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Improved Holographic QCD

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Bibliography

• The work has appeared in

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Introduction

♠ Finding alternative ways to compute in QCD is still a respectable goal.

♠ Our goal is to use input from both string theory and the gauge theory (QCD) in order to provide an improved phenomenological holographic model for real-world QCD.

♠ This is an exploratory adventure, and we will short-circuit several obstacles on the way.

♠ As we will see, we will get an interesting perspective on the physics of pure glue as well as on the quark sector.

Results

♠ The input: a potential for the dilaton in five dimensions: $V(\Phi)$.

- $V(\Phi)$ is in one to one correspondence with the YM β-function, $\beta(\lambda)$.

- The UV geometry is log-corrected AdS_5 capturing asymptotic freedom.

- We classify all IR confining geometries, and show that they are singular (and good!).

- All of them have a mass-gap and a discreet spectrum

- Only one has asymptotically-linear Regge trajectories.

- In the meson sector we use N_f flavor $D_4 + \bar{D}_4$ probe branes, with a tachyon and $U(N_f)_L \times U(N_f)_R$ vectors. We can introduce quark masses, show that chiral symmetry is broken, and determine dynamically the chiral condensate.

- We find that the YM θ-angle runs. It always renormalizes to zero at low energy. This suggests a possible intrinsic resolution to the strong CP problem.

Motivating the effective action

♠ The starting point of pure YM: Non-critical string theory in five dimensions. Shortcut: a two-derivative action in 5d involving (Modulo $B_{\mu\nu}$)

$$g_{\mu\nu} \leftrightarrow T_{\mu\nu} \quad , \quad \phi \leftrightarrow Tr[F^2] \quad , \quad a \leftrightarrow Tr[F \wedge F]$$

- The basic string motivated action for the 5d theory is

$$S_5 = M^3 \int d^5x \sqrt{g} \left[e^{-2\phi} \left(R + 4(\partial\phi)^2 + \frac{\delta c}{\ell_s^2} \right) - \frac{1}{2 \cdot 5!} F_5^2 - \frac{1}{2}(da)^2 \right]$$

$F_5 = dC_4$ seeds the D_3 branes that generate the $U(N_c)$ group. ($a \equiv C_0$, C_4 and C_2 are the RR states obtained from the bispinor decomposition in d=5)

- The C_4 equation of motion gives

$$^*F_5 = N_c$$

and the dual action in the Einstein frame $g_E = e^{\frac{4}{3}\phi}\, g_s$

$$S_E = M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\phi)^2 - \frac{e^{2\phi}}{2}(\partial a)^2 + V_s(\phi) \right] \quad , \quad V_s(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2} \left[\delta c - \frac{N_c^2}{2} e^{2\phi} \right]$$

-

♠ Higher derivative corrections involving the F_5 upon dualization provide further terms in the dilaton potential

$$V_s(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2}\left[\delta c + \sum_{n=1}^{\infty} a_n \; (N_c e^{\phi})^{2n}\right]$$

♠ This potential does not have the requisite properties for QCD. (More info)

♠ We need a potential that in the Einstein frame asymptotes to a constant $V_0 = \frac{12}{\ell^2}$ as $\lambda \to 0$.

♠ The constant can be generated by higher-derivative corrections. `Here we postulate it.`

♠ The five form will then generate a series of (perturbative) terms in λ:

$$V(\lambda) = V_0 \left(1 + \sum_{n=1}^{\infty} a_n \lambda^{a\ n}\right)$$

we will take $a = 1$ for simplicity (by adjusting the kinetic term).

♠ This matches the weak coupling expansion of perturbative QCD and will generate the perturbative β-function expansion.

♠ We will ignore higher-derivative terms associated with R and $(\partial\Phi)^2$.
(Motivated partly by the success of SVZ sum rules)

♠ The "resumed" two-derivative action reads

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x\sqrt{g}\left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda)\right] \quad , \quad \lambda = N_c\, e^{\phi}$$

after redefining the kinetic terms.

• We must choose the holographic energy: the natural choice is

$$ds^2 = e^{2A(r)}(dr^2 + dx^{\mu}dx_{\mu}) \quad , \quad E = e^{A}$$

in the Einstein frame as it is monotonic and will end at zero in the IR.

• We may now solve the equations perturbatively in λ around $\lambda = 0$ and $r = 0$ (this is a weak coupling expansion) to find

$$\frac{1}{\lambda} = L - \frac{b_1}{b_0}\log L + \frac{b_1^2}{b_0^2}\frac{\log L}{L} + \left(\frac{b_1^2}{b_0^2} + \frac{b_2}{b_0}\right)\frac{1}{L} + \frac{b_1^3}{2b_0^3}\frac{\log^2 L}{L^2} + \cdots \quad , \qquad L \equiv -b_0 \log(r\Lambda)$$

with

$$\frac{d\lambda}{d\log E} \equiv \beta(\lambda) = -b_0\lambda^2 + b_1\lambda^3 + b_2\lambda^4 + \cdots$$

$$e^A = \left[1 + \frac{4}{9\log r\Lambda} + \mathcal{O}\left(\frac{\log\log r\Lambda}{\log^2 r\Lambda}\right)\right]\frac{\ell}{r}$$

$$V = \frac{12}{\ell^2}\left[1 + \frac{8}{9}(b_0\lambda) + \frac{23 - 36\frac{b_1}{b_0^2}}{3^4}(b_0\lambda)^2 - 2\frac{324\frac{b_2}{b_0^3} + 124 + 189\frac{b_1}{b_0^2}}{3^7}(b_0\lambda)^3 + \mathcal{O}(\lambda^4)\right]$$

♠ One-to-one correspondence with the perturbative β-function, and the perturbative potential.

• α' corrections affect scheme dependence (More Info)

Organizing the vacuum solutions

- The β-function can be mapped uniquely to the dilaton potential $V(\lambda)$.

- A useful variable is the phase variable

$$X \equiv \frac{\Phi'}{3A'} = \frac{\beta(\lambda)}{3\lambda} \quad , \quad e^{\Phi} \equiv \lambda$$

- We can introduce a (pseudo)superpotential

$$W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \Phi}\right)^2 = \left(\frac{3}{4}\right)^3 V(\Phi).$$

and write the equations in a first order form:

$$A' = -\frac{4}{9}W \quad , \quad \Phi' = \frac{dW}{d\Phi} \quad , \quad \beta(\lambda) = -\frac{9}{4}\lambda\frac{d\log W}{d\log \lambda}$$

♠ The equations have three integration constants: (two for Φ and one for A) One is fixed by $\lambda \to 0$ in the UV. The other is Λ. The one in A is the choice of energy scale.

The IR regime

For any asymptotically AdS$_5$ solution ($e^A \sim \frac{\ell}{r}$):

• The scale factor $e^{A(r)}$ is monotonically decreasing

Girardelo+Petrini+Porrati+Zaffaroni

• Moreover, there are only three possible, mutually exclusive IR asymptotics:

♠ *there is another asymptotic AdS_5 region, at $r \to \infty$, where* exp $A(r) \sim \ell'/r$, *and* $\ell' \leq \ell$ (equality holds if and only if the space is exactly AdS_5 everywhere);

♠ there is a curvature singularity at some finite value of the radial coordinate, $r = r_0$;

♠ **there is a curvature singularity at $r \to \infty$, where the scale factor vanishes and the space-time shrinks to zero size.**

◇ In all the singular backgrounds the 't Hooft coupling increases without bound in the IR.

General criterion for confinement

• Relies on calculation of the static quark potential using the vev of the Wilson loop calculated via an F-string world-sheet.

Rey+Yee, Maldacena

$$T\ E(L) = S_{minimal}(X)$$

• **the geometric version**:

A geometry that shrinks to zero size in the IR is dual to a confining 4D theory if and only if the Einstein metric in conformal coordinates vanishes as (or faster than) e^{-Cr} as $r \to \infty$, for some $C > 0$.

• It is understood here that a metric vanishing at finite $r = r_0$ also satisfies the above condition.

♠ **the superpotential**

A 5D background is dual to a confining theory if the superpotential grows as (or faster than)

$$W \ \sim \ (\log \lambda)^{P/2} \lambda^{2/3} \quad \text{as} \quad \lambda \to \infty$$

for some $P \geq 0$.

the β-function

A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda\to\infty} \left(\frac{\beta(\lambda)}{3\lambda} + \frac{1}{2} \right) \log \lambda = K, \qquad -\infty \leq K \leq 0$$

(No explicit reference to any coordinate system)

• We can determine the geometry if we specify K:

Comments on confining backgrounds

• For all confining backgrounds with $r_0 = \infty$, although the space-time is singular in the Einstein frame, the string frame geometry is asymptotically flat for large r. Therefore only λ grows indefinitely.

• String world-sheets do not probe the strong coupling region, at least classically. The string stays away from the strong coupling region.

• Therefore: singular confining backgrounds have generically the property that the singularity is *repulsive*, i.e. only highly excited states can probe it. This will also be reflected in the analysis of the particle spectrum (to be presented later)

• The confining backgrounds must also screen magnetic color charges. This can be checked by calculating 't Hooft loops using D_1 probes:

♠ All confining backgrounds with $r_0 = \infty$ screen properly

♠ at finite r_0 backgrounds with $e^A \sim (r - r_0)^\delta$ with $0 < \delta < 1$ do not screen. All others OK.

♠ In particular "hard-wall" AdS/QCD confines also the magnetic quarks.

Particle Spectra: generalities

- Linearized equation:

$$\ddot{\xi} + 2\dot{B}\dot{\xi} + \Box_4 \xi = 0 \quad , \quad \xi(r,x) = \xi(r)\xi^{(4)}(x), \qquad \Box \xi^{(4)}(x) = m^2 \xi^{(4)}(x)$$

- Can be mapped to Schrodinger problem

$$-\frac{d^2}{dr^2}\psi + V(r)\psi = m^2\psi \quad , \quad V(r) = \frac{d^2B}{dr^2} + \left(\frac{dB}{dr}\right)^2 \quad , \quad \xi(r) = e^{-B(r)}\psi(r)$$

- Mass gap and discrete spectrum visible from the asymptotics of the potential.

- Large n asymptotics of masses obtained from WKB

$$n\pi = \int_{r_1}^{r_2} \sqrt{m^2 - V(r)}\; dr$$

- Spectrum depends only on initial condition for λ ($\sim \Lambda_{QCD}$) and an overall energy scale (e^A) that must be fixed.

• scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log\frac{\beta(\lambda)^2}{9\lambda^2}$$

• tensor glueballs

$$B(r) = \frac{3}{2}A(r)$$

• pseudo-scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log Z(\lambda)$$

• Universality of asymptotics

$$\frac{m^2_{n\to\infty}(0^{++})}{m^2_{n\to\infty}(2^{++})} \to 1 \quad , \quad \frac{m^2_{n\to\infty}(0^{++})}{m^2_{n\to\infty}(1^{--})} = \frac{m^2_{n\to\infty}(2^{++})}{m^2_{n\to\infty}(1^{--})} = \frac{36}{25}$$

• Only one IR background gives linear trajectories: $\lambda \sim e^{Cr^2}$ corresponding to $V(\lambda) \sim \lambda^{\frac{4}{3}}\sqrt{\log\lambda}$.

$$\frac{m^2_{n\to\infty}(0^{+-})}{m^2_{n\to\infty}(0^{++})} = \frac{1}{2}(d-2)^2 \quad \text{predicts} \quad d = 2+\sqrt{2}$$

via $\frac{m^2}{2\pi\sigma_a} = 2n + J + c,$

The meson sector ($N_f << N_c$)

• Flavor is introduced via the introduction of N_f pairs of space filling $D_4+\bar{D}_4$ branes.

• The important world-volume fields are the tachyon T_{ij} in $(N_f, \bar{N}_f)$ and the $U(N_f)_L \times U(N_f)_R$ vectors:

Casero+Kiritsis+Paredes

$$T_{ij} \leftrightarrow \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \quad , \quad A^{ij\,L,R}_{\mu} \leftrightarrow \bar{q}^i_a \frac{1 \pm \gamma^5}{2} \gamma^{\mu} q^j_a$$

They generate the $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

• The UV mass matrix m_{ij} corresponds to the source term of the Tachyon field.

• The D-WZW sector depends nontrivial on T and realizes properly the P and C symmetries. It generates the appropriate gauge and global flavor anomalies.

• We can dynamically determine the chiral condensate as function of the bare UV masses.

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• We have naturally the χSB breaking order parameter T, and consistency with anomalies implies that it is non-zero and proportional to the identity (Holographic Coleman+Witten theorem).

• The pions appear as Goldstone bosons when $m_q = 0$.

• The correct GOR relation is obtained.

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2} \langle \bar{q} q \rangle \quad , \quad m_q \to 0$$

• There is linear confinement ($M_n^2 \sim n$) associated with the vanishing of the tachyon potential at $T \to \infty$.

• We obtain the correct Stuckelberg coupling mixing with 0^{+-} and the mass for the η'.

Tachyon dynamics

• In the vacuum the gauge fields vanish and $T \sim \mathbf{1}$. Only DBI survives

$$S[\tau] = T_{D_4} \int drd^4x \, \frac{e^{4A_s(r)}}{\lambda} \, V(\tau) \, \sqrt{e^{2A_s(r)} + \dot{\tau}(r)^2} \quad , \quad V(\tau) = e^{-\frac{\mu^2}{2}\tau^2}$$

• We obtain the nonlinear field equation:

$$\ddot{\tau} + \left(3\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right)\dot{\tau} + e^{2A_S}\mu^2\tau + e^{-2A_S}\left[4\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right]\dot{\tau}^3 + \mu^2\tau\;\dot{\tau}^2 = 0.$$

• In the UV we expect

$$\tau = m_q \; r + \sigma \; r^3 + \cdots \quad , \quad \mu^2\ell^2 = 3$$

• We expect that the tachyon must diverge before or at $r = r_0$. We find that indeed it does so `at the singularity`. For the $r_0 = \infty$ backgrounds

$$\tau \sim \exp\left[\frac{2}{a}\,\frac{R}{\ell^2}\; r\right] \qquad \text{as} \qquad r \to \infty$$

• Generically the solutions have spurious singularities: $\tau(r_*)$ stays finite but its derivatives diverges as:

$$\tau \sim \tau_* + \gamma\sqrt{r_* - r}.$$

The condition that they are absent determines σ as a function of m_q.

• The easiest spectrum to analyze is that of vector mesons. We find $(r_0 = \infty)$

$$\Lambda_{glueballs} = \frac{1}{R}, \qquad \Lambda_{mesons} = \frac{3}{\ell}\left(\frac{\alpha\ell^2}{2R^2}\right)^{(\alpha-1)/2} \propto \frac{1}{R}\left(\frac{\ell}{R}\right)^{\alpha-2}.$$

This suggests that $\alpha = 2$ is preferred also from the glue sector.

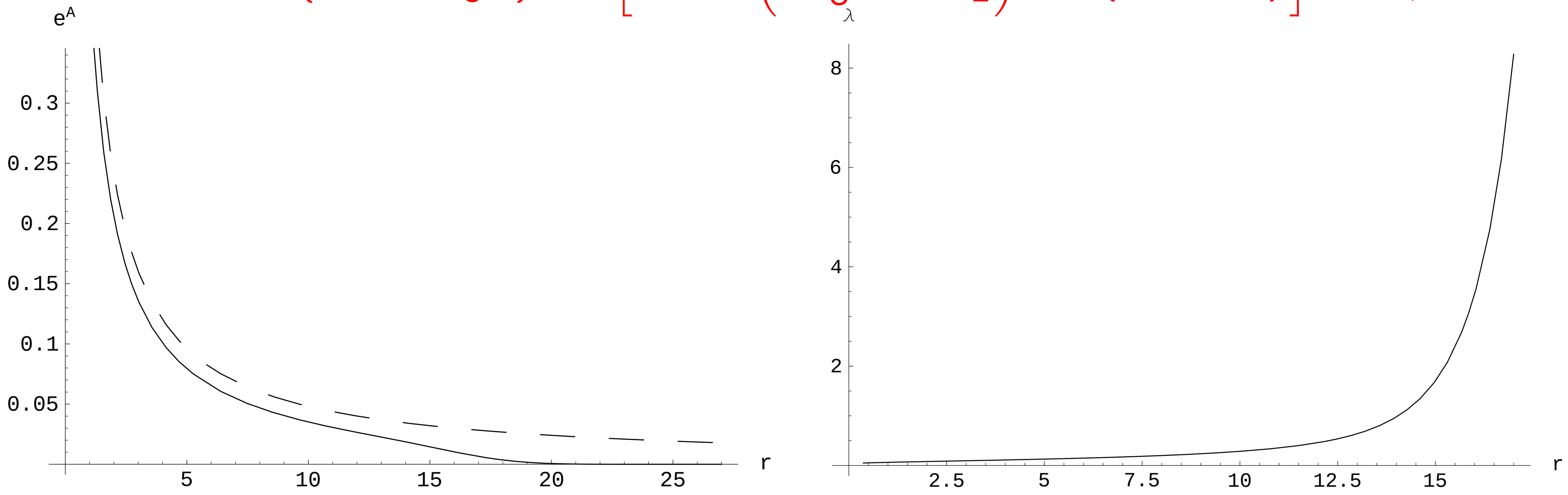
Concrete model: I

- $r_0 = \infty$ and $a = 2$:

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3+2b_0\lambda} - \frac{3a(2b_0^2+3b_1^2)\lambda^3}{(1+\lambda^2)\left(9a+\left(2b_0^2+3b_1^2\right)\log(1+\lambda^2)\right)}$$

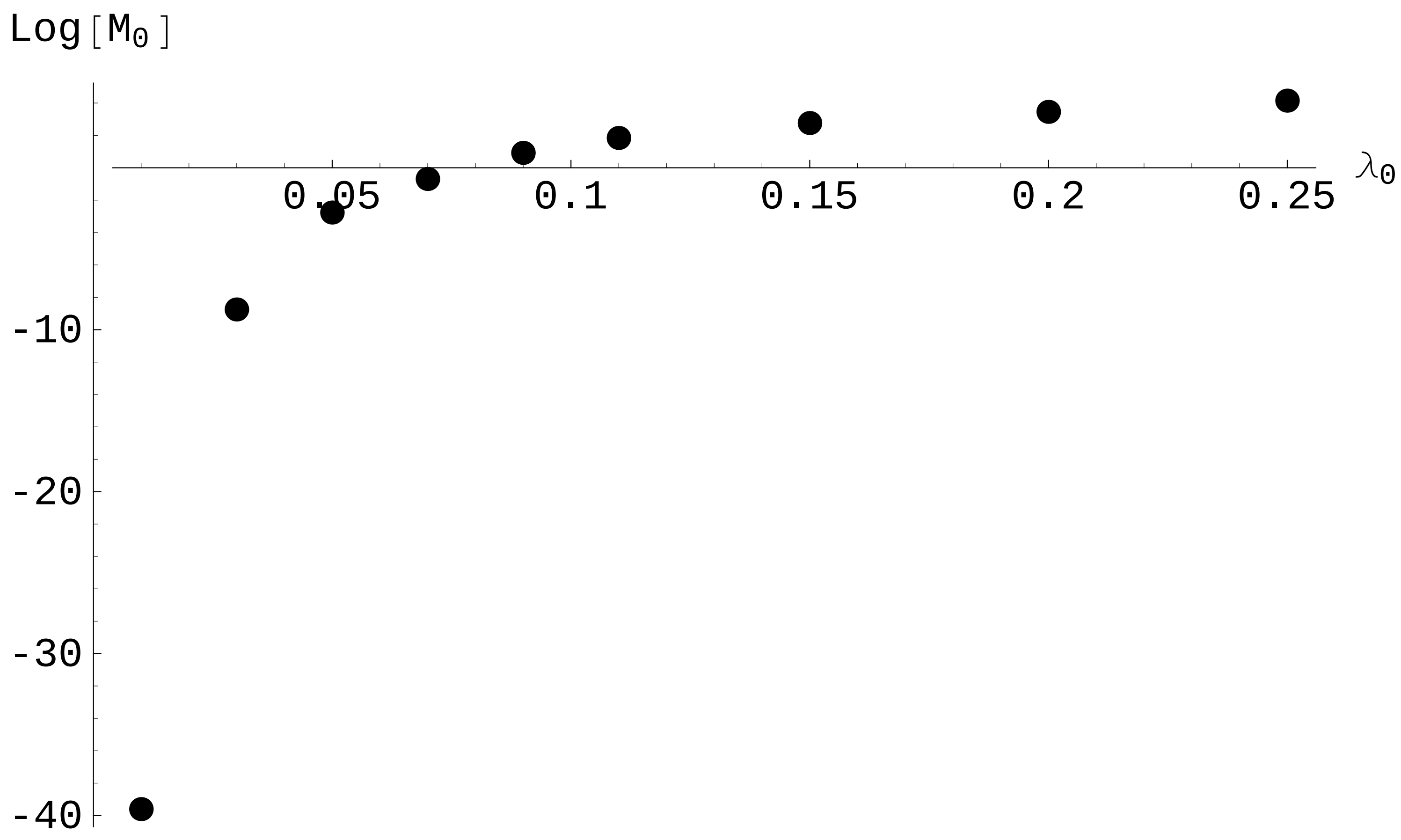
is everywhere regular and has the correct UV and IR asymptotics.

$$W = (3+2b_0\lambda)^{2/3}\left[9a+\left(2b_0^2+3b_1\right)\log(1+\lambda^2)\right]^{2a/3},$$



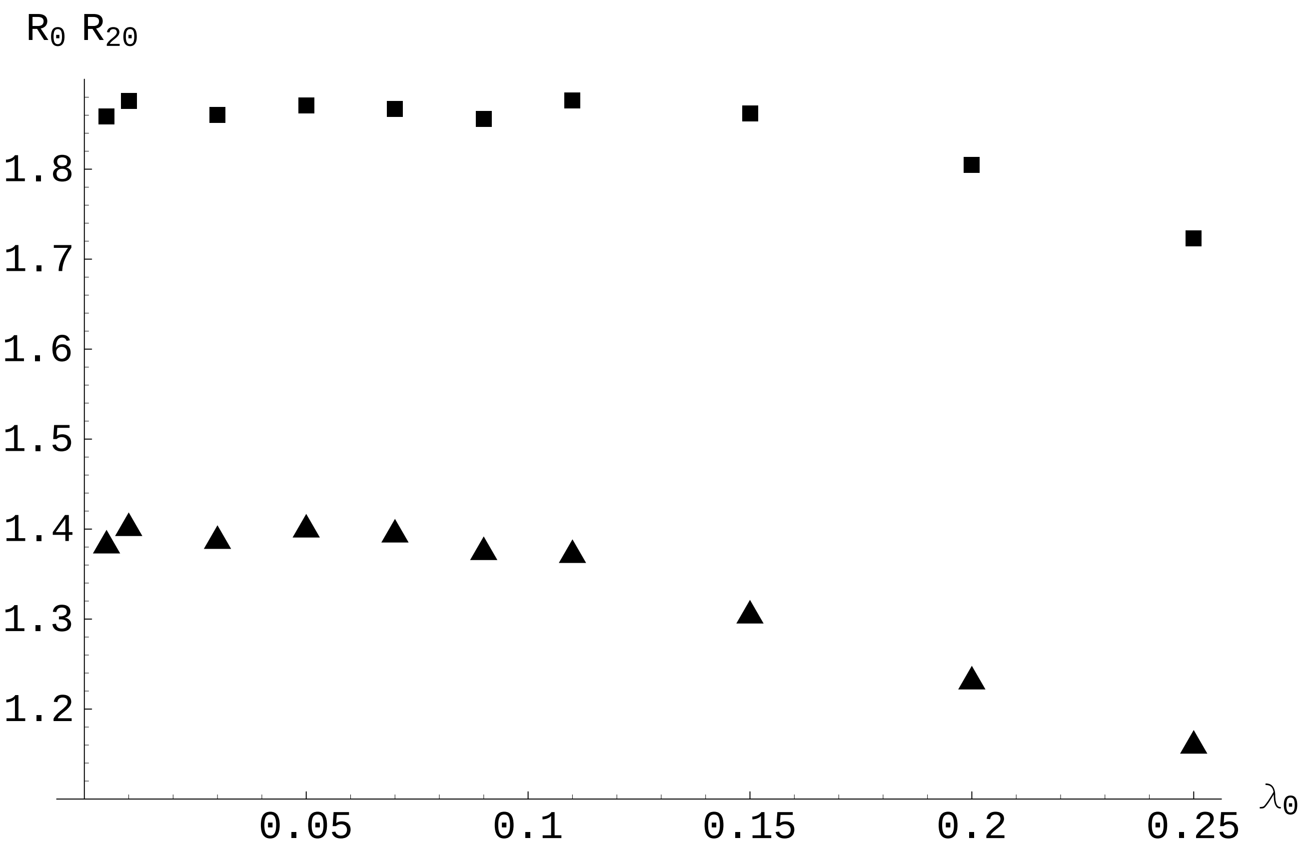
The scale factor and 't Hooft coupling that follow from β. $b_0 = 4.2$, $\lambda_0 = 0.05$, $A_0 = 0$. The units are such that $\ell = 0.5$. The dashed line represents the scale factor for pure AdS.

Dependence of absolute mass scale on λ_0



Dependence on initial condition λ_0 of the absolute scale of the lowest lying glueball (shown in Logarithmic scale)

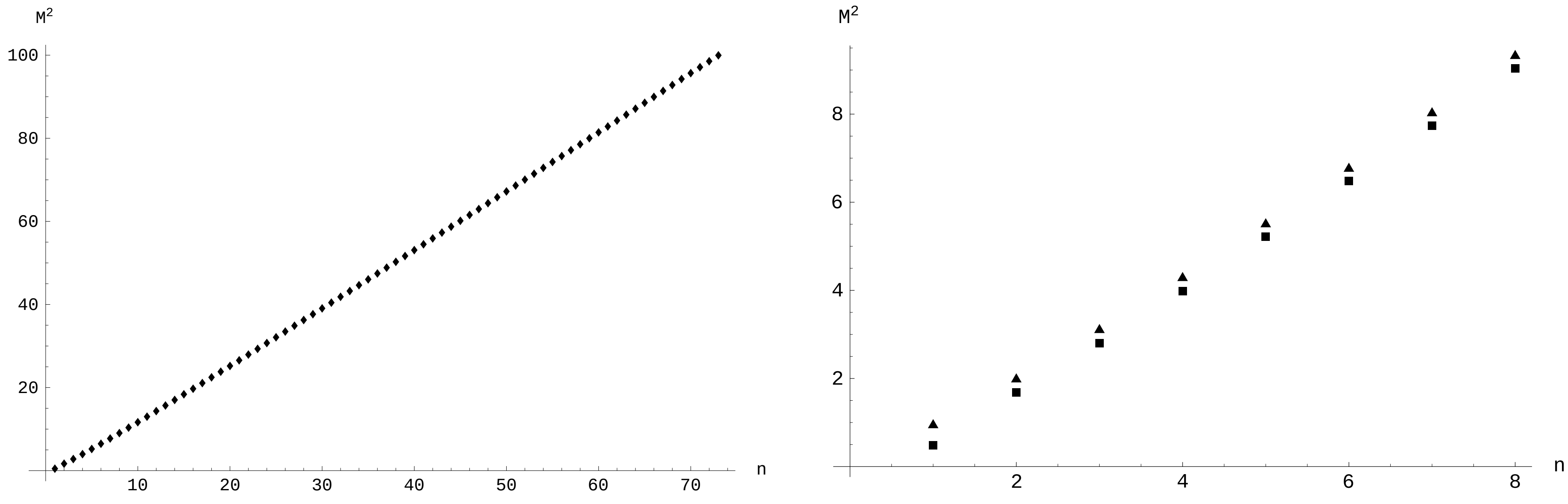
Dependence of mass ratios on λ_0



The mass ratios R_{00} (squares) and R_{20} (triangles).

$$R_{00} = \frac{m_{0^*++}}{m_{0++}}, \qquad R_{20} = \frac{m_{2++}}{m_{0++}}.$$

Linearity of the glueball spectrum

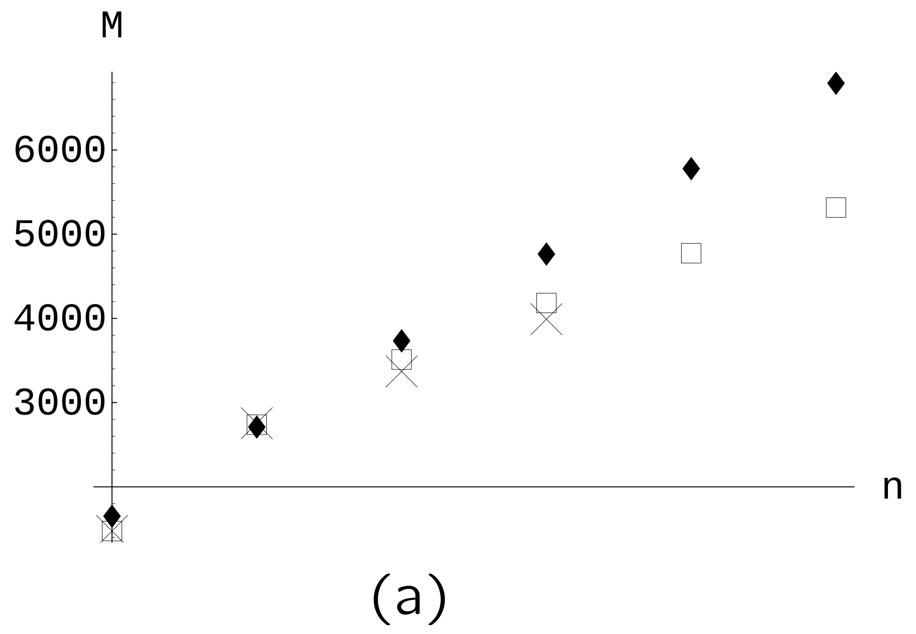


(a) (b)

(a) Linear pattern in the spectrum for the first 40 0^{++} glueball states. M^2 is shown units of $0.015\ell^{-2}$.

(b) The first 8 0^{++} (squares) and the 2^{++} (triangles) glueballs. These spectra are obtained in the background I with $b_0 = 4.2, \lambda_0 = 0.05$.

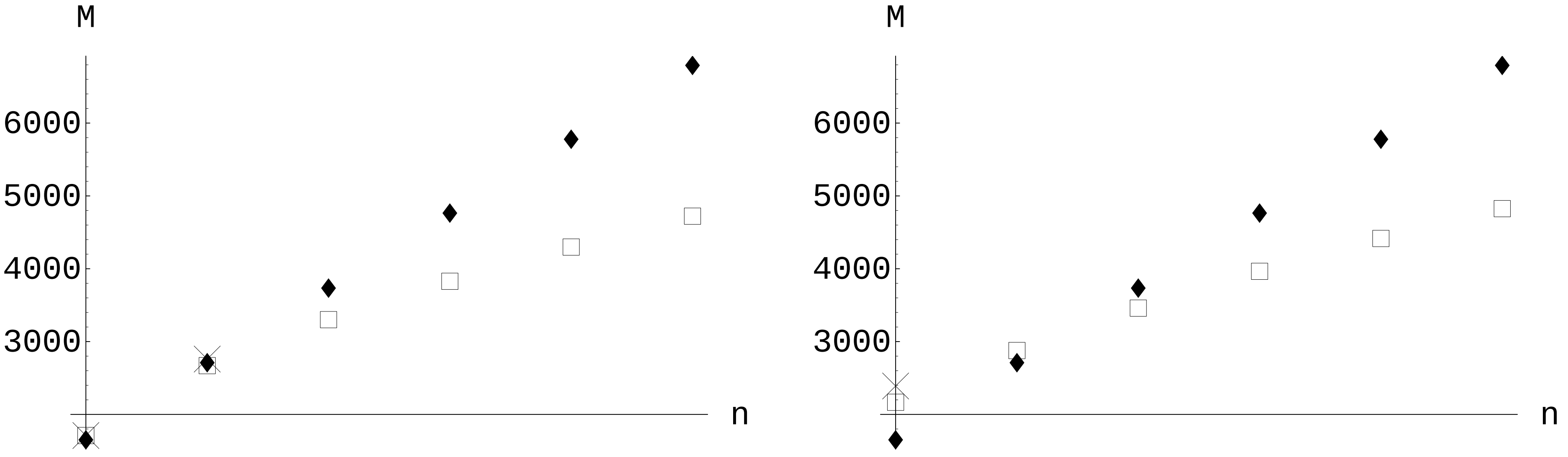
Comparison with lattice data: Ref I



Comparison of glueball spectra from our model with $b_0 = 4.2, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. I (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. I.

$$\ell_{\text{AdS}} = 6.57\ \ell_s \quad , \quad \ell_s^2\ R = -0.46$$

Comparison with lattice data: Ref II

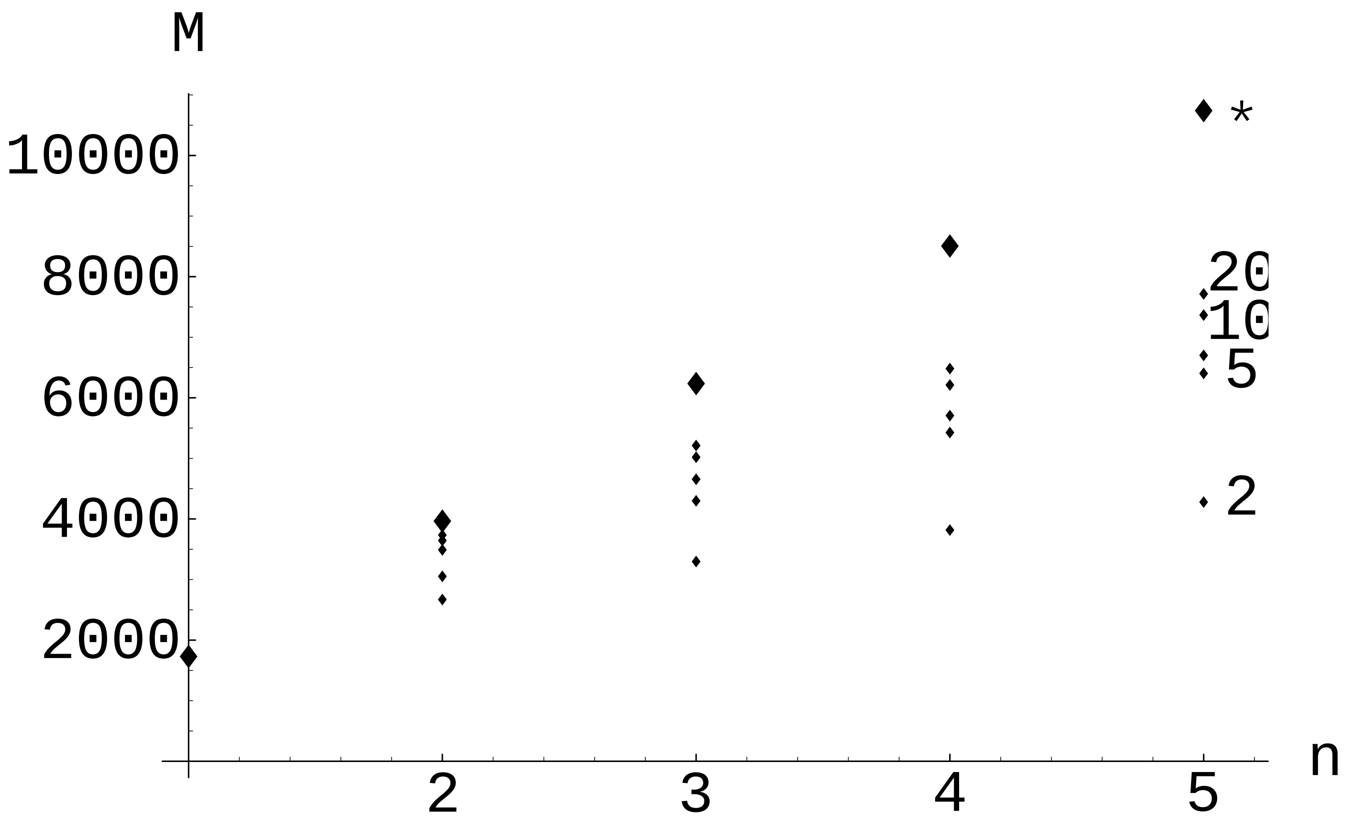


(a) (b)

Comparison of glueball spectra from our model with $b_0 = 2.55, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. II (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. II.

α-dependence of scalar spectrum

$$\lambda(E) \sim E^{-\frac{3}{2}} \; (\log(E))^{\frac{3}{4}\frac{\alpha-1}{\alpha}}$$



The 0^{++} spectra for varying values of α that are shown at the right end of the plot. The symbol $*$ denotes the AdS/QCD result.

Confining background II: r_0 =finite

- We choose a regular β-function with appropriate asymptotics:

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3+2b_0\lambda} - \frac{3\eta(2b_0^2+3b_1^2)\lambda^3}{9\eta+2\left(2b_0^2+3b_1^2\right)\lambda^2}, \qquad \eta \equiv \sqrt{1+\delta^{-1}} - 1$$

- Confining backgrounds with r_0 =finite have a hard time to match the lattice results, even for the first few glueballs.

The QCD axion background

- The kinetic term of the axion is suppressed by $1/N_c^2$. (it is an angle in the gauge theory, it is RR in string theory)

$$S_{\rm axion} = \frac{M^3}{2N_c^2}\int d^5x\ \sqrt{g}\ Z(\lambda)\ (\partial a)^2 \quad , \quad \dot{a} = \frac{C\ e^{-3A}}{Z(\lambda)}$$

with $Z(\lambda) = \lambda^2 + \cdots$ as $\lambda \to 0$. It can be interpreted as the flow equation of the effective θ-angle.

$$a(r) = \theta_{UV} + C\ \int_0^r r\frac{e^{-3A}}{Z(\lambda)} \quad , \quad C = \langle Tr[F\wedge F]\rangle$$

- The vacuum energy is

$$E(\theta_{UV}) = \frac{M^3}{2N_c^2}\int d^5x\sqrt{g}Z(\lambda)(\partial a)^2 = \frac{M^3}{2N_c^2}Ca(r)\Big|_{r=0}^{r=r_0}$$

- Consistency requires to impose that $a(r_0) = 0$. This determines C and

$$E(\theta_{UV}) = -\frac{M^3}{2N_c^2\int_0^{r_0}\frac{dr}{e^{3A}Z(\lambda)}}\theta_{UV}^2 \quad , \quad a(r) = \frac{\theta_{UV}}{\int_0^{r_0}\frac{dr}{e^{3A}Z(\lambda)}}\int_r^{r_0}\frac{dr}{e^{3A}Z(\lambda)}$$

A minimal solution to the strong CP problem?

- The IR effective θ-angle vanishes, independent of θ_{UV}!

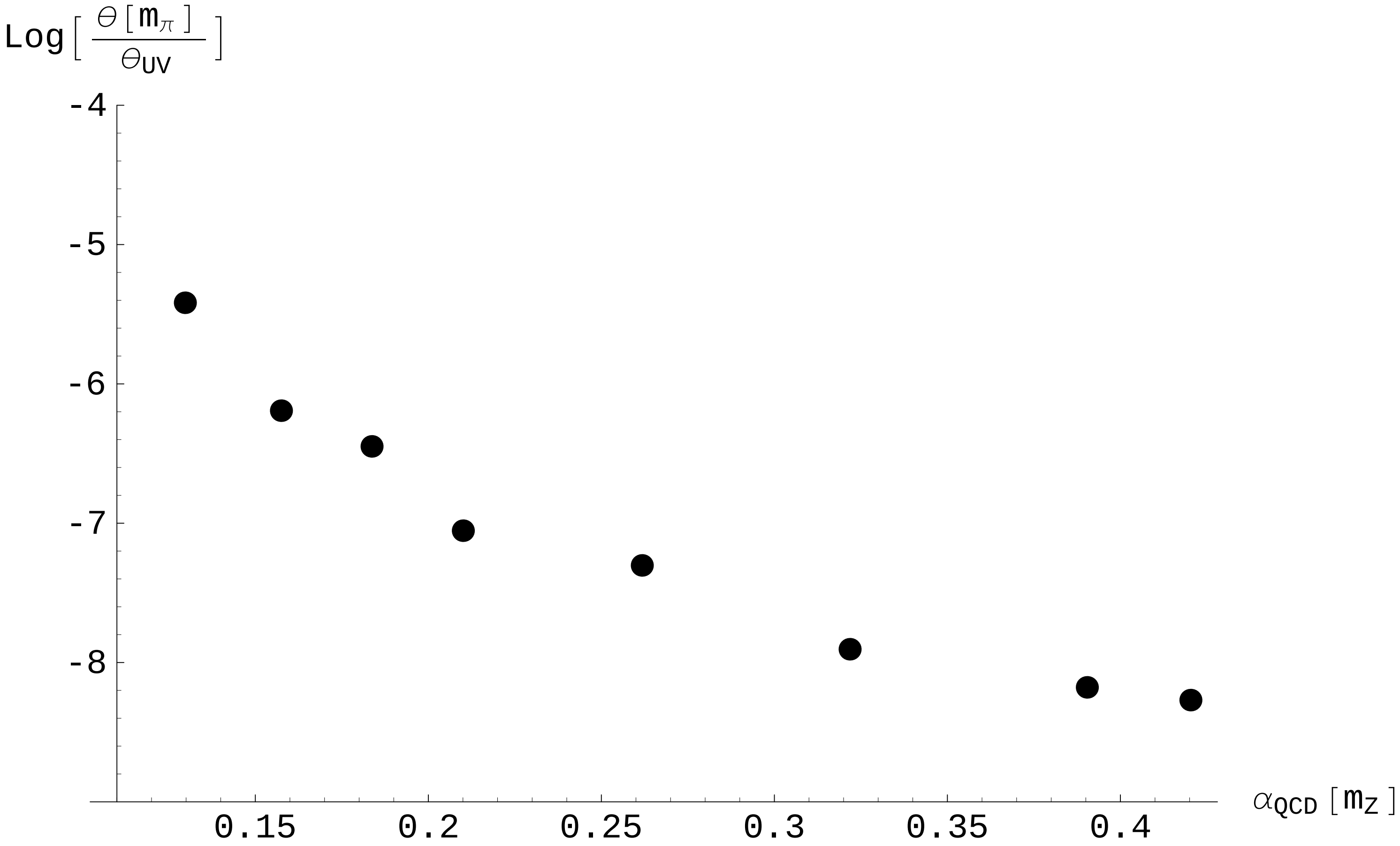
- For $Z(\lambda) \sim \lambda^d$ as $\lambda \to \infty$

$$a(E) \sim E^{\frac{3}{2}(d-2)} \, (\log E)^a \qquad \text{as} \qquad E \to 0$$

- The presence of a discrete gapped 0^{+-} spectrum implies that $d > 2$. Universality of the adjoint string tension gives $d = 2 + \sqrt{2}$.

- We know that $\theta < 10^{-8}$ from electric dipole of the neutron d_n. This assumes that θ does not run.

- It is an interesting possibility that d_n is very small because a(E) vanishes fast in the IR.



θ(100 MeV) (in logarithmic scale) vs. the strong coupling constant at 90 GeV.

Open ends

• This phenomenological approach towards an improved holographic QCD model is preliminary but seems promising. Several open paths:

♠ Determine the finite temperature solutions and the resulting deconfining transitions. The shear viscosity ratio is still $1/4\pi$.

♠ Calculate the meson spectrum and compare with data.

♠ Explore the baryon spectrum

♠ Diagonalize the $\eta' - 0^{+-}$ system and compare with data.

♠ Recalculate the dipole moment of the neutron in connection with the strong CP problem.

♠ Turn on finite baryon and isospin chemical potential and study the associated phase diagram.

♠ Calculate RHIC/LHC finite T observables.

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Thank you for your patience!

A preview of the results: pure glue

♠ The starting point of pure QCD: a two-derivative action in 5d involving

$$g_{\mu\nu} \leftrightarrow T_{\mu\nu} \quad , \quad \phi \leftrightarrow Tr[F^2] \quad , \quad a \leftrightarrow Tr[F \wedge F]$$

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} - \frac{Z(\lambda)}{2N_c^2}(\partial a)^2 + V(\lambda) \right] \quad , \quad \lambda = N_c\, e^{\phi}$$

with

$$V(\lambda) = V_0 \left(1 + \sum_{n=1}^{\infty} V_n \lambda^n \right) = -\frac{4}{3}\lambda^2 \left(\frac{dW}{d\lambda} \right)^2 + \frac{64}{27} W^2.$$

- There is a one-to one-correspondence between the QCD $\beta(\lambda)$ and W:

$$\beta(\lambda) = -\frac{9}{4}\lambda^2 \, \frac{d \log W(\lambda)}{d\lambda}$$

- There is a similar statement between $Z(\lambda)$ and the (non-perturbative) β-function for the θ-angle.

• The space is asymptotically AdS_5 in the UV ($r \to 0$) modulo log corrections (in the Einstein frame):

$$ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu) \quad , \quad E \equiv e^{A(r)}$$

• There are various extra α' corrections to the potential ($\sim \beta$-function). They only correct the non-universal terms. Moreover, α' corrections to E can be set to zero in a special scheme (the "holographic" scheme).

• ALL confining backgrounds have an IR singularity at $r = r_0$. There are two classes: r_0=finite and $r_0 = \infty$. The singularity is always "good": all spectra are well defined without extra input.

• For regular $V(\lambda)$, $\lambda \to \infty$ at the IR singularity.

• In the $r_0 = \infty$ class of backgrounds, the curvature (in the string frame) vanishes in the neighborhood of the IR singularity.

Classification of confining superpotentials $W(\lambda)$ as $\lambda \to \infty$ in IR:

$$W(\lambda) \sim (\log \lambda)^{\frac{P}{2}} \lambda^Q \quad , \quad \lambda \sim E^{-\frac{9}{4}Q} \left(\log \frac{1}{E}\right)^{\frac{P}{2Q}} , \qquad E \to 0.$$

- $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

$$e^A(r) \sim \begin{cases} (r_0 - r)^{\frac{4}{9Q^2-4}} & Q > \frac{2}{3} \\ \exp\left[-\frac{C}{(r_0-r)^{1/(P-1)}}\right] & Q = \frac{2}{3} \end{cases}$$

- $Q = 2/3$, and $0 \leq P < 1$ leads to confinement and a singularity at $r = \infty$ The scale factor e^A vanishes there as

$$e^A(r) \sim \exp[-C r^{1/(1-P)}].$$

- $Q = 2/3, P = 1$ leads to confinement but the singularity may be at a finite or infinite value of r depending on subleading asymptotics of the superpotential.

♠ If $Q < 2\sqrt{2}/3$, no *ad hoc* boundary conditions are needed to determine the glueball spectrum $\to$ One-to-one correspondence with the β-function This is unlike standard AdS/QCD and other approaches.

- when $Q > 2\sqrt{2}/3$, the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

• For all potentials that confine, the spectrum of 0^{++} and 2^{++} glueballs has a mass gap and is purely discrete. For the 0^{+-} glueballs this is the case if

$$Z(\lambda) \sim \lambda^d \quad , \quad d > 2 \quad \text{as} \quad \lambda \to \infty.$$

• In all physically interesting confining backgrounds the magnetic color charges are screened. This is an improvement with respect to AdS/QCD models (magnetic quarks are also confined instead) .

• Of all the possible confining asymptotics, there is a unique one that guarantees "linear confinement" for all glueballs. It corresponds to the case $Q = 2/3, P = 1/2$, i.e.

$$W(\lambda) \sim (\log\lambda)^{\frac{1}{4}}\, \lambda^{\frac{2}{3}} \quad , \quad \beta(\lambda) = -\frac{3}{2}\lambda\left[1 + \frac{3}{8\log\lambda} + \cdots\right] \quad , \quad \lambda \sim E^{-\frac{3}{2}}\left(\log\frac{1}{E}\right)^{\frac{3}{8}}$$

This choice also seems to be preferred from considerations of the meson sector as discussed below.

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- Numerical calculation of the 0^{++} and 2^{++} glueball spectra and comparison with lattice data gives a clear preference for the $r_0 = \infty$ asymptotics.

- We can find the background solution for the axion:

$$a(r) = \theta_{UV} \int_r^{r_0} \frac{dr}{e^{3A} Z(\lambda)} \Big/ \int_0^{r_0} \frac{dr}{e^{3A} Z(\lambda)}$$

written in terms of the axion coupling function $Z(\lambda)$. This provides the "running" of the effective QCD θ angle.

It gives $E(\theta_{UV}) \sim \theta_{UV}^2$.

- Note that always $a(E = 0) = 0$. This suggests a possible intrinsic resolution of the strong CP problem.

Preview: quarks ($N_f \ll N_c$) and mesons

• Flavor is introduced by N_f $D_4 + \bar{D}_4$ branes pairs inside the bulk background. Their back-reaction on the bulk geometry is suppressed by N_f/N_c.

• The important world-volume fields are

$$T_{ij} \leftrightarrow \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \quad , \qquad A^{ij\,L,R}_{\mu} \leftrightarrow \bar{q}^i_a \frac{1 \pm \gamma^5}{2} \gamma^{\mu} q^j_a$$

Generating the $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

• The UV mass matrix m_{ij} corresponds to the source term of the Tachyon field. It breaks the chiral (gauge) symmetry. The normalizable mode corresponds to the vev $\langle \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \rangle$.

• We show that the expectation value of the tachyon is non-zero and $T \sim \mathbf{1}$, breaking chiral symmetry $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$. The anomaly plays an important role in this (holographic Coleman-Witten)

Non-supersymmetric backgrounds with abelian flavor branes

- D_7 brane in deformed AdS_5.

- Only abelian axial symmetry $U(1)_A$ realized geometrically as an isometry.

- A quark mass can be introduced, and a quark condensate can be calculated.

- $U(1)_A$ is spontaneously broken du to the embedding.

- Correct GOR relation

- Qualitatively correct η' mass.

- No non-abelian flavor symmetry (due to N=2 Yukawas)

• The fact that the tachyon diverges in the IR (fusing D with $\bar{D}$) constraints the UV asymptotics and determines the quark condensate $\langle\bar{q}q\rangle$ in terms of m_q. A GOR relation is satisfied (for an asymptotic AdS$_5$ space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\,\langle\bar{q}q\rangle \quad , \quad m_q \to 0$$

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

• Studying the spectrum of highly excited mesons, we find the expected property of linear confinement: $m_n^2 \sim n$.

The Sakai-Sugimoto model

- D4 on non-susy S^1 plus $D8$ branes.
- The flavor symmetry is realized on world-volume
- Full $U(N_f)_L \times U(N_f)_R$ symmetry broken to $U(N_f)_V$.
- Chiral symmetry breaking as brane-antibrane recombination.
- Quark constituent mass
- Qualitatively correct η' mass
- No quark mass parameter, nor chiral condensate.

Non-Critical holography

♠ Non-critical string theories have been explored in order to avoid the KK problem.

Kuperstein+Sonnenschein,
Klebanov+Maldacena,

Bigazzi+Casero+Cotrone+Kiritsis+Paredes

♠ They are expected to involve large curvatures (due to the δc term) and the supergravity approximation seems problematic.

♠ They may provide reliable information on some quantities despite the strong curvature (cf. WZW CFTs).

♠ Recent progress in solving exactly for probe D-branes in non-critical backgrounds has provided important insights for non-critical holography.

Fotopoulos+Niarchos+Prezas,

Ashok+Murthy+Troost

♠ It is fair to say that non-critical holography is so far largely unexplored.

AdS/QCD

- Crude model: AdS$_5$ with a UV and IR cutoff.
- Addition of $U(N_f)_L \times U(N_f)_R$ vectors and a $(N_f, \bar{N}_f)$ scalar T.
- Chiral symmetry broken by hand via IR boundary conditions.
- Vector meson dominance and GOR relation incorporated.
- Chiral condensate not determined.
- Gluon sector problematic.

AdS/QCD-II

♠ A basic phenomenological approach: use a slice of AdS_5, with a UV cutoff, and an IR cutoff.

Polchinski+Strassler

♠ It successfully exhibits confinement (trivially via IR cutoff), and power-like behavior in hard scattering amplitudes

♠ It may be equipped with a bifundamental scalar, T, and $U(N_f)_L \times U(N_f)_R$, gauge fields to describe mesons.

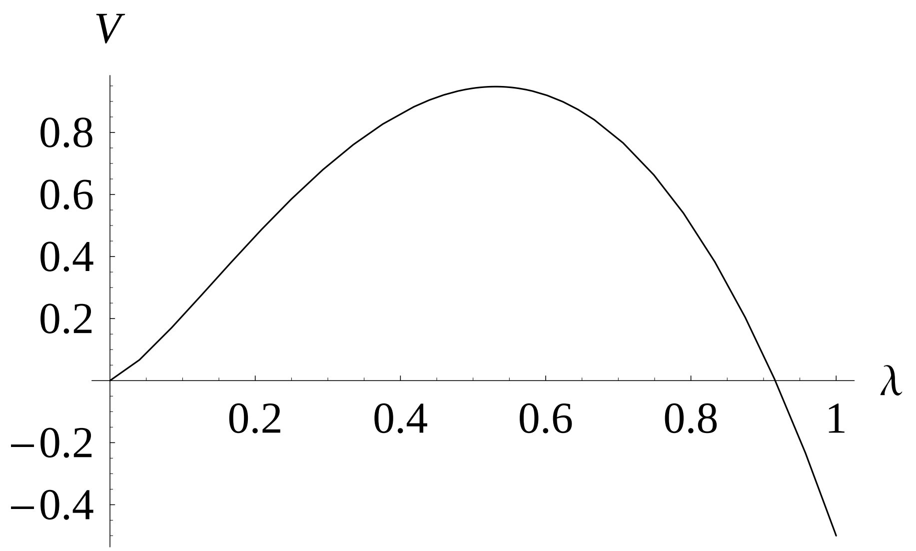
Erlich+Katz+Son+Stepanov, DaRold+Pomarol

Chiral symmetry can be broken, by IR boundary conditions. The low-lying meson spectrum looks "reasonable".

♠ Shortcomings:

- The glueball spectrum fits badly the lattice calculations. It is has the wrong behavior $m_n^2 \sim n^2$ at large n.
- Chiral symmetry breaking is input by hand.
- The meson spectrum has also the wrong UV asymptotics $m_n^2 \sim n^2$.

The "bare" string theory potential



• In QCD we expect that

$$\frac{1}{\lambda} = \frac{1}{N_c e^{\phi}} \sim \frac{1}{\log r} \quad , \quad ds^2 \sim \frac{1}{r^2}(dr^2 + dx_\mu dx^\mu) \quad \text{as} \quad r \to 0$$

• Any potential with $V(\lambda) \sim \lambda^a$ when $\lambda \ll 1$ gives a power different that of AdS_5

• There is an AdS_5 minimum at a finite value λ_*. This cannot be the UV of QCD as dimensions do not match.

RETURN MORE INFO

Fluctuations around the AdS$_5$ extremum

Near an AdS extremum

$$V = \frac{12}{\ell^2} - \frac{16\xi}{3\ell^2}\phi^2 + \mathcal{O}(\phi^3) \quad , \quad \frac{18}{\ell}\delta A' = \delta\phi'^2 - \frac{4}{\ell^2}\phi^2 = \mathcal{O}(\delta\phi^2) \quad , \quad \delta\phi'' - \frac{4}{\ell}\delta\phi' - \frac{4\xi}{\ell^2}\delta\phi = 0$$

where $\phi << 1$. The general solution of the second equation is

$$\delta\phi = C_+ e^{\frac{(2+2\sqrt{1+\xi})u}{\ell}} + C_- e^{\frac{(2-2\sqrt{1+\xi})u}{\ell}}$$

For the potential in question

$$V(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2}\left[5 - \frac{N_c^2}{2}e^{2\phi} - N_f\ e^{\phi}\right] \quad , \quad \lambda_0 \equiv N_c e^{\phi_0} = \frac{-7x + \sqrt{49x^2+400}}{10} \quad , \quad x \equiv \frac{N_f}{N_c}$$

$$\xi = \frac{5}{4}\left[\frac{400 + 49x^2 - 7x\sqrt{49x^2+400}}{100 + 7x^2 - x\sqrt{49x^2+400}}\right] \quad , \quad \frac{\ell_s^2}{\ell^2} = e^{\frac{4}{3}\phi_0}\left[\frac{100 + 7x^2 - x\sqrt{49x^2+400}}{400}\right]$$

The associated dimension is $\Delta = 2 + 2\sqrt{1+\xi}$ and satisfies

$$2 + 3\sqrt{2} < \Delta < 2 + 2\sqrt{6} \qquad \text{or equivalently} \qquad 6.24 < \Delta < 6.90$$

It corresponds to an irrelevant operator. It is probably relevant for the Banks-Zaks fixed points.

Bigazzi+Casero+Cotrone+Kiritsis+Paredes

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Further α' corrections

There are further dilaton terms generated by the 5-form in:

• The kinetic terms of the graviton and the dilaton $\sim \lambda^{2n}$.

• The kinetic terms on probe D_3 branes that affect the identification of the gauge-coupling constant, $\sim \lambda^{2n+1}$. There is also a multiplicative factor relating g_{YM}^2 to e^{ϕ}, (not known). Can be traded for b_0.

• Corrections to the identification of the energy. At $r=0$, $E=1/r$. There can be log corrections to our identification $E=e^A$, and these are a power series in $\sim \lambda^{2n}$.

• It is a remarkable fact that all such corrections affect the higher that the first two terms in the β-function (or equivalently the potential), that are known to be non-universal!

the metric is also insensitive to the change of b_0 by changing Λ.

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Holographic meson dynamics: the models

- Flavor is obtained by adding $N_f << N_C$ D+$\bar{D}$ pairs

- There are several working models of flavor:

♠ Non-supersymmetric backgrounds with abelian D_7flavor brane.

Babington+Erdmenger+Evans+Guralnic+Kirsch

Kruczenski+Mateos+Myers+Winters

♠ Non-supersymmetric $D4+$ $D_8 + \bar{D}_8$

Sakai+Sugimoto

♠ Hard-wall AdS/QCD plus a scalar, plus $U(N_f)_L \times U(N_f)_R$ vectors

Erlich+Katz+son+Stephanov, DaRold+Pomarol

Wilson-Loops and confinement

• Calculation of the static quark potential using the vev of the Wilson loop calculated via an F-string world-sheet.

Rey+Yee, Maldacena

$$T\ E(L) = S_{minimal}(X)$$

We calculate

$$L = 2\int_0^{r_0} dr \frac{1}{\sqrt{e^{4A_S(r)-4A_S(r_0)}-1}}.$$

It diverges when e^{A_s} has a minimum (at $r = r_*$). Then

$$E(L) \sim T_f\ e^{2A_S(r_*)}\ L$$

• Confinement $\to$ $A_s(r_*)$ is finite. This is a more general condition that considered before as A_S `is not monotonic in general`.

• Effective string tension

$$T_{\text{string}} = T_f\ e^{2A_S(r_*)}$$

β-function versus IR geometry

- $K = -\infty$: the scale factor goes to zero at some finite r_0, not faster than a power-law.

- $-\infty < K < -3/8$: the scale factor goes to zero at some finite r_0 faster than any power-law.

- $-3/8 < K < 0$: the scale factor goes to zero as $r \to \infty$ faster than $e^{-Cr^{1+\epsilon}}$ for some $\epsilon > 0$.

- $K = 0$: the scale factor goes to zero as $r \to \infty$ as e^{-Cr} (or faster), but slower than $e^{-Cr^{1+\epsilon}}$ for any $\epsilon > 0$.

The borderline case, $K = -3/8$, is certainly confining (by continuity), but whether or not the singularity is at finite r depends on the subleading terms.

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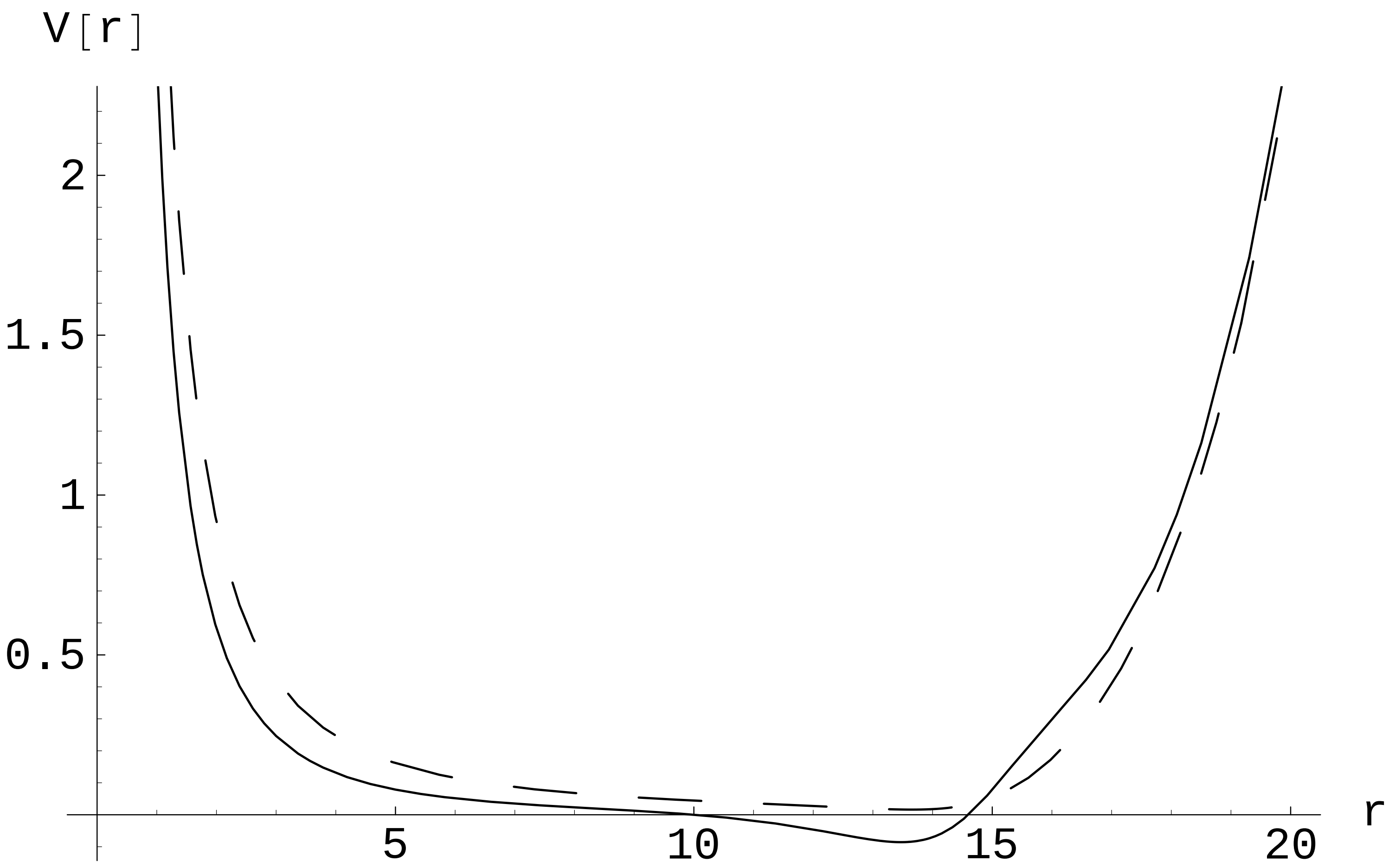
Comparison of scalar and tensor potential



Effective Schrödinger potentials for scalar (solid line) and tensor (dashed line) glueballs. The units are chosen such that $\ell = 0.5$.

The lattice glueball data

J^{++}	Ref. I $(m/\sqrt{\sigma})$	Ref. I (MeV)	Ref. II (mr_0)	Ref. II (MeV)	$N_c \to \infty (m/\sqrt{\sigma})$
0	3.347(68)	1475(30)(65)	4.16(11)(4)	1710(50)(80)	3.37(15)
0^*	6.26(16)	2755(70)(120)	6.50(44)(7)	2670(180)(130)	6.43(50)
0^{**}	7.65(23)	3370(100)(150)	NA	NA	NA
0^{***}	9.06(49)	3990(210)(180)	NA	NA	NA
2	4.916(91)	2150(30)(100)	5.83(5)(6)	2390(30)(120)	4.93(30)
2^*	6.48(22)	2880(100)(130)	NA	NA	NA
R_{20}	1.46(5)	1.46(5)	1.40(5)	1.40(5)	1.46(11)
R_{00}	1.87(8)	1.87(8)	1.56(15)	1.56(15)	1.90(17)

Available lattice data for the scalar and the tensor glueballs. Ref. I =H. B. Meyer, [arXiv:hep-lat/0508002]. and Ref. II = C. J. Morningstar and M. J. Peardon, [arXiv:hep-lat/9901004] + Y. Chen *et al.*, [arXiv:hep-lat/0510074]. The first error corresponds to the statistical error from the the continuum extrapolation. The second error in Ref.I is due to the uncertainty in the string tension $\sqrt{\sigma}$. (Note that this does not affect the mass ratios). The second error in the Ref. II is the estimated uncertainty from the anisotropy. In the last column we present the available large N_c estimates according to B. Lucini and M. Teper, [arXiv:hep-lat/0103027]. The parenthesis in this column shows the total possible error following by the estimations in the same reference. Return to comparison with Ref I or Ref II

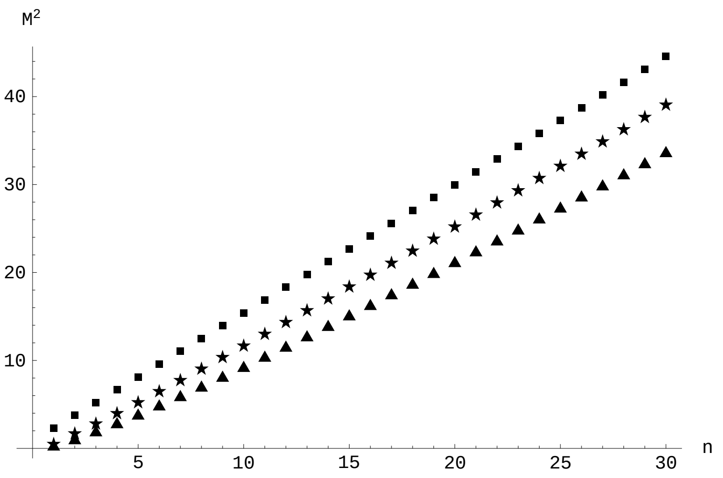
Estimating the importance of logarithmic scaling

We keep the IR asymptotics of background II,but change the UV to power asymptoting AdS_5, with a small λ_*.

$$e^A(r) = \frac{\ell}{r} e^{-(r/R)^2}, \qquad \Phi(r) = \Phi_0 + \frac{3}{2}\frac{r^2}{R^2}\sqrt{1+3\frac{R^2}{r^2}} + \frac{9}{4}\log\frac{2\frac{r}{R}+2\sqrt{\frac{r^2}{R^2}+\frac{3}{2}}}{\sqrt{6}}.$$

$$W_{conf} = W_0\left(9+4b_0^2(\lambda-\lambda_*)^2)^{1/3}\right)\left(9a+(2b_0^2+3b_1)\log\left[1+(\lambda-\lambda_*^2)\right]\right)^{2a/3}.$$

We fix parameters so that the physical QCD scale is the same (as determined from asymptotic slope of Regge trajectories.



The stars correspond to the asymptotically free background I with $b_0 = 4.2$ and $\lambda_0 = 0.05$; the squares correspond the results obtained in the first background with $R = 11.4\ell$; the triangles denote the spectrum in the second background with $b_0 = 4.2$, $li = 0.071$ and $l_* = 0.01$. These values are chosen so that the slopes coincide asymptotically for large n.

Detailed plan of the presentation

- Dependence of absolute mass scale on λ_0 37 minutes
- Dependence of mass ratios on λ_0 38 minutes
- Linearity of the glueball spectrum 39 minutes
- Comparison with lattice data: Ref I 41 minutes
- Comparison with lattice data: Ref II 42 minutes
- α-dependence of scalar spectrum 43 minutes
- Confining background II:r_0 =finite 44 minutes
- The axion background 47 minutes
- A minimal solution to the strong CP problem? 49 minutes
- Open ends 51 minutes