Jerushalem workshop, 9 April 2008

What (we think) we know about Holographic QCD?

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Introduction

♠ QCD is a very successful theory for the strong interactions.

♠ Remarkably, we do not have analytical control over most of the energy regime. Even numerically (lattice), many aspects of the theory are still beyond reach.

♠ Were it not phenomenological models most accelerator data n the past 20 years would have been useless.

♠ What we cannot reliably calculate:

- Observable rates for accelerator experiments. In particular, structure functions have to be measured. Hadronization is done by the Lund Monte Carlo model or the fragmentation model.

- Glueball spectra for higher glueballs (third and up), mesons (3-4-th and up and baryons (2nd and up). Decay widths for all of the above.

- There are at least two weak matrix elements that cannot be computed so far reliably enough by lattice computations: The $\Delta I = \frac{1}{2}$ matrix elements of type $\langle K|\mathcal{O}_{\Delta I=1/2,3/2}|\pi\pi\rangle$, and the $B_K \sim \langle K|\mathcal{O}_{\Delta S=2}|\bar{K}\rangle$.

- Data associated to the chiral symmetry breaking (like the quark condensate), or its restauration at higher temperatures.

- In general matrix elements with at least two particle final states.

- Real time finite temperature correlation functions (associated to QGP dynamics)

- Finite temperature physics at finite baryon density.

♠ Several complementary semi-phenomenological techniques have been developed to deal with the above (chiral perturbation theory, perturbation theory resummation schemes, SD equations, bag models, etc.) with varied success.

◇ There are two competing (rather successful) phenomenological models of QCD (Lund and fragmentation models) that are essential today in interpreting collider QCD data.

AdS/CFT and holography

♠ The large N_c approximation to QCD has promised a string theory description of the color singlet sector of gauge theories.

't Hooft

♠ The nature of this string theory became more palpable with the formulation of the AdS/CFT correspondence for $\mathcal{N} = 4_4$ sYM.

Maldacena, Witten, Gubser+Klebanov+Polyakov

The surprise involved the emergence of an extra holographic dimension.

♠ This has started an effort to extend it to theories as close to QCD as possible.

♠ The original and most controlled approaches relied on "perturbing" the original AdS/CFT correspondence in ten-dimensional (critical) string theory.

♠ More recent attempts dared to use a non-critical string framework.

♠ Some holographic-inspired phenomenological models also popped up (AdS/QCD).

A string theory for QCD:basic expectations

• Pure SU(N_c) d=4 YM is expected to be dual to a string theory in 5 dimensions only. Essentially a single adjoint field $\to$ a single extra dimension.

• The theory becomes asymptotically free and conformal at high energy $\to$ we expect the classical saddle point solution to asymptote to AdS_5.

♠ Which bulk fields are expected to be non-trivial in the large-N_c saddle-point?

• scalar YM operators with $\Delta_{UV} > 4 \to m^2 > 0$ fields near the AdS_5 boundary $\to$ most probably not excited in the vacuum as suggested by a fit of the low energy SVZ sum rules to data.

• bulk fields with zero or negative masses near the boundary may get vevs. Advantage here is that the theory is weakly coupled at UV so UV dimensions are reliably known.

- What are all gauge invariant YM operators of dimension 4 or less?

- They are given by $Tr[F_{\mu\nu}F_{\rho\sigma}]$.

Decomposing into U(4) reps:

$$(\yng(1) \otimes \yng(1))_{\text{symmetric}} = \yng(2,2) \oplus \yng(1,1,1,1) \tag{1}$$

We must remove traces to construct the irreducible representations of O(4):

$$\yng(2,2) = \underline{\yng(2,2)} \oplus \underline{\yng(2)} \oplus \bullet \quad , \quad \yng(1,1,1,1) = \bullet$$

The two singlets are the scalar (dilaton) and pseudoscalar (axion)

$$\phi \leftrightarrow Tr[F^2] \quad , \quad a \leftrightarrow Tr[F \wedge F]$$

The traceless symmetric tensor

$$\underline{\yng(2)} \quad \to \quad T_{\mu\nu} = Tr\left[F^2_{\mu\nu} - \frac{1}{4} g_{\mu\nu} F^2\right]$$

is the conserved stress tensor dual to a massless graviton in 5d reflecting the translational symmetry of YM.

$$\underline{\yng(2,2)} \quad \to \quad T^4_{\mu\nu;\rho\sigma} = Tr[F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2}(g_{\mu\rho}F^2_{\nu\sigma} - g_{\nu\rho}F^2_{\mu\sigma} - g_{\mu\sigma}F^2_{\nu\rho} + g_{\nu\sigma}F^2_{\mu\rho}) + \frac{1}{6}(g_{\mu\rho}g_{\nu\sigma} - g_{\nu\rho}g_{\mu\sigma})F^2]$$

It has 10 independent d.o.f, it is not conserved and it should correspond to a similar massive tensor in 5d. We do not expect it to play an non-trivial role in the large-N_c, YM vacuum also for reasons of Lorentz invariance.

- Therefore the nontrivial fields are expected to be: $g_{\mu\nu}, \phi, a$

bosonic string or superstring?

• Consider the axion a dual to $Tr[F \wedge F]$. We can show that it must come from a RR sector.

In large-N$_c$ YM, the proper scaling of couplings is obtained from

$$\mathcal{L}_{YM} = N_c \, Tr \left[\frac{1}{\lambda} F^2 + \frac{\theta}{N_c} F \wedge F \right] \quad , \quad \zeta \equiv \frac{\theta}{N_c} \sim \mathcal{O}(1)$$

$$E_{YM}(\theta) \simeq N_c^2 \; \min_{k \in Z} \; H\left(\frac{\theta + 2\pi k}{N_c}\right) \quad , \quad H(\theta) = H(-\theta) \quad (CP)$$

Therefore, $H(x) = C_0 + C_1 x^2 + C_2 x^4 + \cdots$ and

Witten

$$E_{YM}(\theta) \simeq C_0 \; N_c^2 + C_1 \theta^2 + C_2 \frac{\theta^4}{N_c^2} + \cdots$$

In the string theory action

$$S \sim \int e^{-2\phi} [R + \cdots] + (\partial a)^2 + e^{2\phi} (\partial a)^4 + \cdots \quad , \quad e^{\phi} \sim g_{YM}^2$$

$$\sim \int \frac{N_c^2}{\lambda^2} [R + \cdots] + (\partial a)^2 + \frac{\lambda^2}{N_c^2} (\partial a)^4 + \cdots \quad , \quad \lambda \sim N_c e^{\phi} \quad , \quad a = \theta [1 + \cdots]$$

bosonic string or superstring? (continued)

• The string theory must have no on-shell fermionic states at all because there are no gauge invariant fermionic operators in pure YM.

• Therefore the string theory must be a 5d-superstring theory resembling the II-0 class.

♠ Another RR field we expect to have is the RR 4-form, as it is necessary to "seed" the D_3 branes responsible for the gauge group.

• It is non-propagating in 5D

• We will see later however that it is responsible for the non-trivial IR structure of the gauge theory vacuum.

The minimal effective string theory spectrum

- NS-NS $\rightarrow$ $g_{\mu\nu}$, $B_{\mu\nu}$, ϕ

- RR $\rightarrow$ Spinor$_5 \times$Spinor$_5 = F_0 + F_1 + F_2 + (F_3 + F_4 + F_5)$

♠ $F_0 \leftrightarrow F_5 \rightarrow$ C_4, background flux $\rightarrow$ no propagating degrees of freedom.

♠ $F_1 \leftrightarrow F_4 \rightarrow$ $C_3 \leftrightarrow C_0$: C_0 is the axion, C_3 its 5d dual that couples to domain walls separating oblique confinement vacua.

♠ $F_2 \leftrightarrow F_3 \rightarrow$ $C_1 \leftrightarrow C_2$: They are associated with baryon number (as we will see later when we add flavor). Dual operators are a mystery.

- In an ISO(3,1) invariant vacuum solution, only $g_{\mu\nu}, \phi, C_0 = a$ can be non-trivial.

$$ds^2 = e^{2A(r)}(dr^2 + dx_4^2) \quad , \quad a(r), \phi(r)$$

The relevant "defects"

• $B_{\mu\nu} \to$ Fundamental string (F_1). This is the QCD (glue) string: fundamental tension $\ell_s^2 \sim \mathcal{O}(1)$

• Its dual $\tilde{B}_\mu \to NS_0$: Tension is $\mathcal{O}(N_c^2)$. It is an effective magnetic baryon vertex binding N_c magnetic quarks.

• $C_4 \to D_3$ branes generating the gauge symmetry.

• $C_3 \to D_2$ branes : domain walls separating different oblique confinement vacua (where $\theta_{k+1} = \theta_k + 2\pi$). Its tension is $\mathcal{O}(N_c)$

.

• $C_2 \to D_1$ branes: These are the magnetic strings (strings attached to magnetic quarks) with tension $\mathcal{O}(N_c)$

• $C_5 \to D_4$: Space filling flavor branes. They must be introduced in pairs: $D_4 + \bar{D}_4$ for charge neutrality/tadpole cancellation.

• $C_1 \to D_0$ branes. These are the baryon vertices: they bind N_c quarks, and their tension is $\mathcal{O}(N_c)$.

• $C_0 \to D_{-1}$ branes: These are the Yang-Mills instantons.

The effective action, I

- as $N_c \to \infty$, only string tree-level is dominant.

- Relevant field for the vacuum solution: $g_{\mu\nu}, a, \phi, F_5$.

- by definition the vev of $F_5 \sim N_c\ \epsilon_5$. It appears always in the combination $e^{2\phi}F_5^2 \sim \lambda^2$, with $\lambda \sim N_c\ e^{\phi}$ All higher derivative corrections $(e^{2\phi}F_5^2)^n$ are $\mathcal{O}(1)$ at large N$_c$.

- This is not the case for all other RR fields: in particular for the axion as $a \sim \mathcal{O}(1)$

$$(\partial a)^2 \sim \mathcal{O}(1) \quad , \quad e^{2\phi}(\partial a)^4 = \frac{\lambda^2}{N_c^2}(\partial a)^4 \sim \mathcal{O}\left(N_c^{-2}\right)$$

Therefore to leading order $\mathcal{O}(N_c^2)$ we can neglect the axion.

The effective action II

Therefore:

$$S_{eff} = M_p^3 N_c^2 \int d^5x \, \frac{1}{\lambda^2} \, Z\left(R, (\partial\phi)^2, \frac{\lambda^2}{N_c^2} F_5^2\right)$$

• Moreover as the F_5 is non-propagating its equation of motion gives

$$G\left(R, (\partial\phi)^2, \frac{\lambda^2}{N_c^2} F_5^2\right) = 0$$

which if solved implicitly and substituted back in the effective action gives

$$S_{eff} = M_p^3 N_c^2 \int d^5x \, \frac{1}{\lambda^2} \, \tilde{Z}\left(R, (\partial\phi)^2, \lambda^2\right)$$

• In particular, a potential for the dilaton, λ will be generated from the higher derivative terms due to F_5.

The UV regime

- In the far UV, the space should asymptote to AdS$_5$.

- The 't Hooft coupling should behave as ($r \to 0$)

$$\lambda \sim \frac{1}{\log(r\Lambda)} + \cdots \quad \to \quad 0$$

- Therefore, as $r \to 0$

$$\text{Curvature} \to \text{finite} \quad , \quad (\partial\phi)^2 \sim \frac{(\partial\lambda)^2}{\lambda^2} \sim \frac{1}{\log^2(r\Lambda)} \to 0 \quad , \quad \lambda^2 \to 0$$

- For $\lambda \to 0$ the potential in the Einstein frame starts as $V(\lambda) \sim \lambda^{\frac{4}{3}}$ and cannot support the asymptotic AdS_5 solution.

- Therefore asymptotic AdS_5 must arise from

$$S_{eff} = M_p^3 N_c^2 \int d^5x \, \frac{1}{\lambda^2} \, Z\,(R, 0, 0)$$

.

• Such AdS_5 solutions are "easy to find" as in the example

$$S = \int d^5x \, \sqrt{g} \; Z(R)$$

There is a a constant curvature solution with R_* that is a zero of the function

$$xZ'(x) - \frac{5}{2} Z(x)$$

• This must involve at least two such terms and therefore, we do not expect a "good" derivative expansion for finding the solution.

• There is however a "good" (but hard to derive) perturbative expansion around this asymptotic AdS_5 solution by perturbing around it:

$$e^A = \frac{\ell}{r}[1 + \delta A] \quad , \quad \lambda = \frac{1}{b_0 \log(r\Lambda)} + \cdots$$

• This turns out to be an regular expansion of the solutions in powers of

$$\frac{P_n(\log\log(r\Lambda))}{(log(r\Lambda))^{-n}}$$

• The coefficients of that expansion are various functions of the constant curvature invariants of AdS_5 that are (unknown) numbers.

• Effectively this can be rearranged as a "perturbative" expansion in $\lambda(r)$.

• It is even cleaner to use λ as the radial coordinate instead of r and write the metric solution in terms of λ.

• The solution for the metric can be written

$$E \equiv e^A = \frac{\ell}{r(\lambda)}\left[1 + c_1\lambda + c_2\lambda^2 + \cdots\right] = \ell\,(e^{-\frac{b_0}{\lambda}})\left[1 + c_1'\lambda + c_2'\lambda^2 + \cdots\right] \quad , \quad \lambda \to 0$$

The IR regime

• Here the situation is a bit more obscure. The constraints/input are: confinement and mass gap.

• We do expect that $\lambda \to \infty$ at the IR bottom.

• This is a "singularity" in the conventional sense: it must be "repulsive", ie the string theory, and even better some effective field theory should not break down there.

• (Very) naive intuition from N=4 and other 10d strongly coupled theories suggests that in this regime there should be a two derivative description of the physics.

• At the IR bottom the space must end (singularity) where the scale factor vanishes.

♠ If it happens very slowly, we loose confinement

♠ if it happens very fast, the singularity is strong and the theory is incomplete (boundary conditions are needed at the singularity.

An assessment of IR asymptotics

• As $\lambda\to\infty$ we assume that the potential terms dominate and we parameterize the effective action in the IR as

$$S_{eff}\sim\int\sqrt{g}\left[R+\frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2}+V(\lambda)=\right]\quad,\quad V(\lambda)-\frac{4}{3}\lambda^2\left(\frac{dW}{d\lambda}\right)^2+\frac{64}{27}W^2$$

Parameterize the IR asymptotics ($\lambda\to\infty$) as

$$W(\lambda)\sim(\log\lambda)^{\frac{P}{2}}\;\lambda^Q$$

• $Q>2/3$ or $Q=2/3$ and $P>1$ leads to confinement and a singularity at finite $r=r_0$.

$$e^A(r)\sim\begin{cases}(r_0-r)^{\frac{4}{9Q^2-4}} & Q>\frac{2}{3}\\ \exp\left[-\frac{C}{(r_0-r)^{1/(P-1)}}\right] & Q=\frac{2}{3}\end{cases}$$

• $Q = 2/3$, and $0 \leq P < 1$ leads to confinement and a singularity at $r = \infty$
The scale factor e^A vanishes there as

$$e^A(r) \sim \exp[-Cr^{1/(1-P)}].$$

The asymptotic spectrum of glueballs is linear only if $P = \frac{1}{2}$

• $Q = 2/3, P = 1$ leads to confinement but the singularity may be at a finite or infinite value of r depending on subleading asymptotics of the superpotential.

♠ If $Q < 2\sqrt{2}/3$, no *ad hoc* boundary conditions are needed to determine the glueball spectrum: the singularity is "good" (repulsive).

♠ when $Q > 2\sqrt{2}/3$, the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

Selecting the IR asymptotics

Only the $Q = 2/3$, $0 \leq P < 1$ is compatible with

- Confinement (it happens non-trivially: a minimum in the string frame scale factor)
- Mass gap+discrete spectrum (except P=0)
- good singularity
- $R \to 0$ partly justifying the original assumption. More precisely: the string frame metric becomes flat at the IR . But $(\partial\phi)^2 \sim V(\lambda)$.

♠ It is interesting that the lower endpoint: P=0 corresponds to linear dilaton and flat space (string frame). It is confining with a mass gap but continuous spectrum.

- For linear asymptotic trajectories for fluctuations (glueballs) we must choose $P = 1/2$

$$V(\lambda) = \lambda^{\frac{4}{3}} \left[1 + c_1\lambda^2 + c_2\lambda^4 + \cdots\right] \sim \lambda^{\frac{4}{3}}\sqrt{\log\lambda} + \text{subleading} \qquad \text{as} \qquad \lambda \to \infty$$

Improved Holographic QCD: a model

The simplification in this model relies on writing down a two-derivative action

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right]$$

with

$$\lim_{\lambda\to 0} V(\lambda) = \frac{12}{\ell^2}\left(1 + \sum_{n=1}^{\infty} c_n \lambda^n\right) \quad , \quad \lim_{\lambda\to\infty} V(\lambda) = \lambda^{\frac{4}{3}}\sqrt{\log\lambda} + \text{subleading}$$

The small λ asymptotics "simulate" the UV expansion around AdS_5.

- There is a 1-1 correspondence between the YM β-function, $\beta(\lambda)$ and W:

$$\beta(\lambda) = -\frac{9}{4}\lambda^2 \, \frac{d\log W(\lambda)}{d\lambda}$$

once a choice of energy is made (here $E = A_E$). Renormalization and other choices modify $\beta(\lambda)$ beyond two-loop level

Shortcomings localized at the UV

- Conformal anomaly is incorrect.

- Shear viscosity ratio is constant and equal to that of N=1 sYM

both of the above need Riemann curvature corrections.

Many other observables though are coming out very well both at T=0 and finite T

Nitti's talk at this meeting

♠ The axion contribution

$$\delta S = M_p^3 \int d^5x \sqrt{g} Z(\lambda)(\partial a)^2$$

with

$$\lim_{\lambda\to 0} Z(\lambda) = c_0 + c_1\lambda + c_2\lambda^2 + \cdots \quad , \quad \lim_{\lambda\to\infty} Z(\lambda) = C_\infty \lambda^4 + \cdots$$

$$a(r) = \theta_{UV} \frac{\int_r^\infty \frac{dr}{e^{3A}Z}}{\int_0^\infty \frac{dr}{e^{3A}Z}}$$

Parameters

• All dimensionless coefficients of the potential are a priori parameters. However, a simple form is typically chosen for simplicity. In our example we fit only one parameter.

• We also have M_p, and the AdS length, ℓ. Asking correct $T \to \infty$ thermodynamics fixes $(M_p \ell)^3 = \frac{1}{45\pi^2}$. ℓ is not a parameter but a unit of length.

• We have 3 initial conditions in the system of graviton-dilaton equations:

One is fixed by picking the branch that corresponds asymptotically to $\lambda \sim \frac{1}{\log(r\Lambda)}$

The other fixes $\Lambda \to \Lambda_{QCD}$.

The third is a gauge artifact as it corresponds to a choice of the origin of the radial coordinate.

Quarks ($N_f \ll N_c$) and mesons

• Flavor is introduced by N_f $D_4 + \bar{D}_4$ branes pairs inside the bulk background. Their back-reaction on the bulk geometry is suppressed by N_f/N_c.

• The important world-volume fields are

$$T_{ij} \leftrightarrow \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \quad , \quad A^{ij\,L,R}_{\mu} \leftrightarrow \bar{q}^i_a \frac{1 \pm \gamma^5}{2} \gamma^\mu q^j_a$$

Generating the $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

• The UV mass matrix m_{ij} corresponds to the source term of the Tachyon field. It breaks the chiral (gauge) symmetry. The normalizable mode corresponds to the vev $\langle \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \rangle$.

• We show that the expectation value of the tachyon is non-zero and $T \sim \mathbf{1}$, breaking chiral symmetry $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$. The anomaly plays an important role in this (holographic Coleman-Witten)

• The fact that the tachyon diverges in the IR (fusing D with $\bar{D}$) constraints the UV asymptotics and determines the quark condensate $\langle\bar{q}q\rangle$ in terms of m_q. A GOR relation is satisfied (for an asymptotic AdS$_5$ space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\,\langle\bar{q}q\rangle \quad , \quad m_q \to 0$$

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

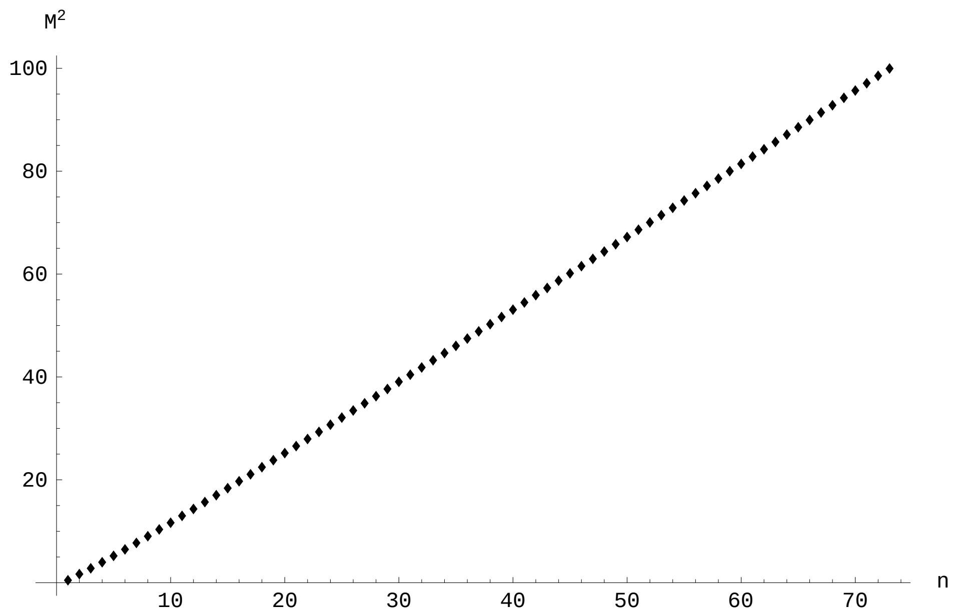
• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

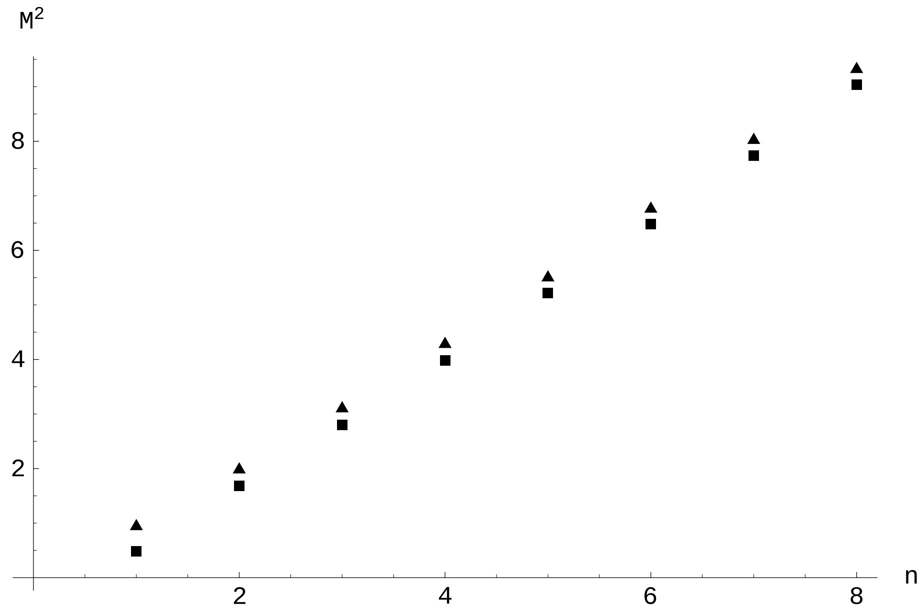
• Studying the spectrum of highly excited mesons, we find the expected property of linear confinement: $m_n^2 \sim n$.

• The detailed spectrum of mesons remains to be worked out

Linearity of the glueball spectrum



(a)

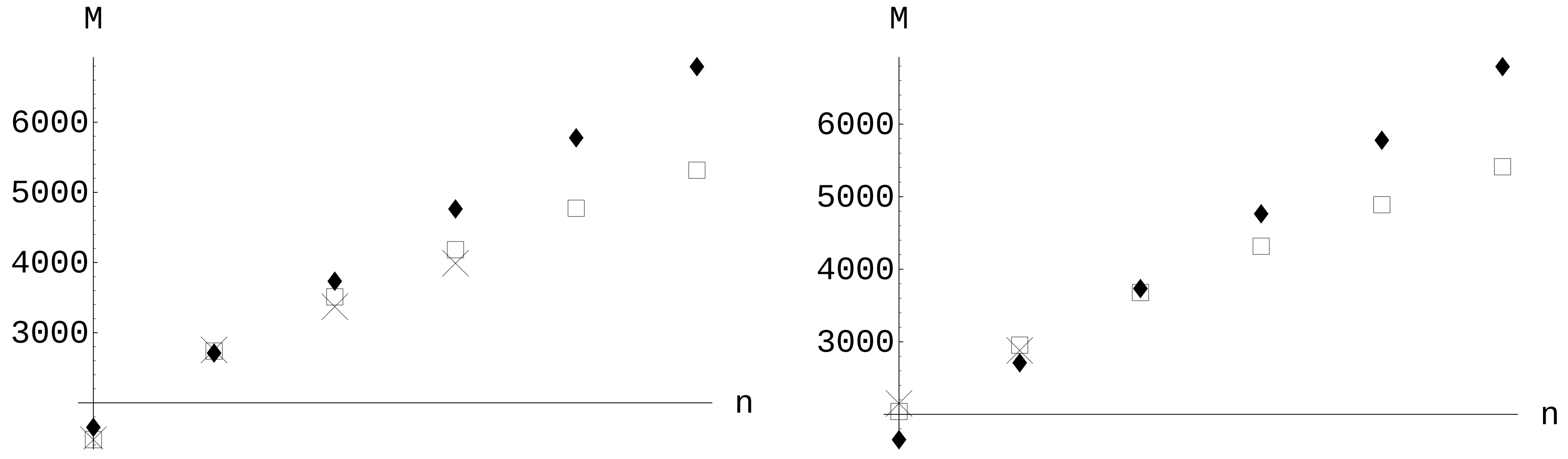


(b)

(a) Linear pattern in the spectrum for the first 40 0^{++} glueball states. M^2 is shown units of $0.015\ell^{-2}$.

(b) The first 8 0^{++} (squares) and the 2^{++} (triangles) glueballs. These spectra are obtained in the background I with $b_0 = 4.2, \lambda_0 = 0.05$.

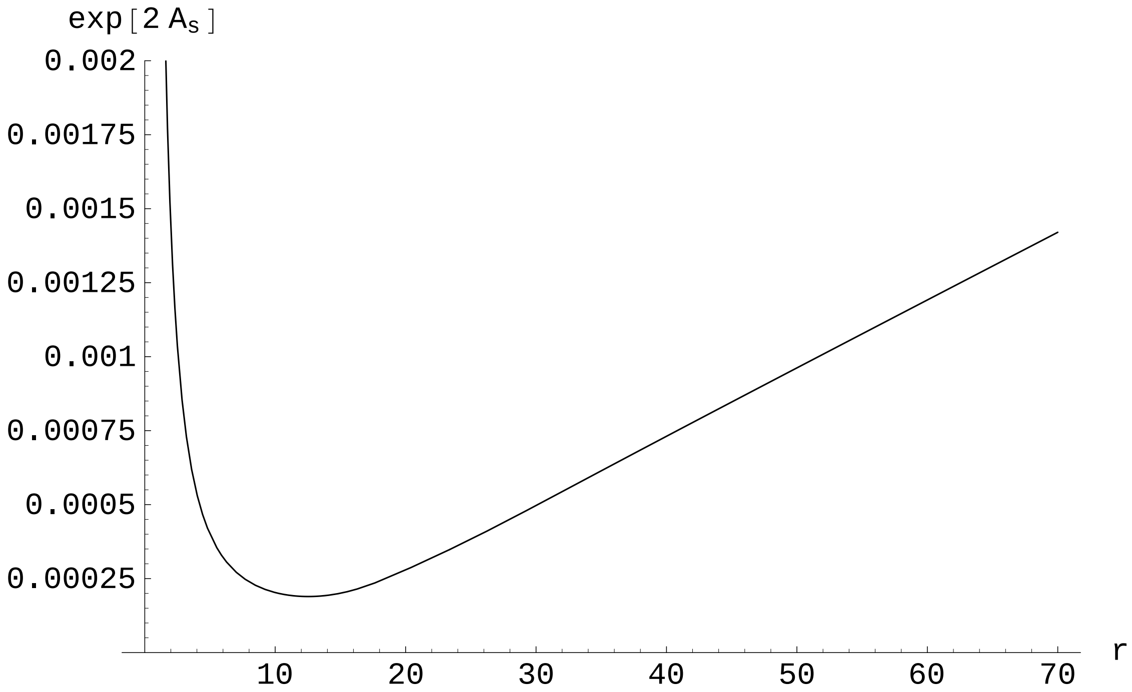
Comparison with lattice data: Ref I



(a) (b)

Comparison of glueball spectra from our model with $b_0 = 4.2, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. I (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. I.

$$\ell_{eff}^2 = 6.88\ \ell^2$$



The string frame scale factor in background I with $b_0 = 4.2$, $\lambda_0 = 0.05$.

We can "measure"

$$\frac{\ell}{\ell_s} \simeq 6.26 \quad , \quad \ell_s^2 R \simeq -0.5 \tag{2}$$

and predict

$$\alpha_s(1.2GeV) = 0.34,$$

which is within the error of the quoted experimental value $\alpha_s^{(exp)}(1.2GeV) = 0.35 \pm 0.01$

The fit to Ref I

J^{PC}	Ref I (MeV)	Our model (MeV)	Mismatch	$N_c \to \infty$ [?]	Mismatch
0^{++}	**1475 (4%)**	**1475**	0	**1475**	0
2^{++}	2150 (5%)	2055	4%	2153 (10%)	5%
0^{-+}	**2250 (4%)**	**2243**	0		
0^{++*}	**2755 (4%)**	**2753**	0	2814 (12%)	2%
2^{++*}	2880 (5%)	2991	4%		
0^{-+*}	3370 (4%)	3288	2%		
0^{++**}	3370 (4%)	3561	5%		
0^{++***}	3990 (5%)	4253	6%		

Comparison between the glueball spectra in Ref. I and in our model. The states we use as input in our fit are marked in red. The parenthesis in the lattice data indicate the percent accuracy.

Open ends

• This phenomenological approach towards an improved holographic QCD model seems promising

• With a few choices of parameters the physics of pure glue at $T = 0$ and $T > 0$ is mostly reproduced correctly.

♠ Calculate the meson spectrum and compare with data.

♠ Explore the baryon spectrum

♠ Recalculate the dipole moment of the neutron in connection with the strong CP problem.

♠ Calculate RHIC/LHC finite T observables (like jet quenching and bulk viscosity)

.

Thank you for your patience!

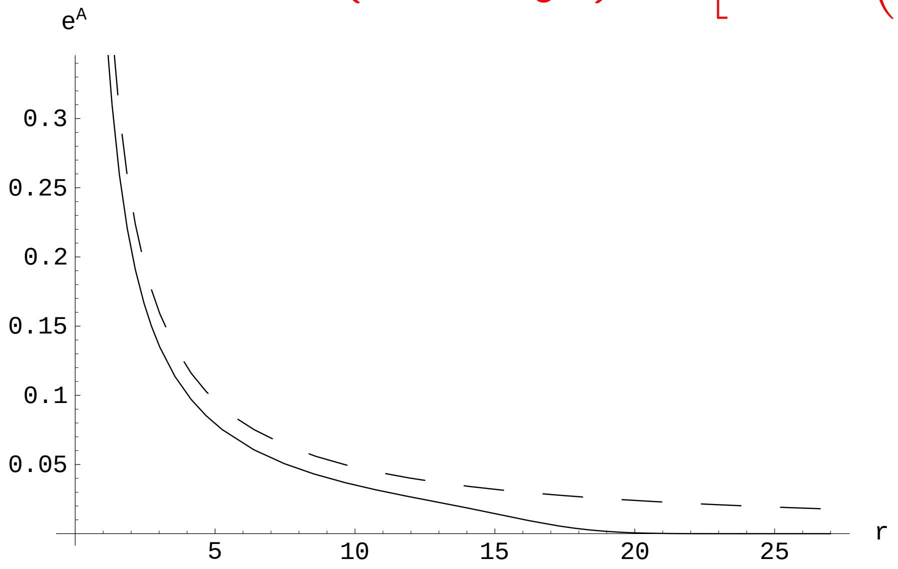
Concrete models: I

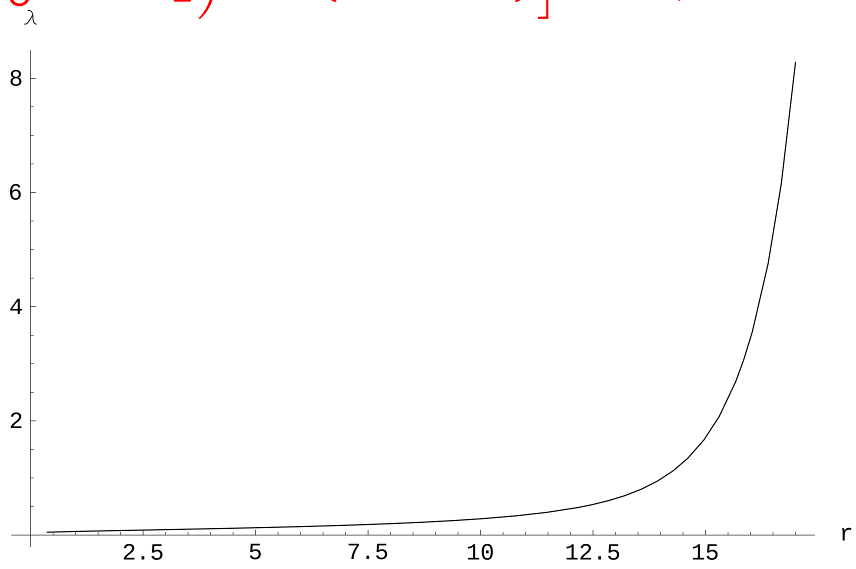
- $r_0 = \infty$ and $a = 2$:

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3+2b_0\lambda} - \frac{3a(2b_0^2+3b_1^2)\lambda^3}{(1+\lambda^2)\left(9a+\left(2b_0^2+3b_1^2\right)\log(1+\lambda^2)\right)}$$

is everywhere regular and has the correct UV and IR asymptotics.

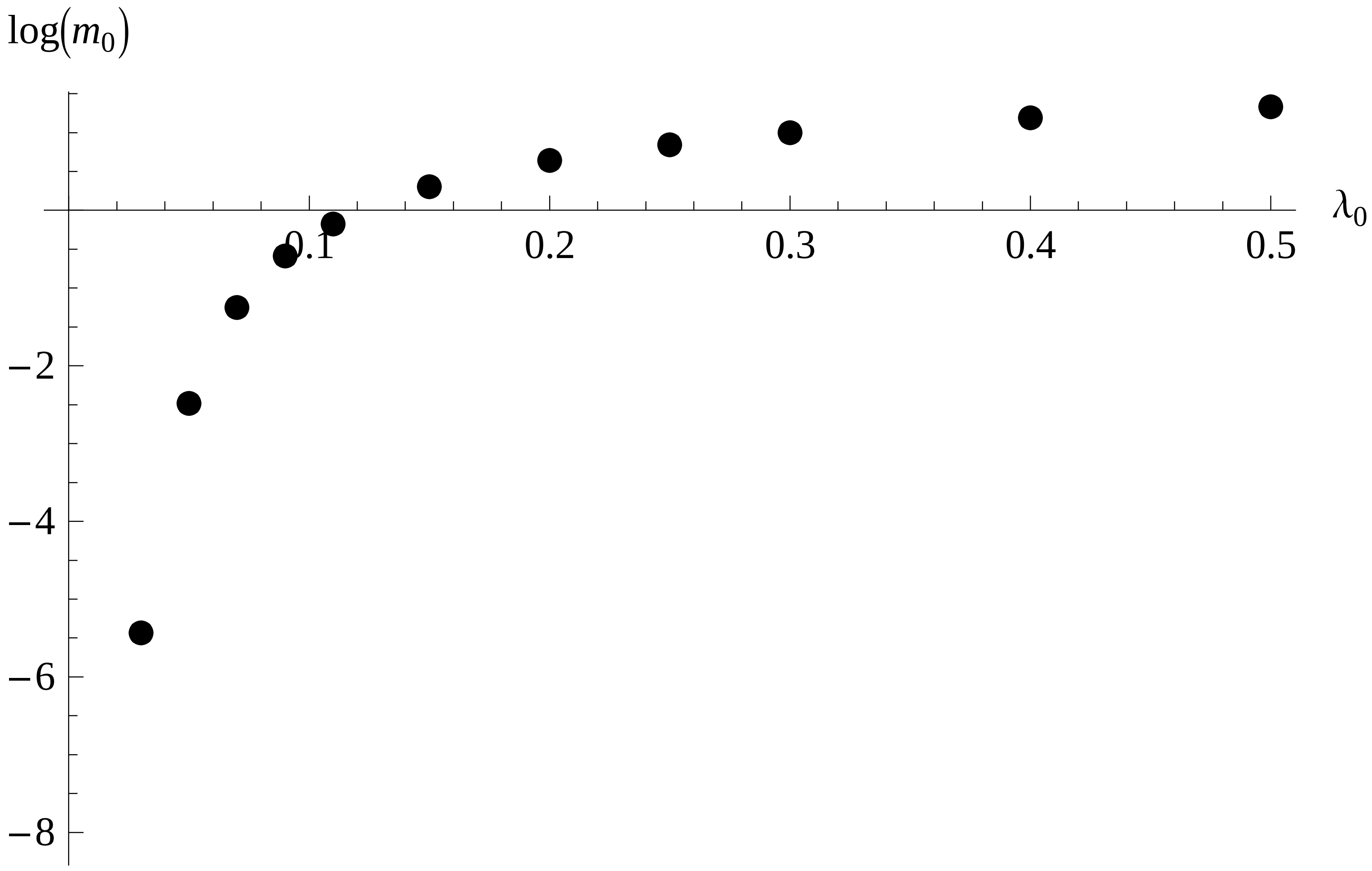
$$W = (3+2b_0\lambda)^{2/3}\left[9a+\left(2b_0^2+3b_1\right)\log(1+\lambda^2)\right]^{2a/3},$$





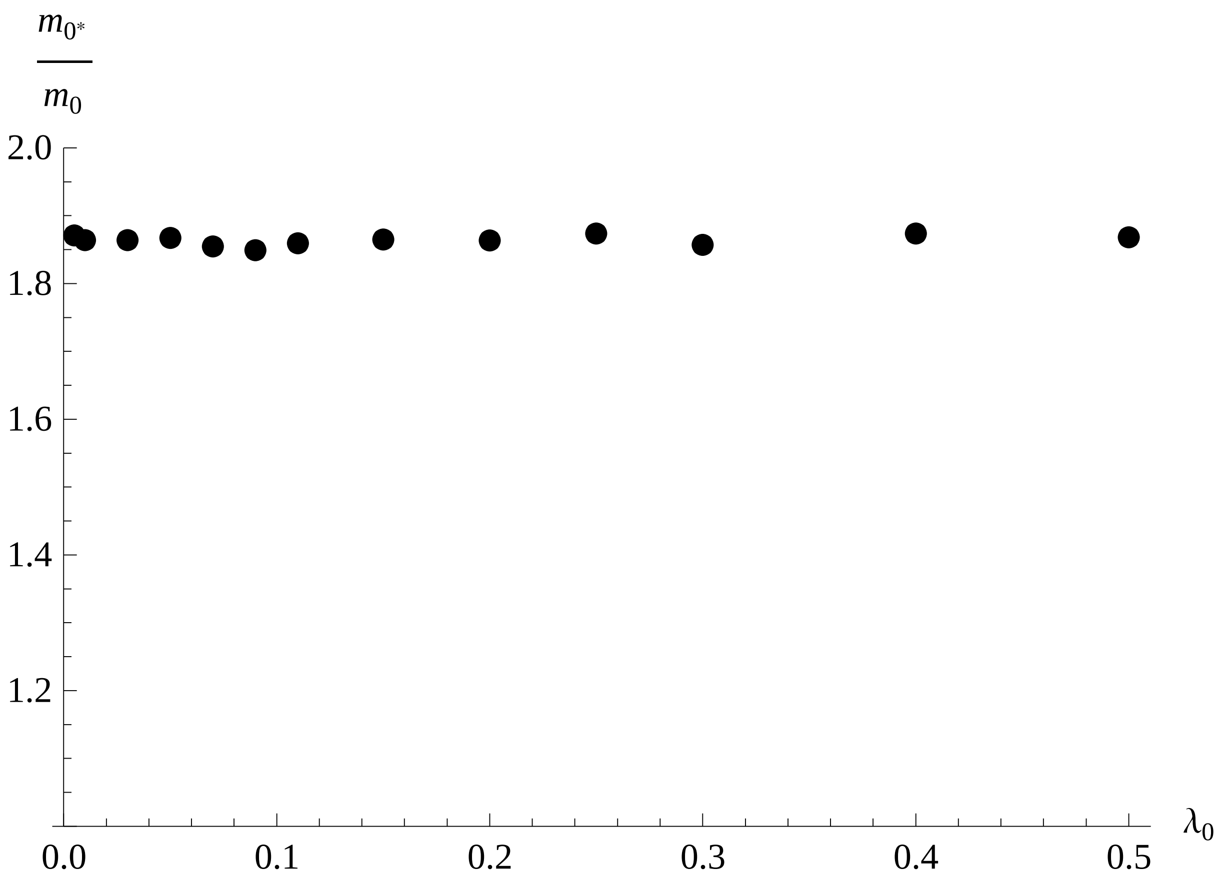
The scale factor and 't Hooft coupling that follow from β. $b_0 = 4.2$, $\lambda_0 = 0.05$, $A_0 = 0$. The units are such that $\ell = 0.5$. The dashed line represents the scale factor for pure AdS.

Dependence of absolute mass scale on λ_0



Dependence on initial condition λ_0 of the absolute scale of the lowest lying glueball (shown in Logarithmic scale)

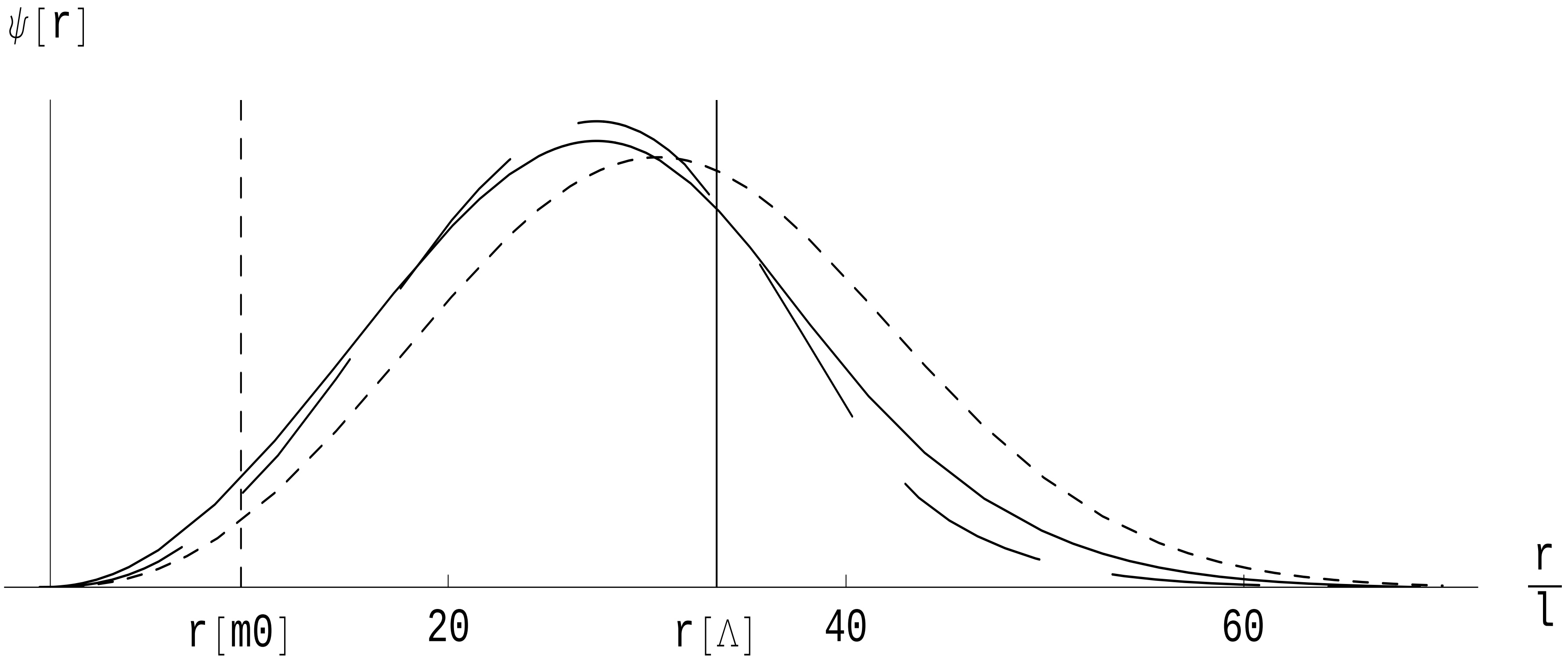
Dependence of mass ratios on λ_0



The mass ratios R_{20}

$$R_{20} = \frac{m_{2++}}{m_{0++}}.$$

The glueball wavefunctions



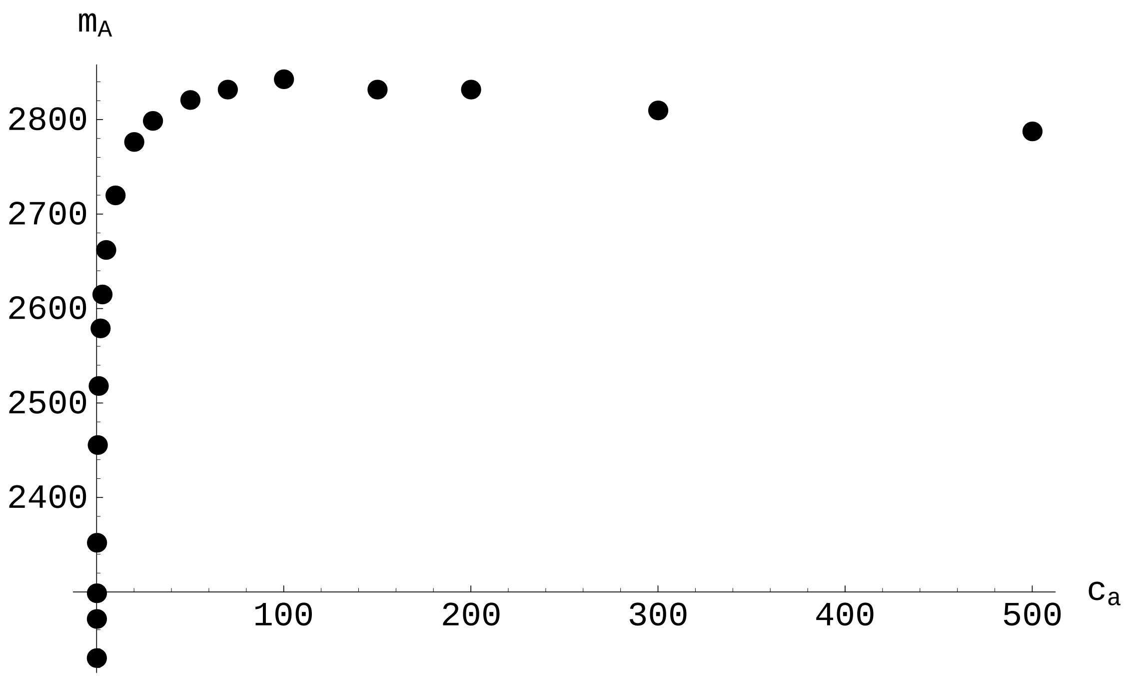
Normalized wave-function profiles for the ground states of the 0^{++} (solid line) ,0^{-+} (dashed line), and 2^{++} (dotted line) towers, as a function of the radial conformal coordinate. The vertical lines represent the position corresponding to $E = m_{0^{++}}$ and $E = \Lambda_p$.

The lattice glueball data

J^{++}	Ref. I $(m/\sqrt{\sigma})$	Ref. I (MeV)	Ref. II (mr_0)	Ref. II (MeV)	$N_c \to \infty(m/\sqrt{\sigma})$
0	3.347(68)	1475(30)(65)	4.16(11)(4)	1710(50)(80)	3.37(15)
0^*	6.26(16)	2755(70)(120)	6.50(44)(7)	2670(180)(130)	6.43(50)
0^{**}	7.65(23)	3370(100)(150)	NA	NA	NA
0^{***}	9.06(49)	3990(210)(180)	NA	NA	NA
2	4.916(91)	2150(30)(100)	5.83(5)(6)	2390(30)(120)	4.93(30)
2^*	6.48(22)	2880(100)(130)	NA	NA	NA
R_{20}	1.46(5)	1.46(5)	1.40(5)	1.40(5)	1.46(11)
R_{00}	1.87(8)	1.87(8)	1.56(15)	1.56(15)	1.90(17)

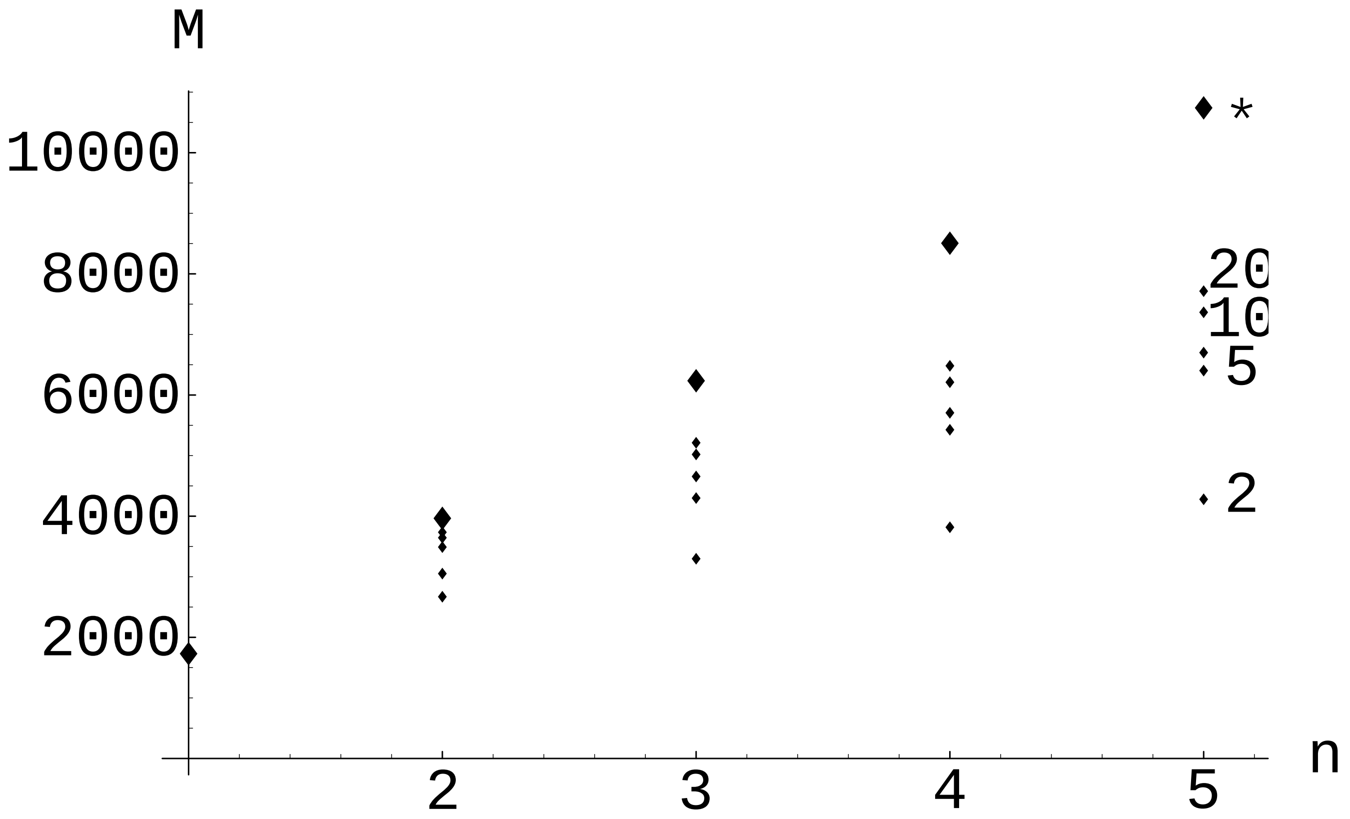
Available lattice data for the scalar and the tensor glueballs. Ref. I =H. B. Meyer, [arXiv:hep-lat/0508002]. and Ref. II = C. J. Morningstar and M. J. Peardon, [arXiv:hep-lat/9901004] + Y. Chen *et al.*, [arXiv:hep-lat/0510074]. The first error corresponds to the statistical error from the the continuum extrapolation. The second error in Ref.I is due to the uncertainty in the string tension $\sqrt{\sigma}$. (Note that this does not affect the mass ratios). The second error in the Ref. II is the estimated uncertainty from the anisotropy. In the last column we present the available large N_c estimates according to B. Lucini and M. Teper, [arXiv:hep-lat/0103027]. The parenthesis in this column shows the total possible error followed by the estimations in the same reference.

Pseudoscalar glueballs



Lowest 0^{-+} glueball mass in MeV as a function of c_a in $Z(\lambda) = Z_a(1+c_a\lambda^4)$.

α-dependence of scalar spectrum



The 0^{++} spectra for varying values of α that are shown at the right end of the plot. The symbol $*$ denotes the AdS/QCD result.

QCD at finite temperature

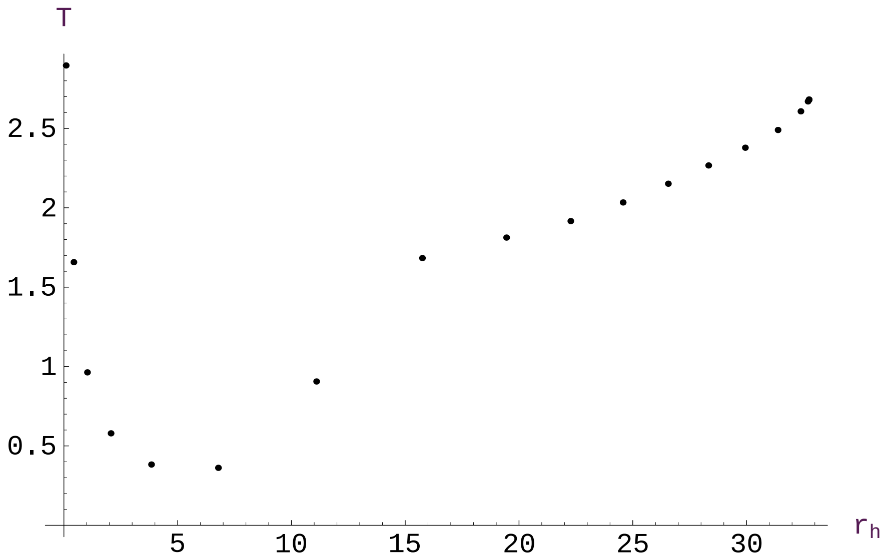
The thermal vacuum can be described by
(1) The "thermal vacuum solution". This is the zero temperature solution we desribed so far with time periodically identified with period β.

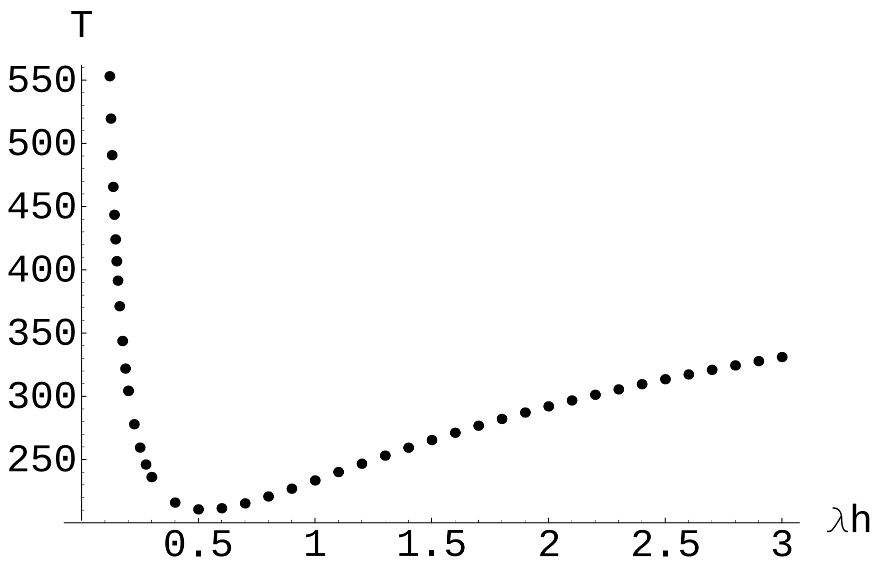
(2) The "black-hole solution"

$$ds^2 = b(r)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx^i\right], \qquad \Phi = \Phi(r)$$

We can show the following:

• For $T > T_{\text{min}}$ there are two black-hole solutions with the same temperature but different horizon positions. One is a "large" BH the other is "small".





• When $T < T_{\text{min}}$ only the "thermal vacuum solution" exists: it describes the confined phase at finite temperature.

• When $T > T_{\text{min}}$ three competing solutions exist. The large BH has the lowest free energy. It describes the deconfined QGP phase.

• The minimum temperature for the black-holes is $T_{\text{min}} \simeq 210$ MeV with $\lambda_h = 0.34$. The critical temperature is

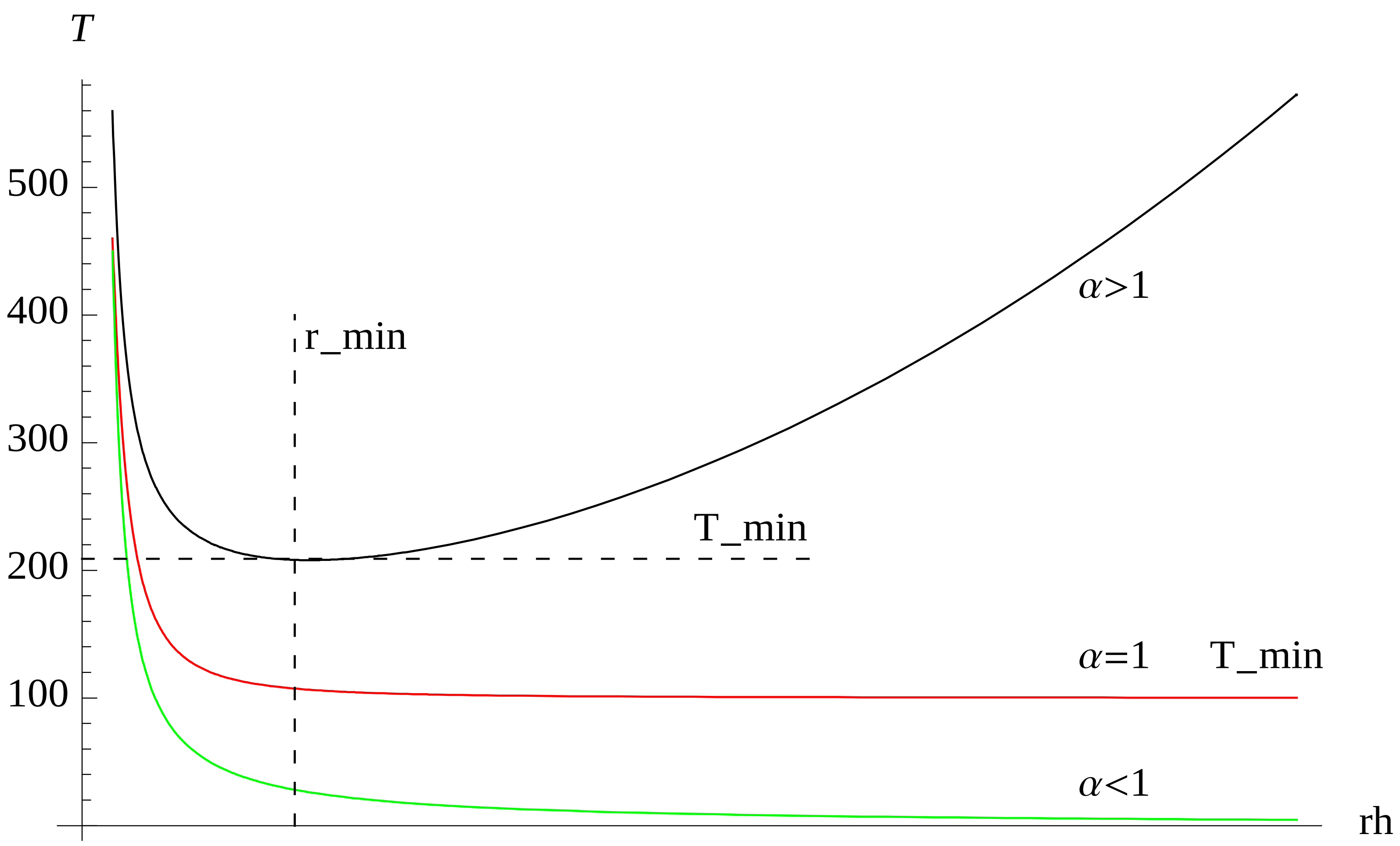
$$T_c \simeq 240 \quad \text{MeV} \quad , \quad \lambda_h = 0.54$$

• The specific heat for the QGP solution is positive as it should:

$$\frac{dE}{dT} = \frac{E}{T + \frac{3}{4\pi}\frac{\partial \log b}{\partial r_h}}$$

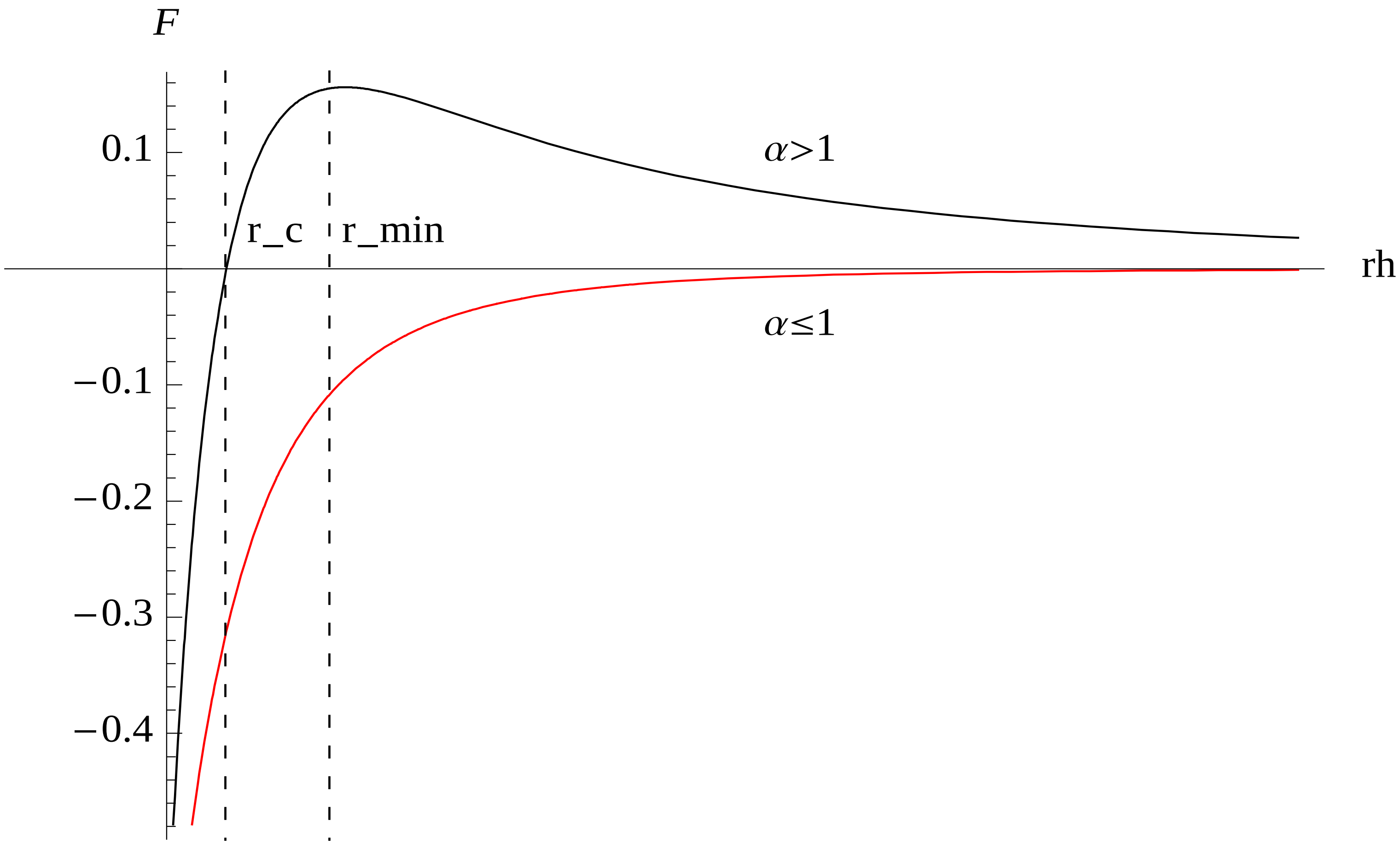
• In the QGP phase, the $q\bar{q}$ potential is screened. This is better than lattice results.

Temperature versus horizon position

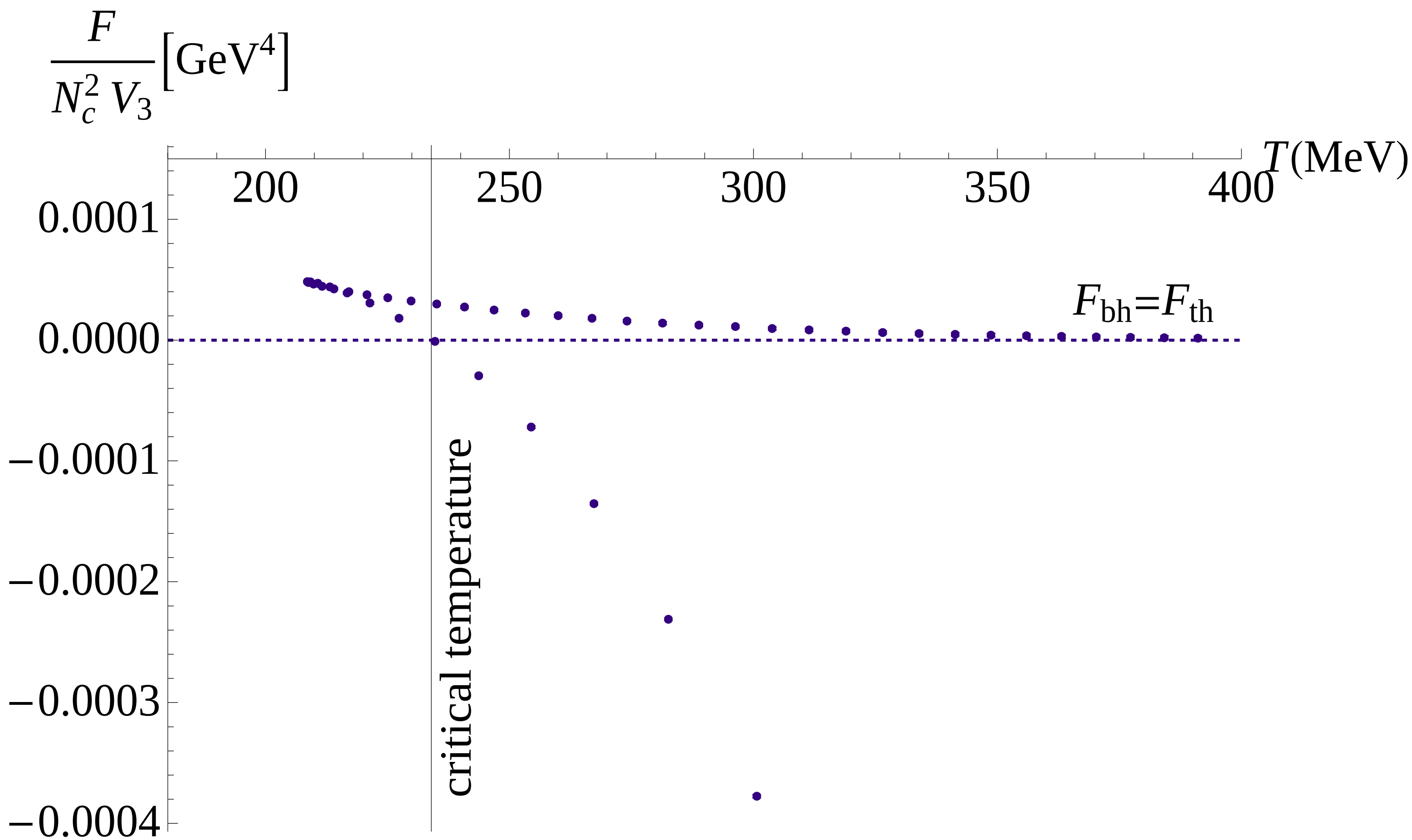


We plot the relation $T(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is only one branch of black-hole solutions.

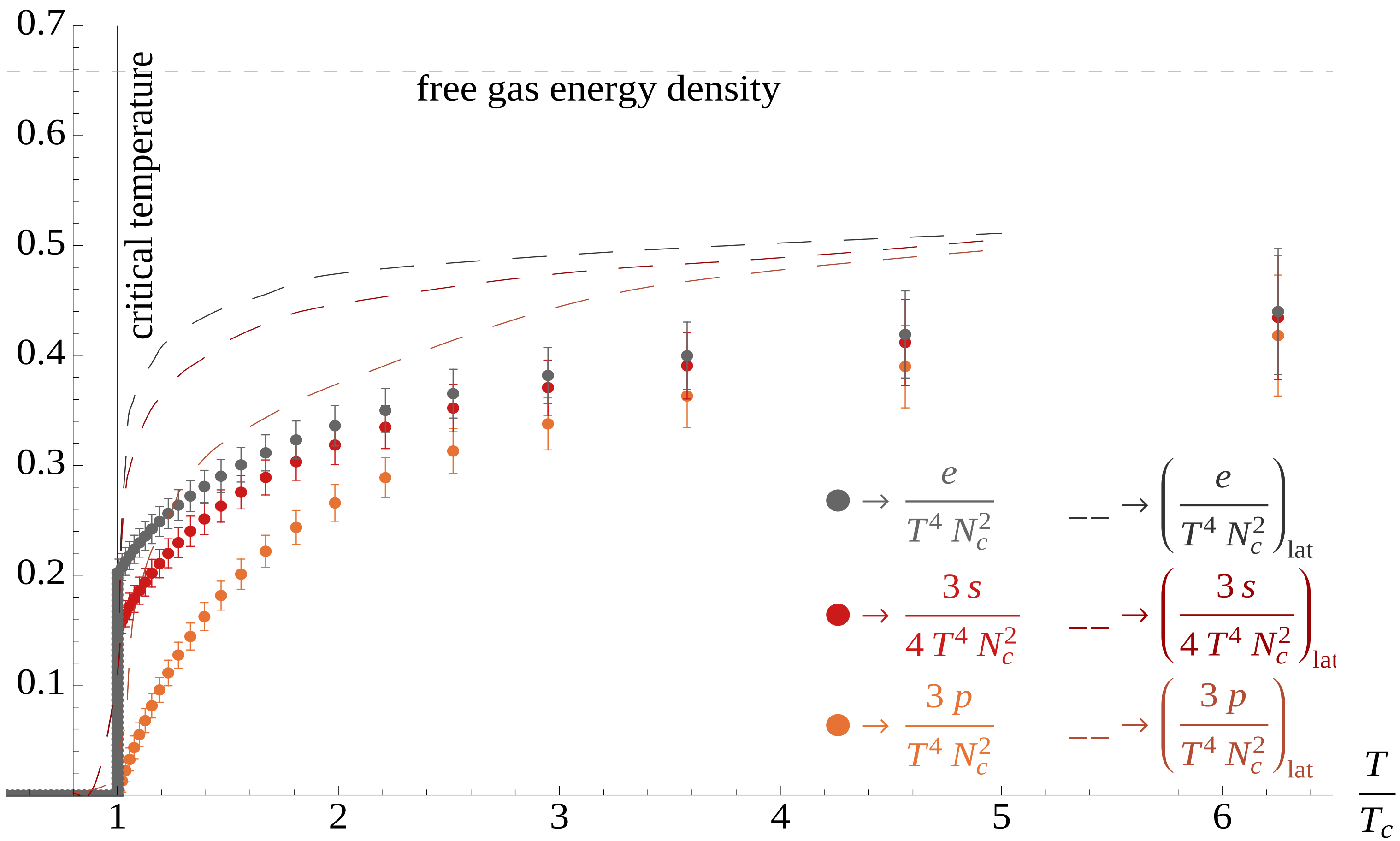
The free energy as a function of r_h



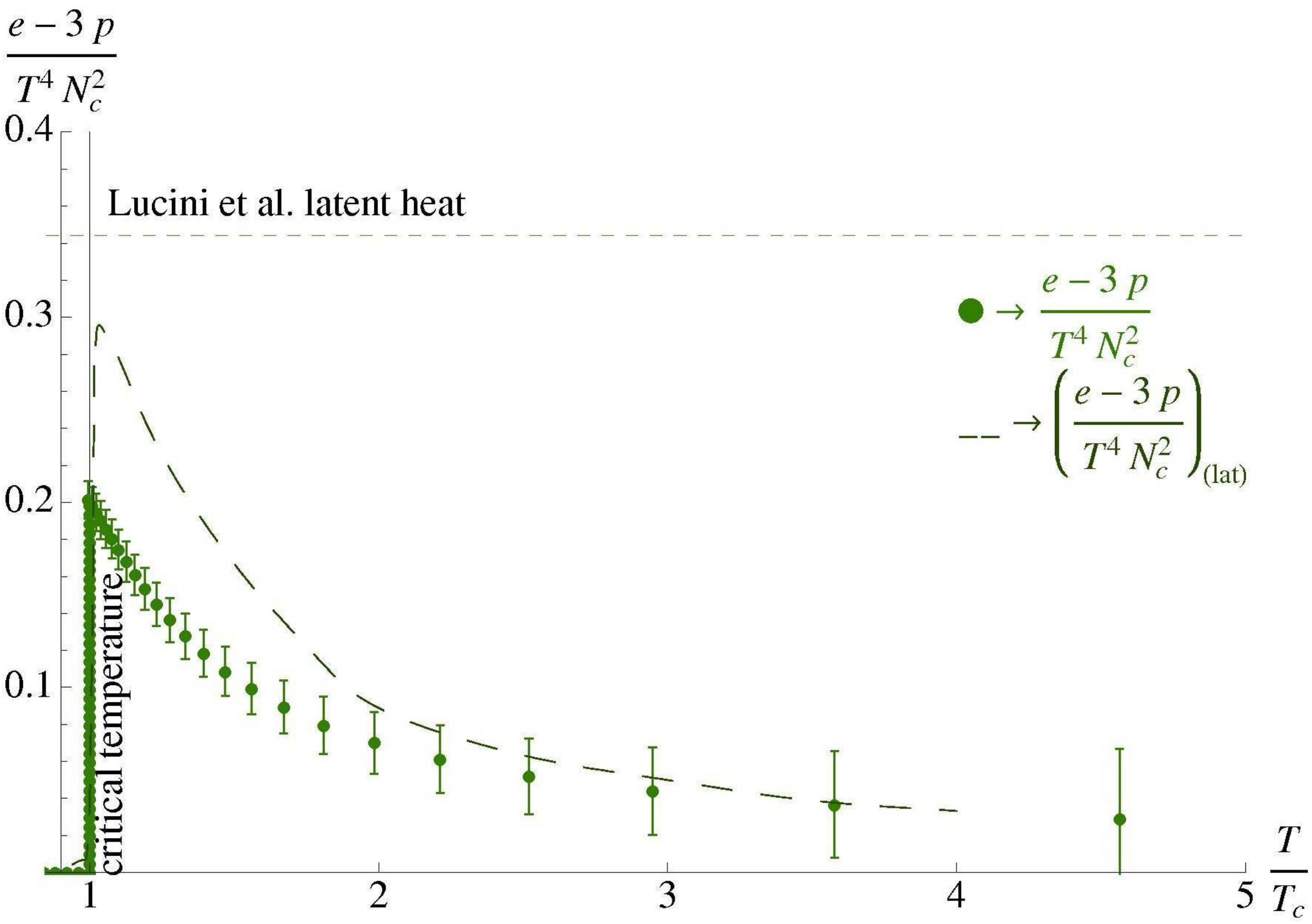
The transition in the free energy



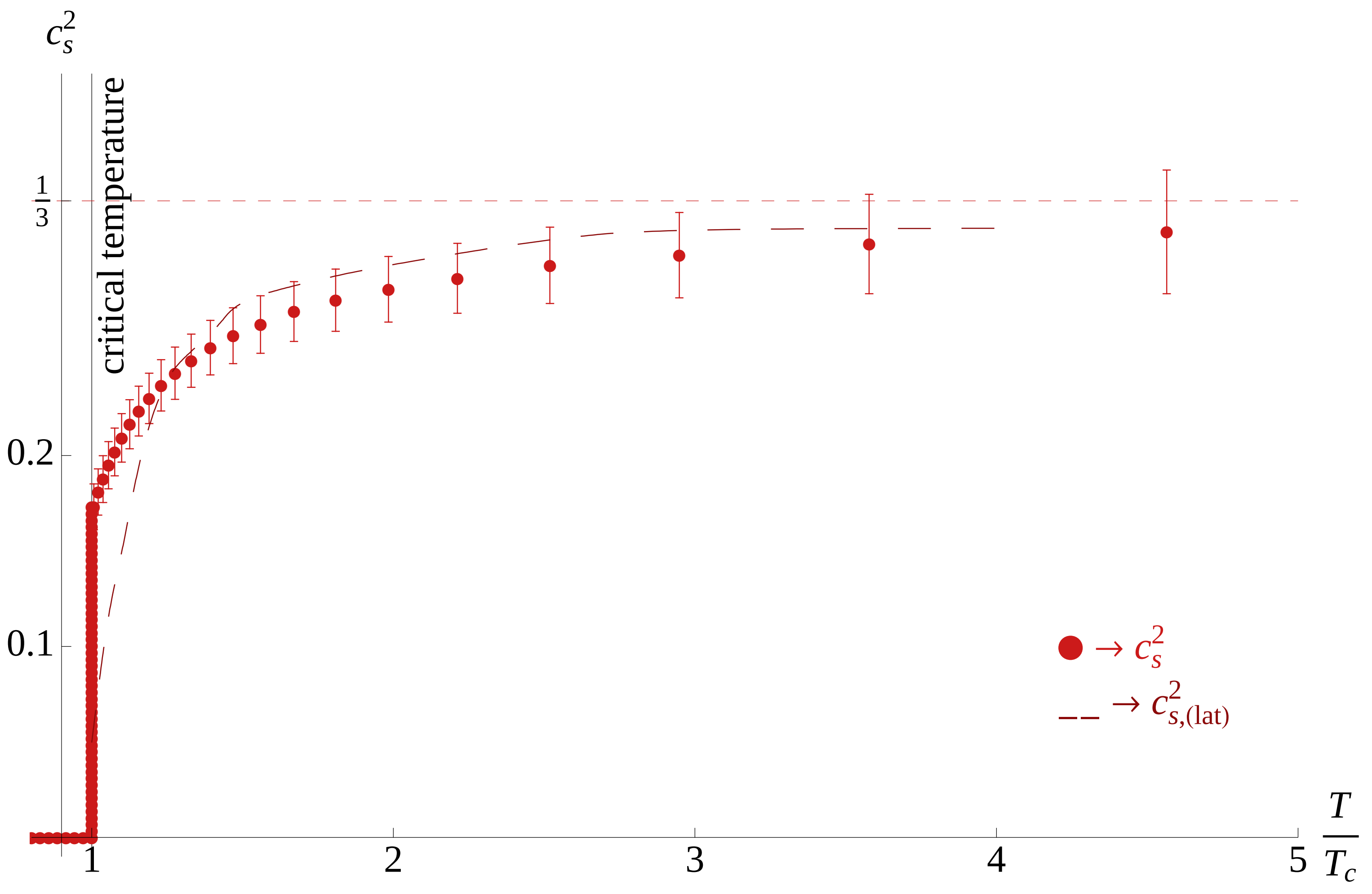
The thermodynamic quantities



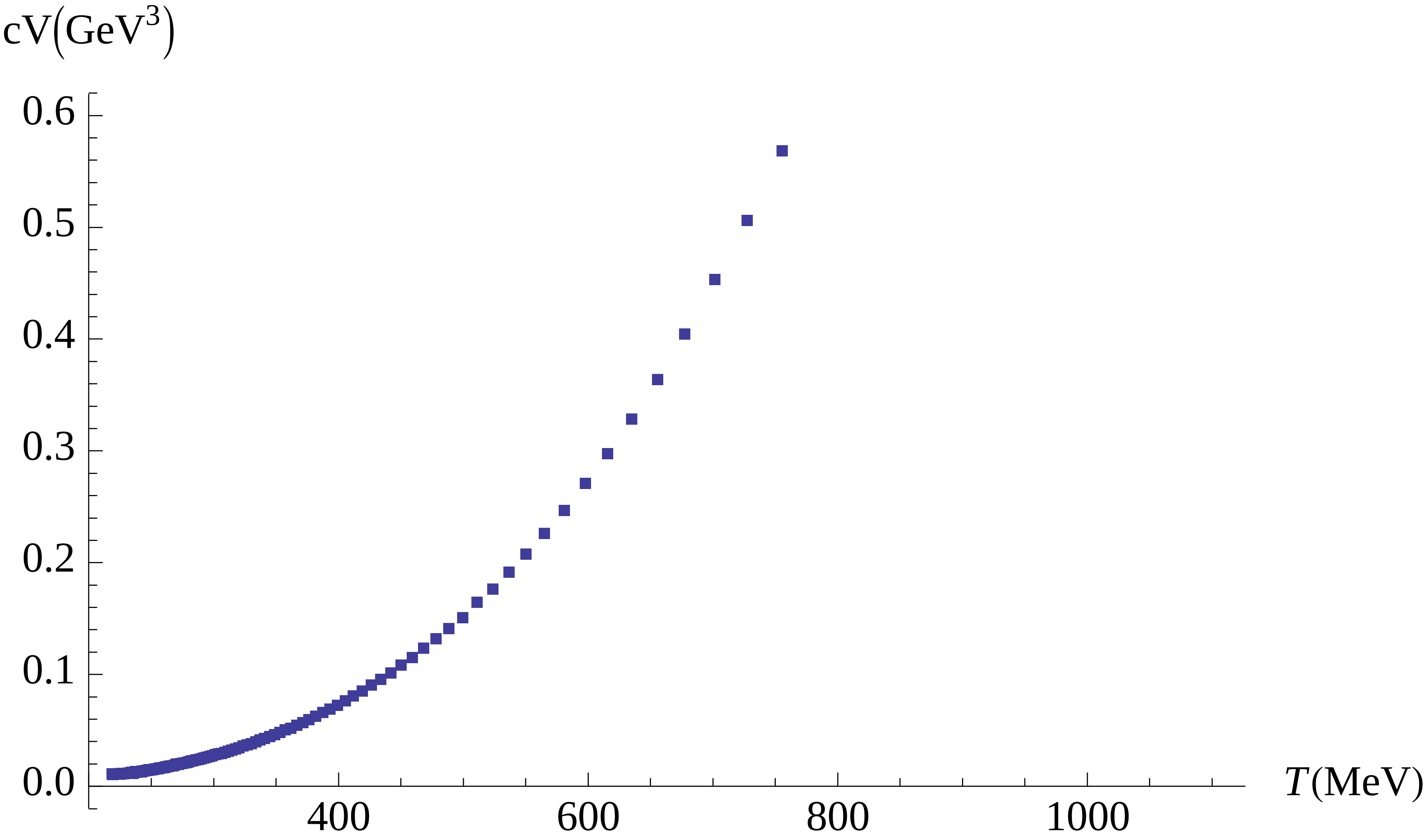
Equation of state



The speed of sound (bulk viscosity)



The specific heat



AdS/QCD

♠ A basic phenomenological approach: use a slice of AdS_5, with a UV cutoff, and an IR cutoff.

Polchinski+Strassler

♠ It successfully exhibits confinement (trivially via IR cutoff), and power-like behavior in hard scattering amplitudes

♠ It may be equipped with a bifundamental scalar, T, and $U(N_f)_L \times U(N_f)_R$, gauge fields to describe mesons.

Erlich+Katz+Son+Stepanov, DaRold+Pomarol

Chiral symmetry is be broken by hand, via IR boundary conditions. The low-lying meson spectrum looks "reasonable".

♠ Shortcomings:

- The glueball spectrum fits badly the lattice calculations. It has the wrong behavior $m_n^2 \sim n^2$ at large n.
- Magnetic quarks are confined instead of screened.
- Chiral symmetry breaking is input by hand.
- The meson spectrum has also the wrong UV asymptotics $m_n^2 \sim n^2$.

Improving AdS/QCD

♠ The goal is to use input from both string theory and the gauge theory (QCD) in order to provide an improved phenomenological holographic model for real world QCD.

♠ This is an exploratory adventure, and we will short-circuit several obstacles on the way.

♠ As we will see, we will get an interesting perspective on the physics of pure glue as well as on the quark sector.

A preview of the results: pure glue

♠ The starting point of pure QCD: a two-derivative action in 5d involving

$$g_{\mu\nu} \leftrightarrow T_{\mu\nu} \quad , \quad \phi \leftrightarrow Tr[F^2] \quad , \quad a \leftrightarrow Tr[F \wedge F]$$

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} - \frac{Z(\lambda)}{2N_c^2}(\partial a)^2 + V(\lambda) \right] \quad , \quad \lambda = N_c\, e^{\phi}$$

with

$$V(\lambda) = V_0 \left(1 + \sum_{n=1}^{\infty} V_n \lambda^n \right) = -\frac{4}{3}\lambda^2 \left(\frac{dW}{d\lambda} \right)^2 + \frac{64}{27} W^2.$$

• There is a 1-1 correspondence between the QCD β-function, $\beta(\lambda)$ and W:

$$\beta(\lambda) = -\frac{9}{4}\lambda^2 \, \frac{d \log W(\lambda)}{d\lambda}$$

• There is a similar statement between $Z(\lambda)$ and the (non-perturbative) β-function for the θ-angle.

• The space is asymptotically AdS$_5$ in the UV ($r \to 0$) modulo log corrections (in the Einstein frame):

$$ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu}dx^\mu dx^\mu) \quad , \quad E \equiv e^{A(r)}$$

• There are various extra α' corrections to the potential ($\sim \beta$-function). They only correct the non-universal terms. Moreover, α' corrections to the energy definition E can be set to zero in a special scheme (the "holographic" scheme).

• ALL confining backgrounds have an IR singularity at $r = r_0$. There are two classes: r_0=finite and $r_0 = \infty$. The singularity is always "good": all spectra are well defined without extra input.

• $\lambda \to \infty$ at the IR singularity.

• In the $r_0 = \infty$ class of backgrounds, the curvature (in the string frame) vanishes in the neighborhood of the IR singularity.

Classification of confining superpotentials $W(\lambda)$ as $\lambda \to \infty$ in IR:

$$W(\lambda) \sim (\log\lambda)^{\frac{P}{2}}\ \lambda^{Q} \quad , \quad \lambda \sim E^{-\frac{9}{4}Q}\left(\log\frac{1}{E}\right)^{\frac{P}{2Q}}, \qquad E \to 0.$$

• $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

• $Q = 2/3$, and $0 \leq P < 1$ leads to confinement and a singularity at $r = \infty$ The scale factor e^{A} vanishes there exponentially in the r coordinate.

• For all potentials that confine, the spectrum of 0^{++} and 2^{++} glueballs has a mass gap and is purely discrete. For the 0^{+-} glueballs this is the case if

$$Z(\lambda) \sim \lambda^{d} \quad , \quad d > 2 \quad \text{as} \quad \lambda \to \infty.$$

We will later derive that $d = 4$.

• In all physically interesting confining backgrounds the magnetic color charges are screened. This is an improvement with respect to AdS/QCD models (magnetic quarks are also confined instead) .

• Of all the possible confining asymptotics, there is a unique one that guarantees "linear confinement" for all glueballs. It corresponds to the case $Q = 2/3, P = 1/2$, i.e.

$$W(\lambda) \sim (\log\lambda)^{\frac{1}{4}}\,\lambda^{\frac{2}{3}} \quad , \quad \beta(\lambda) = -\frac{3}{2}\lambda\left[1 + \frac{3}{8\log\lambda} + \cdots\right] \quad , \quad \lambda \sim E^{-\frac{3}{2}}\left(\log\frac{1}{E}\right)^{\frac{3}{8}}$$

This choice also seems to be preferred from considerations of the meson sector as discussed below.

• Numerical calculation of the 0^{++} and 2^{++} glueball spectra and comparison with lattice data gives a clear preference for the $r_0 = \infty$ asymptotics.

.

- We can find the background solution for the axion:

$$a(r) = (\theta_{UV} + 2\pi k) \int_r^{r_0} \frac{dr}{e^{3A} Z(\lambda)} \Big/ \int_0^{r_0} \frac{dr}{e^{3A} Z(\lambda)}$$

written in terms of the axion coupling function $Z(\lambda)$ and the scale factor e^A. This provides the "running" of the effective QCD θ angle.

- A direct holographic calculation of the θ-dependent vacuum energy gives

$$E(\theta_{UV}) \sim \text{Min}_k (\theta_{UV} + 2\pi k)^2$$

.

- Note that always $a(E = 0) = 0$. This suggests that the θ angle is screened in the IR.

Preview: quarks ($N_f \ll N_c$) and mesons

- Flavor is introduced by N_f $D_4 + \bar{D}_4$ branes pairs inside the bulk background. Their back-reaction on the bulk geometry is suppressed by N_f/N_c.

- The important world-volume fields are

$$T_{ij} \leftrightarrow \bar{q}_a^i \frac{1+\gamma^5}{2} q_a^j \quad , \quad A_\mu^{ij\,L,R} \leftrightarrow \bar{q}_a^i \frac{1 \pm \gamma^5}{2} \gamma^\mu q_a^j$$

Generating the $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

- The UV mass matrix m_{ij} corresponds to the source term of the Tachyon field. It breaks the chiral (gauge) symmetry. The normalizable mode corresponds to the vev $\langle \bar{q}_a^i \frac{1+\gamma^5}{2} q_a^j \rangle$.

- We show that the expectation value of the tachyon is non-zero and $T \sim \mathbf{1}$, breaking chiral symmetry $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$. The anomaly plays an important role in this (holographic Coleman-Witten)

• The fact that the tachyon diverges in the IR (fusing D with $\bar{D}$) constraints the UV asymptotics and determines the quark condensate $\langle\bar{q}q\rangle$ in terms of m_q. A GOR relation is satisfied (for an asymptotic AdS_5 space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\ \langle\bar{q}q\rangle \quad , \quad m_q \to 0$$

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

• Studying the spectrum of highly excited mesons, we find the expected property of linear confinement: $m_n^2 \sim n$.

Motivating the effective action

- The basic string motivated action for the 5d theory is

$$S_5 = M^3 \int d^5x \sqrt{g} \left[e^{-2\phi} \left(R + 4(\partial\phi)^2 + \frac{\delta c}{\ell_s^2} \right) - \frac{1}{2 \cdot 5!} F_5^2 - \frac{1}{2}(da)^2 \right]$$

$F_5 = dC_4$ seeds the D_3 branes that generate the $U(N_c)$ group.

- The C_4 equation of motion gives

$$^*F_5 = N_c$$

and the dual action in the Einstein frame $g_E = e^{\frac{4}{3}\phi}\ g_s$

$$S_E = M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\phi)^2 - \frac{e^{2\phi}}{2}(\partial a)^2 + V_s(\phi) \right] \quad , \quad V_s(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2} \left[\delta c - \frac{N_c^2}{2} e^{2\phi} \right]$$

- Higher derivative corrections involving the F_5 upon dualization provide further terms in the dilaton potential

$$V_s(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2} \left[\delta c + \sum_{n=1}^{\infty} a_n \ (N_c e^{\phi})^{2n} \right]$$

MORE INFO

♠ This potential is very good for the IR behavior but in the UV it vanishes with λ and this is not the correct behavior.

♠ We need a potential that in the Einstein frame asymptotes to a constant $V_0 = \frac{12}{\ell^2}$ as $\lambda \to 0$.

♠ This is generated by higher-derivative corrections in the curvature. Here we postulate it.

♠ The five form will then generate a series of (perturbative) terms in λ:

$$V(\lambda) = V_0 \left(1 + \sum_{n=1}^{\infty} a_n \lambda^{a\ n} \right)$$

we will take $a = 1$ for simplicity (by adjusting the kinetic term).

♠ This matches the weak coupling expansion of perturbative QCD and will give the perturbative β-function expansion.

♠ We will ignore other effects of higher-derivative terms associated with R and $(\partial\Phi)^2$. Motivated partly by the success of SVZ sum rules

♠ The "resumed" two-derivative action reads

$$S_{\rm Einstein} = M^3 N_c^2 \int d^5x\sqrt{g}\left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda)\right] \quad , \quad \lambda = N_c\, e^{\phi}$$

after redefining the kinetic terms.

• We must choose the holographic energy: the natural choice is $E = e^{A_E}$ frame as it is monotonic and end at zero in the IR singularity.

• We may now solve the equations perturbatively in λ around $\lambda = 0$ and $r = 0$ (this is a weak coupling expansion) to find

$$\frac{d\lambda}{d\log E} \equiv \beta(\lambda) = -b_0\lambda^2 + b_1\lambda^3 + b_2\lambda^4 + \cdots$$

with

$$\frac{1}{\lambda} = L - \frac{b_1}{b_0}\log L + \mathcal{O}\left(\frac{\log L}{L}\right) \quad , \quad L \equiv -b_0\log(r\Lambda)$$

$$e^{2A} = \frac{\ell^2}{r^2}\left[1 + \frac{8}{3^2\log r\Lambda} + \cdots\right]$$

$$V = \frac{12}{\ell^2}\left[1 + \frac{8}{9}(b_0\lambda) + \frac{23 - 36\frac{b_1}{b_0^2}}{3^4}(b_0\lambda)^2 + \cdots\right]$$

♠ One-to-one correspondence with the perturbative β-function, and the perturbative potential.

Organizing the vacuum solutions

A useful variable is the phase variable

$$X \equiv \frac{\Phi'}{3A'} = \frac{\beta(\lambda)}{3\lambda} \quad , \quad e^{\Phi} \equiv \lambda$$

and a superpotential

$$W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \Phi}\right)^2 = \left(\frac{3}{4}\right)^3 V(\Phi).$$

with

$$A' = -\frac{4}{9}W \quad , \quad \Phi' = \frac{dW}{d\Phi}$$

$$X = -\frac{3}{4}\frac{d\log W}{d\log\lambda} \quad , \quad \beta(\lambda) = -\frac{9}{4}\lambda\frac{d\log W}{d\log\lambda}$$

♠ The equations have three integration constants: (two for Φ and one for A) One corresponds to the "gluon condensate" in the UV. It must be set to zero otherwise the IR behavior is unacceptable. The other is Λ. The third one is a gauge artifact (corresponds to overall translation in the radial coordinate).

The IR regime

For any asymptotically AdS$_5$ solution ($e^A \sim \frac{\ell}{r}$):

- The scale factor $e^{A(r)}$ is monotonically decreasing

Girardelo+Petrini+Porrati+Zaffaroni
Freedman+Gubser+Pilch+Warner

- Moreover, there are only three possible, mutually exclusive IR asymptotics:

♠ *there is another asymptotic AdS_5 region, at $r \to \infty$, where* $\exp A(r) \sim \ell'/r$, *and $\ell' \leq \ell$* (equality holds if and only if the space is exactly AdS_5 everywhere);

♠ there is a curvature singularity at some finite value of the radial coordinate, $r = r_0$;

♠ `there is a curvature singularity at` $r \to \infty$`, where the scale factor vanishes and the space-time shrinks to zero size.`

Wilson-Loops and confinement

• Calculation of the static quark potential using the vev of the Wilson loop calculated via an F-string worldsheet.

Rey+Yee, Maldacena

$$T\ E(L) = S_{minimal}(X)$$

We calculate

$$L = 2\int_0^{r_0} dr \frac{1}{\sqrt{e^{4A_S(r)-4A_S(r_0)}-1}}.$$

It diverges when e^{A_s} has a minimum (at $r = r_*$). Then

$$E(L) \sim T_f\ e^{2A_S(r_*)}\ L$$

• Confinement $\to$ $A_s(r_*)$ is finite. This is a more general condition that considered before as A_S `is not monotonic in general`.

• Effective string tension

$$T_{\text{string}} = T_f\ e^{2A_S(r_*)}$$

General criterion for confinement

• **the geometric version**:
A geometry that shrinks to zero size in the IR is dual to a confining 4D theory if and only if the Einstein metric in conformal coordinates vanishes as (or faster than) e^{-Cr} as $r \to \infty$, for some $C > 0$.

• It is understood here that a metric vanishing at finite $r = r_0$ also satisfies the above condition.

♠ **the superpotential**
A 5D background is dual to a confining theory if the superpotential grows as (or faster than)

$$W \sim (\log \lambda)^{P/2} \lambda^{2/3} \quad \text{as} \quad \lambda \to \infty \quad , \quad P \geq 0$$

♠ **the β-function** A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda \to \infty} \left(\frac{\beta(\lambda)}{3\lambda} + \frac{1}{2} \right) \log \lambda = K, \qquad -\infty \leq K \leq 0$$

(No explicit reference to any coordinate system) Linear trajectories correspond to $K = -\frac{3}{16}$

Comments on confining backgrounds

• For all confining backgrounds with $r_0 = \infty$, although the space-time is singular in the Einstein frame, the string frame geometry is asymptotically flat for large r. Therefore only λ grows indefinitely.

• String world-sheets do not probe the strong coupling region, at least classically. The string stays away from the strong coupling region.

• Therefore: singular confining backgrounds have generically the property that the singularity is *repulsive*, i.e. only highly excited states can probe it. This will also be reflected in the analysis of the particle spectrum (to be presented later)

• The confining backgrounds must also screen magnetic color charges. This can be checked by calculating 't Hooft loops using D_1 probes:

♠ All confining backgrounds with $r_0 = \infty$ and most at finite r_0 screen properly

♠ In particular "hard-wall" AdS/QCD confines also the magnetic quarks.

Particle Spectra: generalities

- Linearized equation:

$$\ddot{\xi} + 2\dot{B}\dot{\xi} + \Box_4 \xi = 0 \quad , \quad \xi(r,x) = \xi(r)\xi^{(4)}(x), \qquad \Box \xi^{(4)}(x) = m^2 \xi^{(4)}(x)$$

- Can be mapped to Schrodinger problem

$$-\frac{d^2}{dr^2}\psi + V(r)\psi = m^2\psi \quad , \quad V(r) = \frac{d^2B}{dr^2} + \left(\frac{dB}{dr}\right)^2 \quad , \quad \xi(r) = e^{-B(r)}\psi(r)$$

- Mass gap and discrete spectrum visible from the asymptotics of the potential.

- Large n asymptotics of masses obtained from WKB

$$n\pi = \int_{r_1}^{r_2} \sqrt{m^2 - V(r)}\ dr$$

- Spectrum depends only on initial condition for λ ($\sim \Lambda_{QCD}$) and an overall energy scale (e^A) that must be fixed.

- scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log\frac{\beta(\lambda)^2}{9\lambda^2}$$

- tensor glueballs

$$B(r) = \frac{3}{2}A(r)$$

- pseudo-scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log Z(\lambda)$$

- Universality of asymptotics

$$\frac{m^2_{n\to\infty}(0^{++})}{m^2_{n\to\infty}(2^{++})} \to 1 \quad , \quad \frac{m^2_{n\to\infty}(0^{+-})}{m^2_{n\to\infty}(0^{++})} = \frac{1}{4}(d-2)^2$$

predicts $d = 4$ via

$$\frac{m^2}{2\pi\sigma_a} = 2n + J + c,$$

The axion background

• The kinetic term of the axion is suppressed by $1/N_c^2$. (it is an angle in the gauge theory, it is RR in string theory)

$$\ddot{a} + \left(3\dot{A} + \frac{\dot{Z}(\lambda)}{Z(\lambda)}\right)\dot{a} = 0 \quad \to \quad \dot{a} = \frac{C\ e^{-3A}}{Z(\lambda)}$$

It can be interpreted as the flow equation of the effective θ-angle.

• The full solution is

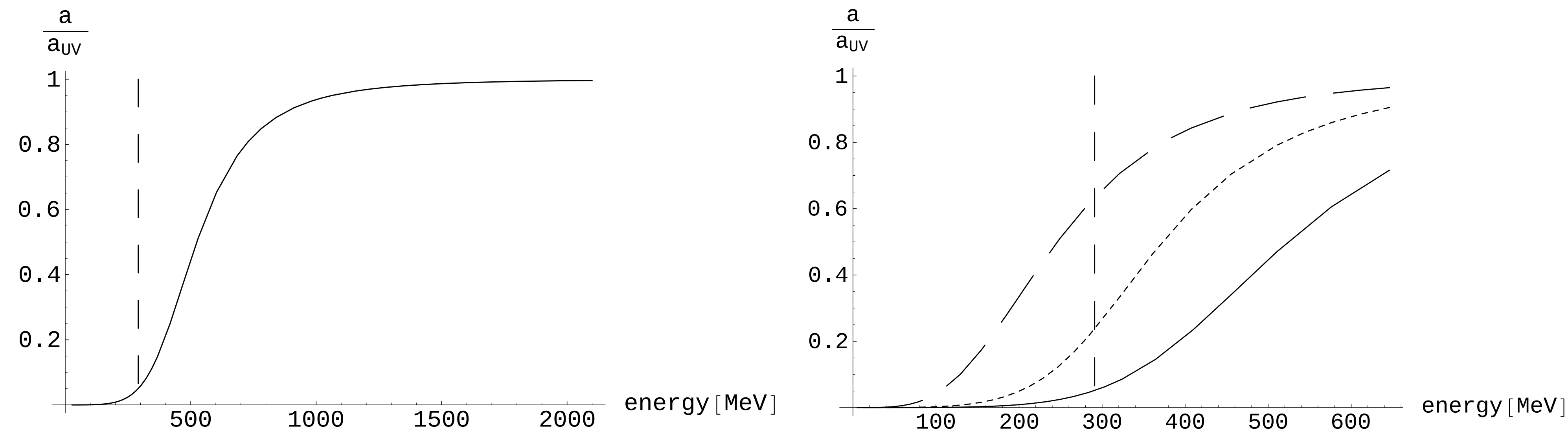
$$a(r) = \theta_{UV} + 2\pi k + C\ \int_0^r r\frac{e^{-3A}}{Z(\lambda)} \quad , \quad C = \langle Tr[F\wedge F]\rangle$$

• The vacuum energy is

$$E(\theta_{UV}) = \frac{M^3}{2N_c^2}\int d^5x\sqrt{g}Z(\lambda)(\partial a)^2 = \frac{M^3}{2N_c^2}Ca(r)\Big|_{r=0}^{r=r_0}$$

• Consistency requires to impose that $a(r_0) = 0$. This determines C and

$$E(\theta_{UV}) = -\frac{M^3}{2}\mathrm{Min}_k\ \frac{(\theta_{UV}+2\pi k)^2}{\int_0^{r_0}\frac{dr}{e^{3A}Z(\lambda)}} \quad , \quad \frac{a(r)}{\theta_{UV}+2\pi k} = \frac{\int_r^{r_0}\frac{dr}{e^{3A}Z(\lambda)}}{\int_0^{r_0}\frac{dr}{e^{3A}Z(\lambda)}}$$



(a) (b)

(a) An example of the axion profile (normalized to one in the UV) as a function of energy, in one of the explicit cases we treat numerically. The energy scale is in MeV, and it is normalized to match the mass of the lowest scalar glueball from lattice data, $m_0 = 1475 MeV$. The axion kinetic function is taken as $Z(\lambda) = Z_a(1 + c_a\lambda^4)$, with $c_a = 100$ (the masses do not depend on the value of Z_a). The vertical dashed line corresponds to $\Lambda_p \equiv \frac{1}{\ell}\frac{\exp\left[A(\lambda_0) - \frac{1}{b_0\lambda_0}\right]}{(b_0\lambda_0)^{b1/b_0^2}}$. In this particular case $\Lambda = 290 MeV$.

(b)A detail showing the different axion profiles for different values of c_a. The values are $c_a = 0.1$ (dashed line), $c_a = 10$ (dotted line) and $c_a = 100$ (solid line).

Critical string theory holography

♠ Several "successful" holographic models of non-trivial gauge dynamics

- The non-supersymmetric D_4 solution,due to Witten, dual to $\mathcal{N} = 4_5$ sYM on a circle, whose supersymmetry is broken by the boundary conditions of the fermions. It exhibits confinement in the IR.

- Flavor has been successfully incorporated by Sakai+Sugimoto by adding D_7 (dipole) branes.

- The Chamseddine-Volkov solution interpreted by Maldacena and Nuñes as the dual of a confining compactified gauge theory (emerging by wrapping NS_5 branes on a two-cycle).

- The Klebanov-Strassler solution corresponding to a cascade of quiver gauge theories, that confine in the IR.

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♠ In all of the above, confinement related quantities (string tension, glue-ball, masses etc, finite temperature effects) can be calculated analytically.

♠ The same applies to the Sakai-Sugimoto model for flavor, except two major drawbacks:

The absence of bare quark masses and the chiral-symmetry-breaking condensate.

♠ In all the above solutions, the scale of KK excitations is of the same order as Λ of the confining gauge theory.

♠ None so far has managed to overcome this obstacle in critical string theory models.

Non-Critical holography

♠ Non-critical string theories have been explored in order to avoid the KK problem.

Kuperstein+Sonnenschein, Klebanov+Maldacena, Bigazzi+Casero+Cotrone+Kiritsis+Paredes

♠ They are expected to involve large curvatures (due to the δc term) and the supergravity approximation seems problematic.

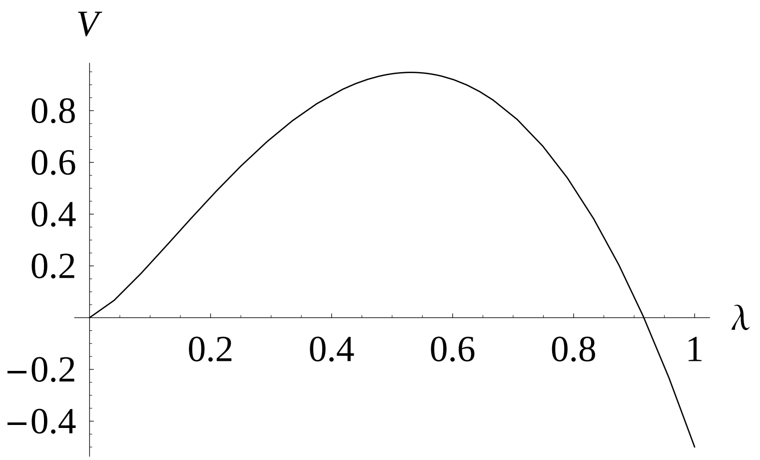
♠ They may provide reliable information on some quantities despite the strong curvature (cf. WZW CFTs).

♠ Recent progress in solving exactly for probe D-branes in non-critical backgrounds has provided important insights for non-critical holography.

Fotopoulos+Niarchos+Prezas, Ashok+Murthy+Troost

♠ It is fair to say that non-critical holography is so far largely unexplored.

Fluctuations around the AdS$_5$ extremum



• In QCD we expect that

$$\frac{1}{\lambda} = \frac{1}{N_c e^{\phi}} \sim \frac{1}{\log r} \quad , \quad ds^2 \sim \frac{1}{r^2}(dr^2 + dx_\mu dx^\mu) \qquad \text{as} \qquad r \to 0$$

• Any potential with $V(\lambda) \sim \lambda^a$ when $\lambda \ll 1$ gives a power different that of AdS$_5$

• There is an AdS$_5$ minimum at a finite value λ_*. This cannot be the UV of QCD as dimensions do not match.

Near an AdS extremum

$$V=\frac{12}{\ell^2}-\frac{16\xi}{3\ell^2}\phi^2+\mathcal{O}(\phi^3)\quad,\quad \frac{18}{\ell}\delta A'=\delta\phi'^2-\frac{4}{\ell^2}\phi^2=\mathcal{O}(\delta\phi^2)\quad,\quad \delta\phi''-\frac{4}{\ell}\delta\phi'-\frac{4\xi}{\ell^2}\delta\phi=0$$

where $\phi << 1$. The general solution of the second equation is

$$\delta\phi=C_+e^{\frac{(2+2\sqrt{1+\xi})u}{\ell}}+C_-e^{\frac{(2-2\sqrt{1+\xi})u}{\ell}}$$

For the potential in question

$$V(\phi)=\frac{e^{\frac{4}{3}\phi}}{\ell_s^2}\left[5-\frac{N_c^2}{2}e^{2\phi}-N_f\;e^{\phi}\right]\quad,\quad \lambda_0\equiv N_c e^{\phi_0}=\frac{-7x+\sqrt{49x^2+400}}{10}\quad,\quad x\equiv\frac{N_f}{N_c}$$

$$\xi=\frac{5}{4}\left[\frac{400+49x^2-7x\sqrt{49x^2+400}}{100+7x^2-x\sqrt{49x^2+400}}\right]\quad,\quad \frac{\ell_s^2}{\ell^2}=e^{\frac{4}{3}\phi_0}\left[\frac{100+7x^2-x\sqrt{49x^2+400}}{400}\right]$$

The associated dimension is $\Delta=2+2\sqrt{1+\xi}$ and satisfies

$$2+3\sqrt{2}<\Delta<2+2\sqrt{6}\qquad \text{or equivalently}\qquad 6.24<\Delta<6.90$$

It corresponds to an irrelevant operator. It is most probably relevant for the Banks-Zaks fixed points.

Bigazzi+Casero+Cotrone+Kiritsis+Paredes

RETURN

Further α' corrections

There are further dilaton terms generated by the 5-form in:

• The kinetic terms of the graviton and the dilaton $\sim \lambda^{2n}$.

• The kinetic terms on probe D_3 branes that affect the identification of the gauge-coupling constant, $\sim \lambda^{2n+1}$. There is also a multiplicative factor relating g_{YM^2} to e^{ϕ}, (not known). Can be traded for b_0.

• Corrections to the identification of the energy. At $r = 0$, $E = 1/r$. There can be log corrections to our identification $E = e^A$, and these are a power series in $\sim \lambda^{2n}$.

• It is a remarkable fact that all such corrections affect the higher that the first two terms in the β-function (or equivalently the potential), that are known to be non-universal!

the metric is also insensitive to the change of b_0 by changing Λ.

Holographic meson dynamics: the models

- Flavor is obtained by adding $N_f << N_C$ D+$\bar{D}$ pairs

- There are several working models of flavor:

♠ Non-supersymmetric backgrounds with abelian D_7flavor brane.

Babington+Erdmenger+Evans+Guralnic+Kirsch

Kruczenski+Mateos+Myers+Winters

♠ Non-supersymmetric $D4+$ $D_8 + \bar{D}_8$

Sakai+Sugimoto

♠ Hard-wall AdS/QCD plus a scalar, plus $U(N_f)_L \times U(N_f)_R$ vectors

Erlich+Katz+son+Stephanov, DaRold+Pomarol

Classification of confining superpotentials

Classification of confining superpotentials $W(\lambda)$ as $\lambda \to \infty$ in IR:

$$W(\lambda) \sim (\log \lambda)^{\frac{P}{2}} \lambda^Q \quad , \quad \lambda \sim E^{-\frac{9}{4}Q} \left(\log \frac{1}{E}\right)^{\frac{P}{2Q}}, \qquad E \to 0.$$

• $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

$$e^A(r) \sim \begin{cases} (r_0 - r)^{\frac{4}{9Q^2-4}} & Q > \frac{2}{3} \\ \exp\left[-\frac{C}{(r_0-r)^{1/(P-1)}}\right] & Q = \frac{2}{3} \end{cases}$$

• $Q = 2/3$, and $0 \leq P < 1$ leads to confinement and a singularity at $r = \infty$ The scale factor e^A vanishes there as

$$e^A(r) \sim \exp[-Cr^{1/(1-P)}].$$

• $Q = 2/3, P = 1$ leads to confinement but the singularity may be at a finite or infinite value of r depending on subleading asymptotics of the superpotential.

♠ If $Q < 2\sqrt{2}/3$, no *ad hoc* boundary conditions are needed to determine the glueball spectrum $\to$ One-to-one correspondence with the β-function This is unlike standard AdS/QCD and other approaches.

• when $Q > 2\sqrt{2}/3$, the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

Confining β-functions

A 5D background is dual to a confining theory if and only if

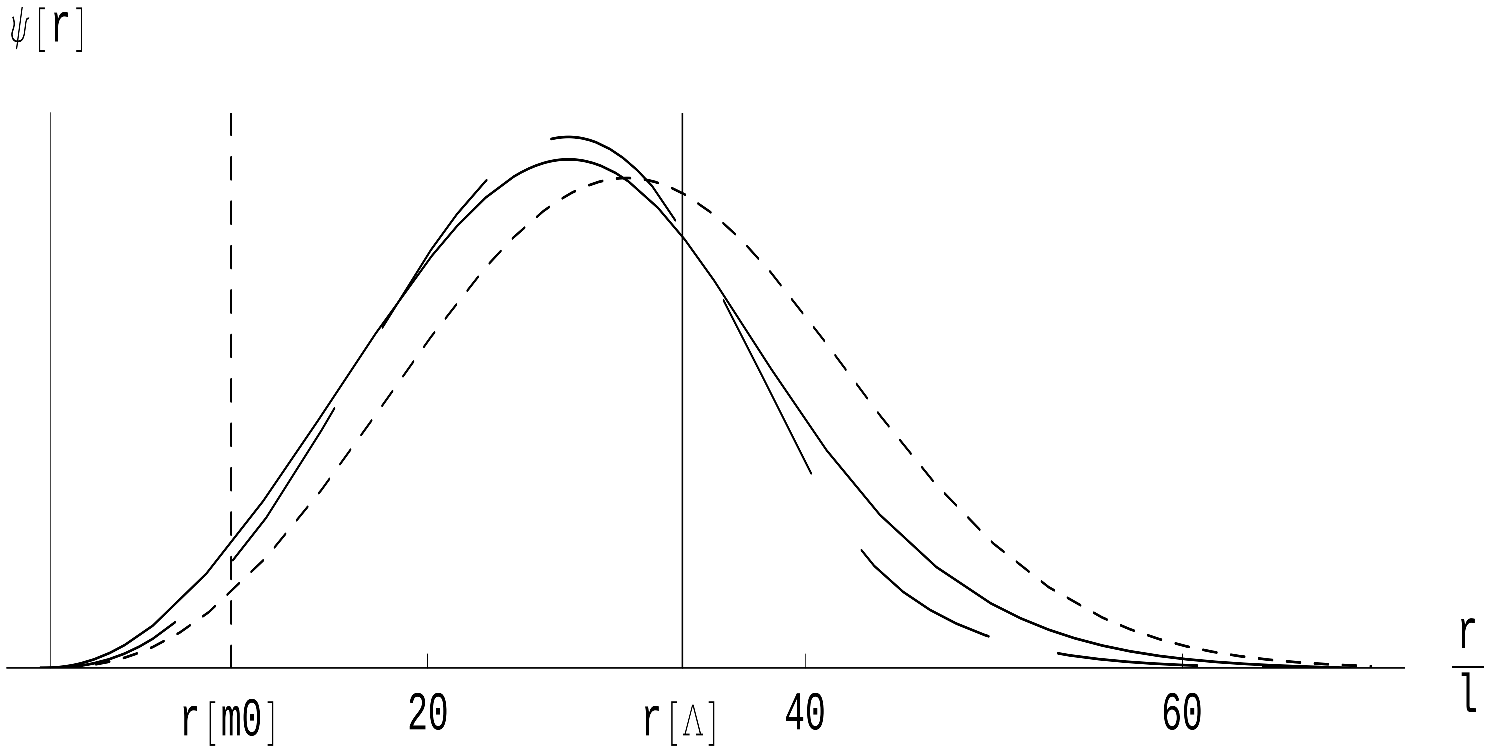
$$\lim_{\lambda\to\infty}\left(\frac{\beta(\lambda)}{3\lambda}+\frac{1}{2}\right)\log\lambda = K, \qquad -\infty \leq K \leq 0$$

(No explicit reference to any coordinate system). Linear trajectories correspond to $K = -\frac{3}{16}$

- We can determine the geometry if we specify K:
- $K = -\infty$: the scale factor goes to zero at some finite r_0, not faster than a power-law.
- $-\infty < K < -3/8$: the scale factor goes to zero at some finite r_0 faster than any power-law.
- $-3/8 < K < 0$: the scale factor goes to zero as $r \to \infty$ faster than $e^{-Cr^{1+\epsilon}}$ for some $\epsilon > 0$.
- $K = 0$: the scale factor goes to zero as $r \to \infty$ as e^{-Cr} (or faster), but slower than $e^{-Cr^{1+\epsilon}}$ for any $\epsilon > 0$.

The borderline case, $K = -3/8$, is certainly confining (by continuity), but whether or not the singularity is at finite r depends on the subleading terms.

The wave-functions of low-lying glueballs



Normalized wave-function profiles for the ground states of the 0^{++} (solid line),0^{-+} (dashed line), and 2^{++} (dotted line) towers, as a function of the radial conformal coordinate. The vertical lines represent the position corresponding to $E = m_{0^{++}}$ and $E = \Lambda_p$.

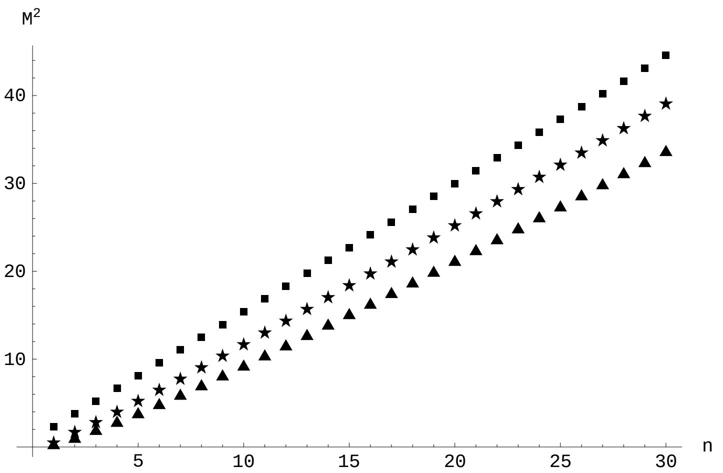
Estimating the importance of logarithmic scaling

We keep the IR asymptotics of background II,but change the UV to power asymptoting AdS_5, with a small λ_*.

$$e^A(r) = \frac{\ell}{r} e^{-(r/R)^2}, \qquad \Phi(r) = \Phi_0 + \frac{3}{2}\frac{r^2}{R^2}\sqrt{1+3\frac{R^2}{r^2}} + \frac{9}{4}\log\frac{2\frac{r}{R} + 2\sqrt{\frac{r^2}{R^2}+\frac{3}{2}}}{\sqrt{6}}.$$

$$W_{conf} = W_0 \left(9 + 4b_0^2(\lambda - \lambda_*)^2)^{1/3}\right) \left(9a + (2b_0^2 + 3b_1)\log\left[1 + (\lambda - \lambda_*^2)\right]\right)^{2a/3}.$$

We fix parameters so that the physical QCD scale is the same (as determined from asymptotic slope of Regge trajectories.



The stars correspond to the asymptotically free background I with $b_0 = 4.2$ and $\lambda_0 = 0.05$; the squares correspond the results obtained in the first background with $R = 11.4\ell$; the triangles denote the spectrum in the second background with $b0 = 4.2$, $li = 0.071$ and $l_* = 0.01$. These values are chosen so that the slopes coincide asymptotically for large n.

Non-supersymmetric backgrounds with abelian flavor branes

- D_7 brane in deformed AdS_5.

- Only abelian axial symmetry $U(1)_A$ realized geometrically as an isometry.

- A quark mass can be introduced, and a quark condensate can be calculated.

- $U(1)_A$ is spontaneously broken du to the embedding.

- Correct GOR relation

- Qualitatively correct η' mass.

- No non-abelian flavor symmetry (due to N=2 Yukawas)

The Sakai-Sugimoto model

- D4 on non-susy S^1 plus $D8$ branes.

- The flavor symmetry is realized on world-volume

- Full $U(N_f)_L \times U(N_f)_R$ symmetry broken to $U(N_f)_V$.

- Chiral symmetry breaking as brane-antibrane recombination.

- Quark constituent mass

- Qualitatively correct η' mass

- No quark mass parameter, nor chiral condensate.

AdS/QCD

- Crude model: AdS_5 with a UV and IR cutoff.
- Addition of $U(N_f)_L \times U(N_f)_R$ vectors and a $(N_f, \bar{N}_f)$ scalar T.
- Chiral symmetry broken by hand via IR boundary conditions.
- Vector meson dominance and GOR relation incorporated.
- Chiral condensate not determined.
- Gluon sector problematic.

The meson sector ($N_f << N_c$)

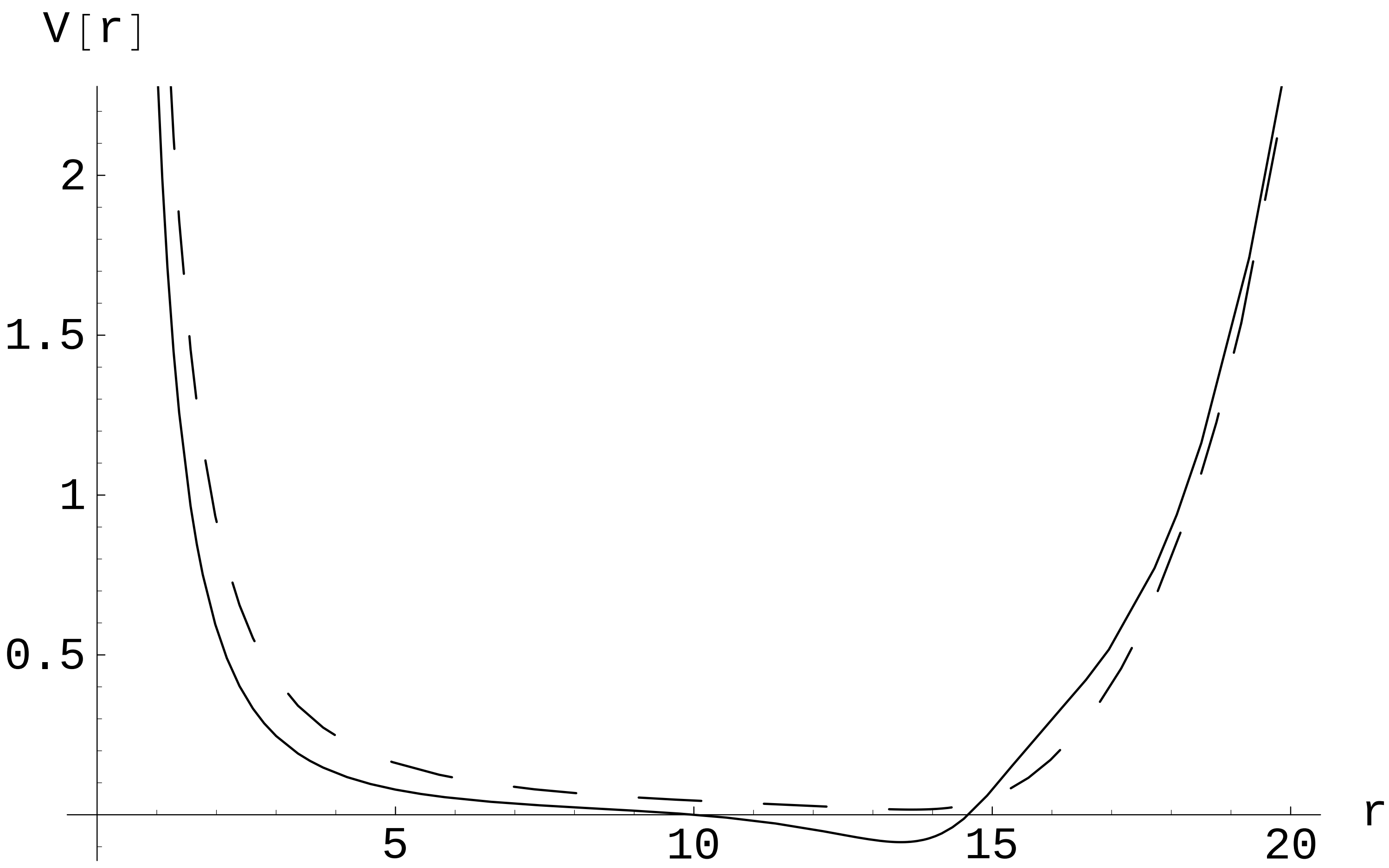
- Flavor is introduced via the introduction of N_f pairs of space filling $D_4+\bar{D}_4$ branes.

- The crucial world volume fields are the tachyon T_{ij} in $(N_f, \bar{N}_f)$ and the $U(N_f)_L \times U(N_f)_R$ vectors.

- The D-WZW sector depends nontrivial on T and realizes properly the P and C symmetries. It generates the appropriate gauge and global flavor anomalies.

- We can introduce explicitly mass matrices for the quarks, and we can dynamically determine the chiral condensate.

Comparison of scalar and tensor potential



Effective Schrödinger potentials for scalar (solid line) and tensor (dashed line) glueballs. The units are chosen such that $\ell = 0.5$.

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• We have naturally the χSB breaking order parameter T, and consistency with anomalies implies that it is non-zero and proportional to the identity (Holographic Coleman+Witten theorem).

• The pions appear as Goldstone bosons when $m_q = 0$.

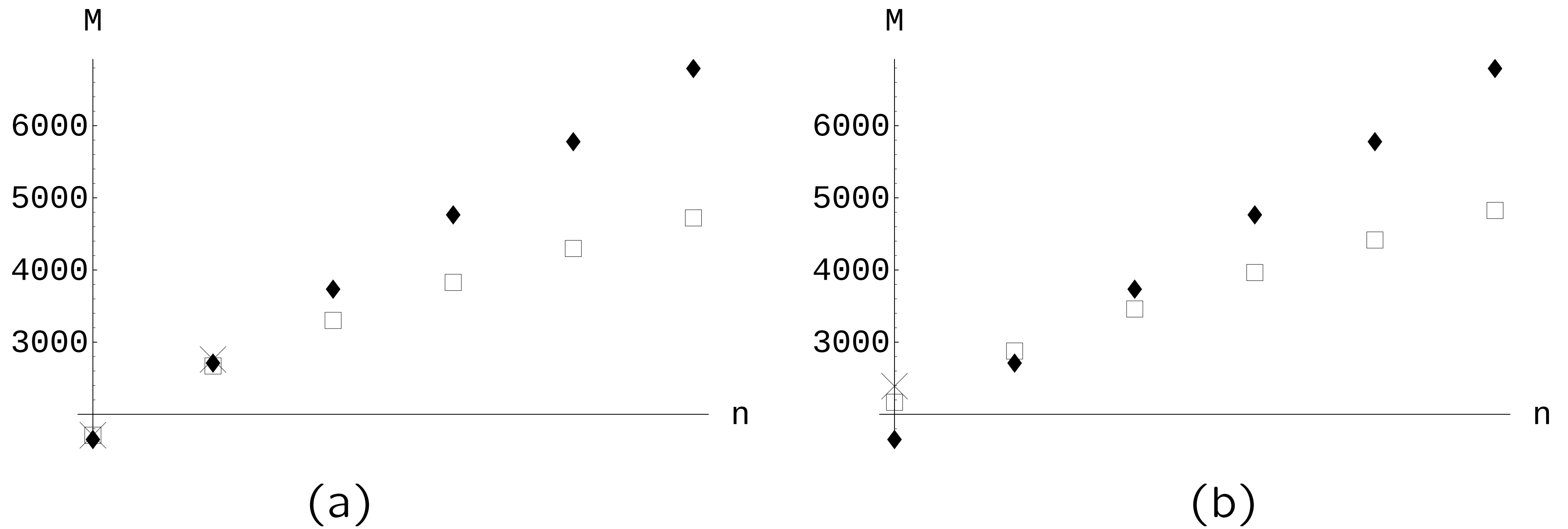
• The correct GOR relation is obtained.

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\,\langle\bar{q}q\rangle \quad , \quad m_q \to 0$$

• There is linear confinement $(M_n^2 \sim n)$ associated with the vanishing of the tachyon potential at $T \to \infty$.

• We obtain the correct Stuckelberg coupling mixing with 0^{+-} and and mass for the η'.

Comparison with lattice data: Ref II



Comparison of glueball spectra from our model with $b_0 = 2.55, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. II (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. II.

Confining background II: r_0 =finite

• We choose a regular β-function with appropriate asymptotics:

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3+2b_0\lambda} - \frac{3\eta(2b_0^2+3b_1^2)\lambda^3}{9\eta+2\left(2b_0^2+3b_1^2\right)\lambda^2}, \qquad \eta \equiv \sqrt{1+\delta^{-1}} - 1$$

• Confining backgrounds with r_0 =finite have a hard time to match the lattice results, even for the first few glueballs.

Tachyon dynamics

• In the vacuum the gauge fields vanish and $T \sim 1$. Only DBI survives

$$S[\tau] = T_{D_4} \int dr d^4x \, \frac{e^{4A_s(r)}}{\lambda} \, V(\tau) \, \sqrt{e^{2A_s(r)} + \dot{\tau}(r)^2} \quad , \quad V(\tau) = e^{-\frac{\mu^2}{2}\tau^2}$$

• We obtain the nonlinear field equation:

$$\ddot{\tau} + \left(3\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right)\dot{\tau} + e^{2A_S}\mu^2\tau + e^{-2A_S}\left[4\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right]\dot{\tau}^3 + \mu^2\tau \; \dot{\tau}^2 = 0.$$

• In the UV we expect

$$\tau = m_q \; r + \sigma \; r^3 + \cdots \quad , \quad \mu^2\ell^2 = 3$$

• We expect that the tachyon must diverge before or at $r = r_0$. We find that indeed it does `at the singularity`. For the $r_0 = \infty$ backgrounds

$$\tau \sim \exp\left[\frac{2}{a}\,\frac{R}{\ell^2}\;r\right] \qquad \text{as} \qquad r \to \infty$$

• Generically the solutions have spurious singularities: $\tau(r_*)$ stays finite but its derivatives diverges as:

$$\tau \sim \tau_* + \gamma\sqrt{r_* - r}.$$

The condition that they are absent determines σ as a function of m_q.

• The easiest spectrum to analyze is that of vector mesons. We find $(r_0 = \infty)$

$$\Lambda_{glueballs} = \frac{1}{R}, \qquad \Lambda_{mesons} = \frac{3}{\ell}\left(\frac{\alpha\ell^2}{2R^2}\right)^{(\alpha-1)/2} \propto \frac{1}{R}\left(\frac{\ell}{R}\right)^{\alpha-2}.$$

This suggests that $\alpha = 2.$ preferred also from the glue sector.

Detailed plan of the presentation

- QCD at finite temperature 60 minutes
- Temperature versus horizon position 61 minutes
- The free energy as a function of r_h 62 minutes
- The transition in the free energy 63 minutes
- The thermodynamic quantities 64 minutes
- Equation of state 65 minutes
- The speed of sound (bulk viscosity) 66 minutes
- The specific heat 67 minutes