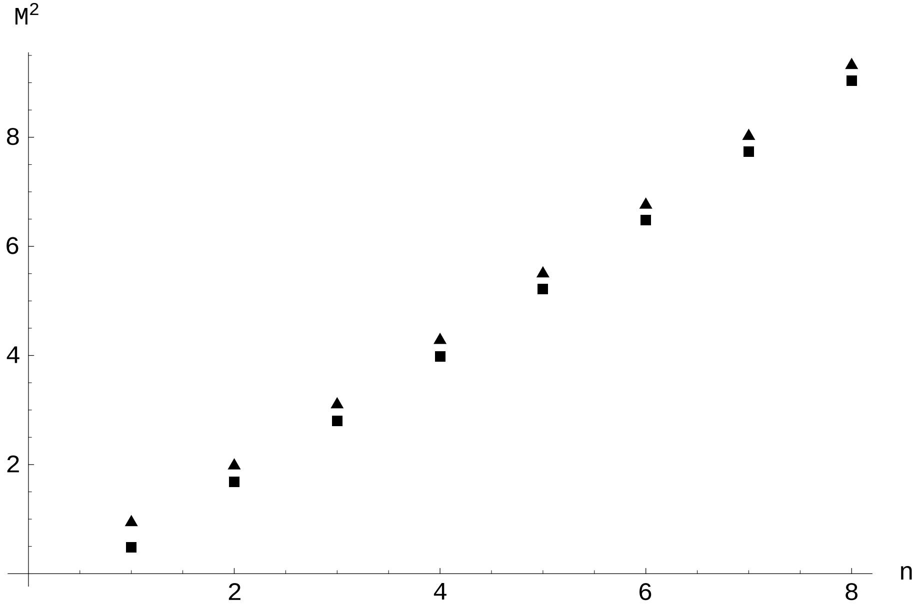
which is everywhere regular and has the correct UV and IR asymptotics.

- b_0 is a free parameter and b_1/b_0^2 is taken from the QCD β-function

Linearity of the glueball spectrum



(a)

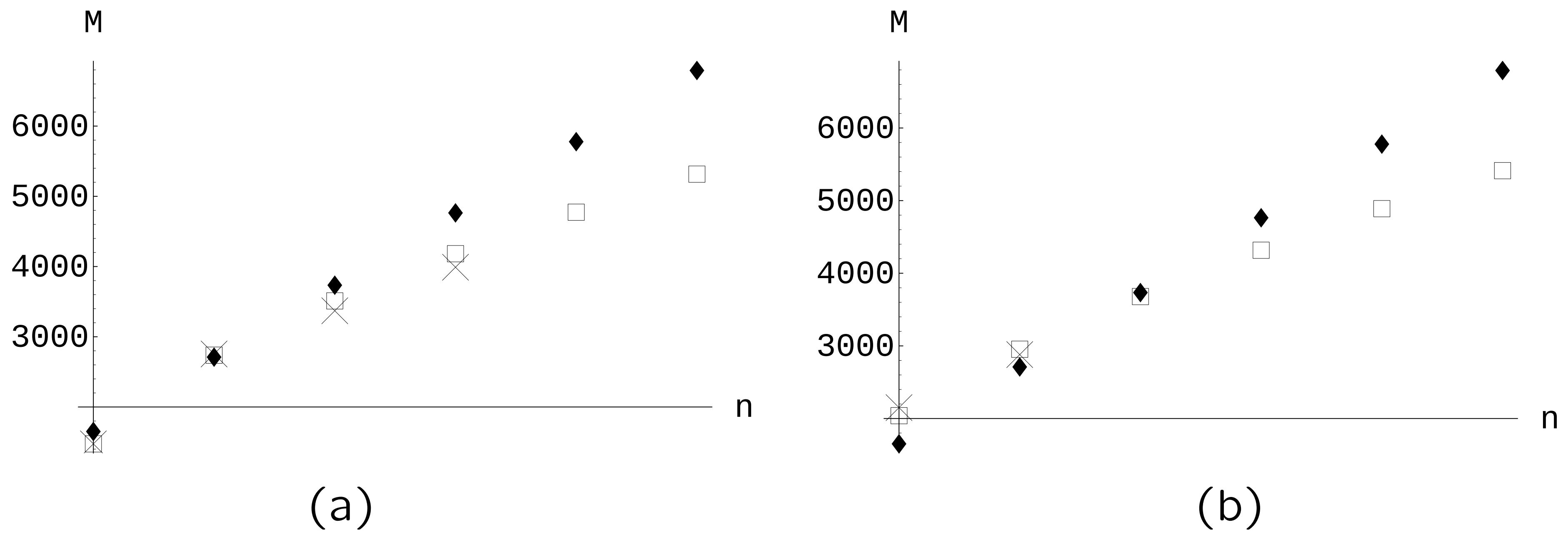


(b)

(a) Linear pattern in the spectrum for the first 40 0^{++} glueball states. M^2 is shown units of $0.015\ell^{-2}$.

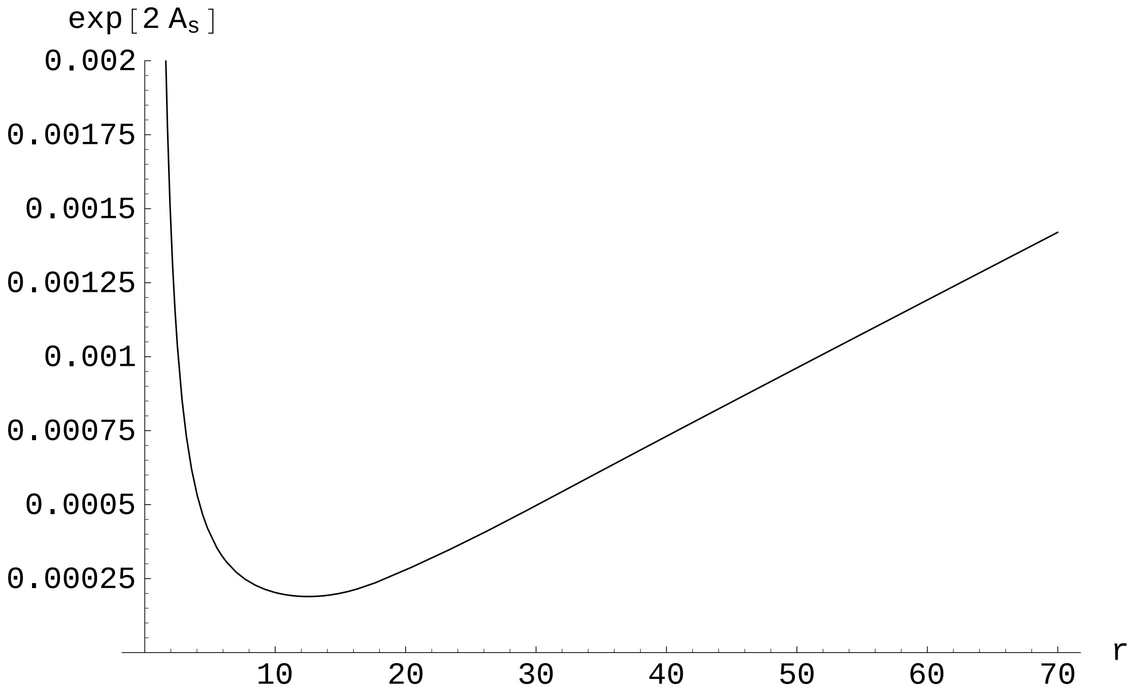
(b) The first 8 0^{++} (squares) and the 2^{++} (triangles) glueballs. These spectra are obtained in the background I with $b_0 = 4.2, \lambda_0 = 0.05$.

Comparison with lattice data (Meyer)



Comparison of glueball spectra from our model with $b_0 = 4.2, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. I (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. I.

$$\ell_{eff}^2 = 6.88\ \ell^2$$



The string frame scale factor in background I with $b_0 = 4.2$, $\lambda_0 = 0.05$.

We can "measure"

$$\frac{\ell}{\ell_s} \simeq 6.26 \quad , \quad \ell_s^2 R \simeq -0.5 \tag{2}$$

and predict

$$\alpha_s(1.2GeV) = 0.34,$$

which is within the error of the quoted experimental value $\alpha_s^{(exp)}(1.2GeV) = 0.35 \pm 0.01$

The fit to Meyer lattice data

J^{PC}	Ref I (MeV)	Our model (MeV)	Mismatch	$N_c \to \infty$ [?]	Mismatch
0^{++}	**1475 (4%)**	**1475**	0	**1475**	0
2^{++}	2150 (5%)	2055	4%	2153 (10%)	5%
0^{-+}	**2250 (4%)**	**2243**	0		
0^{++*}	**2755 (4%)**	**2753**	0	2814 (12%)	2%
2^{++*}	2880 (5%)	2991	4%		
0^{-+*}	3370 (4%)	3288	2%		
0^{++**}	3370 (4%)	3561	5%		
0^{++***}	3990 (5%)	4253	6%		

Comparison between the glueball spectra in Ref. I and in our model. The states we use as input in our fit are marked in red. The parenthesis in the lattice data indicate the percent accuracy.

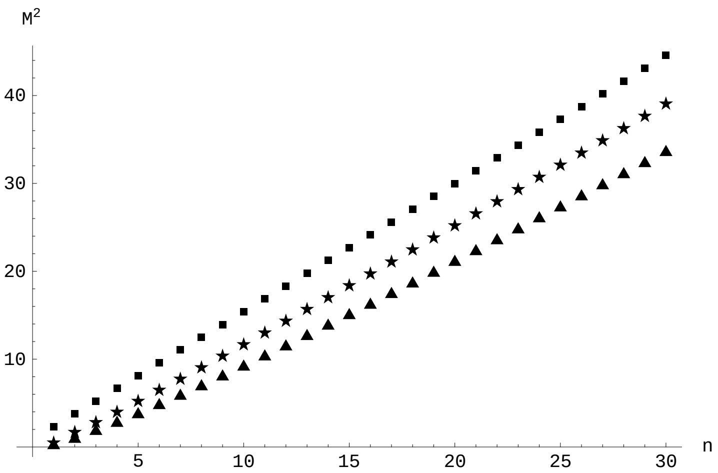
Estimating the importance of logarithmic scaling

We keep the IR asymptotics of background II,but change the UV to power asymptoting AdS$_5$, with a small λ_*.

$$e^A(r) = \frac{\ell}{r} e^{-(r/R)^2}, \qquad \Phi(r) = \Phi_0 + \frac{3}{2}\frac{r^2}{R^2}\sqrt{1+3\frac{R^2}{r^2}} + \frac{9}{4}\log\frac{2\frac{r}{R}+2\sqrt{\frac{r^2}{R^2}+\frac{3}{2}}}{\sqrt{6}}.$$

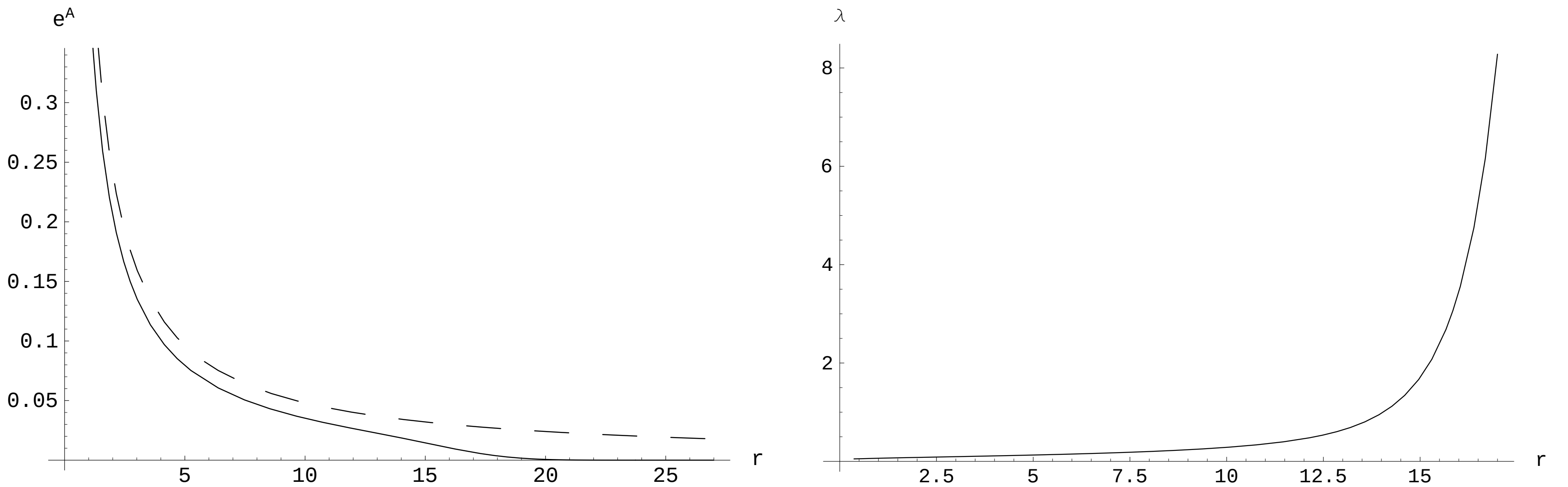
$$W_{conf} = W_0\left(9+4b_0^2(\lambda-\lambda_*)^2)^{1/3}\right)\left(9a+(2b_0^2+3b_1)\log\left[1+(\lambda-\lambda_*^2)\right]\right)^{2a/3}.$$

We fix parameters so that the physical QCD scale is the same (as determined from asymptotic slope of Regge trajectories.



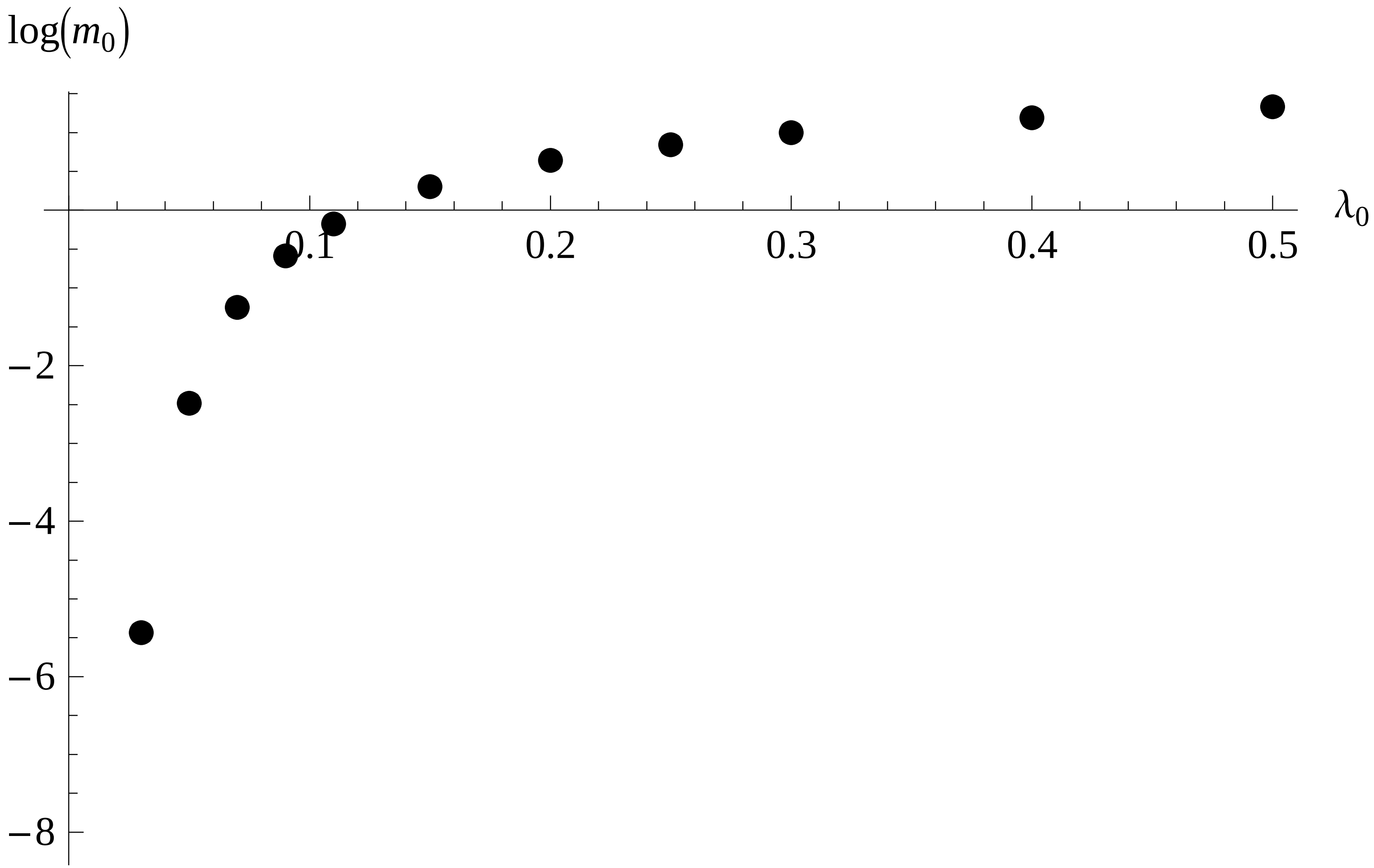
The stars correspond to the asymptotically free background I with $b_0 = 4.2$ and $\lambda_0 = 0.05$; the squares correspond the results obtained in the first background with $R = 11.4\ell$; the triangles denote the spectrum in the second background with $b0 = 4.2$, $li = 0.071$ and $l_* = 0.01$. These values are chosen so that the slopes coincide asymptotically for large n.

Profile of coupling and scale factor



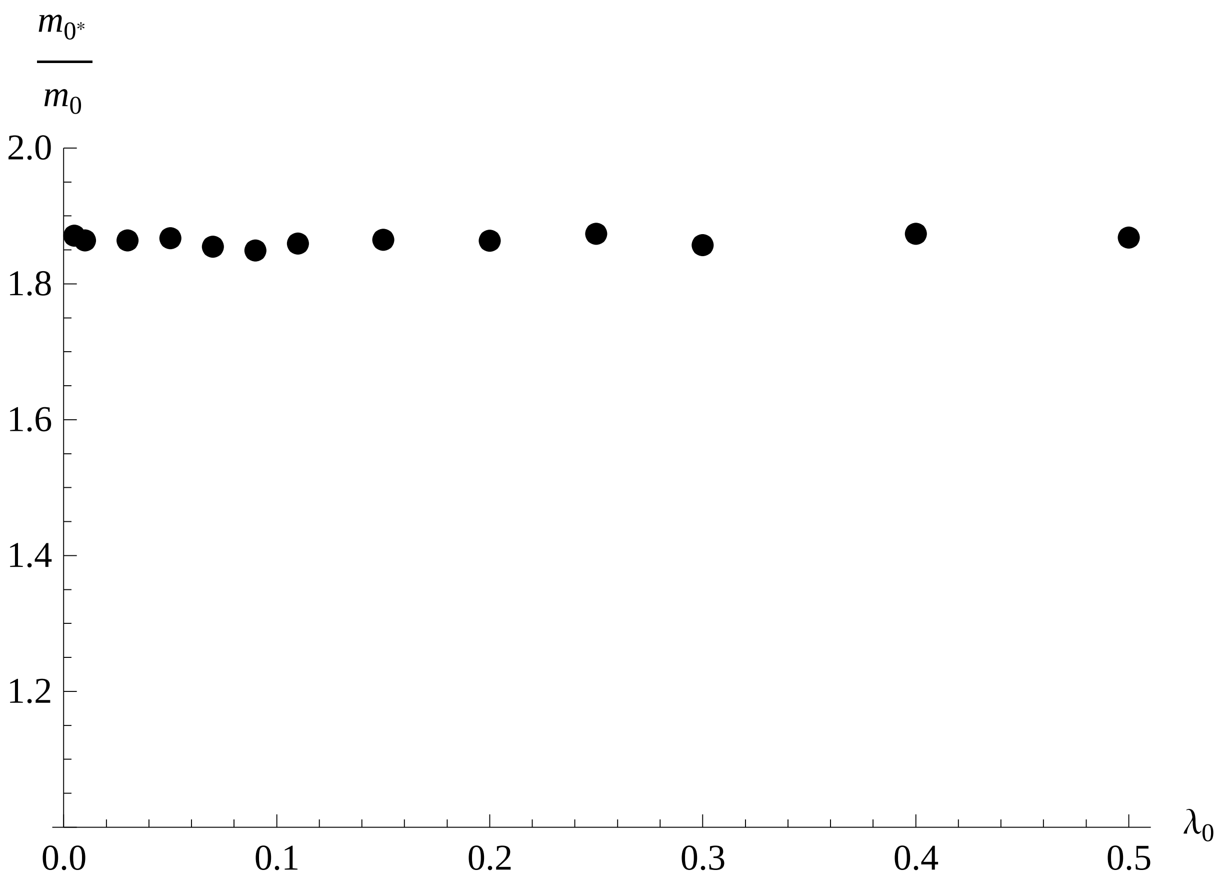
The scale factor and 't Hooft coupling that follow from β. $b_0 = 4.2$, $\lambda_0 = 0.05$, $A_0 = 0$. The units are such that $\ell = 0.5$. The dashed line represents the scale factor for pure AdS.

Dependence of absolute mass scale on λ_0



Dependence on initial condition λ_0 of the absolute scale of the lowest lying glueball (shown in Logarithmic scale)

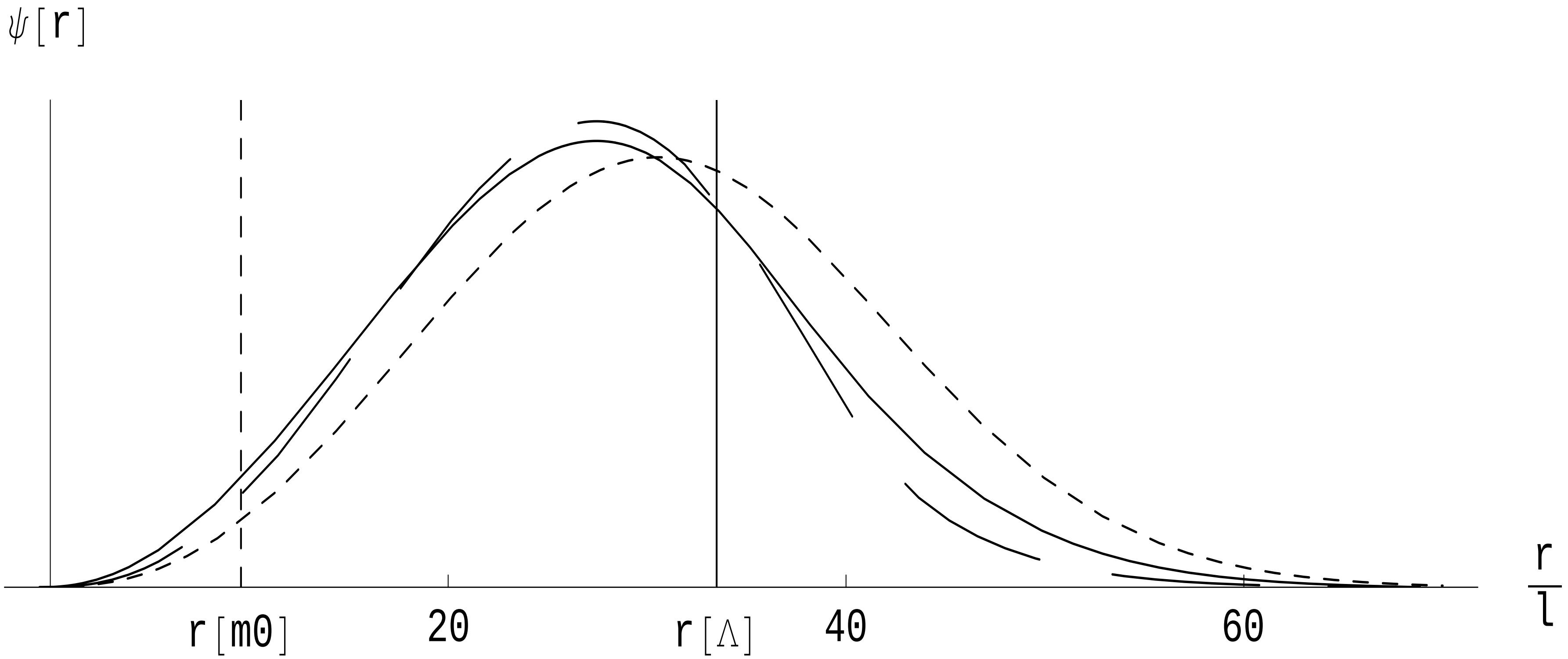
Dependence of mass ratios on λ_0



The mass ratios R_{20}

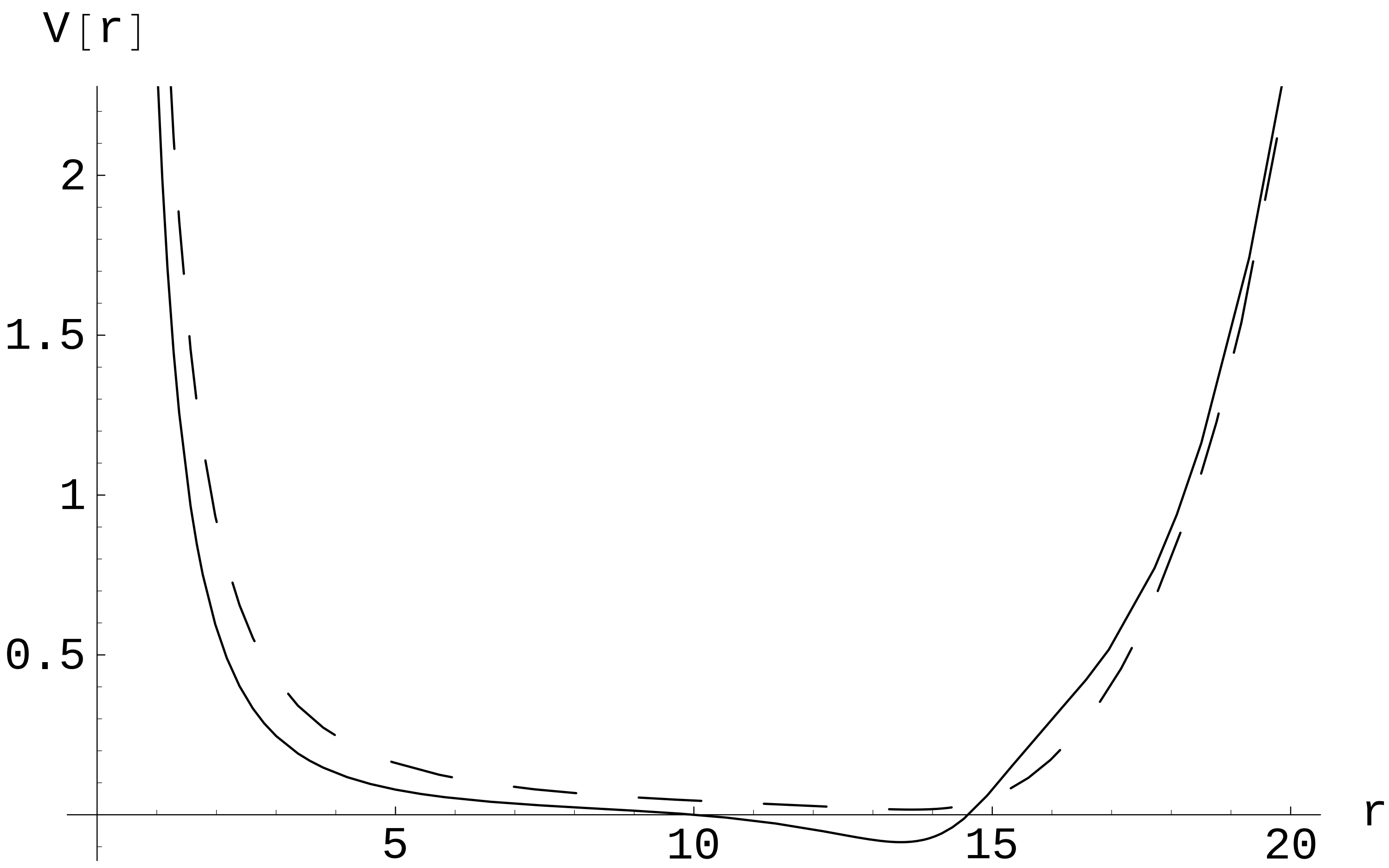
$$R_{20} = \frac{m_{2++}}{m_{0++}}.$$

The glueball wavefunctions



Normalized wave-function profiles for the ground states of the 0^{++} (solid line) ,0^{-+} (dashed line), and 2^{++} (dotted line) towers, as a function of the radial conformal coordinate. The vertical lines represent the position corresponding to $E = m_{0^{++}}$ and $E = \Lambda_p$.

Comparison of scalar and tensor potential



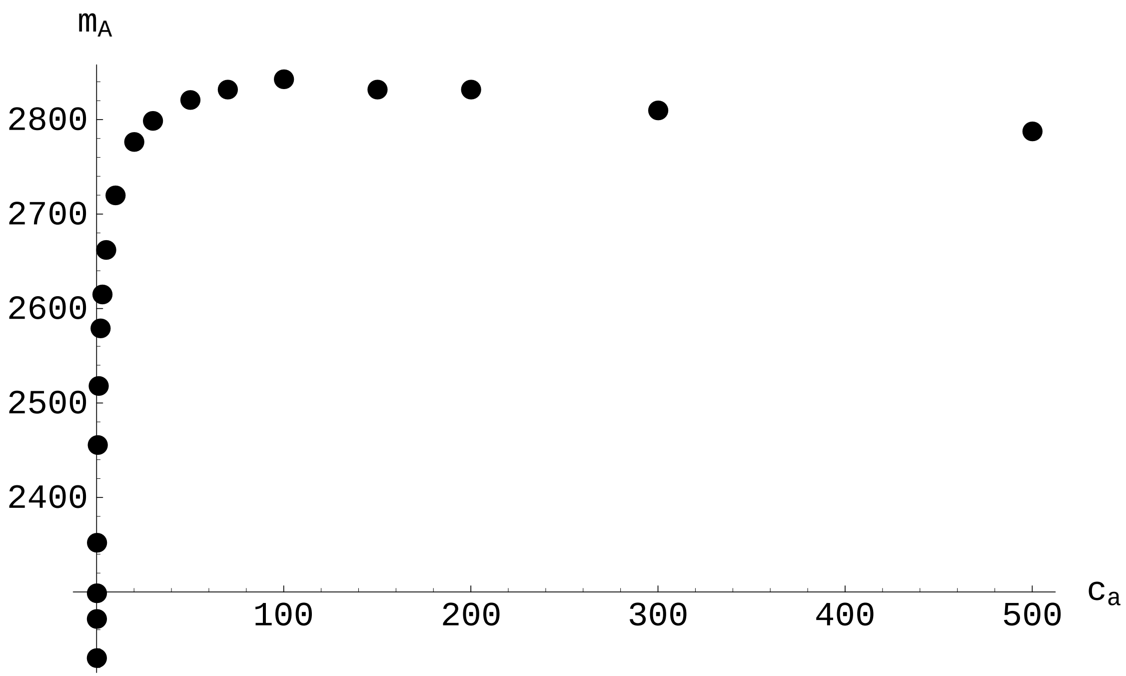
Effective Schrödinger potentials for scalar (solid line) and tensor (dashed line) glueballs. The units are chosen such that $\ell = 0.5$.

The lattice glueball data

J^{++}	Ref. I $(m/\sqrt{\sigma})$	Ref. I (MeV)	Ref. II (mr_0)	Ref. II (MeV)	$N_c \to \infty (m/\sqrt{\sigma})$
0	3.347(68)	1475(30)(65)	4.16(11)(4)	1710(50)(80)	3.37(15)
0^*	6.26(16)	2755(70)(120)	6.50(44)(7)	2670(180)(130)	6.43(50)
0^{**}	7.65(23)	3370(100)(150)	NA	NA	NA
0^{***}	9.06(49)	3990(210)(180)	NA	NA	NA
2	4.916(91)	2150(30)(100)	5.83(5)(6)	2390(30)(120)	4.93(30)
2^*	6.48(22)	2880(100)(130)	NA	NA	NA
R_{20}	1.46(5)	1.46(5)	1.40(5)	1.40(5)	1.46(11)
R_{00}	1.87(8)	1.87(8)	1.56(15)	1.56(15)	1.90(17)

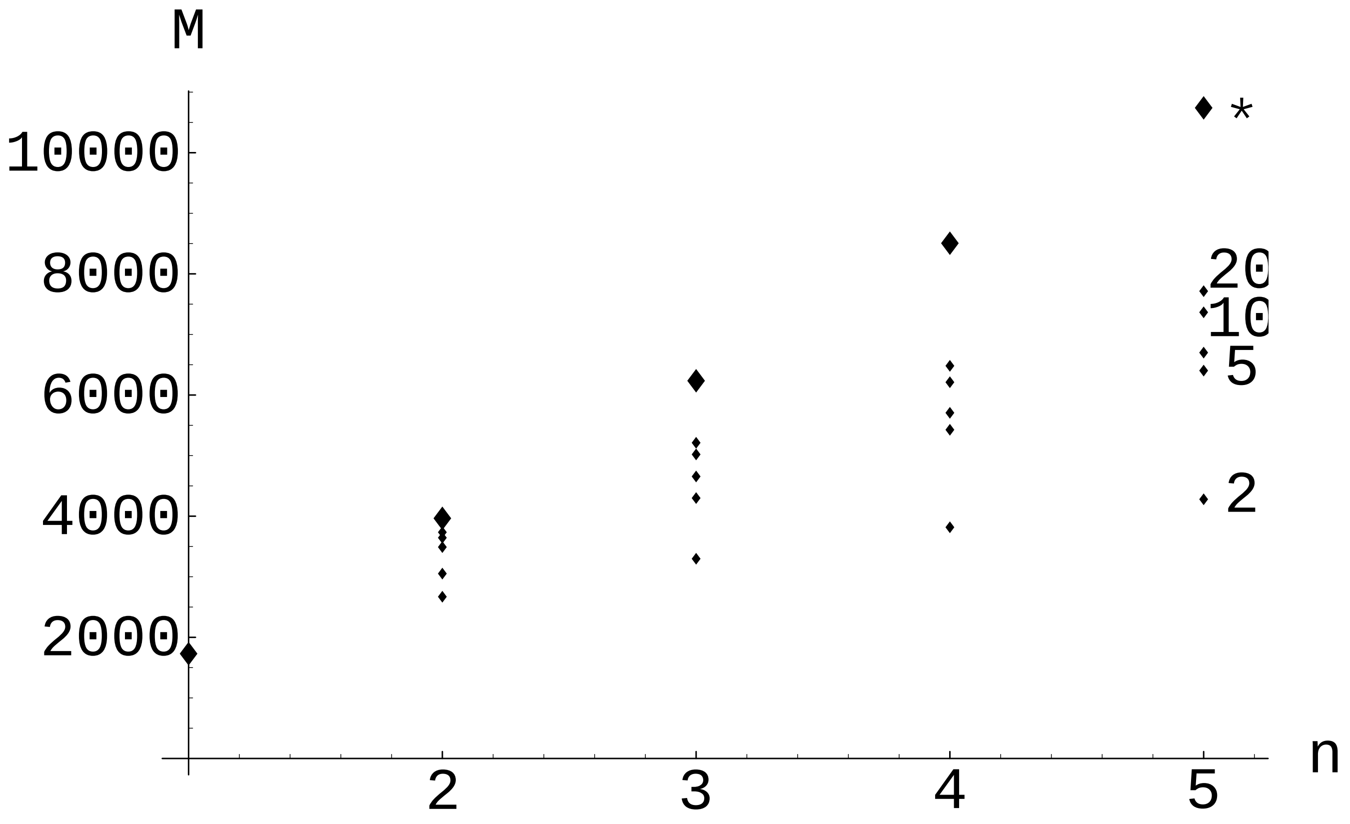
Available lattice data for the scalar and the tensor glueballs. Ref. I =H. B. Meyer, [arXiv:hep-lat/0508002]. and Ref. II = C. J. Morningstar and M. J. Peardon, [arXiv:hep-lat/9901004] + Y. Chen *et al.*, [arXiv:hep-lat/0510074]. The first error corresponds to the statistical error from the the continuum extrapolation. The second error in Ref.I is due to the uncertainty in the string tension $\sqrt{\sigma}$. (Note that this does not affect the mass ratios). The second error in the Ref. II is the estimated uncertainty from the anisotropy. In the last column we present the available large N_c estimates according to B. Lucini and M. Teper, [arXiv:hep-lat/0103027]. The parenthesis in this column shows the total possible error followed by the estimations in the same reference.

Pseudoscalar glueballs



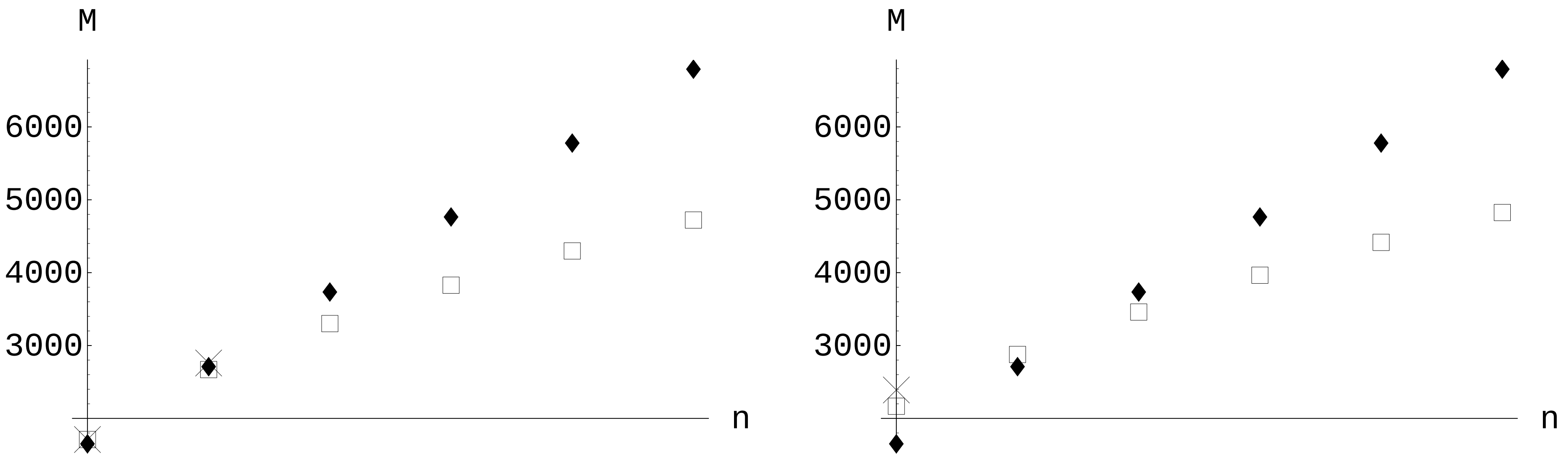
Lowest 0^{-+} glueball mass in MeV as a function of c_a in $Z(\lambda) = Z_a(1+c_a\lambda^4)$.

α-dependence of scalar spectrum



The 0^{++} spectra for varying values of α that are shown at the right end of the plot. The symbol * denotes the AdS/QCD result.

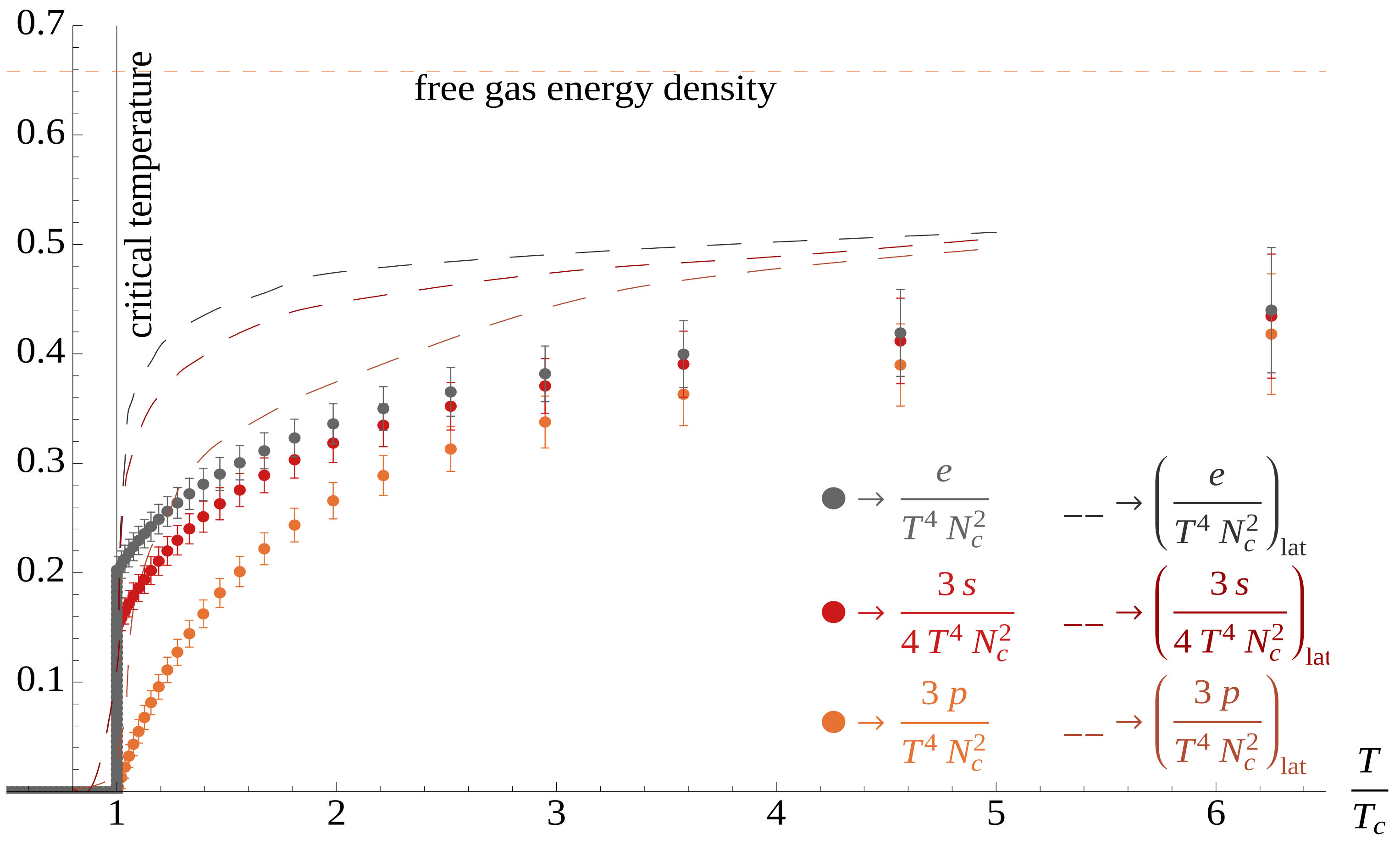
Comparison with lattice data: Ref II



(a) (b)

Comparison of glueball spectra from our model with $b_0 = 2.55, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. II (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. II.

The thermodynamic quantities



The bulk viscosity: theory

It is defines from the Kubo formula

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\ \frac{1}{\omega} Im\ G_R(\omega) \quad , \quad G_R(\omega) \equiv \int d^3x \int dt\ e^{i\omega t}\theta(t)\ \langle 0|[T_{ii}(\vec{x},t), T_{ii}(\vec{0},0)]|0\rangle$$

Using a parametrization $ds^2 = e^{2A}(f dt^2 + d\vec{x}^2) + \frac{e^{2B}}{f}dr^2$ in a special gauge $\phi = r$ the relevant metric perturbation decouples

Gubser+Nellore+Pufu+Rocha

$$h''_{11} = -\left(-\frac{1}{3A'} - 4A' + 3B' - \frac{f'}{f}\right) h'_{11} + \left(-\frac{e^{2B-2A}}{f^2}\omega^2 + \frac{f'}{6fA'} - \frac{f'}{f}B'\right) h_{11}$$

with

$$h_{11}(0) = 1 \quad , \quad h_{11}(r_h) \simeq C\ e^{i\omega t}\left|\log\frac{\lambda}{\lambda_h}\right|^{-\frac{i\omega}{4\pi T}}$$

The correlator is given by the conserved number of h-quanta

$$Im\ G_R(\omega) = -4M^3\mathcal{G}(\omega) \quad , \quad \mathcal{G}(\omega) = \frac{e^{4A-B}f}{4A'^2}|Im[h^*_{11}h'_{11}]|$$

finally giving

$$\frac{\zeta}{s} = \frac{C^2}{4\pi}\ \frac{V'(\lambda_h)^2}{V(\lambda_h)^2}$$

Detailed plan of the presentation

- Dependence of mass ratios on λ_0 104 minutes
- The glueball wavefunctions 105 minutes
- Comparison of scalar and tensor potential 106 minutes
- The lattice glueball data 107 minutes
- Pseudoscalar Glueballs 108 minutes
- α-dependence of scalar spectrum 109 minutes
- Comparison with lattice data: Ref II n110 minutes
- The thermodynamic quantities 111 minutes
- Bulk viscosity: theory 113 minutes