Eurostrings, 1-7 July 2007

Improved Holographic QCD

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Introduction

♠ QCD has been a very successful theory for the strong interactions

♠ Remarkably, we do not have analytical control over most of the energy regime. Even numerically (lattice), many aspects of the theory are still beyond reach

♠ What we can calculate:

- Hard (sub) cross sections and perturbative evolution equations (perturbative QCD)

- Glueball Spectra (a few glueballs), meson spectra (a dozen mesons per tower), baryon spectra (a few low lying baryons).
  "improved" lattice computations, most done in quenched approximation.

- Some weak matrix elements (twisted lattice computations, quenched)

- Topological susceptibility $= \frac{\delta^2 E_{QCD}}{\delta\theta^2}$ (lattice)

- Sum Rules for various observables (perturbative QCD plus general QFT principles)

♠ What we cannot reliably calculate:

- Observable rates for accelerator experiments. In particular, structure functions have to be measured. Hadronization is done by the Lund Monte Carlo model.

- Glueball spectra for higher glueballs mesons and baryons. Decay widths for all of the above.

- There are at least two weak matrix elements that cannot be computed so far reliably enough by lattice computations: The $\Delta I = \frac{1}{2}$ matrix elements of type $\langle K|\mathcal{O}_{\Delta I=1/2,3/2}|\pi\pi\rangle$, and the $B_K \sim \langle K|\mathcal{O}_{\Delta S=2}|\bar{K}\rangle$.

- Data associated to the chiral symmetry breaking (like the quark condensate), or its restauration at higher temperatures.

- In general matrix elements with at least two particle final states.

- Real time finite temperature correlation functions (associated to QGP dynamics)

- Finite temperature physics at finite baryon density.

♠ Several complementary semi-phenomenological techniques have been developed to deal with the above (chiral perturbation theory, perturbation theory resummation schemes, SD equations, bag models, etc.) with varied success.

AdS/CFT and holography

♠ The large N_c approximation to QCD has promised a string theory description of the color singlet sector of gauge theories.

't Hooft

♠ The nature of this string theory became more palpable with the formulation of the AdS/CFT correspondence for $\mathcal{N} = 4_4$ sYM.

Maldacena, Witten, Gubser+Klebanov+Polyakov

The surprise involved the emergence of an extra holographic dimension.

♠ This has started a rush to extend it to theories as close to QCD as possible.

♠ The original and most controlled approaches relied on "perturbing" the original AdS/CFT correspondence in ten-dimensional (critical) string theory.

♠ More recent attempts dared to use a non-critical string framework.

♠ Some holographic-inspired phenomenological models also popped up (AdS/QCD).

Critical string theory holography

♠ Several "successful" holographic models of non-trivial gauge dynamics

- The non-supersymmetric D_4 solution,due to Witten, dual to $\mathcal{N} = 4_5$ sYM on a circle, whose supersymmetry is broken by the boundary conditions of the fermions. It exhibits confinement in the IR.

- Flavor has been successfully incorporated by Sakai+Sugimoto by adding D_7 (dipole) branes.

- The Chamseddine-Volkov solution interpreted by Maldacena and Nuñes as the dual of a confining compactified gauge theory (emerging by wrapping NS_5 branes on a two-cycle).

- The Klebanov-Strassler solution corresponding to a cascade of quiver gauge theories, that confine in the IR.

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♠ In all of the above, confinement related quantities (string tension, glue-ball, masses etc, finite temperature effects) can be calculated analytically.

♠ The same applies to the Sakai-Sugimoto model for flavor, except two major drawbacks:

The absence of bare quark masses and the chiral-symmetry-breaking condensate.

♠ In all the above solutions, the scale of KK excitations is of the same order as Λ of the confining gauge theory.

♠ None so far has managed to overcome this obstacle in critical string theory models.

Non-Critical holography

♠ Non-critical string theories have been explored in order to avoid the KK problem.

Kuperstein+Sonnenschein, Klebanov+Maldacena, Bigazzi+Casero+Cotrone+Kiritsis+Paredes

♠ They are expected to involve large curvatures (due to the δc term) and the supergravity approximation seems problematic.

♠ They may provide reliable information on some quantities despite the strong curvature (cf. WZW CFTs).

♠ Recent progress in solving exactly for probe D-branes in non-critical backgrounds has provided important insights for non-critical holography.

Fotopoulos+Niarchos+Prezas, Ashok+Murthy+Troost

♠ It is fair to say that non-critical holography is so far largely unexplored.

AdS/QCD

♠ A basic phenomenological approach: use a slice of AdS_5, with a UV cutoff, and an IR cutoff.

Polchinski+Strassler

♠ It successfully exhibits confinement (trivially via IR cutoff), and power-like behavior in hard scattering amplitudes

♠ It may be equipped with a bifundamental scalar, T, and $U(N_f)_L \times U(N_f)_R$, gauge fields to describe mesons.

Erlich+Katz+Son+Stepanov, DaRold+Pomarol

Chiral symmetry can be broken, by IR boundary conditions. The low-lying meson spectrum looks "reasonable".

♠ Shortcomings:

- The glueball spectrum fits badly the lattice calculations. It is has the wrong behavior $m_n^2 \sim n^2$ at large n.
- Chiral symmetry breaking is input by hand.
- The meson spectrum has also the wrong UV asymptotics $m_n^2 \sim n^2$.

Improving AdS/QCD

♠ The goal is to use input from both string theory and the gauge theory (QCD) in order to provide an improved phenomenological holographic model for real world QCD.

♠ This is an exploratory adventure, and we will short-circuit several obstacles on the way.

♠ As we will see, we will get an interesting perspective on the physics of pure glue as well as on the quark sector.

A preview of the results: pure glue

♠ The starting point of pure QCD: a two-derivative action in 5d involving

$$g_{\mu\nu} \leftrightarrow T_{\mu\nu} \quad , \quad \phi \leftrightarrow Tr[F^2] \quad , \quad a \leftrightarrow Tr[F \wedge F]$$

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} - \frac{Z(\lambda)}{2N_c^2} (\partial a)^2 + V(\lambda) \right] \quad , \quad \lambda = N_c \, e^{\phi}$$

with

$$V(\lambda) = V_0 \left(1 + \sum_{n=1}^{\infty} V_n \lambda^n \right) = -\frac{4}{3} \lambda^2 \left(\frac{dW}{d\lambda} \right)^2 + \frac{64}{27} W^2.$$

• There is a one-to one-correspondence between the QCD $\beta(\lambda)$ and W:

$$\beta(\lambda) = -\frac{9}{4} \lambda^2 \, \frac{d \log W(\lambda)}{d\lambda}$$

• There is a similar statement between $Z(\lambda)$ and the (non-perturbative) β-function for the θ-angle.

• The space is asymptotically AdS$_5$ in the UV ($r \to 0$) modulo log corrections (in the Einstein frame):

$$ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu}dx^\mu dx^\mu) \quad , \quad E \equiv e^{A(r)}$$

• There are various extra α' corrections to the potential ($\sim \beta$-function). They only correct the non-universal terms. Moreover, α' corrections to E can be set to zero in a special scheme (the "holographic" scheme).

• ALL confining backgrounds have an IR singularity at $r = r_0$. There are two classes: r_0=finite and $r_0 = \infty$. The singularity is always "good": all spectra are well defined without extra input.

• For regular $V(\lambda)$, $\lambda \to \infty$ at the IR singularity.

• In the $r_0 = \infty$ class of backgrounds, the curvature (in the string frame) vanishes in the neighborhood of the IR singularity.

Classification of confining superpotentials $W(\lambda)$ as $\lambda \to \infty$ in IR:

$$W(\lambda) \sim (\log \lambda)^{\frac{P}{2}} \ \lambda^{Q} \quad , \quad \lambda \sim E^{-\frac{9}{4}Q} \left(\log \frac{1}{E}\right)^{\frac{P}{2Q}} , \qquad E \to 0.$$

- $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

$$e^A(r) \sim \begin{cases} (r_0 - r)^{\frac{4}{9Q^2-4}} & Q > \frac{2}{3} \\ \exp\left[-\frac{C}{(r_0-r)^{1/(P-1)}}\right] & Q = \frac{2}{3} \end{cases}$$

- $Q = 2/3$, and $0 \le P < 1$ leads to confinement and a singularity at $r = \infty$ The scale factor e^A vanishes there as

$$e^A(r) \sim \exp[-C r^{1/(1-P)}].$$

- $Q = 2/3, P = 1$ leads to confinement but the singularity may be at a finite or infinite value of r depending on subleading asymptotics of the superpotential.

♠ If $Q < 2\sqrt{2}/3$, no *ad hoc* boundary conditions are needed to determine the glueball spectrum $\to$ One-to-one correspondence with the β-function This is unlike standard AdS/QCD and other approaches.

- when $Q > 2\sqrt{2}/3$, the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

• For all potentials that confine, the spectrum of 0^{++} and 2^{++} glueballs has a mass gap and is purely discrete. For the 0^{+-} glueballs this is the case if

$$Z(\lambda) \sim \lambda^d \quad , \quad d > 2 \quad \text{as} \quad \lambda \to \infty.$$

• In all physically interesting confining backgrounds the magnetic color charges are screened. This is an improvement with respect to AdS/QCD models (magnetic quarks are also confined instead) .

• Of all the possible confining asymptotics, there is a unique one that guarantees "linear confinement" for all glueballs. It corresponds to the case $Q = 2/3, P = 1/2$, i.e.

$$W(\lambda) \sim (\log \lambda)^{\frac{1}{4}} \lambda^{\frac{2}{3}} \quad , \quad \beta(\lambda) = -\frac{3}{2}\lambda \left[1 + \frac{3}{8 \log \lambda} + \cdots\right] \quad , \quad \lambda \sim E^{-\frac{3}{2}} \left(\log \frac{1}{E}\right)^{\frac{3}{8}}$$

This choice also seems to be preferred from considerations of the meson sector as discussed below.

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- Numerical calculation of the 0^{++} and 2^{++} glueball spectra and comparison with lattice data gives a clear preference for the $r_0 = \infty$ asymptotics.

- We can find the background solution for the axion:

$$a(r) = \theta_{UV} \int_r^{r_0} \frac{dr}{e^{3A} Z(\lambda)} \Big/ \int_0^{r_0} \frac{dr}{e^{3A} Z(\lambda)}$$

written in terms of the axion coupling function $Z(\lambda)$. This provides the "running" of the effective QCD θ angle.

It gives $E(\theta_{UV}) \sim \theta_{UV}^2$.

- Note that always $a(E = 0) = 0$. This suggests a possible intrinsic resolution of the strong CP problem.

Preview: quarks ($N_f \ll N_c$) and mesons

• Flavor is introduced by N_f $D_4 + \bar{D}_4$ branes pairs inside the bulk background. Their back-reaction on the bulk geometry is suppressed by N_f/N_c.

• The important world-volume fields are

$$T_{ij} \leftrightarrow \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \quad , \quad A_\mu^{ij\,L,R} \leftrightarrow \bar{q}^i_a \frac{1\pm\gamma^5}{2} \gamma^\mu q^j_a$$

Generating the $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

• The UV mass matrix m_{ij} corresponds to the source term of the Tachyon field. It breaks the chiral (gauge) symmetry. The normalizable mode corresponds to the vev $\langle \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \rangle$.

• We show that the expectation value of the tachyon is non-zero and $T \sim \mathbf{1}$, breaking chiral symmetry $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$. The anomaly plays an important role in this (holographic Coleman-Witten)

• The fact that the tachyon diverges in the IR (fusing D with $\bar{D}$) constraints the UV asymptotics and determines the quark condensate $\langle\bar{q}q\rangle$ in terms of m_q. A GOR relation is satisfied (for an asymptotic AdS$_5$ space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\ \langle\bar{q}q\rangle \quad , \quad m_q \to 0$$

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

• Studying the spectrum of highly excited mesons, we find the expected property of linear confinement: $m_n^2 \sim n$.

Motivating the effective action

- The basic string motivated action for the 5d theory is

$$S_5 = M^3 \int d^5x \sqrt{g} \left[e^{-2\phi} \left(R + 4(\partial\phi)^2 + \frac{\delta c}{\ell_s^2} \right) - \frac{1}{2 \cdot 5!} F_5^2 - \frac{1}{2}(da)^2 \right]$$

$F_5 = dC_4$ seeds the D_3 branes that generate the $U(N_c)$ group.

- The C_4 equation of motion gives

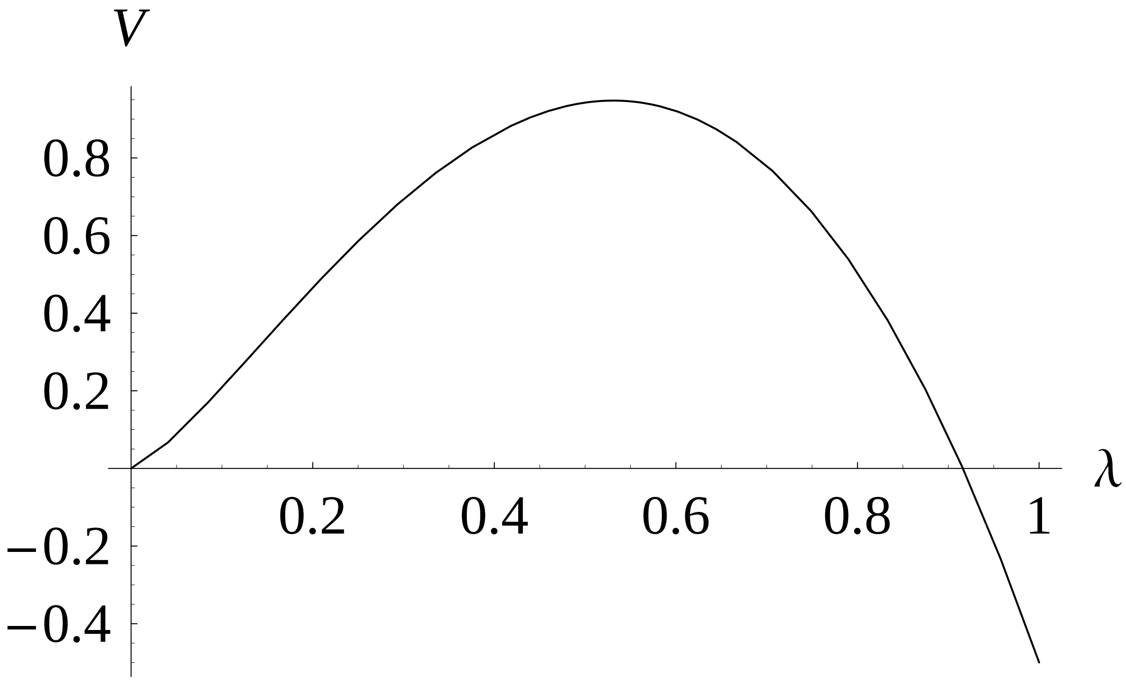
$$^*F_5 = N_c$$

and the dual action in the Einstein frame $g_E = e^{\frac{4}{3}\phi} \, g_s$

$$S_E = M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\phi)^2 - \frac{e^{2\phi}}{2}(\partial a)^2 + V_s(\phi) \right] \quad , \quad V_s(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2} \left[\delta c - \frac{N_c^2}{2} e^{2\phi} \right]$$

- Higher derivative corrections involving the F_5 upon dualization provide further terms in the dilaton potential

$$V_s(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2} \left[\delta c + \sum_{n=1}^{\infty} a_n \, (N_c e^{\phi})^{2n} \right]$$



• In QCD we expect that

$$\frac{1}{\lambda} = \frac{1}{N_c e^{\phi}} \sim \frac{1}{\log r} \quad , \quad ds^2 \sim \frac{1}{r^2}(dr^2 + dx_\mu dx^\mu) \qquad \text{as} \qquad r \to 0$$

• Any potential with $V(\lambda) \sim \lambda^a$ when $\lambda \ll 1$ gives a power different that of AdS_5

• There is an AdS_5 minimum at a finite value λ_*. This cannot be the UV of QCD as dimensions do not match.

MORE INFO

♠ Therefore we need a potential that in the Einstein frame asymptotes to a constant $V_0 = \frac{12}{\ell^2}$ as $\lambda \to 0$.

♠ This can be generated by higher-derivative corrections. `We postulate it.`

♠ The five form will then generate a series of (perturbative) terms in λ:

$$V(\lambda) = V_0 \left(1 + \sum_{n=1}^{\infty} a_n \lambda^{a\ n}\right)$$

we will take $a = 1$ for simplicity (by adjusting the kinetic term).

♠ **This matches the weak coupling expansion of perturbative QCD and will give the perturbative β-function expansion.**

♠ We will ignore higher-derivative terms associated with R and $(\partial\Phi)^2$.
Motivated partly by the success of SVZ sum rules

♠ The "resumed" two-derivative action reads

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda)\right] \quad , \quad \lambda = N_c\, e^{\phi}$$

after redefining the kinetic terms.

• We must choose the holographic energy: the natural choice is $E = e^{A_E}$ frame as it is monotonic and end at zero in the IR singularity.

• We may now solve the equations perturbatively in λ around $\lambda = 0$ and $r = 0$ (this is a weak coupling expansion) to find

$$\frac{1}{\lambda} = L - \frac{b_1}{b_0}\log L + \frac{b_1^2}{b_0^2}\frac{\log L}{L} + \left(\frac{b_1^2}{b_0^2} + \frac{b_2}{b_0}\right)\frac{1}{L} + \frac{b_1^3}{2b_0^3}\frac{\log^2 L}{L^2} + \cdots \quad , \qquad L \equiv -b_0 \log(r\Lambda)$$

with

$$\frac{d\lambda}{d\log E} \equiv \beta(\lambda) = -b_0\lambda^2 + b_1\lambda^3 + b_2\lambda^4 + \cdots$$

$$e^{2A} = \left[1 + \frac{8}{3^2 \log r\Lambda} + \frac{4\left(26 + 9\frac{b_1}{b_0^2} - 18\frac{b_1}{b_0^2}\log(b_0 \log\frac{1}{r\Lambda})\right)}{3^4 \log^2 r\Lambda} + \mathcal{O}\left(\frac{\log^2\log r\Lambda}{\log^3 r\Lambda}\right)\right]\frac{\ell^2}{r^2}$$

$$V = \frac{12}{\ell^2}\left[1 + \frac{8}{9}(b_0\lambda) + \frac{23 - 36\frac{b_1}{b_0^2}}{3^4}(b_0\lambda)^2 - 2\frac{324\frac{b_2}{b_0^3} + 124 + 189\frac{b_1}{b_0^2}}{3^7}(b_0\lambda)^3 + \mathcal{O}(\lambda^4)\right]$$

♠ One-to-one correspondence with the perturbative β-function, and the perturbative potential.

Further α' corrections

There are further dilaton terms generated by the 5-form in:

- The kinetic terms of the graviton and the dilaton $\sim \lambda^{2n}$.

- The kinetic terms on probe D_3 branes that affect the identification of the gauge-coupling constant, $\sim \lambda^{2n+1}$. There is also a multiplicative factor relating g_{YM^2} to e^{ϕ}, (not known). Can be traded for b_0.

- Corrections to the identification of the energy. At $r=0$, $E=1/r$. There can be log corrections to our identification $E=e^A$, and these are a power series in $\sim \lambda^{2n}$.

- It is a remarkable fact that all such corrections affect the higher that the first two terms in the β-function (or equivalently the potential), that are known to be non-universal!

the metric is also insensitive to the change of b_0 by changing Λ.

Organizing the vacuum solutions

A useful variable is the phase variable

$$X \equiv \frac{\Phi'}{3A'} = \frac{\beta(\lambda)}{3\lambda} \quad , \quad e^{\Phi} \equiv \lambda$$

and a superpotential

$$W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \Phi}\right)^2 = \left(\frac{3}{4}\right)^3 V(\Phi).$$

with

$$A' = -\frac{4}{9}W \quad , \quad \Phi' = \frac{dW}{d\Phi}$$

$$X = -\frac{3}{4}\frac{d\log W}{d\log\lambda} \quad , \quad \beta(\lambda) = -\frac{9}{4}\lambda\frac{d\log W}{d\log\lambda}$$

♠ The equations have three integration constants: (two for Φ and one for A) One is fixed by $\lambda \to 0$ in the UV. The other is Λ. The one in A is the choice of energy scale.

The IR regime

For any asymptotically AdS$_5$ solution ($e^A \sim \frac{\ell}{r}$):

- The scale factor $e^{A(r)}$ is monotonically decreasing

Freedman+Gubser+Pilch+Warner

- Moreover, there are only three possible, mutually exclusive IR asymptotics:

♠ *there is another asymptotic AdS_5 region, at $r \to \infty$, where* $\exp A(r) \sim \ell'/r$, *and $\ell' \leq \ell$* (equality holds if and only if the space is exactly AdS_5 everywhere);

♠ there is a curvature singularity at some finite value of the radial coordinate, $r = r_0$;

♠ there is a curvature singularity at $r \to \infty$, where the scale factor vanishes and the space-time shrinks to zero size.

Wilson-Loops and confinement

• Calculation of the static quark potential using the vev of the Wilson loop calculated via an F-string worldsheet.

Rey+Yee, Maldacena

$$T\ E(L) = S_{minimal}(X)$$

We calculate

$$L = 2\int_0^{r_0} dr \frac{1}{\sqrt{e^{4A_S(r)-4A_S(r_0)}-1}}.$$

It diverges when e^{A_s} has a minimum (at $r = r_*$). Then

$$E(L) \sim T_f\ e^{2A_S(r_*)}\ L$$

• Confinement $\rightarrow$ $A_s(r_*)$ is finite. This is a more general condition that considered before as A_S **is not monotonic in general**.

• Effective string tension

$$T_{\text{string}} = T_f\ e^{2A_S(r_*)}$$

General criterion for confinement

• `the geometric version`:

A geometry that shrinks to zero size in the IR is dual to a confining 4D theory if and only if the Einstein metric in conformal coordinates vanishes as (or faster than) e^{-Cr} as $r \to \infty$, for some $C > 0$.

• It is understood here that a metric vanishing at finite $r = r_0$ also satisfies the above condition.

♠ `the superpotential`

A 5D background is dual to a confining theory if the superpotential grows as (or faster than)

$$W \sim (\log \lambda)^{P/2} \lambda^{2/3} \quad \text{as} \quad \lambda \to \infty$$

for some $P \geq 0$.

A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda\to\infty}\left(\frac{\beta(\lambda)}{3\lambda}+\frac{1}{2}\right)\log\lambda = K, \qquad -\infty \le K \le 0$$

(No explicit reference to any coordinate system)

- We can determine the geometry if we specify K:
- $K = -\infty$: the scale factor goes to zero at some finite r_0, not faster than a power-law.
- $-\infty < K < -3/8$: the scale factor goes to zero at some finite r_0 faster than any power-law.
- $-3/8 < K < 0$: the scale factor goes to zero as $r \to \infty$ faster than $e^{-Cr^{1+\epsilon}}$ for some $\epsilon > 0$.
- $K = 0$: the scale factor goes to zero as $r \to \infty$ as e^{-Cr} (or faster), but slower than $e^{-Cr^{1+\epsilon}}$ for any $\epsilon > 0$.

The borderline case, $K = -3/8$, is certainly confining (by continuity), but whether or not the singularity is at finite r depends on the subleading terms.

Comments on confining backgrounds

• For all confining backgrounds with $r_0 = \infty$, although the space-time is singular in the Einstein frame, the string frame geometry is asymptotically flat for large r. Therefore only λ grows indefinitely.

• String world-sheets do not probe the strong coupling region, at least classically. The string stays away from the strong coupling region.

• Therefore: singular confining backgrounds have generically the property that the singularity is *repulsive*, i.e. only highly excited states can probe it. This will also be reflected in the analysis of the particle spectrum (to be presented later)

• The confining backgrounds must also screen magnetic color charges. This can be checked by calculating 't Hooft loops using D_1 probes:

♠ All confining backgrounds with $r_0 = \infty$ screen properly

♠ at finite r_0 backgrounds with $e^A \sim (r - r_0)^\delta$ with $0 < \delta < 1$ do not screen. All others OK.

♠ In particular "hard-wall" AdS/QCD confines also the magnetic quarks.

Particle Spectra: generalities

- Linearized equation:

$$\ddot{\xi} + 2\dot{B}\dot{\xi} + \Box_4 \xi = 0 \quad , \quad \xi(r,x) = \xi(r)\xi^{(4)}(x), \qquad \Box \xi^{(4)}(x) = m^2 \xi^{(4)}(x)$$

- Can be mapped to Schrodinger problem

$$-\frac{d^2}{dr^2}\psi + V(r)\psi = m^2\psi \quad , \quad V(r) = \frac{d^2B}{dr^2} + \left(\frac{dB}{dr}\right)^2 \quad , \quad \xi(r) = e^{-B(r)}\psi(r)$$

- Mass gap and discrete spectrum visible from the asymptotics of the potential.

- Large n asymptotics of masses obtained from WKB

$$n\pi = \int_{r_1}^{r_2} \sqrt{m^2 - V(r)}\, dr$$

- Spectrum depends only on initial condition for λ ($\sim \Lambda_{QCD}$) and an overall energy scale (e^A) that must be fixed.

• scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log\frac{\beta(\lambda)^2}{9\lambda^2}$$

• tensor glueballs

$$B(r) = \frac{3}{2}A(r)$$

• pseudo-scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log Z(\Phi)$$

• Universality of asymptotics

$$\frac{m^2_{n\to\infty}(0^{++})}{m^2_{n\to\infty}(2^{++})} \to 1 \quad , \quad \frac{m^2_{n\to\infty}(0^{+-})}{m^2_{n\to\infty}(0^{++})} = \frac{1}{2}(d-2)^2$$

$$\frac{m^2_{n\to\infty}(0^{++})}{m^2_{n\to\infty}(1^{--})} = \frac{m^2_{n\to\infty}(2^{++})}{m^2_{n\to\infty}(1^{--})} = \frac{36}{25}$$

predicts $d = 2 + \sqrt{2}$ via $\frac{m^2}{2\pi\sigma_a} = 2n + J + c,$

The meson sector ($N_f << N_c$)

- Flavor is introduced via the introduction of N_f pairs of space filling $D_4+\bar{D}_4$ branes.

- The crucial world volume fields are the tachyon T_{ij} in $(N_f, \bar{N}_f)$ and the $U(N_f)_L \times U(N_f)_R$ vectors.

- The D-WZW sector depends nontrivial on T and realizes properly the P and C symmetries. It generates the appropriate gauge and global flavor anomalies.

- We can introduce explicitly mass matrices for the quarks, and we can dynamically determine the chiral condensate.

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• We have naturally the χSB breaking order parameter T, and consistency with anomalies implies that it is non-zero and proportional to the identity (Holographic Coleman+Witten theorem).

• The pions appear as Goldstone bosons when $m_q = 0$.

• The correct GOR relation is obtained.

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2} \langle \bar{q}q \rangle \quad , \quad m_q \to 0$$

• There is linear confinement $(M_n^2 \sim n)$ associated with the vanishing of the tachyon potential at $T \to \infty$.

• We obtain the correct Stuckelberg coupling mixing with 0^{+-} and and mass for the η'.

Tachyon dynamics

- In the vacuum the gauge fields vanish and $T \sim 1$. Only DBI survives

$$S[\tau] = T_{D_4} \int dr d^4x \, \frac{e^{4A_s(r)}}{\lambda} \, V(\tau) \, \sqrt{e^{2A_s(r)} + \dot{\tau}(r)^2} \quad , \quad V(\tau) = e^{-\frac{\mu^2}{2}\tau^2}$$

- We obtain the nonlinear field equation:

$$\ddot{\tau} + \left(3\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right)\dot{\tau} + e^{2A_S}\mu^2\tau + e^{-2A_S}\left[4\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right]\dot{\tau}^3 + \mu^2\tau \; \dot{\tau}^2 = 0.$$

- In the UV we expect

$$\tau = m_q \; r + \sigma \; r^3 + \cdots \quad , \quad \mu^2\ell^2 = 3$$

- We expect that the tachyon must diverge before or at $r = r_0$. We find that indeed it does `at the singularity`. For the $r_0 = \infty$ backgrounds

$$\tau \sim \exp\left[\frac{2}{a} \, \frac{R}{\ell^2} \, r\right] \qquad \text{as} \qquad r \to \infty$$

• Generically the solutions have spurious singularities: $\tau(r_*)$ stays finite but its derivatives diverges as:

$$\tau \sim \tau_* + \gamma\sqrt{r_* - r}.$$

The condition that they are absent determines σ as a function of m_q.

• The easiest spectrum to analyze is that of vector mesons. We find ($r_0 = \infty$)

$$\Lambda_{glueballs} = \frac{1}{R}, \qquad \Lambda_{mesons} = \frac{3}{\ell}\left(\frac{\alpha\ell^2}{2R^2}\right)^{(\alpha-1)/2} \propto \frac{1}{R}\left(\frac{\ell}{R}\right)^{\alpha-2}.$$

This suggests that $\alpha = 2$. preferred also from the glue sector.

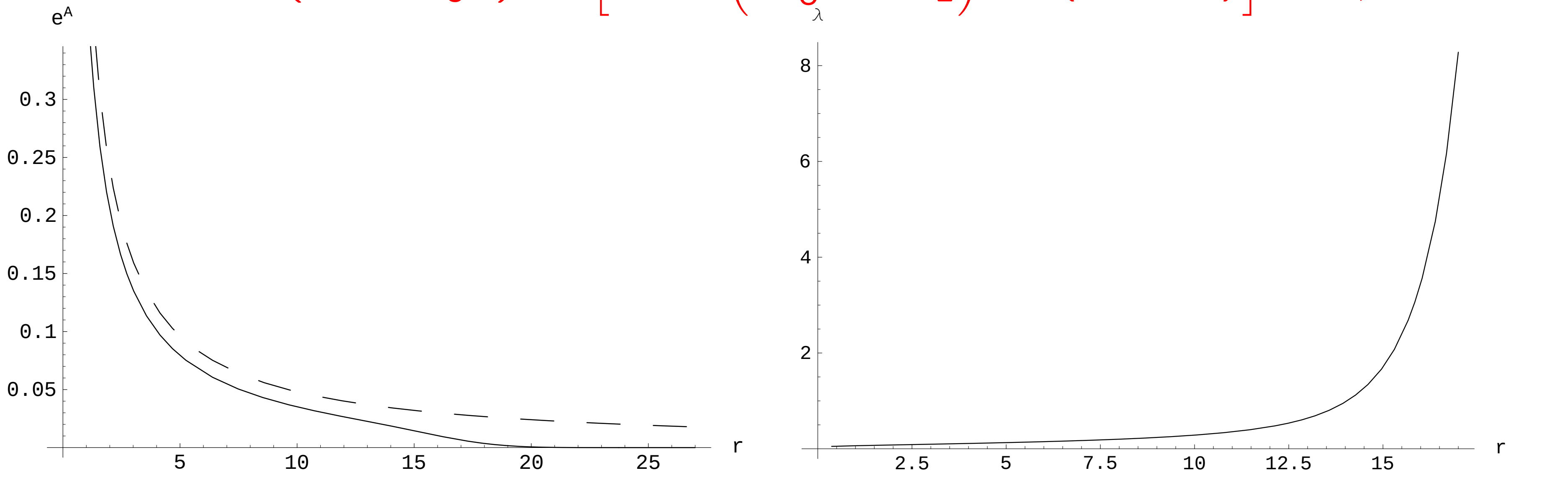
Concrete models: I

- $r_0 = \infty$ and $a = 2$:

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3+2b_0\lambda} - \frac{3a(2b_0^2+3b_1^2)\lambda^3}{(1+\lambda^2)\left(9a+\left(2b_0^2+3b_1^2\right)\log(1+\lambda^2)\right)}$$

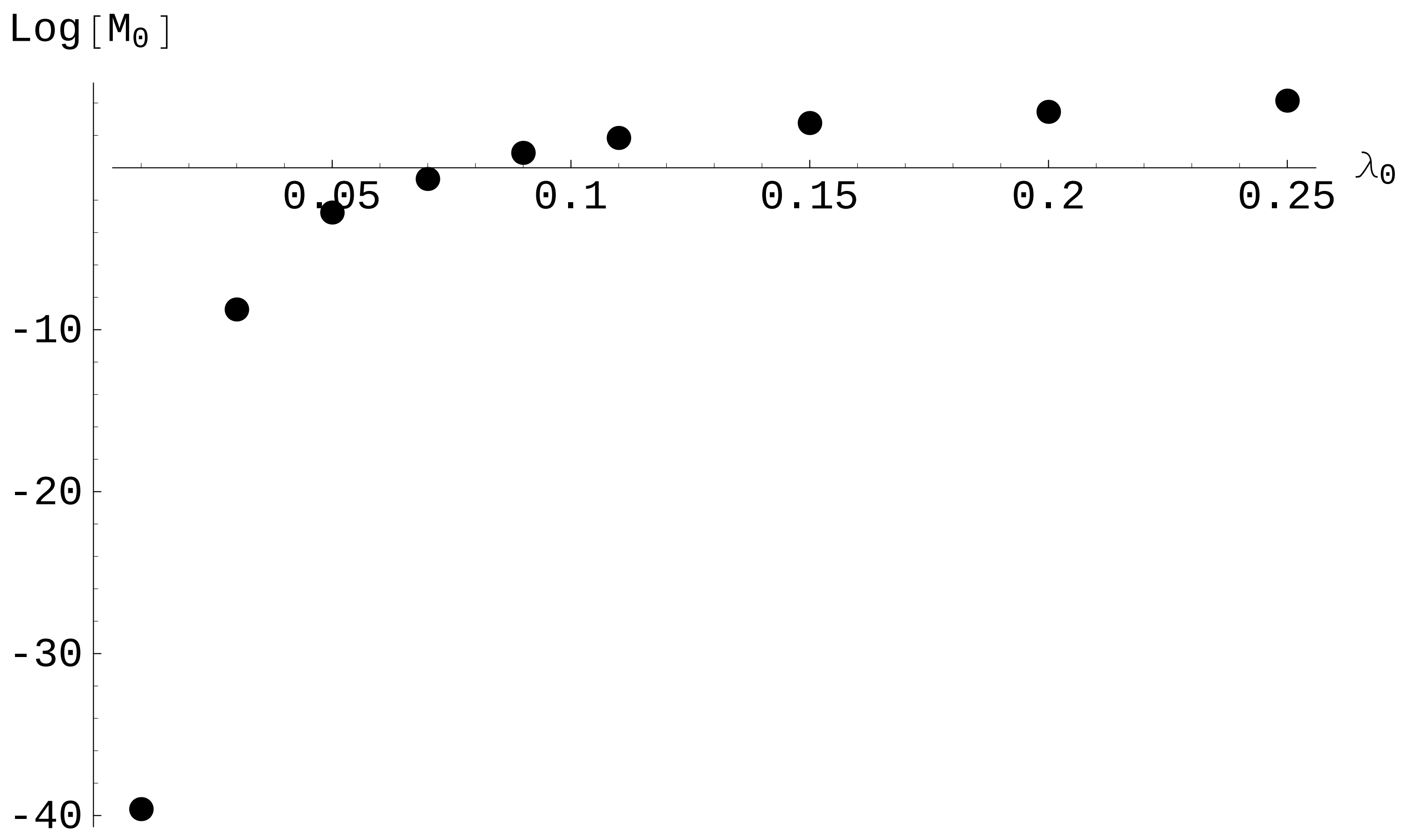
is everywhere regular and has the correct UV and IR asymptotics.

$$W = (3+2b_0\lambda)^{2/3}\left[9a+\left(2b_0^2+3b_1\right)\log(1+\lambda^2)\right]^{2a/3},$$



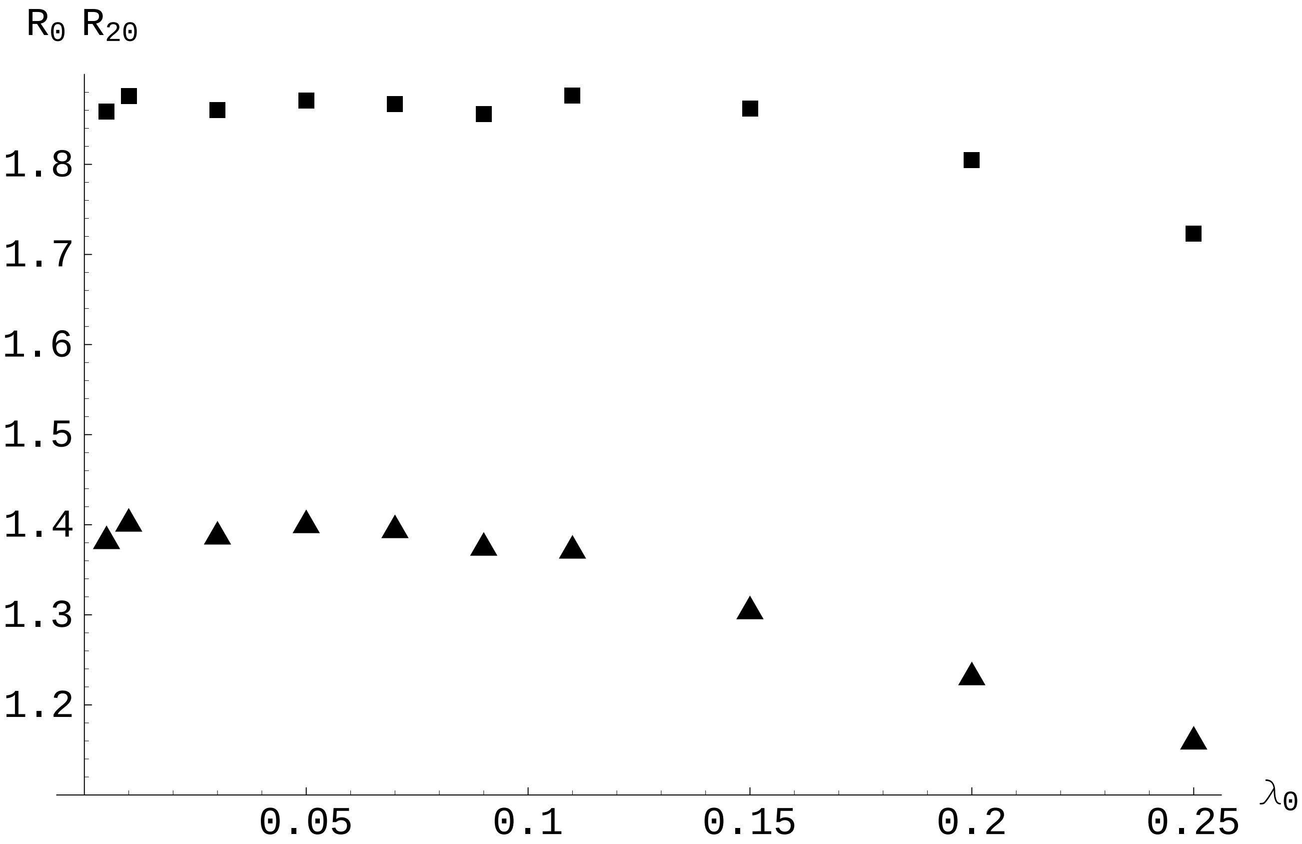
The scale factor and 't Hooft coupling that follow from β. $b_0 = 4.2$, $\lambda_0 = 0.05$, $A_0 = 0$. The units are such that $\ell = 0.5$. The dashed line represents the scale factor for pure AdS.

Dependence of absolute mass scale on λ_0



Dependence on initial condition λ_0 of the absolute scale of the lowest lying glueball (shown in Logarithmic scale)

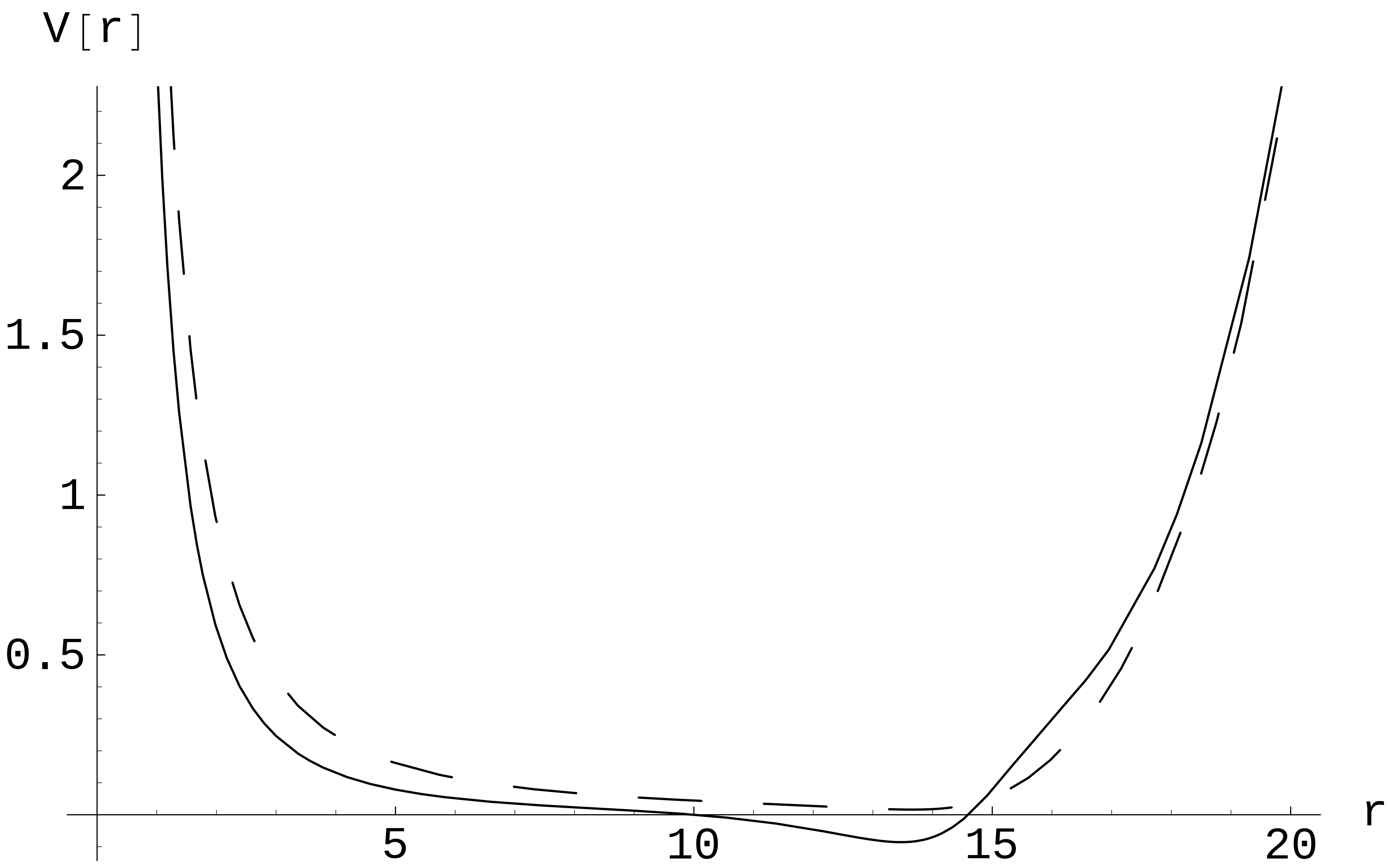
Dependence of mass ratios on λ_0



The mass ratios R_{00} (squares) and R_{20} (triangles).

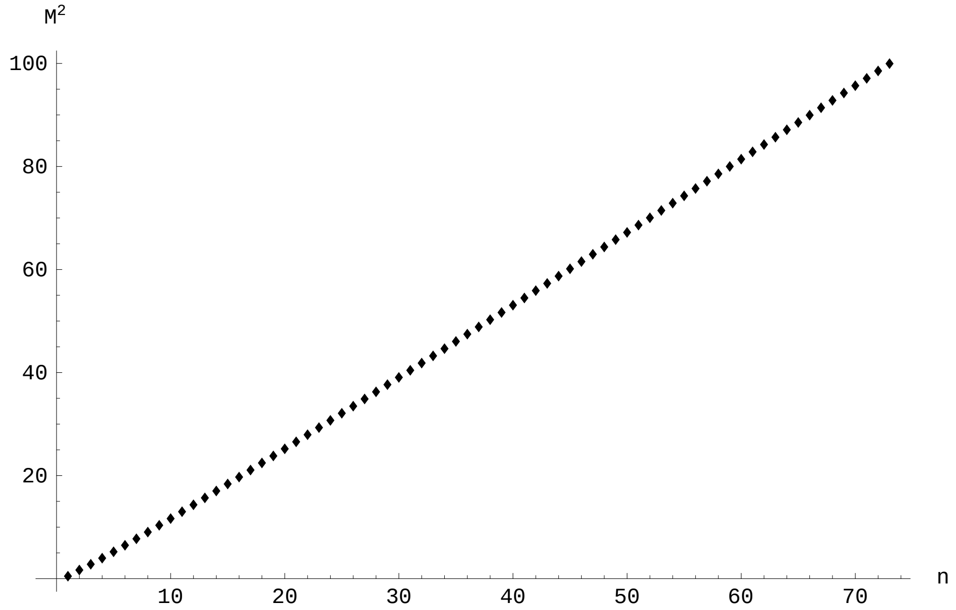
$$R_{00} = \frac{m_{0*++}}{m_{0++}}, \qquad R_{20} = \frac{m_{2++}}{m_{0++}}.$$

Comparison of scalar and tensor potential

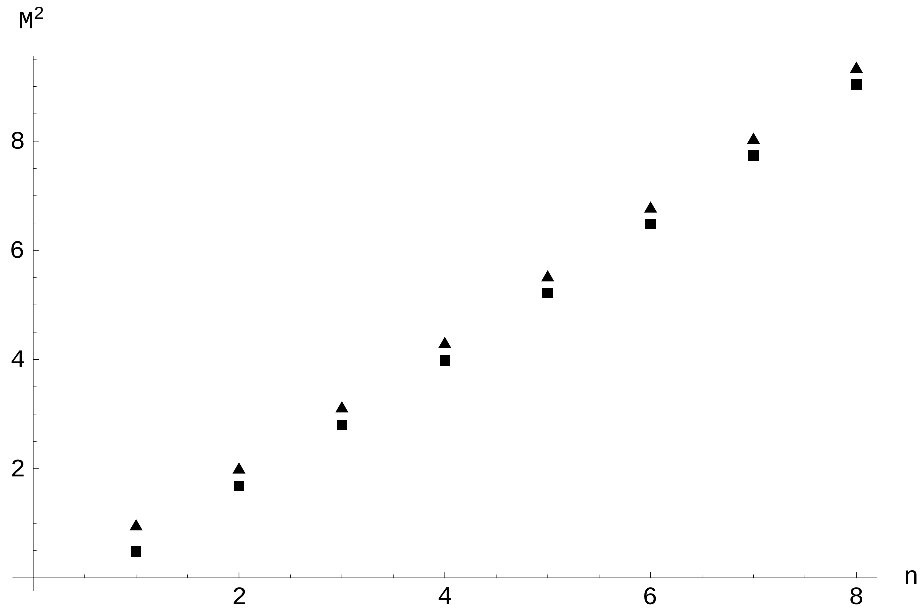


Effective Schrödinger potentials for scalar (solid line) and tensor (dashed line) glueballs. The units are chosen such that $\ell = 0.5$.

Linearity of the glueball spectrum



(a)



(b)

(a) Linear pattern in the spectrum for the first 40 0^{++} glueball states. M^2 is shown units of $0.015\ell^{-2}$.

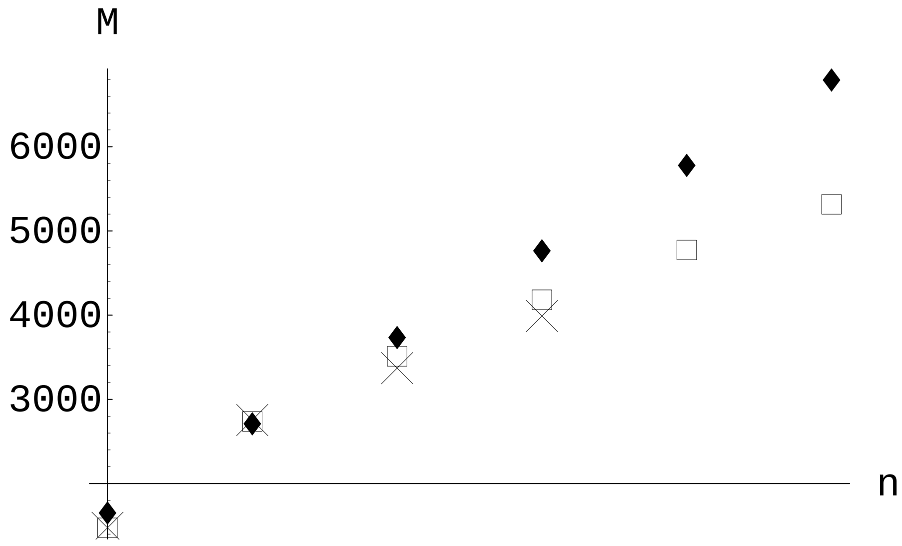
(b) The first 8 0^{++} (squares) and the 2^{++} (triangles) glueballs. These spectra are obtained in the background I with $b_0 = 4.2, \lambda_0 = 0.05$.

The lattice glueball data

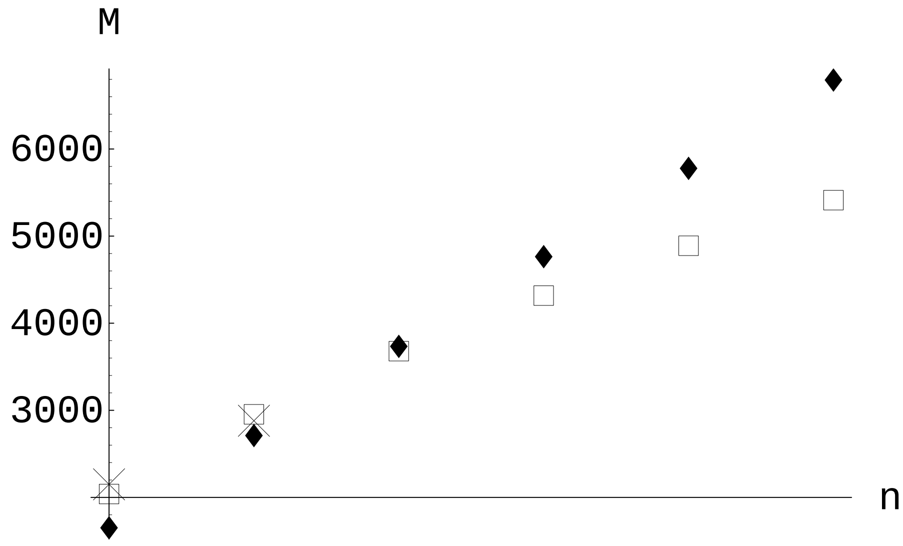
J^{++}	Ref. I $(m/\sqrt{\sigma})$	Ref. I (MeV)	Ref. II (mr_0)	Ref. II (MeV)	$N_c \to \infty (m/\sqrt{\sigma})$
0	3.347(68)	1475(30)(65)	4.16(11)(4)	1710(50)(80)	3.37(15)
0^*	6.26(16)	2755(70)(120)	6.50(44)(7)	2670(180)(130)	6.43(50)
0^{**}	7.65(23)	3370(100)(150)	NA	NA	NA
0^{***}	9.06(49)	3990(210)(180)	NA	NA	NA
2	4.916(91)	2150(30)(100)	5.83(5)(6)	2390(30)(120)	4.93(30)
2^*	6.48(22)	2880(100)(130)	NA	NA	NA
R_{20}	1.46(5)	1.46(5)	1.40(5)	1.40(5)	1.46(11)
R_{00}	1.87(8)	1.87(8)	1.56(15)	1.56(15)	1.90(17)

Available lattice data for the scalar and the tensor glueballs. Ref. I =H. B. Meyer, [arXiv:hep-lat/0508002]. and Ref. II = C. J. Morningstar and M. J. Peardon, [arXiv:hep-lat/9901004] + Y. Chen *et al.*, [arXiv:hep-lat/0510074]. The first error corresponds to the statistical error from the the continuum extrapolation. The second error in Ref.I is due to the uncertainty in the string tension $\sqrt{\sigma}$. (Note that this does not affect the mass ratios). The second error in the Ref. II is the estimated uncertainty from the anisotropy. In the last column we present the available large N_c estimates according to B. Lucini and M. Teper, [arXiv:hep-lat/0103027]. The parenthesis in this column shows the total possible error following by the estimations in the same reference.

Comparison with lattice data: Ref I



(a)

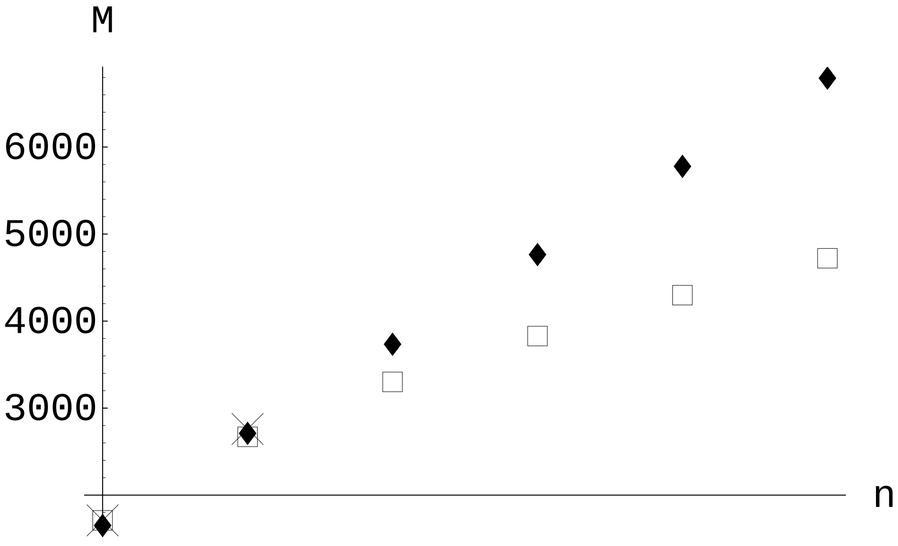


(b)

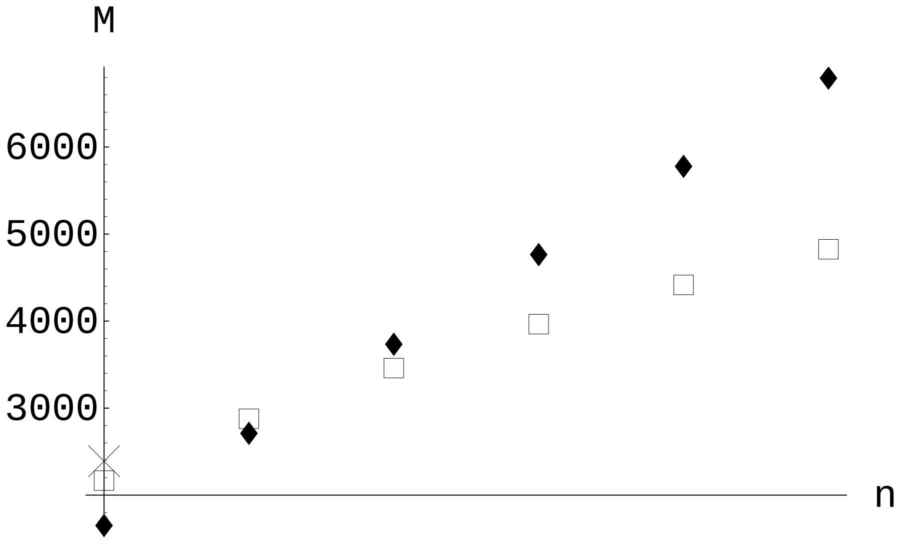
Comparison of glueball spectra from our model with $b_0 = 4.2, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. I (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. I.

$$\ell_{eff}^2 = 6.88 \ \ell^2$$

Comparison with lattice data: Ref II



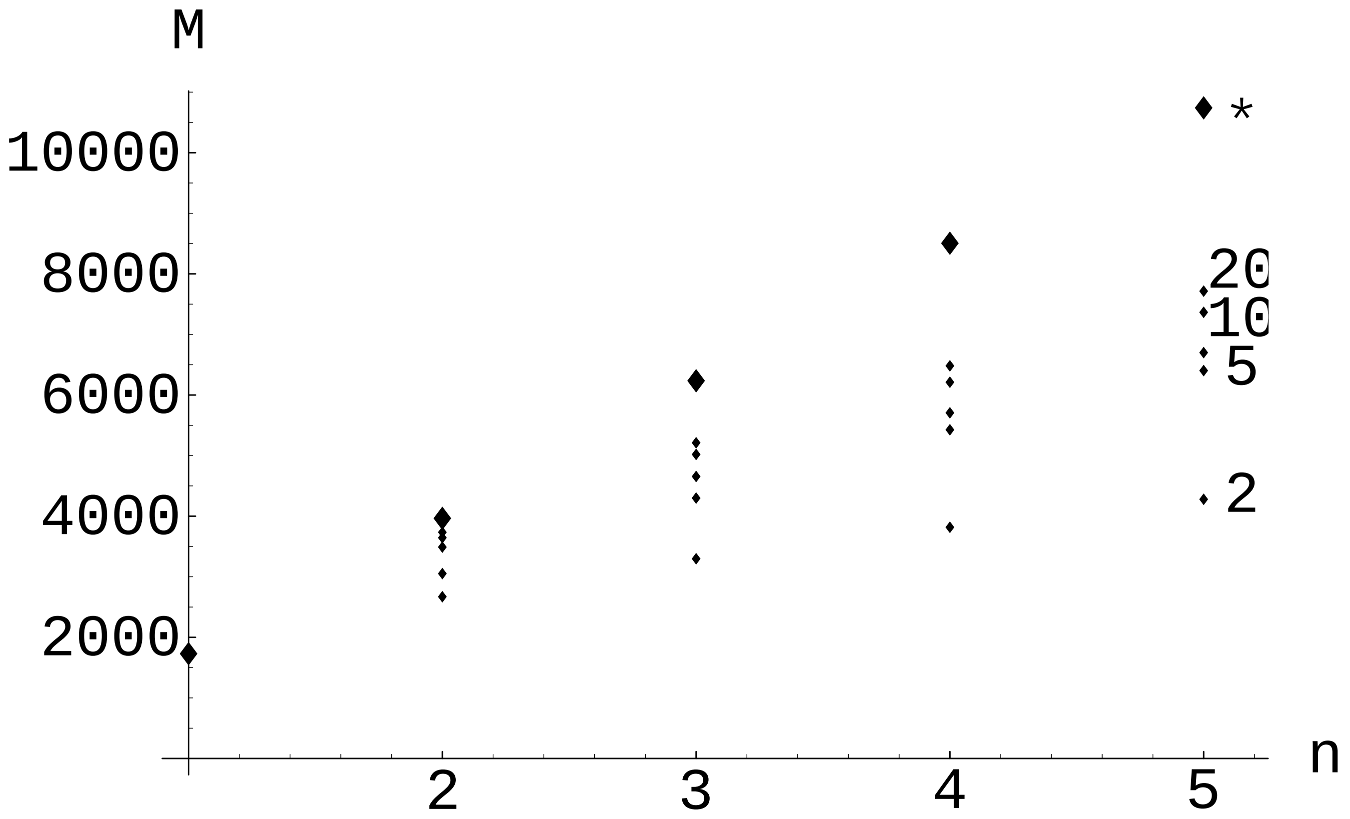
(a)



(b)

Comparison of glueball spectra from our model with $b_0 = 2.55, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. II (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. II.

α-dependence of scalar spectrum



The 0^{++} spectra for varying values of α that are shown at the right end of the plot. The symbol * denotes the AdS/QCD result.

Confining background II: r_0 =finite

- We choose a regular β-function with appropriate asymptotics:

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3+2b_0\lambda} - \frac{3\eta(2b_0^2+3b_1^2)\lambda^3}{9\eta+2\left(2b_0^2+3b_1^2\right)\lambda^2}, \qquad \eta \equiv \sqrt{1+\delta^{-1}} - 1$$

- Confining backgrounds with r_0 =finite have a hard time to match the lattice results, even for the first few glueballs.

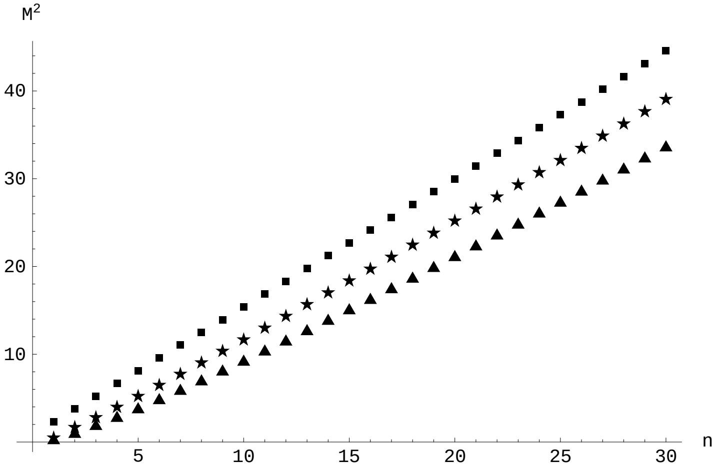
Estimating the importance of logarithmic scaling

We keep the IR asymptotics of background II,but change the UV to power asymptoting AdS_5, with a small λ_*.

$$e^A(r) = \frac{\ell}{r} e^{-(r/R)^2}, \qquad \Phi(r) = \Phi_0 + \frac{3}{2}\frac{r^2}{R^2}\sqrt{1+3\frac{R^2}{r^2}} + \frac{9}{4}\log\frac{2\frac{r}{R} + 2\sqrt{\frac{r^2}{R^2}+\frac{3}{2}}}{\sqrt{6}}.$$

$$W_{conf} = W_0\left(9 + 4b_0^2(\lambda-\lambda_*)^2)^{1/3}\right)\left(9a + (2b_0^2+3b_1)\log\left[1+(\lambda-\lambda_*^2)\right]\right)^{2a/3}.$$

We fix parameters so that the physical QCD scale is the same (as determined from asymptotic slope of Regge trajectories.



The stars correspond to the asymptotically free background I with $b_0 = 4.2$ and $\lambda_0 = 0.05$; the squares correspond the results obtained in the first background with $R = 11.4\ell$; the triangles denote the spectrum in the second background with $b0 = 4.2$, $li = 0.071$ and $l_* = 0.01$. These values are chosen so that the slopes coincide asymptotically for large n.

Open ends

• This phenomenological approach towards an improved holographic QCD model is preliminary but seems promising

• Several immediate problems:

♠ Determine the finite temperature solutions and the resulting deconfining transitions

♠ Calculate the meson spectrum and compare with data.

♠ Explore the baryon spectrum

♠ Calculate the $\Delta I = 1/2$ and B_k data.

♠ Diagonalize the $\eta' - 0^{+-}$ system and compare with data.

♠ Recalculate the dipole moment of the neutron in connection with the strong CP problem.

♠ Calculate RHIC/LHC finite T observables

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Thank you for your patience!

Fluctuations around the AdS$_5$ extremum

Near an AdS extremum

$$V = \frac{12}{\ell^2} - \frac{16\xi}{3\ell^2}\phi^2 + \mathcal{O}(\phi^3) \quad , \quad \frac{18}{\ell}\delta A' = \delta\phi'^2 - \frac{4}{\ell^2}\phi^2 = \mathcal{O}(\delta\phi^2) \quad , \quad \delta\phi'' - \frac{4}{\ell}\delta\phi' - \frac{4\xi}{\ell^2}\delta\phi = 0$$

where $\phi << 1$. The general solution of the second equation is

$$\delta\phi = C_+ e^{\frac{(2+2\sqrt{1+\xi})u}{\ell}} + C_- e^{\frac{(2-2\sqrt{1+\xi})u}{\ell}}$$

For the potential in question

$$V(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2}\left[5 - \frac{N_c^2}{2}e^{2\phi} - N_f\ e^{\phi}\right] \quad , \quad \lambda_0 \equiv N_c e^{\phi_0} = \frac{-7x + \sqrt{49x^2+400}}{10} \quad , \quad x \equiv \frac{N_f}{N_c}$$

$$\xi = \frac{5}{4}\left[\frac{400 + 49x^2 - 7x\sqrt{49x^2+400}}{100 + 7x^2 - x\sqrt{49x^2+400}}\right] \quad , \quad \frac{\ell_s^2}{\ell^2} = e^{\frac{4}{3}\phi_0}\left[\frac{100 + 7x^2 - x\sqrt{49x^2+400}}{400}\right]$$

The associated dimension is $\Delta = 2 + 2\sqrt{1+\xi}$ and satisfies

$$2 + 3\sqrt{2} < \Delta < 2 + 2\sqrt{6} \qquad \text{or equivalently} \qquad 6.24 < \Delta < 6.90$$

It corresponds to an irrelevant operator. It is probably relevant for the Banks-Zaks fixed points.

Bigazzi+Casero+Cotrone+Kiritsis+Paredes

RETURN

Holographic meson dynamics: the models

- Flavor is obtained by adding $N_f << N_C$ D+$\bar{D}$ pairs

- There are several working models of flavor:

♠ Non-supersymmetric backgrounds with abelian D_7flavor brane.

Babington+Erdmenger+Evans+Guralnic+Kirsch

Kruczenski+Mateos+Myers+Winters

♠ Non-supersymmetric $D4+$ $D_8 + \bar{D}_8$

Sakai+Sugimoto

♠ Hard-wall AdS/QCD plus a scalar, plus $U(N_f)_L \times U(N_f)_R$ vectors

Erlich+Katz+son+Stephanov, DaRold+Pomarol

The axion background

- The kinetic term of the axion is suppressed by $1/N_c^2$. (it is an angle in the gauge theory, it is RR in string theory)

$$\ddot{a} + \left(3\dot{A} + \frac{\dot{Z}(\lambda)}{Z(\lambda)}\right)\dot{a} = 0 \quad \to \quad \dot{a} = \frac{C\ e^{-3A}}{Z(\lambda)}$$

It can be interpreted as the flow equation of the effective θ-angle.

- The full solution is

$$a(r) = \theta_{UV} + C \int_0^r r \frac{e^{-3A}}{Z(\lambda)} \quad , \quad C = \langle Tr[F \wedge F] \rangle$$

- The vacuum energy is

$$E(\theta_{UV}) = \frac{M^3}{2N_c^2} \int d^5x \sqrt{g} Z(\lambda)(\partial a)^2 = \frac{M^3}{2N_c^2} C a(r) \Big|_{r=0}^{r=r_0}$$

- Consistency requires to impose that $a(r_0) = 0$. This determines C and

$$E(\theta_{UV}) = -\frac{M^3}{2N_c^2} \frac{\theta_{UV}^2}{\int_0^{r_0} \frac{dr}{e^{3A}Z(\lambda)}} \quad , \quad a(r) = \theta_{UV} \frac{\int_r^{r_0} \frac{dr}{e^{3A}Z(\lambda)}}{\int_0^{r_0} \frac{dr}{e^{3A}Z(\lambda)}}$$

A minimal solution to the strong CP problem?

- The IR effective θ-angle vanishes, independent of θ_{UV}!

- For $Z(\lambda) \sim \lambda^d$ as $\lambda \to \infty$

$$a(E) \sim E^{\frac{3}{2}(d-2)} (\log E)^a \qquad \text{as} \qquad E \to 0$$

- The presence of a discrete gapped 0^{+-} spectrum implies that $d > 2$.

- We know that $\theta < 10^{-8}$ from electric dipole of the neutron d_n. This assumes that θ does not run.

- It is an interesting possibility that d_n is very small because a(E) vanishes fast in the IR.

Non-supersymmetric backgrounds with abelian flavor branes

- D_7 brane in deformed AdS_5.

- Only abelian axial symmetry $U(1)_A$ realized geometrically as an isometry.

- A quark mass can be introduced, and a quark condensate can be calculated.

- $U(1)_A$ is spontaneously broken du to the embedding.

- Correct GOR relation

- Qualitatively correct η' mass.

- No non-abelian flavor symmetry (due to N=2 Yukawas)

The Sakai-Sugimoto model

- D4 on non-susy S^1 plus $D8$ branes.

- The flavor symmetry is realized on world-volume

- Full $U(N_f)_L \times U(N_f)_R$ symmetry broken to $U(N_f)_V$.

- Chiral symmetry breaking as brane-antibrane recombination.

- Quark constituent mass

- Qualitatively correct η' mass

- No quark mass parameter, nor chiral condensate.

AdS/QCD

- Crude model: AdS_5 with a UV and IR cutoff.

- Addition of $U(N_f)_L \times U(N_f)_R$ vectors and a $(N_f, \bar{N}_f)$ scalar T.

- Chiral symmetry broken by hand via IR boundary conditions.

- Vector meson dominance and GOR relation incorporated.

- Chiral condensate not determined.

- Gluon sector problematic.

Detailed plan of the presentation