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”*Interacting String Multi-verses and holographic instabilities of Massive Gravity* ”

Elias Kiritsis

University of Crete and Ecole Polytechnique

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Based on previous work:

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Introduction

• One of the great surprises of modern physics is the observation of the acceleration of the observable universe.

• It seems to require a vacuum/dark energy that today is dominating the energy balance of the universe ($\sim$ 74%)

• All models that have been proposed so far are highly fine-tuned: Cosmological constant, quintessence, IR modified gravity.

• It is not clear if in any of the above the fine tuning is at least "technically stable".

♠The most conservative approach involves an attempt to modify gravity in the IR. There are two classes of modifications:

• Introducing a mass (or potential) for the graviton.

• Brane-induced gravity, where the massive graviton is a resonance embedded in a (higher-d) continuum. The best example in this class is the DGP model.

Massive Gravitons

• Fierz and Pauli introduced them many decades ago. They found that a generic mass term contains a scalar ghost. A tuning at the quadratic level can get rid of it.

Fierz+Pauli (1939)

• Re-analyzed by Boulware and Deser. They confirmed Fierz+Pauli at linearized level but discovered that the ghost reappears at the non-linear level and cannot be avoided.

Boulware+Deser (1972)

• This sets gravity apart from any other IR QFT. In some sense gravity seems to react to the brutal breaking of diffeomorphism invariance (unlike standard gauge theories)

• Van Dam, Veltman and Zakharov pointed out that in the limit $m_g \to 0$, the effects of the scalar graviton mode, do not decouple and pose a real phenomenological threat.

Van Dam+Veltman (1970), Zakharov (1970)

• Vainshtein pointed out the at the non-linear level the story can be different as linearized expansion break down in general backgrounds.

Vainstein, (1972)

Massive Gravitons and cosmology

Consider the cosmology of massive gravity: *Babak+Grischuk (2002)*

$$L_{GR} = -\frac{M_P^2}{2}\sqrt{-g}\ R \quad , \quad L_{mass} = -\frac{1}{2\kappa^2}\sqrt{-\eta}\left[k_1 h^{\mu\nu}h_{\mu\nu} + k_2(h^{\mu\nu}\eta_{\mu\nu})^2\right]$$

$$k_1 = \frac{m_g^2}{4} \quad , \quad k_2 = -\frac{m_g^2}{8}\ \frac{m_g^2 + 2m_0^2}{2m_g^2 + m_0^2} \quad , \quad \zeta = \frac{m_0^2}{m_g^2} \to \infty$$

We make the cosmological ansatz

$$ds^2 = b^2(t)dt^2 - a^2(t)dx^i dx_i$$

The Einstein equations determine $b(t)$ in terms of $a(t)$:

$$4a^2b^2 - 4a^4 + 8\frac{a^3}{b} - 8 = 0 \quad , \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_P^2} - \frac{m_g^2}{4}\left(2\frac{b}{a} + \frac{1}{b^2} - 3\frac{1}{a^2}\right) \simeq \frac{\rho}{3M_P^2} + \frac{m_g^2}{2} + \mathcal{O}\left(\frac{1}{a^2}\right)$$

Kiritsis (2003)

- $\Lambda_{eff} \sim m_g^2$. If the graviton has a horizon-size wavelength, then the cosmological constant has the correct order of magnitude today

$$m_g \sim \frac{1}{H_0} \sim 10^{-32}\ \text{eV} \quad , \quad (\text{vaccuum} - \text{energy})^4 \sim m_g^2 M_P^2 \sim (10^{-3}\ \text{eV})^4$$

- But...., higher terms in the graviton potential give more important contributions unless they are fine-tuned away

Massive Gravitons and strong coupling

• Introducing Stuckelberg Fields $(A_\mu,\ \phi)$ in order to reinstate the diffeomorphism invariance can provide a clearer view of the intricacies of massive gravity

Arkhani-Hamed+Georgi+Schwartz (2002)

• The scalar mode becomes generically strongly coupled around flat space at

$$E \sim \Lambda_V = (m_g^4 M_P)^{\frac{1}{5}} \quad , \quad m_g \ll \Lambda_V \ll M_P$$

• "Fine-Tuning" the graviton potential one can push the strong coupling to

$$E \sim \Lambda_{AGS} = (m_g^2 M_P)^{\frac{1}{3}} \quad , \quad m_g \ll \Lambda_V \ll \Lambda_{AGS} \ll M_P$$

For $m_g \sim H_0^{-1}$ $\quad \frac{1}{\Lambda_V} \sim 10^{13}$ Km , $\frac{1}{\Lambda_{AGS}} \sim 10^4$ Km

• A cutoff $\Lambda_* \sim \sqrt{m_g\ M_P} \sim 10^{-3}$ eV would put the strong coupling problem at the millimeter scale. This so far has been elusive.

• Strong coupling effects depend on the background: they also happen around masses giving rise to the Vainshtein radius

$$R_V \sim \left(M/(M_P^2 m_g^4)\right)^{\frac{1}{5}}$$

Vainshtein (1972)

♠ Naive UV-IR decoupling fails in massive gravity.

Simple examples of massive gravitons

• Stringy massive gravitons: at higher levels of flat space string theory. They cannot become arbitrarily light.

$$m_g \sim M_s \quad , \quad m_g \quad \ll \quad \Lambda_V \quad \ll \quad \Lambda_{AGS} \quad \ll \quad M_{10P}$$

No signal of strong coupling in string theory.

• Massive KK gravitons: they can become arbitrarily light, but they carry a whole KK tower along with them.

$$\frac{1}{R} \quad \ll \quad \Lambda_V \quad \ll \quad \Lambda_{AGS} \quad \ll \quad M_{d+1} \quad \ll \quad M_d$$

In the interesting regime, they are described by a "massless" $d+1$-graviton. No strong coupling problem is expected and indeed this is the case.

Schwartz (2003)

• Massive gravitons on a codimension-one subspace: They can be induced on a defect inside AdS_d

Karch-Randall (2001), Porrati (2003)

A Holographic approach to massive gravity

- Our goal will be to find models of massive gravity that have gauge-theory duals

- The existence of the gauge-theory dual provides a "definition" and a UV completion for the gravitational theory.

- As our best understood holographic examples are in asymptotically AdS spaces our massive graviton theories will be such.

- Although AdS is curved, such cases do not preclude subspaces with very weak curvature.

- We will study the relevant CFTs and their RG flows and we will translate our findings in the gravitational language

Holographic String (Multi)verses

• A large-N CFT^d is dual to a string theory on $AdS_{d+1} \times X$.

• The tensor product CFT$_1^d \times$CFT$_2^d$ is dual to a product of string theories one in $AdS_{d+1} \times X_1$ the other in $AdS_{d+1} \times X_2$.

• The two AdS-spaces share a common boundary.

• There are two non-interacting “massless” $d+1$-dimensional gravitons.

• We now couple the two CFTs via a marginal/relevant coupling: $\sim \mathcal{O}_1 \, \mathcal{O}_2$. This is necessarily a double-trace coupling.

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• We expect that from the two stress tensors T_1, T_2 only one will remain conserved: $T = T_1 + T_2$ as the theories can exchange energy via the boundary.

• The other will acquire an anomalous dimension at one-loop: the dual graviton $h_1 - h_2$ with acquire a one-loop mass.

Kiritsis (2006), Aharony+Clark+Karch (2006)

• Corollary: Multiple massless gravitons are necessarily non-interacting

Bachas+Petropoulos (1992)

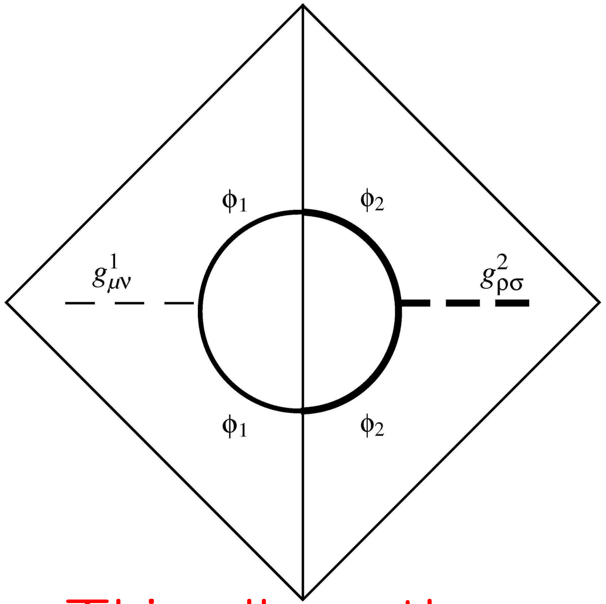
Boulanger+Damour+Gualtieri+Henneaux, (2000)

The graviton mass

A direct calculation in the field theory $CFT_1^d + CFT_2^d + h\int d^dx\, \mathcal{O}_1(x)\, \mathcal{O}_2(x)$ gives

$$\langle T_1(x)T_2(y)\rangle \sim h^2 \int d^dz_1 d^dz_2 \;\langle T_1(x)\mathcal{O}_1(z_1)\mathcal{O}_1(z_2)\rangle\; \langle T_2(y)\mathcal{O}_2(z_1)\mathcal{O}_2(z_2)\rangle$$

$$m_g^2 = \frac{d(\Delta_T - d)}{\ell^2} = \left(\frac{1}{c_1}+\frac{1}{c_2}\right)\frac{d\Delta(d-\Delta)}{(d+2)(d-1)}\,\frac{h^2}{\ell^2} \sim \mathcal{O}\left(\frac{h^2}{N^2}\right)$$



- In the bulk theory, $\mathcal{O}_1 \sim \Phi_1$ and $\mathcal{O}_2 \sim \Phi_2$, with the same mass.

- The double trace deformation induces mixed boundary conditions for Φ_1, Φ_2

Witten (2001), Berkooz+Sever+Shomer (2001), Muck (2002)

- This allows the one-loop diagram that provides a term $g_1^{\mu\nu} g_{2,\mu\nu}$ mixing the two gravitons.

The gravitational side

- Before coupling the two theories we have two non-interacting actions

$$S = S_1 + S_2 = M_1^{d-1}\int d^{d+1}x\sqrt{g_1}\left[R_1 + (\partial\Phi_1)^2 + \cdots\right] +$$

$$+ M_2^{d-1}\int d^{d+1}y\sqrt{g_2}\left[R_2 + (\partial\Phi_2)^2 + \cdots\right]$$

- After coupling the theories we have also an interaction

$$S_{12} = (\ell_1\ell_2)^{d+1}\int d^{d+1}x \int d^{d+1}y I(g_1(x), g_2(y))$$

- This interaction is non-local at the AdS scale.
- At much larger distances there is a local expansion and the theory is of the bi-gravity type. Such classical theories have been studied extensively.

Isham+Salam+Strandee, (1971)
Damour+Kogan+Papazoglou, (2002)

- To study the phase space of solutions nearby and therefore the stability of these theories we will look at renormalization group flows in the dual FT language.

Double-trace Renormalization Group flows

- Once we turn-on a double-trace coupling term between two distinct CFT's other operators turn-on too:

$$S = S_1 + S_2 + \int d^d x \left[\quad N(g_1 \mathcal{O}_1 + g_2 \mathcal{O}_2) + g_{11}(\mathcal{O}_1)^2 + g_{22}(\mathcal{O}_2)^2 + g_{12}\mathcal{O}_1\mathcal{O}_2 + \right.$$

$$\left. + \frac{1}{N} \sum_{i,j,k=1,2} g_{ijk}\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k + \cdots \right]$$

- We will calculate the RG flow to leading order in N ($\langle \mathcal{O}^{n+2} \rangle \sim \mathcal{O}(N^{-n})$) This can be done independent of the type of CFT.

- We can set the single-trace couplings $g_1 = g_2 = 0$.

- If there is a fixed point solution for g_{ij}, then all higher-trace couplings can be determined without extra conditions.

- The fixed-point conditions become:

$$(d - 2\Delta_1)g_{11} - 8g_{11}^2 - 8g_{12}^2 = 0 \quad , \quad (d - 2\Delta_2)g_{22} - 8g_{22}^2 - 8g_{12}^2 = 0$$

$$(d - \Delta_1 - \Delta_2)g_{12} - 8g_{12}(g_{11} + g_{22}) = 0$$

- There is no (non-trivial) solution with $g_{11} = g_{22} = 0$.

- There is no solution unless $\Delta_1 + \Delta_2 = d$ or $\Delta_1 = \Delta_2$ except $\Delta_1 = \Delta_2 = \frac{d}{2}$ (asymptotically-free case).

Double-trace RG flows, part II

• If $\Delta_1 + \Delta_2 = d$ there is a one-parameter family (circle) of fixed points:

$$g_{11} = -g_{22} \quad , \quad 4g_{11}^2 - a g_{11} + 4g_{12}^2 = 0$$

with $a \equiv \frac{d}{2} - \Delta_1$ as described in the graph.

• Apart from the trivial solution $g_{11} = g_{22} = g_{12} = 0$ all other points of the fixed circle have $g_{12} \neq 0$ and represent a product of AdS spaces with a massive graviton.

• The one-loop approximation is valid for the whole circle, if $a \ll 1$. However at large N, our β-functions are all-loop exact.

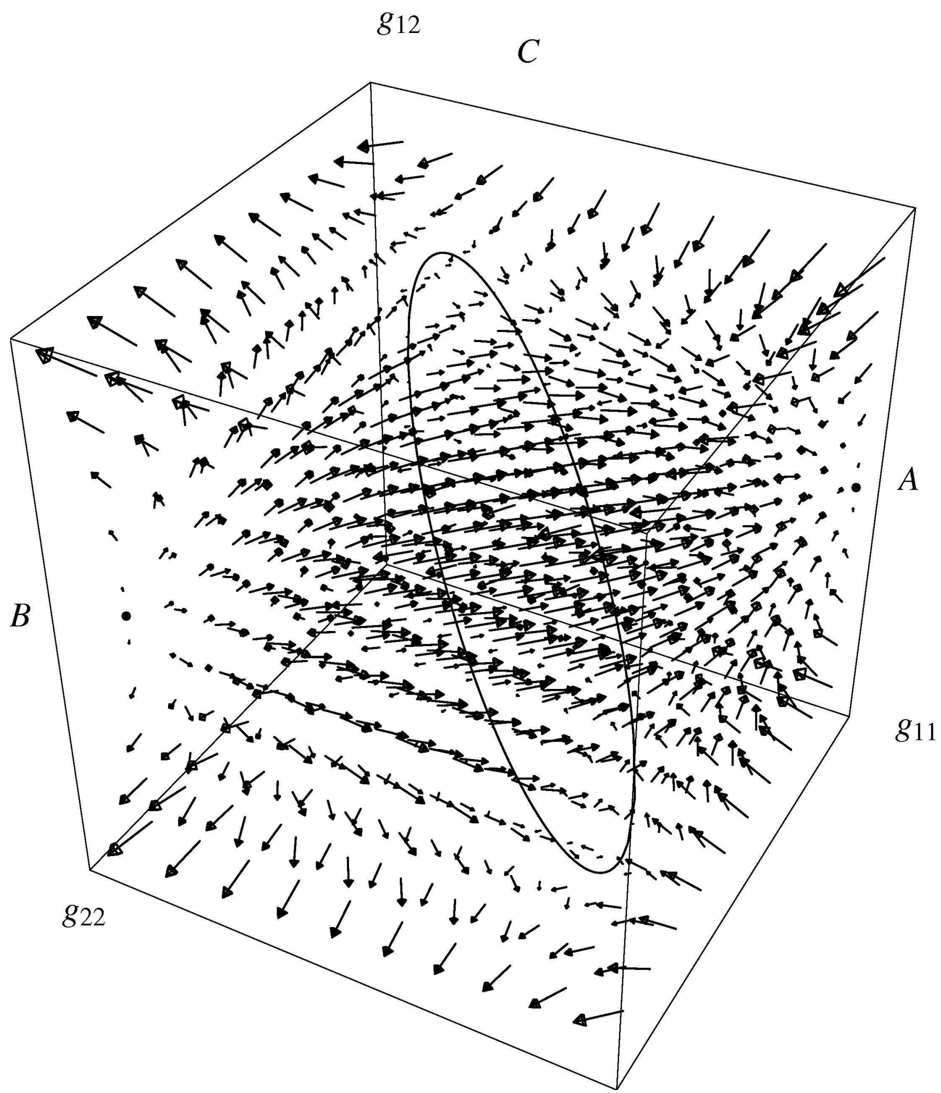
• For arbitrary a there is always a part of the circle that is perturbative (when $g_{11} \ll 1$)

♠The fixed point theories at $g_{12} = 0$, $g_{ii} \neq 0$ are dual to the "non-local" string theories of

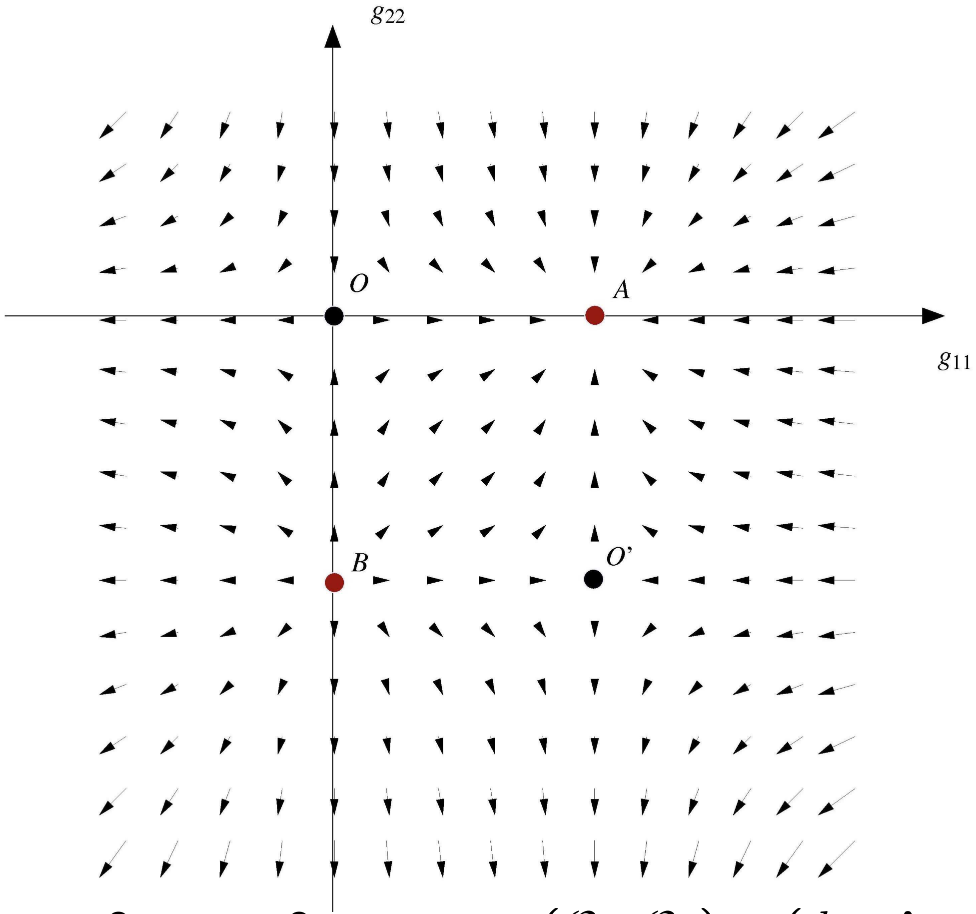
Aharony+Berkooz+Silverstein (2001)

♠All fixed points except the trivial one are repellors of the RG flow!

RG flows: the map



RG flows: the $g_{12} = 0$ slice



Stable: A : $g_{11} = \frac{a}{4}, g_{22} = 0, g_{12} = 0$ $(\mathcal{O}_1, \mathcal{O}_2) \sim (d - \Delta_1, d - \Delta_1)$

Unstable: B : $g_{11} = 0, g_{22} = -\frac{a}{4}, g_{12} = 0$ $(\mathcal{O}_1, \mathcal{O}_2) \sim (\Delta_1, \Delta_1)$

Saddle: $O = (0, 0, 0)$ $(\mathcal{O}_1, \mathcal{O}_2) \sim (\Delta_1, d - \Delta_1)$

Saddle: $O' = (\frac{a}{4}, -\frac{a}{4}, 0)$ $(\mathcal{O}_1, \mathcal{O}_2) \sim (d - \Delta_1, \Delta_1)$

RG flow: Next to leading in $1/N$

- To next order, the fixed point values of g_i are non-zero

$$g_i^{(0)} = \mathcal{O}\left(\frac{(g_{ij}^{(0)})^2}{N^2}\right)$$

- This represents a source for the bulk scalars $\Phi_{1,2}$ in the dual string theory.

- The fixed circle stays put but its shape is slightly deformed.

- The same applies to the pattern of the RG flows.

Examples in 0 and 1 dimension (Matrix Models)

The simplest example

$$\mathcal{Z} = \int D\Phi_1 D\Phi_2 \;\; e^{-N_1 \mathrm{tr}\left[\frac{1}{2}\Phi_1^2+\lambda_1\Phi_1^4\right]-N_2\left[\frac{1}{2}\Phi_2^2+\lambda_2\Phi_2^4\right]-\left[g_{11}(\mathrm{tr}\Phi_1^4)^2+g_{22}(\mathrm{tr}\Phi_2^4)^2+2g_{12}\mathrm{tr}\Phi_1^4\mathrm{tr}\Phi_2^4\right]}$$

• Analogue of conformal invariance → continuum limit

• When the deforming operators have the same scaling dimension there is a circle of fixed points.

• When not on fixed circle the theory flows to:

1. Back to the decoupled theory (weak coupling)
2. To a branched polymer phase ("strong coupling")

• At leading order in 1/N the theory on the fixed circle is a coupled Legendre transform of the decoupled theories, in agreement with the higher-d case. *Klebanov (1994)*

• Beyond the leading order the theory is genuinely non-local although in a computable way:

$$e^{\tilde{F}(\tilde{t})} = \int_{-\infty}^{\infty} dt \; e^{\tilde{t}t+F(t)} \tag{1}$$

• The old interpretation must be changed to agree with AdS-CFT: the fixed circle theories are Liouville theories with the same action containing both cosmological constants but with different interpretation of sources.

• Similar remarks apply to matrix QM.

Examples in 2 and 4 dimensions

• In 2d we can find large-N CFTs with single-trace scalar operators of arbitrary low dimension.

• The analogue of CS terms without F^2 terms in 3d CFTs is here the WZW-gauging of a (chiral) SU(N) current algebra. Example:

$$\frac{SU(N)_{k_1} \times SU(N)_{k_2}}{SU(N)_{k_1+k_2}} \quad , \quad N \to \infty \quad , \quad \lambda_1 = \frac{N}{k_1} \, , \lambda_2 = \frac{N}{k_2} \quad fixed$$

• Operators associated to $(\Box, \bar{\Box}, 1)$ are single-trace and have large-N scaling dimension

$$\Delta_{\Box,\bar{\Box},1} = \frac{\lambda_1}{1+\lambda_1} + \frac{\lambda_2}{1+\lambda_2}$$

IN FOUR DIMENSIONS:

• The Klebanov-Witten $SU(N) \times SU(N)$ theory with $\Delta_{tr[A_k B_l]} = \frac{3}{2}$.

• SQCD $N_c, N_f \to \infty$ with $x = \frac{N_c}{N_f}$ fixed. In the conformal window $\frac{1}{3} \leq x < \frac{2}{3}$ the meson operators have scaling dimension $\Delta = 3 - 3x$ and therefore satisfy $1 < \Delta \leq 2$.

• $SQCD$ with an extra adjoint chiral multiplet X in the conformal window. The relevant operators are $tr[X^k]$.

Multiply coupled CFTs

In general we can consider M CFTs, CFT_i, and various multitrace interactions coupling at time two or more CFTs

$$\delta S = \sum_I \int d^d x \, \frac{h_I}{N^{m_I - 2}} \prod_{i=1}^{m_I} \mathcal{O}_i^I(x)$$

• The general RG analysis remains valid.

• It is possible both in matrix models and in higher-d CFTs ($d < 6$) to have fixed points with triple and higher interactions only.

• Can be used to deconstruct gravitational dimensions à la Arkani-Hamed and Schwartz.

The bulk picture

• At string tree level single-trace couplings are unaffected. Therefore the scalar background solution remain trivial and the space $AdS_d \times AdS_d$.

• If the double-trace couplings are tuned, then we are at the fixed circle and the theory is unaffected. If they are not tuned then there is double-trace flow whose effects are visible in the correlation functions of the scalars.

• At one-loop one of the two gravitons gets a mass if $g_{12} \neq 0$ at tree level. The single trace couplings change. This implies that the scalars have non-trivial $\mathcal{O}(1/N^2)$ solutions

• On the (deformed) fixed circle they are constant and non-trivial. Otherwise they "run".

• The Effective field theory at one-loop is an interacting bi-gravity theory à la Damour-Kogan.. We cannot decouple the massless graviton.

• To one-loop order the space remains $AdS_d \times AdS_d$, but if we are not on the fixed circle, at two loops even this is modified.

• In general a fine-tuning is needed to achieve a massive graviton. This fine-tuning is unstable under RG flow.

• Solutions close to AdS tend to “erase” the effects of the mass.

Strong coupling, ghosts and non-locality

- Using $m_g \sim \frac{1}{N\ell}$ we find

$$\Lambda_{min} \sim \Lambda_V \sim \Lambda_{AGS} \sim \frac{1}{\ell} \sim \left(m_g M_P^{\frac{d-2}{2}}\right)^{\frac{2}{d}}$$

The same is true in $M2$ and $M5$ CFTs

- In $d = 4$ $\Lambda_{min} = \Lambda_* = \sqrt{m_g M_P}$

- There is no signal of strong coupling at $E \sim \frac{1}{\ell}$ in the bulk theory. This is also visible from the boundary QFT.

- A similar mechanism should be at work as in the case of KK gravitons: The one-loop scalar diagram in AdS has (instead of a branch cut) an infinite sequence of poles, signalling bound states with masses $\sim \frac{n}{\ell}$.

Porrati (2003)

- The lowest ones are the Stuckelberg degrees of freedom for the massive graviton.

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• The next massive ones combine with the massless graviton to cancel the strong interaction of the scalar graviton mode.

• The validity of the string effective field theory is extended to the string scale.

• In the absence of strong coupling we do not expect a ghost problem.

Deffayet+Rombouts (2005)

• The poles induce a non-locality at the AdS length scale.

• The non-locality of the bulk theory beyond tree-level is clearly visible in matrix model examples.

Conclusions

- Interacting product large-N CFTs provide a holographic setup involving interacting distinct stringy universes.

- The interaction is via boundary conditions at tree level, and induce non-trivial communication at loop level.

- Such theories contain gravitons that acquire a non-zero mass at one-loop

- They provide UV completions of massive (multi)-gravity theories.

- Inducing a graviton mass requires fine tuning in the space of back-rounds/boundary conditions.

- This fine-tuning is unstable under RG flow.

- The theories are free of strong-coupling and ghost problems for small deformations (but may not be for large deformations)

- They are non-local at the AdS scale.

Interacting Multi-verses vs Multi-throat backgrounds

- Multithroat backgrounds in a single string theory have effective field theories that contain several interacting (massive) gravitons.

- The prime example is $U(2N) \to U(N) \times U(N)$ by a Higgs vev. Integrating out the massive vector multiplets we end up with an IR $U(N) \times U(N)$ theory interacting via multitrace interactions.

- Taking $|\Phi| \to \infty$ the two $U(N)$ theories completely decouple.

- Can the interacting CFTs via UV marginal or relevant operators be considered as limits of multithroat compactifications?

- No, for relevant and marginally relevant perturbations.

- We do not know the answer for marginal perturbations.

UV-complete gravitational deconstruction

- We consider $\hat{N} \gg 1$ large-N CFTs in $d-1$ dimensions, coupled pairwise with couplings $\sim h \int \mathcal{O}_i \mathcal{O}_{i+1}$ in order to form a discrete circle.

- The dual bulk theory consists of $\hat{N}$ copies of string theory on AdS_d coupled pairwise at their common boundary.

- There is one overall massless graviton and $\hat{N}-1$ massive gravitons with masses that are multiples of $\frac{h}{N\ \ell}$.

- At large distance this is effectively the same as a single theory on $AdS_d \times S^1$ with radius

$$R = \frac{\hat{N}}{m_g} \sim \frac{N\hat{N}}{h}\ell \geq \ell \quad , \quad (M_{d+1})^{d-1} = (M_d)^{d-2}\ m_g \sim \frac{hN}{\ell^{d-1}}$$

which establish the hierarchy

$$\frac{1}{R} \quad \ll \quad \frac{1}{\ell} \quad \ll \quad M_{d+1} \quad \ll \quad M_d$$

- In $\frac{1}{R} \quad \ll \quad E \quad \ll \quad \frac{1}{\ell}$ the theory is (d+1)-dimensional and weakly coupled.

- In $\frac{1}{\ell} \quad \ll \quad E \quad \ll \quad M_{d+1}$ the theory is non-local and weakly coupled. In analogy with KK compactification it may be made local by adding more that nearest neighbor interactions

- $M_{d+1} \quad \ll \quad E \quad \ll \quad M_d$ the higher dimensional (emergent) gravity seem strongly coupled. We do not understand yet if this is real.

- This setup may allow to describe higher-dimensional gravity theories in terms of lower dimensional theories that have better UV behavior.

Open Problems

• Determine the effective multigravity theory of interacting multi-verses

• Study "non-Lorentz invariant" situations: Product CFTs in a R^d with lower-dimensional defects and multitrace couplings that involve also boundary operators.

• Investigate in more depth the relation to multi-throat geometries

• In some cases there are non-perturbative instabilities in coupled CFTs: $\mathcal{N} = 4_4$ sYM is an example. Are such instabilities generic and what is their interpretation on the string theory side? This is difficult to address even in matrix model examples.

• Study the non-locality at the AdS scale and its removal via a judicious choice of inter-universe interactions.

• Study the application of such ideas to (quasi)-realistic cosmologies.

• Investigate multi-universe generalization of "designer gravity".

• Consider similar couplings in asymptotically flat string theories. They should correspond to universes that interact via boundary conditions in the infinite past and which generate quantum correlations among them during the cosmological evolution.

Double-trace Renormalization Group flows for n CFT copies

- Consider n not necessarily identical CFTs coupled with double trace interactions

$$S = \sum_{i=1}^{n} S_i + \sum_{i,j=1}^{n} g_{ij} O_i O_j$$

where g_{ij} are symmetric and real

- The fixed point equations read

$$(d - \Delta_i - \Delta_j) g_{ij} - 8 \sum_{k=1}^{n} g_{ik} g_{jk} = 0 \quad , \quad \forall i, j$$

- When the copies are identical, $\Delta_i = \Delta_j$ then the general solution is $g = O\ D\ O^T$ where O and arbitrary $n \times n$ orthogonal matrix and D is a diagonal matrix whose diagonal entries are either 0 or $\frac{d-2\Delta}{8}$. There are n non-trivial solutions with moduli spaces isomorphic to $O(n)/O(n-i)$. A special one is the replica fixed-point

$$g_{ij} = \frac{d - 2\Delta}{8n} \quad , \quad \forall \ \ i, j = 1, 2, \cdots, n$$

• When we have n identical copies (Δ), and n conjugate identical copies ($d-\Delta$) then the general solution is

$$g = \begin{pmatrix} x_1 \mathbf{1} & x_2\, O \\ x_2\, O^T & -x_1 \mathbf{1} \end{pmatrix} \quad , \quad x_1 = \frac{d-2\Delta}{16}\cos\theta \quad , \quad x_2 = \frac{d-2\Delta}{16}(1+\sin\theta) \tag{2}$$

where O is an arbitrary $m \times m$ orthogonal matrix and $\mathbf{1}$ is the unit $m \times m$ matrix. The solution has continuous parameters isomorphic to $O(m) \times O(2)$.

• In the general case the general fixed point solution is obtained by discarding self-conjugate entries, and spliting the rest into groups that contain the same dimensions and their conjugates and then using the previous solutions.

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