CERN Summer lectures, July-August, 2006

POSSIBLE PHYSICS BEYOND THE STANDARD MODEL

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Quote

"Understanding nature is one of the noblest endeavors the human race has ever undertaken"

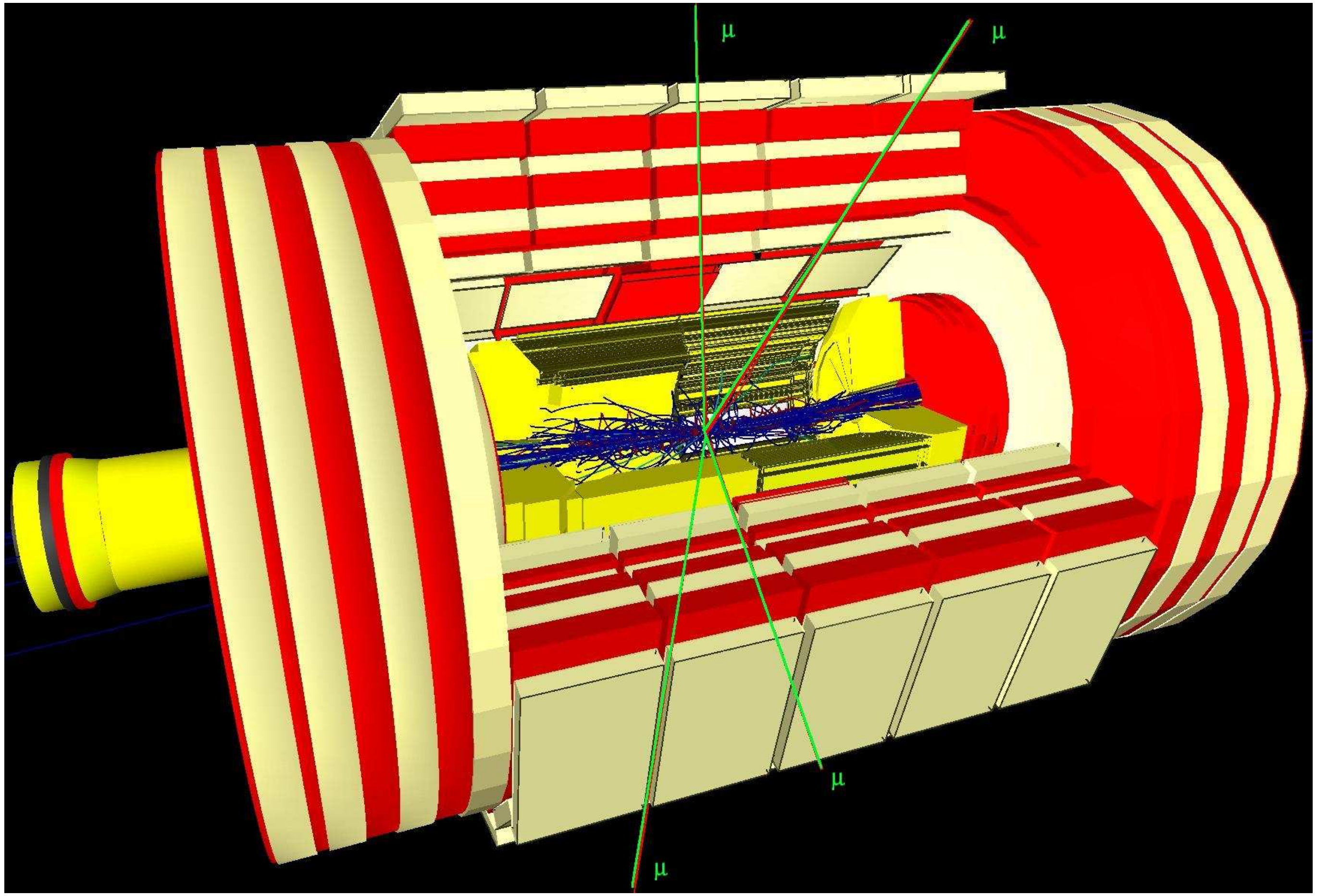
Steven Weinberg

Preview

- What do we expect to see at LHC?

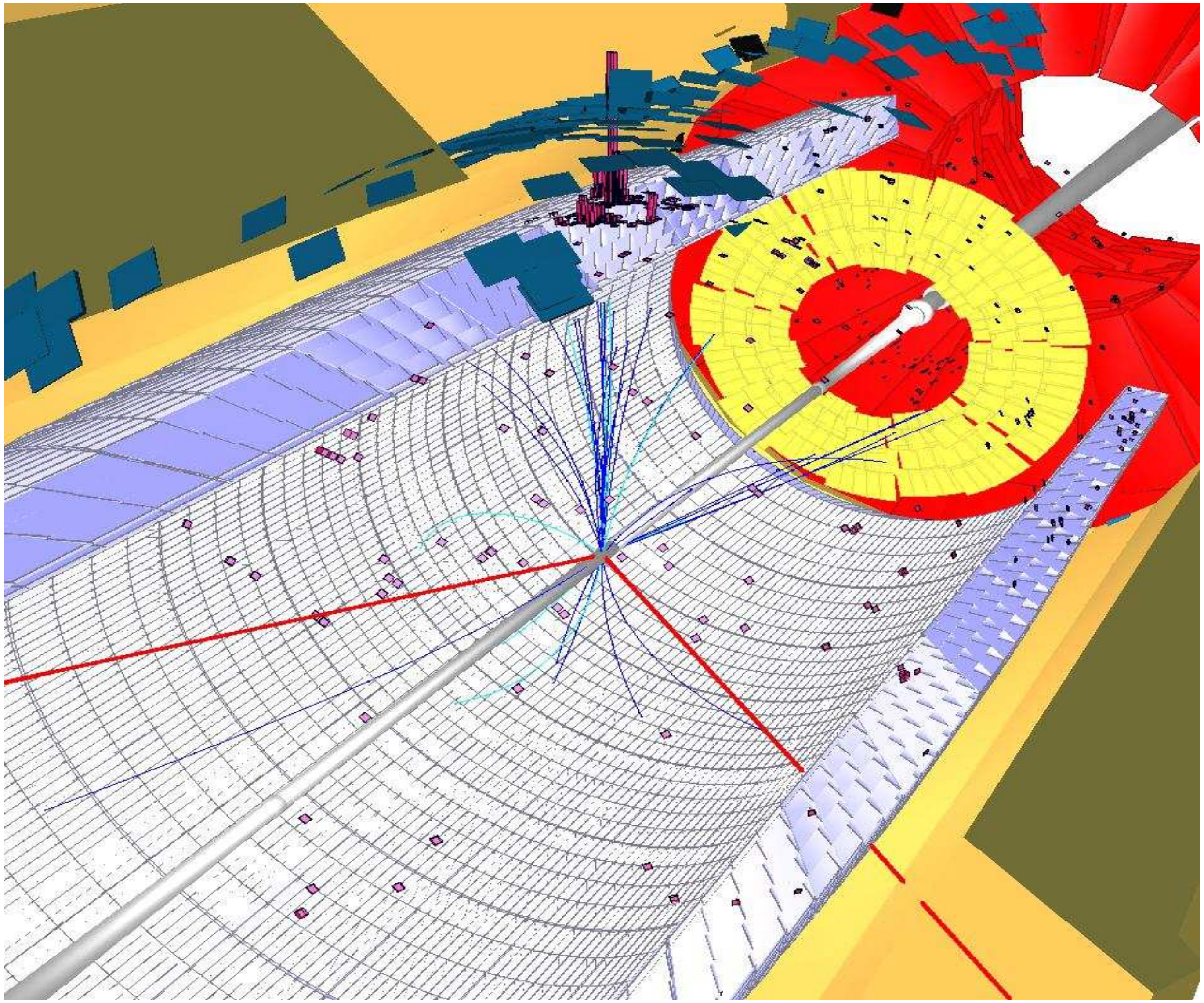
Preview: Higgs

Possible exciting physics we are preparing to search for: Higgs particle



Requested by the SM, might tell us a lot about the hierarchy puzzle.

Preview: Supersymmetry



SUSY event: A decay of a neutralino into Z + LSP, the Z decays into two muons.

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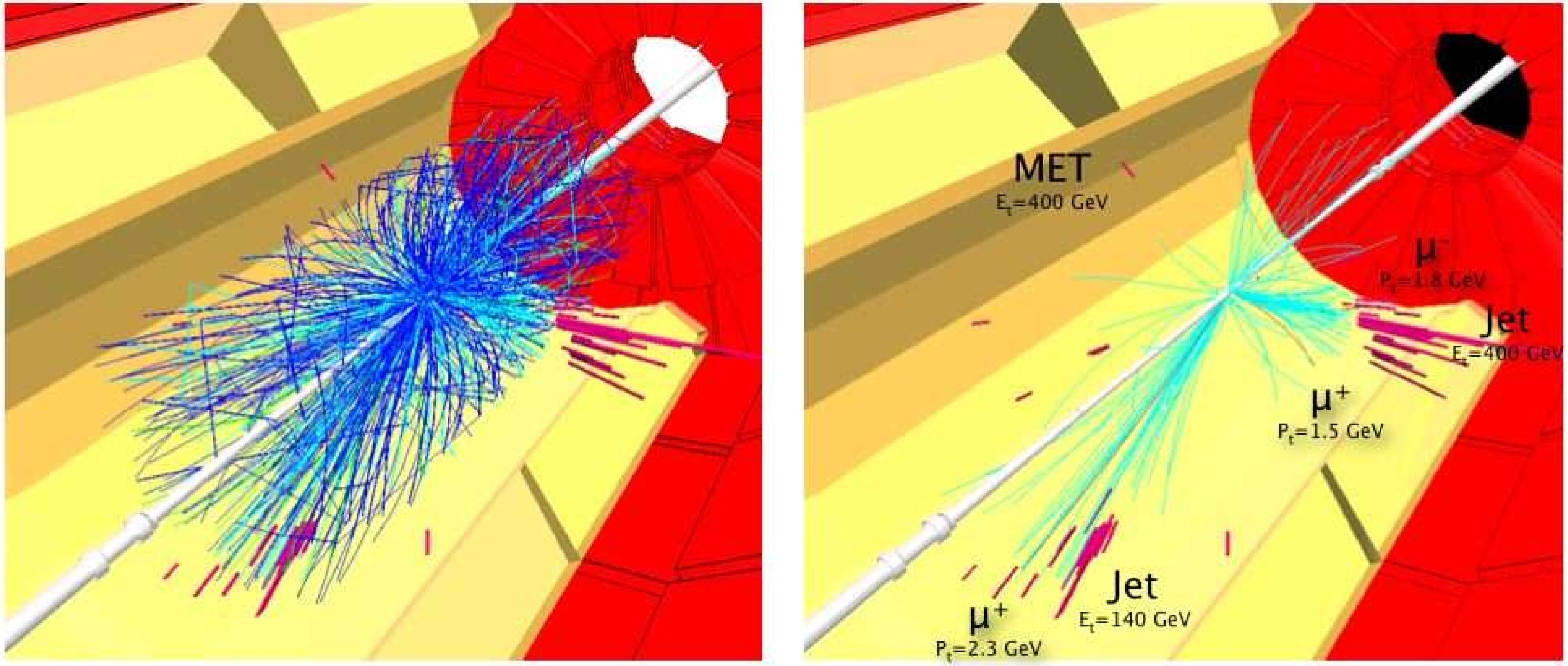
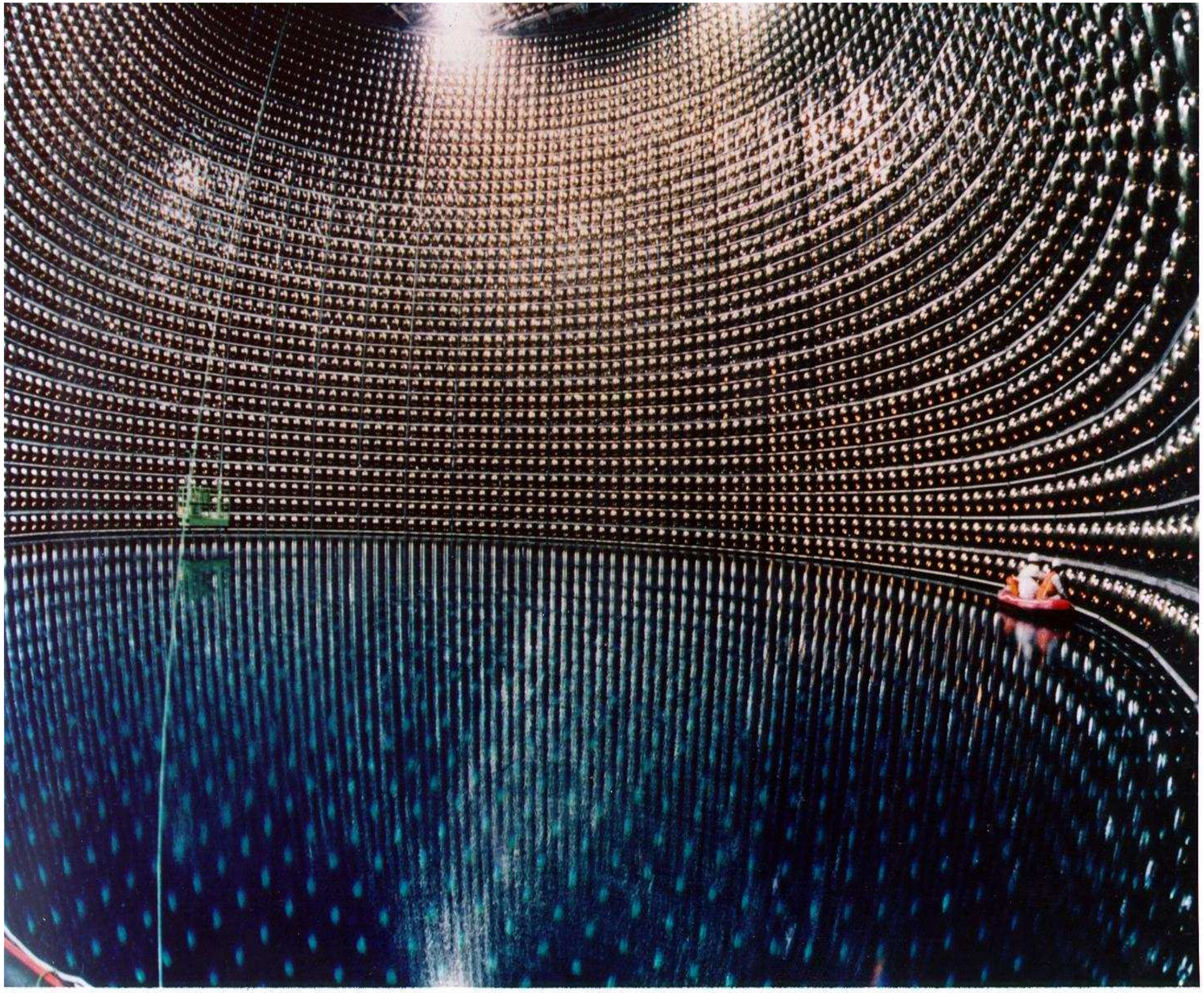


Missing transverse energy susy event at high luminosity

Preview: Proton decay

Large detectors (neutrino telescopes) search for signals from the decay of protons.



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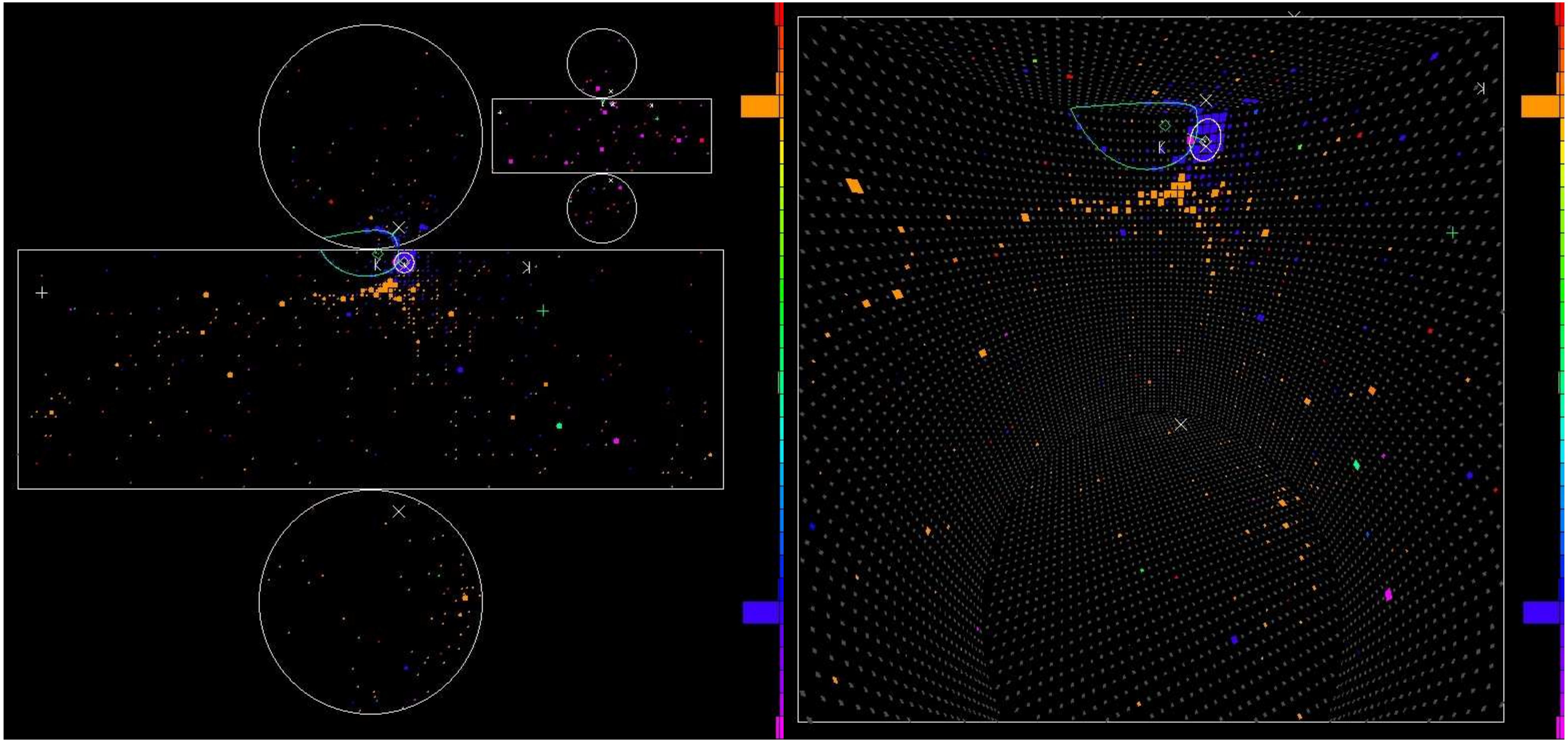
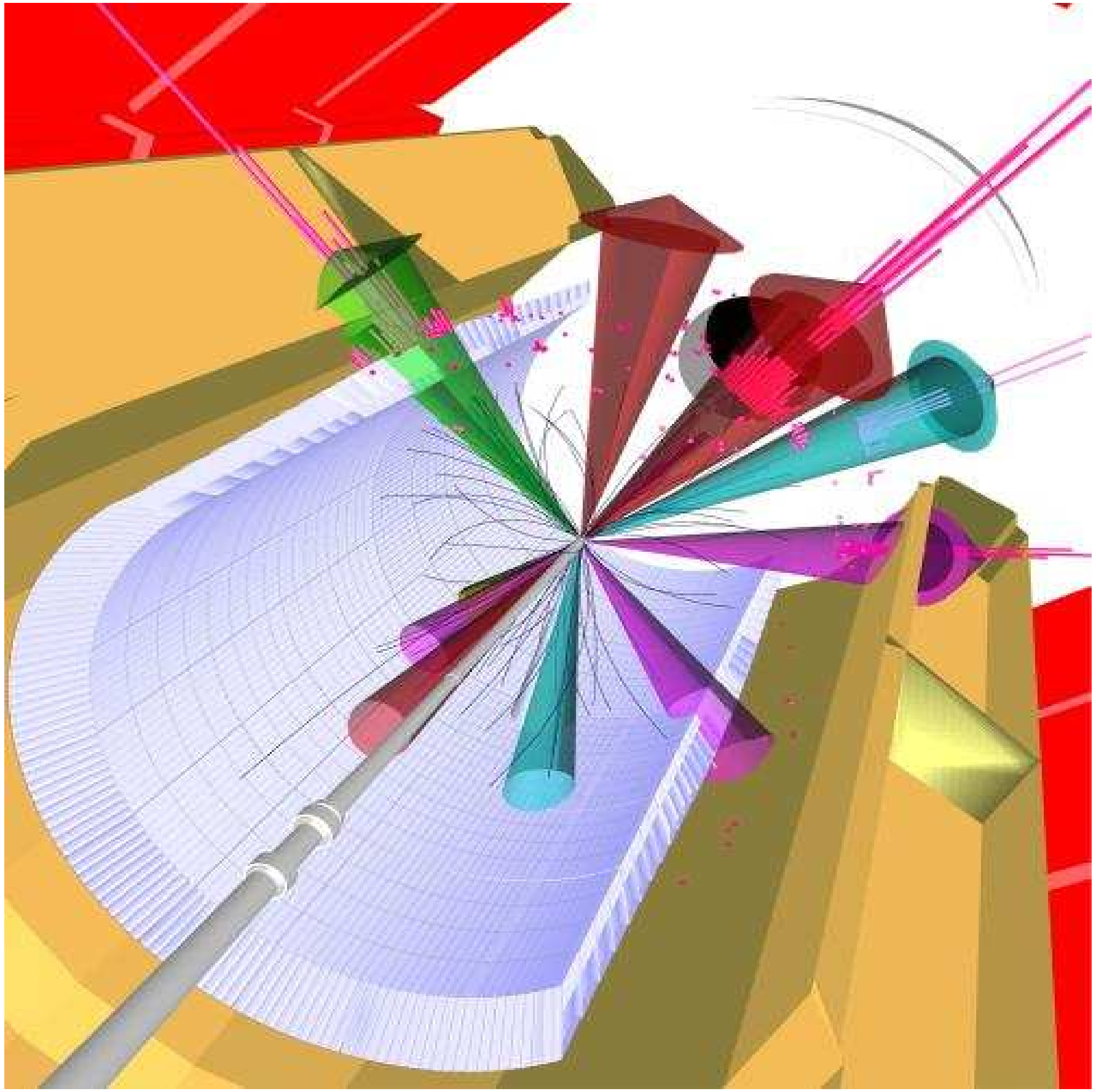
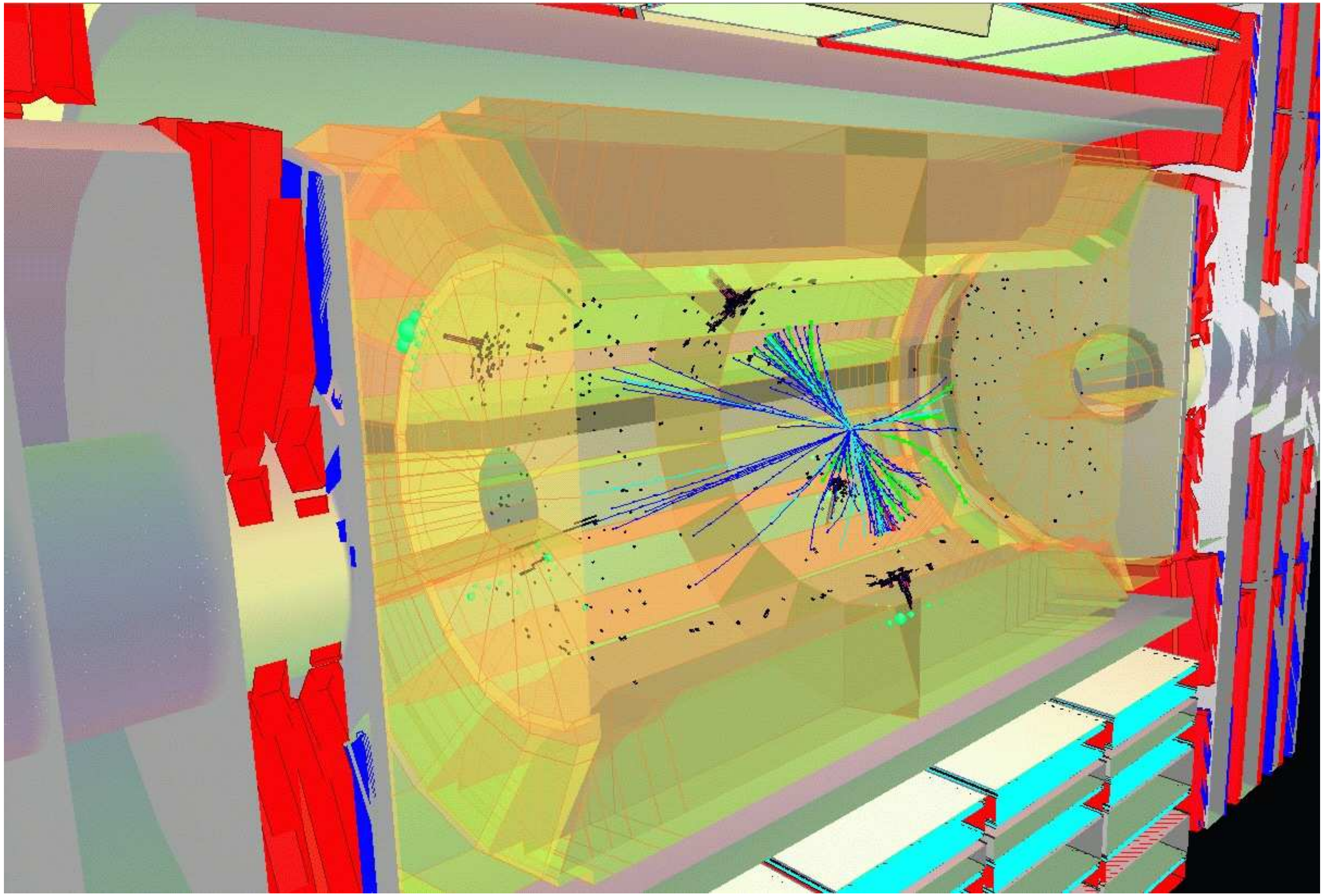


Neutrino event inside the SKM detector. It could come from one of the potential decay channels of the proton.

Preview: Small black-Hole production

Small black-hole may be produced and decay via Hawking radiation at LHC, if the scale of (quantum) gravity is low.





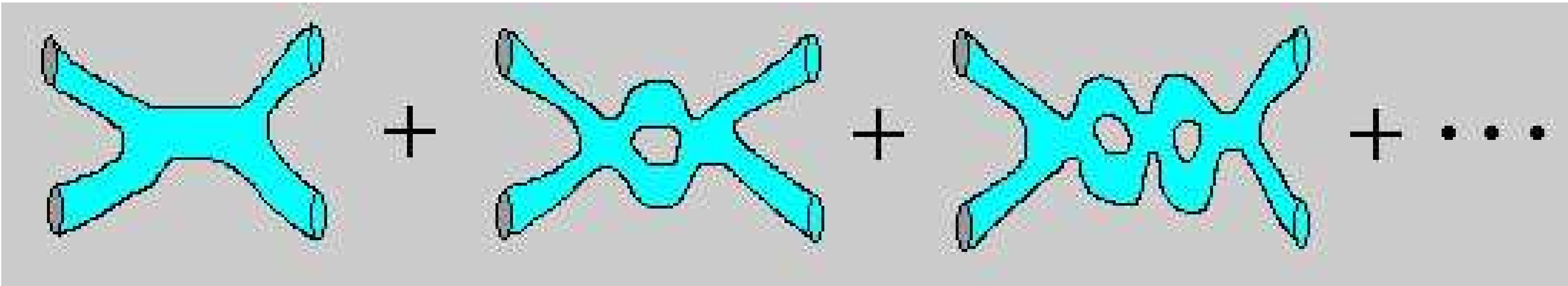
Event of BH production with $M_P = 1$ TeV and two extra dimensions.

See the Charybdis site

http://www.ippp.dur.ac.uk/montecarlo/leshouches/generators/charybdis/manual.html

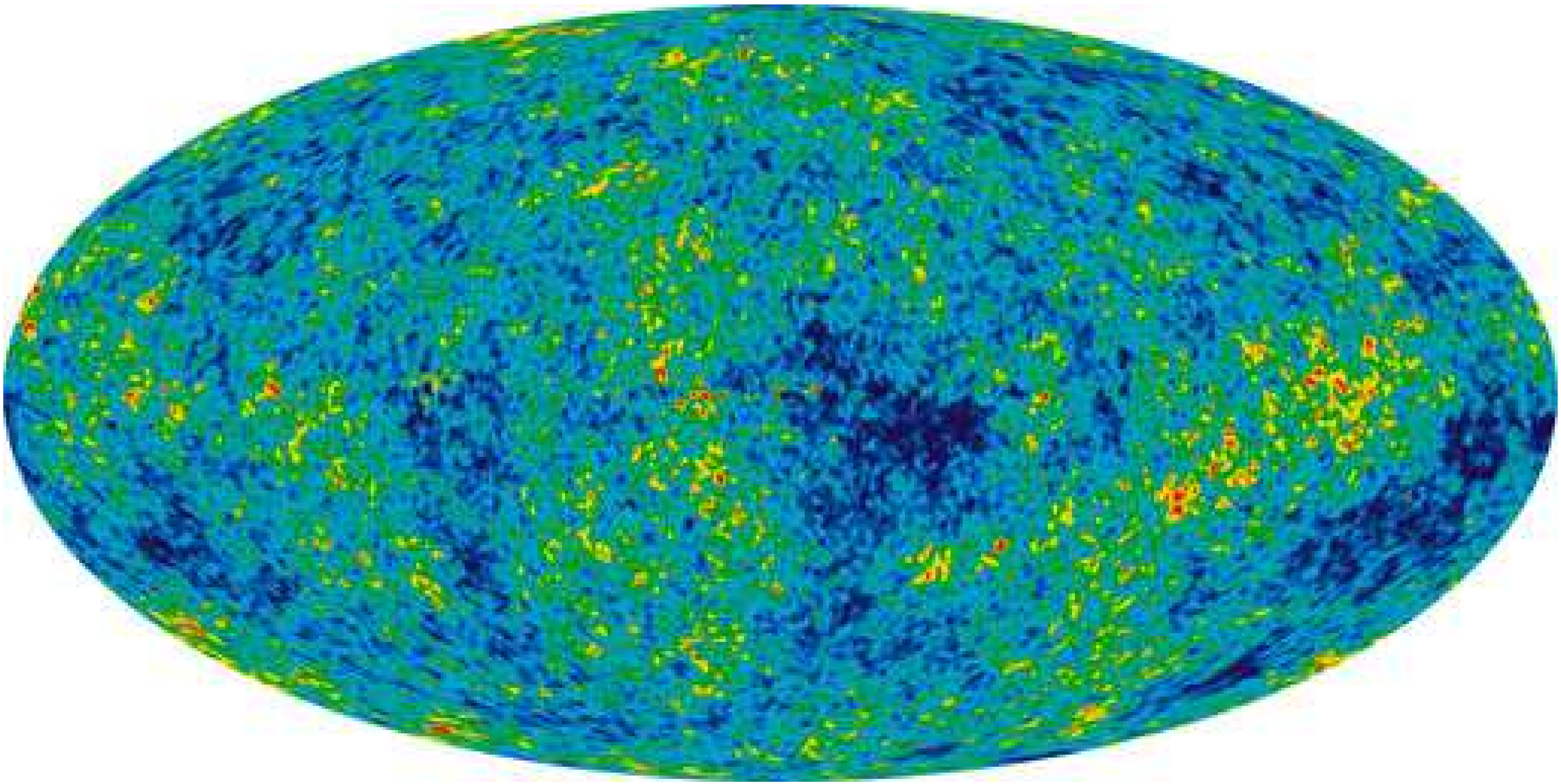
Preview: Strings?

String excited modes can also be produced if the string scale is sufficiently low.

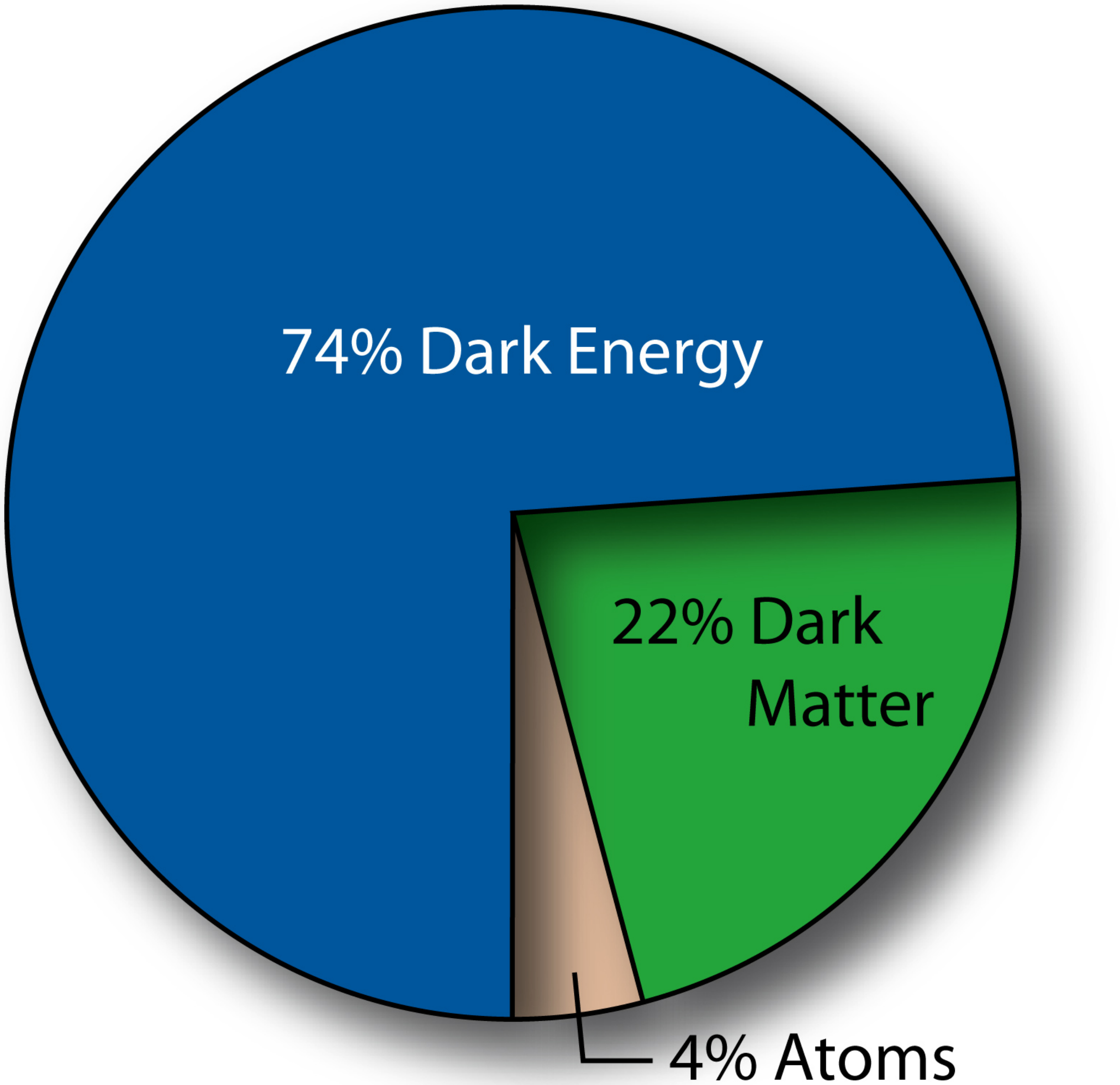


Preview: Dark Matter+Dark Energy

Can particle physics provide a candidate for the dark matter of the universe?



Can particle physics provide explanations for the primordial spectrum of cosmological fluctuations?



What is the dark energy?

The purpose of these lectures

- Why we believe the Standard Model is not the final (fundamental) theory of the world?

- Why do we believe that there is new physics around the TeV range?

- What types of new physics at shorter distances theorists have guessed during the past twenty years and why?

♡ This is an exciting period because it is some of this new physics that we are going to test at LHC

♠ We are also living in an era where similar, revolutionary data are coming from cosmology and they also probe the nature of the fundamental theory

♣ Most probably it will be some of you that will solve the puzzles and nail down the fundamental theory that extends and completes the SM!

Suggested reading

- “Beyond the Standard Model” by Fabio Zwirner, http://doc.cern.ch/cernrep/1998/98-03/98-03.html

- “Supersymmetry Phenomenology” by Hitoshi Murayama [arXiv:hep-ph/0002232]

- “Physics beyond the Standard Model” by Gian Giudice, Lect. Notes Phys. **591**:294-327,2002

- “Beyond the Standard Model” by John Iliopoulos http://preprints.cern.ch/cernrep/2004/2004-001/2004-001.html

- “Phenomenological guide to physics beyond the standard model”, by Stefan Pokorski, [arXiv:hep-ph/0502132]

- “Phenomenology beyond the Standard Model” by Joe Lykken, [arXiv:hep-ph/0503148]

Most obtainable from the archive http://xxx.arxiv.cornell.edu/

A tentative plan

- The Standard Model and its problems: Why do we expect new physics?

- Supersymmetry

- Grand Unification

- Gravity and String Theory

- The physics of extra dimensions.

High Energy Units

We use

$$h = 1 \quad , \quad c = 1$$

$$[Energy] \sim [Mass] \sim \frac{1}{[Length]} \sim \frac{1}{[Time]}$$

The Standard Model: principles

• The Standard Model of the Electroweak and Strong interactions has been a very successful theory.

• Effort started at the beginning of the twentieth century. Consolidated by the establishment of Quantum Field Theory.

QFT=Special Relativity+ Quantum Mechanics

• All interactions are based on the "gauge principle" (including gravity) ⇛ invariance under local (independent) symmetry transformations. (the first model for this was electromagnetism)

• Renormalizability was another principle. Today's vision (due to Wilson) is broader and gives a clearer physical picture.

• Other important principles are: Locality, Unitarity.

The Standard Model: ingredients

A review of the ingredients

Gauge groups

♣ Strong force: $SU(3)_{\text{color}} \rightarrow$ three colors.

Carriers: gluons are spin-one octets

$\rightarrow$ (color/anti-color) combinations. ($SU(N) \rightarrow N^2$-1 gauge bosons)

They are confined inside hadrons $\Rrightarrow$ "glue".

♠ The electroweak force:$SU(2)\times U(1)_Y$, it is spontaneously broken to $U(1)_{EM}$ by the Higgs effect.

Carriers: $W^{\pm}$, Z^0 (massive), γ (massless)

Standard Model: the quarks

Left-handed: $\begin{pmatrix} U_L \\ D_L \end{pmatrix}^a_{\frac{1}{6}}$, $\begin{pmatrix} C_L \\ S_L \end{pmatrix}^a_{\frac{1}{6}}$, $\begin{pmatrix} T_L \\ B_L \end{pmatrix}^a_{\frac{1}{6}}$,

a=red, blue, green

Right-handed: $\left(\quad U_R \quad \right)^a_{\frac{2}{3}}$, $\left(\quad D_R \quad \right)^a_{-\frac{1}{3}}$, $\left(\quad C_R \quad \right)^a_{\frac{2}{3}}$, $\left(\quad S_R \quad \right)^a_{-\frac{1}{3}}$, $\left(\quad T_R \quad \right)^a_{\frac{2}{3}}$, $\left(\quad B_R \quad \right)^a_{-\frac{1}{3}}$

The SM is a chiral theory.

Standard Model: the leptons

Left-handed: $\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}_{-\frac{1}{2}}$, $\begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}_{-\frac{1}{2}}$, $\begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}_{-\frac{1}{2}}$

Right-handed $\left(\quad e_R \quad \right)_{-1}$, $\left(\quad \mu_R \quad \right)_{-1}$, $\left(\quad \tau_R \quad \right)_{-1}$

and

$$\left(\quad \nu_e^R \quad \right)_0 \quad , \quad \left(\quad \nu_\mu^R \quad \right)_0 \quad , \quad \left(\quad \nu_\tau^R \quad \right)_0$$

For the neutrino sector you will learn the recent developments from the forthcoming lectures of J. Gomez-Cadenas

All fermions come in three copies called families.

Standard Model: the Higgs

• So far we have seen the interaction carriers with spin-one and "matter" with spin-$\frac{1}{2}$

• A spin-0 player as well: the Higgs. It is a (complex)-scalar SU(2) doublet with hypercharge $\frac{1}{2}$. Its "raison d'être" : break the electroweak symmetry spontaneously. As a result it gives masses to matter particles.

• Three of its components

$$H^{\pm} \quad , \quad Im(H^0)$$

become the third components of the massive gauge bosons

$$W^{\pm} \quad , \quad Z^0$$

after electro-weak symmetry breaking.

• The fourth, Re(H^0) $\Rightarrow$ physical neutral scalar that we expect to see at LHC.

Standard Model: Open problems

The standard model was constructed as a renormalizable theory $\longrightarrow$ the low-energy interactions hint on its behavior at high energy.

Why do we believe that there is more to know beyond the Standard Model?

Three sets of data that are not accounted for by the SM:

♣ Neutrinos have masses and they mix.

♠ There is a lot (22%) of dark (non-SM) matter in the universe. Neutrinos cannot account for it.

♠ There is another source of energy in the universe (74%) , known as "dark energy". It looks like vacuum energy. This gives $|V_{vac}| \sim 10^{-12}$ eV^4.

$$\text{In the SM} \quad |V_{vac}| \gtrsim 10^{44} \quad \text{eV}^4$$

Standard Model: Open problems II

• Quantum Gravity is not part of the Standard Model. One of the deepest questions of modern theoretical physics is:why the characteristic scale of gravity

$$M_{\text{Planck}} = \frac{1}{\sqrt{G_N}} \simeq 10^{19} \quad \text{GeV}$$

is so much higher than the other scales of particle physics?

• The Standard model alone contains IR-free couplings $\Rightarrow$ strongly-coupled UV physics.

• The SM has many unexplained parameters and patterns.

THEREFORE: SM is an Effective Field Theory (EFT) valid below 100 GeV. Must be replaced by a more fundamental theory at a higher scale Λ.

How big is Λ?

• Λ must be small: $\Lambda \sim$ a few TeV. Otherwise we suffer from a technical (fine-tuning) problem: the hierarchy problem (see later).

SM patterns and parameters

• The standard model group $SU(3)\times SU(2)\times U(1)$ is not "unified" :coupling constants $g_s^2 \simeq 1.5$, $g_W^2 \simeq 0.42$, $g_Y^2 \simeq 0.13$ are independent parameters. This can be improved if the fundamental theory has a simple gauge group, like SU(5) that contains the SM gauge group.

• The matter content and representations seems not very "regular". Why not higher representations? Hypercharges seem also bizarre. (but up to normalizations they are determined by the absence of gauge anomalies (BIM)

• Why three families? ("Who ordered that?")

• What decides the scale of Electroweak symmetry breaking

$$v_F \simeq 174 \quad \text{GeV ?}$$

What decides the mass of the Higgs?

The pattern of masses

- The pattern of SM masses is mysterious at least:

family \ type	ups	downs	leptons
3rd	$m_t = 175$	$m_b = 4.2$	$m_\tau = 1.7$
2nd	$m_c = 1.2$	$m_s = 0.1$	$m_\mu = 0.1$
1st	$m_u = 3 \times 10^{-3}$	$m_d = 5 \times 10^{-3}$	$m_e = 5 \times 10^{-4}$

- Neutrino masses seem to be in the $10^{-12} - 10^{-14}$ GeV range. SM masses span 16 orders of magnitude.

This is a question for the Yukawa couplings λ_i: $m_i = \lambda_i \, v_F$.

We want to explain their ratios and the absolute normalization, (as we can do it for the spectral lines of atoms.)

Other parameters

There are others parameters, measured in the SM, whose values are not explained:

• The elements of the Kobayashi-Maskawa matrix: three mixing angles and a phase that controls CP violation. There is a similar one for the Neutrino sector.

• A non-perturbative parameter: the θ-angle of QCD: $\sim \theta \int d^4x \, \epsilon^{\mu\nu\rho\sigma} Tr[F_{\mu\nu}F_{\rho\sigma}]$

A non-zero value breaks CP in the strong interactions (contrary to observations.) This is the "strong CP-problem"

Experimentally

$$d_n \lesssim 10^{-25} \;\; e \text{ cm} \quad \rightarrow \quad \theta \lesssim 2 \times 10^{-10}$$

How parameters affect us?

How academic is the issue of such parameters?

Most of them are crucial to the existence of our universe as we know it, and the existence of humans as we know them.

- $v_F \to 0$ then p is unstable to decay to neutrons $\to$ no Hydrogen. ($m_n - m_p \simeq m_d - m_u + E_n^{\text{EM}} - E_p^{\text{EM}}$. $E_n^{\text{EM}} - E_p^{\text{EM}} \sim -1.7$ MeV is independent of v_F. $m_d - m_u \simeq (\lambda_d - \lambda_u)v_F > 0$)

- $v_F >> 170 GeV$ n-p mass difference is very large and the nuclear force becomes of shorter range $\to$ nothing but hydrogen in the universe.

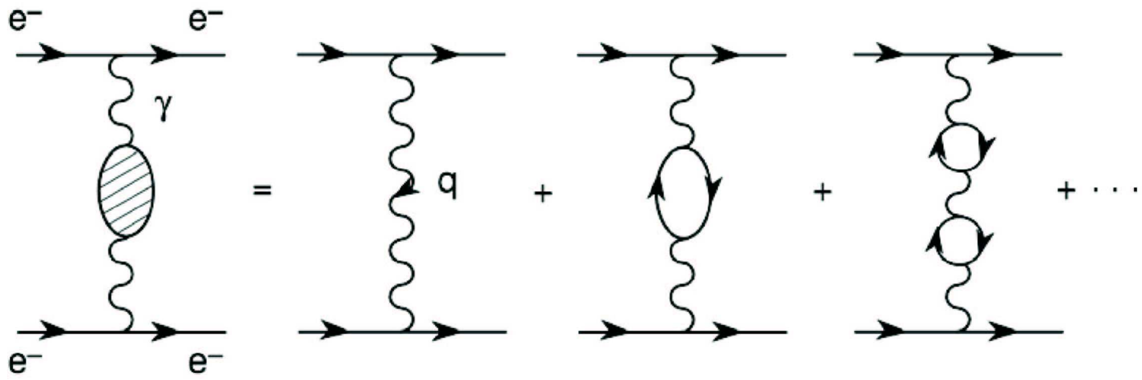
- changing the α_{em} $\to$ no C^{12} resonance $\to$ no carbon in our universe.

etc.

See [arXiv:hep-ph/9801253] for more information.

Running Couplings

You have learned that coupling constants "run" with energy. The reason is : quantum effects

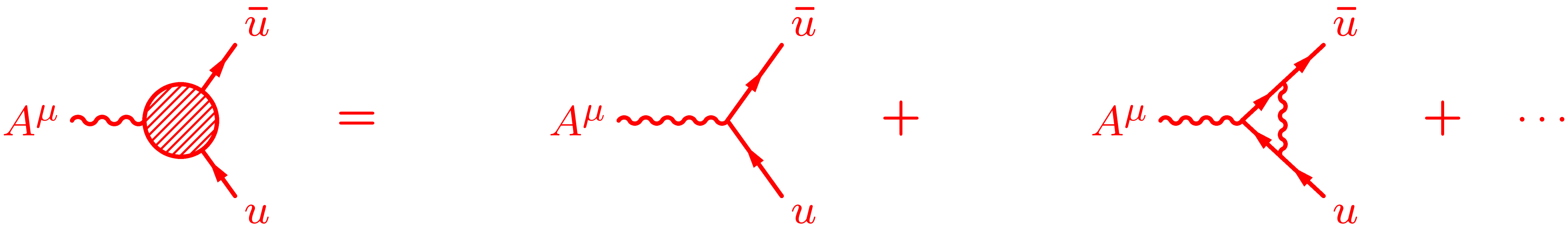


In electromagnetism we have "screening": $e^+ - e^-$ pairs have the tendency to screen lone charges.
higher the energy = more $e^+ - e^-$ pairs = more charge screening.
Result: charge is a function of the energy= $\frac{1}{\text{distance}}$:

$$\alpha_{em} \equiv \frac{e^2}{2(\hbar c)} \quad , \quad \alpha_{em}(E) \simeq \frac{\alpha_{em}(m_e)}{1 - \frac{\alpha_{em}(m_e)}{3\pi} \log \frac{E^2}{m^2}}$$

• After taking into account these quantum effects on the coupling, we may replace the EM interaction by its corrected value:

$$\delta S \quad \sim \quad e(E) \int \; A_\mu \; \bar{\psi}_e \gamma^\mu \psi_e$$



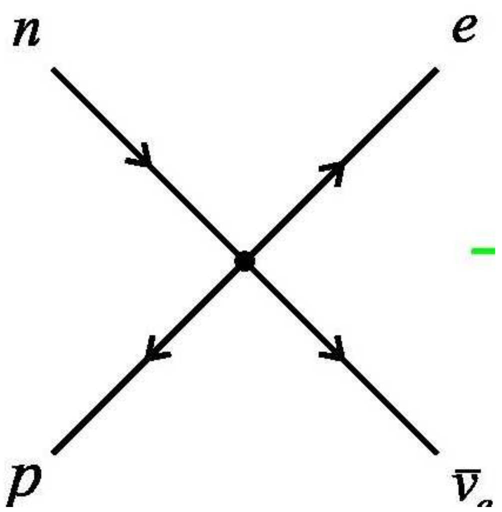
This is the effective interaction valid at energy E.

Effective couplings: the Fermi theory paradigm

The Fermi theory described the decay of neutrons:

$$n \to p + e + \bar{\nu}_e$$

via a four-fermion (dimension-6=non-renormalizable interaction)

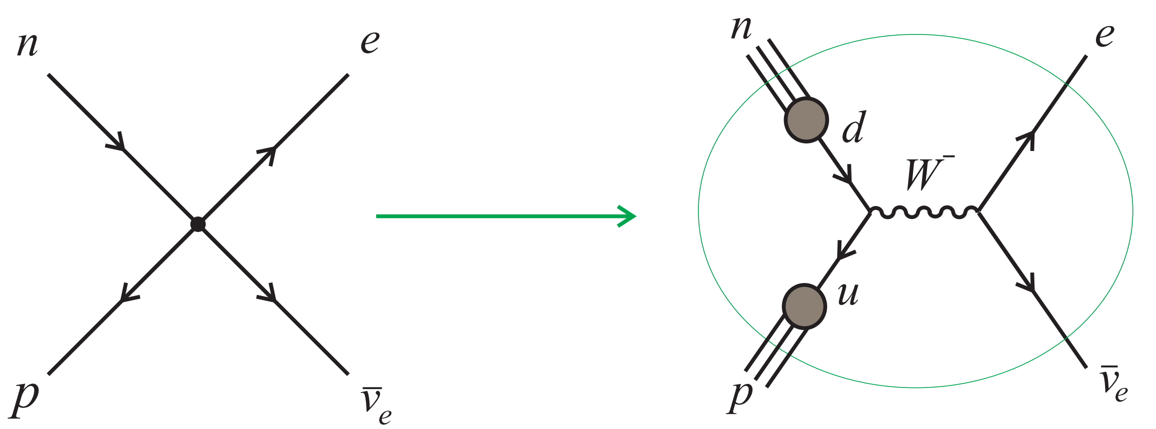


$$L_{\text{interaction}} = G_F\ (\bar{p}\ \gamma^\mu\ n)(\bar{\nu}_e\ \gamma_\mu\ e)$$

with $G_F \simeq \frac{1}{(300\ GeV)^2}$

This descriptions is very accurate for energies $E << 100GeV$.

However, with a better magnifying glass the four-fermi interaction originates from standard gauge interactions



Effective interaction :

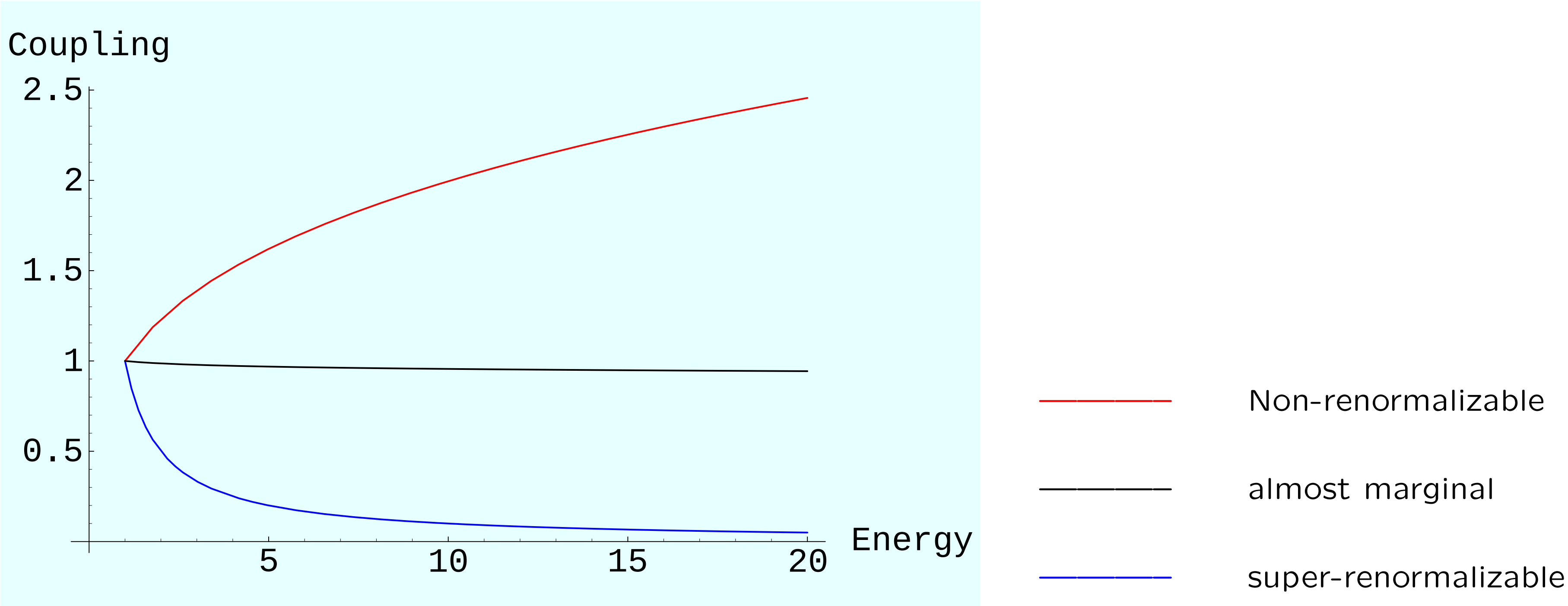
$$p = (uud) \quad , \quad n = (udd) \quad , \quad d \to W^- + u \to (e^- + \bar{\nu}_e) + u$$

$$\frac{g_W^2}{p^2 + M_W^2} \simeq \frac{g_W^2}{M_W^2} = G_F \quad , \quad p^2 << M_W^2$$

The effective interaction is dimension 6. It is the result of interactions with dimension 4 (renormalizable) interactions at higher energy.

Renormalization

• Couplings λ_i depend on energy, E: $\lambda_i(E)$. This dependence is the result of including virtual effects at higher energies than E (shorter distances).



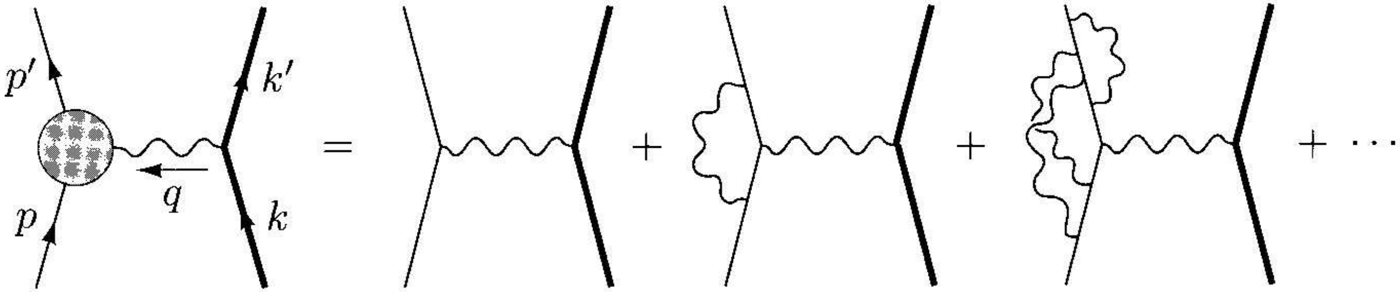
• Couplings can be irrelevant (non-renormalizable),

$$\lambda(E) = \lambda(E_0)\left(\frac{E}{E_0}\right)^{\Delta-4} \quad , \quad \Delta > 4$$

relevant (super-renormalizable), or almost marginal (renormalizable, log running).

$$\lambda(E) = \frac{1}{\frac{1}{\lambda(E_0)} + b_0 \log\frac{E}{E_0}}$$

• We define a theory (via its couplings) at $E = \infty$ and uniquely determine (by calculating the quantum effects) the couplings at any lower energy (renormalization group flow).



• The reverse is not possible: we cannot guess the high-energy theory from the low-energy couplings. Two distinct theories can have the same low energy theory (universality).

• The couplings $\lambda_i(E)$ define the EFFECTIVE theory at energy E. It captures the low energy physics.

• When we make measurements at accelerators , we measure the effective couplings $\lambda_i(E_{\text{experiment}})$.

• Our goal is to find the UV couplings (complete specification of the theory).

Renormalization Summary

• The fundamental theory is defined at a high-energy scale $\Lambda \to \infty$.

• What we measure are effective interactions at low(er) energy (larger distance)

• Knowledge of the high-energy (short-distance) theory defines completely the low energy theory. It does not work the other way around!

• At low energy , all possible dimension interactions (allowed by symmetries) are generated.
Their effective couplings scale generically as

$$\lambda_d \simeq \Lambda^{4-d}\left[1+\mathcal{O}\left(\frac{E}{\Lambda}\right)\right]$$

where Λ is the characteristic high energy scale.

• The (old) wisdom: a quantum theory must be renormalizable $\Rightarrow$
Only renormalizable theories are definable at $\Lambda = \infty$.
(They depend on a finite number of fundamental parameters)
(Exercise: give a qualitative demonstration)

The hierarchy problem:introduction

We are now ready to understand the nature of the hierarchy problem.

According to our previous discussion operators of dimension two and three (mass terms for bosons and fermions) should have at low energy their coefficients scale as

$$m_i^2 \sim \Lambda^2 \quad \Rightarrow \quad m_i \sim \Lambda$$

If we want the SM to make sense up to $\Lambda \simeq 10^{18}$ GeV, then either:

- All masses are enormous (excluded from experiment)

or

- We must fine-tune the high-energy masses and couplings (technically very difficult)

There are exceptions to $m_i \sim \Lambda$ and these are due to symmetry.

Fermion masses

Consider the electron Lagrangian written in terms of the left- and right-handed components of the electron,

$$e_{L,R} = \frac{1 \pm \gamma^5}{2}\, e$$

$$S = i\left[\bar{e}_R(\not{\partial} + \not{A})e_R + \bar{e}_L(\not{\partial} + \not{A})e_L\right] + m_e\, (\bar{e}_L e_R + \bar{e}_R e_L)$$

The theory has the usual vector U(1) symmetry:

$$e_{L,R} \to e^{i\epsilon}\, e_{L,R} \quad , \quad \bar{e}_{L,R} \to e^{-i\epsilon}\, \bar{e}_{L,R}$$

When $m_e = 0$ there is more symmetry: chiral symmetry,

$$e_L \to e^{i\epsilon} e_L \quad , \quad e_L \to e^{-i\epsilon} e_L$$

Inversely: chiral symmetry forbids a mass.

The quantum corrections to the fermion mass coming from the diagrams

It must have the following form (to leading order)

$$m_{eff}(E) = m_e + \frac{3\alpha_{em}}{4\pi} m_e \log\frac{E}{\Lambda} = m_e\left[1 + \frac{3\alpha_{em}}{4\pi}\log\frac{E}{\Lambda}\right]$$

Therefore, **it is very insensitive to the high-energy scale Λ.** ($\sim$ 4% for $E = 1$ GeV and $\Lambda = 10^{19}$ GeV).

Gauge boson masses

Unbroken gauge symmetry forbids gauge bosons to have a mass. Upon spontaneous breaking of the gauge symmetry gauge bosons acquire masses.

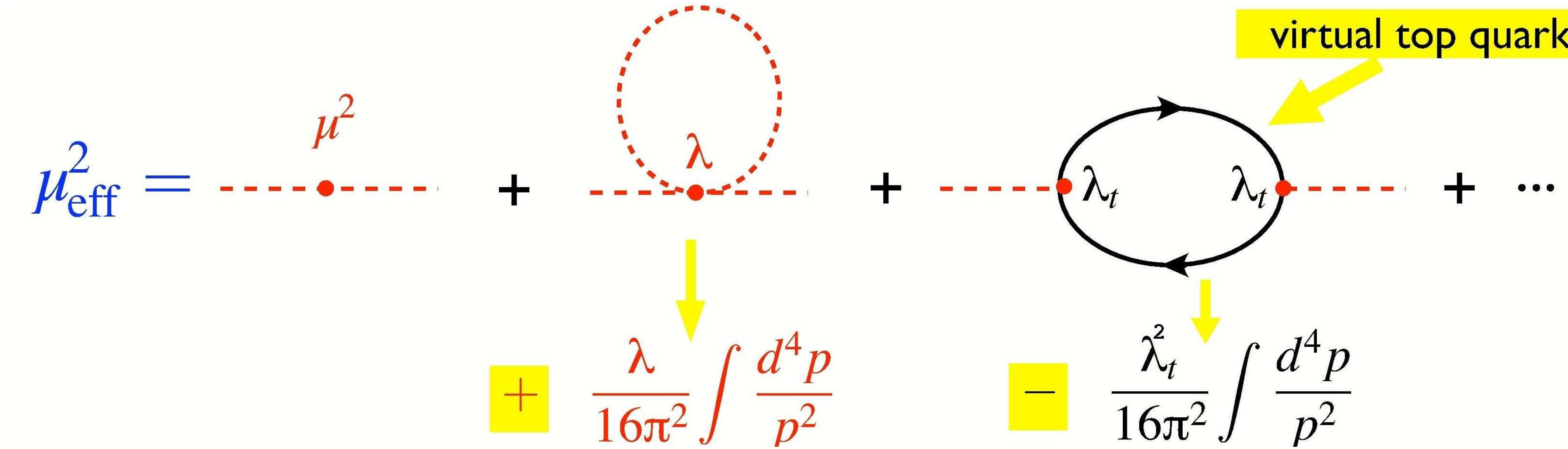
$$M_{Z,W^{\pm}} \sim g\ v_F \quad v_F \sim \frac{\mu}{\sqrt{\lambda}} \quad , \quad V = -\frac{\mu^2}{2}H^2 + \lambda H^4$$

Dimensionless couplings run logarithmically $\sim \log \frac{E}{\Lambda}$ and therefore are not very sensitive to Λ.

The important sensitivity comes from the renormalization of the mass-term of the Higgs, μ.

The Higgs mass term

We have seen that the sensitivity of SM masses depends on the behavior of a single parameter: the mass term μ of the Higgs scalar.



$$\mu^2_{eff}(E) = \mu^2 + \frac{\lambda - \lambda_t^2}{4\pi^2}(\Lambda^2 - E^2)$$

The Higgs mass, and therefore many other SM masses depend quadratically on the UV scale Λ.

The hierarchy problem

We found that:

• All dimensionless couplings of the SM run logarithmically and are therefore not very sensitive to the UV scale of the theory.

• The Higgs quadratic term $\mu \Rightarrow$ the Higgs expectation value $v_F \Rightarrow$ Fermion and gauge-boson masses are quadratically sensitive to Λ.

• The SM physics is therefore technically hard to calculate.

This is the hierarchy problem:**It is very difficult in a theory where parameters run polynomially with the cutoff Λ to extend it to hierarchically higher energies.**

End of first act

Avoiding the hierarchy problem

Very SPECIAL theories may avoid the hierarchy problem.

- “Technicolor”

- “Supersymmetry”

- Large dimensions

- Pseudo-Goldstone particles (little Higgs)

See [arXiv:hep-ph/0512128] and [arXiv:hep-ph/0502182]

Technicolor

• The idea, known under the name of "technicolor", is to assume that all particles in the fundamental theory except the gauge bosons are fermions.

• And the Higgs? It could be a bound state of two fermions (like mesons are bound states of quarks and anti-quarks).

• This needs a new gauge interaction (technicolor) that becomes strong at an energy $\Lambda_T > v_F$.

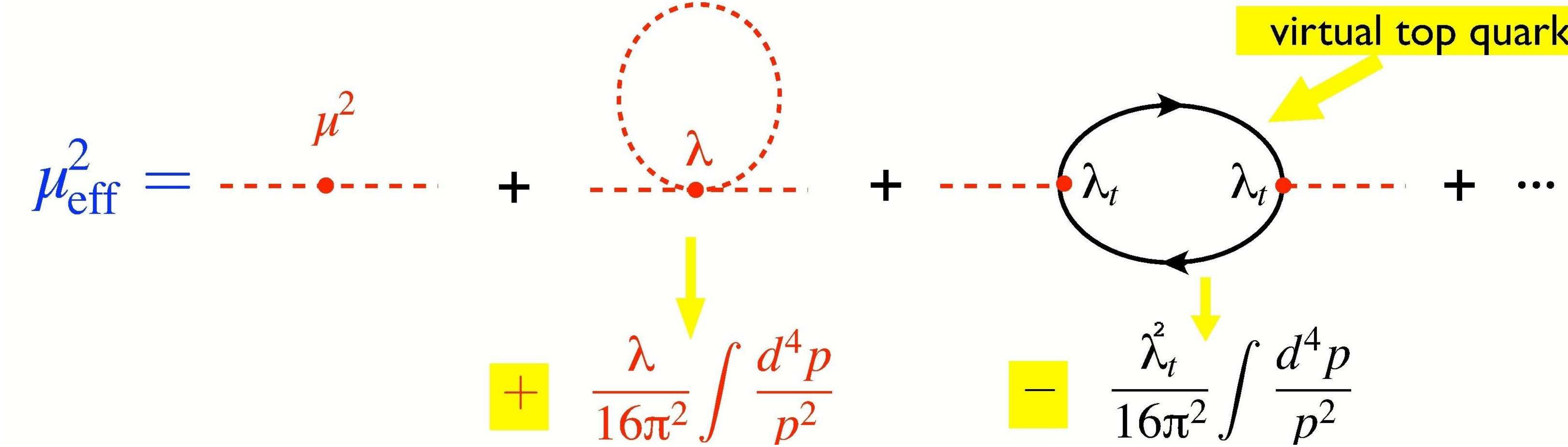
• For $\infty >> E >> \Lambda_T$ the theory is a theory of fermions and all masses run logarithmically.

• For $E << \Lambda_T$ the theory looks like the SM.

♣ Unfortunately detailed models that satisfy known experimental constraints are very difficult to construct.

Supersymmetry

Another SPECIAL class of theories:



If $\lambda = \lambda_t^2$ then the quadratic divergence will cancel. Fermion and boson loops cancel each other.

The symmetry that imposes such relations is known as

supersymmetry (SUSY) .

Supersymmetry, Vol II

$$\delta_{\text{SUSY}}\,(\text{Boson}) = \epsilon \cdot (\text{Fermion})$$

$$\delta_{\text{SUSY}}\,(\text{Fermion}) = \epsilon \cdot \partial\,(\text{Boson})$$

Therefore

$$\delta_{\text{SUSY}} \cdot \delta_{\text{SUSY}} \sim \partial$$

and in this sense SUSY is a "square root" of a translation.

Supersymmetry pairs a particle with spin j will another with spin $j \pm \frac{1}{2}$

Then the Higgs will have a fermionic partner (the Higgsino) whose effect will be to cancel the quadratic terms in the running of the mass.

The Supersymmetric Multiplets (representations)

• In the SSM we have vector multiplets containing a vector (gauge boson) and a Majorana fermion (gaugino) in the adjoint of the gauge group $\to \quad (A^a_\mu, \lambda^a)$.

• We also have chiral multiplets containing a complex scalar and a Weyl fermion, in some appropriate representation of the gauge group $\to \quad (\phi^i, \psi^i)$.

♠ In the SSM we have such multiplets for each quark and lepton as well as two conjugate multiplets $H_1^{-\frac{1}{2}}$, $H_2^{+\frac{1}{2}}$ for the Higgs.

♣ This is to avoid $U(1)_Y$ anomalies and give masses to both up and down quarks (see later).

The Supersymmetric Standard Model

particles				Sparticles			
quarks	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R	squarks	$\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$	$\tilde{u}_R$	$\tilde{d}_R$
leptons	$\begin{pmatrix} e_L \\ \nu_L \end{pmatrix}$	e_R		sleptons	$\begin{pmatrix} \tilde{e}_L \\ \tilde{\nu}_L \end{pmatrix}$	$\tilde{e}_R$	
Higgs doublets	H_1 (hypercharge $= -1$) H_2 (hypercharge $= +1$)			Higgsinos	$\tilde{H}_1$ $\tilde{H}_2$		
	W^+, H^+			charginos	χ_1^+, χ_2^+		
	Z, γ, H_i^0	$(i = 1, 2, 3)$		neutralinos	χ_a^0	$(a = 1, 2, 3, 4)$	
	g			gluino	$\tilde{g}$		

Exercise: Show that no particle of the SM can be a susy partner: a full doubling of the spectrum is necessary.

The quantum numbers

chiral supermultiplet	SU(3)	SU(2)	$U(1)_Y$
Q	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$
U^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$
D^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$
L	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
E^c	$\mathbf{1}$	$\mathbf{1}$	1
H_1	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
H_2	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$

Note that L and H_1 are indistinguishable in terms of gauge quantum numbers.

The supersymmetric interactions

• The renormalizable interactions of a gauge theory are encoded in gauge couplings, Yukawa couplings and the potential for the scalars.

• In a supersymmetric renormalizable theory, the interactions are encoded into the gauge couplings and the super-potential W, a gauge-invariant function of the scalar fields, (but not of their complex conjugates). For renormalizability it must be at most cubic.

• The kinetic terms of the fields and their couplings to the gauge bosons are standard and determined by the representations/charges and the gauge couplings.

• The Yukawa couplings are as follows:

$$L_{\text{Yukawa}} = [i\sqrt{2}\ g\ \bar{\psi}_i\ \lambda^a (T^a\phi)^i + h.c.] - \frac{1}{2}\left[\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\ \bar{\psi}^i\psi^j + h.c.\right]$$

- Finally there is the scalar potential

$$V=\sum_i F_i^* F_i+\frac{g^2}{2}\sum_a D^a D^a \geq 0$$

$$F_i=\frac{\partial W}{\partial \phi^i} \quad , \quad D^a=\phi_i^*(T^a)^i{}_j\phi^j$$

- The most general cubic, gauge invariant superpotential is

$$W=h^U\ QU^cH_2+h^D\ QD^cH_1+h^E\ LE^cH_1+$$

$$+\mu H_1H_2+$$

$$+\lambda QD^cL+\lambda' LLE^c+\mu' LH_2+\lambda'' U^cD^cD^c$$

Exercise: Show this!

- μ' has one family index (h^U,h^D,h^E) have two such indices and $(\lambda,\lambda',\lambda'')$ have three.
- The last four terms violate baryon and lepton number.

$$\lambda,\lambda',\mu'\neq 0 \quad\to\quad \Delta B=0,|\Delta L|=1 \quad , \quad \lambda''\neq 0 \quad\to\quad \Delta B=1,|\Delta L|=0$$

A symmetry (R-parity) must be imposed to forbid them.

Exercise: Why such offending terms are absent in the SM?

R-parity

To avoid problems with fast proton decay and lepton number violation: assume the existence of an extra Z_2 symmetry

$$\text{R} - \text{parity} \quad = \quad (-1)^{2S+3B+L} \quad = \quad (-1)^{\text{number of Sparticles}}$$

- Sparticles can only be produced or annihilated in pairs.

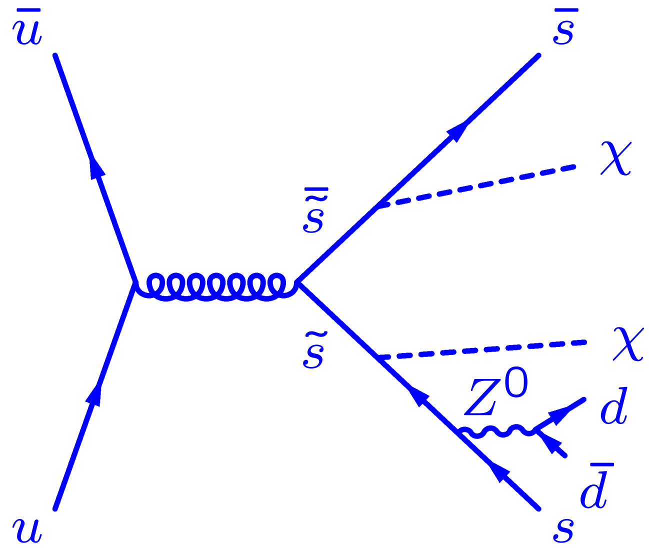
- The lightest Sparticle (LSP) is absolutely stable.

- It is almost always a neutralino → it has only weak interactions → it is not directly visible in experiments → missing energy.

This is a characteristic SUSY signal at LHC.

- When supersymmetry breaks, R-parity must remain intact!

This is an example of a possible event that can be seen at LHC:

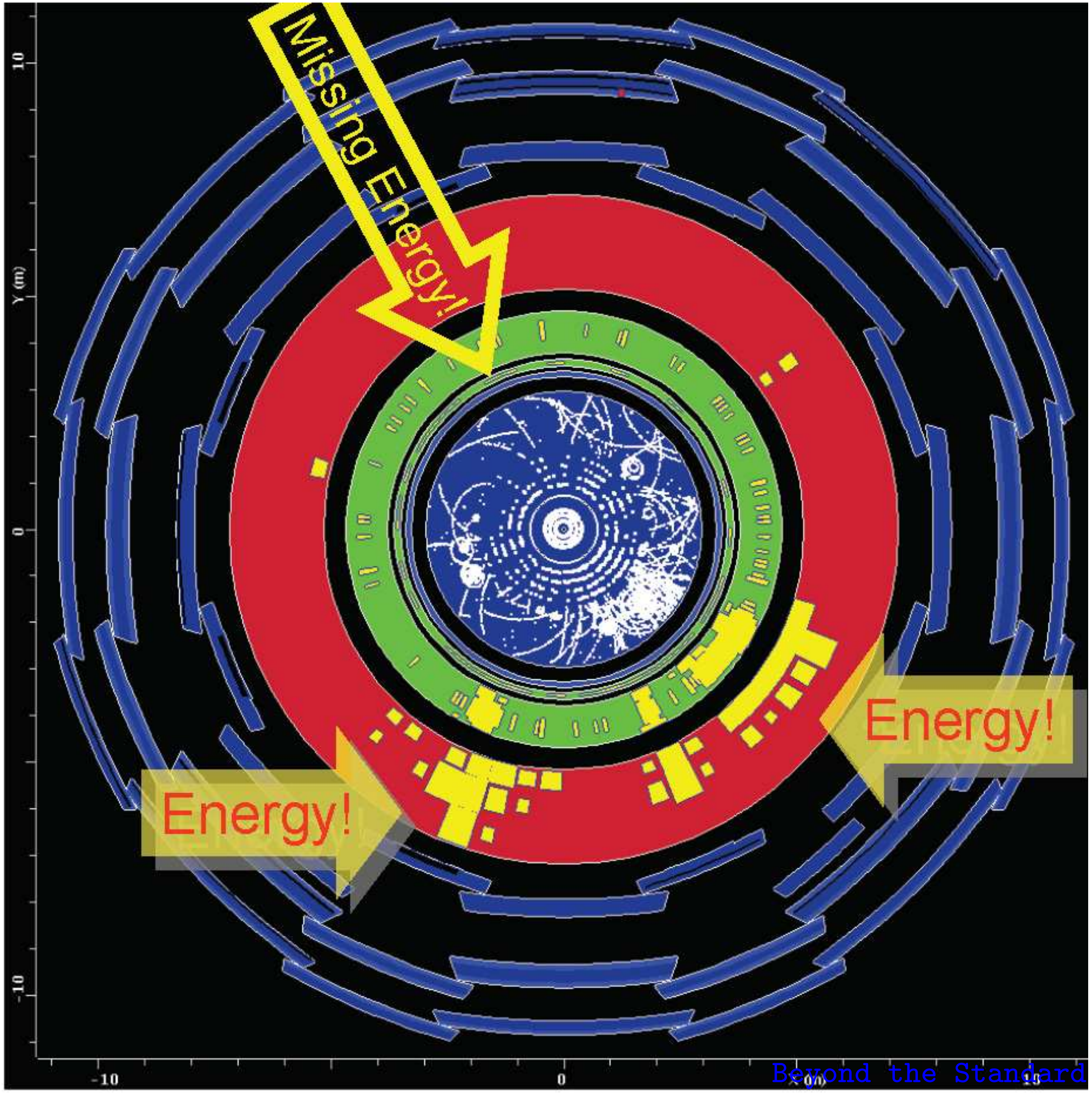


where:

$$\tilde{q} \quad = \quad squark$$

$$\chi \quad = \quad \text{LSP}$$

Missing Energy (Atlas simulation)



A link to the dark matter of the Universe

- The universe contains an important fraction (22%) of non-relativistic, non-SM matter. This is known as Dark Matter.

- Its presence is crucial for structure formation in the universe.

- It is mostly composed of Weakly Interacting (very) Massive Particles: WIMPS.

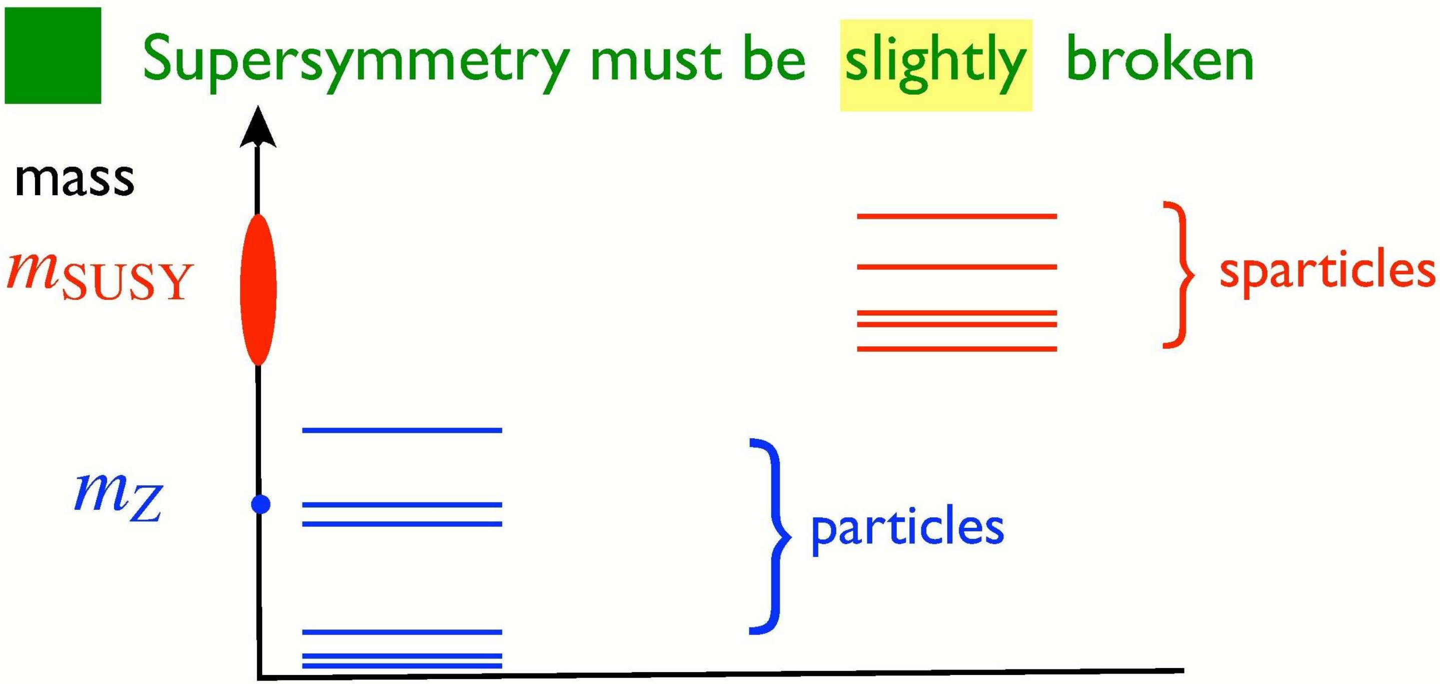
- The supersymmetric LSP, is an excellent candidate for forming the dark matter of our universe.

Supersymmetry breaking

So far we have neglected the fact that exact supersymmetry forces the superpartners to have the same mass as the SM particles, e.g.

$$m_e = m_{\tilde{e}} \quad , \quad \text{etc.}$$

It is unavoidable to conclude that:



Supersymmetry breaking, II

We must ensure that SUSY breaking does not destroy the good properties of SUSY:

• Like gauge symmetry breaking, supersymmetry breaking must be spontaneous (soft). Then the Higgs mass runs logarithmically like that of the fermions!

• M_{SUSY} must not be very far from v_F. It should be $1-10$ TeV.

If $M_{\text{SUSY}} >> v_F$ the hierarchy problem resurfaces.

• Therefore, naturalness tells us that the superpartners must be in the TeV range.

• If this idea is correct, most probably the superpartners will be found at LHC.

The soft supersymmetry breaking terms

We can parameterize all possible extra terms that can appear because of supersymmetry breaking ($d \leq 4$) These are scalar masses, gaugino masses, and cubic scalar interactions proportional to the ones present in the superpotential:

$$L_{\rm soft} = \sum_i \tilde{m}_i^2 |\phi_i|^2 + \frac{1}{2}\sum_A M_A \bar{\lambda}^A \lambda^A +$$

$$+ \left(h^U A^U \; Q U^c H_2 + h^D A^D \; Q D^c H_1 + h^E A^E \; L E^c H_1 + m_3^2 \; H_1 H_2 + h.c.\right)$$

• Since A^i are matrices in flavor space, we have a large number of parameters. For generic values of such parameters there are phenomenological problems (like flavor changing neutral currents).

• There are several simple choices of soft parameters that are motivated by (i) simplicity (ii) some concrete Susy-breaking mechanism.

The tree-level MSSM potential is:

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 H_2 + h.c.) + \frac{g_2^2}{8}\left(H_2^\dagger \vec{\sigma} H_2 + H_1^\dagger \vec{\sigma} H_1\right)^2 + \frac{g_Y^2}{8}\left(|H_2|^2 - |H_1|^2\right)^2$$

$$m_1^2 = \mu^2 + \tilde{m}_{H_1}^2 \quad , \quad m_2^2 = \mu^2 + \tilde{m}_{H_2}^2$$

Without any extra input, there are no UV constraints on the MSSM parameters. Simple ansatz (compatible with data so far, and can arise from supergravity/string theory). Imposed at some UV scale Λ:

Gaugino masses and soft scalar masses are universal

$$M_3 = M_2 = M_Y \equiv m_{1/2}$$

$$\tilde{m}_Q = \tilde{m}_{U^c} = \tilde{m}_{D^c} = \tilde{m}_L = \tilde{m}_{E^c} = \tilde{m}_{H_1} = \tilde{m}_{H_2} \equiv m_0$$

So are the soft scalar couplings

$$A^U = A^D = A^E \equiv A_0$$

If we now include the μ-term coefficient, μ and the soft breaking term m_3 we end up with 5 extra parameters on top of the SM ones:

$$\mu 1q \quad , \quad m_{1/2} \quad , \quad m_0 \quad , \quad A_0 \quad , \quad m_3$$

• After minimization of the Higgs potential with $\langle H_1 \rangle = \binom{v_1}{0}$, $\langle H_2 \rangle = \binom{0}{v_2}$ we can trade μ and m_3 with sign(μ) and $\tan\beta \equiv \frac{v_1}{v_2}$.

$$\text{sign}(\mu) \quad , \quad m_{1/2} \quad , \quad m_0 \quad , \quad A_0 \quad , \quad \tan\beta$$

This is known as the mSUGRA parametrization of the MSSM.

• The parameters, $m_{1/2}, m_0, A_0$, must be evolved to low energy using the RGE equations and eventually compared to data. $\tan\beta$ is already a low energy parameter.

Spontaneous supersymmetry breaking

Spontaneous supersymmetry breaking is an important problem. There are many different classes of models.

• Global supersymmetry breaks spontaneously, when $\langle V \rangle > 0$. Since $V \sim |F_i|^2 + |D^a|^2$, this implies that if some $\langle F_i \rangle$ or $\langle D^a \rangle$ are non-zero susy is broken.

• Like standard global symmetries, there is a massless fermion, the Goldstino, $\tilde{G} = \langle F_i \rangle \psi^i + \langle D_a \rangle \lambda^a$, associated with spontaneous global supersymmetry breaking.

• Supersymmetry can be promoted into a local symmetry. The appropriate theory then contains also gravity and is known as supergravity.

• In particular, the "gauge-field" associated to local supersymmetry is a spin-3/2 fermion known as the gravitino. It is the supersymmetric partner of the graviton. Like the graviton it is massless when supersymmetry is unbroken.

• When supersymmetry breaks spontaneously, the gravitino acquires a non-zero mass $m_{3/2}$. It becomes massive by combining with the Goldstino field. This is the super-Higgs mechanism.

• The supersymmetry breaking scale Λ_S is related to the gravitino mass in a universal fashion:

$$\Lambda_S = \sqrt{3\ m_{3/2}\ M_P}$$

• The superpartner mass splittings depend on the sector I of the theory as:

$$(\Delta m^2)_I \quad \sim \quad \lambda_I\ \Lambda_S^2$$

where λ_I is the (renormalized) Goldstino/gravitino coupling to sector I.

There are two rough avenues to arrange for $\Delta m \sim$ TeV:

(A) Heavy gravitino mass $\rightarrow$ large Λ_S, but very small λ_I.

(B) Light gravitino mass, and $\lambda_I \sim 1$.

SSB: Heavy gravitino mass

• Here the supersymmetry breakings happens in a “hidden sector”.

• It is communicated to the observable sector by the gravitational interaction

$$\lambda_I \sim \frac{\Lambda_S^2}{M_P^2} \quad , \quad \Lambda_S \sim \sqrt{(\Delta m) M_P} \sim 10^{10} - 10^{11} \ \text{GeV} \quad , \quad m_{3/2} \sim 1 \ \text{TeV}$$

Taking the limit $M_P \to \infty$ to recover the EFT, we obtain the MSSM with typically universal soft terms.

• Such breaking can be realized in supergravity and in superstring vacua where susy is broken by hidden gaugino condensation.

• The EFT is MSSM and is valid up to close the Planck scale.

• There is another “mechanism” in this class: Anomaly Mediated Susy Breaking.

Further reading: http://doc.cern.ch/cernrep/1998/98-03/98-03.html

SSB: Light gravitino mass

This may be realized when supersymmetry is broken in a hidden sector, and is communicated to the observable sector by gauge or Yukawa interactions. Here $\lambda_I \sim \mathcal{O}(1)$.

• To obtain the desired mass splittings, $\lambda_S \sim$ TeV and therefore $m_{3/2} \sim 10^3 - 10^{-5}$ eV.

• A class of models realizing this supersymmetry breaking pattern are known as messenger or gauge mediated supersymmetry breaking models. They contain apart from the observable sector, the "messenger" sector and the "hidden" sector.

• Here the gravitino is part of the low energy spectrum and its Goldstino component couples to the low energy fields with strength that ranges from order the gauge couplings to several orders smaller.

• Such theories have new physics well below the Planck scale.

• The LSP is the gravitino.

Further reading: http://doc.cern.ch/cernrep/1998/98-03/98-03.html

Grand Unification: The idea

The Standard Model gauge group is not "fully unified". At higher energy, the symmetry becomes larger. At lower energies it breaks spontaneously to the standard model group: $SU(3) \times SU(2) \times U(1)_Y$

$$SU(3) \Longrightarrow U_3 U_3^\dagger = 1 \quad , \quad det(U_3) = 1$$

$$SU(2) \Longrightarrow U_2 U_2^\dagger = 1 \quad , \quad det(U_2) = 1$$

We can include $SU(3) \times SU(2) \times U(1)_Y$ inside SU(5)

$$SU(5) \Longrightarrow U_5 U_5^\dagger = 1 \quad , \quad det(U_5) = 1$$

subgroups of SU(5)

$$SU(3) \rightarrow \left(\begin{array}{ccc|cc} & U_3 & & 0 & 0 \\ & & & 0 & 0 \\ & & & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} U_3 & 0 \\ \hline 0 & \mathbf{1}_{2\times 2} \end{array}\right)$$

$$SU(2) \rightarrow \left(\begin{array}{ccc|cc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & & \\ 0 & 0 & 0 & & U_2 \end{array}\right) = \left(\begin{array}{c|c} \mathbf{1}_{3\times 3} & 0 \\ \hline 0 & U_2 \end{array}\right)$$

$$U(1)_Y \sim \frac{1}{6}\begin{pmatrix} \mathbf{2} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{2} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{2} & 0 & 0 \\ 0 & 0 & 0 & -\mathbf{3} & 0 \\ 0 & 0 & 0 & 0 & -\mathbf{3} \end{pmatrix}$$

Since dim($SU(5)$)=24, there are 12 extra gauge bosons apart from the SM ones.

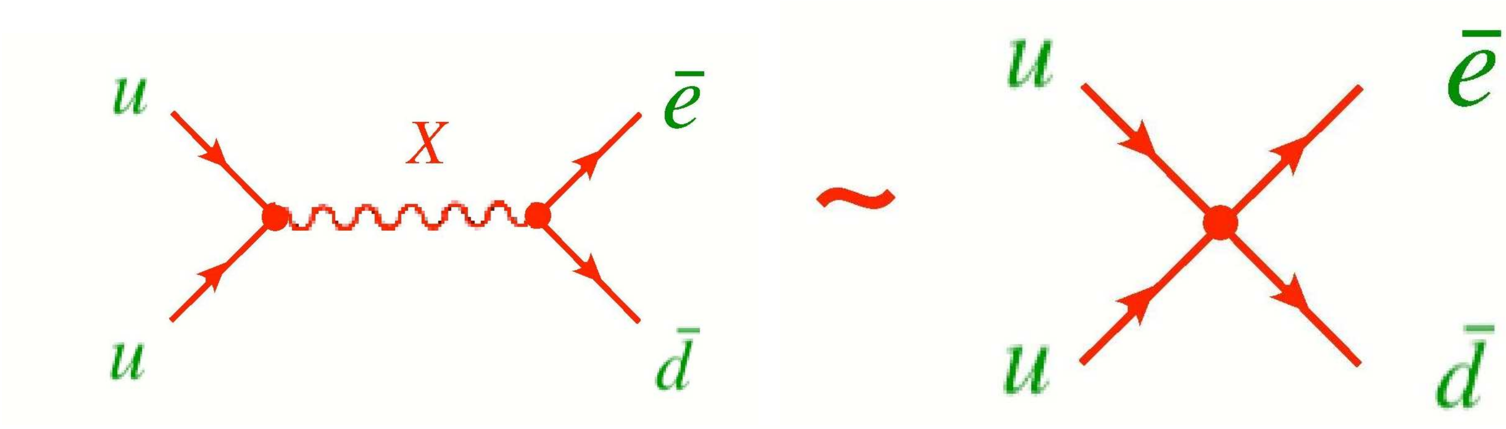
SU(5): the matter

$$\mathbf{10} = \begin{pmatrix} 0 & \bar{u}_3 & -\bar{u}_2 & u_1 & d_1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & u_2 & d_2 \\ \bar{u}_2 & -\bar{u}_1 & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & \bar{e} \\ -d_1 & -d_2 & -d_3 & -\bar{e} & 0 \end{pmatrix} \quad \rightarrow \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \rightarrow \bar{e}_R \qquad \bar{\mathbf{5}} = \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ e_L \\ \nu_L \end{pmatrix}$$

There should also be a singlet to accommodate ν_R.

The new, larger symmetry mixes quarks and leptons:We expect baryon and lepton number to be violated by the new gauge interactions.

Proton decay



As with the Fermi example this four-fermion effective interaction has a coupling $\sim \frac{g_5^2}{M_X^2}$

From experiment we obtain that $\tau_p > 2.6 \times 10^{33}$ years. This implies

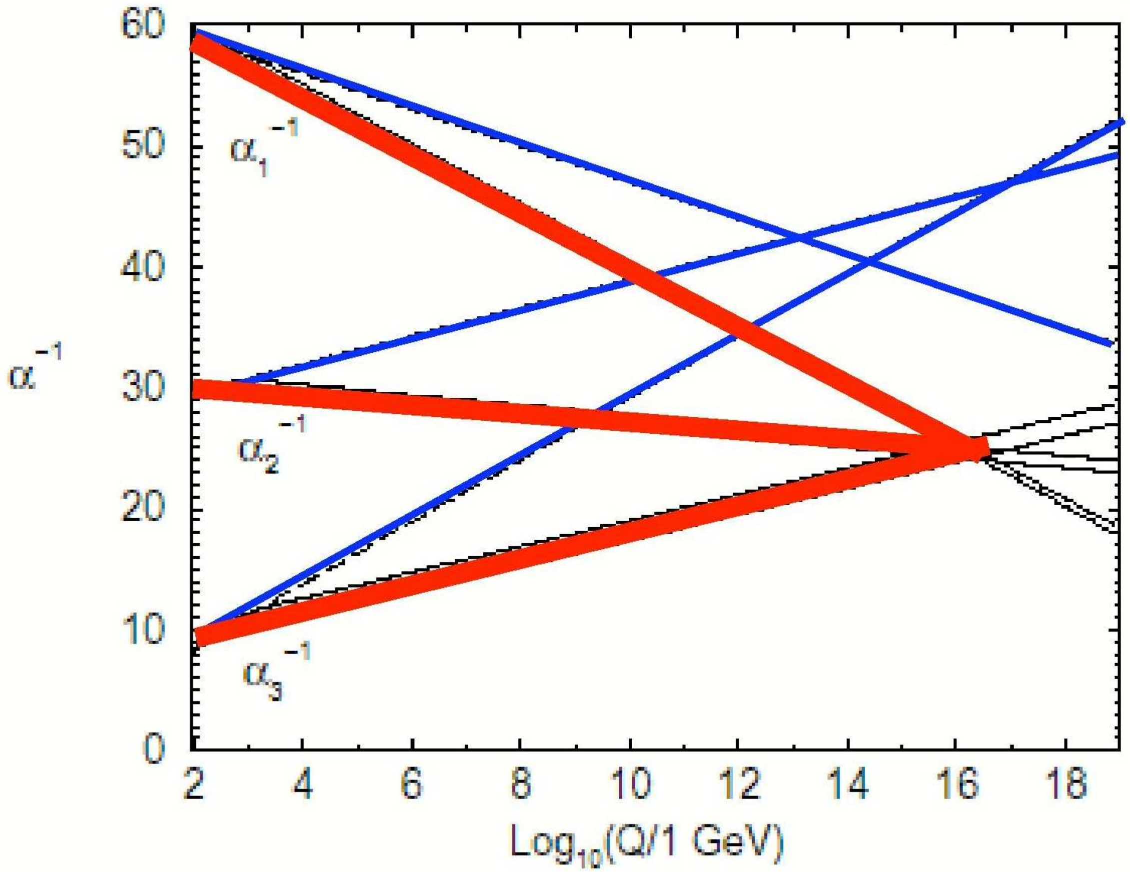
$$M_X > 10^{15} \text{ GeV}$$

Coupling unification

We have coupling unification at the scale $\Lambda = M_X$

$$g_3 = g_2 = \sqrt{\frac{5}{3}}\, g_Y = g_5 \equiv g_{GUT}$$

This seems in good agreement with the data if we allow for the renormalization group running



The gravitational coupling

The coupling of gravity, Newton's constant G_N has dimensions M^{-2}. This is how we define the Planck Mass : $G_N = M_{\text{Planck}}^{-2}$.

Gravitational force:

$$F = G_N \, \frac{M_1 \, M_2}{R^2} \sim G_N \, \frac{E_1 \, E_2}{R^2}$$

The dimensionless gravitational coupling runs fast with energy:

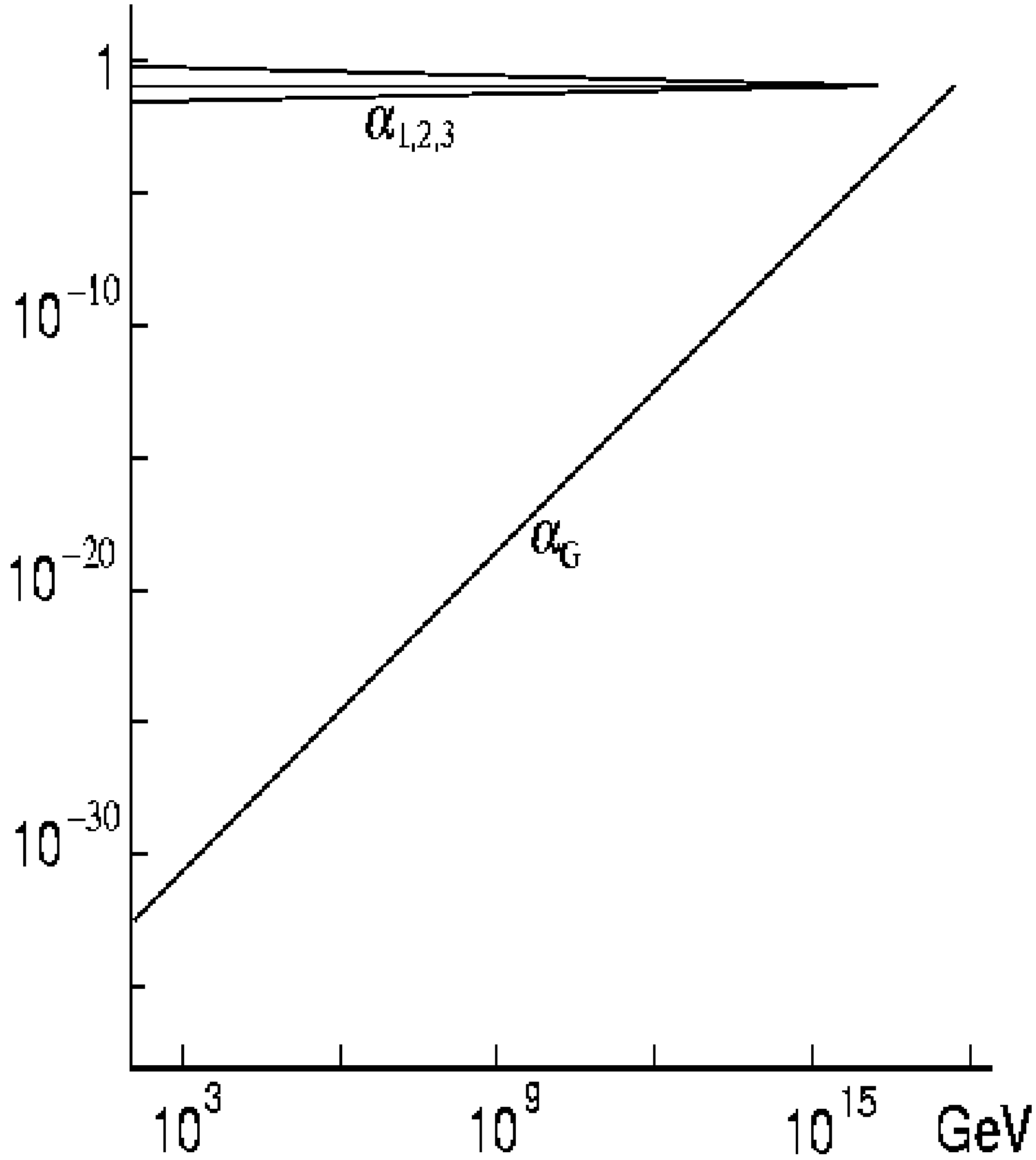
$$\alpha_{grav} \equiv G_N \, E^2 = \frac{E^2}{M_{\text{Planck}}^2}$$

Gravity versus other interactions

interaction	dimensionless coupling	strength
Strong	$\alpha_s = g_s^2/4\pi\hbar c$	~ 1
Electromagnetic	$\alpha_{\text{em}} = e^2/4\pi\hbar c$	$\sim \frac{1}{137}$
Weak	$G_F\ m_p^2$	$\sim 10^{-5}$
Gravity	$G_N\ m_p^2/\hbar c$	$\sim 10^{-36}$

Therefore until now gravity has been safely neglected in particle physics.

The running of all couplings



Gravity and the SM

The existence of gravity is the most solid argument that the SM is not the final theory.

• Gravity interacts with SM fields.

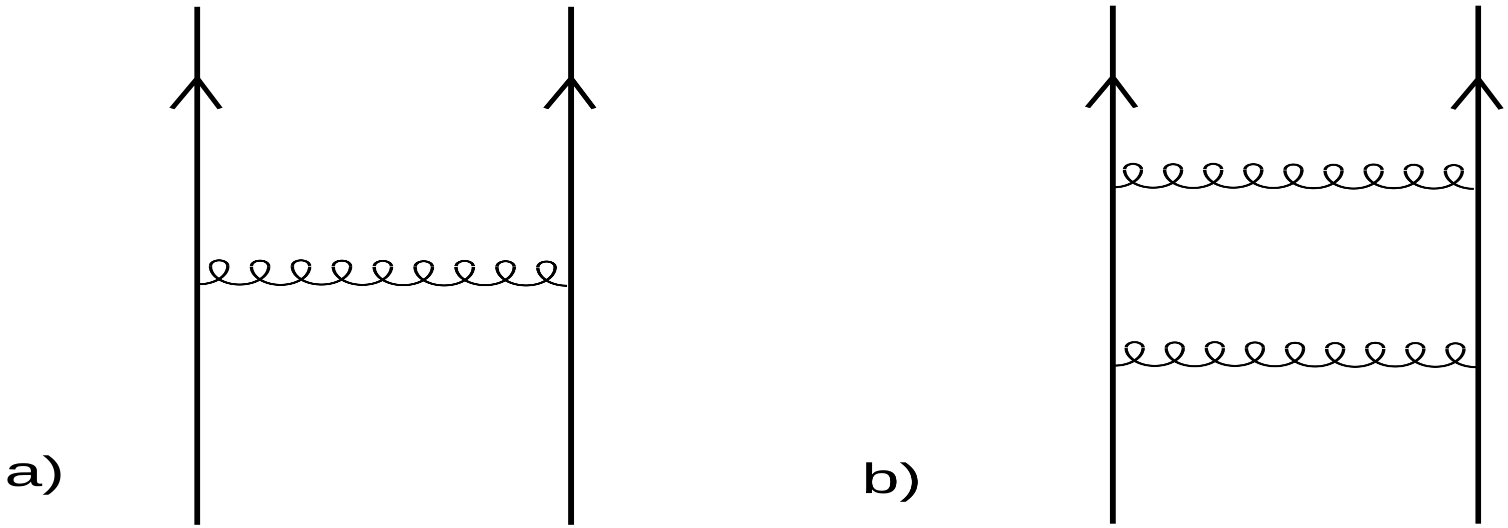
• At some high energy scale, Λ_P gravity will become strong, and quantum effects must be incorporated. This scale could be $M_P \sim 10^{19}$ GeV but (as we will see later) it could also be much lower.

• This fundamental theory, would look like classical gravity plus the SM at energies $E \ << \ \Lambda_P$.

• In this sense the SM is an effective theory, valid (at most) up to Λ_P.

• Things look bad, since classical gravity (general relativity) is a non-renormalizable theory.

Gravity at short distances?



- The classical gravitational theory is non-renormalizable

$$(b) \quad \sim \quad \frac{E^2}{M^4_{\text{Planck}}} \int_0^{\Lambda} dp \ p \quad \sim \quad \frac{\Lambda^2 \ E^2}{M^4_{\text{Planck}}} \ ,$$

- At higher orders it gets worse and worse.
- No clue as to what the short distance theory is.
- This has been an open problem for more than 50 years.

Gravity and String Theory

• String theory is a different framework for describing and unifying all interactions.

• It has become popular because it always includes quantum gravity, without UV problems (divergences)

• Moreover it also includes the other ingredients of the SM: Gauge interactions, chiral matter (fermions) and if needed, supersymmetry.

• It offers some conceptual features that are appealing to physicists:

(a) String theory ALWAYS contains gravity

(b) The existence of fermions implies supersymmetry at high energy.

(c) It has a priori no fundamental parameters but only one dimensionfull scale: the size of the strings. All dimensionless parameters of a given ground state of the theory are "dynamical" (expectation values of scalar fields).

(d) It contains solitonic extended objects (known as branes) that provide an incredible richness to the theory as well as a deep link between gauge theories and gravity.

What is String Theory?

Shift in paradigm: from point particle to a closed string.

• In QFT fields are "point-like". In string theory, they depend not on a point of space-time but a loop in space-time (the position of a closed string).

What is the difference between a closed "fundamental" string and a loop of wire?

(A) The fundamental string is much smaller: its size is definitely smaller than 10^{-18} m. This would explain why we have not seen one so far.

(B) Apart from the usual degrees of freedom (their coordinates in space-time), fundamental strings have also fermionic degrees of freedom. There a kind of supersymmetry relating the coordinates to such fermionic degrees of freedom.

Since the smallest length we can see today (with accelerators) is approximately 10^{-18} m strings would appear in experiments so far as point-like objects.

• Fundamental strings, like the analogous classical objects, can vibrate in an infinite possible number of harmonics.

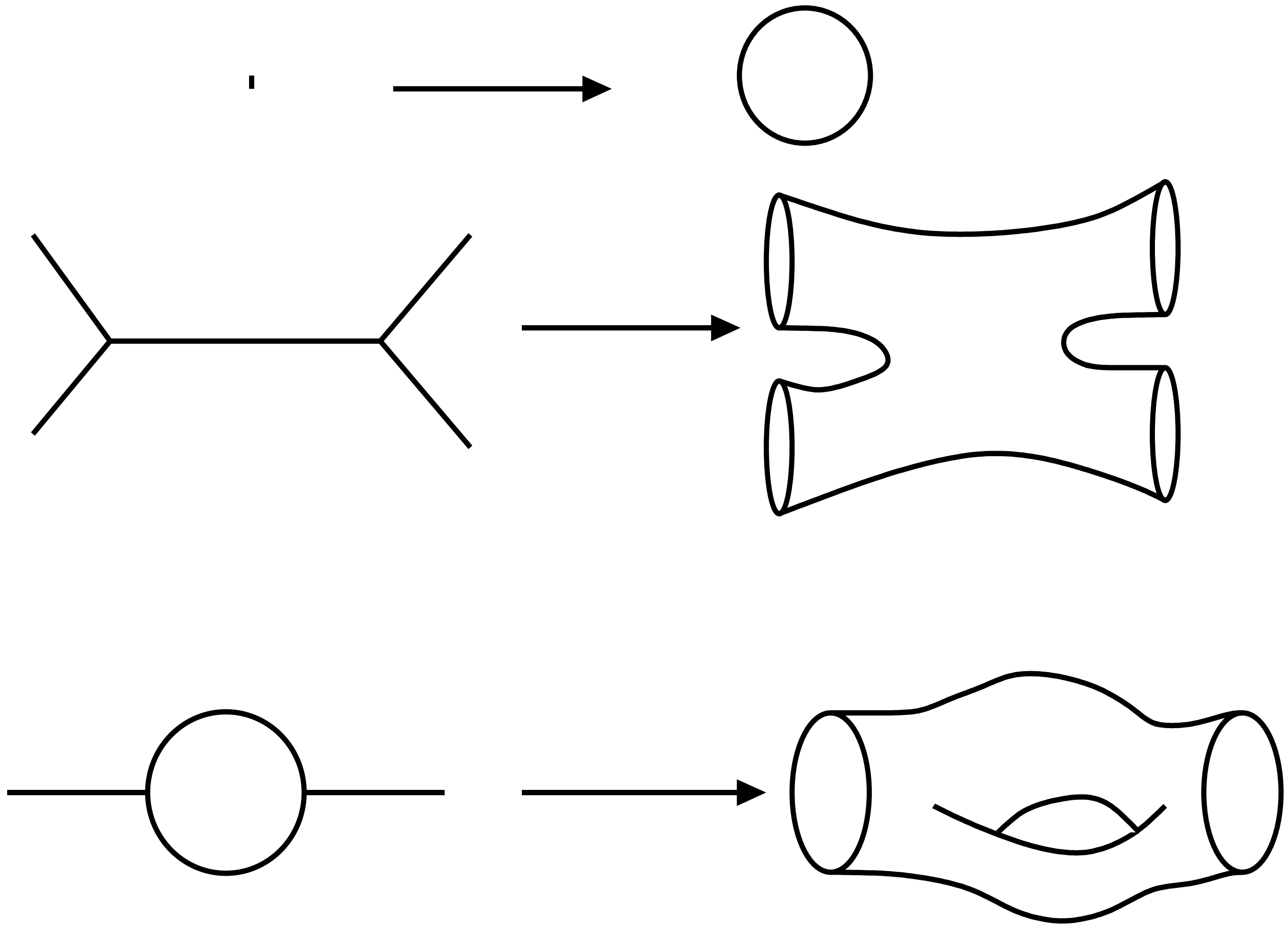
• Upon quantization, these harmonics behave like different particles in space-time.

A single string upon quantization $\Longrightarrow$ an infinite number of particles with ever increasing mass.

• Infinity of particles is responsible for the unusual properties of string theory (and its complicated structure).

• Strings live in diverse dimensions. Lorentz invariances $\Leftrightarrow$ 9+1 dimensions. Although this seems to contradict common experience it can be compatible under certain circumstances. How do we see the extra dimensions? More on this later .....

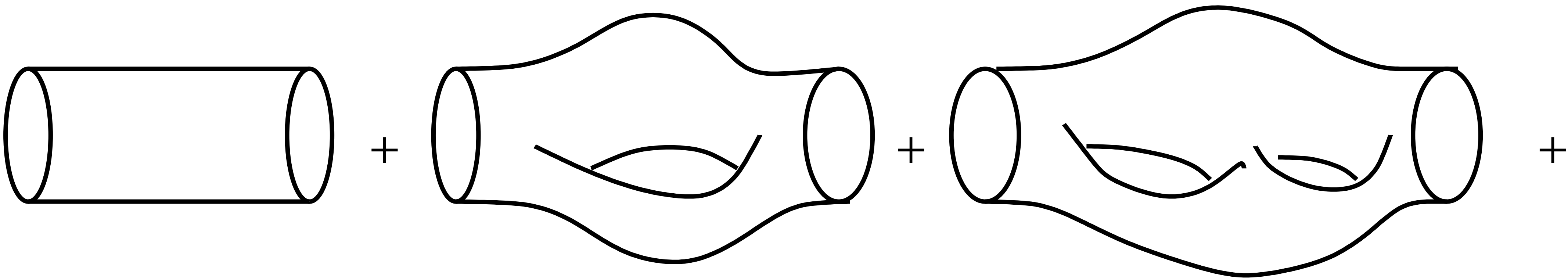
• In perturbation theory, standard QFT Feynman diagrams are replaced with string diagrams (two-dimensional surfaces)



String perturbation theory

♣ In QFT perturbation theory is formulated using Feynman diagrams.

♠ In string theory we have Riemann surfaces. For closed strings, each order contains a single diagram. At low energy, they reduce to the (many) QFT Feynman diagrams.



• String theory diagrams, when appropriately defined, give finite amplitudes in the UV. Quantum gravity, which is part of string theory is essentially finite.

Extra space dimensions

- The idea that space has extra, hitherto unobservable dimensions goes back to the beginning of the twentieth century, with Nordström (1914), Kaluza (1925) and Klein (1926).

- It comes naturally in string theory.

How come they are not visible today?

(A) Because they compact and sufficiently small.

(B) Because we are "stuck" on the four-dimensional world.

(C) Because they are of a more bizarre kind (for example, they are discretized appropriately)

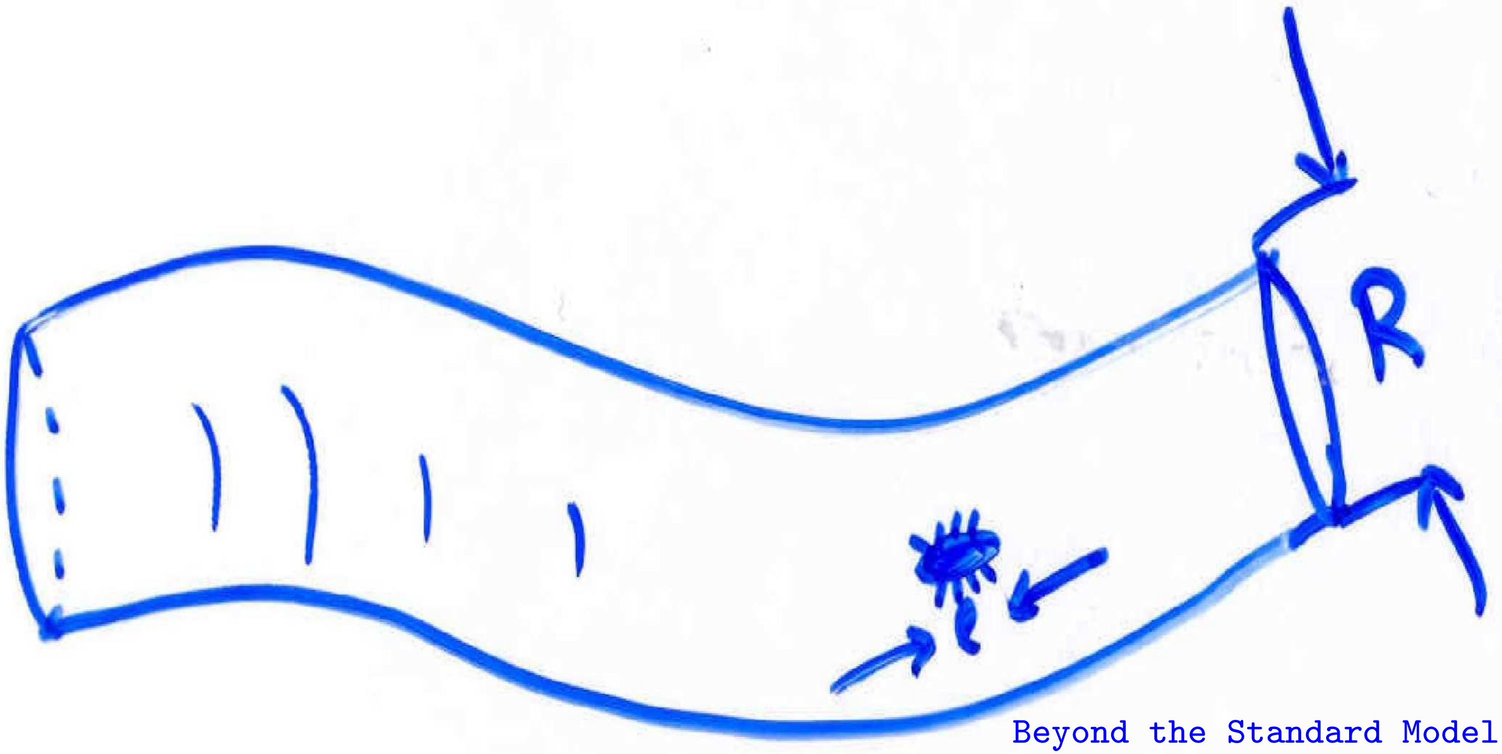
"Small" compact dimensions

A compact, sufficiently small extra dimension is not visible !

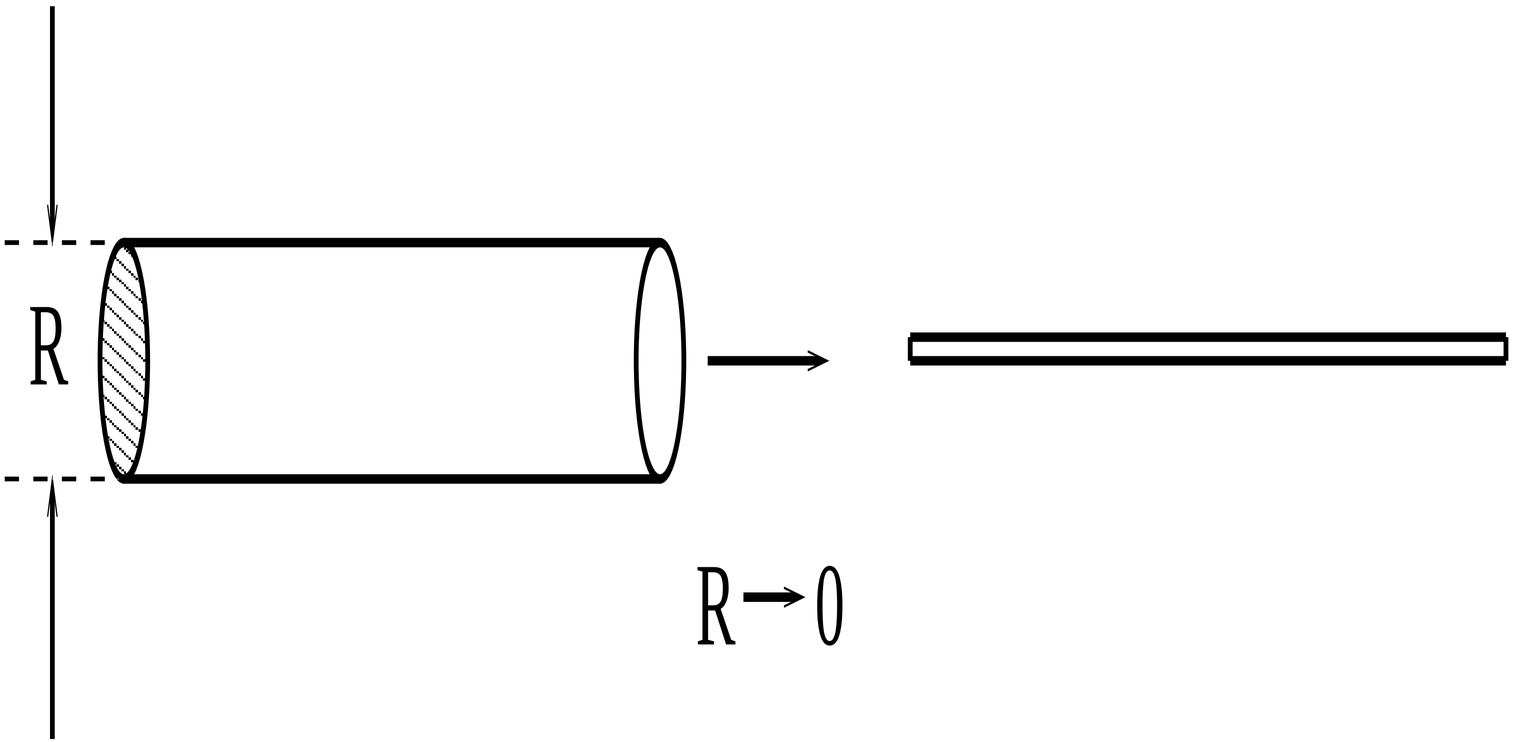
A simple example of a space with one compact (circle) and one non-compact (real line) dimension: a hose of infinite length and radius R.

There are two regimes:

(A) At distance $<<$ R the space looks like an (infinite) two-dimensional plane.



(B) At distance $>>$ R the compact direction of the hose is invisible. The hose looks one-dimensional.



We will now make this intuition more precise.

Kaluza-Klein states

Consider the usual 3+1 dimensional space-time and a fifth dimension that is a circle of radius R. Consider also a free massless scalar field in this 5d space-time.

- From QM: the momentum on a circle is quantized.

$$e^{ip_4\ (x^4+2\pi R)} = e^{ip_4\ x^4} \quad \longrightarrow \quad p_4 = \frac{n}{R} \quad , \quad n \in Z$$

From the mass-less condition in 5 dimensions:

$$E^2 - p_1^2 - p_2^2 - p_3^2 - p_4^2 = 0 \to E^2 - p_1^2 - p_2^2 - p_3^2 = \frac{n^2}{R^2}$$

Compare with four-dimensional relation for massive particles:

$$E^2 - p_1^2 - p_2^2 - p_3^2 = M^2$$

This is equivalent to an infinite tower of four-dimensional particles (KK states) with masses

$$M_n = \frac{|n|}{R}$$

This result is generic and applies also to massive fields or fields with spin. (Exercise: Derive the KK masses for a massive 5d scalar. Derive the analogous result for a 5-dimensional gauge field. What is the spin of 4-dimensional fields that are obtained and what are their masses?)

♣ If at low energy, our available energy in accelerators is

$$E \lesssim \frac{1}{R}$$

none of the massive KK-states can be produced ("seen"). The extra dimension is invisible!

♠ When $E >> \frac{1}{R}$ several KK states can be produced and studied. When many have been seen the extra compact dimension can be reconstructed.

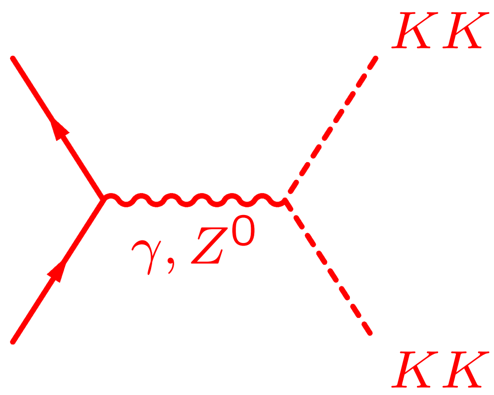
♢ The fact that till today in colliders we have not seen such states (with SM charges) gives a limit on R:

$$\frac{1}{R} > 300 \quad GeV$$

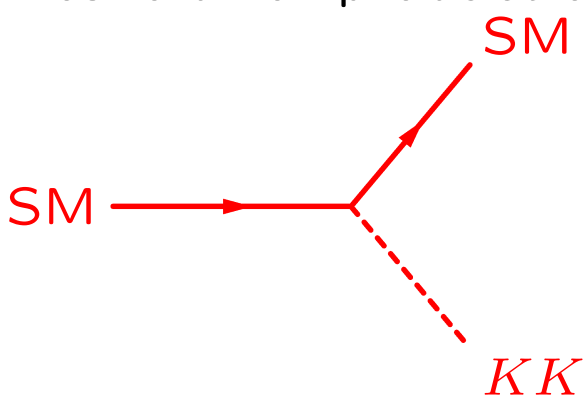
In LHC, there will be searches for KK states.

• Since a circle is translationally invariant, p_4 is conserved. n is therefore like a conserved KK U(1) charge.

• Therefore KK-states must be pair produced, so the threshold for their production is $\frac{2}{R}$.



• There are cases where the extra dimension is not translationally invariant. (e.g. a finite interval) Then KK-charge is not conserved, KK states can be singly produced and the threshold for production is $\frac{1}{R}$.



Kaluza-Klein states in string theory

In string theory the KK spectrum is more complex: beyond the usual KK states, the string can wind around the circle, m times. This gives an extra contribution to the energy:

$$\sim T\ (2\pi m\ R) \quad , \quad m \in Z \quad , \quad T = \frac{1}{2\pi\ \ell_s^2}$$

The spectrum of KK masses now becomes

$$M^2 = \frac{n^2}{R^2} + (2\pi T R)^2 m^2 = \frac{n^2}{R^2} + \ell_s^4 R^2 m^2 \quad , \quad m, n \in Z$$

- The spectrum of stringy KK states is invariant under T-duality

$$m \leftrightarrow n \quad , \quad R \leftrightarrow \frac{\ell_s^2}{R}$$

There is no circle with $R < \ell_s$ in string theory!

Branes and large extra dimensions

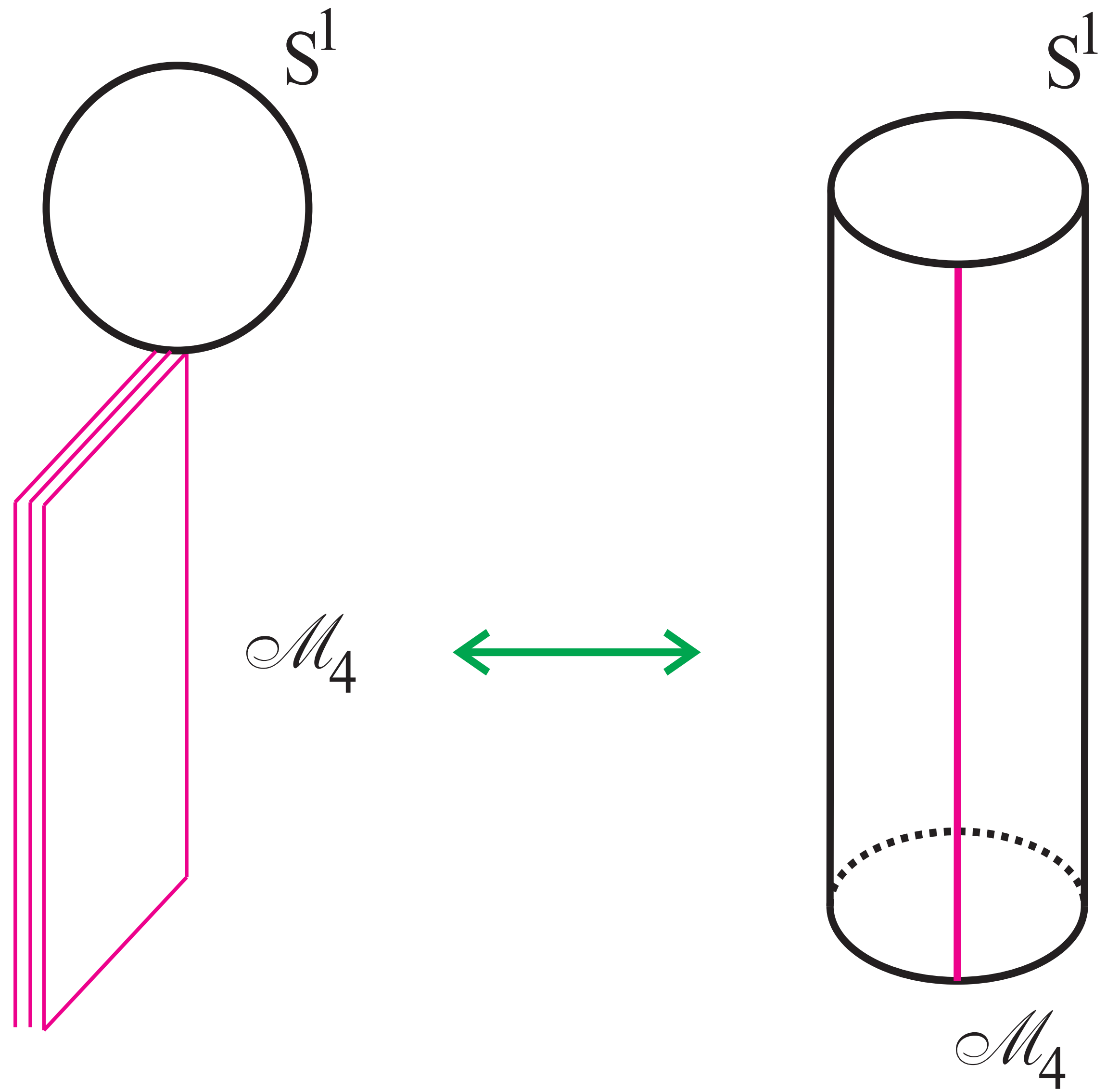
The collider bound on R: $1/R > 300$ GeV can be simply evaded if the KK states carry no SM charges. In the simplest case they couple gravitationally. This setup is possible using the idea of branes.

Consider $M_4 \times (S^1)^N$, with all circles of radius R.

A 3-brane is a (hyper)-membrane with 3 spacial dimensions. We can imagine such a 3-brane embedded inside our (4+N)-dimensional space. Such branes are part of string theory. They have fluctuating fields that live on them.

Such localized fields are typically gauge fields, fermions and scalars. We may therefore arrange that the SM fields live on such a 3-brane and cannot propagate in the rest n dimensions (the "bulk")

The gravitational field on the other hand can propagate in all (4+N) directions.


S^1
$\mathcal{M}_4$
S^1
$\mathcal{M}_4$

Consider the Newton constant and Planck mass in the (4+n)-dimensional theory:

$$G_{4+N} \sim M_*^{-(N+2)}$$

- At distances $l << R$ gravity is effectively (4+N)-dimensional.

- At large distances $l >> R$ gravity is four-dimensional with effective Newton constant

$$\frac{1}{G_4} \sim M_P^2 \sim M_*^{(N+2)} R^n$$

This can be easily seen from the gravitational action

$$M_*^{N+2} \int d^{4+N}x \sqrt{g}\ R_{4+N} \quad \sim \quad M_*^{N+2} R^N \int d^4x \sqrt{g}\ R_4$$

By choosing appropriately the size of extra dimensions

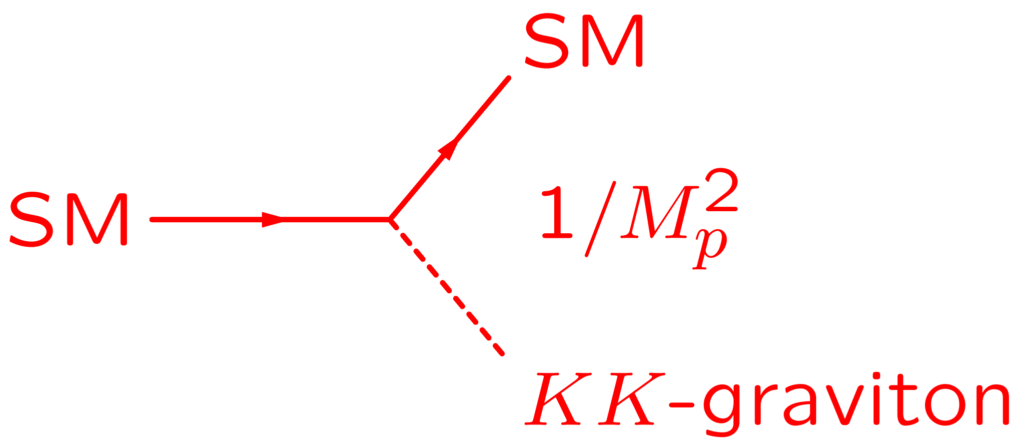
$$R \sim 10^{\frac{32}{N}} \quad \text{TeV}^{-1} \quad \sim \quad 10^{\frac{32}{N}-12} \ \text{eV}^{-1} \sim 10^{\frac{32}{N}-16} \ \text{mm}$$

It may be that the quantum gravity scale of the full theory M_* is as low as 1 TeV while $M_P = 10^{19}$ GeV.

SM particles have no KK descendants (no bulk propagation). They do not directly feel the extra dimensions. The collider bound on R is not relevant here.

The graviton has KK descendants, with the usual masses $\frac{|n|}{R}$.
They couple to SM matter gravitationally.

Each KK graviton couples with strength M_P^{-2} which is very weak.



However, the existence of many KK-gravitons enhances this coupling (more later).

For two extra dimensions their size can be 0.1 mm !!! How come we have not seen such a "large dimension"?

• It cannot be seen at accelerators because of the weak coupling of KK gravitons. (It becomes substantial at 1 TeV).

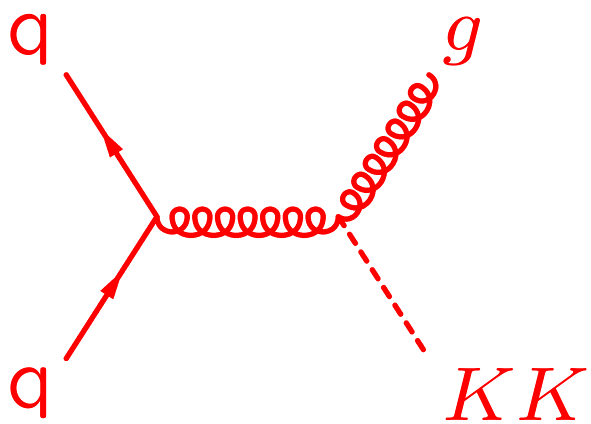
• For distances smaller than 0.1 mm gravity becomes five-dimensional : $F \sim 1/r^3$. (Exercise: The compact Newton's law $F = G_* M^2 \sum_{\vec{n}\in Z^N} \frac{1}{|\vec{r}+2\pi R\vec{n}|^{(N+2)}}$)

Surprisingly, until recently the gravitational law has been measured only up to distances of 1 mm! Today, the limiting distance has gone down to 10 μm.

Where can we see signals for all this?

(A) From tabletop short distance experiments

(B) At LHC. The signal is missing energy due to brehmstralung into KK gravitons that escape undetected in the bulk.



For $E >> \frac{1}{R}$

$$\sigma \sim \frac{1}{M_P^2}(\#\ of\ KK\ gravitons) \sim \frac{1}{M_P^2}(ER)^N \sim \frac{1}{M_*^2}(EM_*)^N$$

(Exercise: calculate the number $(ER)^N$ of KK states that can contribute to this process.)

Further reading:[arXiv:hep-ph/0503148]

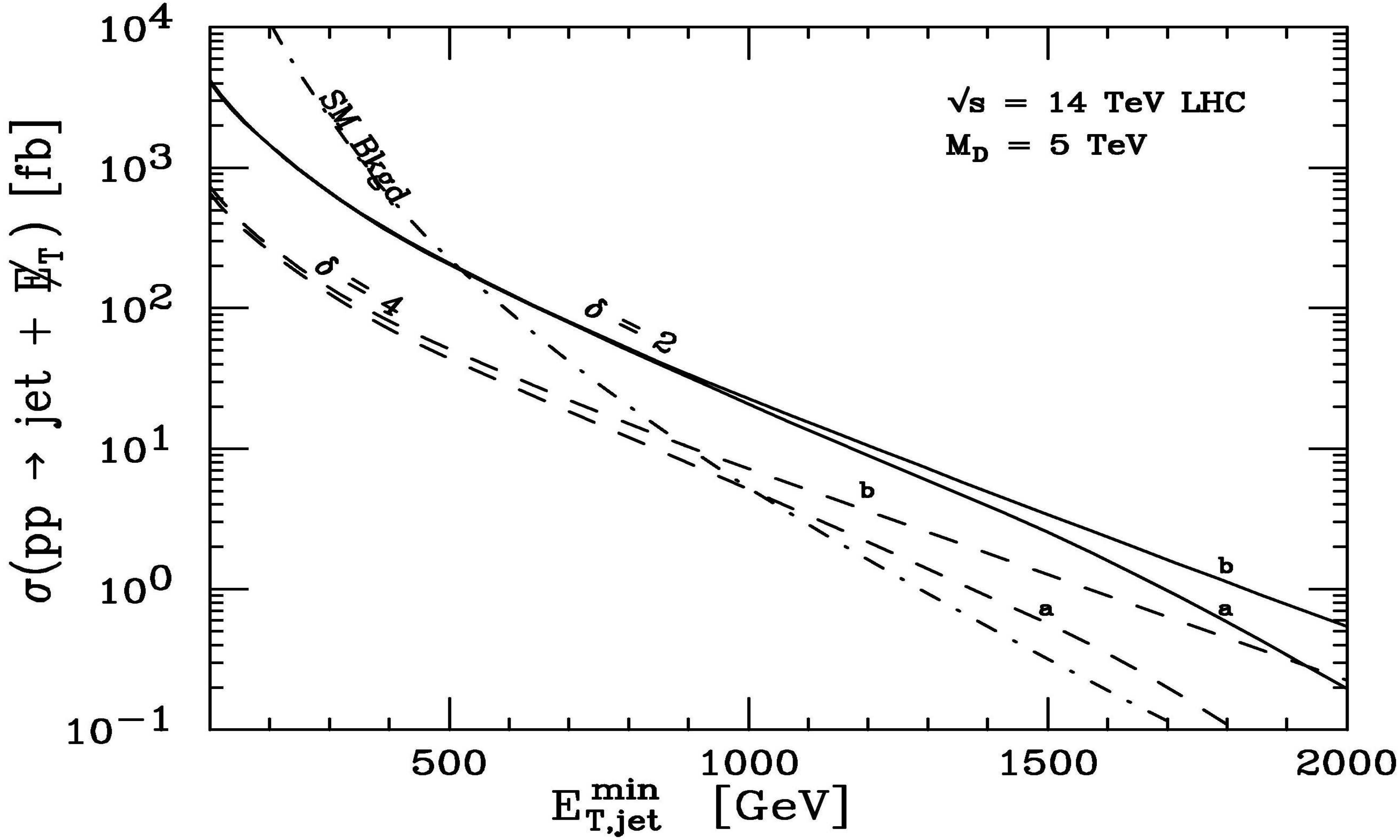


Figure 3: The total jet + nothing cross-section at the LHC integrated for all $E_{T,\text{jet}} > E^{\text{min}}_{T,\text{jet}}$ with the requirement that $|\eta_{\text{jet}}| < 3.0$. The Standard Model background is the dash-dotted line, and the signal is plotted as solid and dashed lines for fixed $M_D = 5$ TeV with $\delta = 2$ and 4 extra dimensions. The **a** (**b**) lines are constructed by integrating the cross-section over $\hat{s} < M_D^2$ (all $\hat{s}$).

From Giudice, Rattazzi and Wells [arXiv:hep-ph/9811291]

Black holes at colliders?

Black holes are very special (and singular) solutions of GR

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta\ d\phi^2)$$

$$f(r) = 1 - \frac{2M}{M_P^2\ r} \quad , \quad f(R) = 0 \quad \to \quad R = 2\frac{M}{M_P^2} = 2\frac{M}{M_P}\ \ell_P$$

Far away, $r \to \infty$ the space is flat

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\ d\phi^2) = -dt^2 + dx^i dx^i$$

- $r = R$ is the horizon, $r = 0$ is the singularity.

- Black holes are classically stable (and "hungry").

- In the quantum theory they decay via Hawking radiation.

- Particles with masses $M << M_P$ have an invisible horizon size: $R \ll \ell_P$

But very massive particles $M >> M_P$ have a macroscopic horizon: $R \gg \ell_P$
They should be treated as black holes

• In the case of large extra dimensions, the higher-dimensional Planck scale M_* is much smaller than the four-dimensional one M_P.

$$\frac{M_P^2}{M_*^2} \sim (M_* R)^n \sim \left(\frac{R}{\ell_*}\right)^N \gg 1$$

• If $M_* \sim 1$ TeV then multi-TeV particles will behave as (higher-dimensional*) black holes.
They will be created during a collision, and they will decay (democratically) via Hawking radiation.

• Although we do not yet control the details of such processes at LHC energies, we may be faced with such events at LHC

Further reading: start from hep-ph/0111230

*Exercise: Derive the higher-dimensional black-hole solution, by thinking simply about its asymptotic properties. In particular it must satisfy the (higher-dimensional) Poisson equation.

Conclusions

We have seen that we already have experimental data that cannot be explained in the context of the Standard Model,

- Neutrino masses and mixings.
- Dark matter.
- Dark Energy.

We have also seen many ideas that attempt to unify the forces, make a UV stable theory, incorporate gravity, and try to explain the data above.

No theory so far can successfully accommodate all three data above.

♣ We must think harder!

♠ We need input from experiments!

Happily, data are still flowing in from cosmological observations, and accelerators like LHC are expected to provide complementary views of the fundamental physical theory.

The (old) quest for understanding nature is still on!

The compact Newton's law

Assume 3+1 non-compact dimensions, and a single compact direction of radius R ($x^4 \to x^4 + 2\pi R$). The Newton's law, obtained by the method of images is

$$F = \frac{M_1 M_2}{M_*^3} \sum_{n\in Z} \frac{1}{[(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4 + 2\pi n R)^2]^{\frac{3}{2}}}$$

$$r = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}$$

is the usual distance in 3+1 dimensions.

• When $r << R$, all other images $n \neq 0$ are far away and can be neglected. Therefore we have 5d gravity.

$$F(r << R) \simeq \frac{M_1 M_2}{M_*^3 \; r^3}$$

• When $r >> R$ all images give equally important contributions. The result can be obtained by a Poisson resummation

$$\sum_{n\in Z} \frac{1}{[r^2 + (x^4 + 2\pi n R)^2]^{\frac{3}{2}}} \simeq \frac{1}{\pi R r^2} \left[1 + \mathcal{O}\left(\frac{R}{r}\right)\right]$$

$$F(r >> R) \simeq \frac{M_1 M_2}{\pi M_*^3 \; R} \frac{1}{r^2} \quad , \quad M_P^2 = \pi M_*^3 R$$

mGMSB

• There is a source of supersymmetry breaking due to a vev $\langle X\rangle = M + \theta^2\ F$ in a hidden (secluded) sector.

• There are messenger superfields in complete SU(5) reps (not to upset gauge coupling unification) Φ_i that couple as $\lambda_{ij}\ \bar{\Phi}_i\ X\ \Phi_j$ both to the secluded and the SSM sector. They modify the GUT scale coupling as

$$\delta\alpha^{-1}_{GUT} = -\frac{N}{2\pi}\log\frac{M_{GUT}}{M} \quad , \quad N = \sum_i n_i$$

• Diagonalize and absorb λ's into (M, F) $\to (M_i, F_i)$. Then the gaugino and scalar masses are given by

$$M_a = k_a\frac{\alpha_a}{4\pi}\Lambda_G \quad , \quad \Lambda_G = \sum_i n_i\frac{F_i}{M_i} \quad , \quad k_Y = \frac{5}{3}\ ,\ k_2 = k_3 = 1$$

$$m_i^2(t) = 2\sum_{a=1}^{3} C_a^i\ k_a\frac{\alpha_a^2(0)}{(4\pi)^2}\left[\Lambda_S^2 + h_a\Lambda_G^2\right] \quad , \quad h_a = \frac{k_a}{b_a}\left[1 - \frac{\alpha_a^2(t)}{\alpha_a^2(0)}\right] \quad , \quad \Lambda_S^2 = N\frac{F^2}{M^2}$$

• The MSSM soft parameters are here parameterized by $(M, N, \Lambda_G, \tan\beta, \text{sign}(\mu))$

Further reading: [arXiv:hep-ph/9801271]

mAMSB

• The idea of anomaly mediated supersymmetry breaking comes from brane realizations of the SM.

• The "hidden" sector where supersymmetry breaks spontaneously is localized on a brane different from the SSM-brane.

• The breaking of supersymmetry is communicated to the SSM via the Weyl anomaly.

• The form of the gaugino and scalar soft masses is of the form

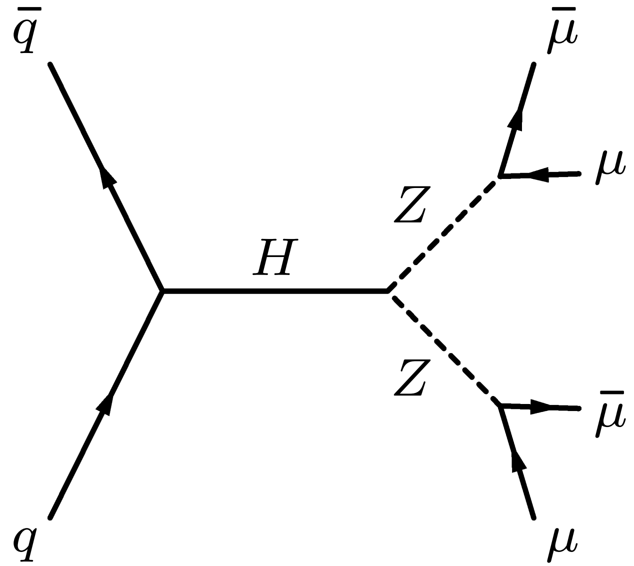
$$M_a = \beta_a \; M \quad , \quad m_i^2 = m_0^2 - C_i^a \; \beta_a \; M^2$$

where M is a characteristic energy scale and m_0 a phenomenological parameter and β_a the gauge β-functions.

• This mechanism is still in its infancy and has many obscure points. It is known as mAMSB and characterized by the parameters $(m_0, M, \tan\beta, \text{sign}(\mu))$

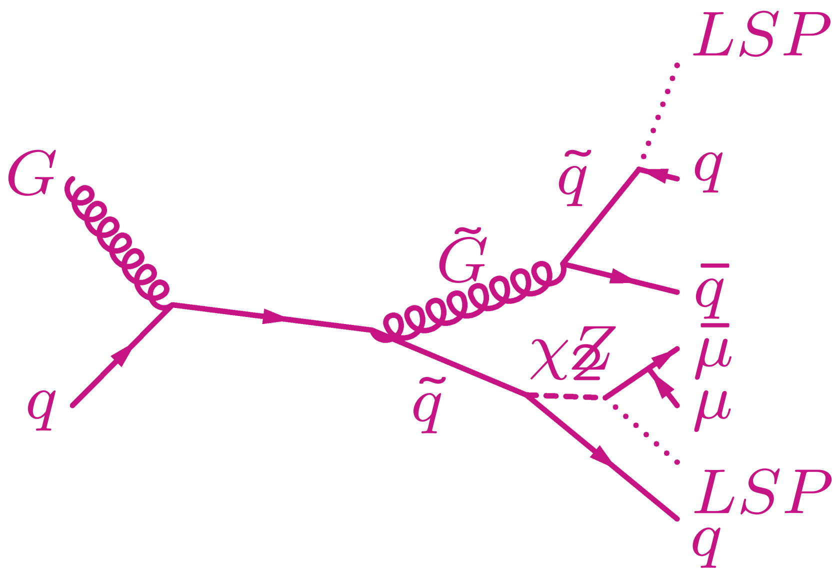
Further reading:[arXiv:hep-th/9810155], [arXiv:hep-ph/9810442]

Higgs Event



Higgs→ZZ→ $\mu\mu\mu\mu$ event

First SUSY event



The events were generated by Maria Spiropulu for the following SUSY mSUGRA parameters:

$$\tan\beta = 10 \quad , \quad m_{\frac{1}{2}} = 285 \quad \text{GeV} \quad , \quad m_0 = 210 \quad \text{GeV} \quad , \quad A = 0, \text{sign}(\mu) = +$$

This is known as the LM4 mSUGRA Point.

For these parameters the squark (gluino) masses are about 600 (700) GeV and the lightest neutralino, which escapes direct detection, has a mass of 114 GeV.

See http://iguanacms.web.cern.ch/iguanacms/gallery-page4.html

Hypercharge normalization

We have seen that

$$Y=\frac{1}{6}\begin{pmatrix}2&0&0&0&0\\0&2&0&0&0\\0&0&2&0&0\\0&0&0&-3&0\\0&0&0&0&-3\end{pmatrix}\quad,\quad T_Y=\xi\ Y\quad,\quad Tr[T_YT_Y]=\frac{1}{2}\quad,\quad \xi=\sqrt{\frac{3}{5}}$$

$$A_\mu=T_Y\ B_\mu\quad,\quad \delta L=-\frac{1}{2}Tr[F_A]^2+g_{GUT}Tr[A_\mu J^\mu]=-\frac{1}{4}F_B^2+\xi g_{\text{GUT}}Tr[Y\ B_\mu J^\mu]]$$

$$g_Y=\xi\ g_{\text{GUT}}=\sqrt{\frac{3}{5}}\ g_{\text{GUT}}$$

Proton decay channels

In standard GUTs the nucleon decay channels are as follows:

$$p \to \pi^0 + e^+ \quad \text{or} \quad p \to \pi^0 + \mu^+$$

$$p \to K^+ + \bar{\nu}$$

$$p \to K^0 + e^+ \quad \text{or} \quad p \to K^0 + \mu^+$$

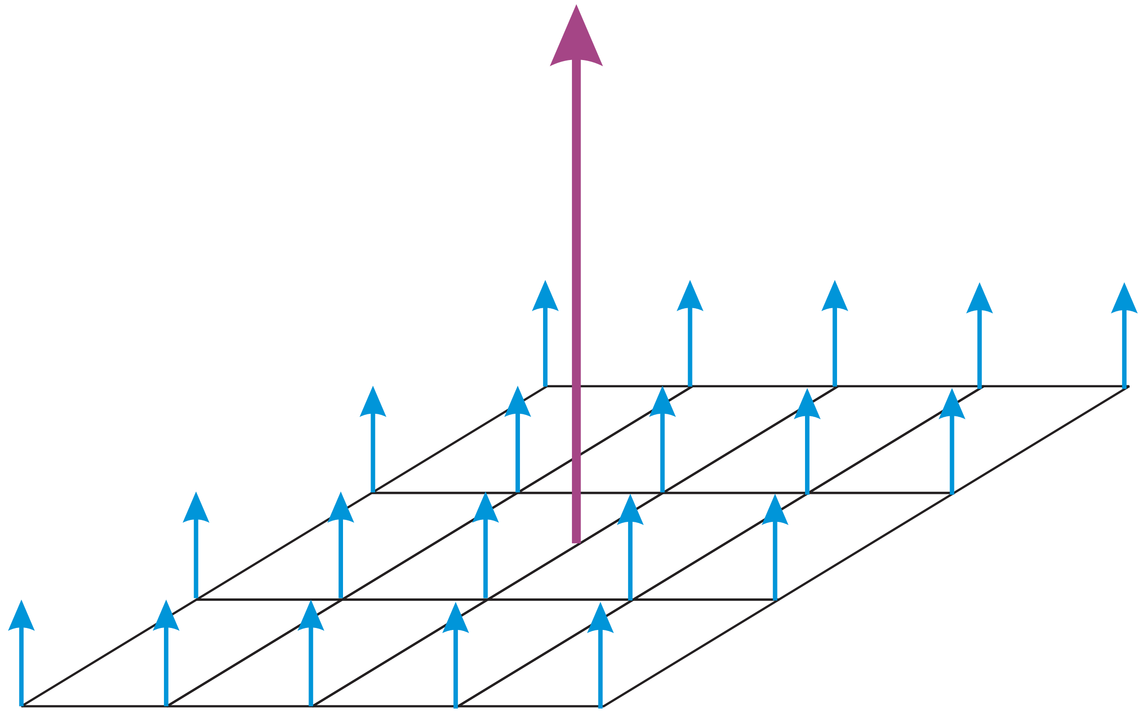
$$n \to K^0 + \bar{\nu}$$

See http://arxiv.org/pdf/hep-ph/0211024

Effective Field Theory: Wilson's View

♣ A Quantum Field Theory is defined (by e.g. a Lagrangian) at a high energy scale Λ, usually called the "cut-off" scale.

In standard space-time, this means that it is defined at some minimum distance, e.g. on a space-time lattice with a minimum length.



♠ Once it is defined at this high scale we can compute its (quantum) physics at any lower energy scale E.

This is done by "averaging" over the quantum effects associated with all energies between the cut-off Λ and the energy of interest E. in practice and in perturbation theory this amounts of integrating the internal momenta of Feynman diagrams between Λ and E

Let us take a simple example of a scalar theory that at the high energy cutoff has a Lagrangian

$$L_\Lambda = \frac{1}{2}(\partial\phi)^2 + \lambda_2\phi^2 + \lambda_1\phi^3 + \lambda_0\phi^4 + \lambda_{-1}\phi^5 + \cdots$$

$$dim(\phi) = 1 \qquad \Rightarrow \qquad \lambda_n \simeq M^n$$

Note that $n \geq 0$ corresponds to what we call renormalizable interactions.

We now want to find the effective theory at a much lower energy scale $E << \Lambda$. By doing the (quantum computations) we can determine the interactions of the effective theory at the energy scale E:

$$L_E = \frac{1}{2}(\partial\phi)^2 + \tilde{\lambda}_2\phi^2 + \tilde{\lambda}_1\phi^3 + \tilde{\lambda}_0\phi^4 + \tilde{\lambda}_{-1}\phi^5 + \cdots$$

All possible terms are generated, compatible with the symmetries!

The $\tilde{\lambda}_n$ are functions of all the λ_n and Λ, E. The transformation that brings us from the high energy couplings to the low energy effective couplings is called a renormalization group transformation. It is determined by the dynamics of the high energy (short distance) theory.

Moreover, generically (exercise),

$$\tilde{\lambda}_n \sim \Lambda^n$$

Therefore et low energy $E << \Lambda$, high-dimension (non-renormalizable interactions) are suppressed as inverse powers of the high energy scale Λ. For $\Lambda = \infty$, $\tilde{\lambda}_{n<0} = 0$.

On the other hand, (modulo exceptions), renormalizable interactions become large.

The measurable ("physical") quantities are the low energy ones: $\tilde{\lambda}_n$.

Note an **asymmetry** in this picture:

- A high-energy theory determines all lower-energy theories.

- A low-energy theory DOES NOT determine the higher-energy theories.

There is a similarity between the irreversibility of renormalization group flow and irreversibility in thermodynamics.

Unfortunately, we are constrained to to try to built the theory starting from low energy. And for this we built it step by step in energy.

We can equivalently phrase the problem as: We know the theory at energies 200 GeV (and below). What can we say about higher energies?

Renormalization: the old view

In the traditional approach:

- $\Lambda = \infty$. The theory is defined to make sense at all possible energies.

- $\lambda_{n>0}(\infty) = 0$

Since

$$\tilde{\lambda}_n(E) \sim \Lambda^n$$

- Effective dimension$>$ 4 interactions are insensitive to high energy physics.

- Effective dimension$\leq$ 4 couplings are infinite. We must choose carefully the $\lambda_n(\infty)$ so that this infinity cancels.

$$\tilde{\lambda}_2(E) = \lambda_2 + a\Lambda^2 + b\lambda_2 \log\frac{E^2}{\Lambda^2} + \text{finite} \quad \text{as} \quad \Lambda \to \infty$$

Choose

$$\lambda_2 = -a\Lambda^2 - b\lambda_2 \log\frac{E^2}{\Lambda^2}$$

Renormalized couplings: a concrete example

Consider that there exists at low energy a single scalar ϕ and we write the basic interactions at a high scale Λ. (this is the definition scale. At higher scales the theory may change):

$$S = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{m_0^2}{2}\phi^2 + \lambda_0\phi^4$$

We now calculate various low energy parameters, at a given scale $E_0 << \Lambda$.

$$m^2(E_0) = m_0^2 - \xi_1\Lambda^2 + \cdots \quad , \quad \frac{1}{\lambda(E_0)} = \frac{1}{\lambda_0} - b_0 \log\frac{E_0}{\Lambda} + \cdots$$

These are obtained from the two and four-point functions or equivalently from $\sigma_{2\to 2}$. If we now compute a $2 \to 4$ scattering cross section

$$\sigma_{2\to 4}(m(E_0), \lambda(E_0), \Lambda) \simeq \frac{\lambda_0^2}{m_0^2} \sim \frac{\lambda(E_0)^2}{m(E_0)^2 + \xi_1\Lambda^2}$$

From this we can "measure" $\Lambda = \Lambda_*$. If $\Lambda_* \neq \infty$ then the theory must change at $E \sim \Lambda_*$.

If instead we look at a theory like EM with a massive fermion, then both the gauge coupling constant and mass run logarithmically. Smaller sensitivity at the Λ scale.

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