

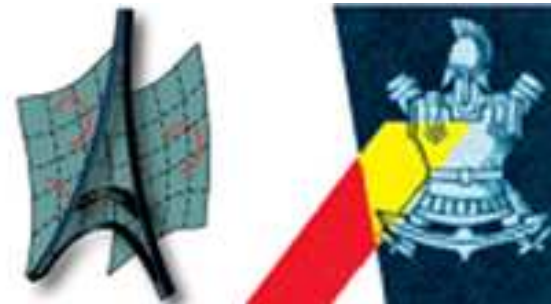
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# *Introduction to (Improved) Holographic QCD*

Elias Kiritsis



University of Crete



and

Ecole Polytechnique

# Bibliography

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- Ongoing work with:

U. Gursoy, L. Mazzanti, G. Michalogiorgakis, F. Nitti

Based on previous work

- U. Gursoy and E. Kiritsis, *“Exploring improved holographic theories for QCD: Part I,”* JHEP **0802** (2008) 032 [ArXiv:0707.1324][hep-th].

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# Introduction

- AdS/CFT has provided so far controlled/computable examples of confinement, chiral symmetry breaking and hadron spectra of concrete gauge theories
- Its direct application to QCD is marred by two (related) problems:
  - ♠ The KK problem of critical examples
  - ♠ The strong curvature problem of non-critical (or hierarchically separated critical examples)
- We will follow the non-critical road, try to understand what we expect from the string theory dual to QCD, and motivate a phenomenological approximation (model).
- We will then use the model to compute experimentally interesting non-perturbative quantities like transport coefficients.

# What are we after?

- Interactions of hadrons at medium or low energy (little or no help from lattice, partial help from chiral perturbation theory)
- Transport coefficients of the deconfined phase (not computable directly from lattice, crucial for understanding current (RHIC) and future (LHC) heavy-ion data)
- The phase structure and properties of dense matter (not computable from lattice, important for understanding properties of nuclei, and dense nuclear matter, like neutron stars)
- Exploring the strong dynamics of other QCD-like theories, eg.
  - ♠ N=1 super- QCD. (a very interesting toy model and may be relevant for nature)
  - ♠ Technicolor theories

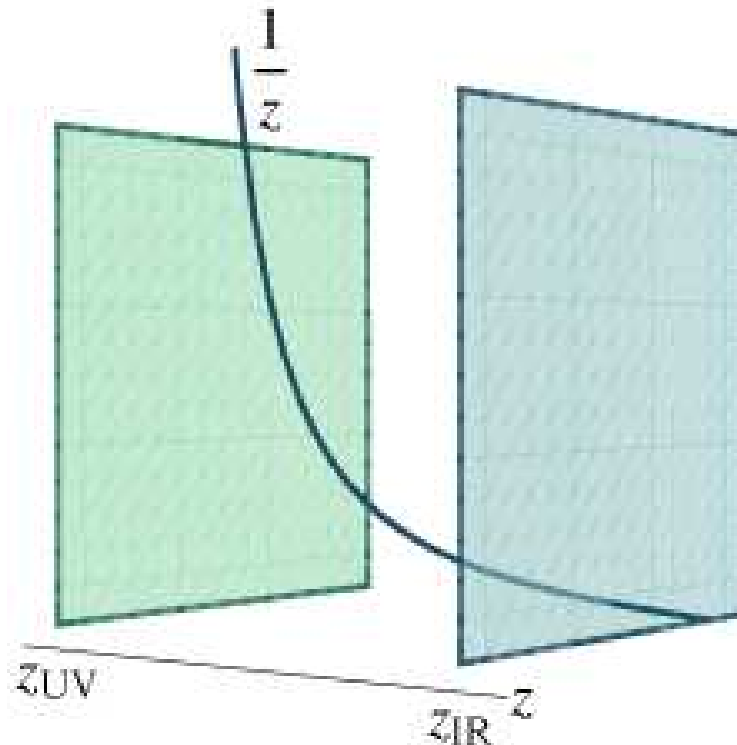
# AdS/QCD

♠ A basic phenomenological approach: use a slice of  $\text{AdS}_5$ , with a UV cutoff, and an IR cutoff. *Polchinski+Strassler*

♠ It successfully exhibits confinement (trivially via IR cutoff), and power-like behavior in hard scattering amplitudes

♠ It may be equipped with a bifundamental scalar,  $T$ , and  $U(N_f)_L \times U(N_f)_R$ , gauge fields to describe mesons. *Erlich+Katz+Son+Stepanov, DaRold+Pomarol*

Chiral symmetry is broken by hand, via IR boundary conditions. The low-lying meson spectrum looks "reasonable".



## ♠ Shortcomings:

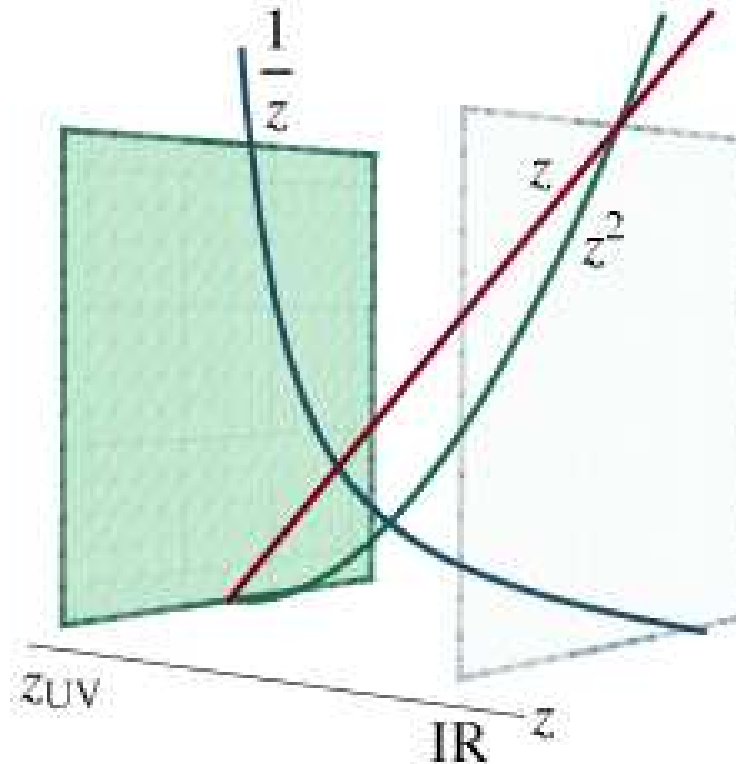
- The glueball spectrum does not fit very well the lattice calculations. It has the wrong asymptotic behavior  $m_n^2 \sim n^2$  at large  $n$ .
- Magnetic quarks are confined instead of screened.
- Chiral symmetry breaking is input by hand.
- The meson spectrum has also the wrong UV asymptotics  $m_n^2 \sim n^2$ .
- at finite temperature there is a deconfining transition but the equation of state is trivial (conformal) (e-2p) and the speed of sound is  $c_s^2 = \frac{1}{3}$ .

# The “soft wall”

♠ The asymptotic spectrum can be fixed by introducing a **non-dynamical** dilaton profile  $\Phi \sim r^2$  (soft wall)

*Karch+Katz+Son+Stephanov*

● It is not a solution of equations of motion: the metric is still AdS: Neither  $g_{\mu\nu}$  nor  $\Phi$  solves the equations of motion.



# A string theory for QCD: basic expectations

- Pure  $SU(N_c)$   $d=4$  YM at large  $N_c$  is expected to be dual to a string theory in 5 dimensions only. Essentially a single adjoint field  $\rightarrow$  a single extra dimension.
- The theory becomes asymptotically free and conformal at high energy  $\rightarrow$  we expect the classical saddle point solution to asymptote to  $AdS_5$ .
- ♠ Operators with lowest dimension (or better: lowest bulk masses) are expected to be the only important non-trivial bulk fields in the large- $N_c$  saddle-point
- Scalar YM operators with  $\Delta_{UV} > 4 \rightarrow m^2 > 0$  fields near the  $AdS_5$  boundary  $\rightarrow$  vanish fast in the UV regime and do not affect correlators of low-dimension operators.



• Their dimension may grow large in the IR so they are also irrelevant there. The large 't Hooft coupling is expected to suppress the effects of such operators.

• This is suggested by the success of low-energy SVZ sum rules as compared to data.

♠ Therefore we will consider

$$T_{\mu\nu} \leftrightarrow g_{\mu\nu}, \quad \text{tr}[F^2] \leftrightarrow \phi, \quad \text{tr}[F \wedge F] \leftrightarrow a$$

# Bosonic string or superstring?

- Consider the axion  $a$  dual to  $Tr[F \wedge F]$ . We can show that it must come from a RR sector.

In large- $N_c$  YM, the proper scaling of couplings is obtained from

$$\mathcal{L}_{YM} = N_c Tr \left[ \frac{1}{\lambda} F^2 + \frac{\theta}{N_c} F \wedge F \right] , \quad \zeta \equiv \frac{\theta}{N_c} \sim \mathcal{O}(1)$$

It can be shown

$$E_{YM}(\theta) \simeq C_0 N_c^2 + C_1 \theta^2 + C_2 \frac{\theta^4}{N_c^2} + \dots \quad \text{Witten}$$

In the string theory action

$$S \sim \int e^{-2\phi} [R + \dots] + (\partial a)^2 + e^{2\phi} (\partial a)^4 + \dots , \quad e^\phi \sim g_{YM}^2 , \quad \lambda \sim N_c e^\phi$$
$$\sim \int \frac{N_c^2}{\lambda^2} [R + \dots] + (\partial a)^2 + \frac{\lambda^2}{N_c^2} (\partial a)^4 + \dots , \quad a = \theta [1 + \dots]$$

## bosonic string or superstring? (continued)

- The string theory must have **no on-shell fermionic states at all** because there are no gauge invariant fermionic operators in pure YM. (even with quarks modulo baryons).
- Therefore the string theory must be a 5d-superstring theory resembling the II-0 class.
- ♠ Another RR field we expect to have is the RR 4-form, as it is necessary to “seed” the  $D_3$  branes responsible for the gauge group.
- It is non-propagating in 5D
- We will see later however that it is responsible for the non-trivial IR structure of the gauge theory vacuum.

# The minimal effective string theory spectrum

- NS-NS  $\rightarrow g_{\mu\nu} \leftrightarrow T_{\mu\nu}$  ,  $B_{\mu\nu} \leftrightarrow Tr[F]^3$  ,  $\phi \leftrightarrow Tr[F^2]$
- RR  $\rightarrow Spinor_5 \times Spinor_5 = F_0 + F_1 + F_2 + (F_3 + F_4 + F_5)$
- ♠  $F_0 \leftrightarrow F_5 \rightarrow C_4$ , background flux  $\rightarrow$  no propagating degrees of freedom.
- ♠  $F_1 \leftrightarrow F_4 \rightarrow C_3 \leftrightarrow C_0$ :  $C_0$  is the axion,  $C_3$  its 5d dual that couples to domain walls separating **oblique confinement vacua**.
- ♠  $F_2 \leftrightarrow F_3 \rightarrow C_1 \leftrightarrow C_2$ : They are associated with baryon number (as we will see later when we add flavor). Dual operators are a mystery (**topological currents?**).
- In an ISO(3,1) invariant vacuum solution, only  $g_{\mu\nu}, \phi, C_0 = a$  can be non-trivial.

$$ds^2 = e^{2A(r)}(dr^2 + dx_4^2) \quad , \quad a(r), \phi(r)$$

## The relevant “defects”

- $B_{\mu\nu} \rightarrow$  Fundamental string ( $F_1$ ). This is the QCD (glue) string: fundamental tension  $\ell_s^2 \sim \mathcal{O}(1)$
- Its dual  $\tilde{B}_\mu \rightarrow NS_0$ : Tension is  $\mathcal{O}(N_c^2)$ . It is an effective magnetic baryon vertex binding  $N_c$  magnetic quarks.
- $C_5 \rightarrow D_4$ : Space filling flavor branes. They must be introduced in pairs:  $D_4 + \bar{D}_4$  for charge neutrality/tadpole cancelation  $\rightarrow$  gauge anomaly cancelation in QCD.
- $C_4 \rightarrow D_3$  branes generating the gauge symmetry.

- $C_3 \rightarrow D_2$  branes : domain walls separating different oblique confinement vacua (where  $\theta_{k+1} = \theta_k + 2\pi$ ). Its tension is  $\mathcal{O}(N_c)$
- $C_2 \rightarrow D_1$  branes: These are the magnetic strings: (strings attached to magnetic quarks) with tension  $\mathcal{O}(N_c)$
- $C_1 \rightarrow D_0$  branes. These are the baryon vertices: they bind  $N_c$  quarks, and their tension is  $\mathcal{O}(N_c)$ .  
Its instantonic source is the (solitonic) baryon in the string theory.
- $C_0 \rightarrow D_{-1}$  branes: These are the Yang-Mills instantons.

# The effective action, I

- as  $N_c \rightarrow \infty$ , only string tree-level is dominant.
- Relevant field for the vacuum solution:  $g_{\mu\nu}, a, \phi, F_5$ .
- The vev of  $F_5 \sim N_c \epsilon_5$ . It appears always in the combination  $e^{2\phi} F_5^2 \sim \lambda^2$ , with  $\lambda \sim N_c e^\phi$ . All higher derivative corrections  $(e^{2\phi} F_5^2)^n$  are  $\mathcal{O}(1)$ .  
A non-trivial potential for the dilaton will be generated already at string tree-level.
- This is not the case for all other RR fields: in particular for the axion as  $a \sim \mathcal{O}(1)$

$$(\partial a)^2 \sim \mathcal{O}(1) \quad , \quad e^{2\phi} (\partial a)^4 = \frac{\lambda^2}{N_c^2} (\partial a)^4 \sim \mathcal{O}(N_c^{-2})$$

Therefore to leading order  $\mathcal{O}(N_c^2)$  we can neglect the axion.

# The UV regime

- In the far UV, the space should asymptote to  $\text{AdS}_5$ .
- The 't Hooft coupling should behave as ( $r \rightarrow 0$ )

$$\lambda \sim \frac{1}{\log(r\Lambda)} + \dots \rightarrow 0, \quad r \sim \frac{1}{E}$$

The effective action to leading order in  $N_c$  is

$$S_{eff} \sim \int d^5x \sqrt{g} e^{-2\phi} Z( \ell_s^2 R, \ell_s^2 (\partial\phi)^2, e^{2\phi} \ell_s^2 F_5^2 )$$

Solving the equation of motion of  $F_5$  amounts to replacing

$$e^{2\phi} \ell_s^2 F_5^2 \sim e^{2\phi} N_c^2 \equiv \lambda^2$$

$$S_{eff} \sim N_c^2 \int d^5x \sqrt{g} \frac{1}{\lambda^2} H( \ell_s^2 R, \ell_s^2 (\partial\lambda)^2, \lambda^2 )$$



- As  $r \rightarrow 0$

$$\text{Curvature} \rightarrow \text{finite} \quad , \quad \square\phi \sim (\partial\phi)^2 \sim \frac{(\partial\lambda)^2}{\lambda^2} \sim \lambda^2 \sim \frac{1}{\log^2(r\Lambda)} \rightarrow 0$$

- For  $\lambda \rightarrow 0$  the potential in the Einstein frame starts as  $V(\lambda) \sim \lambda^{\frac{4}{3}}$  and cannot support the asymptotic  $AdS_5$  solution.

- Therefore asymptotic  $AdS_5$  must arise from curvature corrections:

$$S_{eff} \simeq \int d^5x \frac{1}{\lambda^2} H(\ell_s^2 R, 0, 0)$$

- Setting  $\lambda = 0$  at leading order we can generically get an  $AdS_5$  solution coming from balancing the higher curvature corrections.

$$H(x, 0, 0) \equiv f(x) \quad , \quad x_* f'(x_*) + \frac{5}{2} f(x_*) = 0 \quad , \quad x_* = \ell_s^2 R = 12 \frac{\ell_s^2}{\ell^2}$$

**INTERESTING QUESTION:** Is there a good toy example of string vacuum (CFT) which is not Ricci flat, and is supported only by a metric?

- There is a "good" (but hard to derive the coefficients) perturbative expansion around this asymptotic  $AdS_5$  solution by perturbing inwards :

$$e^A = \frac{\ell}{r} [1 + \delta A(r)] \quad , \quad \lambda = \frac{1}{b_0 \log(r\Lambda)} + \dots$$

- This turns out to be a regular expansion of the solution in powers of

$$\frac{P_n(\log \log(r\Lambda))}{(\log(r\Lambda))^{-n}}$$

- Effectively this can be rearranged as a "perturbative" expansion in  $\lambda(r)$ . In the case of running coupling, the radial coordinate can be substituted by  $\lambda(r)$ .

- Using  $\lambda$  as a radial coordinate the solution for the metric can be written

$$E \equiv e^A = \frac{\ell}{r(\lambda)} [1 + c_1 \lambda + c_2 \lambda^2 + \dots] = \ell (e^{-\frac{b_0}{\lambda}}) [1 + c'_1 \lambda + c'_2 \lambda^2 + \dots] \quad , \quad \lambda \rightarrow 0$$

Conclusion: The asymptotic  $AdS_5$  is stringy, but the rest of the geometry is "perturbative around the asymptotics". We cannot however do computations even if we seen the structure.

QUESTION: Can one constrain  $H(R, 0, 0)$  by asking the stress tensor correlators asymptote to free correlators near the boundary?

# The IR regime

- Here the situation is more obscure. The constraints/input will be: confinement and mass gap.
- We do expect that  $\lambda \rightarrow \infty$  (or becomes large) at the IR bottom.
- Intuition from N=4 and other 10d strongly coupled theories suggests that in this regime there should be an (approximate) two-derivative description of the physics.
- From the string  $\sigma$ -model we can write

*Tseytlin*

$$S_{eff} \sim N_c^2 \int d^5x \sqrt{g} \frac{1}{\lambda^2} \left[ \frac{(\partial\lambda)^2}{\lambda^2} + H(\ell_s^2 R, \lambda^2) \right]$$

- For the theory to reduce to two derivatives,  $R \rightarrow 0$  in the IR (string frame). Then

$$H(\ell_s^2 R, \lambda^2) \sim \ell_s^2 R + V(\lambda) + \mathcal{O}(R^2)$$

- The simplest solution linear dilaton with this property is the linear dilaton solution with

$$\lambda \sim e^{Qr} \quad , \quad V(\lambda) \sim \delta c = 10 - D \quad \rightarrow \quad \text{constant} \quad , \quad R = 0$$

- As we shall see this property persists with potentials  $V(\lambda) \sim (\log \lambda)^P$ . Moreover all such cases have confinement, a mass gap and a discrete spectrum (except the  $P=0$  case).
- At the IR bottom (in the string frame) the scale factor vanishes, and 5D space becomes (asymptotically) flat.

# Improved Holographic QCD: a model

The simplification in this model relies on writing down a two-derivative action

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5 x \sqrt{g} \left[ R - \frac{4(\partial\lambda)^2}{3\lambda^2} + V(\lambda) \right]$$

with

$$\lim_{\lambda \rightarrow 0} V(\lambda) = \frac{12}{\ell^2} \left( 1 + \sum_{n=1}^{\infty} c_n \lambda^n \right), \quad \lim_{\lambda \rightarrow \infty} V(\lambda) = \lambda^{\frac{4}{3}} \sqrt{\log \lambda} + \text{subleading}$$

- The small  $\lambda$  asymptotics “simulate” the UV expansion around  $AdS_5$ .
- There is a 1-1 correspondence between the YM  $\beta$ -function,  $\beta(\lambda)$  and  $W$ :

$$\left(\frac{3}{4}\right)^3 V(\lambda) = W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \log \lambda}\right)^2, \quad \beta(\lambda) = -\frac{9}{4} \lambda^2 \frac{d \log W(\lambda)}{d \lambda}$$

once a choice of energy is made (here  $\log E = A_E$ ).

Not everything is perfect: There are some shortcomings localized at the UV

- The conformal anomaly is incorrect.
- Shear viscosity ratio is constant and equal to that of N=4 sYM.  
(This is not expected to be a serious error in the experimentally interesting  $T_c \leq T \leq 4T_c$  range.)

Both of the above need Riemann curvature corrections.

- We shall see that other observables can come out very well both at  $T=0$  and finite  $T$

# An assessment of IR asymptotics

- We define the superpotential  $W$  as

$$V(\lambda) = \frac{4}{3}\lambda^2 \left(\frac{dW}{d\lambda}\right)^2 + \frac{64}{27}W^2$$

- We parameterize the UV ( $\lambda \rightarrow 0$ ) and IR asymptotics ( $\lambda \rightarrow \infty$ ) as

$$V(\lambda) = \frac{12}{\ell^2} [1 + \mathcal{O}(\lambda)] \quad , \quad V(\lambda) \sim V_\infty \lambda^Q (\log \lambda)^P$$

There are three types of solution for  $W$ :

- The "Good type" (single solution)

$$W(\lambda) \sim (\log \lambda)^{\frac{P}{2}} \lambda^{\frac{Q}{2}}$$

It leads to a "good" IR singularity, confinement, a mass gap, discrete spectrum of glueballs and screening of magnetic charges if

$$\frac{8}{3} > Q > \frac{4}{3} \quad \text{or} \quad Q = \frac{4}{3} \quad \text{and} \quad P > 0$$

- The asymptotic spectrum of glueballs is linear if  $Q = \frac{2}{3}$  and  $P = \frac{1}{2}$ .

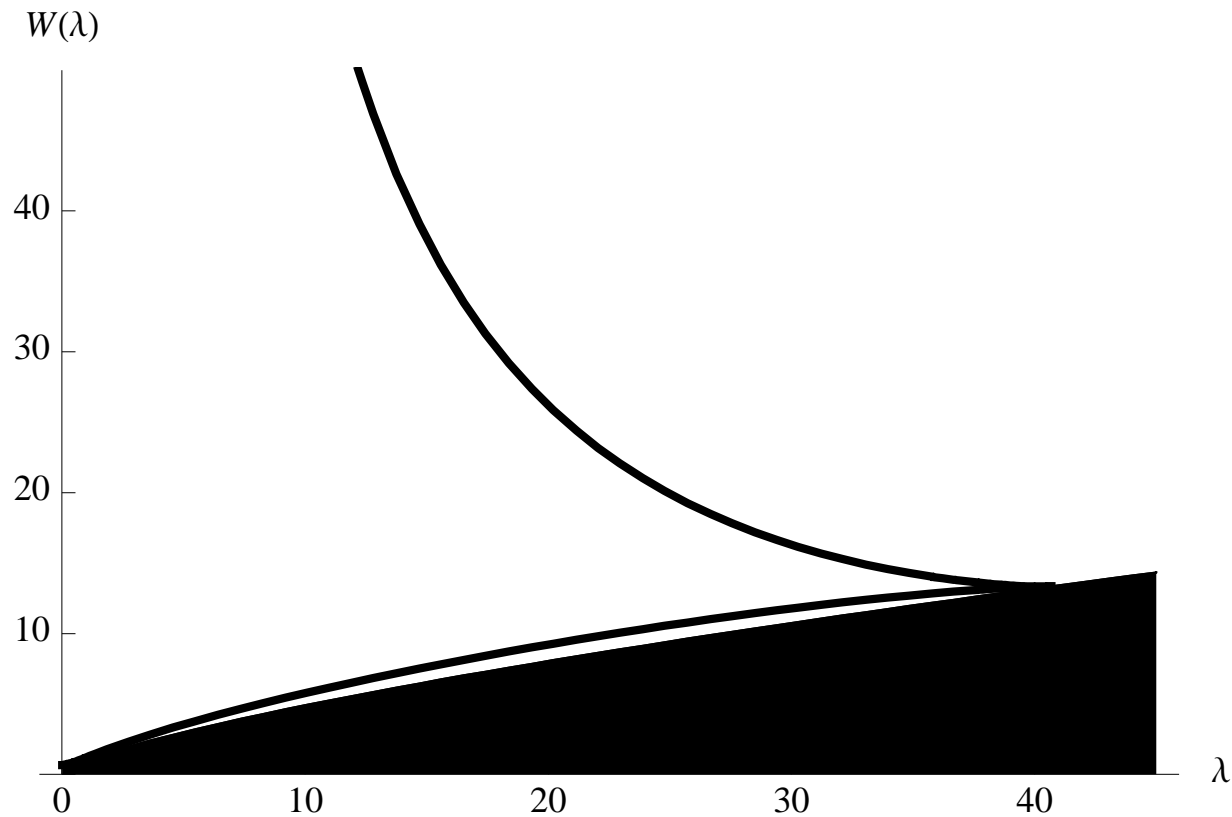


- The **Bad type**. This is a one parameter family of solutions with

$$W(\lambda) \sim \lambda^{\frac{4}{3}}$$

It has a **bad IR singularity**.

- ♠ The **Ugly type**. This is a one parameter family of solutions. In such solutions there are two branches but they never reach the IR  $\lambda \rightarrow \infty$ . Instead  $\lambda$  goes back to zero



# Selecting the IR asymptotics

Only the  $Q = 2/3$ ,  $0 \leq P < 1$  is compatible with

- Confinement (it happens non-trivially: a minimum in the string frame scale factor )
- Mass gap+discrete spectrum (except  $P=0$ )
- good singularity
- $R \rightarrow 0$  justifying the original assumption. More precisely: the string frame metric becomes flat at the IR .

♠ It is interesting that the lower endpoint:  $P=0$  corresponds to linear dilaton and flat space (string frame). It is confining with a mass gap but continuous spectrum.

- For linear asymptotic trajectories for fluctuations (glueballs) we must choose  $P = 1/2$

$$V(\lambda) = \lambda^{\frac{4}{3}} [1 + c_1 \lambda^2 + c_2 \lambda^4 + \dots] \sim \lambda^{\frac{4}{3}} \sqrt{\log \lambda} + \text{subleading} \quad \text{as} \quad \lambda \rightarrow \infty$$

# Parameters

- We have 3 initial conditions in the system of graviton-dilaton equations:

- ♠ One is fixed by picking the branch that corresponds asymptotically to

$$\lambda \sim \frac{1}{\log(r\Lambda)}$$

- ♠ The other fixes  $\Lambda \rightarrow \Lambda_{QCD}$ .

- ♠ The third is a gauge artifact as it corresponds to a choice of the origin of the radial coordinate.

- We also have the Planck scale  $M_p$ , and the AdS length,  $\ell$ .

Asking for correct  $T \rightarrow \infty$  thermodynamics (free gas) fixes

$$(M_p \ell)^3 = \frac{1}{45\pi^2} \quad , \quad M_{\text{physical}} = M_p N_c^{\frac{2}{3}} = \left( \frac{8}{45\pi^2 \ell^3} \right)^{\frac{1}{3}} \simeq 4.6 \text{ GeV}$$

$\ell$  is not a parameter but a unit of length.

- All dimensionless coefficients of the potential are a priori parameters. However, a simple form is typically chosen for simplicity.
- At  $T = 0$  we fit only one parameter: the normalization of the 't Hooft coupling.
- At  $T > 0$  we fit one more parameter: the coefficient of the leading strong coupling term.
- We choose a dilaton potential with large- $\lambda$  asymptotics  $V(\lambda) \sim \sqrt{\log \lambda}$  to obtain linear asymptotic glueball spectrum.

## Adding flavor

- To add  $N_f$  quarks  $q_L^I$  and antiquarks  $q_R^{\bar{I}}$  we must add (in 5d) space-filling  $N_f$   $D_4$  and  $N_f$   $\bar{D}_4$  branes.  
(tadpole cancellation=gauge anomaly cancellation)

- The  $q_L^I$  are the “zero modes” of the  $D_3 - D_4$  strings while  $q_R^{\bar{I}}$  are the “zero modes” of the  $D_3 - \bar{D}_4$

- The low-lying fields on the  $D_4$  branes ( $D_4 - D_4$  strings) are  $U(N_f)_L$  gauge fields  $A_\mu^L$ . The low-lying fields on the  $\bar{D}_4$  branes ( $\bar{D}_4 - \bar{D}_4$  strings) are  $U(N_f)_R$  gauge fields  $A_\mu^R$ . They are dual to the  $J_L^\mu$  and  $J_R^\mu$

$$\delta S_A \sim \bar{q}_L^I \gamma^\mu (A_\mu^L)^{IJ} q_L^J + \bar{q}_R^{\bar{I}} \gamma^\mu (A_\mu^R)^{\bar{I}\bar{J}} q_R^{\bar{J}} = \text{Tr}[J_L^\mu A_\mu^L + J_R^\mu A_\mu^R]$$

- There are also the low lying fields of the ( $D_4 - \bar{D}_4$  strings), essentially the string-theory “tachyon”  $T_{I\bar{J}}$  transforming as  $(N_f, \bar{N}_f)$  under the chiral symmetry  $U(N_f)_L \times U(N_f)_R$ . It is dual to the mass terms

$$\delta S_T \sim \bar{q}_L^I T_{I\bar{J}} q_R^{\bar{J}} + \text{complex conjugate}$$

- The interactions on the flavor branes are weak, so that  $A_\mu^{L,R}, T$  are as sources for the quarks.

- Integrating out the quarks, generates an effective action  $S_{flavor}(A_\mu^{L,R}, T)$ , so that  $A_\mu^{L,R}, T$  can be thought as effective  $q\bar{q}$  composites, that is : mesons

- On the string theory side: integrating out  $D_3 - D_4$  and  $D_3 - \bar{D}_4$  strings gives rise to the DBI action for the  $D_4 - \bar{D}_4$  branes in the  $D_3$  background:

$$S_{flavor}(A_\mu^{L,R}, T) \longleftrightarrow S_{DBI}(A_\mu^{L,R}, T) \quad \text{holographically}$$

- In the "vacuum" only  $T$  can have a non-trivial profile:  $T^{I\bar{J}}(r)$ . Near the  $AdS_5$  boundary ( $r \rightarrow 0$ )

$$T^{I\bar{J}}(r) = M_{I\bar{J}} r + \dots + \langle \bar{q}_L^I q_R^{\bar{J}} \rangle r^3 + \dots$$

- A typical solution is  $T$  vanishing in the UV and  $T \rightarrow \infty$  in the IR. At the point  $r = r_*$  where  $T = \infty$ , the  $D_4$  and  $\bar{D}_4$  branes “fuse”. The true vacuum is a brane that enters folds on itself and goes back to the boundary. A non-zero  $T$  breaks chiral symmetry.

- A GOR relation is satisfied (for an asymptotic AdS<sub>5</sub> space)

$$m_\pi^2 = -2 \frac{m_q}{f_\pi^2} \langle \bar{q}q \rangle \quad , \quad m_q \rightarrow 0$$

- We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

- When  $m_q = 0$ , the meson spectrum contains  $N_f^2$  massless pseudoscalars, the  $U(N_f)_A$  Goldstone bosons.

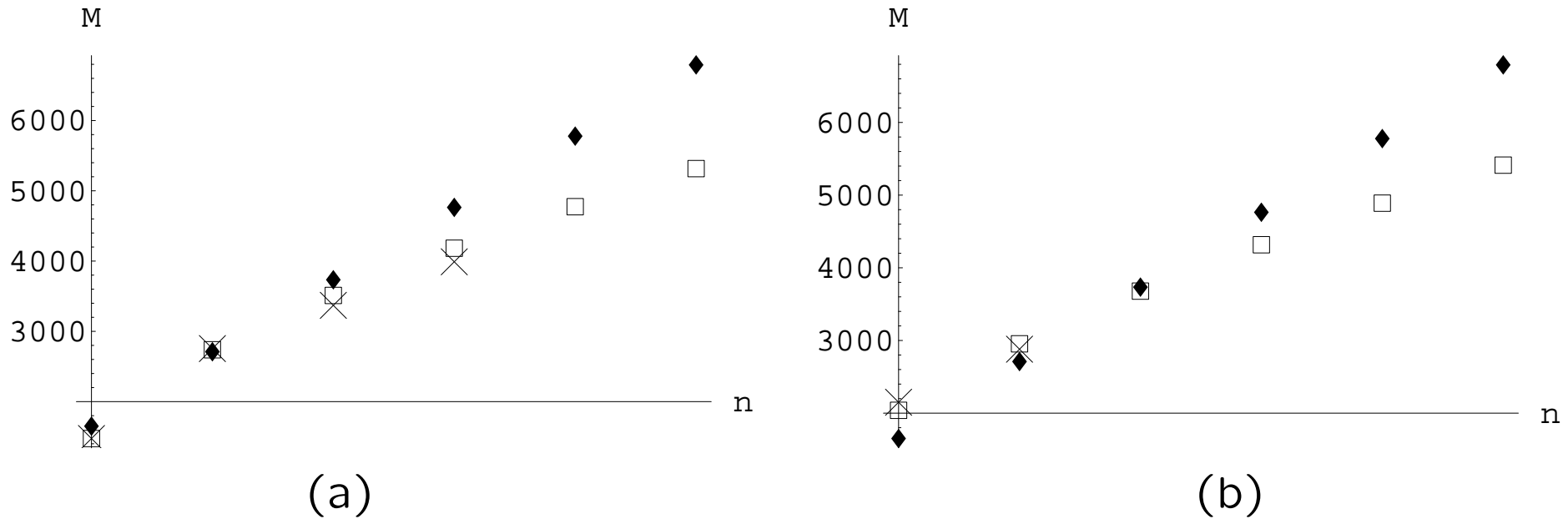
- The WZ part of the flavor brane action gives the Adler-Bell-Jackiw  $U(1)_A$  axial anomaly and an associated Stueckelberg mechanism gives an  $O\left(\frac{N_f}{N_c}\right)$  mass to the would-be Goldstone boson  $\eta'$ , in accordance with the Veneziano-Witten formula.

- Fluctuations around the  $T$  solution for  $T, A_\mu^{L,R}$  give the spectra (and interactions) of various meson trajectories.

- Studying the spectrum of highly excited mesons, we find the expected property of linear confinement:  $m_n^2 \sim n$ .

- The detailed spectrum of mesons remains to be worked out

# Comparison with lattice data



Comparison of glueball spectra from our model with  $b_0 = 4.2, \lambda_0 = 0.05$  (boxes), with the lattice QCD data from Meyer (crosses) and the AdS/QCD computation (diamonds), for (a)  $0^{++}$  glueballs; (b)  $2^{++}$  glueballs. The masses are in MeV, and the scale is normalized to match the lowest  $0^{++}$  state.

We measure :  $\frac{l_{eff}}{l} = 2.62$  ,  $\frac{l_s}{l} \simeq 0.16$

and post-dict

$$\alpha_s(1.2 \text{ GeV}) = 0.34,$$

which is within the error of the quoted experimental value  $\alpha_s^{(exp)}(1.2 \text{ GeV}) = 0.35 \pm 0.01$



# Finite temperature

The theory at finite temperature can be described by:

(1) The “thermal vacuum solution”. This is the zero-temperature solution we described so far with time periodically identified with period  $\beta$ .

(2) “black-hole” solutions

$$ds^2 = b(r)^2 \left[ \frac{dr^2}{f(r)} - f(r) dt^2 + dx^i dx^i \right], \quad \lambda = \lambda(r)$$

♠ We need VERY UNUSUAL boundary conditions: The dilaton (scalar) is diverging at the boundary so that  $\lambda \sim e^\phi \rightarrow \frac{1}{\log r} \rightarrow 0$

♠ The boundary AdS is NOT at a minimum of the potential.

• No such type of solutions have been analyzed so far in the literature.

# General phase structure

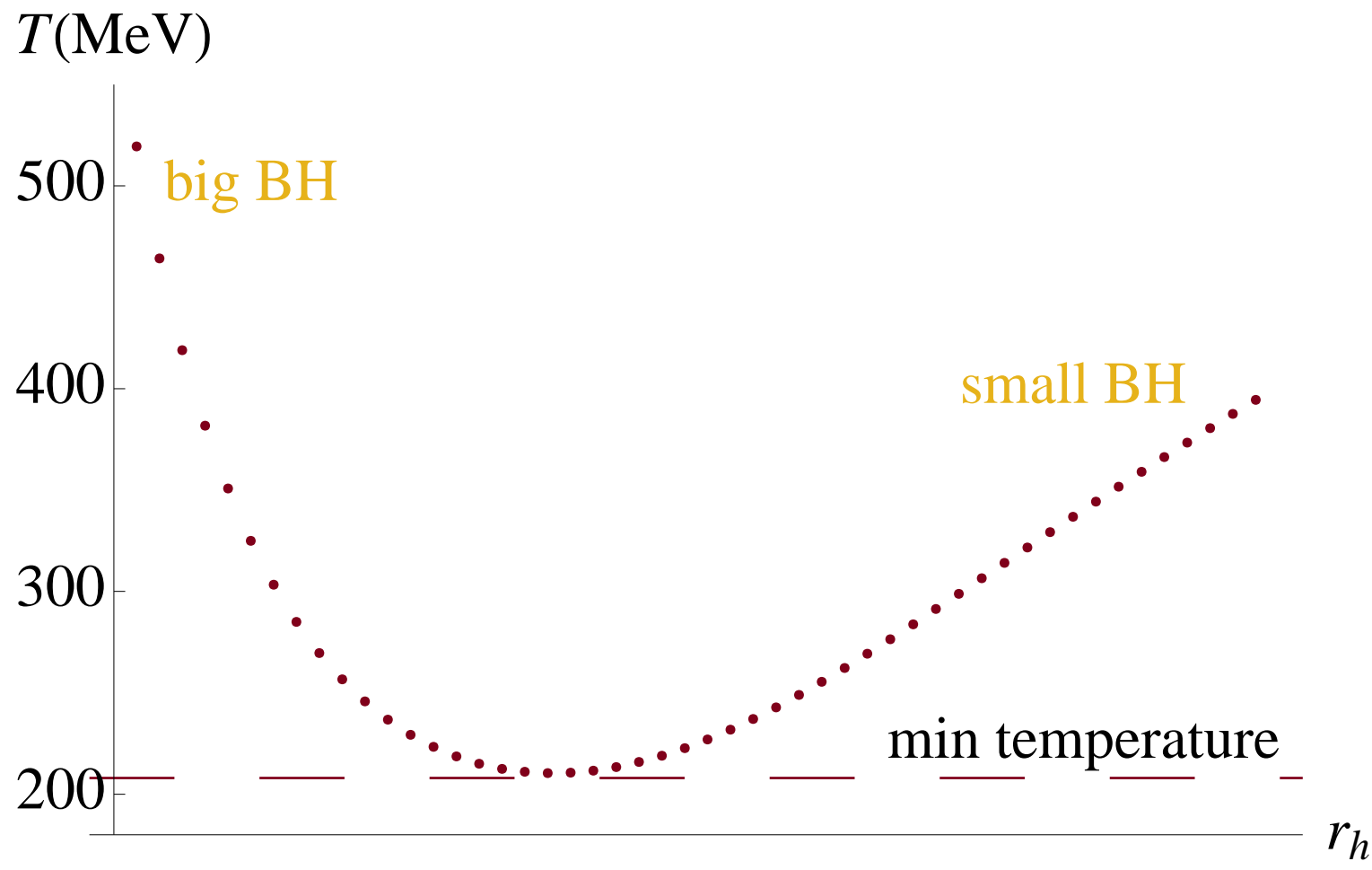
- For a general potential (with no minimum) the following can be shown :
  - There exists a phase transition at finite  $T = T_c$ , if and only if the zero- $T$  theory confines.*
  - This transition is of the first order for all of the confining geometries, with a single exception described in iii:*
  - In the limit confining geometry  $b_0(r) \rightarrow e^{-Cr}$ ,  $\lambda_0 \rightarrow e^{\frac{3}{2}Cr}$ , (as  $r \rightarrow \infty$ ), the phase transition is of the second order and happens at  $T = 3C/4\pi$ .*

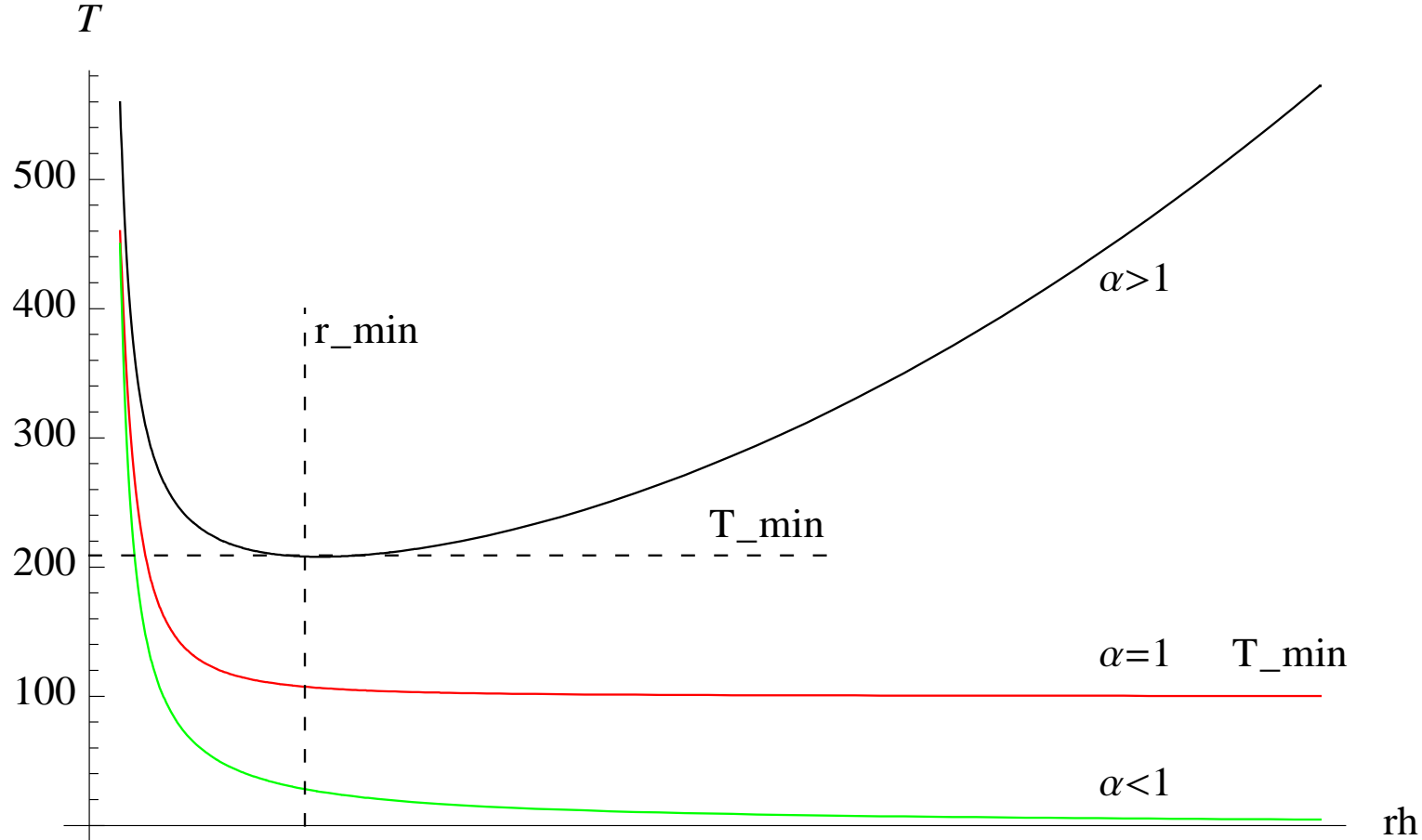
This is the linear dilaton vacuum solution in the IR.
  - All of the non-confining geometries at zero  $T$  are always in the black hole phase at finite  $T$ . They exhibit a second order phase transition at  $T = 0^+$ .*

# Finite-T Confining Theories

- There is a minimal temperature  $T_{min}$  for the existence of Black-hole solutions
- When  $T < T_{min}$  only the “thermal vacuum solution” exists: it describes the confined phase at small temperatures.
- For  $T > T_{min}$  there are two black-hole solutions with the same temperature but different horizon positions. One is a “large” BH the other is “small”.
- When  $T > T_{min}$  three competing solutions exist. The large BH has the lowest free energy for  $T > T_c > T_{min}$ . It describes the deconfined “Gluon-Glass” phase.

# Temperature versus horizon position





We plot the relation  $T(r_h)$  for various potentials parameterized by  $a$ .  $a = 1$  is the critical value below which there is only one branch of black-hole solutions.

# The free energy

- The free energy is calculated from the action as a boundary term for both the black-holes and the thermal vacuum solution. They are all UV divergent but their differences are finite.

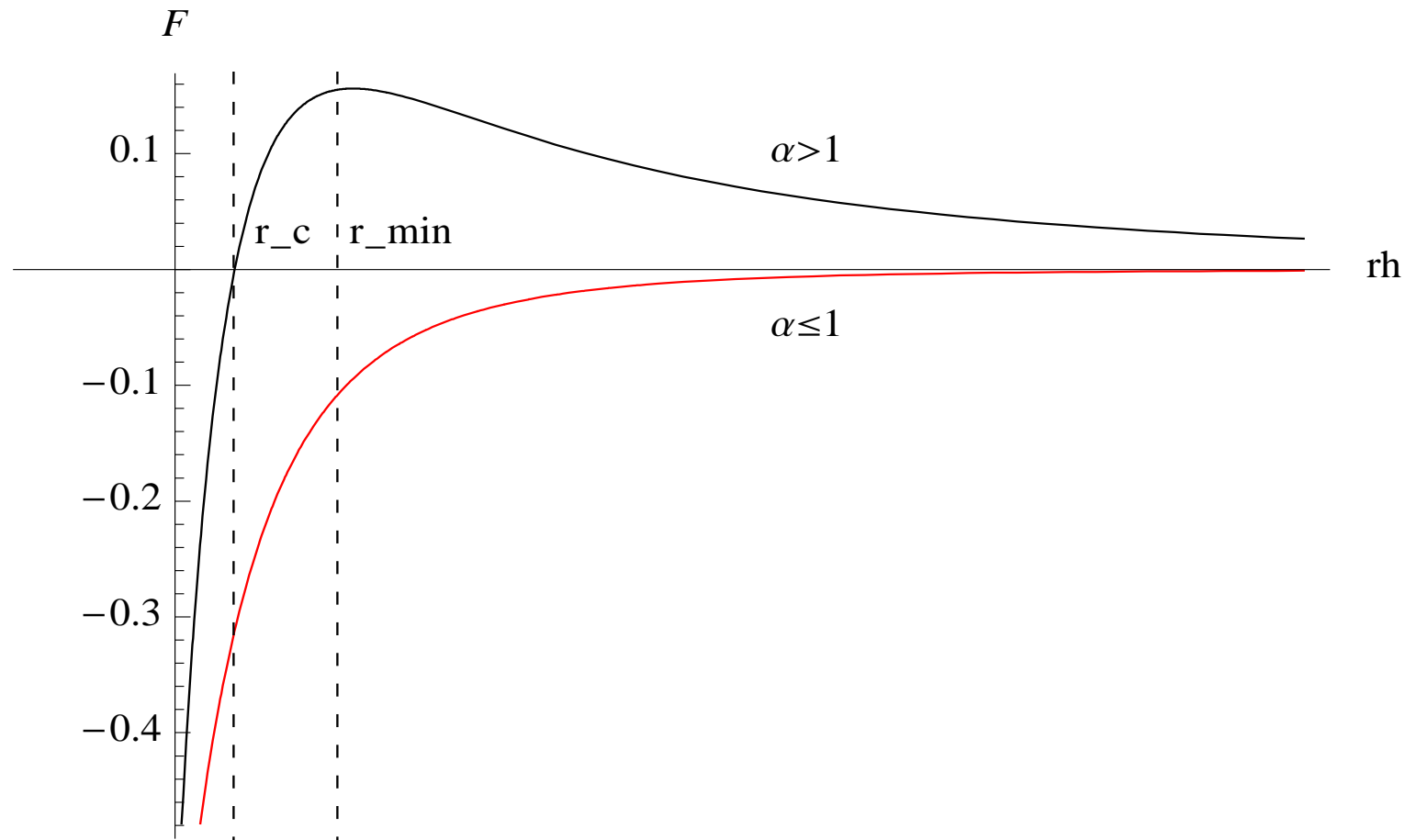
$$\frac{\mathcal{F}}{M_p^3 V_3} = 12\mathcal{G}(T) - T S(T)$$

- $\mathcal{G}$  is the temperature-depended gluon condensate  $\langle Tr[F^2] \rangle_T - \langle Tr[F^2] \rangle_{T=0}$  defined as

$$\lim_{r \rightarrow 0} \lambda_T(r) - \lambda_{T=0}(r) = \mathcal{G}(T) r^4 + \dots$$

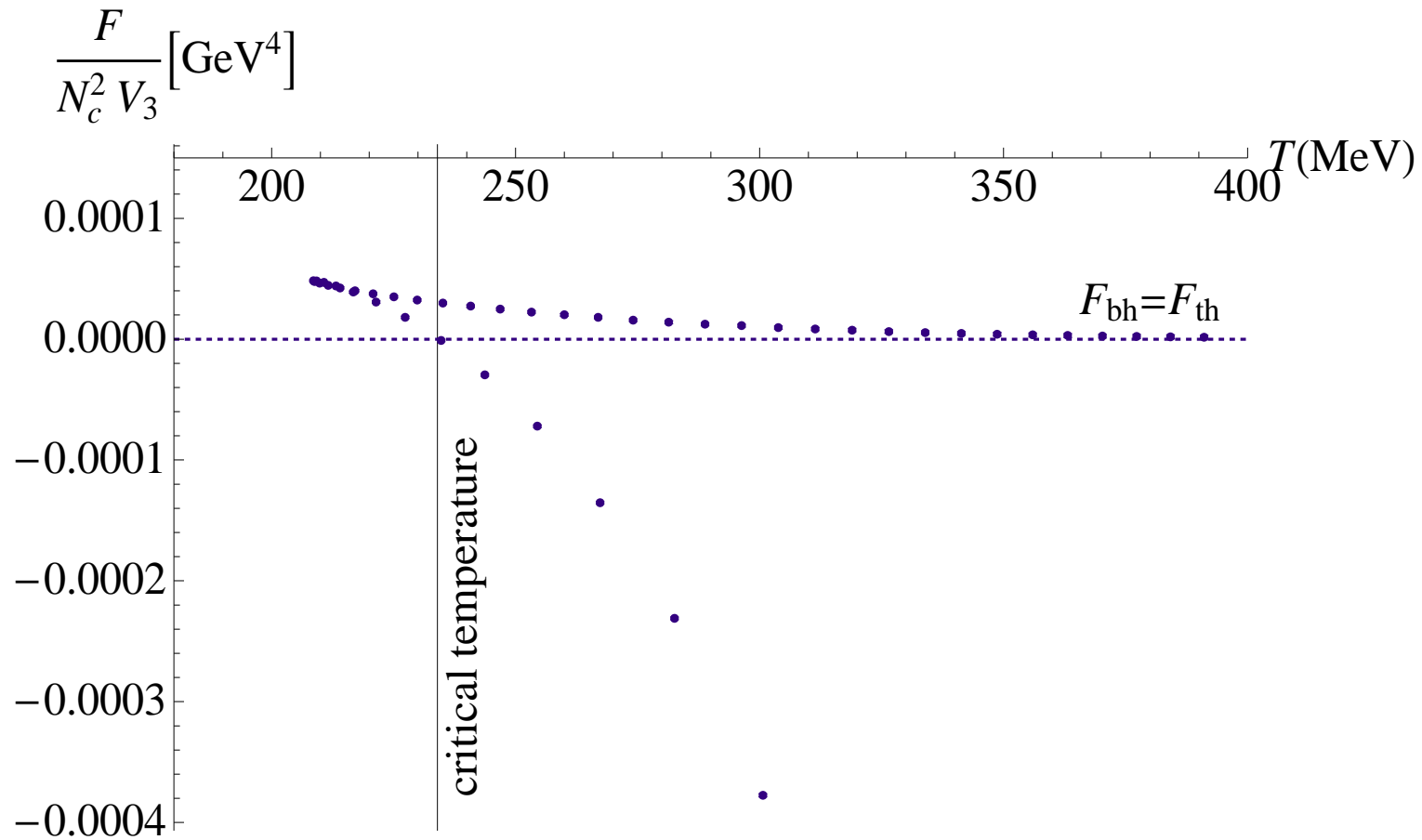
- It is  $\mathcal{G}$  the breaks conformal invariance essentially and leads to a non-trivial deconfining transition (as  $S > 0$  always)

# Free energy versus horizon position



We plot the relation  $\mathcal{F}(r_h)$  for various potentials parameterized by  $a$ .  $a = 1$  is the critical value below which there is no first order phase transition .

# The transition in the free energy





# Critical temperature and latent heat

- For the YM potential the minimum temperature for the black-holes is  $T_{\min} \simeq 210$  MeV with  $\lambda_h \simeq 12$ . The critical temperature is

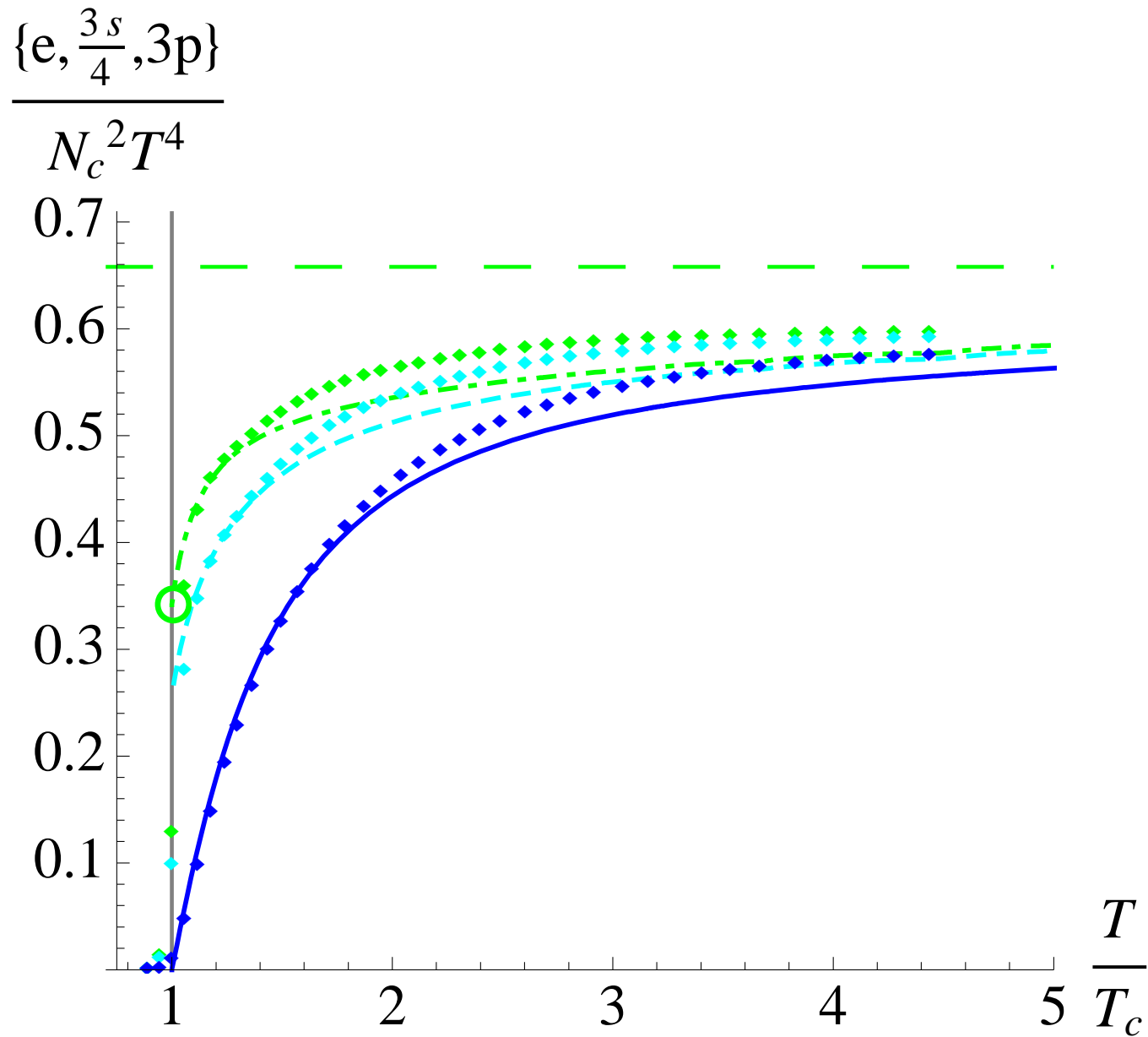
$$T_c \simeq 235 \pm 15 \text{ MeV} \quad \text{with} \quad \lambda_h \simeq 8 \quad , \quad \frac{L_h^{\frac{1}{4}}}{T_c} = 0.65\sqrt{N_c}$$

to be compared with  $260 \pm 11$  MeV and  $0.77\sqrt{N_c}$

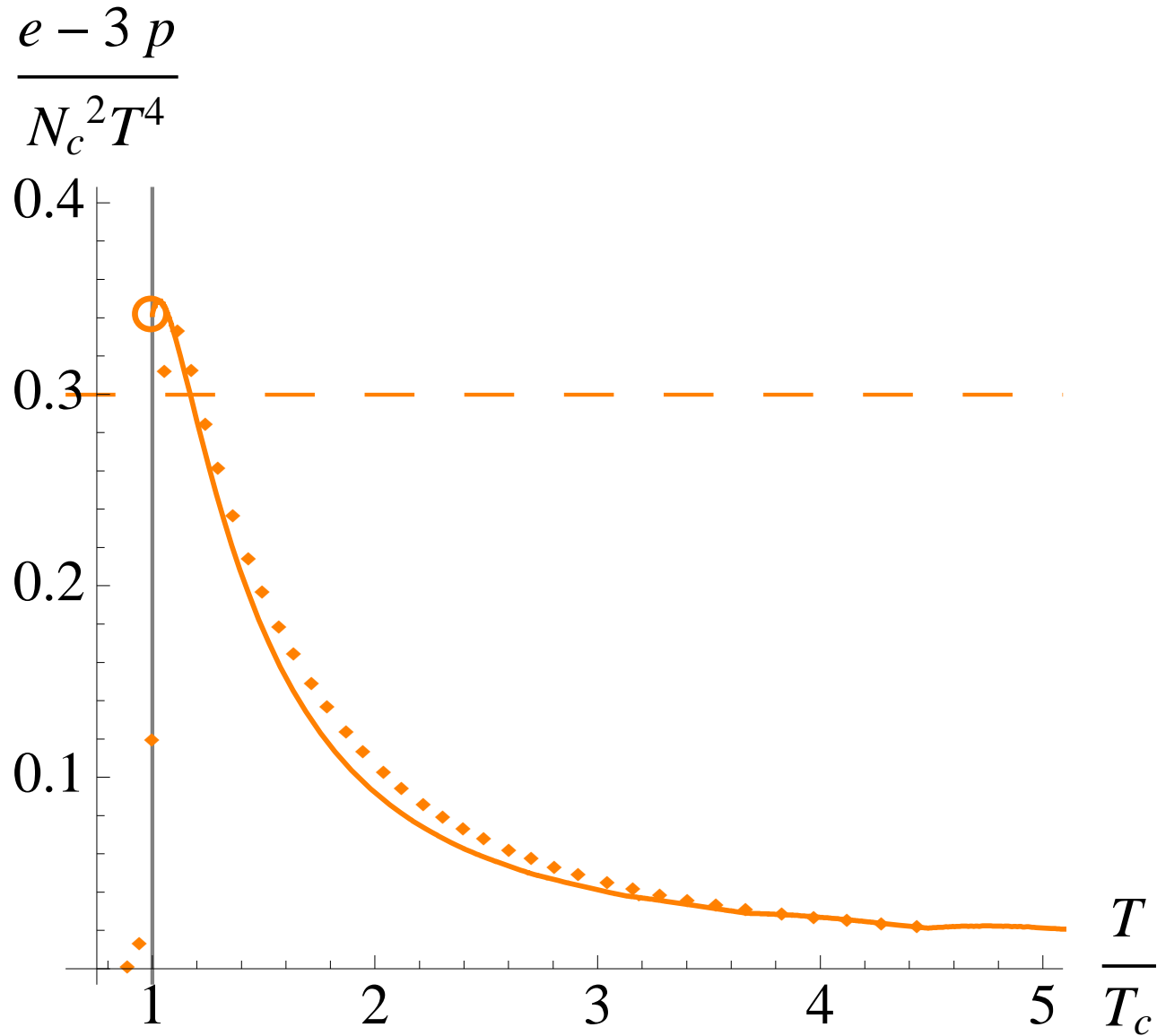
*Lucini+Teper, Lucini+Teper+Wenger*

- The specific heat for the QGP solution is positive as it should. For the small black-hole it is negative.
- In the QGP phase, the  $q\bar{q}$  potential is screened.

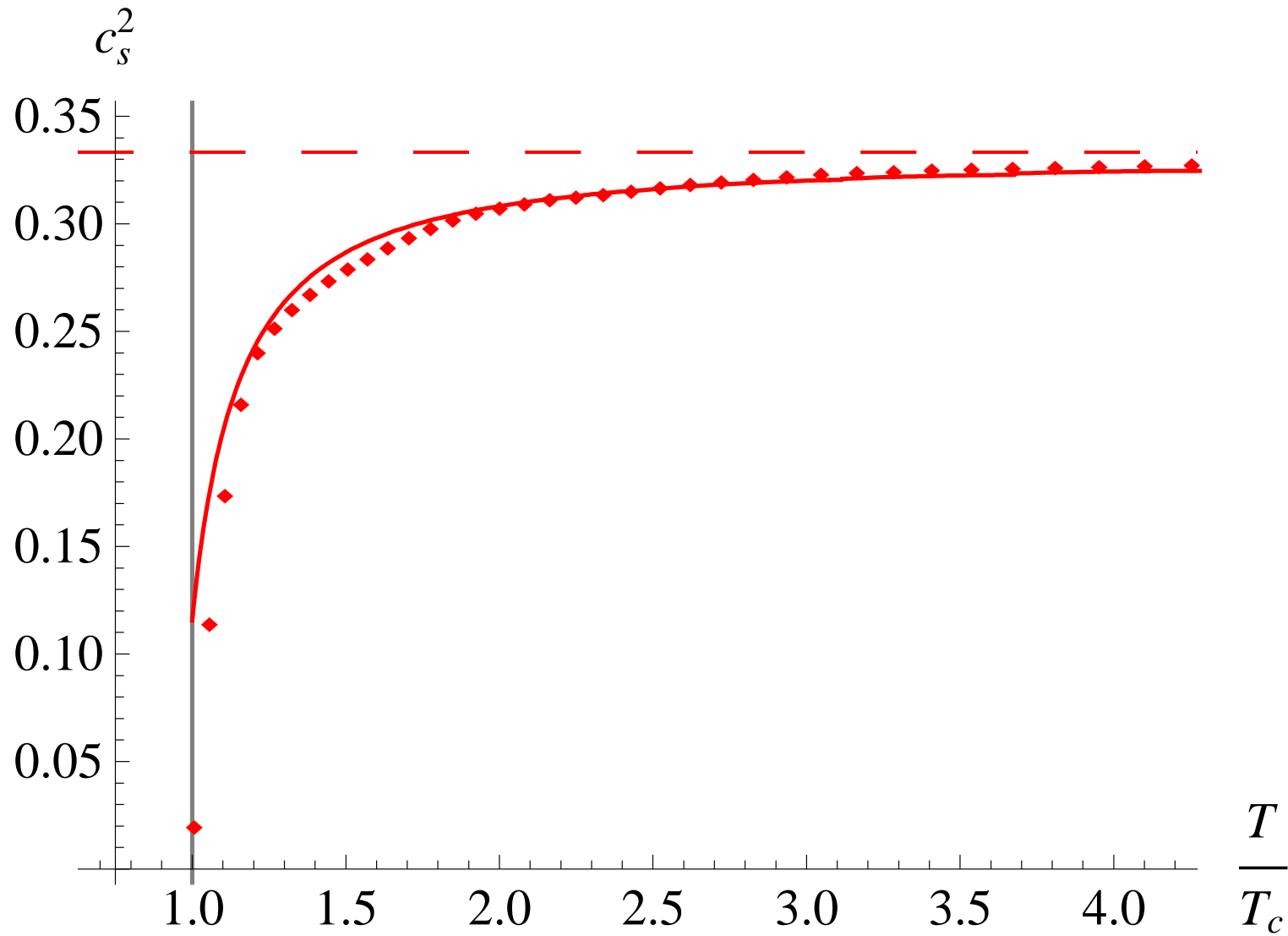
# Thermodynamic variables



# Equation of state



# The speed of sound



# The bulk viscosity: theory

- It is one of the important parameters for QGP hydrodynamics (along with the shear viscosity).
- It is related to entropy production (measurable at RHIC and LHC)
- It is defined from the Kubo formula

$$\zeta = \frac{1}{9} \lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} G_R(\omega) \quad , \quad G_R(\omega) \equiv \int d^3x \int dt e^{i\omega t} \theta(t) \langle 0 | [T_{ii}(\vec{x}, t), T_{ii}(\vec{0}, 0)] | 0 \rangle$$

Using a parametrization  $ds^2 = e^{2A}(f dt^2 + d\vec{x}^2 + \frac{dr^2}{f})$  in a special gauge  $\phi = r$  the relevant metric perturbation decouples

*Gubser+Nellore+Pufu+Rocha*

$$h''_{11} = - \left( -\frac{1}{3A'} - A' - \frac{f'}{f} \right) h'_{11} + \left( -\frac{\omega^2}{f^2} + \frac{f'}{6fA'} - \frac{f'}{f} A' \right) h_{11}$$

with

$$h_{11}(0) = 1 \quad , \quad h_{11}(r_h) \simeq C e^{i\omega t} \left| \log \frac{\lambda}{\lambda_h} \right|^{-\frac{i\omega}{4\pi T}}$$

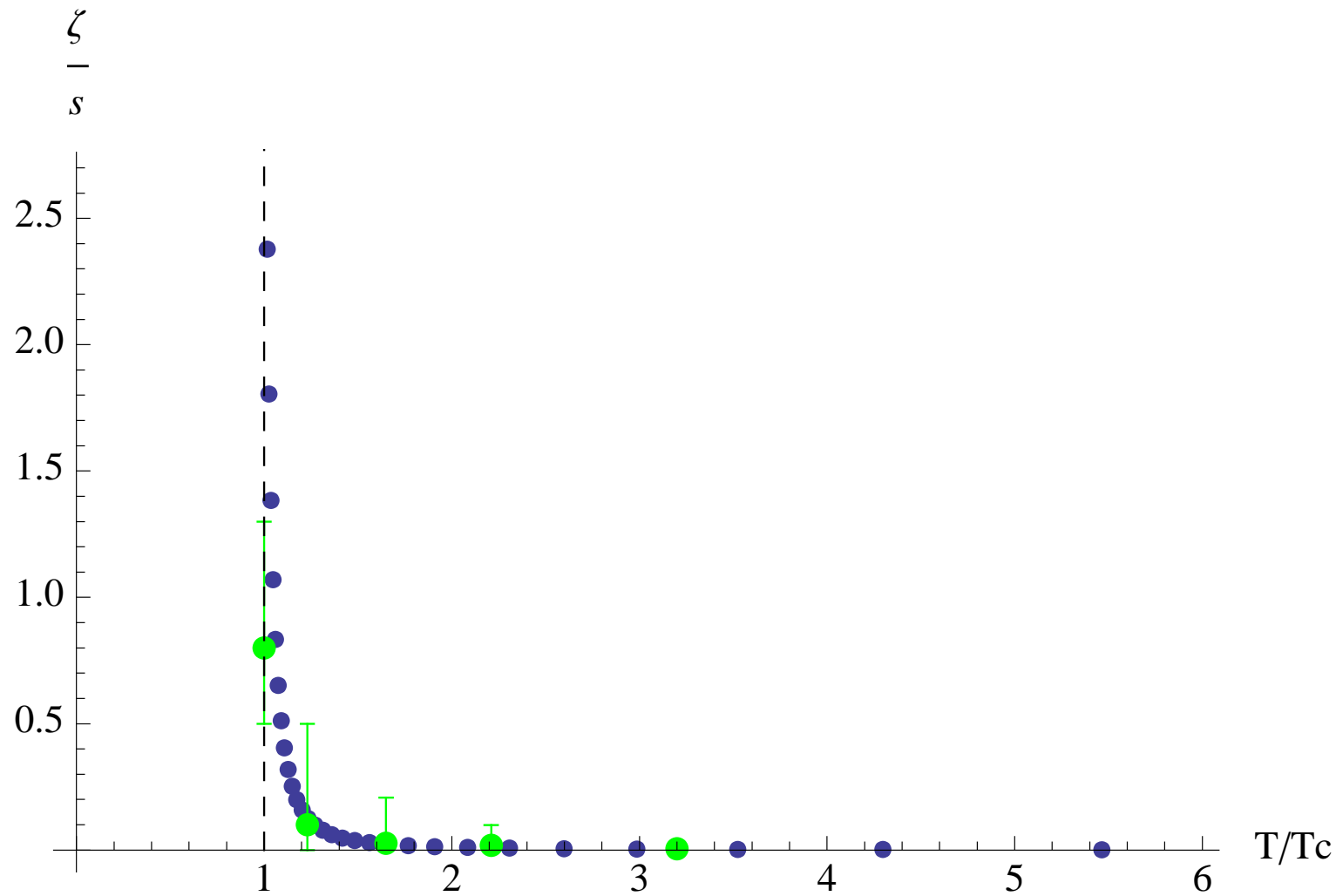
The correlator is given by the conserved number of h-quanta

$$\text{Im} G_R(\omega) = -4M^3 \mathcal{G}(\omega) \quad , \quad \mathcal{G}(\omega) = \frac{e^{3A} f}{4A'^2} |\text{Im}[h_{11}^* h'_{11}]|$$

finally giving

$$\frac{\zeta}{s} = \frac{C^2}{4\pi} \left( \frac{V'(\lambda_h)}{V(\lambda_h)} \right)^2$$

# The bulk viscosity: data



# Open problems

## THEORETICAL:

- Investigate further the structure of the string dual of QCD. Try to control the UV physics (to which RR flux plays little role).

## MORE PRACTICAL:

- Re-Calculate quantities relevant for heavy ion collisions: jet quenching parameter, drag force etc.
- Calculate the finite-temperature Polyakov loops and Debye screening lengths in various symmetry channels.
- Investigate quantitatively the meson sector
- Calculate the phase diagram in the presence of baryon number.

# The low dimension spectrum

- What are all gauge invariant YM operators of dimension 4 or less?
  - They are given by  $Tr[F_{\mu\nu}F_{\rho\sigma}]$ .
- Decomposing into U(4) reps:

$$(\square \otimes \square)_{\text{symmetric}} = \square\square \oplus \begin{array}{|c|} \hline \square \\ \hline \end{array} \quad (1)$$

We must remove traces to construct the irreducible representations of O(4):

$$\square\square = \underline{\square\square} \oplus \square\square \oplus \bullet, \quad \begin{array}{|c|} \hline \square \\ \hline \end{array} = \bullet$$

The two singlets are the scalar (dilaton) and pseudoscalar (axion)

$$\phi \leftrightarrow Tr[F^2], \quad a \leftrightarrow Tr[F \wedge F]$$

The traceless symmetric tensor

$$\underline{\square\square} \rightarrow T_{\mu\nu} = Tr \left[ F_{\mu\nu}^2 - \frac{1}{4} g_{\mu\nu} F^2 \right]$$

is the conserved stress tensor dual to a massless graviton in 5d reflecting the translational symmetry of YM.

$$\underline{\square\square} \rightarrow T_{\mu\nu;\rho\sigma}^4 = Tr \left[ F_{\mu\nu} F_{\rho\sigma} - \frac{1}{2} (g_{\mu\rho} F_{\nu\sigma}^2 - g_{\nu\rho} F_{\mu\sigma}^2 - g_{\mu\sigma} F_{\nu\rho}^2 + g_{\nu\sigma} F_{\mu\rho}^2) + \frac{1}{6} (g_{\mu\rho} g_{\nu\sigma} - g_{\nu\rho} g_{\mu\sigma}) F^2 \right]$$



It has 10 independent d.o.f, it is not conserved and it should correspond to a similar **massive** tensor in 5d. We do not expect it to play an non-trivial role in the large- $N_c$ , YM vacuum also for reasons of Lorentz invariance.

- Therefore the nontrivial fields are expected to be:

$$g_{\mu\nu}, \phi, a$$

## Further $\alpha'$ corrections

There are further dilaton terms generated by the 5-form in:

- The kinetic terms of the graviton and the dilaton  $\sim \lambda^{2n}$ .
- The kinetic terms on probe  $D_3$  branes that affect the identification of the gauge-coupling constant,  $\sim \lambda^{2n+1}$ . There is also a multiplicative factor relating  $g_{YM^2}$  to  $e^\phi$ , (not known). Can be traded for  $b_0$ .
- Corrections to the identification of the energy. At  $r = 0$ ,  $E = 1/r$ . There can be log corrections to our identification  $E = e^A$ , and these are a power series in  $\sim \lambda^{2n}$ .
- It is a remarkable fact that all such corrections affect the higher that the first two terms in the  $\beta$ -function (or equivalently the potential), that are known to be non-universal!

the metric is also insensitive to the change of  $b_0$  by changing  $\Lambda$ .

# Organizing the vacuum solutions

A useful variable is the phase variable

$$X \equiv \frac{\Phi'}{3A'} = \frac{\beta(\lambda)}{3\lambda} \quad , \quad e^\Phi \equiv \lambda$$

and a superpotential

$$W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \Phi}\right)^2 = \left(\frac{3}{4}\right)^3 V(\Phi).$$

with

$$A' = -\frac{4}{9}W \quad , \quad \Phi' = \frac{dW}{d\Phi}$$

$$X = -\frac{3 d \log W}{4 d \log \lambda} \quad , \quad \beta(\lambda) = -\frac{9}{4} \lambda \frac{d \log W}{d \log \lambda}$$

♠ The equations have three integration constants: (two for  $\Phi$  and one for  $A$ ) One corresponds to the “gluon condensate” in the UV. It must be set to zero otherwise the IR behavior is unacceptable. The other is  $\Lambda$ . The third one is a gauge artifact (corresponds to overall translation in the radial coordinate).

# The IR regime

For any asymptotically  $AdS_5$  solution ( $e^A \sim \frac{\ell}{r}$ ):

- The scale factor  $e^{A(r)}$  is monotonically decreasing

*Girardello+Petrini+Porrati+Zaffaroni  
Freedman+Gubser+Pilch+Warner*

- Moreover, there are only three possible, mutually exclusive IR asymptotics:

♠ *there is another asymptotic  $AdS_5$  region, at  $r \rightarrow \infty$ , where  $\exp A(r) \sim \ell'/r$ , and  $\ell' \leq \ell$  (equality holds if and only if the space is exactly  $AdS_5$  everywhere);*

♠ there is a curvature singularity at some finite value of the radial coordinate,  $r = r_0$ ;

♠ there is a curvature singularity at  $r \rightarrow \infty$ , where the scale factor vanishes and the space-time shrinks to zero size.

# Wilson-Loops and confinement

- Calculation of the static quark potential using the vev of the Wilson loop calculated via an F-string worldsheet.

*Rey+Yee, Maldacena*

$$T E(L) = S_{\text{minimal}}(X)$$

We calculate

$$L = 2 \int_0^{r_0} dr \frac{1}{\sqrt{e^{4A_S(r)} - 4A_S(r_0)} - 1}.$$

It diverges when  $e^{A_S}$  has a minimum (at  $r = r_*$ ). Then

$$E(L) \sim T_f e^{2A_S(r_*)} L$$

- **Confinement**  $\rightarrow A_S(r_*)$  is finite. This is a more general condition that considered before as  $A_S$  is not monotonic in general.
- Effective string tension

$$T_{\text{string}} = T_f e^{2A_S(r_*)}$$

# General criterion for confinement

- the geometric version:

A geometry that shrinks to zero size in the IR is dual to a confining 4D theory if and only if the Einstein metric in conformal coordinates vanishes as (or faster than)  $e^{-Cr}$  as  $r \rightarrow \infty$ , for some  $C > 0$ .

- It is understood here that a metric vanishing at finite  $r = r_0$  also satisfies the above condition.

- ♠ the superpotential

A 5D background is dual to a confining theory if the superpotential grows as (or faster than)

$$W \sim (\log \lambda)^{P/2} \lambda^{2/3} \quad \text{as } \lambda \rightarrow \infty, \quad P \geq 0$$

- ♠ the  $\beta$ -function A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda \rightarrow \infty} \left( \frac{\beta(\lambda)}{3\lambda} + \frac{1}{2} \right) \log \lambda = K, \quad -\infty \leq K \leq 0$$

(No explicit reference to any coordinate system) Linear trajectories correspond to  $K = -\frac{3}{16}$

# Classification of confining superpotentials

Classification of confining superpotentials  $W(\lambda)$  as  $\lambda \rightarrow \infty$  in IR:

$$W(\lambda) \sim (\log \lambda)^{\frac{P}{2}} \lambda^Q, \quad \lambda \sim E^{-\frac{9}{4}Q} \left( \log \frac{1}{E} \right)^{\frac{P}{2Q}}, \quad E \rightarrow 0.$$

- $Q > 2/3$  or  $Q = 2/3$  and  $P > 1$  leads to confinement and a singularity at finite  $r = r_0$ .

$$e^A(r) \sim \begin{cases} (r_0 - r)^{\frac{4}{9Q^2 - 4}} & Q > \frac{2}{3} \\ \exp \left[ -\frac{C}{(r_0 - r)^{1/(P-1)}} \right] & Q = \frac{2}{3} \end{cases}$$

- $Q = 2/3$ , and  $0 \leq P < 1$  leads to confinement and a singularity at  $r = \infty$  The scale factor  $e^A$  vanishes there as

$$e^A(r) \sim \exp[-Cr^{1/(1-P)}].$$

- $Q = 2/3, P = 1$  leads to confinement but the singularity may be at a finite or infinite value of  $r$  depending on subleading asymptotics of the superpotential.

♠ If  $Q < 2\sqrt{2}/3$ , no *ad hoc* boundary conditions are needed to determine the glueball spectrum  $\rightarrow$  One-to-one correspondence with the  $\beta$ -function This is unlike standard AdS/QCD and other approaches.

- when  $Q > 2\sqrt{2}/3$ , the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

## Confining $\beta$ -functions

A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda \rightarrow \infty} \left( \frac{\beta(\lambda)}{3\lambda} + \frac{1}{2} \right) \log \lambda = K, \quad -\infty \leq K \leq 0$$

(No explicit reference to any coordinate system). Linear trajectories correspond to  $K = -\frac{3}{16}$

- We can determine the geometry if we specify  $K$ :
- $K = -\infty$ : the scale factor goes to zero at some finite  $r_0$ , not faster than a power-law.
- $-\infty < K < -3/8$ : the scale factor goes to zero at some finite  $r_0$  faster than any power-law.
- $-3/8 < K < 0$ : the scale factor goes to zero as  $r \rightarrow \infty$  faster than  $e^{-Cr^{1+\epsilon}}$  for some  $\epsilon > 0$ .
- $K = 0$ : the scale factor goes to zero as  $r \rightarrow \infty$  as  $e^{-Cr}$  (or faster), but slower than  $e^{-Cr^{1+\epsilon}}$  for any  $\epsilon > 0$ .

The borderline case,  $K = -3/8$ , is certainly confining (by continuity), but whether or not the singularity is at finite  $r$  depends on the subleading terms.



# Comments on confining backgrounds

- For all confining backgrounds with  $r_0 = \infty$ , although the space-time is singular in the Einstein frame, the string frame geometry is asymptotically flat for large  $r$ . Therefore only  $\lambda$  grows indefinitely.
- String world-sheets do not probe the strong coupling region, at least classically. The string stays away from the strong coupling region.
- Therefore: singular confining backgrounds have generically the property that the singularity is *repulsive*, i.e. only highly excited states can probe it. This will also be reflected in the analysis of the particle spectrum (to be presented later)
- The confining backgrounds must also screen magnetic color charges. This can be checked by calculating 't Hooft loops using  $D_1$  probes:
  - ♠ All confining backgrounds with  $r_0 = \infty$  and most at finite  $r_0$  screen properly
  - ♠ In particular “hard-wall” AdS/QCD confines also the magnetic quarks.

# Particle Spectra: generalities

- Linearized equation:

$$\ddot{\xi} + 2\dot{B}\dot{\xi} + \square_4\xi = 0 \quad , \quad \xi(r, x) = \xi(r)\xi^{(4)}(x), \quad \square\xi^{(4)}(x) = m^2\xi^{(4)}(x)$$

- Can be mapped to Schrodinger problem

$$-\frac{d^2}{dr^2}\psi + V(r)\psi = m^2\psi \quad , \quad V(r) = \frac{d^2B}{dr^2} + \left(\frac{dB}{dr}\right)^2 \quad , \quad \xi(r) = e^{-B(r)}\psi(r)$$

- Mass gap and discrete spectrum visible from the asymptotics of the potential.

- Large  $n$  asymptotics of masses obtained from WKB

$$n\pi = \int_{r_1}^{r_2} \sqrt{m^2 - V(r)} \, dr$$

- Spectrum depends only on initial condition for  $\lambda$  ( $\sim \Lambda_{QCD}$ ) and an overall energy scale ( $e^A$ ) that must be fixed.

- scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2} \log \frac{\beta(\lambda)^2}{9\lambda^2}$$

- tensor glueballs

$$B(r) = \frac{3}{2}A(r)$$

- pseudo-scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2} \log Z(\lambda)$$

- Universality of asymptotics

$$\frac{m_{n \rightarrow \infty}^2(0^{++})}{m_{n \rightarrow \infty}^2(2^{++})} \rightarrow 1 \quad , \quad \frac{m_{n \rightarrow \infty}^2(0^{+-})}{m_{n \rightarrow \infty}^2(0^{++})} = \frac{1}{4}(d-2)^2$$

predicts  $d = 4$  via

$$\frac{m^2}{2\pi\sigma_a} = 2n + J + c,$$

# The axion background

- The kinetic term of the axion is suppressed by  $1/N_c^2$ . (it is an angle in the gauge theory, it is RR in string theory)

$$\ddot{a} + \left( 3\dot{A} + \frac{\dot{Z}(\lambda)}{Z(\lambda)} \right) \dot{a} = 0 \quad \rightarrow \quad \dot{a} = \frac{C e^{-3A}}{Z(\lambda)}$$

It can be interpreted as the flow equation of the effective  $\theta$ -angle.

- The full solution is

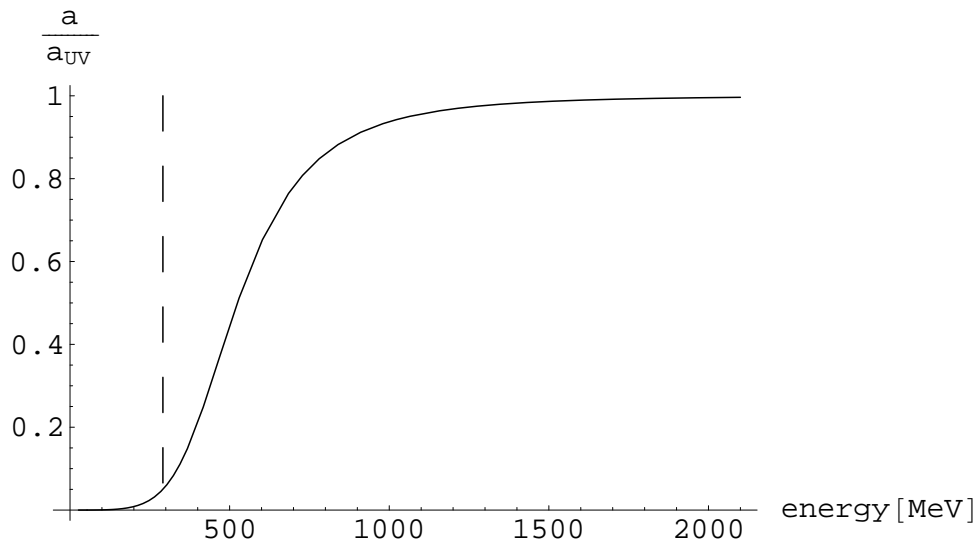
$$a(r) = \theta_{UV} + 2\pi k + C \int_0^r \frac{e^{-3A}}{Z(\lambda)} \, , \quad C = \langle \text{Tr}[F \wedge F] \rangle$$

- The vacuum energy is

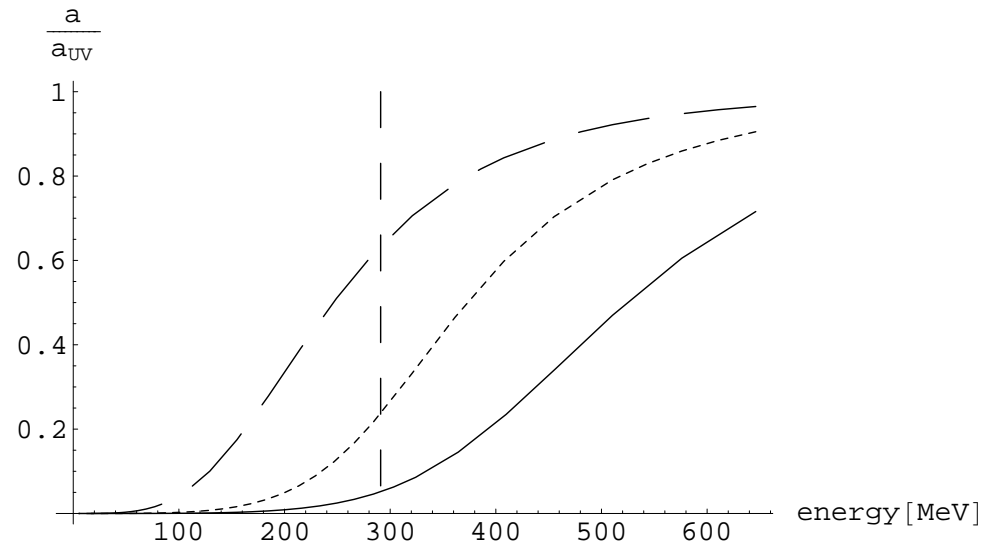
$$E(\theta_{UV}) = \frac{M^3}{2N_c^2} \int d^5x \sqrt{g} Z(\lambda) (\partial a)^2 = \frac{M^3}{2N_c^2} C a(r) \Big|_{r=0}^{r=r_0}$$

- Consistency requires to impose that  $a(r_0) = 0$ . This determines  $C$  and

$$E(\theta_{UV}) = -\frac{M^3}{2} \text{Min}_k \frac{(\theta_{UV} + 2\pi k)^2}{\int_0^{r_0} \frac{dr}{e^{3A} Z(\lambda)}} \, , \quad \frac{a(r)}{\theta_{UV} + 2\pi k} = \frac{\int_r^{r_0} \frac{dr}{e^{3A} Z(\lambda)}}{\int_0^{r_0} \frac{dr}{e^{3A} Z(\lambda)}}$$



(a)



(b)

(a) An example of the axion profile (normalized to one in the UV) as a function of energy, in one of the explicit cases we treat numerically. The energy scale is in MeV, and it is normalized to match the mass of the lowest scalar glueball from lattice data,  $m_0 = 1475 \text{ MeV}$ . The axion kinetic function is taken as  $Z(\lambda) = Z_a(1 + c_a \lambda^4)$ , with  $c_a = 100$  (the masses do not depend on the value of  $Z_a$ ). The vertical dashed line corresponds to

$$\Lambda_p \equiv \frac{1}{\ell} \frac{\exp\left[A(\lambda_0) - \frac{1}{b_0 \lambda_0}\right]}{(b_0 \lambda_0)^{b_1/b_0^2}}. \text{ In this particular case } \Lambda = 290 \text{ MeV}.$$

(b) A detail showing the different axion profiles for different values of  $c_a$ . The values are  $c_a = 0.1$  (dashed line),  $c_a = 10$  (dotted line) and  $c_a = 100$  (solid line).

# Quarks ( $N_f \ll N_c$ ) and mesons

- Flavor is introduced by  $N_f D_4 + \bar{D}_4$  branes pairs inside the bulk background. Their back-reaction on the bulk geometry is suppressed by  $N_f/N_c$ .
- The important world-volume fields are

$$T_{ij} \leftrightarrow \bar{q}_a^i \frac{1 + \gamma^5}{2} q_a^j \quad , \quad A_{\mu}^{ijL,R} \leftrightarrow \bar{q}_a^i \frac{1 \pm \gamma^5}{2} \gamma^{\mu} q_a^j$$

Generating the  $U(N_f)_L \times U(N_f)_R$  chiral symmetry.

- The UV mass matrix  $m_{ij}$  corresponds to the source term of the Tachyon field. It breaks the chiral (gauge) symmetry. The normalizable mode corresponds to the vev  $\langle \bar{q}_a^i \frac{1 + \gamma^5}{2} q_a^j \rangle$ .
- We show that the expectation value of the tachyon is non-zero and  $T \sim 1$ , breaking chiral symmetry  $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$ . The anomaly plays an important role in this (holographic Coleman-Witten)

- The fact that the tachyon diverges in the IR (fusing  $D$  with  $\bar{D}$ ) constraints the UV asymptotics and determines the quark condensate  $\langle \bar{q}q \rangle$  in terms of  $m_q$ . A GOR relation is satisfied (for an asymptotic  $\text{AdS}_5$  space)

$$m_\pi^2 = -2 \frac{m_q}{f_\pi^2} \langle \bar{q}q \rangle \quad , \quad m_q \rightarrow 0$$

- We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.
- When  $m_q = 0$ , the meson spectrum contains  $N_f^2$  massless pseudoscalars, the  $U(N_f)_A$  Goldstone bosons.
- The WZ part of the flavor brane action gives the Adler-Bell-Jackiw  $U(1)_A$  axial anomaly and an associated Stueckelberg mechanism gives an  $O\left(\frac{N_f}{N_c}\right)$  mass to the would-be Goldstone boson  $\eta'$ , in accordance with the Veneziano-Witten formula.
- Studying the spectrum of highly excited mesons, we find the expected property of linear confinement:  $m_n^2 \sim n$ .
- The detailed spectrum of mesons remains to be worked out

# Tachyon dynamics

- In the vacuum the gauge fields vanish and  $T \sim 1$ . Only DBI survives

$$S[\tau] = T_{D_4} \int dr d^4x \frac{e^{4A_s(r)}}{\lambda} V(\tau) \sqrt{e^{2A_s(r)} + \dot{\tau}(r)^2} \quad , \quad V(\tau) = e^{-\frac{\mu^2}{2}\tau^2}$$

- We obtain the nonlinear field equation:

$$\ddot{\tau} + \left(3\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right) \dot{\tau} + e^{2A_S} \mu^2 \tau + e^{-2A_S} \left[4\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right] \dot{\tau}^3 + \mu^2 \tau \dot{\tau}^2 = 0.$$

- In the UV we expect

$$\tau = m_q r + \sigma r^3 + \dots \quad , \quad \mu^2 \ell^2 = 3$$

- We expect that the tachyon must diverge before or at  $r = r_0$ . We find that indeed it does **at the singularity**. For the  $r_0 = \infty$  backgrounds

$$\tau \sim \exp\left[\frac{2}{a} \frac{R}{\ell^2} r\right] \quad \text{as} \quad r \rightarrow \infty$$



- Generically the solutions have spurious singularities:  $\tau(r_*)$  stays finite but its derivatives diverges as:

$$\tau \sim \tau_* + \gamma \sqrt{r_* - r}.$$

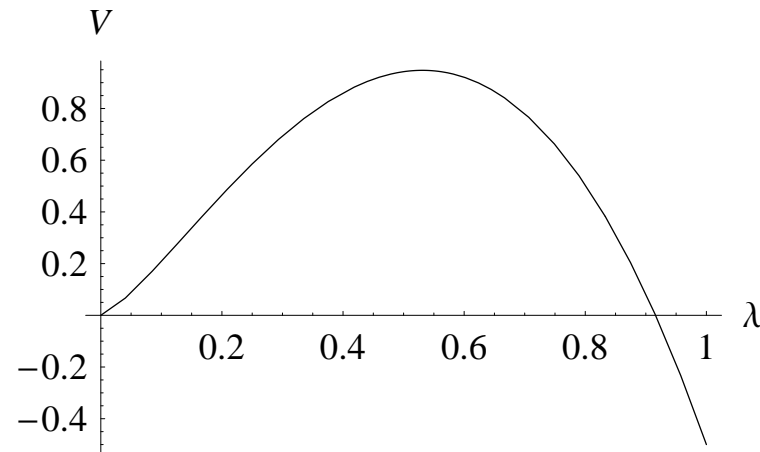
The condition that they are absent determines  $\sigma$  as a function of  $m_q$ .

- The easiest spectrum to analyze is that of vector mesons. We find ( $r_0 = \infty$ )

$$\Lambda_{glueballs} = \frac{1}{R}, \quad \Lambda_{mesons} = \frac{3}{\ell} \left( \frac{\alpha \ell^2}{2R^2} \right)^{(\alpha-1)/2} \propto \frac{1}{R} \left( \frac{\ell}{R} \right)^{\alpha-2}.$$

This suggests that  $\alpha = 2$ . preferred also from the glue sector.

# Fluctuations around the AdS<sub>5</sub> extremum



- In QCD we expect that

$$\frac{1}{\lambda} = \frac{1}{N_c e^{\phi}} \sim \frac{1}{\log r} \quad , \quad ds^2 \sim \frac{1}{r^2} (dr^2 + dx_{\mu} dx^{\mu}) \quad \text{as} \quad r \rightarrow 0$$

- Any potential with  $V(\lambda) \sim \lambda^a$  when  $\lambda \ll 1$  gives a power different that of AdS<sub>5</sub>
- There is an AdS<sub>5</sub> minimum at a finite value  $\lambda_*$ . This cannot be the UV of QCD as dimensions do not match.

Near an AdS extremum

$$V = \frac{12}{\ell^2} - \frac{16\xi}{3\ell^2}\phi^2 + \mathcal{O}(\phi^3) \quad , \quad \frac{18}{\ell}\delta A' = \delta\phi'^2 - \frac{4}{\ell^2}\phi^2 = \mathcal{O}(\delta\phi^2) \quad , \quad \delta\phi'' - \frac{4}{\ell}\delta\phi' - \frac{4\xi}{\ell^2}\delta\phi = 0$$

where  $\phi \ll 1$ . The general solution of the second equation is

$$\delta\phi = C_+ e^{\frac{(2+2\sqrt{1+\xi})u}{\ell}} + C_- e^{\frac{(2-2\sqrt{1+\xi})u}{\ell}}$$

For the potential in question

$$V(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2} \left[ 5 - \frac{N_c^2}{2} e^{2\phi} - N_f e^\phi \right] \quad , \quad \lambda_0 \equiv N_c e^{\phi_0} = \frac{-7x + \sqrt{49x^2 + 400}}{10} \quad , \quad x \equiv \frac{N_f}{N_c}$$

$$\xi = \frac{5}{4} \left[ \frac{400 + 49x^2 - 7x\sqrt{49x^2 + 400}}{100 + 7x^2 - x\sqrt{49x^2 + 400}} \right] \quad , \quad \frac{\ell_s^2}{\ell^2} = e^{\frac{4}{3}\phi_0} \left[ \frac{100 + 7x^2 - x\sqrt{49x^2 + 400}}{400} \right]$$

The associated dimension is  $\Delta = 2 + 2\sqrt{1 + \xi}$  and satisfies

$$2 + 3\sqrt{2} < \Delta < 2 + 2\sqrt{6} \quad \text{or equivalently} \quad 6.24 < \Delta < 6.90$$

It corresponds to an irrelevant operator. It is most probably relevant for the Banks-Zaks fixed points.

*Bigazzi+Casero+Cotrone+Kiritsis+Paredes*

RETURN

# Concrete potential

- The superpotential chosen is

$$W = (3 + 2b_0\lambda)^{2/3} \left[ 18 + (2b_0^2 + 3b_1) \log(1 + \lambda^2) \right]^{4/3},$$

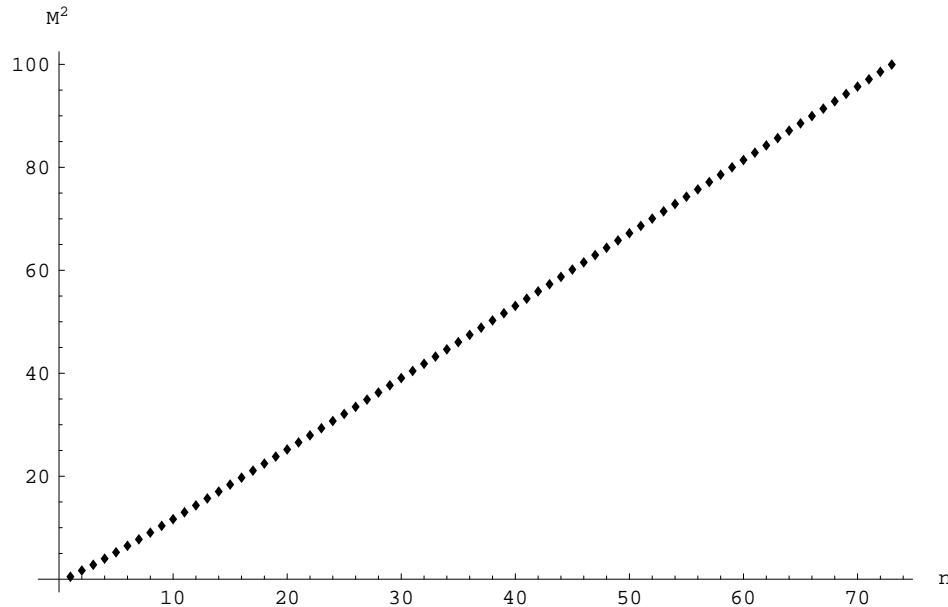
with corresponding potential

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3 + 2b_0\lambda} - \frac{6(2b_0^2 + 3b_1)\lambda^3}{(1 + \lambda^2) \left( 18 + (2b_0^2 + 3b_1) \log(1 + \lambda^2) \right)}$$

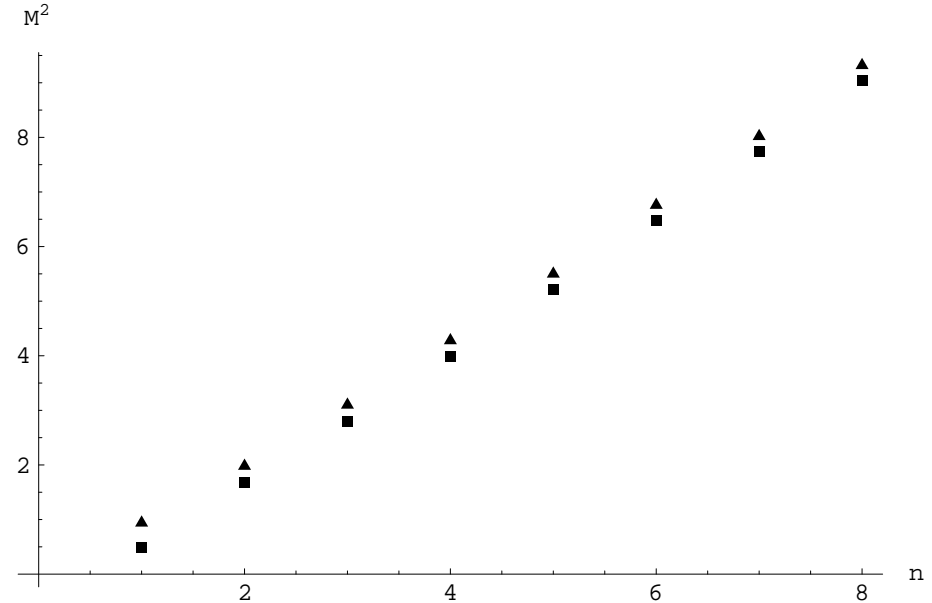
which is everywhere regular and has the correct UV and IR asymptotics.

- $b_0$  is a free parameter and  $b_1/b_0^2$  is taken from the QCD  $\beta$ -function

# Linearity of the glueball spectrum



(a)

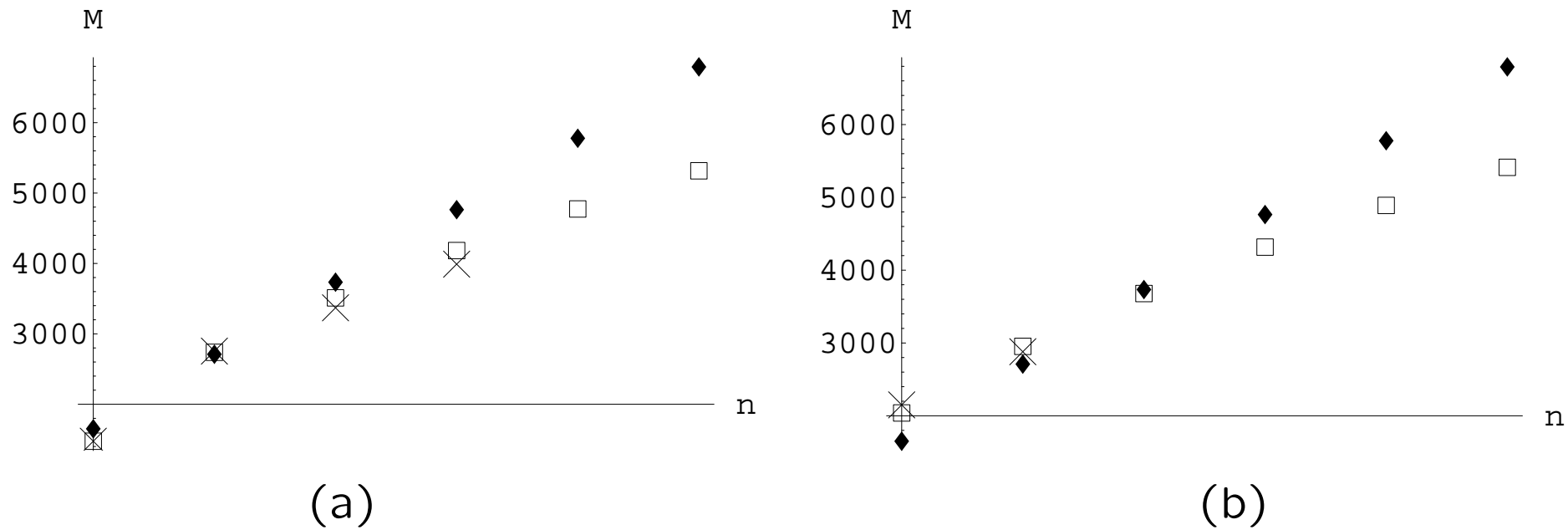


(b)

(a) Linear pattern in the spectrum for the first 40  $0^{++}$  glueball states.  $M^2$  is shown units of  $0.015\ell^{-2}$ .

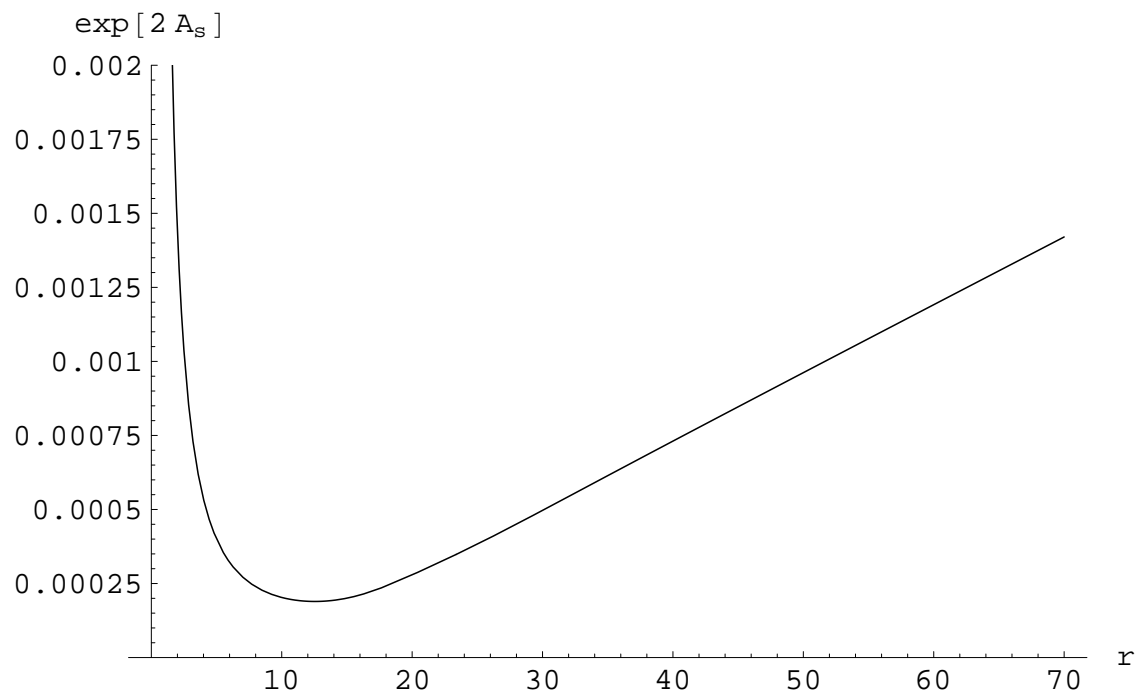
(b) The first 8  $0^{++}$  (squares) and the  $2^{++}$  (triangles) glueballs. These spectra are obtained in the background I with  $b_0 = 4.2, \lambda_0 = 0.05$ .

# Comparison with lattice data (Meyer)



Comparison of glueball spectra from our model with  $b_0 = 4.2$ ,  $\lambda_0 = 0.05$  (boxes), with the lattice QCD data from Ref. I (crosses) and the AdS/QCD computation (diamonds), for (a)  $0^{++}$  glueballs; (b)  $2^{++}$  glueballs. The masses are in MeV, and the scale is normalized to match the lowest  $0^{++}$  state from Ref. I.

$$l_{eff}^2 = 6.88 l^2$$



The string frame scale factor in background I with  $b_0 = 4.2$ ,  $\lambda_0 = 0.05$ .

We can “measure”

$$\frac{\ell}{\ell_s} \simeq 6.26 \quad , \quad \ell_s^2 R \simeq -0.5 \quad (2)$$

and predict

$$\alpha_s(1.2\text{GeV}) = 0.34,$$

which is within the error of the quoted experimental value  $\alpha_s^{(exp)}(1.2\text{GeV}) = 0.35 \pm 0.01$

# The fit to Meyer lattice data

| $J^{PC}$    | Ref I (MeV)      | Our model (MeV) | Mismatch | $N_c \rightarrow \infty$ [?] | Mismatch |
|-------------|------------------|-----------------|----------|------------------------------|----------|
| $0^{++}$    | <b>1475 (4%)</b> | <b>1475</b>     | 0        | <b>1475</b>                  | 0        |
| $2^{++}$    | 2150 (5%)        | 2055            | 4%       | 2153 (10%)                   | 5%       |
| $0^{-+}$    | <b>2250 (4%)</b> | <b>2243</b>     | 0        |                              |          |
| $0^{++*}$   | <b>2755 (4%)</b> | <b>2753</b>     | 0        | 2814 (12%)                   | 2%       |
| $2^{++*}$   | 2880 (5%)        | 2991            | 4%       |                              |          |
| $0^{-+*}$   | 3370 (4%)        | 3288            | 2%       |                              |          |
| $0^{++**}$  | 3370 (4%)        | 3561            | 5%       |                              |          |
| $0^{++***}$ | 3990 (5%)        | 4253            | 6%       |                              |          |

Comparison between the glueball spectra in Ref. I and in our model. The states we use as input in our fit are marked in **red**. The parenthesis in the lattice data indicate the percent accuracy.



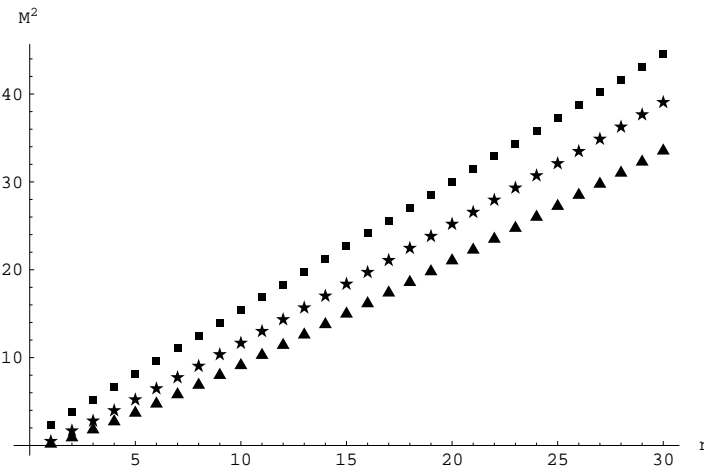
# Estimating the importance of logarithmic scaling

We keep the IR asymptotics of background II, but change the UV to power asymptoting AdS<sub>5</sub>, with a small  $\lambda_*$ .

$$e^A(r) = \frac{\ell}{r} e^{-(r/R)^2}, \quad \Phi(r) = \Phi_0 + \frac{3r^2}{2R^2} \sqrt{1 + 3\frac{R^2}{r^2}} + \frac{9}{4} \log \frac{2\frac{r}{R} + 2\sqrt{\frac{r^2}{R^2} + \frac{3}{2}}}{\sqrt{6}}.$$

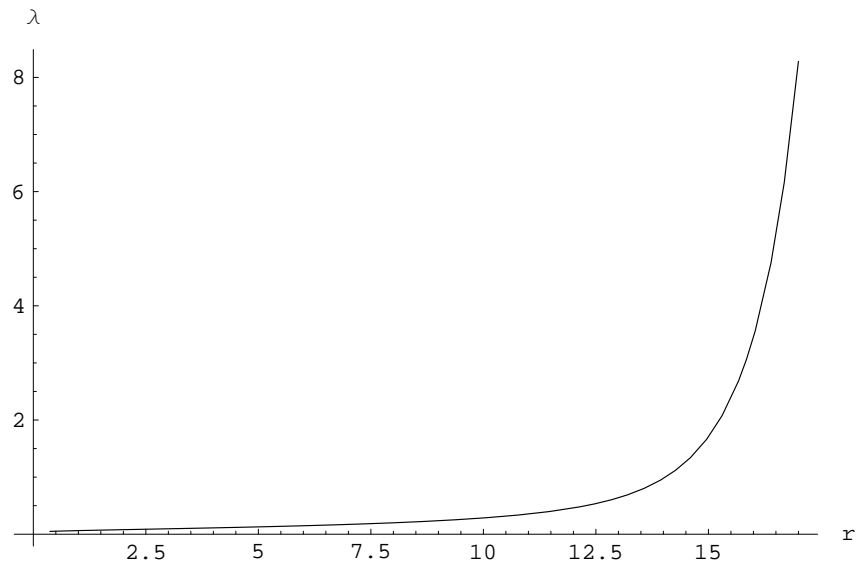
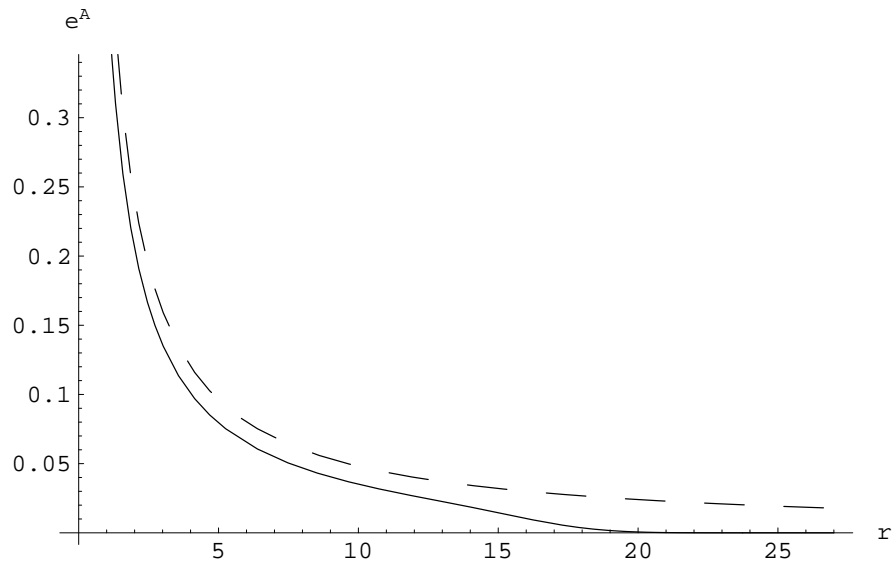
$$W_{conf} = W_0 \left( 9 + 4b_0^2(\lambda - \lambda_*)^2 \right)^{1/3} \left( 9a + (2b_0^2 + 3b_1) \log [1 + (\lambda - \lambda_*^2)] \right)^{2a/3}.$$

We fix parameters so that the physical QCD scale is the same (as determined from asymptotic slope of Regge trajectories).



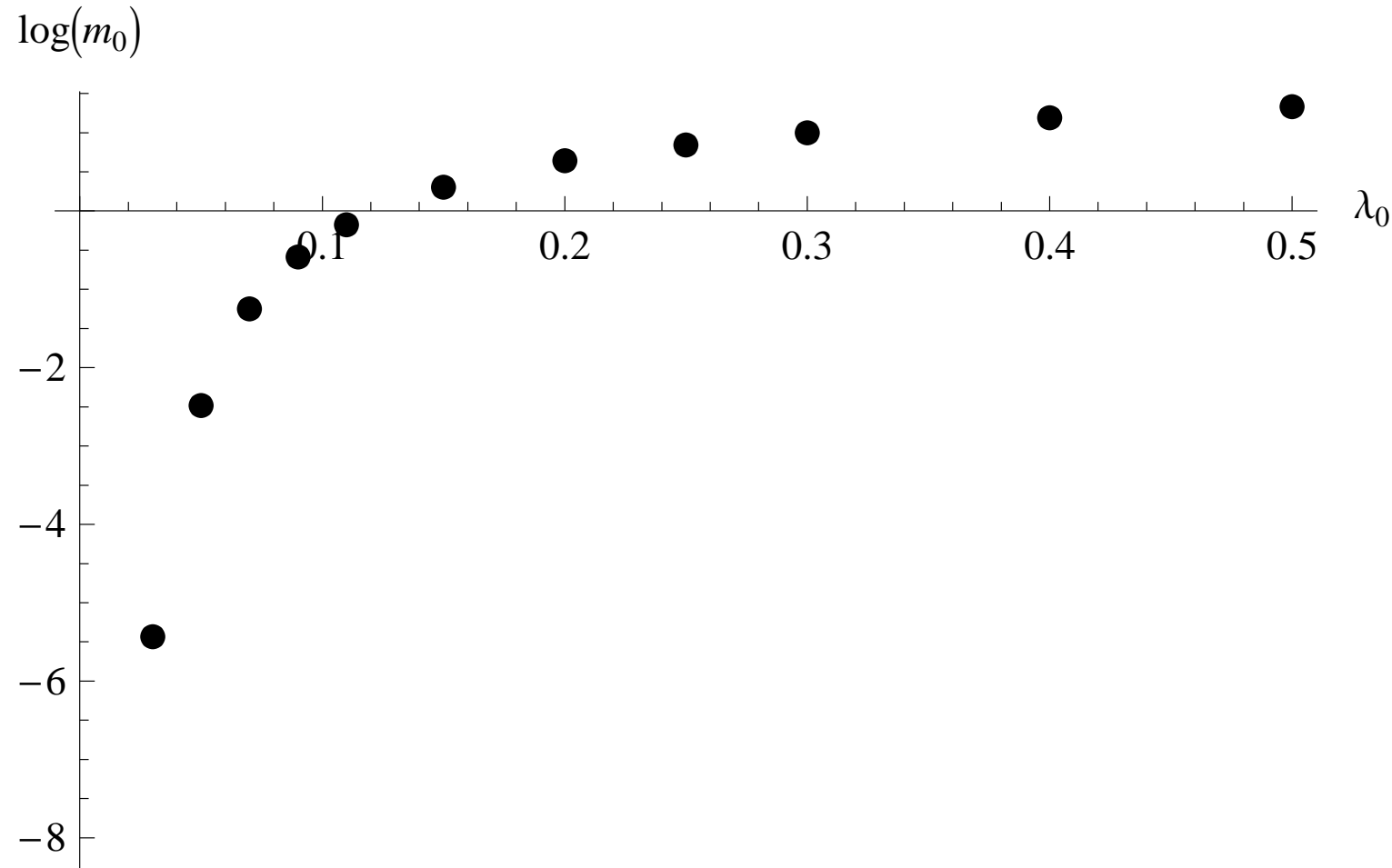
The stars correspond to the asymptotically free background I with  $b_0 = 4.2$  and  $\lambda_0 = 0.05$ ; the squares correspond to the results obtained in the first background with  $R = 11.4\ell$ ; the triangles denote the spectrum in the second background with  $b_0 = 4.2$ ,  $l_i = 0.071$  and  $l_* = 0.01$ . These values are chosen so that the slopes coincide asymptotically for large  $n$ .

# Profile of coupling and scale factor



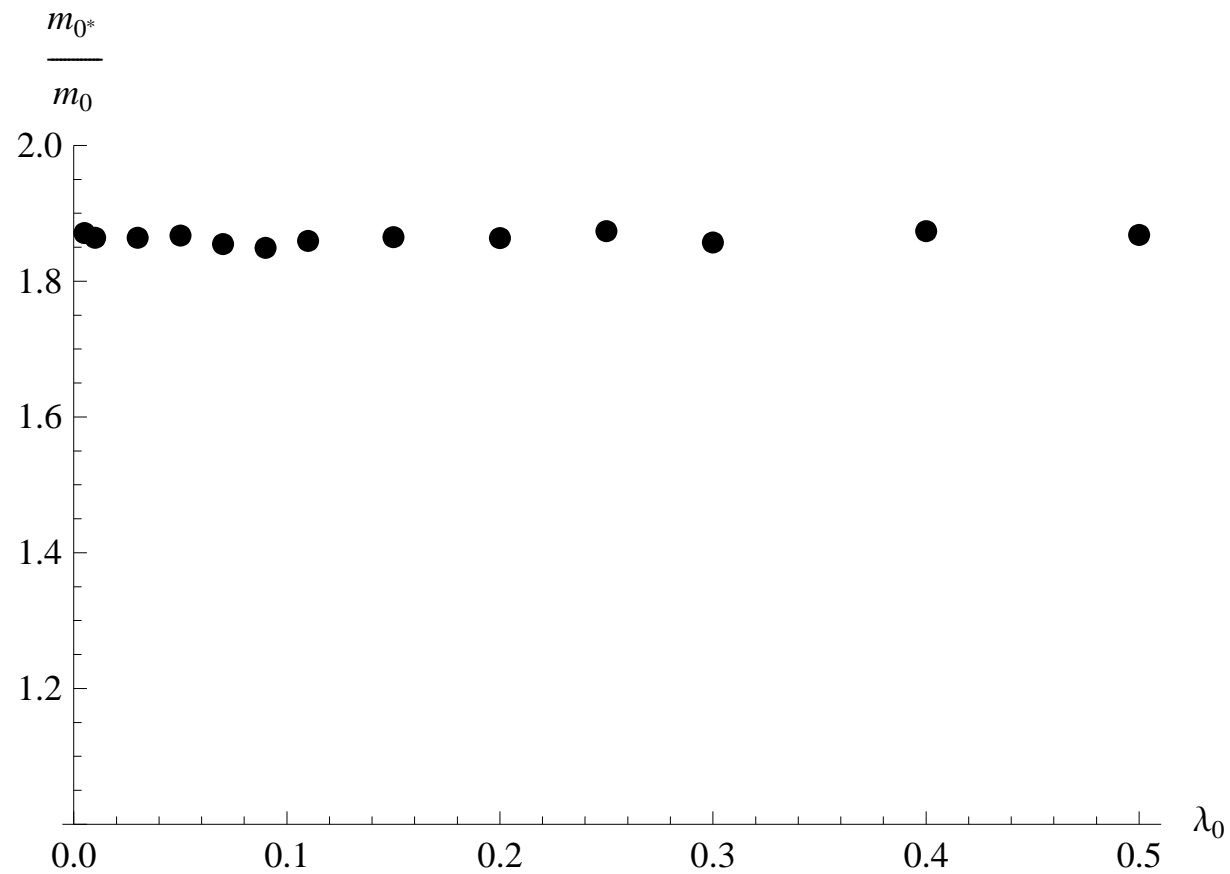
The scale factor and 't Hooft coupling that follow from  $\beta$ .  $b_0 = 4.2$ ,  $\lambda_0 = 0.05$ ,  $A_0 = 0$ . The units are such that  $\ell = 0.5$ . The dashed line represents the scale factor for pure  $AdS$ .

# Dependence of absolute mass scale on $\lambda_0$



Dependence on initial condition  $\lambda_0$  of the absolute scale of the lowest lying glueball (shown in Logarithmic scale)

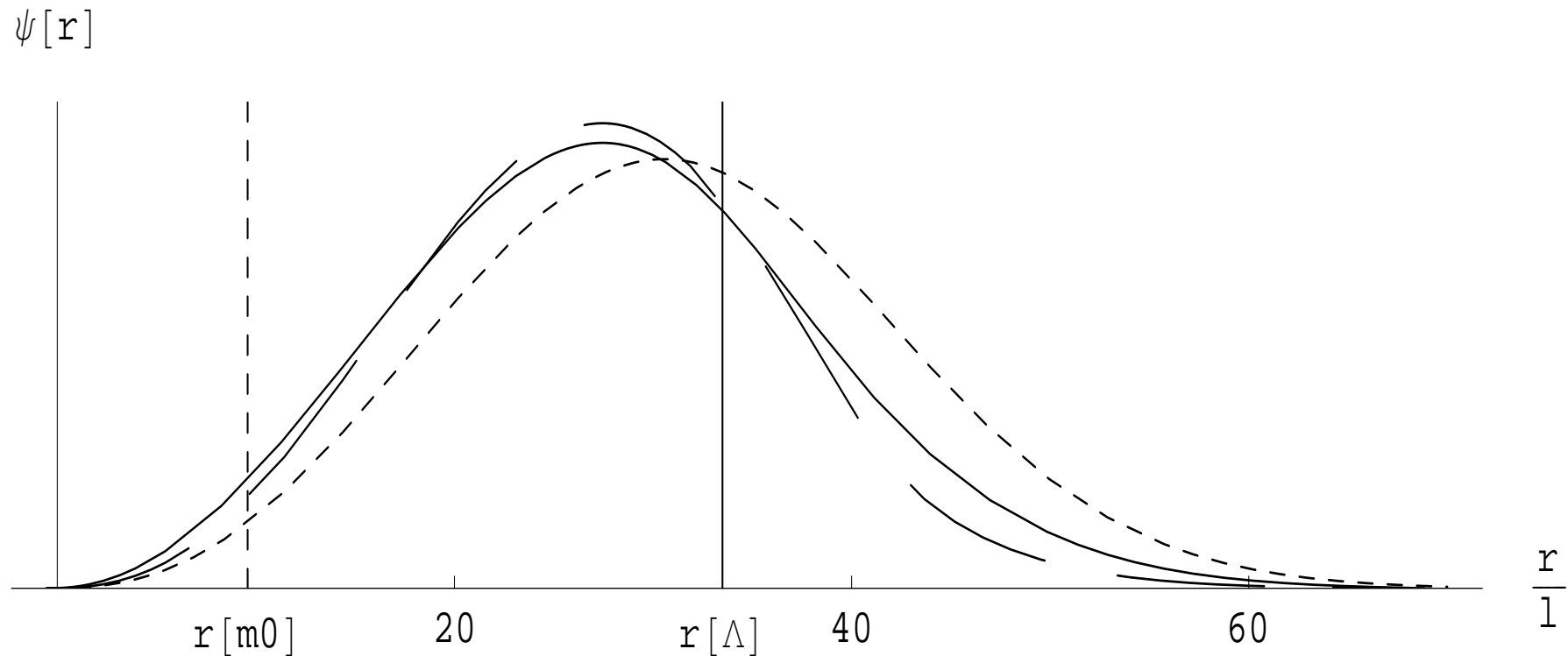
# Dependence of mass ratios on $\lambda_0$



The mass ratios  $R_{20}$

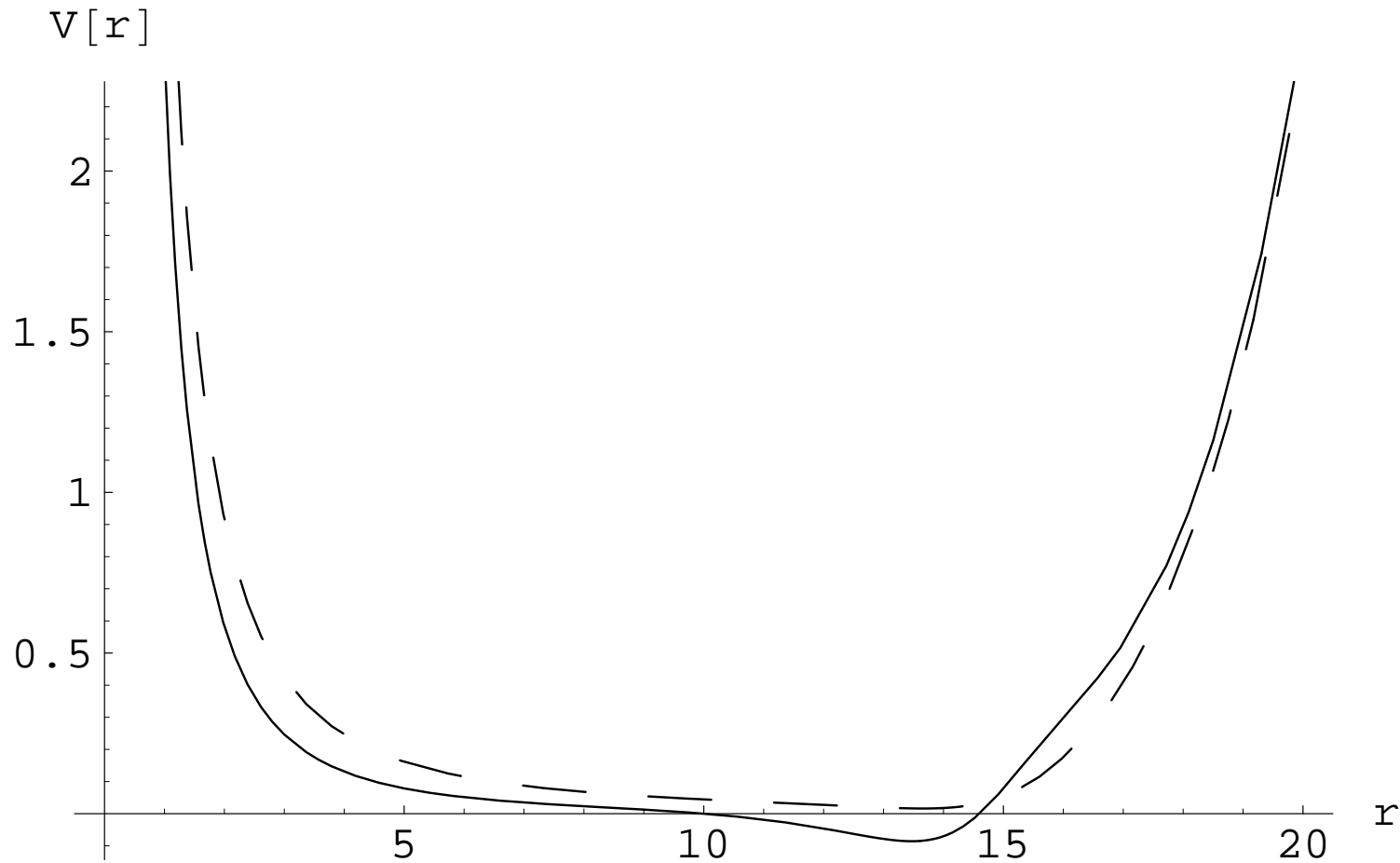
$$R_{20} = \frac{m_{2++}}{m_{0++}}.$$

# The glueball wavefunctions



Normalized wave-function profiles for the ground states of the  $0^{++}$  (solid line),  $0^{-+}$  (dashed line), and  $2^{++}$  (dotted line) towers, as a function of the radial conformal coordinate. The vertical lines represent the position corresponding to  $E = m_{0^{++}}$  and  $E = \Lambda_p$ .

# Comparison of scalar and tensor potential



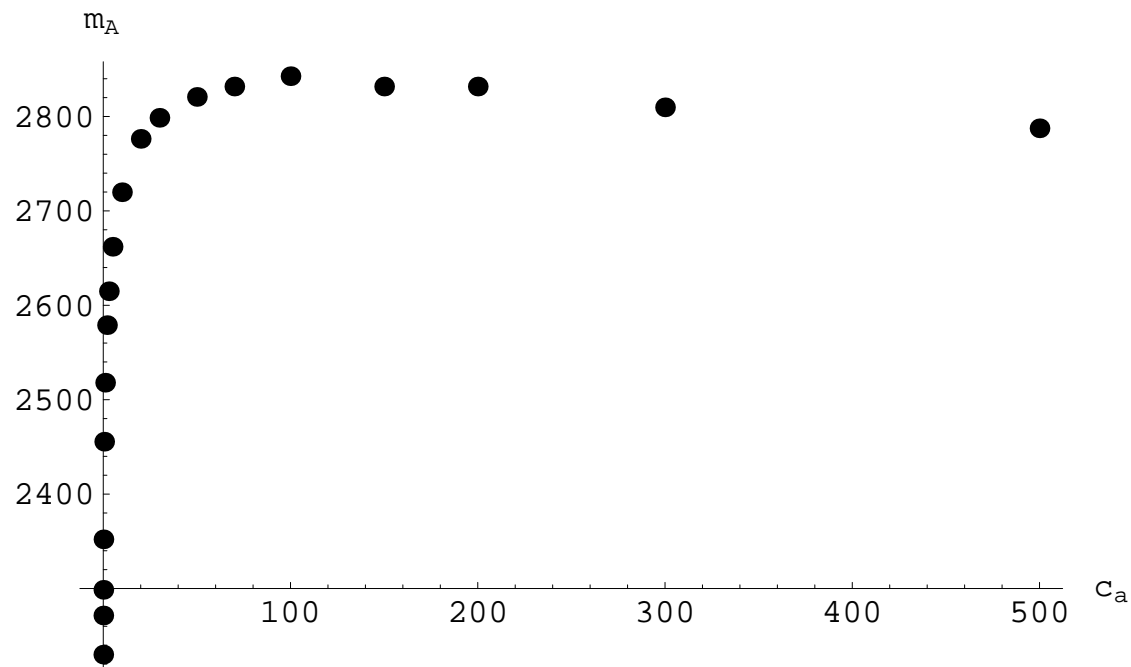
Effective Schrödinger potentials for scalar (solid line) and tensor (dashed line) glueballs. The units are chosen such that  $\ell = 0.5$ .

# The lattice glueball data

| $J^{++}$ | Ref. I ( $m/\sqrt{\sigma}$ ) | Ref. I (MeV)   | Ref. II ( $mr_0$ ) | Ref. II (MeV)  | $N_c \rightarrow \infty(m/\sqrt{\sigma})$ |
|----------|------------------------------|----------------|--------------------|----------------|---|
| 0        | 3.347(68)                    | 1475(30)(65)   | 4.16(11)(4)        | 1710(50)(80)   | 3.37(15)                                  |
| 0*       | 6.26(16)                     | 2755(70)(120)  | 6.50(44)(7)        | 2670(180)(130) | 6.43(50)                                  |
| 0**      | 7.65(23)                     | 3370(100)(150) | NA                 | NA             | NA  |
| 0***     | 9.06(49)                     | 3990(210)(180) | NA                 | NA             | NA  |
| 2        | 4.916(91)                    | 2150(30)(100)  | 5.83(5)(6)         | 2390(30)(120)  | 4.93(30)                                  |
| 2*       | 6.48(22)                     | 2880(100)(130) | NA                 | NA             | NA  |
| $R_{20}$ | 1.46(5)                      | 1.46(5)        | 1.40(5)            | 1.40(5)        | 1.46(11)                                  |
| $R_{00}$ | 1.87(8)                      | 1.87(8)        | 1.56(15)           | 1.56(15)       | 1.90(17)                                  |

Available lattice data for the scalar and the tensor glueballs. Ref. I = [H. B. Meyer, \[arXiv:hep-lat/0508002\]](#). and Ref. II = [C. J. Morningstar and M. J. Peardon, \[arXiv:hep-lat/9901004\]](#) + [Y. Chen et al., \[arXiv:hep-lat/0510074\]](#). The first error corresponds to the statistical error from the continuum extrapolation. The second error in Ref.I is due to the uncertainty in the string tension  $\sqrt{\sigma}$ . (Note that this does not affect the mass ratios). The second error in the Ref. II is the estimated uncertainty from the anisotropy. In the last column we present the available large  $N_c$  estimates according to [B. Lucini and M. Teper, \[arXiv:hep-lat/0103027\]](#). The parenthesis in this column shows the total possible error followed by the estimations in the same reference.

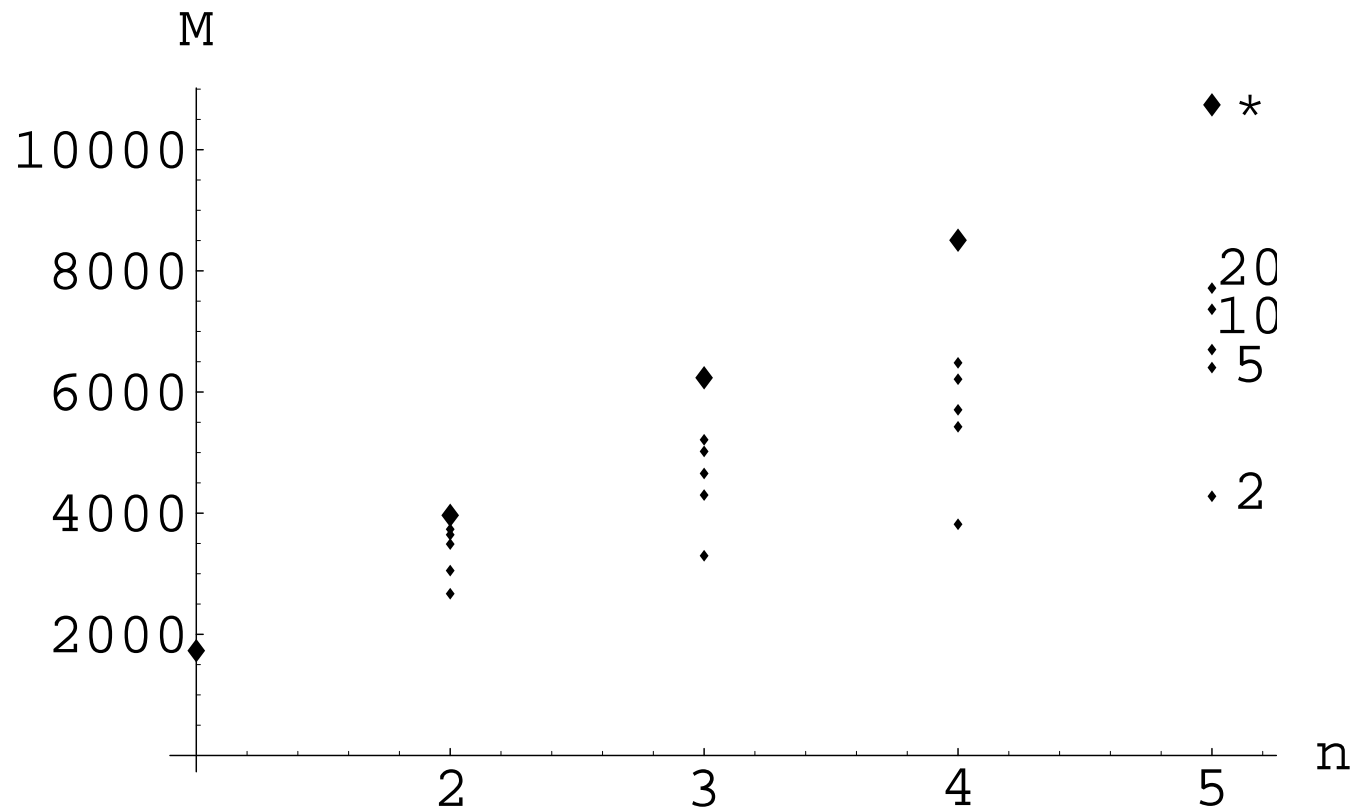
# Pseudoscalar glueballs



Lowest  $0^{-+}$  glueball mass in MeV as a function of  $c_a$  in  $Z(\lambda) = Z_a(1 + c_a\lambda^4)$ .

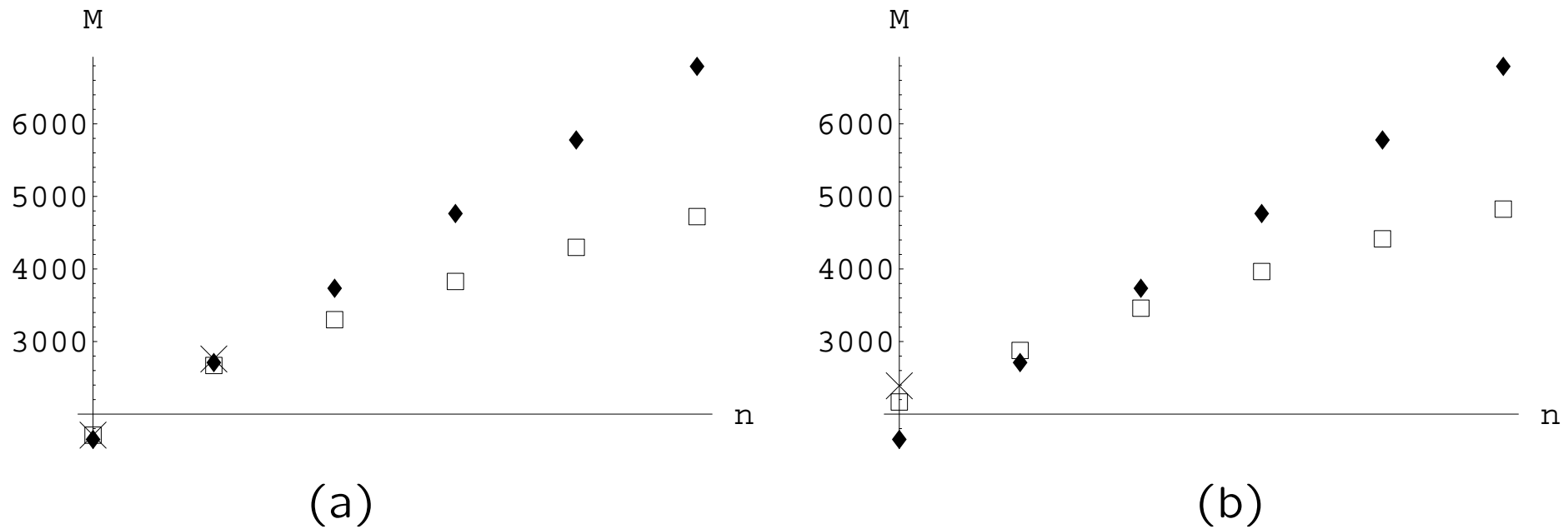


# $\alpha$ -dependence of scalar spectrum



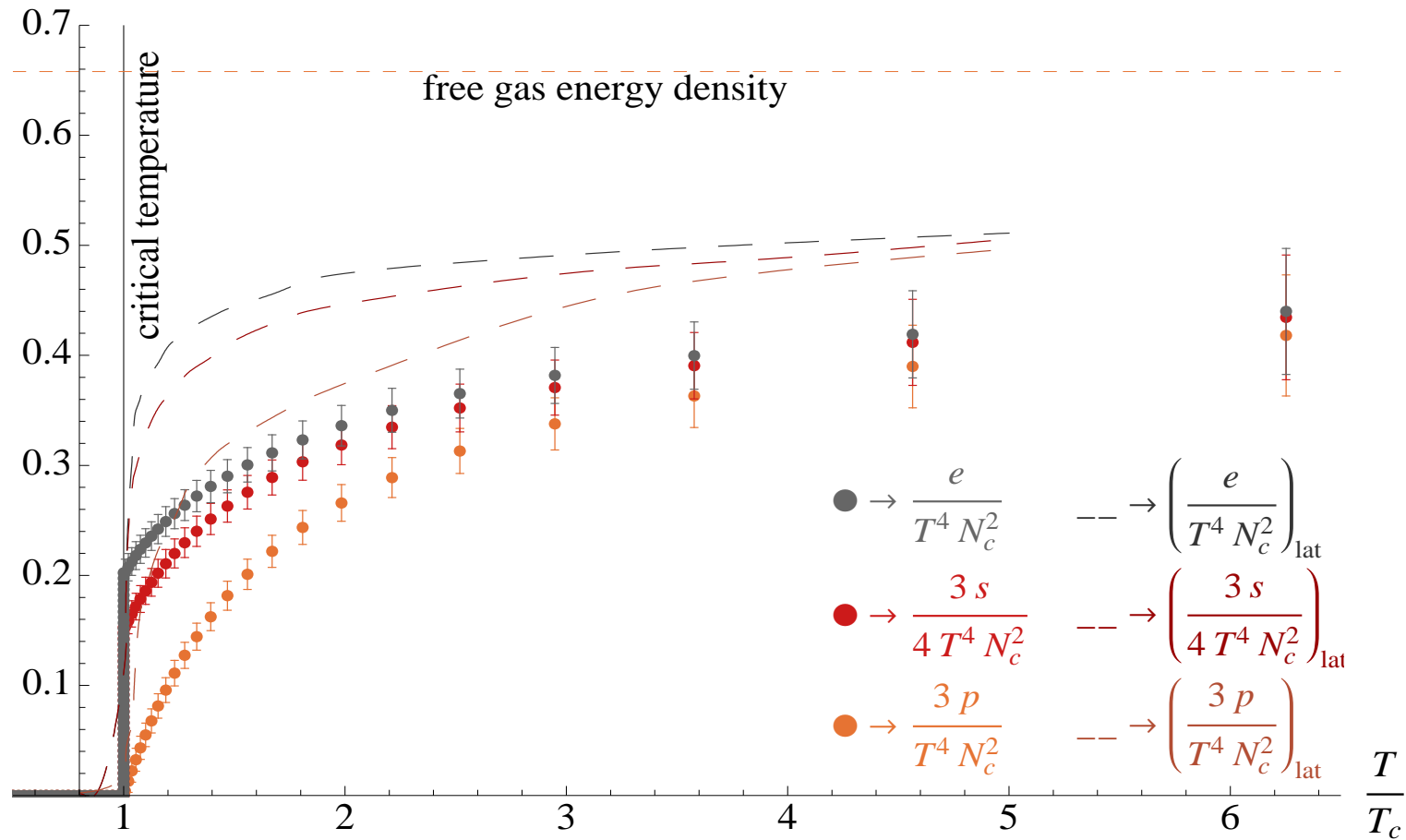
The  $0^{++}$  spectra for varying values of  $\alpha$  that are shown at the right end of the plot. The symbol \* denotes the AdS/QCD result.

# Comparison with lattice data: Ref II



Comparison of glueball spectra from our model with  $b_0 = 2.55, \lambda_0 = 0.05$  (boxes), with the lattice QCD data from Ref. II (crosses) and the AdS/QCD computation (diamonds), for (a)  $0^{++}$  glueballs; (b)  $2^{++}$  glueballs. The masses are in MeV, and the scale is normalized to match the lowest  $0^{++}$  state from Ref. II.

# The thermodynamic quantities



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- The “soft wall” 8 minutes
- A string theory for QCD: basic expectation 11 minutes
- Bosonic string or superstring 13 minutes
- Bosonic string or superstring? (continued) 14 minutes
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### THE $T = 0$ DATA

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### FINITE TEMPERATURE

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